Report for Advanced Topics for Robot Mechanism

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1. Introduction

The aim of the report is to study the kinematics of the manipulator. A 4DOF robot with a 4 revolute joint has been used for this purpose. A modified Elbow manipulator has been chosen for the kinematic model study. The manipulator contains four revolution joints. In this report, the forward kinematics has been performed, DH (Denavit–Hartenberg) parameters has been used to obtain the detailed mathematical solution. The assumed manipulator and its endpoint 3D motion drawn in fig(num) as a hand-drawn punch picture using the technique of technical illustration.

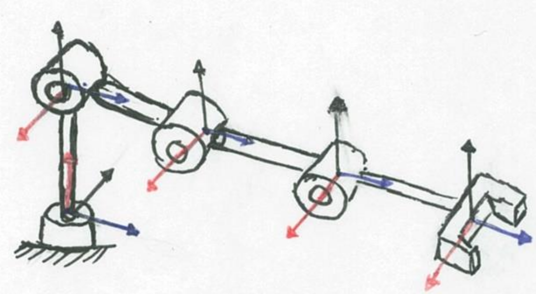

Fig1. 4DOF Elbow manipulator

From the above illustration we can easily see that this robot is capable to move around in the 3D space to achieve the task. The 3D motion is realized through its 4 revolute joints. The first joint is responsible for the main 3D motion around the Z_B axis, it controls link 2, 3 and 4. The second, third and forth joints are rotates the end effector along Y_B to achieve its task. This way the 3D motion can be realized.

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2. DH Notation and Robot Structure

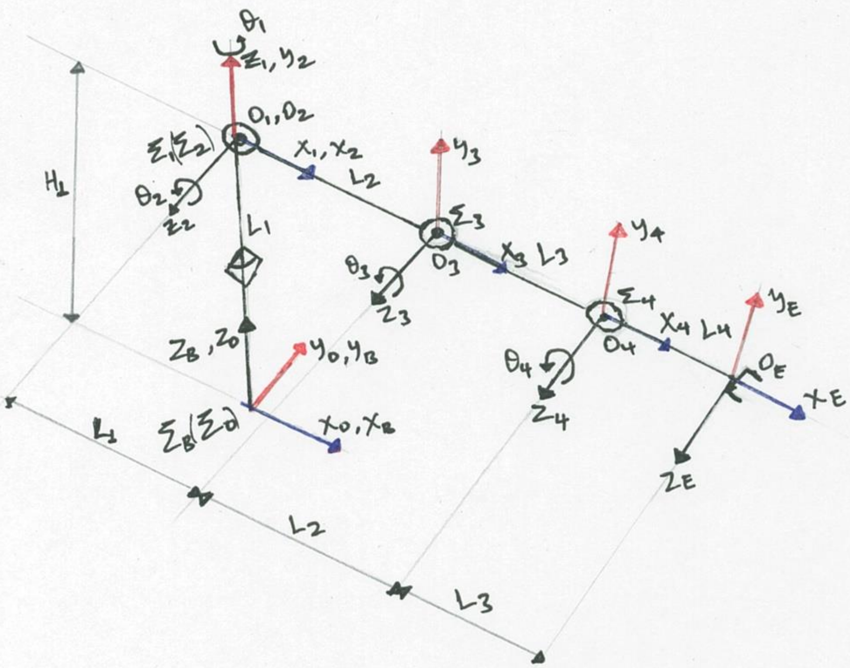

Fig2. Robot structure

The four link parameters are demonstrated in the above hand-draw picture which are associated with each link coordinate system. These parameters are also known as DH link parameters. These are illustrated in the Table [1] below:

Table 1. DH link parameters

$Link\ (i)$	$a_{i-1}, (m)$	$\alpha_{i-1}, (rad)$	$d_i, (m)$	$\theta_i, (rad)$
1	0	0	$H_1 = 0.5$	θ_1
2	0	$\frac{\pi}{2}$	0	θ_2
3	$L_1 = 0.5$	0	0	θ_3
4	$L_2 = 0.5$	0	0	θ_4
5	$L_3 = 0.5$	0	0	0

3. Kinematic Equation Derivation of this Manipulator

The parameters mentioned above in Table (1) are used for kinematic modelling of the robot. In kinematics, modelling the geometry of the robot is represented. Homogenous transformation (of the matrix) is commonly used as the definition of the kinematics model. As described below,

$$^{B}T_E = \ ^{0}T_1\,^{1}T_2\,^{2}T_3\,^{3}T_4 \ldots\ldots\,^{n}T_E$$

where n is the number of links, $^{i-1}T_i$ is the link transformation from the i^{th} joint, and $^{B}T_E$ is the final pose for end-effector relative to the base.

Forward kinematics is the study of the manipulator to find out its tip or end-effector position and orientation by using joint values of the manipulator.

The homogeneous transformation matrix that describes the relationship between Σ_i axis and Σ_{i-1} can be obtained by using the following equation:

$$^{i-1}T_i = T_T(x_{i-1}, a_{i-1})T_R(x_{i-1}, \alpha_{i-1})T_T(z_i, d_i)T_T(z_i, \theta_i)$$

The transformation matrix for a link (i) is described as follows:

$$^{i-1}T_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} & 0 & a_{i-1} \\ C_{\alpha_{i-1}}S_{\theta_i} & C_{\alpha_{i-1}}C_{\theta_i} & -S_{\alpha_{i-1}} & -S_{\alpha_{i-1}}d_i \\ S_{\alpha_{i-1}}S_{\theta_i} & S_{\alpha_{i-1}}C_{\theta_i} & C_{\alpha_{i-1}} & C_{\alpha_{i-1}}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$where:\ C_{\theta_i} = \cos\theta_i\,,\ S_{\theta_i} = \sin\theta_i$$

Now let's find the transformation matrix for each joint by replacing the parameter value from the DH table into equation (num):

The transformation matrix between the base and joint 1 is $\boldsymbol{^{0}T_1}$:

$$^{0}T_1 = \begin{bmatrix} C_1 & -S_2 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & H_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix between joint 1 and joint 2 is $\boldsymbol{^{1}T_2}$:

$$^{1}T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & H_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix between joint 2 and joint 3 is $\boldsymbol{^{2}T_3}$:

$$
{}^{2}T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & L_1 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The transformation matrix between joint 3 and joint 4 is $\boldsymbol{{}^{3}T_4}$:

$$
{}^{3}T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & L_4 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Finally, the transformation matrix from the end effector to link 4 is $\boldsymbol{{}^{4}T_E}$:

$$
{}^{4}T_E = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Now, the resultant transformation matrix between the base axis and the end effector axis can be found by multiplying each matrices:

$$
{}^{B}T_E = {}^{B}T_0\,{}^{0}T_1\,{}^{1}T_2\,{}^{2}T_3\,{}^{3}T_4 \ldots\ldots {}^{n}T_E
$$

$$
(\textit{we can avoid } \boldsymbol{{}^{B}T_0} \textit{ because it is an identity matrix.})
$$

For simplification let's first find the resultant transformation between the base and joint 2 as follow:

$$
{}^{0}\mathrm{T}_2 = {}^{0}\mathrm{T}_1\,{}^{1}\mathrm{T}_2
$$

$$
{}^{0}\mathrm{T}_2 = \begin{bmatrix} \mathrm{C}_1 & -\mathrm{S}_2 & 0 & 0 \\ \mathrm{S}_1 & \mathrm{C}_1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{H}_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{C}_2 & -\mathrm{S}_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \mathrm{S}_2 & \mathrm{C}_2 & 0 & \mathrm{H}_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}\mathrm{T}_2 = \begin{bmatrix} \mathrm{C}_1\,\mathrm{C}_2 & -\mathrm{C}_1\,\mathrm{S}_2 & \mathrm{S}_1 & 0 \\ \mathrm{S}_1\mathrm{C}_2 & -\mathrm{S}_1\,\mathrm{S}_2 & -\mathrm{C}_1 & 0 \\ \mathrm{S}_2 & \mathrm{C}_2 & 1 & \mathrm{H}_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Next, let's find the resultant transformation between the joint 2 and joint 4 as follow:

$$
{}^{2}T_4 = {}^{2}T_3\,{}^{3}T_4
$$

$$
{}^{2}T_4 = \begin{bmatrix} C_3 & -S_3 & 0 & L_1 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & L_4 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_4 = \begin{bmatrix} C_3C_4 - S_3S_4 & -C_3S_4 - S_3C_4 & 0 & L_1 + L_2C_3 \\ S_3C_4 + C_3S_4 & -S_3S_4 + C_3C_4 & 0 & L_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_4 = \begin{bmatrix} C_{34} & -S_{34} & 0 & L_1 + L_2C_3 \\ S_{34} & C_{34} & 0 & L_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
where\ C_{34} = C_3C_4 - S_3S_4, and\ S_{34} = S_3C_4 + C_3S_4
$$

Now, we can find the Transformation matrix from base to joint $\mathbf{4}$ can be found as follow:

$$
{}^{B}T_4 = {}^{0}T_2\, {}^{2}T_4
$$

$$
{}^{B}T_4 = \begin{bmatrix} C_1\,C_2 & -C_1\,S_2 & S_1 & 0 \\ S_1C_2 & -S_1\,S_2 & -C_1 & 0 \\ S_2 & C_2 & 1 & H_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{34} & -S_{34} & 0 & L_1 + L_2C_3 \\ S_{34} & C_{34} & 0 & L_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}T_4 = \begin{bmatrix} C_1\,C_2C_{34} - C_1\,S_2S_{34} & -C_1\,S_2S_{34} - C_1\,S_2C_{34} & S_1 & C_1\,C_2L_1 + C_1\,C_2L_2C_3 - C_1\,S_2L_2S_3 \\ S_1C_2\,C_{34} - S_1\,S_2S_{34} & -S_1C_2\,S_{34} - S_1\,S_2C_{34} & -C_1 & S_1C_2\,L_1 + S_1C_2\,L_2C_3 - S_1\,S_2L_2S_3 \\ S_2C_{34} + C_2S_{34} & -S_2S_{34} + C_2C_{34} & 0 & H_1 + S_2L_1 + S_2L_2C_3 + C_2L_2S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}T_4 = \begin{bmatrix} C_1\,C_{234} & -C_1\,S_{234} & S_1 & C_1\,C_2L_1 + C_1\,C_{23}L_2 \\ C_{234}S_1 & -S_1S_{234} & -C_1 & S_1C_2\,L_1 + S_1C_{23}\,L_2 \\ S_{234} & C_{234} & 0 & H_1 + S_2L_1 + S_{23}L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
where\ C_{234} = -S_2S_{34} + C_2C_{34}, and\ S_{234} = S_2C_{34} + C_2S_{34}
$$

Finally, Transformation matrix from base to end-effector is:

$$
{}^{B}T_E = {}^{B}T_4\, {}^{4}T_E
$$

$$
{}^{B}T_E = \begin{bmatrix} C_1\,C_{234} & -C_1\,S_{234} & S_1 & C_1\,C_2L_1 + C_1\,C_{23}L_2 \\ C_{234}S_1 & -S_1S_{234} & -C_1 & S_1C_2\,L_1 + S_1C_{23}\,L_2 \\ S_{234} & C_{234} & 0 & H_1 + S_2L_1 + S_{23}L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}T_E = \begin{bmatrix} C_1\, C_{234} & -C_1\, S_{234} & S_1 & C_1\,(C_2 L_1 + C_{23} L_2 + C_{234} L_3) \\ C_{234} S_1 & -S_1 S_{234} & -C_1 & S_1 (C_2\, L_1 + C_{23}\, L_2 + C_{234} L_3) \\ S_{234} & C_{234} & 0 & H_1 + S_2 L_1 + S_{23} L_2 + S_{234} L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Where:

$$
{}^{B}P_{E\mathrm{x}} = C_1\,(C_2 L_1 + C_{23} L_2 + C_{234} L_3)
$$

$$
{}^{B}P_{E\mathrm{y}} = S_1\left(C_2\, L_1 + C_{23}\, L_2 + C_{234} L_3\right)
$$

$$
{}^{B}P_{E\mathrm{z}} = H_1 + S_2 L_1 + S_{23} L_2 + S_{234} L_3
$$

$$
\phi = \theta_1 + \theta_2 + \theta_3 + \theta_4
$$

4. Calculation and Verification of 6 pairs of joint variables and endpoint positions

I. $(\theta_1, \theta_2, \theta_3, \theta_4)[rad] = (0,\ 0,\ 0,\ 0\ )$

We are using the at rest orientation to find our whether our calculated transformation matrix right or wrong. Substituting the assumed values into the above obtained transformation matrix, the value below can easily calculate as follow:

$$
{}^{B}T_E = \begin{bmatrix} 1 & -1 & 0 & 1.5 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Where:

$$
{}^{B}P_{Ex} = 1.5\ [m]
$$

$$
{}^{B}P_{Ey} = 0\ [m]
$$

$$
{}^{B}P_{E\mathrm{z}} = 0.5\ [\mathrm{m}]
$$

$$
\phi = 0\ [\mathrm{rad}]
$$

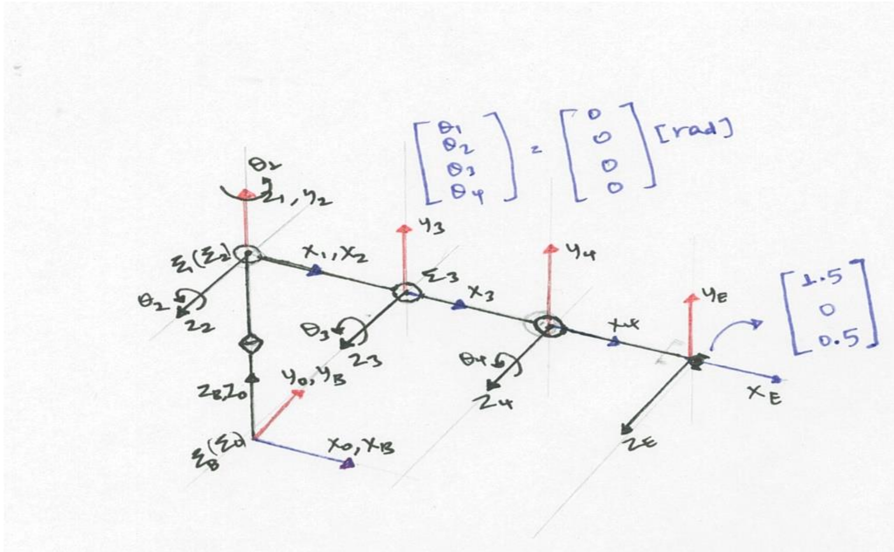

Fig3. Case 1

II. $(\theta_1, \theta_2, \theta_3, \theta_4)[rad] = \left({}^{\pi}/_{4},\ {}^{-\pi}/_{8},\ {}^{\pi}/_{4},\ 0\right)$

$$
{}^{B}T_{E} = \begin{bmatrix} 0.6533 & -0.2706 & \frac{\sqrt{2}}{2} & 0.9799 \\ 0.6533 & -0.2706 & \frac{-\sqrt{2}}{2} & 0.9799 \\ 0.3827 & 0.9239 & 0 & 0.6913 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Where:

$$
{}^{B}P_{Ex} = 0.9799\ [m]
$$

$$
{}^{B}P_{Ey} = 0.9799\ [m]
$$

$$
{}^{B}P_{Ez} = 0.6913\ [\mathrm{m}]
$$

$$
\phi = {}^{3\pi}/_{4}\ [\mathrm{rad}]
$$

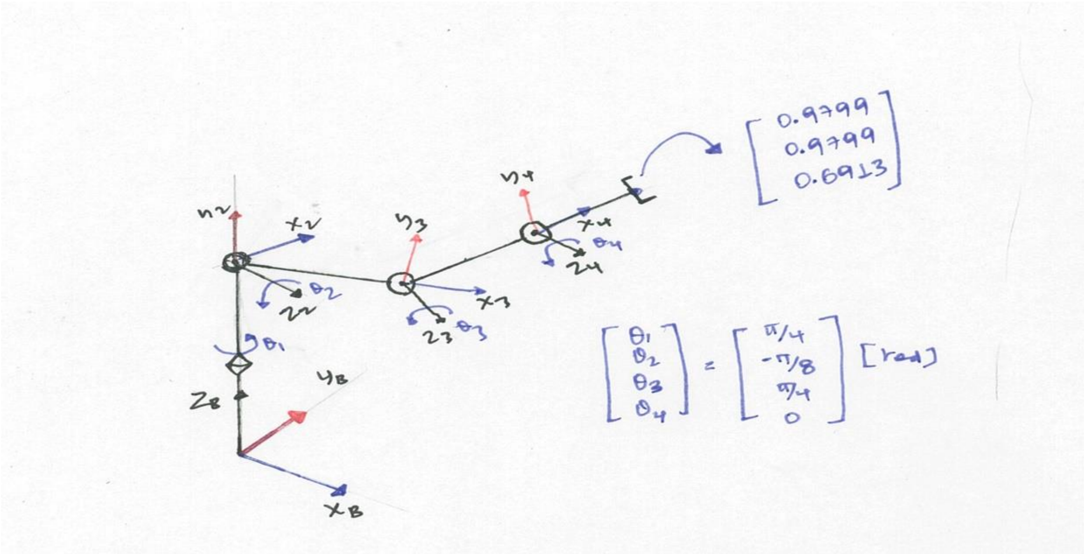

Fig4. Case 2

III. $(\theta_1, \theta_2, \theta_3, \theta_4)[rad] = \left(\pi/_2,\ 0,\ ^{3\pi}/_4,\ ^{-\pi}/_4\right)$

$$^{B}T_E = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0.1465 \\ 1 & 0 & 0 & 1.3536 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where:

$$^{B}P_{Ex} = 0\ [m]$$

$$^{B}P_{Ey} = 0.1465\ [m]$$

$$^{B}P_{Ez} = 1.3536\ [\mathrm{m}]$$

$$\phi = 0\ [\mathrm{rad}]$$

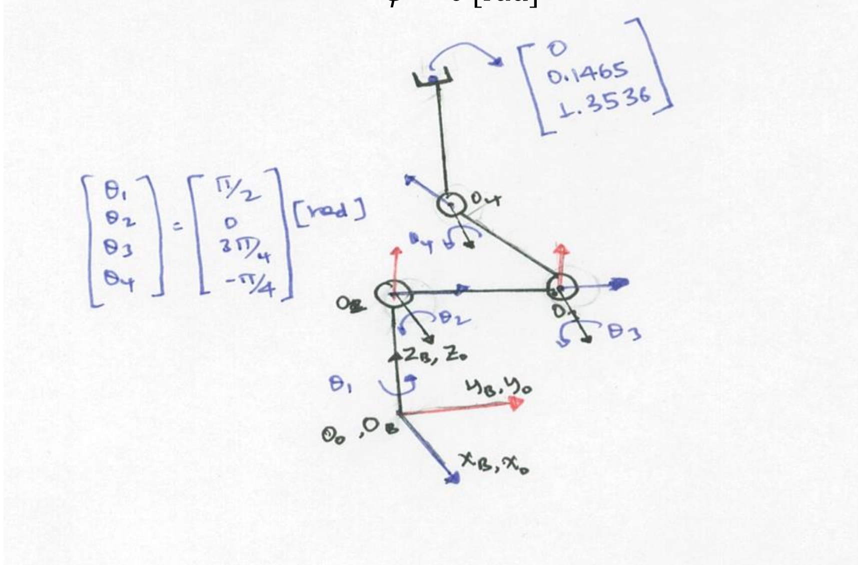

Fig5. Case 3

IV. $(\theta_1, \theta_2, \theta_3, \theta_4)[rad] = \left({}^{\pi}/_{2},\ {}^{\pi}/_{2},\ {}^{\pi}/_{4},\ {}^{\pi}/_{2} \right)$

$$ {}^{B}T_{E} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{-\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Where:

$$ {}^{B}P_{Ex} = 0\ [m] $$

$$ {}^{B}P_{Ey} = \frac{-\sqrt{2}}{2}\ [m] $$

$$ {}^{B}P_{Ez} = 1\ [\mathrm{m}] $$

$$ \phi = {}^{7\pi}/_{4}\ [\mathrm{rad}] $$

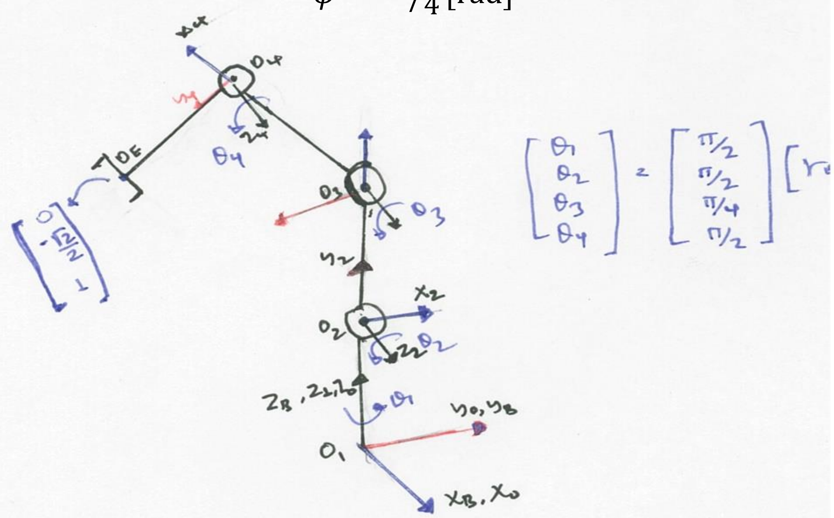

Fig6. Case 4

V. $(\theta_1, \theta_2, \theta_3, \theta_4)[rad] = \left(\,^{-\pi}/_{4},\ ^{\pi}/_{8},\ ^{-\pi}/_{8},\ ^{\pi}/_{2}\right)$

$$^{B}T_{E} = \begin{bmatrix} 0 & \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & 0.6802 \\ 0 & \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & -0.6802 \\ 1 & 0 & 0 & 1.1913 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where:

$$^{B}P_{Ex} = 0.6802\ [m]$$

$$^{B}P_{Ey} = -0.6802\ [m]$$

$$^{B}P_{Ez} = 1.1913[\mathrm{m}]$$

$$\phi = {}^{\pi}/_{4}\ [\mathrm{rad}]$$

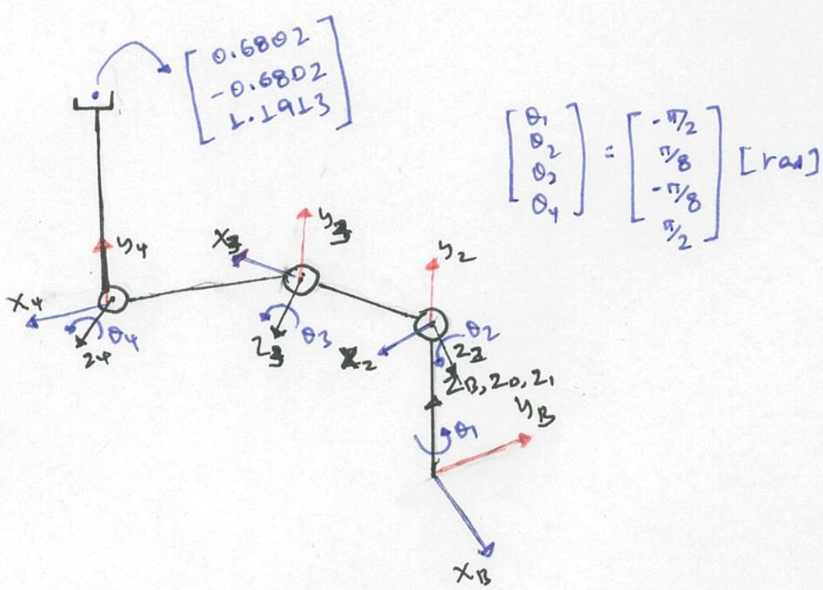

Fig7. Case 5

VI. $(\theta_1, \theta_2, \theta_3, \theta_4)[rad] = \left({}^{3\pi}/_{4},\ 0,\ {}^{\pi}/_{2},\ {}^{-\pi}/_{4}\right)$

$$^{B}T_{E} = \begin{bmatrix} -0.5 & 0.5 & \frac{\sqrt{2}}{2} & -0.6036 \\ 0.5 & -0.5 & \frac{\sqrt{2}}{2} & 0.6036 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 1.3536 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where:

$$^{B}P_{Ex} = -0.6036\ [m]$$

$$^{B}P_{Ey} = 0.6036\ [m]$$

$$^{B}P_{Ez} = 1.3536[\mathrm{m}]$$

$$\phi = \pi[\mathrm{rad}]$$

5. Conclusion

In this small report, modified 4DOF elbow manipulator that moves in a 3D space proposed. The kinematics equation of this manipulator derived by using the DH parameter method. And further each calculation has been shown and verified using hand drawn picture.