2-DOF and 7-DOF Dynamic Modeling and Simulation of Automobiles

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Table of contents:

1. Building a road model

When analyzing the active suspension control process, the road input is an important factor that cannot be ignored. In this paper, the white noise signal is used to stimulate the road input,

$$\dot{x}_g(t) = -2\pi f_0 x_g(t) + 2\pi\sqrt{G_0 U_0}\, w(t)$$

in the formula, f_0 is the lower cutoff frequency, Hz; G_0 is the road roughness coefficient, m^3/cycle; U_0 is the forward speed, m/sec; w is random input unit white noise with mean zero. The above formula shows that the road surface displacement can be expressed as a randomly filtered white noise signal. This representation is derived from the shape of the power spectral density (PSD) curve of road surface roughness measured experimentally. We can add the road surface input to the model in the form of an equation of state:

$$\begin{cases} \dot{X}_{road} = A_{road} X + F_{road} W \\ Y_{road} = C_{road} X \end{cases}$$

$X_{road} = x_g, A_{road} = -2\pi f_0, B_{road} = 2\pi\sqrt{G_0 U_0}, C_{road} = 1$; D=0;

Considering that the road surface is an ordinary road, the road surface roughness coefficient G_0=5e-6m^3/cycle; speed U_0=20m/s; In modeling, road random white noise can be generated by random number generation or limited bandwidth white noise. This paper uses bandwidth white noise generation, and uses MATLAB/simulink to establish a simulation model as follows:

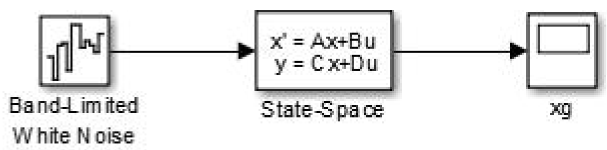

Fig. 1 Pavement model

2. Modeling of a 2-DOF system of a car

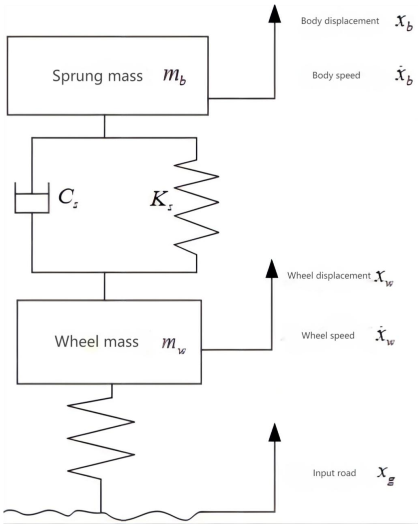

Fig. 2 Model of car 2-DOF system

According to the 2-DOF system model of the car shown in Figure 2, first establish the differential equation of motion:

$$\begin{cases} m_b \ddot{x}_b = -K_s(x_b - x_w) - C_s(\dot{x}_b - \dot{x}_w) \\ m_w \ddot{x}_w = -K_t(x_w - x_g) + K_s(x_b - x_w) + C_s(\dot{x}_b - \dot{x}_w) \end{cases}$$

Simplified：

$$\begin{cases} \ddot{x}_b = \dfrac{-C_s}{m_b} xb + \dfrac{-C_s}{m_b} x_w + \dfrac{-K_s}{m_b} x_b + \dfrac{-K_s}{m_b} x_b \\ \ddot{x}_w = \dfrac{-C_s}{m_w} x_b + \dfrac{-C_s}{m_w} x_b + \dfrac{-K_s}{m_w} x_b + \dfrac{-K_s - K_t}{m_w} x_b + \dfrac{K_t}{m_w} x_g \end{cases}$$

In the formula, C_s is the suspension damping, K_s A is the suspension stiffness, K_t A is tire stiffness, m_b is the body mass, m_w is the wheel mass, x_b、$\dot{x}_b$、$\ddot{x}_b$ are the body displacement, velocity, and acceleration, respectively. x_w、$\dot{x}_w$、$\ddot{x}_w$ are the wheel displacement, velocity, and acceleration, respectively, x_g is the road input.

The state variables and input vectors are selected as:

$$X = [\dot{x}_b \quad \dot{x}_w \quad x_b \quad x_w] \qquad\qquad U = x_g$$

Then the system motion equation and road excitation can be written in the form of state space matrix:

$$\dot{X} = AX + BU$$

In the formula, A is the state matrix, B is the input matrix, and its values are as follows:

$$A = \begin{bmatrix} -\frac{C_s}{m_b} & \frac{C_s}{m_b} & -\frac{K_s}{m_b} & \frac{K_s}{m_b} \\ \frac{C_s}{m_w} & -\frac{C_s}{m_w} & \frac{K_s}{m_w} & \frac{-K_s - K_s}{m_w} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{K_t}{m_w} \\ 0 \\ 0 \end{bmatrix}$$

Taking body acceleration, tire dynamic deformation, and suspension dynamic travel as performance indicators:

$$Y = [x_b \quad x_w - x_g \quad x_b - x_w]^T$$

Write the performance index term as a linear combination of state variables and input signals:

$$Y = CX + DU$$

In the formula:

$$C = \begin{bmatrix} -\frac{Cs}{m_b} & \frac{Cs}{m_b} & -\frac{Ks}{m_b} & \frac{Ks}{m_b} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

The simulation model established by MATLAB/simulink is as follows:

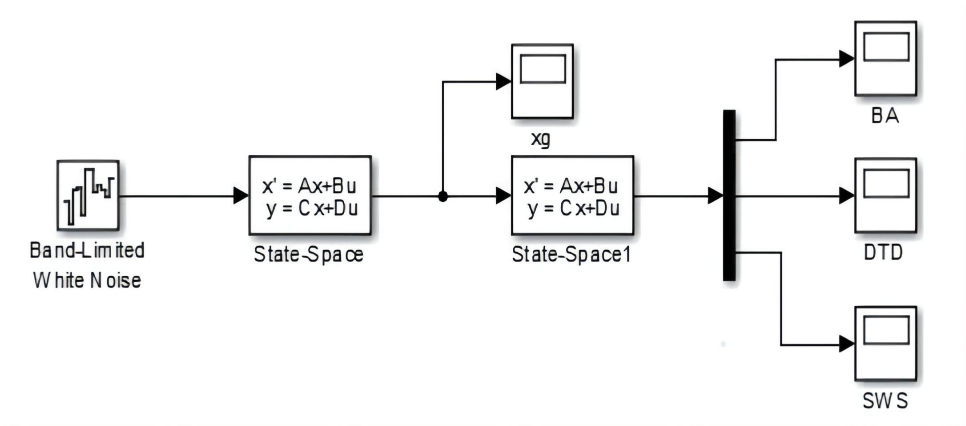

Fig. 3 Simulink model of car with 2 degrees of freedom

3. Modeling of a 7-DOF system of a car

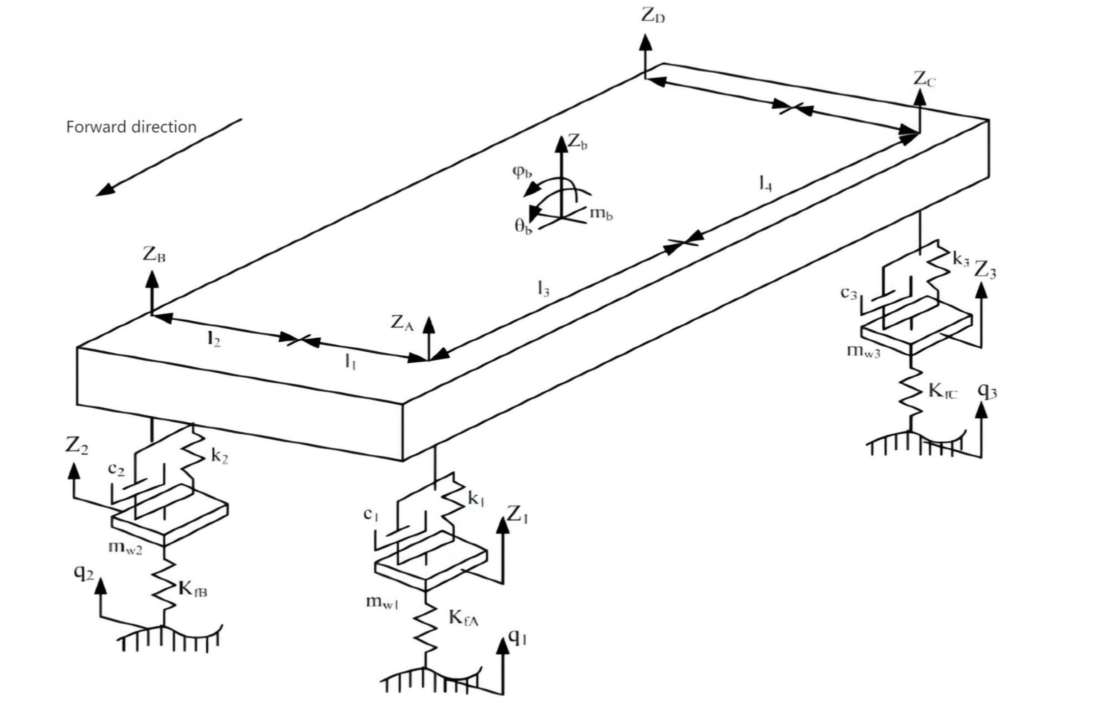

Fig. 4 Model of car 7-DOF system

According to the 2-DOF system model of the car shown in Figure 2, first establish the differential equation of motion.When the pitch angle θ_b and the roll angle ϕ are small, the vertical displacements at the four end points of the vehicle body have the following relationship:

$$z_{bA} = z_b - a\theta_b + \frac{1}{2}B_f\phi \tag{1}$$

$$z_{bB} = z_b - a\theta_b - \frac{1}{2}B_f\phi \tag{2}$$

$$z_{bC} = z_b + b\theta_b + \frac{1}{2}B_r\phi \tag{3}$$

$$z_{bD} = z_b + b\theta_b - \frac{1}{2}B_r\phi \tag{4}$$

Therefore, the equation of vertical motion at the center of mass of the body is:

$$\begin{aligned} m_b\ddot{z}_b &= C_{sA}(\dot{z}_{wA} - \dot{z}_{bA}) + k_{sA}(z_{wA} - z_{bA}) + C_{sB}(\dot{z}_{wB} - \dot{z}_{bB}) + k_{sB}(z_{wB} - z_{bB}) \\ &\quad + C_{sC}(\dot{z}_{wC} - \dot{z}_{bC}) + k_{sC}(z_{wC} - z_{bC}) + C_{sD}(\dot{z}_{wD} - \dot{z}_{bD}) + k_{sD}(z_{wD} - z_{bD}) \end{aligned} \tag{5}$$

The body pitch motion equation is:

$$\begin{aligned} I_p\ddot{\theta}_b &= b[C_{sC}(\dot{z}_{wC} - \dot{z}_{bC}) + k_{sC}(z_{wC} - z_{bC}) + C_{sD}(\dot{z}_{wD} - \dot{z}_{bD}) + k_{sD}(z_{wD} - z_{bD})] \\ &\quad - a[C_{sA}(\dot{z}_{wA} - \dot{z}_{bA}) + k_{sA}(z_{wA} - z_{bA}) + C_{sB}(\dot{z}_{wB} - \dot{z}_{bB}) + k_{sB}(z_{wB} - z_{bB})] \end{aligned} \tag{6}$$

The body roll motion equation is:

$$I_r\ddot{\phi}=[C_{sA}(\dot{z}_{wA}-\dot{z}_{bA})+k_{sA}(z_{wA}-z_{bA})-C_{sB}(\dot{z}_{wB}-\dot{z}_{bB})-k_{sB}(z_{wB}-z_{bB})\frac{B_f}{2} \\ +[C_{sC}(\dot{z}_{wC}-\dot{z}_{bC})+k_{sC}(z_{wC}-z_{bC})-C_{sD}(\dot{z}_{wD}-\dot{z}_{bD})-k_{sD}(z_{wD}-z_{bD})\frac{B_r}{2} \tag{7}$$

The vertical motion equations of the four unsprung masses are:

$$m_{wA}\ddot{z}_{wA}=k_{tA}(z_{gA}-z_{wA})+k_{sA}(z_{bA}-z_{wA})+C_{sA}(\dot{z}_{bA}-\dot{z}_{wA}) \tag{8}$$

$$m_{wB}\ddot{z}_{wB}=k_{tB}(z_{gB}-z_{wB})+k_{sB}(z_{bB}-z_{wB})+C_{sB}(\dot{z}_{bB}-\dot{z}_{wB}) \tag{9}$$

$$m_{wC}\ddot{z}_{wC}=k_{tC}(z_{gC}-z_{wC})+k_{sC}(z_{bC}-z_{wC})+C_{sC}(\dot{z}_{bC}-\dot{z}_{wC}) \tag{10}$$

$$m_{wD}\ddot{z}_{wD}=k_{tD}(z_{gD}-z_{wD})+k_{sD}(z_{bD}-z_{wD})+C_{sD}(\dot{z}_{bD}-\dot{z}_{wD}) \tag{11}$$

The seven differential equations (5) to (11) above represent the vehicle dynamics model with seven degrees of freedom. Taking z_b 、 θ_b 、 ϕ 、 z_{wA} 、 z_{wB} 、 z_{wC} and z_{wD} as state variables to establish a differential matrix equation in the form of $M\ddot{X}+C\dot{X}+KX=K_tZ_g$, we get:

$$\begin{aligned}&m_b\ddot{z}_b\\&+(C_{sA}+C_{sB}+C_{sC}+C_{sD})\dot{z}_b+(-aC_{sA}-aC_{sB}+bC_{sC}+bC_{sD})\dot{\theta}_b+\frac{1}{2}(B_fC_{sA}-B_fC_{sB}+B_rC_{sC}-B_rC_{sD})\dot{\phi}\\&-C_{sA}\dot{z}_{wA}-C_{sB}\dot{z}_{wB}-C_{sC}\dot{z}_{wC}-C_{sD}\dot{z}_{wD}\\&+(K_{sA}+K_{sB}+K_{sC}+K_{sD})z_b+(-aK_{sA}-aK_{sB}+bK_{sC}+bK_{sD})\theta_b+\frac{1}{2}(B_fK_{sA}-B_fK_{sB}+B_rK_{sC}-B_rK_{sD})\phi\\&-K_{sA}z_{wA}-K_{sB}z_{wB}-K_{sC}z_{wC}-K_{sD}z_{wD}=0\end{aligned} \tag{12}$$

$$\begin{aligned}&I_p\ddot{\theta}_b\\&+(-aC_{sA}-aC_{sB}+bC_{sC}+bC_{sD})\dot{z}_b+(a^2C_{sA}+a^2C_{sB}+b^2C_{sC}+b^2C_{sD})\dot{\theta}_b\\&+\frac{1}{2}(-aB_fC_{sA}+aB_fC_{sB}+bB_rC_{sC}-bB_rC_{sD})\dot{\phi}+aC_{sA}\dot{z}_{wA}+aC_{sB}\dot{z}_{wB}-bC_{sC}\dot{z}_{wC}-bC_{sD}\dot{z}_{wD}\\&+(-aK_{sA}-aK_{sB}+bK_{sC}+bK_{sD})z_b+(a^2K_{sA}+a^2K_{sB}+b^2K_{sC}+b^2K_{sD})\theta_b\\&+\frac{1}{2}(-aB_fK_{sA}+aB_fK_{sB}+bB_rK_{sC}-bB_rK_{sD})\phi+aK_{sA}z_{wA}+aK_{sB}z_{wB}-bK_{sC}z_{wC}-bK_{sD}z_{wD}=0\end{aligned} \tag{13}$$

$$
\begin{aligned}
& I_r\ddot{\phi} \\
& +\frac{1}{2}(B_f C_{sA} - B_f C_{sB} + B_r C_{sC} - B_r C_{sD})\dot{z}_b + \frac{1}{2}(-aB_f C_{sA} + aB_f C_{sB} + bB_r C_{sC} - bB_r C_{sD})\dot{\theta}_b \\
& +\frac{1}{4}(B_f^2 C_{sA} + B_f^2 C_{sB} + B_r^2 C_{sC} + B_r^2 C_{sD})\dot{\phi} - \frac{B_f C_{sA}}{2}\dot{z}_{wA} + \frac{B_f C_{sB}}{2}\dot{z}_{wB} - \frac{B_r C_{sC}}{2}\dot{z}_{wC} + \frac{B_r C_{sD}}{2}\dot{z}_{wD} \\
& +\frac{1}{2}(B_f K_{sA} - B_f K_{sB} + B_r K_{sC} - B_r K_{sD})z_b + \frac{1}{2}(-aB_f K_{sA} + aB_f K_{sB} + bB_r K_{sC} - bB_r K_{sD})\theta_b \\
& +\frac{1}{4}(B_f^2 K_{sA} + B_f^2 K_{sB} + B_r^2 K_{sC} + B_r^2 K_{sD})\phi - \frac{B_f K_{sA}}{2}z_{wA} + \frac{B_f K_{sB}}{2}z_{wB} - \frac{B_r K_{sC}}{2}z_{wC} + \frac{B_r K_{sD}}{2}z_{wD} = 0
\end{aligned}
\tag{14}
$$

$$
m_{wA}\ddot{z}_{wA} - C_{sA}\dot{z}_b + aC_{sA}\dot{\theta}_b - \frac{B_f C_{sA}}{2}\dot{\phi} + C_{sA}\dot{z}_{wA} - K_{sA}z_b + aK_{sA}\theta_b - \frac{B_f K_{sA}}{2}\phi + (K_{sA} + K_{tA})z_{wA} = K_{tA}z_{gA} \tag{15}
$$

$$
m_{wB}\ddot{z}_{wB} - C_{sB}\dot{z}_b + aC_{sB}\dot{\theta}_b + \frac{B_f C_{sB}}{2}\dot{\phi} + C_{sB}\dot{z}_{wB} - K_{sB}z_b + aK_{sB}\theta_b + \frac{B_f K_{sB}}{2}\phi + (K_{sB} + K_{tB})z_{wB} = K_{tB}z_{gB} \tag{16}
$$

$$
m_{wC}\ddot{z}_{wC} - C_{sC}\dot{z}_b - bC_{sC}\dot{\theta}_b - \frac{B_r C_{sC}}{2}\dot{\phi} + C_{sC}\dot{z}_{wC} - K_{sC}z_b - bK_{sC}\theta_b - \frac{B_r K_{sC}}{2}\phi + (K_{sC} + K_{tC})z_{wC} = K_{tC}z_{gC} \tag{17}
$$

$$
m_{wD}\ddot{z}_{wD} - C_{sD}\dot{z}_b - bC_{sD}\dot{\theta}_b + \frac{B_r C_{sD}}{2}\dot{\phi} + C_{sD}\dot{z}_{wD} - K_{sD}z_b - bK_{sD}\theta_b + \frac{B_r K_{sD}}{2}\phi + (K_{sD} + K_{tD})z_{wD} = K_{tD}z_{gD} \tag{18}
$$

Taking the coefficients of the differential equations (12) to (18), the mass matrix M, damping matrix C, stiffness matrix K and input matrix K_t:

$$
M = \begin{bmatrix}
m_b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & I_p & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I_r & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_{wA} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_{wB} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_{wC} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_{wD}
\end{bmatrix}
\qquad
K_t = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
K_{tA} & 0 & 0 & 0 \\
0 & K_{tB} & 0 & 0 \\
0 & 0 & K_{tC} & 0 \\
0 & 0 & 0 & K_{tD}
\end{bmatrix}
$$

$$C=\begin{bmatrix} C_{sA}+C_{sB}+C_{sC}+C_{sD} & -aC_{sA}-aC_{sB}+bC_{sC}+bC_{sD} \\ -aC_{sA}-aC_{sB}+bC_{sC}+bC_{sD} & a^2C_{sA}+a^2C_{sB}+b^2C_{sC}+b^2C_{sD} \\ \frac{1}{2}(B_fC_{sA}-B_fC_{sB}+B_rC_{sC}-B_rC_{sD}) & \frac{1}{2}(-aB_fC_{sA}+aB_fC_{sB}+bB_rC_{sC}-bB_rC_{sD}) \\ -C_{sA} & aC_{sA} \\ -C_{sB} & aC_{sB} \\ -C_{sC} & -bC_{sC} \\ -C_{sD} & -bC_{sD} \end{bmatrix.$$

$$\begin{matrix} \frac{1}{2}(B_fC_{sA}-B_fC_{sB}+B_rC_{sC}-B_rC_{sD}) & -C_{sA} & -C_{sB} & -C_{sC} & -C_{sD} \\ \frac{1}{2}(-aB_fC_{sA}+aB_fC_{sB}+bB_rC_{sC}-bB_rC_{sD}) & aC_{sA} & aC_{sB} & -bC_{sC} & -bC_{sD} \\ \frac{1}{4}(B_f^2C_{sA}+B_f^2C_{sB}+B_r^2C_{sC}+B_r^2C_{sD}) & -\frac{B_fC_{sA}}{2} & \frac{B_fC_{sB}}{2} & -\frac{B_rC_{sC}}{2} & \frac{B_rC_{sD}}{2} \\ -\frac{B_fC_{sA}}{2} & C_{sA} & 0 & 0 & 0 \\ \frac{B_fC_{sB}}{2} & 0 & C_{sB} & 0 & 0 \\ -\frac{B_rC_{sC}}{2} & 0 & 0 & C_{sC} & 0 \\ \frac{B_rC_{sD}}{2} & 0 & 0 & 0 & C_{sD} \end{matrix}\right]$$

$$K=\begin{bmatrix} K_{sA}+K_{sB}+K_{sC}+K_{sD} & -aK_{sA}-aK_{sB}+bK_{sC}+bK_{sD} \\ -aK_{sA}-aK_{sB}+bK_{sC}+bK_{sD} & a^2K_{sA}+a^2K_{sB}+b^2K_{sC}+b^2K_{sD} \\ \frac{1}{2}(B_fK_{sA}-B_fK_{sB}+B_rK_{sC}-B_rK_{sD}) & \frac{1}{2}(-aB_fK_{sA}+aB_fK_{sB}+bB_rK_{sC}-bB_rK_{sD}) \\ -K_{sA} & aK_{sA} \\ -K_{sB} & aK_{sB} \\ -K_{sC} & -bK_{sC} \\ -K_{sD} & -bK_{sD} \end{bmatrix.$$

$$\begin{matrix} \frac{1}{2}(B_fK_{sA}-B_fK_{sB}+B_rK_{sC}-B_rK_{sD}) & -K_{sA} & -K_{sB} & -K_{sC} & -K_{sD} \\ \frac{1}{2}(-aB_fK_{sA}+aB_fK_{sB}+bB_rK_{sC}-bB_rK_{sD}) & aK_{sA} & aK_{sB} & -bK_{sC} & -bK_{sD} \\ \frac{1}{4}(B_f^2K_{sA}+B_f^2K_{sB}+B_r^2K_{sC}+B_r^2K_{sD}) & -\frac{B_fK_{sA}}{2} & \frac{B_fK_{sB}}{2} & -\frac{B_rK_{sC}}{2} & \frac{B_rK_{sD}}{2} \\ -\frac{B_fK_{sA}}{2} & K_{sA}+K_{tA} & 0 & 0 & 0 \\ \frac{B_fK_{sB}}{2} & 0 & K_{sB}+K_{tB} & 0 & 0 \\ -\frac{B_rK_{sC}}{2} & 0 & 0 & K_{sC}+K_{tC} & 0 \\ \frac{B_rK_{sD}}{2} & 0 & 0 & 0 & K_{sD}+K_{tD} \end{matrix}\right]$$

In the differential equation, the subscripts A, B, C, and D represent the front left, front right, rear left, and rear right wheels, respectively, z_b is the vertical displacement at the center of mass of the body, z_w is the vertical displacement of the wheel, θ_b is the pitch angle of the vehicle, ϕ

is the roll angle of the vehicle, and z_g is the input vertical displacement of the road surface.In addition, m_b is the sprung mass of the vehicle, m_w is the unsprung mass of the vehicle, a is the distance from the center of mass of the body to the front axle, b is the distance from the center of mass of the body to the rear axle, B_f is the wheel track of the front wheel, B_r is the wheel track of the rear wheel, K_s is the spring stiffness of the suspension, K_t is the tire stiffness, C_s is the damping coefficient of the suspension, I_p is the pitch moment of inertia, and I_r is the roll moment of inertia.

The simulation model established by MATLAB/Simulink is as follows: