Tarefas de Diferenciação Numérica

Tarefa 01: Desenvolva uma fórmula de derivada segunda na filosofia central de tal forma que o erro seja da ordem de $(\Delta x)^4$.

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$\Delta x_{(k)}$	*f(x)*	*f''(x)*	*e(x)*
0.5	*20.47996*	*47.219433*	--
0.25	--	*45.602379*	*0.035460*
0.125	--	*45.205280*	*0.008784*
0.0625	--	*45.106449*	*0.002191*
0.03125	--	*45.081769*	*0.000547*
0.015625	--	*45.075601*	*0.000137*
0.007812	--	*45.074059*	*0.000034*

Pela derivada segunda encontrada analiticamente, temos:

f''(2) = 45.073545

Note: está sendo considerado 6 casas após a vírgula

a) Combinando expansões em série de Taylor de pontos da vizinhança;

1) $$f(x+\Delta x) = f(x) + f'(x)\,\Delta x + \frac{1}{2} f''(x)\,(\Delta x)^2 + \frac{1}{3!} f'''(x)\,(\Delta x)^3 + \frac{1}{4!} f^{iv}(x)\,(\Delta x)^4 + \frac{1}{5!} f^{v}(x)\,(\Delta x)^5 + \frac{1}{6!} f^{vi}(x)\,(\Delta x)^6$$

2) $$f(x+\Delta x) = f(x) - f'(x)\,\Delta x + \frac{1}{2} f''(x)\,(\Delta x)^2 - \frac{1}{3!} f'''(x)\,(\Delta x)^3 + \frac{1}{4!} f^{iv}(x)\,(\Delta x)^4 - \frac{1}{5!} f^{v}(x)\,(\Delta x)^5 + \frac{1}{6!} f^{vi}x(x)\,(\Delta x)^6$$

3) $$f(x+2\Delta x) = f(x) + f'(x)\,2\Delta x + \frac{1}{2} f''(x)\,(2\Delta x)^2 + \frac{1}{3!} f'''(x)\,(2\Delta x)^3 + \frac{1}{4!} f^{iv}(x)\,(2\Delta x)^4 + \frac{1}{5!} f^{v}(x)\,(2\Delta x)^5 + \frac{1}{6!} f^{vi}(x)\,(2\Delta x)^6$$

4) $$f(x+2\Delta x) = f(x) - f'(x)\,2\Delta x + \frac{1}{2} f''(x)\,(2\Delta x)^2 - \frac{1}{3!} f'''(x)\,(2\Delta x)^2 + \frac{1}{4!} f^{iv}(x)\,(2\Delta x)^4 - \frac{1}{5!} f^{v}(x)\,(2\Delta x)^5 + \frac{1}{6!} f^{vi}(x)\,(2\Delta x)^6$$

(1) + α (2) + β (3) + γ (4)

Como queremos eliminar f'(x), f'''(x), $f^{iv}(x), e\ f^{v}(x)$ seguimos os mesmos passos já apresentados em aula, logo: α = 1, β = $\frac{-1}{16}$ e γ = $\frac{-1}{16}$

(1) + α (2) + β (3) + γ (4) = $f(x+\Delta x) + f(x-\Delta x) + f(x+2\Delta x) + f(x-2\Delta x) =$

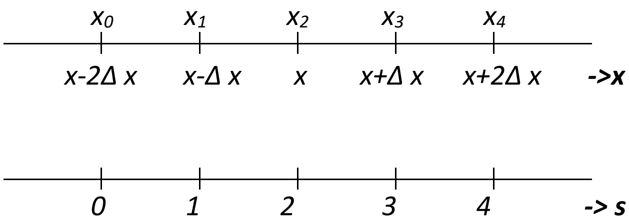
$$f\ (x)(1+\alpha+\beta+\gamma\ ) + \frac{1}{2} f''(x)(\Delta x)^2(1+\alpha+4\beta+\ 4\gamma) + \frac{1}{6!} f^{vi}(x)(\Delta x)^6\ (1+\alpha+32\beta+32\gamma)$$

Isolando $f''(x)$ e substituindo os valores de α, β, γ

Temos:

$$f''(x) = \frac{1}{(\Delta x)^2}\left(\frac{-1}{12} f(x+2\Delta x) + \frac{4}{3} f(x+\Delta x) + \frac{4}{3} f(x-\Delta x) \frac{-1}{12} f(x-2\Delta x) - \frac{-5}{2} f(x)\right) + \frac{8}{6!} f^{vi}(x)(\Delta x)^4$$

b) **Usando o polinômio de interpolação de Newton:**



i. $x(s) = x_0 + (s-2)\Delta x$

ii. $s(x) = \frac{(x-x_0)}{\Delta x} + 2$

Derivando *(ii)* dos dois lados, temos:

$$ds = \frac{dx}{\Delta d} \rightarrow \frac{ds}{dx} = \frac{1}{\Delta x}$$

$$f(x) \approx g(s) = \sum_{j=0}^{n} \binom{s}{j} \Delta^j f_0$$

$$= \binom{s}{1} \Delta^1 f_0 + \binom{s}{2} \Delta^2 f_0 + \binom{s}{3} \Delta^3 f_0 + \binom{s}{4} \Delta^4 f_0$$

$$\Delta^0 f_0 \Delta \Delta^1 f_0 \left(\frac{s^2-s}{2}\right) \Delta^2 f_0 + \left(\frac{s^3-3s^2-2s}{3!}\right) \Delta^3 f_0 + \left(\frac{(s^4-6s^3+11s^2-6s}{4!}\right) \Delta^4 f_0$$

Derivando $f(x)$ 2 vezes e usando a regra da cadeia, além de que $\frac{\mathbf{ds}}{\mathbf{dx}} = \frac{\mathbf{1}}{\mathbf{\Delta x}}$:

$$\mathrm{f}''(\mathrm{x}) = \frac{1}{\Delta \mathrm{x}^2} \mathrm{g}''(\mathrm{s})$$

$$\mathrm{g}''(\mathrm{s}) = \Delta^0 f_0 + (\mathrm{s} - 1)\Delta^3 f_0 + \frac{6\mathrm{s}^2 - 18\mathrm{s} + 11}{12} \Delta^4 f_0$$

Como queremos o ponto central, logo *s* = 2.

$$\mathrm{g}''(2) = \Delta^2 f_0 + \Delta^3 f_0 - \frac{1}{12} \Delta^4 f_0$$

Como:

$$\Delta^2 f_0 = f_2 - 2f_1 + f_0$$

$$\Delta^3 f_0 = f_3 - 3f_2 + 3f_1 + f_0$$

$$\Delta^4 f_0 = f_4 - 4f_3 + 6f_2 - 4f_1 + f_0$$

Logo:

$$\mathrm{g}''(2) = -\frac{5}{2} f_2 + \frac{4}{3} f_1 - \frac{1}{12} f_0 + \frac{4}{3} f_3 - \frac{1}{12} f_4$$

Sendo assim:

$$f''(x) = \frac{1}{(\Delta x)^2} = \left(-\frac{5}{2}f_2 + \frac{4}{3}f_1 - \frac{1}{12}f_0 + \frac{4}{3}f_3 - \frac{1}{12}f_4\right)$$

Substituindo f_0, f_1, f_2, f_3, e f_4 temos:

$$f''(x) = \frac{1}{(\Delta x)^2}\left(\frac{-1}{12}f(x + 2\Delta x) + \frac{4}{3}f(x + \Delta x) + \frac{4}{3}f(x - \Delta x)\frac{-1}{12}f(x - 2\Delta x) - \frac{-5}{2}f(x)\right) + \frac{8}{6!}f^{vi}(x)(\Delta x)^4$$