Chapter Four

The Theory of Production and Cost

Definition of production

- Production is the process of converting inputs such as labor, land, capital and entrepreneurial ability into outputs .

- Production is the process of transforming inputs into outputs. It can also be defined as an act of creating value or utility.

Production function

- Production function is a technical relationship b/n inputs and outputs.
- It shows the maximum output that can be produced with fixed amount of inputs and existing technology.
- A production function may take the form of algebraic equation, table or graph.
- e.g. A general equation for production function can be described as:

$$Q = f(X_1, X_2, X_3, \ldots, X_n)$$

where, *Q is output and* $X_1, X_2, X_3, \ldots, X_n$ *are different types of inputs.*

- Inputs are commonly classified as fixed or variable.
- ***Fixed inputs*** are those inputs **whose quantity cannot readily be changed when market conditions** indicate that an immediate adjustment in output is required.
- In fact, no input is ever absolutely fixed but may be fixed during an immediate requirement.
- E.g. **Buildings, land and machineries** are examples of fixed inputs since their quantity cannot be manipulated easily in a short period of time.
- ***Variable inputs*** are those **inputs whose quantity can be altered** almost instantaneously in response to desired changes in output.

- i.e., their quantities can easily be diminished when the market demand for the product decreases and vice versa. The best example of variable input is unskilled labor.
- **Production period : based on time horizon**
- ***1. short run*** *refers to a period of time in which the quantity of* ***at least one input is*** **fixed.**
- In other words, short run is a time period which is not sufficient to change the quantities of all inputs so that at least one input remains fixed.
- **2. long run** :planning horizon and **sufficient time to adjust the quantities of all inputs** used in production
- **all inputs used in production are variable**
- No fixed input in the long run

***Production in the short run*:**

Consider a firm that uses two inputs: capital (fixed) and labor (variable).

- Given the assumptions of short run production, the firm can increase output only by increasing the amount of labour it uses.
- Hence, its production function can be given by:
  - Q = f (L,k) or Q = f (L) only…in the short run , outputs can changes only by changing /adjusting variable input only …since fixed inputs cannot be adjusted easily.where, Q is output and L is the quantity of labour.
- ***Total, average, and marginal product in the short run***

1. ***Total product (TP)*:** is the **total amount of output** that can be produced by **efficiently utilizing** specific combinations of the variable input and fixed input.

- Increasing the variable input (while some other inputs are fixed) can increase the **total product only up to a certain point.**
- Initially, as we combine more and more units of the variable input with the fixed input, output continues to increase,
- but eventually if we employ more and more unit of the variable input beyond the **carrying capacity of the fixed input,** output tends to decline.

- In general, the TP function in the short-run follows a certain trend:
- it initially increases at an increasing rate, then increases at a decreasing rate, reaches a maximum point and eventually falls as the quantity of the variable input rises.
- This tells us the shape a total product curve assumes.

2. Marginal Product (MP): is the **change in output** attributed to the **addition** of one **unit of the variable input** to the production process, ceteris paribus.

- e.g., the change in total output resulting from employing additional worker (holding other inputs constant) is the marginal product of labour (*MPL*

- In other words, *MPL measures the slope of the total product curve at a given point.*

$$MP_L = dTP/dL = \Delta Q/\Delta L$$

- In the short run, the MP of the variable input first increases, reaches its maximum and then decreases to the extent of being negative.

3. *Average Product (AP)*: AP of an input is the **level of output that each unit of input produces**, on the average.

- It tells us the **mean contribution of each variable input** to the total product.
- Mathematically, it is the ratio of total output to the number of the variable input.
- The average product of labour (APL), for instance, is given by:

$$AP_L = TP/L$$

- Average product of labour first increases, reaches its maximum value and eventually declines.

- The AP curve can be measured by the slope of rays originating from the origin to a point on the TP curve (see figure 4.1).
- e.g., the AP_L at L_2 is the ratio of TP_2 to L_2.
- This is identical to the slope of ray *a.*

Labor Input (days)	(TPP)	(MPP)	(APP)
0	0	-	-
10	100	10	10
20	800	70	40
30	2000	120	67
40	2900	90	73
50	3600	70	72
60	4200	60	70
70	4600	40	66
80	4800	20	60
90	4800	0	53
100	4500	-30	45

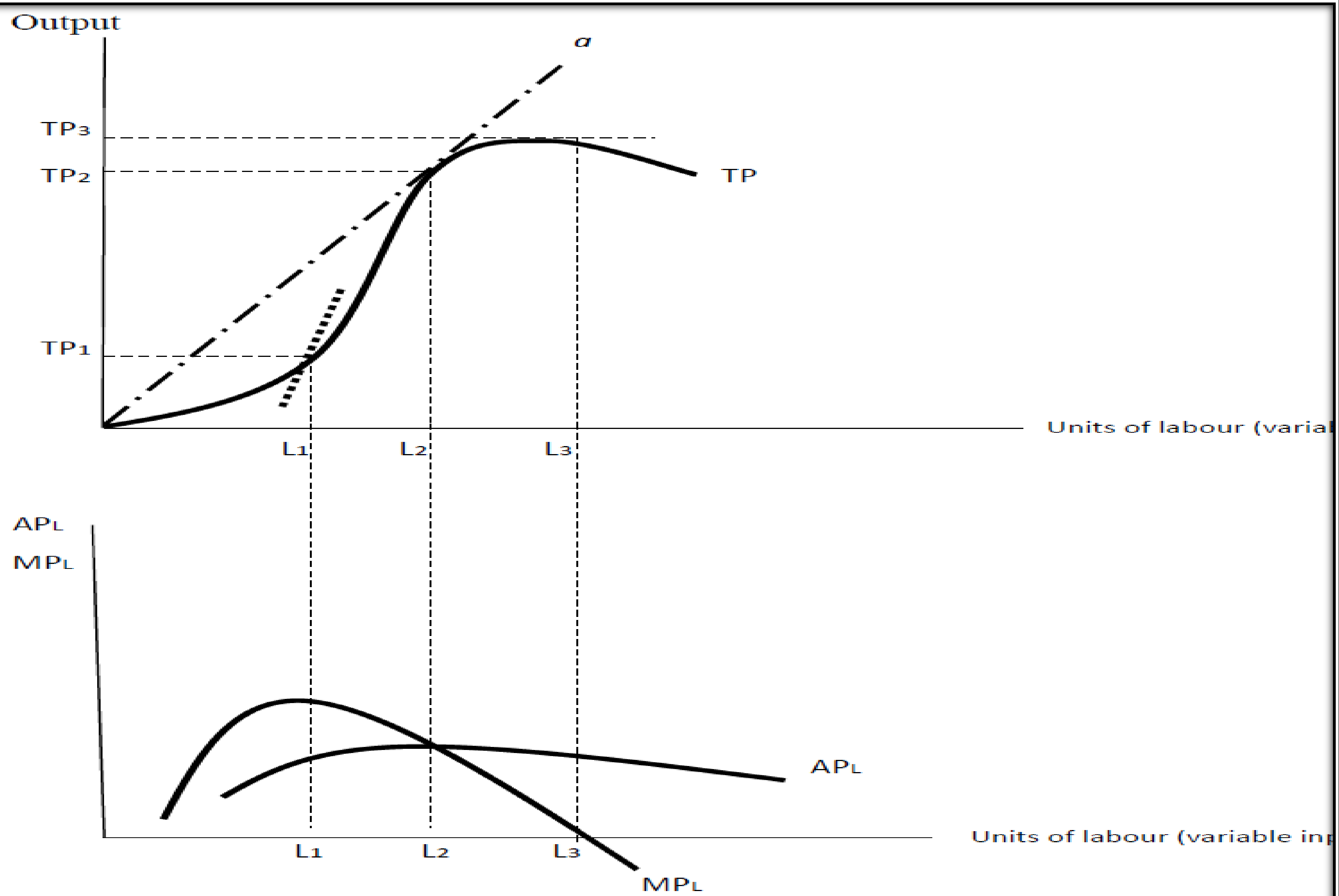


Figure 4.1: Total product, average product and marginal product curves

Stages of production

<u>Stage I:</u> Covers the range of variable input levels over which the AP_L *continues to increase.*

- *It goes from the origin to the point where the* AP_L is maximum, i.e., $MP_L = AP_L$. This stage is **not an efficient region** of production though the MP_L is positive. **fixed input is under-utilized. Its is Increasing marginal returns**

<u>Stage II</u>: It ranges from the point where AP_L *is at its maximum* ($MP_L = AP_L$) *to the point where* MP_L *is zero.*

- *Here, as the labour input increases by one unit, output* still increases but at a decreasing rate.
- Thus, the second stage of production is termed as the stage of ***diminishing marginal returns. APL declines throughout it .MPL is decline but positive . It is an efficient region of production***

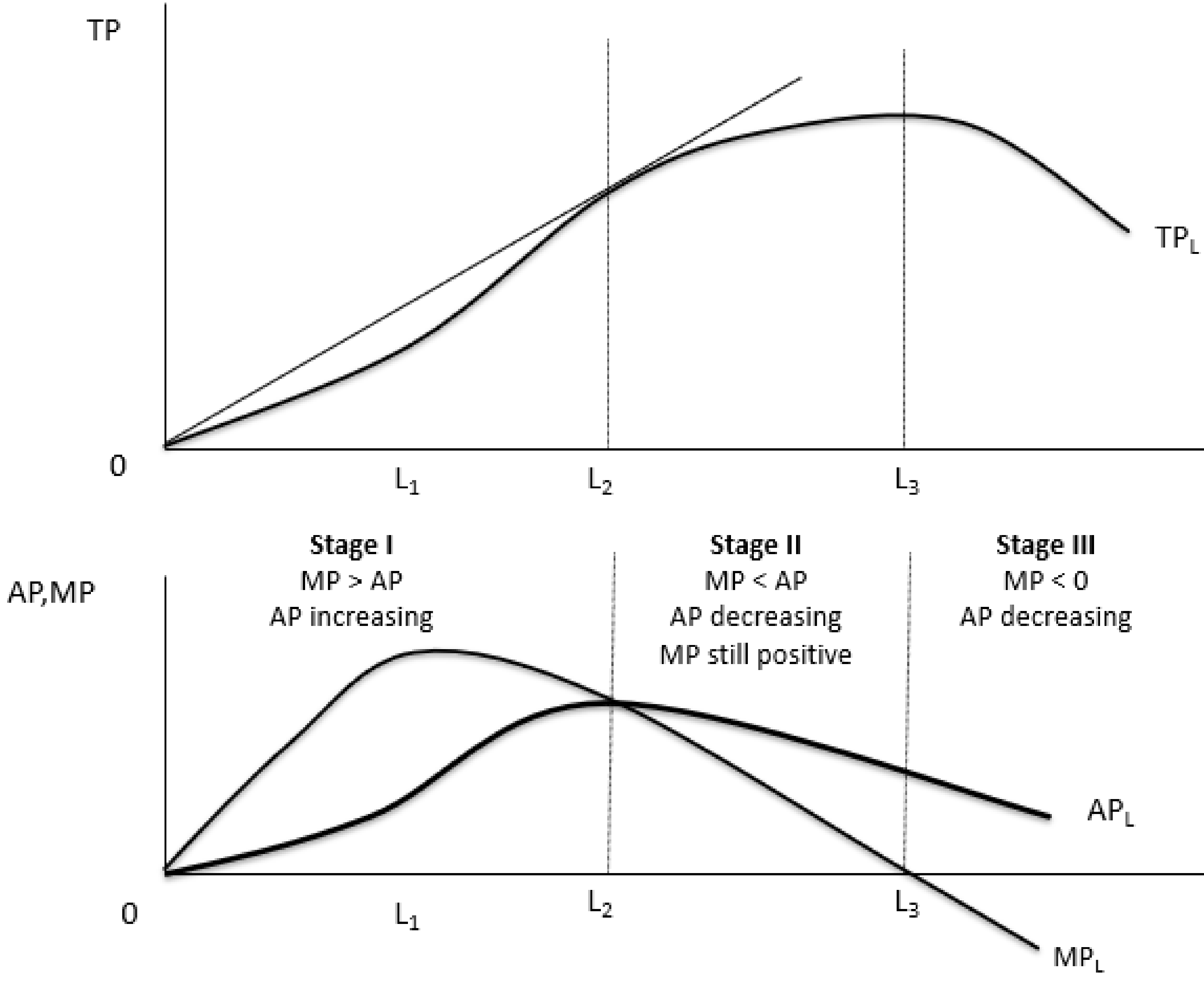
***<u>Stage III</u>*: **In this stage, an increase in the variable input is accompanied by a decline in the TP.**

- Thus, the TP curve slopes downwards, and the marginal product of labor becomes negative.
- This stage is also known as the stage of ***negative marginal returns to the*** variable input.
- Obviously, a rational firm **should not operate in stage III** because additional units of variable input are contributing negatively to the TP (MP_L is negative).
- Under this region fixed input; **capital is over utilized and variable input ; labor is overemployed**

<u>The relationship b/n MP_L and AP_L can be stated as follows.</u>

- When AP_L is increasing, $MP_L > AP_L$.
- When AP_L is at its maximum, $MP_L = AP_L$.
- When AP_L is decreasing, $MP_L < AP_L$.

In figure 4.2, this stage is indicated by the employment of labor beyond L_3.


TP
TP_L
0
L_1
L_2
L_3
L
Stage I
MP > AP
AP increasing
Stage II
MP < AP
AP decreasing
MP still positive
Stage III
MP < 0
AP decreasing
AP,MP
AP_L
0
L_1
L_2
L_3
L
MP_L

Numerical example: Assume that a production function is given as

$$Q = f(L) = 7L + 10L^2 - L^3$$

Determine:

a) Determine the average product of labour (AP_L) *function and* the amount of employment which maximizes average productivity
b) The amount of employment which maximizes total production or at which marginal product is zero.
c) Define the three stages of production for the above production function.

Average product reaches maximum when the slope of average product curve is zero. In other words, average product reaches maximum when marginal product equals to average product.

Slope of $AP_L = \dfrac{dAP_L}{dL} = 0$, and $AP_L = \dfrac{TP}{L} = \dfrac{7L + 10L^2 - L^3}{L} = 7 + 10L - L^2$

Slope of $AP_L = \dfrac{d(7 + 10L - L^2)}{dL} = 0$

$\Rightarrow 10 - 2L = 0 \Rightarrow L = 5$

$\Rightarrow L = 5$

B) $MP_L = \frac{d(TP)}{dL} = 0$

$$MP_L = \frac{d(7L + 10L^2 - L^3)}{dL} = 0$$

$$MP_L = 7 + 20L - 3L^2 = 0$$

$\Rightarrow -7 - 20L + 3L^2 = 0$ Using Quadratic solution method, a= 3, b= -20 and c= -7

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{20 \pm \sqrt{(-20)^2 - 4(3)(-7)}}{2(3)}$$

$$= \frac{20 \pm \sqrt{484}}{6}$$

$$= \frac{20 \pm 22}{6}$$

$$= \frac{20+22}{6} \; or \; \frac{20-22}{6} \Rightarrow 7 \; or \; -0.33$$

c. The three stages of production is defined as follows:

- Stage one from zero labor to 5 units of labor.
- Stage two from 5 labor to 7 units of labor and
- Stage three above 7 units of employment

The law of variable proportions(Law of diminishing marginal returns)

- This law states that as successive units of a variable input(labour) are added to a fixed input (capital or land),
- beyond some point the extra, or marginal, product that can be attributed to each additional unit of the variable resource will decline.
- e.g., if additional workers are hired to work with a constant amount of capital equipment, output will eventually rise by smaller and smaller amounts as more workers are hired. This law assumes that technology is fixed and thus the techniques of production do not change.

- All units of labour are assumed to be of equal quality.
- Each successive worker is presumed to have the same innate ability, education, training, and work experience.
- Marginal product ultimately diminishes not because successive workers are less skilled or less energetic rather it is because more workers are being used relative to the amount of plant and equipment available.
- The law starts to operate after the marginal product curve reaches its maximum (this happens when the number of workers exceeds L_1 in figure 4.1).
- This law is also called the *law of diminishing returns.*

Theory of costs in the short run

Definition and types of costs

- To produce goods and services, firms need factors of production or inputs. To acquire these inputs, they have to buy them from resource suppliers.
- Cost is the monetary value of inputs used in the production of an item.

- ***To the economist***, economic profit is total revenue less economic costs (**explicit and implicit costs**).
- **Accounting cost** is the monetary value of all **purchased inputs** used in production; **it ignores** the cost of non-purchased (**self-owned**) inputs.
- It considers **only direct expenses** such as wages/salaries, cost of raw materials, depreciation allowances, interest on borrowed funds and utility expenses (electricity, water, telephone, etc.).
- These costs are said to *be explicit costs.*
- Explicit costs are **out of pocket expenses** for the purchased inputs.

- If a producer calculates her cost by considering only the costs incurred for purchased inputs, then her profit will be an accounting profit.

Economic cost of producing a commodity considers the monetary value of all inputs (purchased and non-purchased).

- Calculating economic costs will be difficult since it is based on estimation.
- The monetary value of these inputs is obtained by estimating their opportunity costs in monetary terms.
- **The estimated monetary cost for** non-purchased inputs is known as *implicit cost.*
- *e.g.**, if Mr. X resigns a job which pays him*** **Birr 10, 000 per** month to run a firm he has established, then the opportunity cost of his labor is taken to be Birr 10,000 per month (the salary forgone to run his own business).
- **Economic cost** is given by the sum of implicit cost and explicit cost.

Calculation of the firm’s profit

Profit: is the difference between total revenue and total cost

Total Revenue: the product of output and price

Economists use the term —profit differently from the way accountants use it.

To the economists, profit is the firm‘s total revenue less both **explicit costs or accounting costs** and **implicit cost**

To the accountant, profit is the firm‘s total revenue less its explicit costs (accounting costs).

Accounting profit = Total revenue – Accounting cost

= Total revenue – Explicit cost

Economic profit =Total revenue – Economic cost

= TR-Explicit cost plus implicit cost

- Economic profit will give the real profit of the firm since all costs are taken into account.
- Accounting profit of a firm will be greater than economic profit by the amount of implicit cost.
- If all inputs are purchased from the market, accounting and economic profit will be the same.
- If implicit costs exist, then accounting profit will be larger than economic profit.

Firm’s total costs in the short run

- A cost function shows the total cost of producing a given level of output, which can be described using equations, tables or curves.
- A cost function can be represented using an equation as follows.

$$\mathbf{TC = f\ (Q)}$$

where C is the total cost of production and Q is the level of output.

- In the short run, total cost (TC) can be broken down in to two – total fixed cost (TFC) and total variable cost (TVC).
- ***<u>Total fixed costs :</u>*** *costs* which do not vary with the level of output.

- **Fixed costs** are costs which do not vary with the level of output produced.
- They are regarded as fixed because these costs are **unavoidable regardless** of the level of output.
- A firm can avoid fixed costs only if it stops operation.
- Fixed costs: salaries of administrative staff, expenses for building depreciation and repairs, expenses for land maintenance and rent of building used for production.
- ***<u>Total Variable costs</u>*** include all costs which directly vary with the level of output produced.

- e.g., if the firm produces zero output, the variable cost is zero.
- These costs may include the cost of raw materials, the cost of direct labour and the running expenses of fuel, water, electricity, etc.
- In general, the short run total cost is given by the sum of total fixed cost and total variable cost. i.e.,

$$TC = TFC + TVC$$

- Based on the definition of the short run cost functions, let‘s see what their shapes look like.

Total fixed cost (TFC): Total fixed cost is denoted by a straight line parallel to the output axis since such costs do not vary with the level of output.

Total variable cost (TVC): The total variable cost of a firm has an inverse S-shape, which indicates the law of variable proportions in production.

- At the initial stage of production with a given plant, as more of the variable factor is employed, its productivity increases.
- TVC increases at a decreasing rate, which continues until the optimal combination of the fixed and variable factor is reached.

- Beyond this point, as increased quantities of the variable factor are combined with the fixed factor, the productivity of the variable factor declines, and the TVC increases at an increasing rate.

Total Cost (TC): The total cost curve is obtained by vertically adding TFC and TVC at each level of output.

- The shape of the TC curve follows the shape of the TVC curve, i.e. the TC has also an inverse S-shape.
- Note that when the level of output is zero, TVC is also zero which implies TC = TFC.

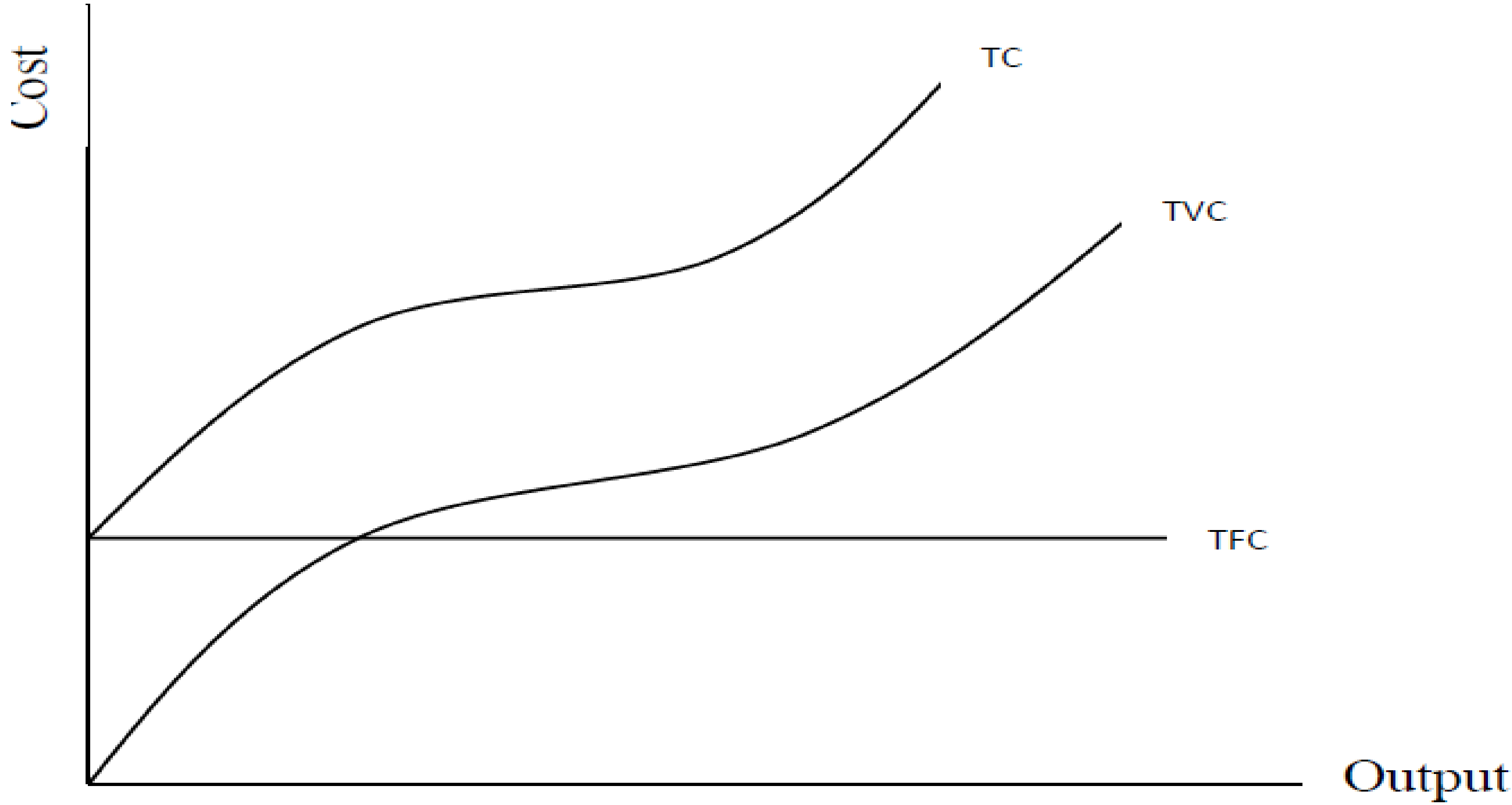


Figure 4.2*: Short run TC, TFC and TVC curves*

Per unit costs(average costs).

- From total costs functions we can derive per-unit costs which are even more important in the short run analysis of the firm.

a) **Average fixed cost (AFC) -** Average fixed cost is total fixed cost per unit of output.

- It is calculated by dividing TFC by the corresponding level of output.
- The curve declines continuously and approaches both axes asymptotically.

$$AFC = TFC/Q$$

b) Average variable cost (AVC) - Average variable cost is total variable cost per unit of output. It is obtained by dividing total variable cost by the level of output.

$$AVC = TVC/Q$$

- The short run AVC falls initially, reaches its minimum, and then starts to increase.
- The AVC curve has U-shape because the law of variable proportions.

c) Average total cost (ATC) or simply Average cost (AC) - Average total cost is the total cost per unit of output. It is calculated by dividing the total cost by the level of output.

$$AC = TC/Q$$

Equivalently, $AC = (TVC + TFC)/Q = TVC/Q + TFC/Q$

$$= AVC + AFC$$

- Thus, AC can also be given by the vertical sum of AVC and AFC.

<u>Marginal Cost (MC)</u>

- Marginal cost is defined as the additional cost that a firm incurs to produce one extra unit of output.
- In other words, it is the change in total cost which results from a unit change in output.
- Graphically, MC is the slope of TC function.

$$MC = dTC/dQ$$

- In fact, MC is also a change in TVC with respect to a unit change in the level of output.

  $MC = (dTFC + dTVC)/dQ = dTVC/dQ$ (since $dTFC/dQ = 0$)

- Given inverse S-shaped TC and TVC curves, MC initially decreases, reaches its minimum and then starts to rise.
- From this, we can infer that the MC exhibits U shape because of the law of variable proportions.
- In summary, AVC, AC and MC curves are all U-shaped due to the law of variable proportions.

Units of output	Total Fixed Cost	Total Variable Cost	Total Cost	Marginal Cost	Average Cost
Q	TFC	TVC	TC	MC	AC
0	100	0	100	-	-
1	100	30	130	30	130
2	100	50	150	20	75
3	100	60	160	10	53.3
4	100	65	165	5	41.25
5	100	75	175	10	35
6	100	95	195	20	32.5
7	100	125	225	30	32.14
8	100	165	265	40	33.12
9	100	215	315	50	35
10	100	275	375	60	37.5

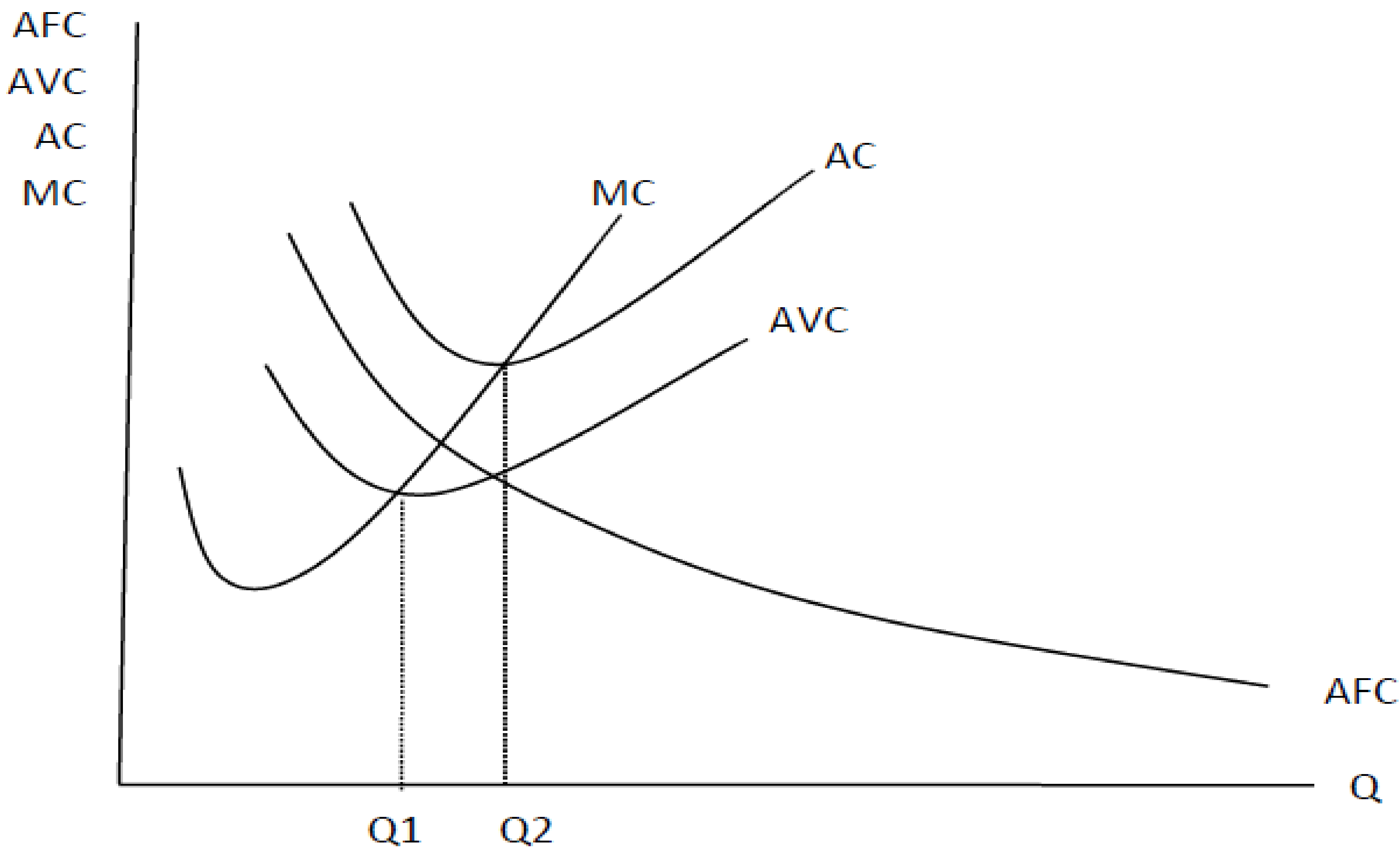


Figure 4.3: ***Short run AFC, AVC, AC and MC Curves***

- In the above figure, the AVC curve reaches its minimum point at Q_1 level of output and AC reaches its minimum point at Q_2 level of output.
- The vertical distance between AC and AVC, i.e., AFC decreases continuously as output increases.
- Note that the MC curve passes through the minimum points of both AVC and AC curves.

Example: Suppose the short run cost function of a firm is given by: $TC = 2Q^3 - 2Q^2 + Q + 10$.

a) Find the expression of TFC & TVC

b) Derive the expressions of AFC, AVC, AC and MC

c) Find the levels of output that minimize MC and AVC and then find the minimum values of MC and AVC

Solution:

Given $TC = 2Q^3 - 2Q^2 + Q + 10$

a) $TFC = 10$, $TVC = 2Q^3 - 2Q^2 + Q$

b) $AFC = TFC/Q = 10/Q$

$AVC = TVC/Q = (2Q^3 - 2Q^2 + Q)/Q = 2Q^2 - 2Q + 1$

$AC = TC/Q = (2Q^3 - 2Q^2 + Q + 10)/Q = 2Q^2 - 2Q + 1 + 10/Q$

$MC = dC/dQ = 6Q^2 - 4Q + 1$

c) To find the minimum value of MC,

$dMC/dQ = 12Q - 4 = 0$

$Q = 1/3$

MC is minimized when $Q = 0.33$

The minimum value of MC will be:

$MC = 6Q2 – 4Q + 1$

$= 6(1/3)2 -4(1/3) + 1 = 0.33$

To find the minimum value of AVC

$dAVC/dQ = 4Q – 2 = 0$

$Q = 0.5$

AVC is minimized at $Q =0.5$

The minimum value of AVC will be:

$AVC = 2Q2 – 2Q + 1$

$AVC = 2 (0.5) 2 - 2(0.5) +1$

$= 0.5 – 1 + 1$

$= 0.5$

The relationship between short run production and cost curves

- Suppose a firm in the short run uses labour as a variable input and capital as a fixed input.
- Let the price of labour be given by **w,** which is constant.
- Given these conditions, we derive the relation b/n: MC and MP_L as well as AVC and AP_L.

i) Marginal Cost and Marginal Product of Labour

$$MC = \frac{\Delta TVC}{\Delta Q}, \text{where } TVC = w.L$$

$$MC = \frac{\Delta(w.L)}{\Delta Q} = w.\frac{\Delta L}{\Delta Q}, \text{ but } \frac{\Delta L}{\Delta Q} = \frac{1}{MP_L}$$

$$\text{Therefore, } MC = \frac{w}{MP_L}$$

- The above expression shows that MC and MP_L are inversely related.
- When initially MP_L increases, MC decreases; when MP_L is at its maximum, MC must be at a minimum and when finally MP_L declines, MC increases.

ii) Average Variable Cost and Average Product of Labour

$$AVC = \frac{TVC}{Q}, \text{ where, } TVC = w.L$$

$$AVC = \frac{w.L}{Q} = w.\frac{L}{Q}, \text{ but } \frac{L}{Q} = \frac{1}{AP_L}$$

$$\text{Therefore, } AVC = \frac{w}{AP_L}$$

- This expression also shows inverse relation b/n AVC and AP_L.
- When AP_L increases, AVC decreases;
- when APL is at a maximum, AVC is at a minimum
- Finally, when AP_L declines, AVC increases.
- We can also sketch the relationship b/n these production and cost curves using graphs.
- From the following figure, we can conclude that the MC curve is the mirror image of MP_L curve and AVC curve is the mirror image of AP_L curve.
- Take look at

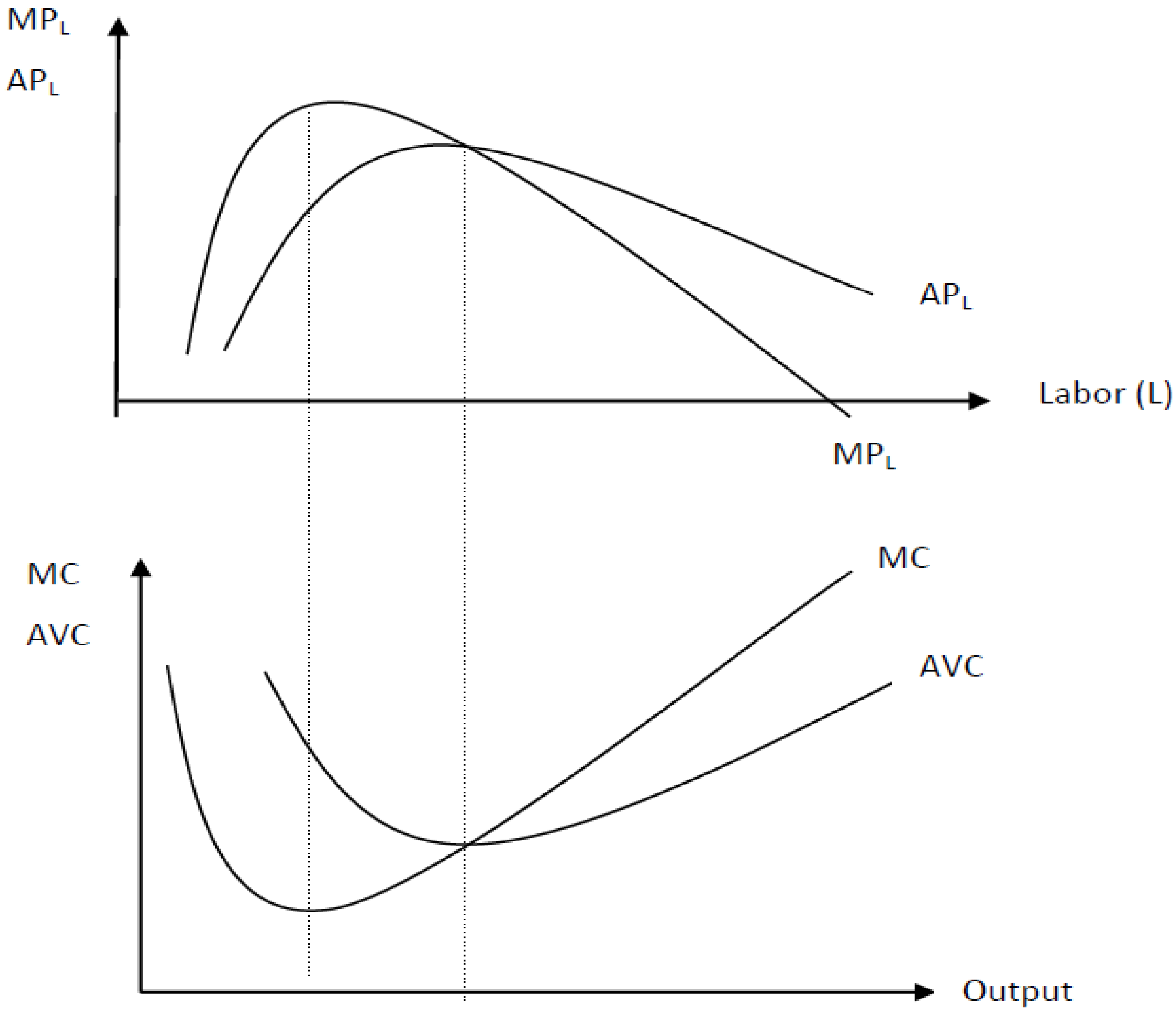


Figure 4.4: relationship between short run production and cost curves

The end of the Chapter 4

Thank you for attention