# Mathematical Modelling, Theory, and Model Fitting in Ecology & Evolution

*A MulQuaBio Lecture*

January 16, 2025

# OUTLINE

- Modelling - what and why
- Types of Models
- How to build 'em
- How to test 'em (AKA Fitting Models to Data)
- Summary and Readings

# MODELS AND MODELLING

*What does “modelling” mean to you?*

# Models and Modelling

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*Caricature of a phenomenon that captures its essence*
*(the model's output reproduces/emulates the phenomenon)*

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- To develop accurate predictions of an observed phenomenon in the future
- To find out what is important to know in an otherwise complex system (cannot measure and monitor everything!)

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*Accuracy is a more fundamental property*

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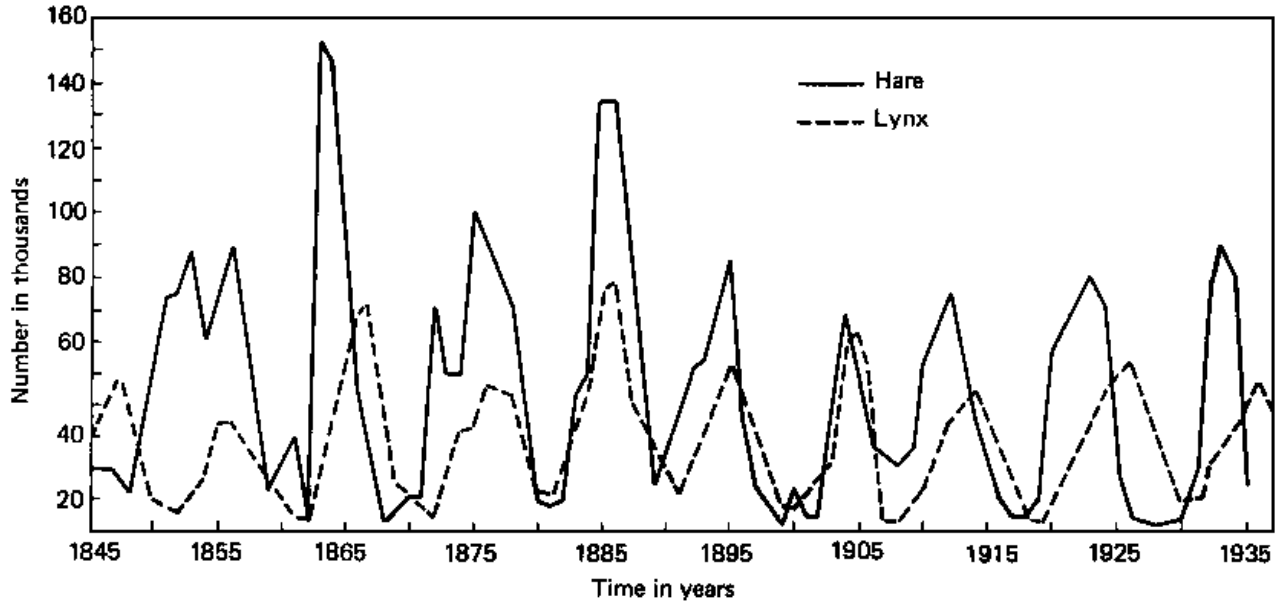
# MECHANISTIC VS. INFERENTIAL MODEL FITTING

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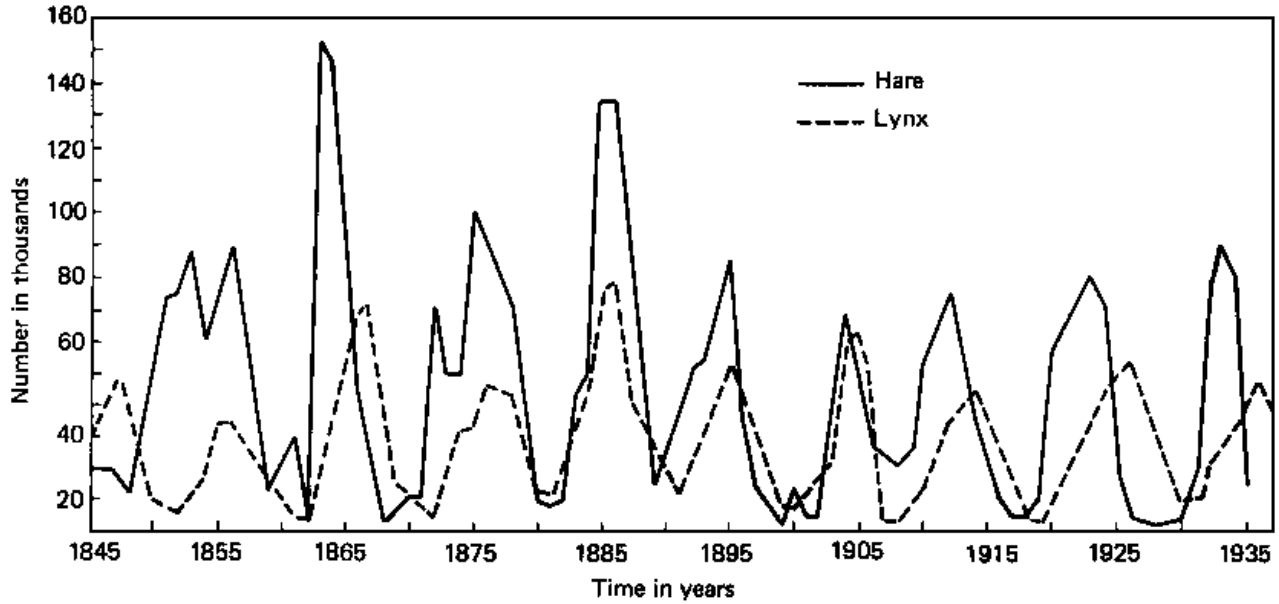


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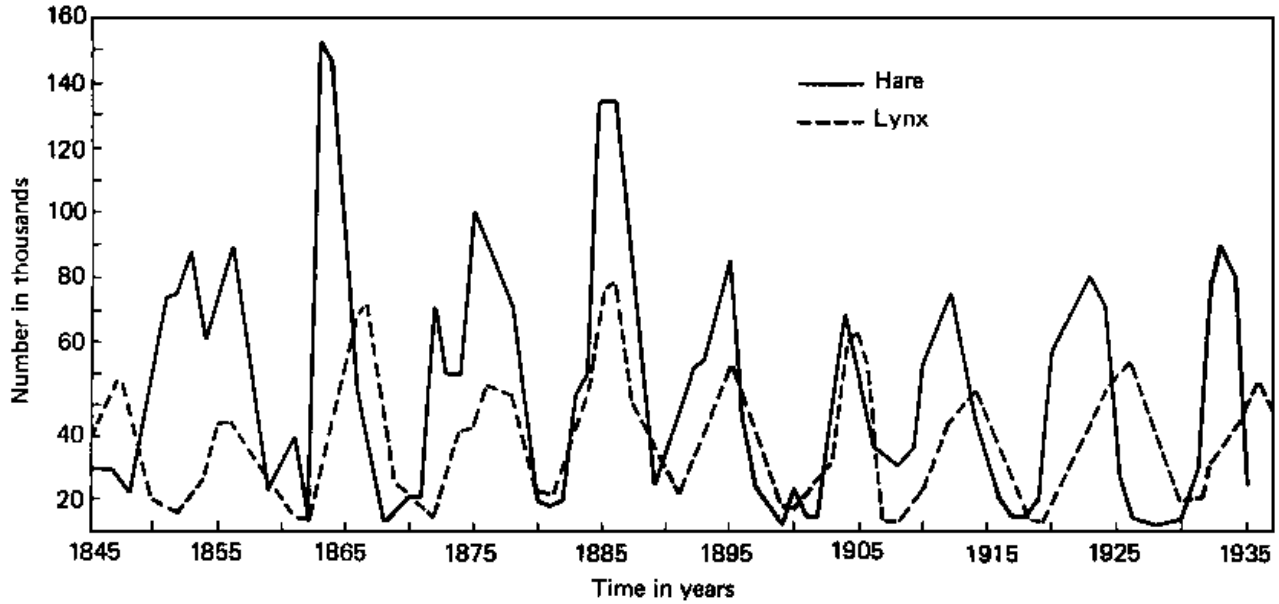


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- **Mechanistic model**: *The Lynx-Hare Cycle is driven by density-dependent population growth in hares*
- **Inferential model**: *The Lynx and Hare Cycles have a significant asynchrony (period shift) of x years*

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  - Example: *How* climatic temperature *drives* the Lynx-Hare cycle
- *Ultimately, successful, EMPIRICALLY-GROUNDED mechanistic models are the best path towards a THEORY in any scientific discipline (including ecology and evolution)*

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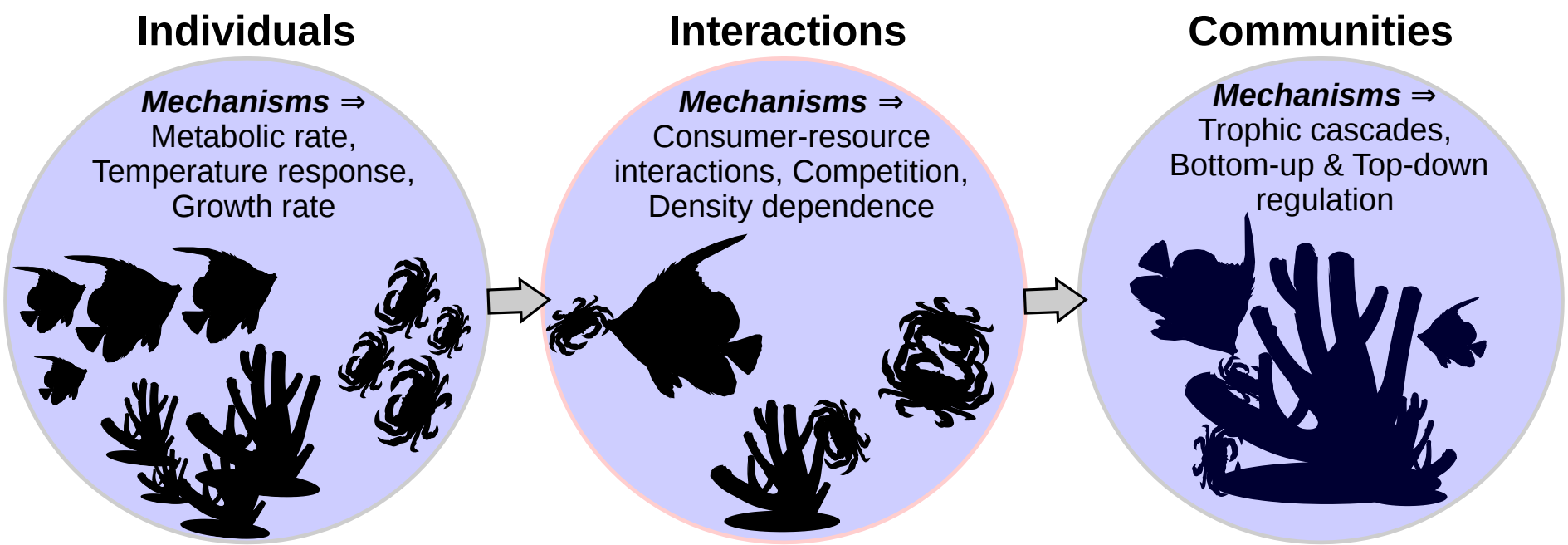

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- *So the big question is, can we FORECAST WITHOUT EXPLAINING?*
  - For example, disease outbreaks: Do we really need to care about the underlying mechanisms if we can predict a future event using Inferential modelling (e.g., Machine-learning of time series patterns)?

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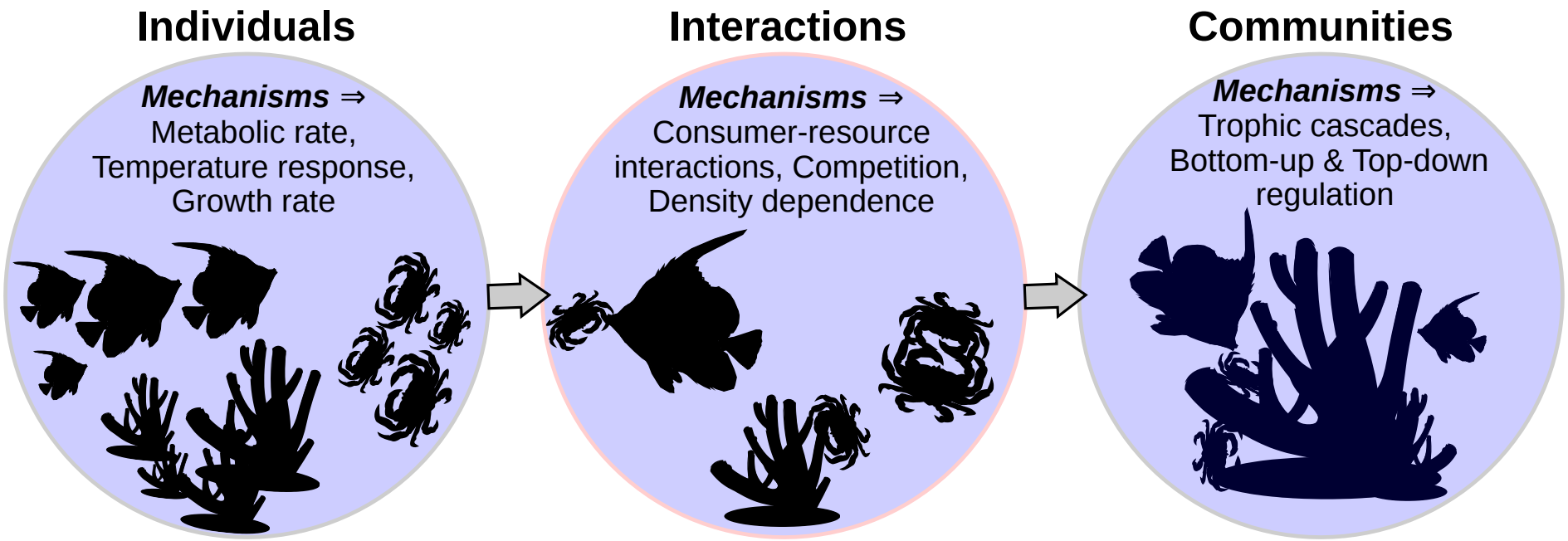
- But is this REALLY mechanistic? What are $r$ and $K$ really?

# EXAMPLE OF A FUNDAMENTAL MECHANISM: METABOLIC RATE

- Proponents of *Ecological Metabolic Theory* (AKA "Metabolic Theory of Ecology") argue that we have not progressed far enough towards mechanistic modelling because metabolism has been ignored



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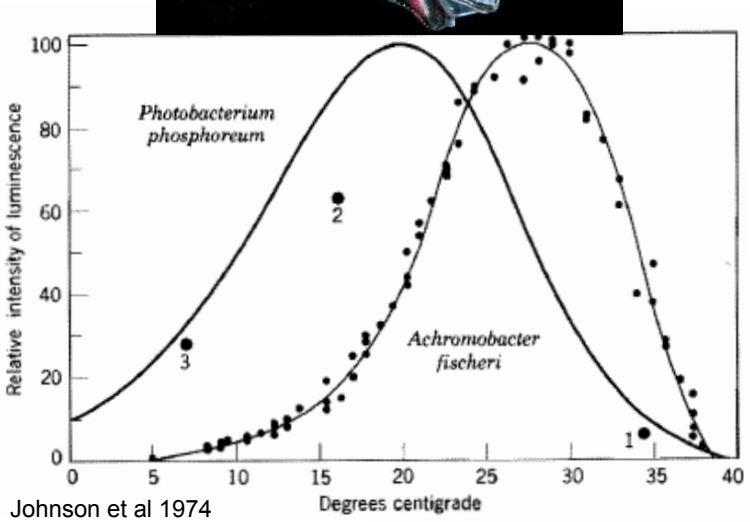


Johnson et al 1974

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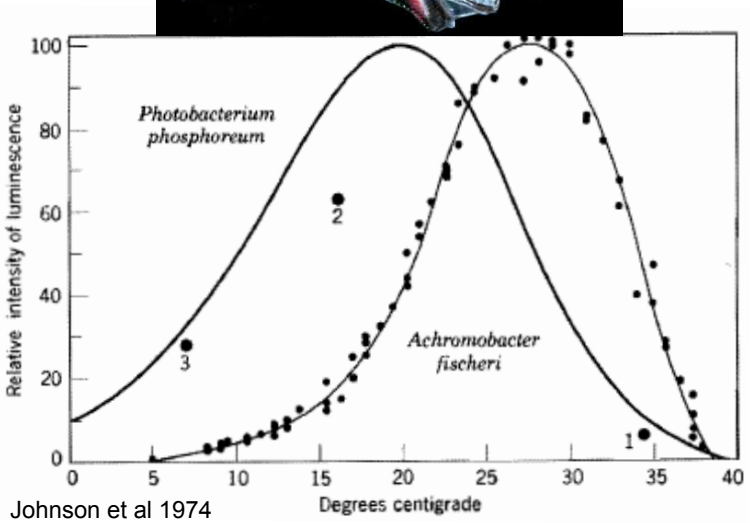


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$$B = B_0 \boxed{e^{-\frac{E}{kT}}} f(T, T_{pk}, E_D)$$

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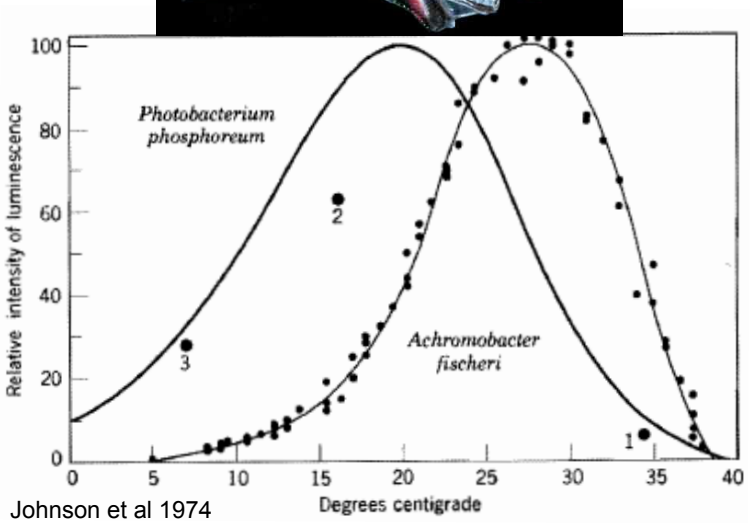


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- Surely there is more to thermal responses?
- *What about alternative models?*

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- Inferential/statistical models often perform better than mechanistic ones. *Why? — because they have lest restrictive assumptions*

# Building Models

- It's an art, takes practice (Levins' paper on the strategy of model building in biology)
- Build models one mechanism at a time — in biology, it means start at the right level of organization!
- Always consider an alternative that is more parsimonious, even if it is an Inferential model!
- For example, the Boltzmann-Arrhenius model is a good first try describe and uncover mechanisms underlying individual level "traits" that are rates (e.g., fecundity or development rate)
- The next step would be to include species interactions with temperature dependence of individuals (or go in an evolutionary direction)

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*Mechanisms are the key to accurate prediction!*

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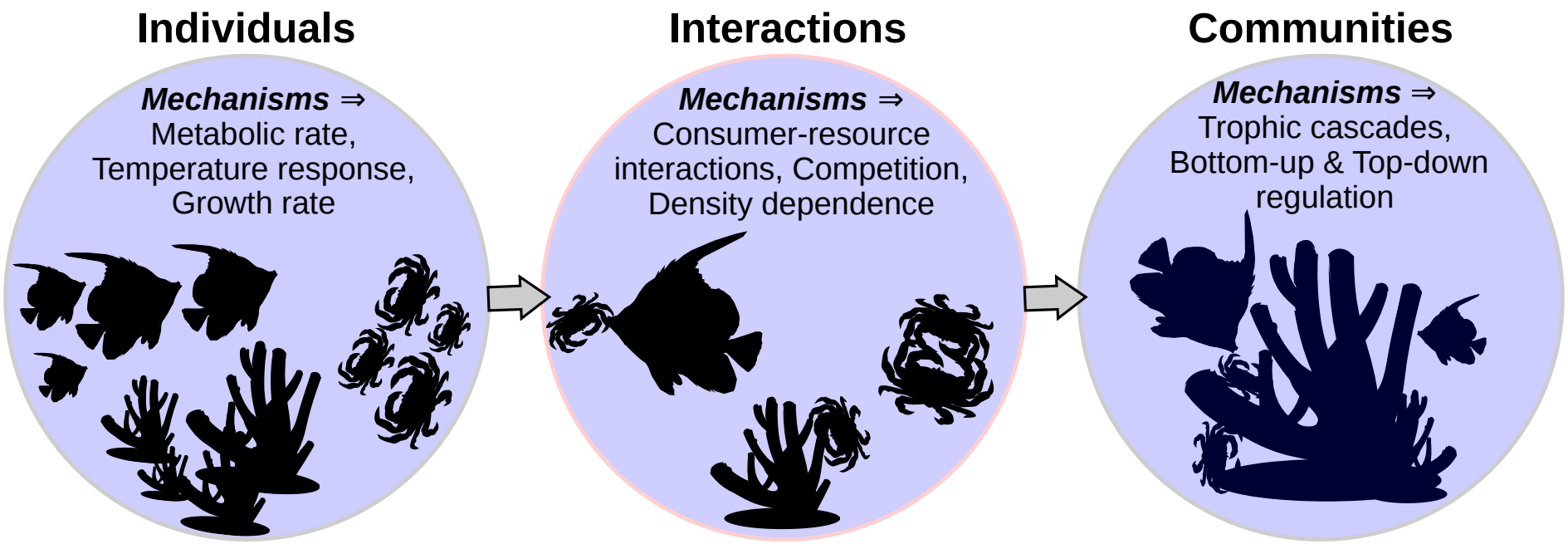


- *You will learn about (deriving and analysing) key models (and theories) at different levels of organization in this part of the course (starting with metabolism)*

# Fitting Models (to data)

- Least Squares methods
  - Linear
  - Non-linear
- Likelihood-based methods
  - Maximum Likelihood Estimation (MLE)
  - Bayesian
- Machine learning and Artificial intelligence

# FITTING MODELS (TO DATA)

- Linear and non-linear least squares model fitting: (mathematically /algorithmically simple) approaches, useful in many scenarios in biology
  - Many mechanisms in biology are inherently non-linear (i.e., r data are better-explained by a non-linear mathematical model)
- Bayesian (and MLE) methods: Versatile and powerful when data are limited and your (e.g., mechanistic) models are complex (many parameters). Bayesian more accurate results than MLE if you have prior info (mechanistic models!).
- AI/machine Learning: most versatile and powerful for large amounts of noisy data, but the focus on maximizing ability to discover pattern and predict comes at the cost of mechanistic insights

# Summary: Model comparison and selection is the key

- Ideally, several competing (meaningful, not just null) hypotheses (mathematical models) should be fitted to data and compared using statistical theory

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- This is an advance over the traditional “null hypothesis” approach in Biology
- Necessary for the advancement of Biology from from an observational and axiomatic discipline to one with general theories
- Necessary for understanding the mechanisms underlying biological patterns/phenomena

# READINGS

- Levins, R. (1966) The strategy of model building in population biology. Am. Sci. 54, 421–431.
- Otto, S.P. and Day, T. (2011) A biologist's guide to mathematical modeling in ecology and evolution. Princeton University Press. (Read Chapters 1-2)
- Kingsland, Sharon E. (1995) Modeling Nature. University of Chicago Press. (Read over this term - and beyond!)
- Additional readings on the MQB git repository (`Modelling` directory)