Unit Three

Dynamics of a particle

3. The concept of force

Force is a pull or push which produce or tends to produce motion in a body at rest, stops or tends to stop a body which is in its state of motion.

- force do not always cause motion.
  - ✓ **E.g: 1.** a person sitting on a chair(a gravitational force acts on his body. Yet he remains stationary.
- A force is a vector quantity which is characterized by its magnitude and direction.
- The unit of a force is called **Newton (N).**
- What force causes the moon to orbit the earth?

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- Force can be classified into **two** classes:

1. **Contact force:-** is they involves physical contact between two objects.
   - ✓ **E.g:** a spring is pulled, a stationary cart is pulled
   - ✓ frictional force
2. **Field forces:-** They do not involves physical contact between two objects but instead act through empty space.
   - ✓ **E.g:** Gravitational force, magnetic force, electrostatic force.

- *There are Four Fundamental forces in nature:*
  1. *Gravitational forces*
  2. *Electromagnetic forces*
  3. *Strong nuclear Forces (e.g:Nuclei)*
  4. *Weak nuclear forces (e.g: Radio active decay process)*

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3.1 Newton's laws of motion

1. <u>Newton's first law (The law of Inertia)</u>

- This law can be stated as follows:

"In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line)."

- **Inertia**: is the tendency of an object to resist any attempt to change its velocity.

<u>2. Newton's second law</u>

- It can be stated as follows:

"The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass".

$$\sum \mathbf{F} = m\mathbf{a}$$

- *In component form:*

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

3. Newton's third law

- It can be stated as follows:

"If two objects interact, the force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1".

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

- The action and reaction forces act on different objects and must be of the same type.

3.2 Mass and weight

- Mass can be described as a measure of the inertia of a body.
- Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it.

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- Mass is scalar quantity and it is constant every body.
- The SI unit of mass is kilogram(kg)
- Weight is a vector quantity and it is not constant.
- The SI unit of weight is Newton.
- Weight is the force of gravity on a body towards its centre.
- A freely falling body has an acceleration equal to "g".

From Newton's law:

$$F_g = m_o g = \frac{G m_e m_o}{r_e^2}$$

$$g = \frac{G m_e}{r_e^2}$$

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Application of Newton's Laws

- When we applying Newton's laws, the following procedures recommended to solve the problems.

1. Draw a simple, neat diagram of the system to help conceptualize the problem.
2. Draw a free body diagram of the object for systems containing more than one object drawn. Draw separate free body diagram for each object and establish convenient coordinate axes for each object.
3. Apply Newton's 2nd law in component form
4. Solve the component equations for the unknowns.
5. Make sure your result are consistent with the free body diagram.

Example:-

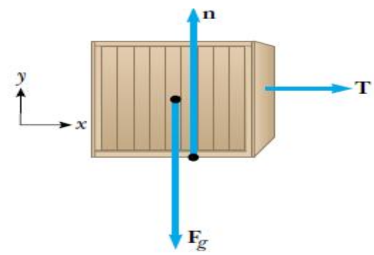
- when we apply Newton's laws to an object, we are interested only in external forces that act on the object. In this case the external forces are the F_N and F_g
- The Force that originates in any body when it is stretched is called **Tension**

- When a rope attached to an object is pulling on the object, the rope exerts a force T on the object, and the magnitude *T of that force is called the* tension in the rope. Let us consider this example

- A free body diagram is shows all the external force acting on the object.

$\Sigma F = ma$
$\Sigma F_x = T = ma_x$
$T - ma_x = 0$
$T = ma_x$

$\Sigma F_y = ma_y$
$F_N - mg = 0$
$F_N = mg$



Example 1: A 3.0 kg mass undergoes an acceleration given by a = (2i + 5j) m/s² . Find the resultant force F and its magnitude.

Newton's Second Law tells us that the resultant (net) force on a mass m is $\sum F = ma$.

- So here we find:

$$\begin{aligned}\mathbf{F}_{\text{net}} &= m\mathbf{a} \\ &= (3.0\,\text{kg})(2.0\mathbf{i} + 5.0\mathbf{j})\,\tfrac{\text{m}}{\text{s}^2} \\ &= (6.0\mathbf{i} + 15.\mathbf{j})\,\text{N}\end{aligned}$$

The *magnitude* of the resultant force is

$$F_{\text{net}} = \sqrt{(6.0\,\text{N})^2 + (15.\,\text{N})^2} = 16.\,\text{N}$$

Example 2: Five forces pull on the 4.0 kg box as shown in the figure below. Find the box's acceleration (a)in unit–vector notation and (b) as a magnitude and direction.

solution:

(a) Newton's Second Law will give the box's acceleration,

but we must first find the sum of the forces on the box.

Adding the x components of the forces gives:

$$\begin{aligned} \sum F_x &= -11\,\mathrm{N} + 14\,\mathrm{N}\cos 30^\circ + 3.0\,\mathrm{N} \\ &= 4.1\,\mathrm{N} \end{aligned}$$

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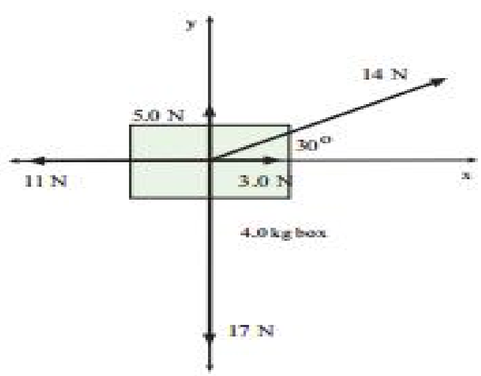

(two of the forces have only y components). Adding the y components of the forces gives:

$$\begin{aligned} \sum F_y &= +5.0\,\mathrm{N} + 14\,\mathrm{N}\sin 30^\circ - 17\,\mathrm{N} \\ &= -5.0\,\mathrm{N} \end{aligned}$$

So the net force on the box (in unit-vector notation) is

$$\sum \mathbf{F} = (4.1\,\mathrm{N})\mathbf{i} + (-5.0\,\mathrm{N})\mathbf{j}\ .$$

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Then we find the x and y components of the box's acceleration using $\mathbf{a} = \sum \mathbf{F}/m$:

$$a_x = \frac{\sum F_x}{m} = \frac{(4.1\,\mathrm{N})}{(4.0\,\mathrm{kg})} = 1.0\,\frac{\mathrm{m}}{\mathrm{s}^2}$$
$$a_y = \frac{\sum F_y}{m} = \frac{(-5.0\,\mathrm{N})}{(4.0\,\mathrm{kg})} = -1.2\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

So in unit–vector form, the acceleration of the box is

$$\mathbf{a} = (1.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2})\mathbf{i} + (-1.2\,\tfrac{\mathrm{m}}{\mathrm{s}^2})\mathbf{j}$$

(b) The acceleration found in part (a) has a magnitude of

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(1.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2})^2 + (-1.2\,\tfrac{\mathrm{m}}{\mathrm{s}^2})^2} = 1.6\,\tfrac{\mathrm{m}}{\mathrm{s}^2}$$

and to find its direction θ we calculate

$$\tan\theta = \frac{a_y}{a_x} = \frac{-1.2}{1.0} = -1.2$$

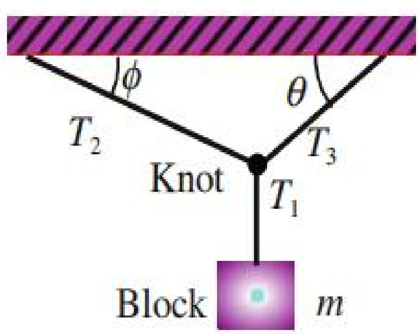
which gives us:

$$\theta = \tan^{-1}(-1.2) = -50^\circ$$

Here, since a_y is negative and a_x is positive, this choice for θ lies in the proper quadrant.

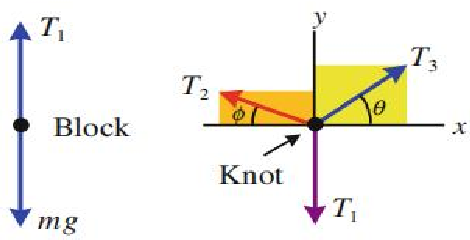
Example:-

1. A block of mass m = 21 kg hangs from three cords as shown in the Fig. below.Taking sinθ = 4/5, cosθ = 3/5, sinφ = 5/13, and cosφ = 12/13, find the tensions in the three cords.



Solution: We construct a free-body diagram for the block and the forces in the cords as shown below:

The tension in the vertical cord balances the weight of the block. Thus,by taking g = 10 m/s², we get: T_1 = mg = (21 kg)(10 m/s²) = 210 N



By applying Newton's second law in the x and y directions for the secod figure, we find that:

$$\Sigma F_x = T_3 \cos\theta - T_2 \cos\phi = 0 \quad \text{and} \quad \Sigma F_y = T_3 \sin\theta + T_2 \sin\phi - T_1 = 0$$

From the x-component equation we get the following relation:

$$T_3 = \frac{\cos\phi}{\cos\theta} T_2 = \frac{12/13}{3/5} T_2 = \frac{20}{13} T_2$$

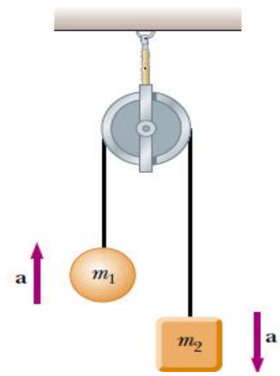
When we substitute the result of T_3 into the y component equation, after putting T_1 = mg = 210 N, we get:

$$\frac{20}{13}\frac{4}{5} T_2 + \frac{5}{13} T_2 - 210 = 0 \quad \Rightarrow \quad \left(\frac{16}{13} + \frac{5}{13}\right) T_2 = 210 \quad \Rightarrow \quad T_2 = 130\text{ N}$$

Consequently, one can find the value of the third tension to be:

$$T_3 = \frac{20}{13} T_2 = \frac{20}{13} \times 130\text{ N} = 200\text{ N}$$

2. When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in the Figure bellow. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.



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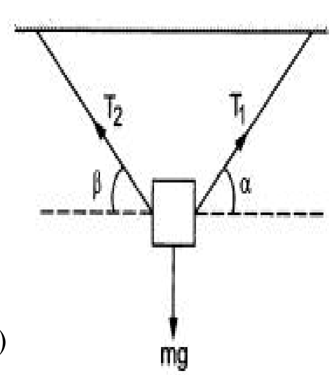
1. A body of mass m is suspended by two strings making angles α and β with the horizontal. Find the tensions in the strings.

Solution : Take the body of mass m as the system. The forces acting on the system are

(i) mg downwards (by the earth),

(ii) T_1 along the first string (by the first string) and

(iii) T_2 along the second string (by the second string).



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As the body is in equilibrium, these forces must add to zero. Taking horizontalcomponents,

$T_1\cos\alpha - T_2\cos\beta = 0$

$T_1\cos\alpha = T_2\cos\beta$.

solving for T_1 :

$T_1 = T_2 \frac{\cos\beta}{\cos\alpha}$..................................(i)

Taking the vertical components,

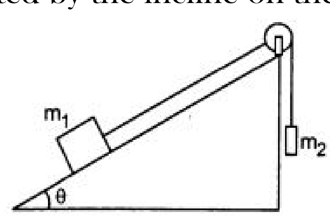
$T_1\sin\alpha + T_2\sin\beta - mg = 0$ ....(ii)

substituting (i)in to (ii),

$T_2 \frac{\cos\beta}{\cos\alpha}\sin\alpha + T_2\sin\beta = mg$

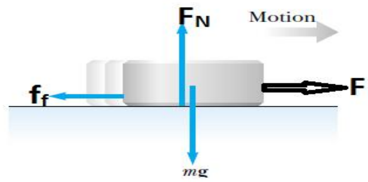
$T_2 = \frac{mg}{\frac{\cos\beta}{\cos\alpha}\sin\alpha + \sin\beta}$

2. Two bodies of masses m_1 and m_2 are connected by a light string going over a smooth light pulley at the end of an incline. The mass m_1 lies on the incline and m_2 hangs vertically. The system is at rest. Find the angle of the incline and the force exerted by the incline on the body of mass m_1



3.3 Force of Friction

- When a body moves over a surface of another body there exists a force that opposes (resists) motion is called **friction force**.



- Friction force is directly proportional to the normal force(F_N)

$$f_f \propto F_N$$

$f_f = \mu F_N$ Where μ - coefficient of friction

There are three kind of friction. Those are

1. **Static friction:-** the maximum force is pulled, when the object starts to move. It is denoted by fs

$$f_s = \mu_s F_N$$

2. **Kinetic friction:-** is the friction exerted on a body, when it is in motion. It is denoted by f_k

$$f_k = \mu_k F_N$$

3. **Rolling friction:-** is the frictional force which arises, when a body rolls on another body. It is denoted by f_r

$$f_r = \mu_r F_n$$

- When we compare the coefficient of friction

$$\mu_s > \mu_k > \mu_r$$

Exercise:- An inclined plan makes 53^0 from the horizontal. If a force of 100 N parallel to the incline is required to cause a 10 kg box to slide up the plane with an acceleration of 2 m/s^2 . What is the coefficient of sliding friction between box and the plane.

2. A block of mass m_1 = 4 kg lying on a rough horizontal surface is connected to asecond block of mass m_2 = 6 kg by a light non-stretchable cord over a massless,frictionless pulley as shown in the figure below. The coefficient of kinetic friction between the block and the surface is μ_k = 0.5.
(a) Find the magnitudes of the acceleration of the system and the tension in the cord.

(b) Find the relation between m_1 and m_2 in the case when the system is on the verge of slipping

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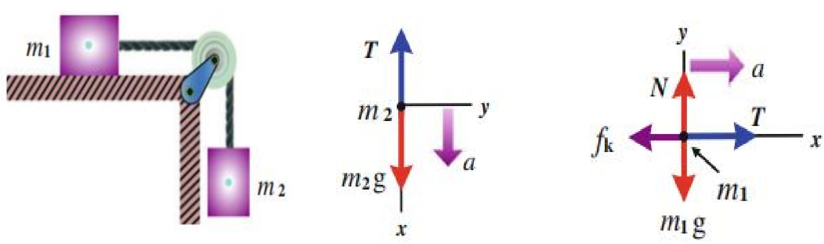

(a) Since the cord is non-stretchable, the two masses have the same magnitude of acceleration. where we take the x axis always along any of the body's motion. In this case a cannot take negative values. When Newton's second law is applied to m_2 in the second figure, we find:

$$(1) \qquad \Sigma F_x = m_2 g - T = m_2 a$$

$$\Sigma F_y = 0$$

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- From (1), we can find the magnitude of the tension in terms of g and a. That is:

(2) $$T = m_2 g - m_2 a$$

Doing the same for m_1 we get:

(3) $$\Sigma F_x = T - f_k = m_1 a$$

(4) $$\Sigma F_y = N - m_1 g = 0$$

Since $f_k = \mu_k N$, and from (4) we have $N = m_1 g$, then:

(5) $$f_k = \mu_k m_1 g$$

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- When this result is substituted into (3), we get:

(6) $$T = \mu_k m_1 g + m_1 a$$

Equating the magnitude of the tension in (2) and (6), we get:

$$\mu_k m_1 g + m_1 a = m_2 (g - a)$$

Solving for a we get:

$$a = \frac{m_2 - \mu_k m_1}{m_1 + m_2} g$$

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Note that, when $m_2 > \mu_k m_1$ we have accelerated motion, and when $m_2 = \mu_k m_1$,we have motion with zero acceleration, i.e. the speed is constant. The value of a can then be evaluated as follows:

$$a = \frac{6\text{ kg} - 0.5(4\text{ kg})}{6\text{ kg} + 4\text{ kg}} \times 9.8\text{ m/s}^2 = 3.92\text{ m/s}^2$$

We can find T by substituting the expression of a into (6), to get:

$$T = \frac{(\mu_k + 1)\, m_1\, m_2}{m_1 + m_2} g$$

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Thus,

(b) When the system is on the verge of slipping, the magnitude of the force T that acts on mass m_1 must equal the maximum static friction $f_{s,max} = \mu_s N$, i.e.

$T = \mu_s N = \mu_s m_1 g$. Also, the weight of the mass m_2 must equal the magnitude of the tension, i.e.

$T = m_2 g$. Thus:

$m_2 g = \mu_s m_1 g$

$$\boxed{m_2 = \mu_s m_1} \quad \text{(On the verge of slipping)}$$

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