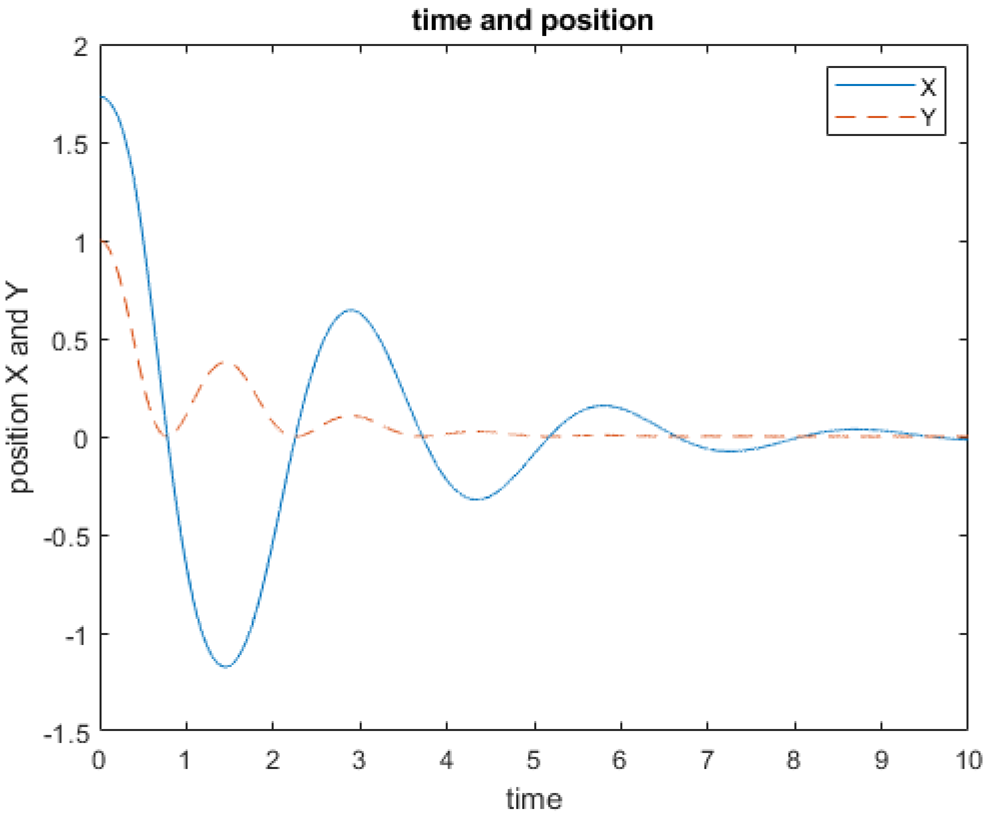
Solution for the first Report

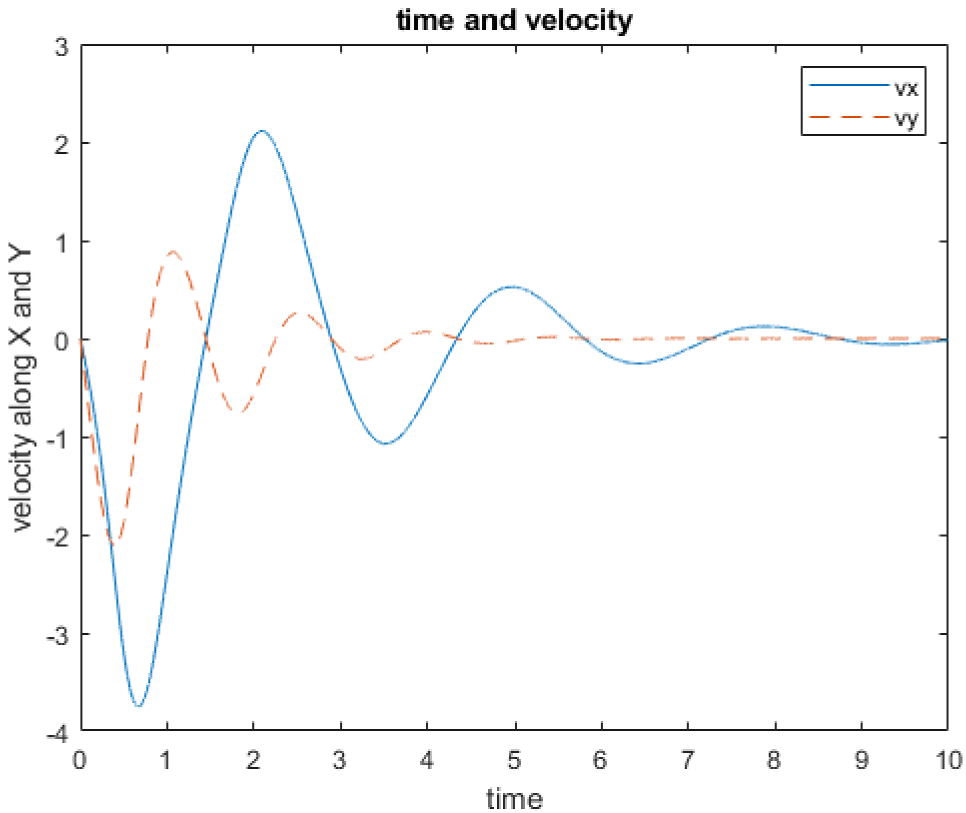
```
% equation of motion of simple pendulum (Cartesian)
function dotq = damped_pendulum_Cartesian (t,q)

    global mass; global length; global grav; global alpha; global
viscous;
    x = q(1); y = q(2); vx = q(3); vy = q(4);

    dotx = vx; doty = vy;
    R = sqrt(x^2+(y-length)^2) - length;
    P = 1/sqrt(x^2+(y-length)^2);
    Rx = x*P; Ry = (y-length)*P;
    Rxx = P - x^2*P^3;
    Ryy = P - (y-length)^2*P^3;
    Rxy = -x*(y-length)*P^3;
    C = [vx,vy]*[Rxx, Rxy; Rxy, Ryy]*[vx;vy] ...
        + 2*alpha*[Rx, Ry]*[vx;vy]...
        + alpha^2*R;
    fvx = - viscous*vx;
    fvy = - viscous*vy;
    A = [mass, 0, -Rx; 0, mass, -Ry; -Rx, -Ry, 0];
    b = [ fvx; -mass*grav - fvy; C ];
    s = A \ b;
    dotvx = s(1); dotvy = s(2);
    dotq = [dotx; doty; dotvx; dotvy];
end
```

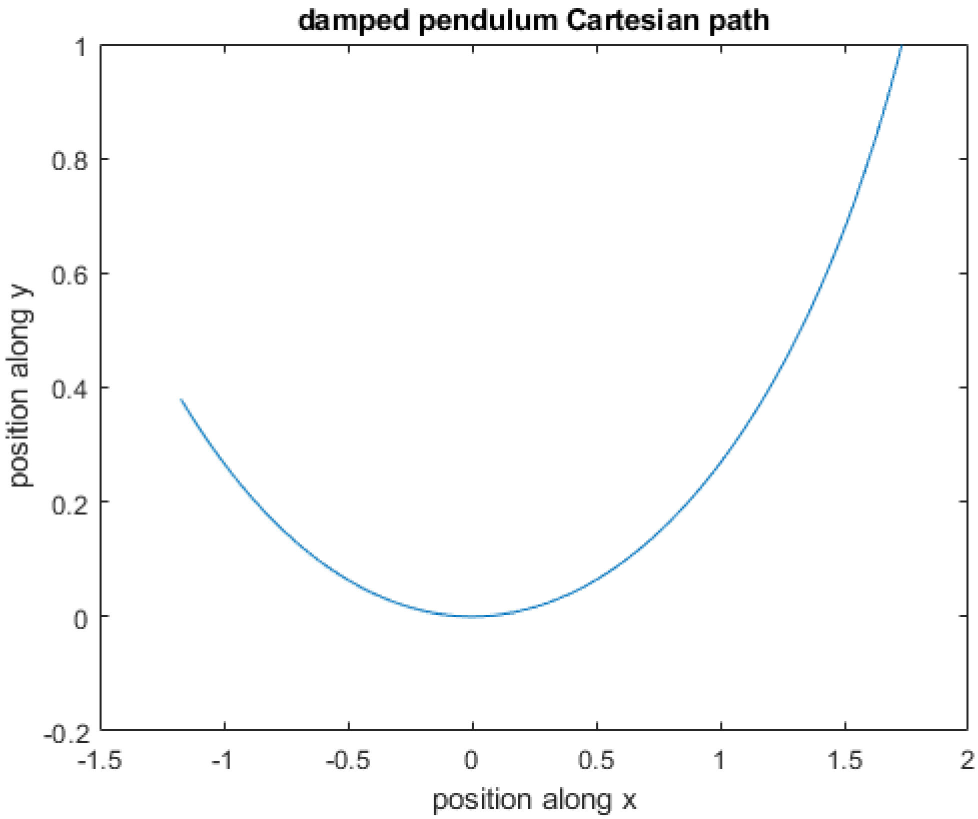
```
% solve the equation of motion of damped pendulum (Cartesian)

global mass; global length; global grav;
mass = 0.01; length = 2.0; grav = 9.8;
global alpha;
alpha = 1000;
global viscous;
viscous = 0.01;

interval = [0, 10];
qinit = [length*sin(pi/3); length*(1-cos(pi/3)); 0; 0];
[time, q] = ode45(@damped_pendulum_Cartesian, interval, qinit);

% time - x and y
plot(time,q(:,1),'-', time,q(:,2),'--');
title('time and position');
xlabel('time');
ylabel('position X and Y');
legend('X','Y');
saveas(gcf, 'damped_pendulum_Cartesian_x_y.png');
figure;

% time - vx and vy
plot(time,q(:,3),'-', time,q(:,4),'--');
title('time and velocity');
xlabel('time');
ylabel('velocity along X and Y');
legend('vx','vy');
saveas(gcf, 'damped_pendulum_Cartesian_vx_vy.png');
figure;

% x - y
plot(q(:,1), q(:,2));
```

```
title('damped pendulum Cartesian path')
xlabel('position along x');
ylabel('position along y');
saveas(gcf, 'damped_pendulum_Cartesian_path.png');
figure;

% computed pendulum angle
angle = atan2(q(:,1), length-q(:,2));
plot(time, angle);
title('damped pendulum Cartesian computed angle');
xlabel('time');
ylabel('angle');
saveas(gcf, 'damped_pendulum_Cartesian_computed_angle.png');
figure;

% constraint
R = sqrt(q(:,1).^2 + (q(:,2)-length).^2)-length;
plot(time, R);
title('damped pendulum cartesian R');
xlabel('time');
ylabel('constraint R');
saveas(gcf, 'damped_pendulum_Cartesian_R.png');
```

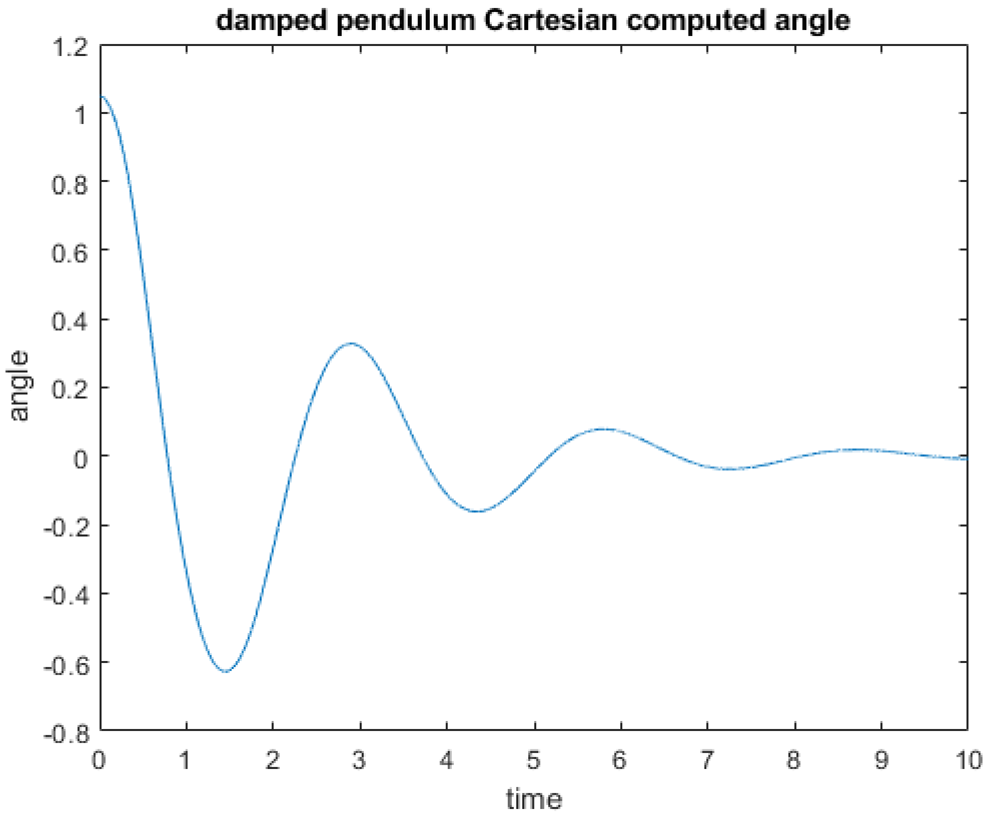


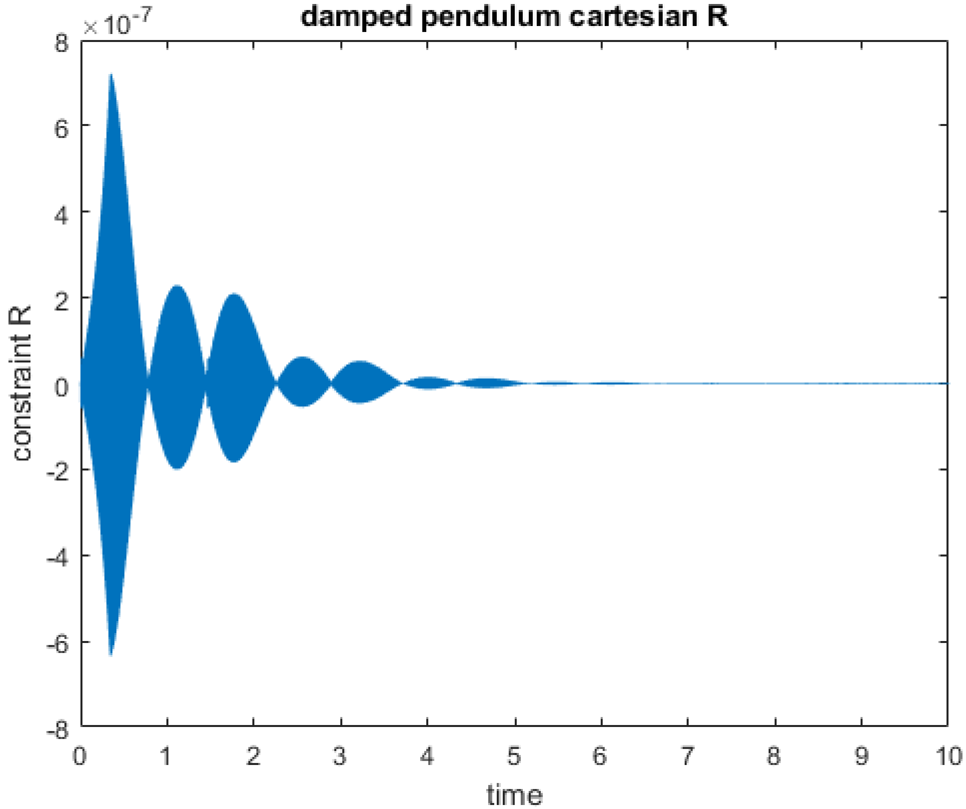
time and position
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7
8
9
10
time
position X and Y
X
Y


time and velocity
vx
vy
velocity along X and Y
time
3
2
1
0
-1
-2
-3
-4
0
1
2
3
4
5
6
7
8
9
10



damped pendulum Cartesian path
1
0.8
0.6
0.4
0.2
0
-0.2
-1.5
-1
-0.5
0
0.5
1
1.5
2
position along y
position along x


damped pendulum Cartesian computed angle
angle
time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1
2
3
4
5
6
7
8
9
10



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Derivation:

$(G.R)$ - distance b/n c and mass $= 0$

$$R \triangleq \{x^2 + (y-l)^2\}^{1/2} - l$$

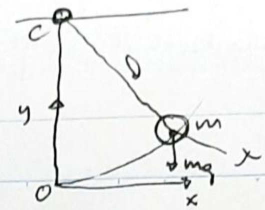
Lagrangian,

$$\mathcal{L} = T - U + W + \lambda R$$

Lagrange eqn of motion:

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

$$\frac{\partial L}{\partial y} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = 0$$



$$T = \frac{1}{2} m \{\dot{x}^2 + \dot{y}^2\}$$

$$U = mgy$$

$$W = f_x x + f_y y + b\{\dot{x} + \dot{y}\}$$

$$\underline{f_v} = -b\{\dot{x} + \dot{y}\}$$

$$R = \{x^2 + (y-l)^2\}^{1/2} - l$$

Lagrange

$$L = \frac{1}{2} m\{\dot{x}^2 + \dot{y}^2\} - mgy + f_x x + f_y y + b\{\dot{x} + \dot{y}\} + \lambda R$$

derivatives:

$$\frac{\partial L}{\partial x} = f_x + \lambda R_x, \quad \frac{\partial L}{\partial \dot{x}} = +m\dot{x} + \boxed{\overset{b}{\cancel{f_{v_x}}}}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} + b\dot{x}$$

$$\frac{\partial L}{\partial y} = -mg + f_y + \lambda R_y, \quad \frac{\partial L}{\partial \dot{y}} = +m\dot{y} + b$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m\ddot{y} + b\dot{y}$$

Lagrange equation of motion becomes :-

$$f_x + \lambda R_x - m\ddot{x} - b\dot{x} = 0$$

$$-mg + f_y + \lambda R_y - m\ddot{y} - b\dot{y} = 0$$

$$\begin{bmatrix} 0 \\ -mg \end{bmatrix} + \begin{bmatrix} f_x \\ f_y \end{bmatrix} + \lambda \begin{bmatrix} R_x \\ R_y \end{bmatrix} + \left\{-m\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}\right\} + \underbrace{\left\{-b\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}\right\}}_{\text{viscous friction}} = 0$$

So we have :-

$$m\ddot{x} = f_x + \lambda R_x - b\dot{x}$$

$$m\ddot{y} = -mg + f_y + \lambda R_y - b\dot{y}$$

$$\{R = 0\} \quad \text{algebraic equation}$$

constraint stabilization

(*) change algebraic equation to most equivalent diff. eqn

$R = 0$

$\Downarrow$

$\ddot{R} + 2\alpha \dot{R} + \alpha^2 R = 0$

$\Rightarrow$ Since R - B in cartesian : $R(x,y) = 0$

$$\begin{cases} \dot{R} = R_x \dot{x} + R_y \dot{y} \\ \ddot{R} = \dot{R}_x \dot{x} + R_x \ddot{x} + \dot{R}_y \dot{y} + R_y \ddot{y} \end{cases}$$

$\Downarrow$

$$\Rightarrow \dot{R}_x = \dot{R}_x(x,y) = \frac{\partial R_x}{\partial x}\frac{dx}{dt} + \frac{\partial R_x}{\partial y}\frac{dy}{dt}$$

$$\dot{R}_x = R_{xx}\dot{x} + R_{xy}\dot{y}$$

$$\Rightarrow \dot{R}_y = \dot{R}_y(x,y) = R_{yy}\dot{y} + R_{yx}\dot{x}$$

$$\therefore \ddot{R} = (R_{xx}\dot{x} + R_{xy}\dot{y})\dot{x} + R_x\ddot{x} + (R_{yy}\dot{y} + R_{yx}\dot{x})\dot{y} + R_y\ddot{y}$$

$$\ddot{R} = [R_x \; R_y]\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} + [\dot{x} \; \dot{y}]\begin{bmatrix}R_{xx} & R_{xy}\\ R_{yx} & R_{yy}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}$$

Since :- $\ddot{R} = -(2\alpha\dot{R} + \alpha^2 R) = -(2\alpha(R_x\dot{x} + R_y\dot{y}) + \alpha^2 R)$

equating the above eqn :-

$$-[R_x \; R_y]\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = [\dot{x} \; \dot{y}]\begin{bmatrix}R_{xx} & R_{xy}\\ R_{yx} & R_{yy}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} + \left\{2\alpha[R_x \; R_y]\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} + \alpha^2 R\right\}$$

$$V_x = \dot{x}, \; V_y = \dot{y}$$

$\Downarrow$

$$-[R_x \; R_y]\begin{bmatrix}\dot{V}_x\\ \dot{V}_y\end{bmatrix} = [V_x \; V_y]\begin{bmatrix}R_{xx} & R_{xy}\\ R_{yx} & R_{yy}\end{bmatrix}\begin{bmatrix}V_x\\ V_y\end{bmatrix} + \left\{2\alpha[R_x \; R_y]\begin{bmatrix}V_x\\ V_y\end{bmatrix} + \alpha^2 R\right\}$$

$-R_x\dot{V}_x - R_y\dot{V}_y = C(x,y,V_x,V_y)$ * Equation for stability constraint $R(x,y) = 0$

Final Equation (by combining equation motion ~~of~~ and equation of constraint stabilization)

$$\begin{bmatrix} m & 0 & -R_x \\ 0 & m & -R_x \\ -R_x & -R_y & 0 \end{bmatrix}\begin{bmatrix}\dot{V}_x\\ \dot{V}_y\\ \lambda\end{bmatrix} = \begin{bmatrix} f_x - bV_x \\ -mg + f_y - bV_y \\ C(x,y,V_x,V_y)\end{bmatrix}$$

$\dot{x} = V_x$

$\dot{y} = V_y$