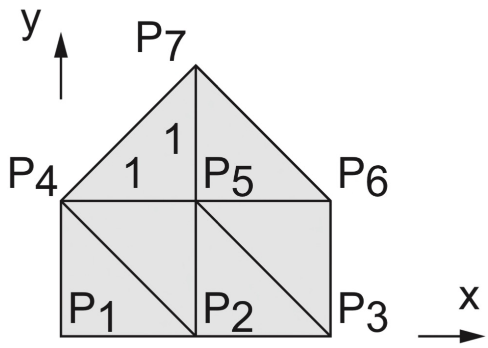
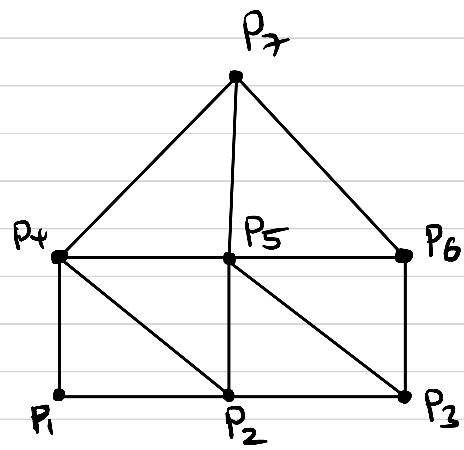
Analytical Mechanics Report #7

Report #7 due date : Jan. 9 (Mon) 1:00 AM

Cauculate inertia matrix M and connection matrices J_λ, J_μ for a two-dimensional object shown in the figure. Length of the orthogonal sides of isosceles right triangles is 1 and thickness h is equal to 2.



Solution :-



$\Rightarrow$ From the kinetic energy of $\Delta = \Delta P_i P_j P_k$

$$T_{i,j,k} = \int_{\Delta} \frac{1}{2} \rho \dot{U}^T U h \, ds$$

We have the inertia matrix :-

$$M_{i,j,k} = \frac{\rho h \Delta}{12} \begin{bmatrix} 2I_{2\times2} & I_{2\times2} & I_{2\times2} \\ I_{2\times2} & 2I_{2\times2} & I_{2\times2} \\ I_{2\times2} & I_{2\times2} & 2I_{2\times2} \end{bmatrix}$$

$\Rightarrow$ From the above figure, the region consists of six triangular regions: $\Delta P_1 P_2 P_4$, $\Delta P_2 P_3 P_5$, $\Delta P_5 P_4 P_2$, $\Delta P_6 P_5 P_3$, $\Delta P_5 P_4 P_7$, and $\Delta P_5 P_6 P_7$.

$\Rightarrow$ Then the partial inertial matrices are given as :-

$$M_{1,2,4} = M_{2,3,5} = M_{5,4,2} = M_{6,5,3} = M_{5,4,7} = M_{5,6,7} = \frac{\rho h \Delta}{12} \begin{bmatrix} 2I_{2\times2} & I_{2\times2} & I_{2\times2} \\ I_{2\times2} & 2I_{2\times2} & I_{2\times2} \\ I_{2\times2} & I_{2\times2} & 2I_{2\times2} \end{bmatrix}$$

(*) we can assume $\frac{\rho h \Delta}{12} = 1$ for simplicity.

$\Rightarrow$ Then the inertial matrix M consists of 7^2 2×2 blocks and is a sparse matrix. It can be described as

$$M = \bigoplus_{i,j,k} M_{i,j,k}$$

(*) where i, j, k represent the nodal point nos.

Now :-

(*) contribution of $M_{1,2,4}$ to M (a 7×7 block inertial matrix)

↳ $(1,2,3)\times(1,2,3)$ block of $M_{1,2,4}$ contributes to $(1,2,4)\times(1,2,4)$ blocks of M

$$\begin{bmatrix} 2I_{2\times2} & I_{2\times2} & & I_{2\times2} & & & \\ I_{2\times2} & 2I_{2\times2} & & I_{2\times2} & & & \\ & & & & & & \\ I_{2\times2} & I_{2\times2} & & 2I_{2\times2} & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

(*) contribution of $M_{2,3,5}$ to M

↳ $(1,2,3)\times(1,2,3)$ block of $M_{2,3,5}$ contributes to $(2,3,5)\times(2,3,5)$ blocks of M, that is :-

$$\begin{bmatrix} & & & & & & \\ & 2I_{2\times2} & I_{2\times2} & & I_{2\times2} & & \\ & I_{2\times2} & 2I_{2\times2} & & I_{2\times2} & & \\ & & & & & & \\ & I_{2\times2} & I_{2\times2} & & 2I_{2\times2} & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

* contribution of $M_{5,4,2}$ to M, similarly

$$\begin{bmatrix} & & & & & & \\ & 2I_{2\times2} & & I_{2\times2} & I_{2\times2} & & \\ & & & & & & \\ & I_{2\times2} & & 2I_{2\times2} & I_{2\times2} & & \\ & I_{2\times2} & & I_{2\times2} & 2I_{2\times2} & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

* contribution of $M_{6,5,3}$ to M :-

$$\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & 2I_{2\times2} & & I_{2\times2} & I_{2\times2} & \\ & & & & & & \\ & & I_{2\times2} & & 2I_{2\times2} & I_{2\times2} & \\ & & I_{2\times2} & & I_{2\times2} & 2I_{2\times2} & \\ & & & & & & \end{bmatrix}$$

(*) contribution of $M_{5,4,7}$ to M :-

$$\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & 2I_{2\times2} & I_{2\times2} & & I_{2\times2} \\ & & & I_{2\times2} & 2I_{2\times2} & & I_{2\times2} \\ & & & & & & \\ & & & I_{2\times2} & I_{2\times2} & & 2I_{2\times2} \end{bmatrix}$$

(*) contribution of $M_{5,6,7}$ to M :-

$$\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & 2I_{2\times2} & I_{2\times2} & I_{2\times2} \\ & & & & I_{2\times2} & 2I_{2\times2} & I_{2\times2} \\ & & & & I_{2\times2} & I_{2\times2} & 2I_{2\times2} \end{bmatrix}$$

$\Rightarrow$ summing up all the contribution, we finally have :-

$$M = \begin{bmatrix} 2I_{2\times2} & I_{2\times2} & O_{2\times2} & I_{2\times2} & O_{2\times2} & O_{2\times2} & O_{2\times2} \\ I_{2\times2} & 6I_{2\times2} & I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & O_{2\times2} & O_{2\times2} \\ O_{2\times2} & I_{2\times2} & 4I_{2\times2} & O_{2\times2} & 2I_{2\times2} & I_{2\times2} & O_{2\times2} \\ I_{2\times2} & 2I_{2\times2} & O_{2\times2} & 6I_{2\times2} & 2I_{2\times2} & O_{2\times2} & I_{2\times2} \\ O_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & 10I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} \\ O_{2\times2} & O_{2\times2} & I_{2\times2} & O_{2\times2} & 2I_{2\times2} & 4I_{2\times2} & I_{2\times2} \\ O_{2\times2} & O_{2\times2} & O_{2\times2} & I_{2\times2} & 2I_{2\times2} & I_{2\times2} & 2I_{2\times2} \end{bmatrix}$$

$\Rightarrow$ From the stiffness matrix, we can find the connection matrices J_λ and J_μ

$$K_{i,j,k} = \lambda J_\lambda^{i,j,k} + \mu J_\mu^{i,j,k}$$

Where :-

$$J_\lambda = \bigoplus_{i,j,k} J_\lambda^{i,j,k} \quad , \quad J_\mu = \bigoplus_{i,j,k} J_\mu^{i,j,k}$$

$\Rightarrow$ 1, 4, 2, 5, 3, 6 rows and columns of $H_\lambda, H_\mu \longrightarrow$ 1, 2, 3, 4, 5, 6 rows and columns of $J_\lambda^{i,j,k}$ and $J_\mu^{i,j,k}$

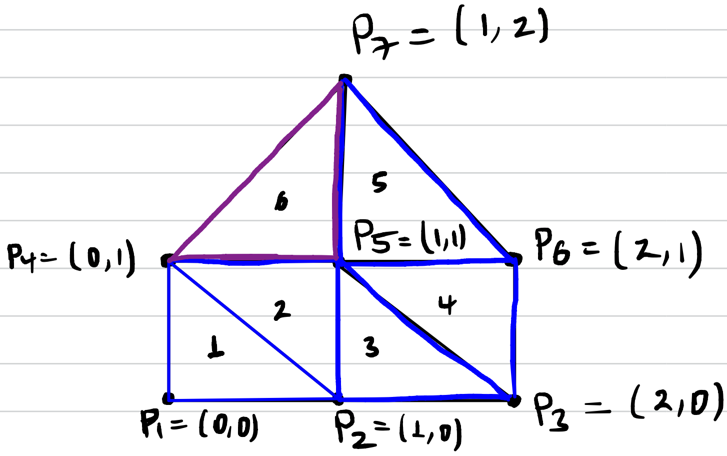
Where :-

$$H_\lambda = \begin{bmatrix} aa^T & ab^T \\ ba^T & bb^T \end{bmatrix} h\Delta$$

$$H_\mu = \begin{bmatrix} 2aa^T + bb^T & ba^T \\ ab^T & 2bb^T + aa^T \end{bmatrix} h\Delta$$

$$a = \frac{1}{2\Delta}\begin{bmatrix} y_j - y_k \\ y_k - y_i \\ y_i - y_j \end{bmatrix} \quad , \quad b = \frac{1}{2\Delta}\begin{bmatrix} x_j - x_k \\ x_k - x_i \\ x_i - x_j \end{bmatrix}$$

$\Rightarrow$ From the above figure we have :-



$\Rightarrow$ Now, let us calculate the partial connection matrices of triangle $P_1 P_2 P_4$, since $\underline{\underline{h = 2}}$

$P_1 = (0,0) = P_i$
$P_2 = (1,0) = P_j$
$P_4 = (0,1) = P_k$

$$a = \begin{bmatrix} 0-1 \\ 1-0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$b = -\begin{bmatrix} 1-0 \\ 0-0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\circledast$ $$aa^T = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\circledast$ $$bb^T = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$\circledast$ $$ab^T = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\circledast$ $$ba^T = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

$$H_\lambda = \begin{bmatrix} aa^T & ab^T \\ ba^T & bb^T \end{bmatrix} h\Delta$$

$$H\lambda = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$J_{\lambda}^{1,2,4} = \left[\begin{array}{cc|cc|cc} 1 & 1 & -1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 0 & 0 & -1 \\ \hline -1 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H\mu = \left[\begin{array}{ccc|ccc} 3 & -2 & -1 & 1 & -1 & 0 \\ -2 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 1 & 0 \\ \hline 1 & 0 & -1 & 3 & -1 & -2 \\ -1 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 \end{array}\right]$$

$$J_{\mu}^{1,2,4} = \left[\begin{array}{cc|cc|cc} 3 & 1 & -2 & -1 & -1 & 0 \\ 1 & 3 & 0 & -1 & -1 & -2 \\ \hline -2 & 0 & 2 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ \hline -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \end{array}\right]$$

$\Rightarrow$ The partial connection matrices for Δ 1, 2, 3, 4, and 5 are the same since the connection matrices are invariant with respect to translational displacement. In addition they posses the same aa^T, bb^T, ab^T, ba^T matrices.

$\Rightarrow$ As a result,

$$J_\lambda^{1,2,4} = J_\lambda^{2,3,5} = J_\lambda^{5,4,2} = J_\lambda^{6,5,3} = J_\lambda^{5,6,7}$$

$$J_\mu^{1,2,4} = J_\mu^{2,3,5} = J_\mu^{5,4,2} = J_\mu^{6,5,3} = J_\mu^{5,6,7}$$

$\Rightarrow$ Now let's calculate the partial connection matrice for the last triangle $\Delta P_5 P_4 P_7$

$P_5 = (1,1) = P_i$
$P_4 = (0,1) = P_j$
$P_7 = (1,2) = P_k$

$$a = \begin{bmatrix} 1-2 \\ 2-1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$b = -\begin{bmatrix} 0-1 \\ 1-1 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

* $$aa^T = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* $$bb^T = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

* $$ab^T = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

* $$ba^T = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$H_\lambda = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$J_\lambda^{5,4,7} = \left[\begin{array}{cc|cc|cc} 1 & -1 & -1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & -1 \\ \hline -1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H_\mu = \left[\begin{array}{ccc|ccc} 3 & -2 & -1 & -1 & 1 & 0 \\ -2 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & -1 & 0 \\ \hline -1 & 0 & 1 & 3 & -1 & -2 \\ 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 \end{array}\right]$$

$$J_\mu^{5,4,7} = \left[\begin{array}{cc|cc|cc} 3 & -1 & -2 & 1 & -1 & 0 \\ -1 & 3 & 0 & -1 & 1 & -2 \\ \hline -2 & 0 & 2 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & -1 & 0 \\ \hline -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 \end{array}\right]$$

⇒ Now, let's calculate the connection matrices J_λ and J_μ of the entire region.

$$J_\lambda = J_\lambda^{1,2,4} \oplus J_\lambda^{2,3,5} \oplus J_\lambda^{5,4,2} \oplus J_\lambda^{6,5,3} \oplus J_\lambda^{5,4,7} \oplus J_\lambda^{5,6,7}$$

$$J_\mu = J_\mu^{1,2,4} \oplus J_\mu^{2,3,5} \oplus J_\lambda^{5,4,2} \oplus J_\mu^{6,5,3} \oplus J_\mu^{5,4,7} \oplus J_\mu^{5,6,7}$$

* $J_\lambda^{1,2,4}$ contribute to $(1,2,4) \times (1,2,4)$ block of J_λ

$$\begin{bmatrix}
\begin{smallmatrix}1 & 1\\ 1 & 1\end{smallmatrix} & \begin{smallmatrix}-1 & 0\\ -1 & 0\end{smallmatrix} & & \begin{smallmatrix}0 & -1\\ 0 & -1\end{smallmatrix} & & & \\
\begin{smallmatrix}-1 & -1\\ 0 & 0\end{smallmatrix} & \begin{smallmatrix}1 & 0\\ 0 & 0\end{smallmatrix} & & \begin{smallmatrix}0 & 1\\ 0 & 0\end{smallmatrix} & & & \\
 & & & & & & \\
\begin{smallmatrix}0 & 0\\ -1 & -1\end{smallmatrix} & \begin{smallmatrix}0 & 0\\ 1 & 0\end{smallmatrix} & & \begin{smallmatrix}0 & 0\\ 0 & 1\end{smallmatrix} & & & \\
 & & & & & & \\
 & & & & & & \\
 & & & & & &
\end{bmatrix}$$

* $J_\lambda^{2,3,5}$ contribution to $(2,3,5) \times (2,3,5)$ block of J_λ

$$\begin{bmatrix}
 & & & & & & \\
 & \begin{smallmatrix}1 & 1\\ 1 & 1\end{smallmatrix} & \begin{smallmatrix}-1 & 0\\ -1 & 0\end{smallmatrix} & & \begin{smallmatrix}0 & -1\\ 0 & -1\end{smallmatrix} & & \\
 & \begin{smallmatrix}-1 & -1\\ 0 & 0\end{smallmatrix} & \begin{smallmatrix}1 & 0\\ 0 & 0\end{smallmatrix} & & \begin{smallmatrix}0 & 1\\ 0 & 0\end{smallmatrix} & & \\
 & & & & & & \\
 & \begin{smallmatrix}0 & 0\\ -1 & -1\end{smallmatrix} & \begin{smallmatrix}0 & 0\\ 1 & 0\end{smallmatrix} & & \begin{smallmatrix}0 & 0\\ 0 & 1\end{smallmatrix} & & \\
 & & & & & & \\
 & & & & & &
\end{bmatrix}$$

(*) $J_\lambda^{5,4,2}$ contribution to $(5,4,2) \times (5,4,2)$ block of J_λ

$$
\left[\begin{array}{c|c|c|c|c|c|c}
 & & & & & & \\
\hline
 & \begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix} & & \begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix} & \begin{smallmatrix} 0 & 0 \\ -1 & -1 \end{smallmatrix} & & \\
\hline
 & & & & & & \\
\hline
 & \begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix} & & \begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix} & \begin{smallmatrix} -1 & -1 \\ 0 & 0 \end{smallmatrix} & & \\
\hline
 & \begin{smallmatrix} 0 & -1 \\ 0 & -1 \end{smallmatrix} & & \begin{smallmatrix} -1 & 0 \\ -1 & 0 \end{smallmatrix} & \begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix} & & \\
\hline
 & & & & & & \\
\hline
 & & & & & &
\end{array}\right]
$$

(*) $J_\lambda^{6,5,3}$ contribution to $(6,5,3) \times (6,5,3)$ block of J_λ

$$
\left[\begin{array}{c|c|c|c|c|c|c}
 & & & & & & \\
\hline
 & & & & & & \\
\hline
 & & \begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix} & & \begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix} & \begin{smallmatrix} 0 & 0 \\ -1 & -1 \end{smallmatrix} & \\
\hline
 & & & & & & \\
\hline
 & & \begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix} & & \begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix} & \begin{smallmatrix} -1 & -1 \\ 0 & 0 \end{smallmatrix} & \\
\hline
 & & \begin{smallmatrix} 0 & -1 \\ 0 & -1 \end{smallmatrix} & & \begin{smallmatrix} -1 & 0 \\ -1 & 0 \end{smallmatrix} & \begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix} & \\
\hline
 & & & & & &
\end{array}\right]
$$

$J_\lambda^{5,4,7}$ contribution to $(5,4,7)\times(5,4,7)$ block of J_λ

$$
\left[\begin{array}{c|c|c|c|c|c|c}
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & \begin{matrix}1 & 0\\ 0 & 0\end{matrix} & \begin{matrix}-1 & 1\\ 0 & 0\end{matrix} & & \begin{matrix}0 & -1\\ 0 & 0\end{matrix} \\ \hline
 & & & \begin{matrix}-1 & 0\\ 1 & 0\end{matrix} & \begin{matrix}1 & -1\\ -1 & 1\end{matrix} & & \begin{matrix}0 & 1\\ 0 & -1\end{matrix} \\ \hline
 & & & & & & \\ \hline
 & & & \begin{matrix}0 & 0\\ 1 & 0\end{matrix} & \begin{matrix}0 & 0\\ 1 & -1\end{matrix} & & \begin{matrix}0 & 0\\ 1 & -1\end{matrix}
\end{array}\right]
$$

$J_\lambda^{5,6,7}$ contribution to $(5,6,7)\times(5,6,7)$ block of J_λ

$$
\left[\begin{array}{c|c|c|c|c|c|c}
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & & \begin{matrix}1 & 1\\ 1 & 1\end{matrix} & \begin{matrix}-1 & 0\\ -1 & 0\end{matrix} & \begin{matrix}0 & -1\\ 0 & -1\end{matrix} \\ \hline
 & & & & \begin{matrix}-1 & -1\\ 0 & 0\end{matrix} & \begin{matrix}1 & 0\\ 0 & 0\end{matrix} & \begin{matrix}0 & 1\\ 0 & 0\end{matrix} \\ \hline
 & & & & \begin{matrix}0 & 0\\ -1 & -1\end{matrix} & \begin{matrix}0 & 0\\ 1 & 0\end{matrix} & \begin{matrix}0 & 0\\ 0 & 1\end{matrix}
\end{array}\right]
$$

$\Rightarrow$ Summing up all the contribution :-

$$J_\lambda = \left[\begin{array}{cc|cc|cc|cc|cc|cc|cc}
1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-1 & -1 & 2 & 1 & -1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & -1 & 0 & 1 & 0 & -1 & -2 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\
\hline
0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & -1 \\
-1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & -1 & 0 & 1 & -2 & 0 & 4 & 1 & -2 & -1 & 0 & 0 \\
0 & 0 & -1 & -2 & 1 & 0 & 0 & 0 & 1 & 4 & -1 & 0 & 0 & -2 \\
\hline
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -2 & -1 & 2 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 & 1 & 0
\end{array}\right]$$

(*) Similarly, J_μ can be found as follow :-

$J_\mu^{1,2,4}$ contribution of $(1,2,4) \times (1,2,4)$ block of J_μ

$$\left[\begin{array}{cc|cc|cc|cc|cc|cc|cc}
3 & 1 & -2 & -1 & & & -1 & 0 & & & & & & \\
1 & 3 & 0 & -1 & & & -1 & -2 & & & & & & \\
\hline
-2 & 0 & 2 & 0 & & & 0 & 0 & & & & & & \\
-1 & -1 & 0 & 1 & & & 1 & 0 & & & & & & \\
\hline
-1 & -1 & 0 & 1 & & & 1 & 0 & & & & & & \\
0 & -2 & 0 & 0 & & & 0 & 2 & & & & & & \\
\hline
& & & & & & & & & & & & & \\
& & & & & & & & & & & & & \\
\hline
& & & & & & & & & & & & & \\
& & & & & & & & & & & & & \\
\hline
& & & & & & & & & & & & & \\
& & & & & & & & & & & & & \\
\hline
& & & & & & & & & & & & & \\
& & & & & & & & & & & & &
\end{array}\right]$$

⊛ $J_\mu^{2,3,5}$ contribution to $(2,3,5) \times (2,3,5)$ block of J_μ

$$
\left[\begin{array}{cc|cc|cc|cc|cc|cc|cc}
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & 3 & 1 & -2 & -1 & & & -1 & 0 & & & & \\
 & & 1 & 3 & 0 & -1 & & & -1 & -2 & & & & \\
\hline
 & & -2 & 0 & 2 & 0 & & & 0 & 0 & & & & \\
 & & -1 & -1 & 0 & 1 & & & 1 & 0 & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & -1 & -1 & 0 & 1 & & & 1 & 0 & & & & \\
 & & 0 & -2 & 0 & 0 & & & 0 & 2 & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\end{array}\right]
$$

⊛ $J_\mu^{5,4,2}$ contribution to $(5,4,2) \times (5,4,2)$ block of J_μ

$$
\left[\begin{array}{cc|cc|cc|cc|cc|cc|cc}
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & 1 & 0 & & & 0 & 1 & -1 & -1 & & & & \\
 & & 0 & 2 & & & 0 & 0 & 0 & -2 & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & 0 & 0 & & & 2 & 0 & -2 & 0 & & & & \\
 & & 1 & 0 & & & 0 & 1 & -1 & -1 & & & & \\
\hline
 & & -1 & 0 & & & -2 & -1 & 3 & 1 & & & & \\
 & & -1 & -2 & & & 0 & -1 & 1 & 3 & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\end{array}\right]
$$

⊛ $J_\mu^{6,5,3}$ contribution to $(6,5,3)\times(6,5,3)$ block of J_μ

$$
\left[\begin{array}{c|c|c|c|c|c|c}
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & \begin{matrix}1&0\\0&2\end{matrix} & & \begin{matrix}0&1\\0&0\end{matrix} & \begin{matrix}-1&-1\\0&-2\end{matrix} & \\ \hline
 & & & & & & \\ \hline
 & & \begin{matrix}0&0\\1&0\end{matrix} & & \begin{matrix}2&0\\0&1\end{matrix} & \begin{matrix}-2&0\\-1&-1\end{matrix} & \\ \hline
 & & \begin{matrix}-1&0\\-1&-2\end{matrix} & & \begin{matrix}-2&-1\\0&-1\end{matrix} & \begin{matrix}3&1\\1&3\end{matrix} & \\ \hline
 & & & & & &
\end{array}\right]
$$

⊛ $J_\mu^{5,4,7}$ contribution to $(5,4,7)\times(5,4,7)$ block of J_μ

$$
\left[\begin{array}{c|c|c|c|c|c|c}
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & & & & \\ \hline
 & & & \begin{matrix}2&0\\0&1\end{matrix} & \begin{matrix}-2&0\\1&-1\end{matrix} & & \begin{matrix}0&0\\-1&0\end{matrix} \\ \hline
 & & & \begin{matrix}-2&1\\0&-1\end{matrix} & \begin{matrix}3&-1\\-1&3\end{matrix} & & \begin{matrix}-1&0\\1&-2\end{matrix} \\ \hline
 & & & & & & \\ \hline
 & & & \begin{matrix}0&-1\\0&0\end{matrix} & \begin{matrix}-1&1\\0&2\end{matrix} & & \begin{matrix}1&0\\0&2\end{matrix}
\end{array}\right]
$$

(7) $J_u^{5,6,7}$ contribution to $(5,6,7) \times (5,6,7)$ block of J_u

$$
\left[\begin{array}{cc|cc|cc|cc|cc|cc|cc}
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
\hline
 & & & & & & & & 3 & 1 & -2 & -1 & -1 & 0 \\
 & & & & & & & & 1 & 3 & 0 & -1 & -1 & -2 \\
\hline
 & & & & & & & & -2 & 0 & 2 & 0 & 0 & 0 \\
 & & & & & & & & -1 & -1 & 0 & 1 & 1 & 0 \\
\hline
 & & & & & & & & -1 & -1 & 0 & 1 & 1 & 0 \\
 & & & & & & & & 0 & -2 & 0 & 0 & 0 & 2
\end{array}\right]
$$

$\Rightarrow$ summing up all the contribution :-

$$
J_u = \left[\begin{array}{cc|cc|cc|cc|cc|cc|cc}
3 & 1 & -2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 3 & 0 & -1 & 0 & 0 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-2 & 0 & 6 & 1 & -2 & -1 & 0 & 1 & -2 & -1 & 0 & 0 & 0 & 0 \\
-1 & -1 & 1 & 6 & 0 & -1 & 1 & 0 & -1 & -4 & 0 & 0 & 0 & 0 \\
\hline
-1 & -1 & -2 & 1 & 3 & 0 & 1 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & -2 & -1 & -1 & 0 & 3 & 0 & 2 & 1 & 0 & 0 & -2 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & -6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & -3 & 0 & 0 & -1 & 0 \\
\hline
0 & 0 & -2 & -1 & 0 & 1 & -4 & 0 & 12 & 1 & -4 & -1 & -2 & 0 \\
0 & 0 & -1 & -4 & 1 & 0 & 0 & -2 & 1 & 12 & -1 & -2 & 0 & -4 \\
\hline
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -4 & -1 & 5 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -2 & 0 & 0 & -1 & -2 & 1 & 4 & 1 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4
\end{array}\right]
$$