Logistic Regression with non-linear features

import library

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from matplotlib import ticker, cm
```

load training data

In [ ]:

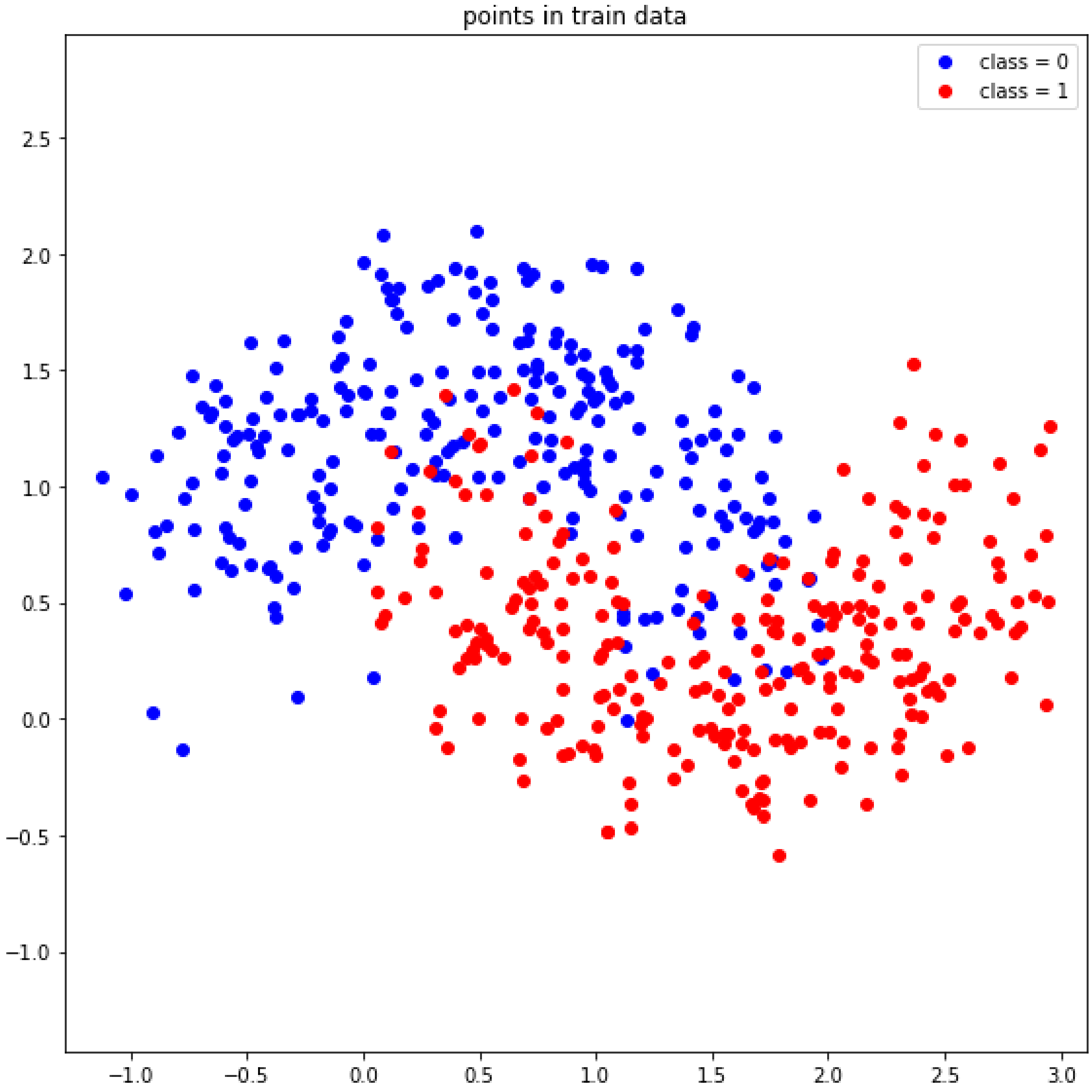
```
fname_data_train    = 'assignment_10_data_train.csv'
fname_data_test     = 'assignment_10_data_test.csv'

data_train          = np.genfromtxt(fname_data_train, delimiter=',')
data_test           = np.genfromtxt(fname_data_test, delimiter=',')

number_data_train   = data_train.shape[0]
number_data_test    = data_test.shape[0]

data_train_point    = data_train[:, 0:2]
data_train_point_x  = data_train_point[:, 0]
data_train_point_y  = data_train_point[:, 1]
data_train_label    = data_train[:, 2]

data_test_point     = data_test[:, 0:2]
data_test_point_x   = data_test_point[:, 0]
data_test_point_y   = data_test_point[:, 1]
data_test_label     = data_test[:, 2]

data_train_label_class_0    = (data_train_label == 0)
data_train_label_class_1    = (data_train_label == 1)

data_test_label_class_0     = (data_test_label == 0)
data_test_label_class_1     = (data_test_label == 1)

data_train_point_x_class_0  = data_train_point_x[data_train_label_class_0]
data_train_point_y_class_0  = data_train_point_y[data_train_label_class_0]

data_train_point_x_class_1  = data_train_point_x[data_train_label_class_1]
data_train_point_y_class_1  = data_train_point_y[data_train_label_class_1]

data_test_point_x_class_0   = data_test_point_x[data_test_label_class_0]
data_test_point_y_class_0   = data_test_point_y[data_test_label_class_0]

data_test_point_x_class_1   = data_test_point_x[data_test_label_class_1]
data_test_point_y_class_1   = data_test_point_y[data_test_label_class_1]

print('shape of point in train data = ', data_train_point.shape)
print('shape of point in test data = ', data_train_point.shape)

print('shape of label in train data = ', data_test_label.shape)
print('shape of label in test data = ', data_test_label.shape)

print('data type of point x in train data = ', data_train_point_x.dtype)
print('data type of point y in train data = ', data_train_point_y.dtype)

print('data type of point x in test data = ', data_test_point_x.dtype)
print('data type of point y in test data = ', data_test_point_y.dtype)
```

```
shape of point in train data =  (500, 2)
shape of point in test data =  (500, 2)
shape of label in train data =  (500,)
shape of label in test data =  (500,)
data type of point x in train data =  float64
data type of point y in train data =  float64
data type of point x in test data =  float64
data type of point y in test data =  float64
```

plot the data

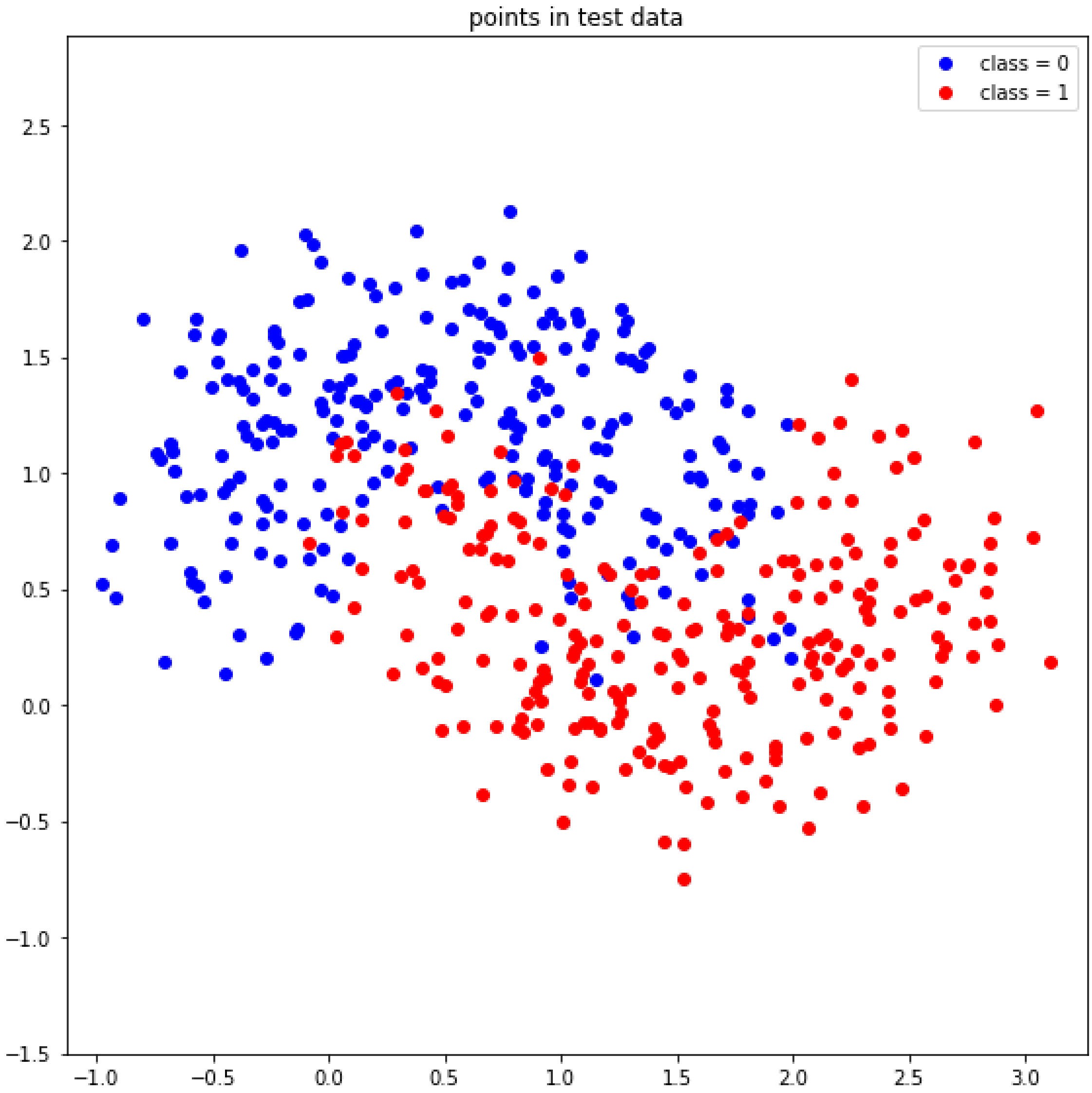
In [ ]:

```
f = plt.figure(figsize=(8,8))

plt.title('points in train data')
plt.plot(data_train_point_x_class_0, data_train_point_y_class_0, 'o', color='blue', label='class = 0')
plt.plot(data_train_point_x_class_1, data_train_point_y_class_1, 'o', color='red', label='class = 1')
plt.axis('equal')
plt.legend()
plt.tight_layout()
plt.show()
```



In [ ]:

```
f = plt.figure(figsize=(8,8))

plt.title('points in test data')
plt.plot(data_test_point_x_class_0, data_test_point_y_class_0, 'o', color='blue', label='class = 0')
plt.plot(data_test_point_x_class_1, data_test_point_y_class_1, 'o', color='red', label='class = 1')
plt.axis('equal')
plt.legend()
plt.tight_layout()
plt.show()
```



define the feature functions

- feature vector is defined by $(1, f_1(x, y), f_2(x, y), \cdots, f_{k-1}(x, y)) \in \mathbb{R}^k$

In [ ]:

```
def compute_feature(point):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    n = np.shape(point)[0]
    x = point[:,0]
    y = point[:,1]

    x_square = np.power(x,2)
    x_cubic = np.power(x,3)
    xy = x*y




    feature = np.column_stack([np.ones(n), x, y, x_square, x_cubic, xy])
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return feature
```

define the linear regression function

- $\theta = (\theta_0, \theta_1, \cdots, \theta_{k-1}) \in \mathbb{R}^k$
- feature = $(1, f_1(x, y), \cdots, f_{k-1}(x, y)) \in \mathbb{R}^k$

In [ ]:

```
def compute_linear_regression(theta, feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    value = np.matmul(theta, feature.T)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return value
```

define sigmoid function with input

- $z \in \mathbb{R}$

In [ ]:

```
def sigmoid(z):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    value = 1 / (1 + np.exp(-z))
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return value
```

define the logistic regression function

- $\theta = (\theta_0, \theta_1, \cdots, \theta_{k-1}) \in \mathbb{R}^k$
- feature $= (1, f_1(x, y), \cdots, f_{k-1}(x, y) \in \mathbb{R}^k$

In [ ]:

```
def compute_logistic_regression(theta, feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    value = sigmoid(compute_linear_regression(theta, feature))
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return value
```

define the residual function

- $\theta = (\theta_0, \theta_1, \cdots, \theta_{k-1}) \in \mathbb{R}^k$
- feature $= (1, f_1(x, y), \cdots, f_{k-1}(x, y) \in \mathbb{R}^k$
- label $= l \in \{0, 1\}^k$

In [ ]:

```
def compute_residual(theta, feature, label):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    h = compute_logistic_regression(theta, feature)
    residual = (-label * np.log(h)) - ((1-label) * np.log(1-h))
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return residual
```

define the loss function for the logistic regression

- $\theta = (\theta_0, \theta_1, \cdots, \theta_{k-1}) \in \mathbb{R}^k$
- feature $= (1, f_1(x, y), \cdots, f_{k-1}(x, y) \in \mathbb{R}^k$
- label $= l \in \{0, 1\}^k$

In [ ]:

```
def compute_loss(theta, feature, label, alpha):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    residual = compute_residual(theta, feature, label)
    loss = (np.sum(residual) / np.shape(residual)[0]) + (np.inner(theta,theta)*alpha/2)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return loss
```

define the gradient of the loss with respect to the model parameter θ

- $\theta = (\theta_0, \theta_1, \cdots, \theta_{k-1}) \in \mathbb{R}^k$
- feature $= (1, f_1(x, y), \cdots, f_{k-1}(x, y) \in \mathbb{R}^k$
- label $= l \in \{0, 1\}^k$

In [ ]:

```
def compute_gradient(theta, feature, label, alpha):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    h = compute_logistic_regression(theta, feature)
    gradient = (np.matmul(feature.T, (h-label)) / np.shape(label)[0]) + (alpha*theta)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return gradient
```

compute the accuracy of the prediction for point with a given model parameter

In [ ]:

```
def compute_accuracy(theta, feature, label):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    n = np.shape(feature)[0]
    pred = np.zeros(n)
    h = compute_logistic_regression(theta, feature)
    sum = 0
    for i in range(n):
        if h[i] >= 0.5:
            pred[i] = 1
        if pred[i] == label[i]:
                sum += 1

    accuracy = sum / n

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return accuracy
```

initialize the gradient descent algorithm

In [ ]:

```
number_iteration    = 20000 # you can change this value as you want
learning_rate       = 0.4 # you can change this value as you want
number_feature      = 6 # you can change this value as you want
alpha               = 0.0001 # you can change this value as you want

theta                   = np.zeros(number_feature)
loss_iteration_train    = np.zeros(number_iteration)
loss_iteration_test     = np.zeros(number_iteration)
accuracy_iteration_train = np.zeros(number_iteration)
accuracy_iteration_test  = np.zeros(number_iteration)
```

run the gradient descent algorithm to optimize the loss function with respect to the model parameter

In [ ]:

```
for i in range(number_iteration):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    theta           = theta - learning_rate*compute_gradient(theta, compute_feature(data_train_point), data_train_label, alpha)
    loss_train      = compute_loss(theta, compute_feature(data_train_point), data_train_label, alpha)
    loss_test       = compute_loss(theta, compute_feature(data_test_point), data_test_label, alpha)
    accuracy_train  = compute_accuracy(theta, compute_feature(data_train_point), data_train_label)
    accuracy_test   = compute_accuracy(theta, compute_feature(data_test_point), data_test_label)

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    loss_iteration_train[i]     = loss_train
    loss_iteration_test[i]      = loss_test
    accuracy_iteration_train[i] = accuracy_train
    accuracy_iteration_test[i]  = accuracy_test

theta_optimal = theta
```

functions for presenting the results

In [ ]:

```
def function_result_01():

    print("final training accuracy = {:13.10f}".format(accuracy_iteration_train[-1]))
```

In [ ]:

```
def function_result_02():

    print("final testing accuracy = {:13.10f}".format(accuracy_iteration_test[-1]))
```

In [ ]:

```
def function_result_03():

    plt.figure(figsize=(8,6))
    plt.title('training loss')

    plt.plot(loss_iteration_train, '-', color='red')
    plt.xlabel('iteration')
    plt.ylabel('loss')

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_04():

    plt.figure(figsize=(8,6))
    plt.title('testing loss')

    plt.plot(loss_iteration_test, '-', color='red')
    plt.xlabel('iteration')
    plt.ylabel('loss')

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_05():

    plt.figure(figsize=(8,6))
    plt.title('training accuracy')

    plt.plot(accuracy_iteration_train, '-', color='red')
    plt.xlabel('iteration')
    plt.ylabel('accuracy')

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_06():

    plt.figure(figsize=(8,6))
    plt.title('testing accuracy')

    plt.plot(accuracy_iteration_test, '-', color='red')
    plt.xlabel('iteration')
    plt.ylabel('accuracy')

    plt.tight_layout()
    plt.show()
```

plot the linear regression values over the 2-dimensional Euclidean space and superimpose the training data

In [ ]:

```
def function_result_07():

    plt.figure(figsize=(8,8))
    plt.title('linear regression values on the training data')

    min_x   = np.min(data_train_point_x)
    max_x   = np.max(data_train_point_x)
    min_y   = np.min(data_train_point_y)
    max_y   = np.max(data_train_point_y)

    X = np.arange(min_x - 0.5, max_x + 0.5, 0.1)
    Y = np.arange(min_y - 0.5, max_y + 0.5, 0.1)

    [XX, YY] = np.meshgrid(X, Y)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    ZZ = (theta_optimal[0] + theta_optimal[1]*XX + theta_optimal[2]*YY + theta_optimal[3]*np.p
ower(XX,2) + theta_optimal[4]*np.power(XX,3) + theta_optimal[5]*XX*YY)



    im = plt.imshow(ZZ, aspect='auto', origin = 'lower', cmap = 'RdBu_r', alpha=0.98, extent=(
min_x-0.5,max_x+0.5,min_y-0.5,max_y+0.5),vmin=-50,vmax=50)
    plt.colorbar(im)
    plt.ylim(min_y - 1.5, max_y+1.5)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.plot(data_train_point_x_class_0, data_train_point_y_class_0, '.', color='blue', label=
'class = 0')
    plt.plot(data_train_point_x_class_1, data_train_point_y_class_1, '.', color='red', label=
'class = 1')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_08():

    plt.figure(figsize=(8,8))
    plt.title('linear regression values on the testing data')

    min_x   = np.min(data_test_point_x)
    max_x   = np.max(data_test_point_x)
    min_y   = np.min(data_test_point_y)
    max_y   = np.max(data_test_point_y)

    X = np.arange(min_x - 0.5, max_x + 0.5, 0.1)
    Y = np.arange(min_y - 0.5, max_y + 0.5, 0.1)

    [XX, YY] = np.meshgrid(X, Y)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    ZZ = (theta_optimal[0] + theta_optimal[1]*XX + theta_optimal[2]*YY + theta_optimal[3]*np.p
ower(XX,2) + theta_optimal[4]*np.power(XX,3) + theta_optimal[5]*XX*YY)



    im = plt.imshow(ZZ, aspect='auto', origin = 'lower', cmap = 'RdBu_r', alpha=0.98, extent=(
min_x-0.5,max_x+0.5,min_y-0.5,max_y+0.5),vmin=-50,vmax=50)
    plt.colorbar(im)
    plt.ylim(min_y - 1.5, max_y+1.5)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.plot(data_test_point_x_class_0, data_test_point_y_class_0, '.', color='blue', label='c
lass = 0')
    plt.plot(data_test_point_x_class_1, data_test_point_y_class_1, '.', color='red', label='cl
ass = 1')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

plot the logistic regression values over the 2-dimensional Euclidean space

In [ ]:

```
def function_result_09():

    plt.figure(figsize=(8,8))
    plt.title('logistic regression values on the training data')

    min_x   = np.min(data_train_point_x)
    max_x   = np.max(data_train_point_x)
    min_y   = np.min(data_train_point_y)
    max_y   = np.max(data_train_point_y)

    X = np.arange(min_x - 0.5, max_x + 0.5, 0.1)
    Y = np.arange(min_y - 0.5, max_y + 0.5, 0.1)

    [XX, YY] = np.meshgrid(X, Y)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    logit_ZZ = sigmoid(theta_optimal[0] + theta_optimal[1]*XX + theta_optimal[2]*YY + theta_op
timal[3]*np.power(XX,2) + theta_optimal[4]*np.power(XX,3) + theta_optimal[5]*XX*YY)



    im = plt.imshow(logit_ZZ, aspect='auto', origin = 'lower', cmap = 'RdBu_r', alpha=0.98, ex
tent=(min_x-0.5,max_x+0.5,min_y-0.5,max_y+0.5))
    plt.colorbar(im)
    plt.ylim(min_y - 1.5, max_y+1.5)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++


    plt.plot(data_train_point_x_class_0, data_train_point_y_class_0, '.', color='blue', label=
'class = 0')
    plt.plot(data_train_point_x_class_1, data_train_point_y_class_1, '.', color='red', label=
'class = 1')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

In [ ]:

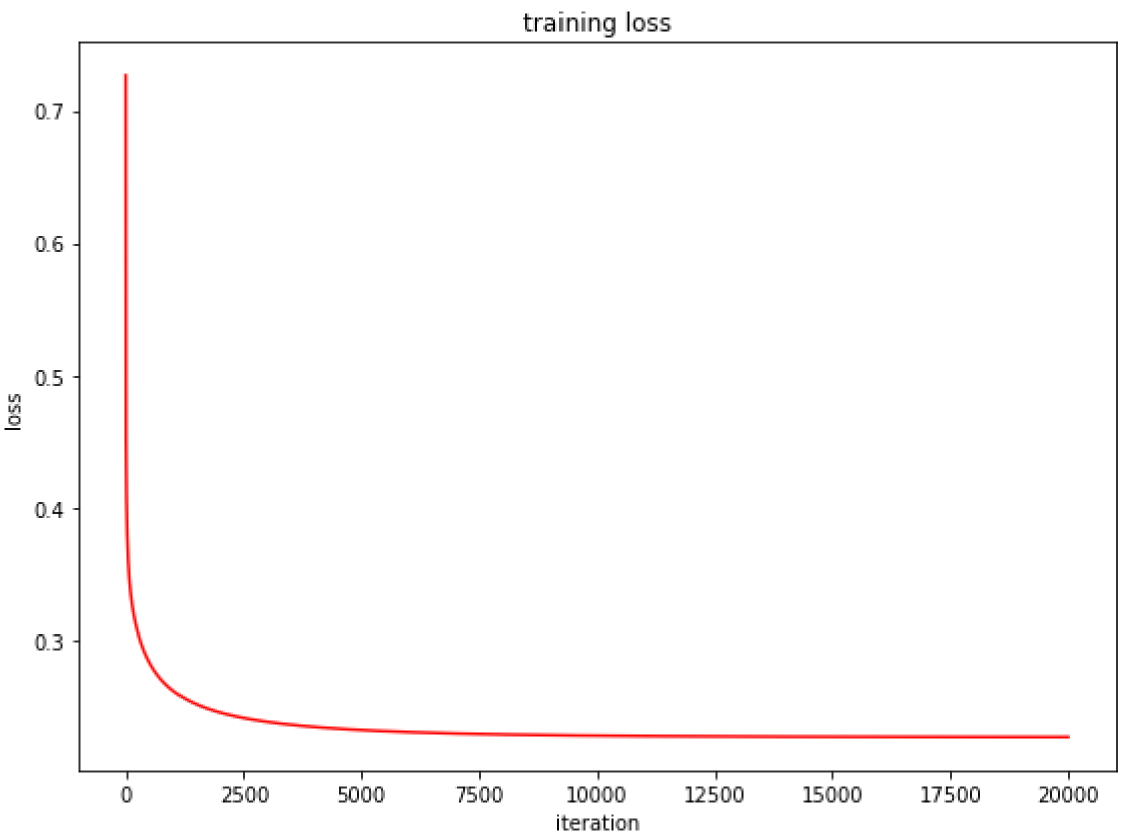
```
def function_result_10():

    plt.figure(figsize=(8,8))
    plt.title('logistic regression values on the testing data')

    min_x   = np.min(data_test_point_x)
    max_x   = np.max(data_test_point_x)
    min_y   = np.min(data_test_point_y)
    max_y   = np.max(data_test_point_y)

    X = np.arange(min_x - 0.5, max_x + 0.5, 0.1)
    Y = np.arange(min_y - 0.5, max_y + 0.5, 0.1)

    [XX, YY] = np.meshgrid(X, Y)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    logit_ZZ = sigmoid(theta_optimal[0] + theta_optimal[1]*XX + theta_optimal[2]*YY + theta_op
timal[3]*np.power(XX,2) + theta_optimal[4]*np.power(XX,3) + theta_optimal[5]*XX*YY)



    im = plt.imshow(logit_ZZ, aspect='auto', origin = 'lower', cmap = 'RdBu_r', alpha=0.98, ex
tent=(min_x-0.5,max_x+0.5,min_y-0.5,max_y+0.5))
    plt.colorbar(im)
    plt.ylim(min_y - 1.5, max_y+1.5)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++


    plt.plot(data_test_point_x_class_0, data_test_point_y_class_0, '.', color='blue', label='c
lass = 0')
    plt.plot(data_test_point_x_class_1, data_test_point_y_class_1, '.', color='red', label='cl
ass = 1')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

results

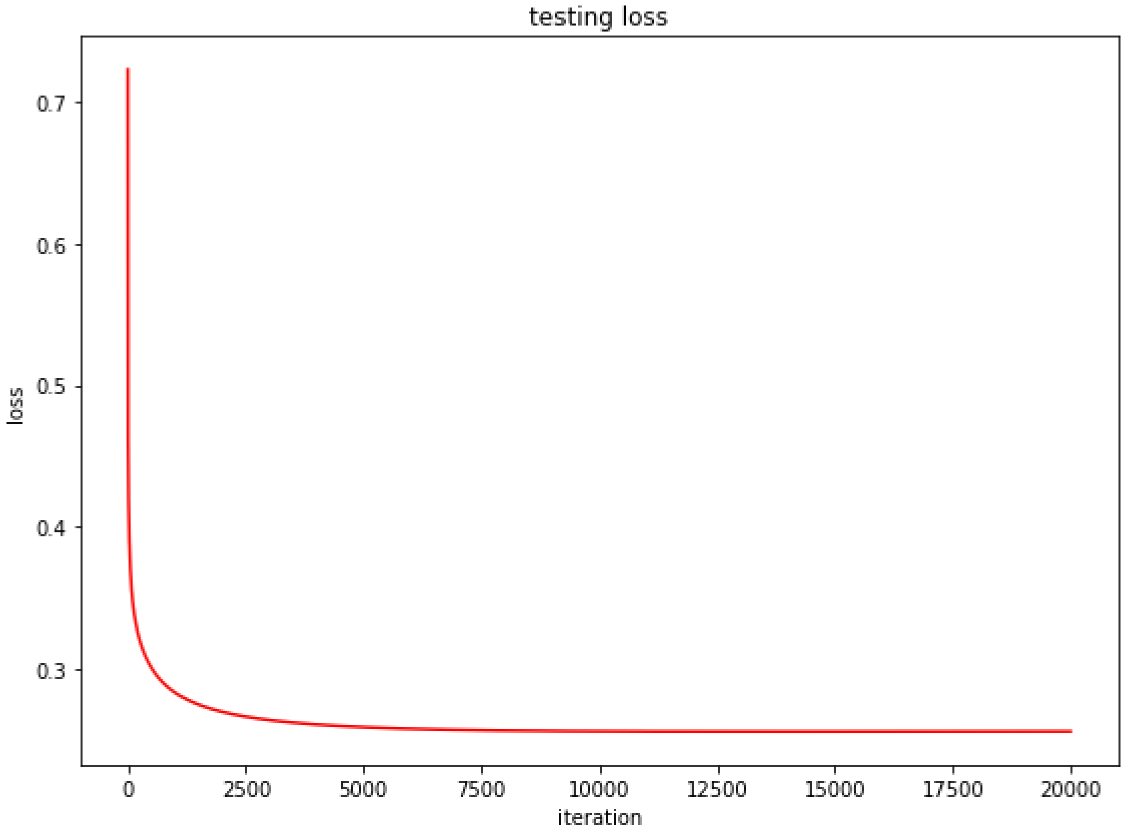
In [ ]:

```
number_result = 10

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

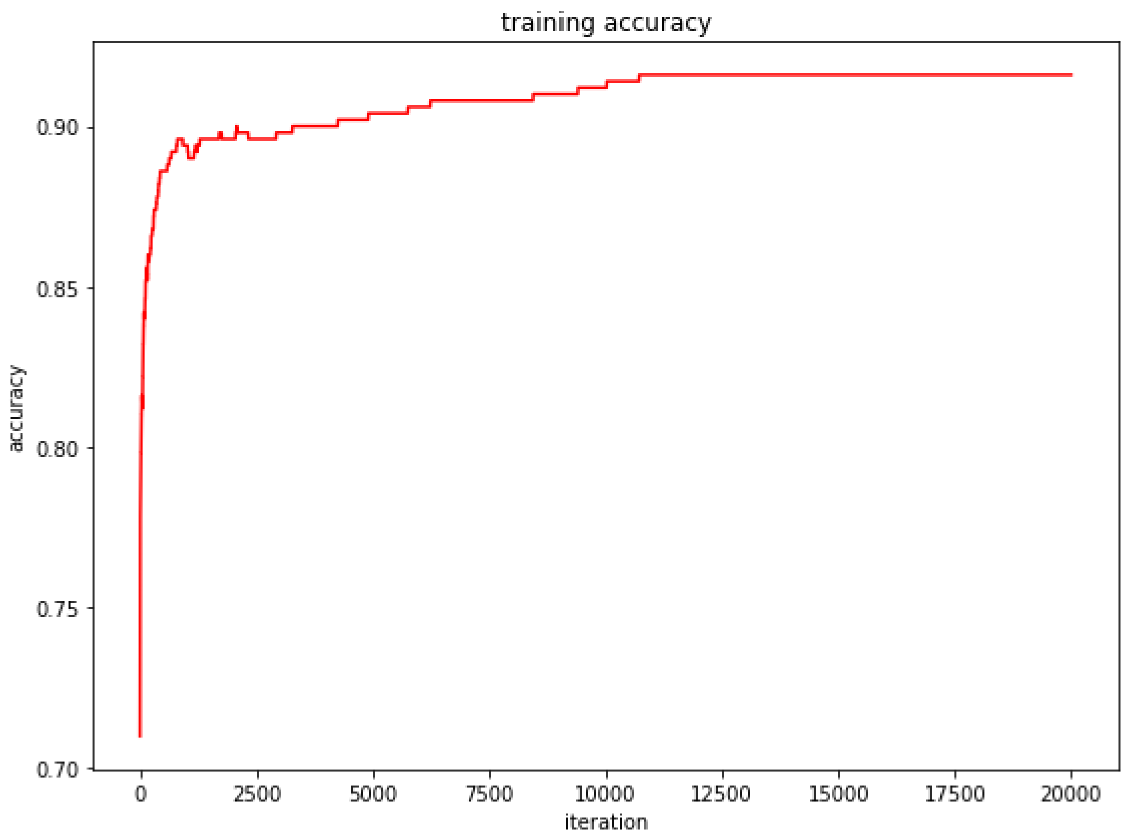
```
**************************************************
## [RESULT 01]
**************************************************
final training accuracy =  0.9160000000
**************************************************
## [RESULT 02]
**************************************************
final testing accuracy =  0.9040000000
**************************************************
## [RESULT 03]
**************************************************
```



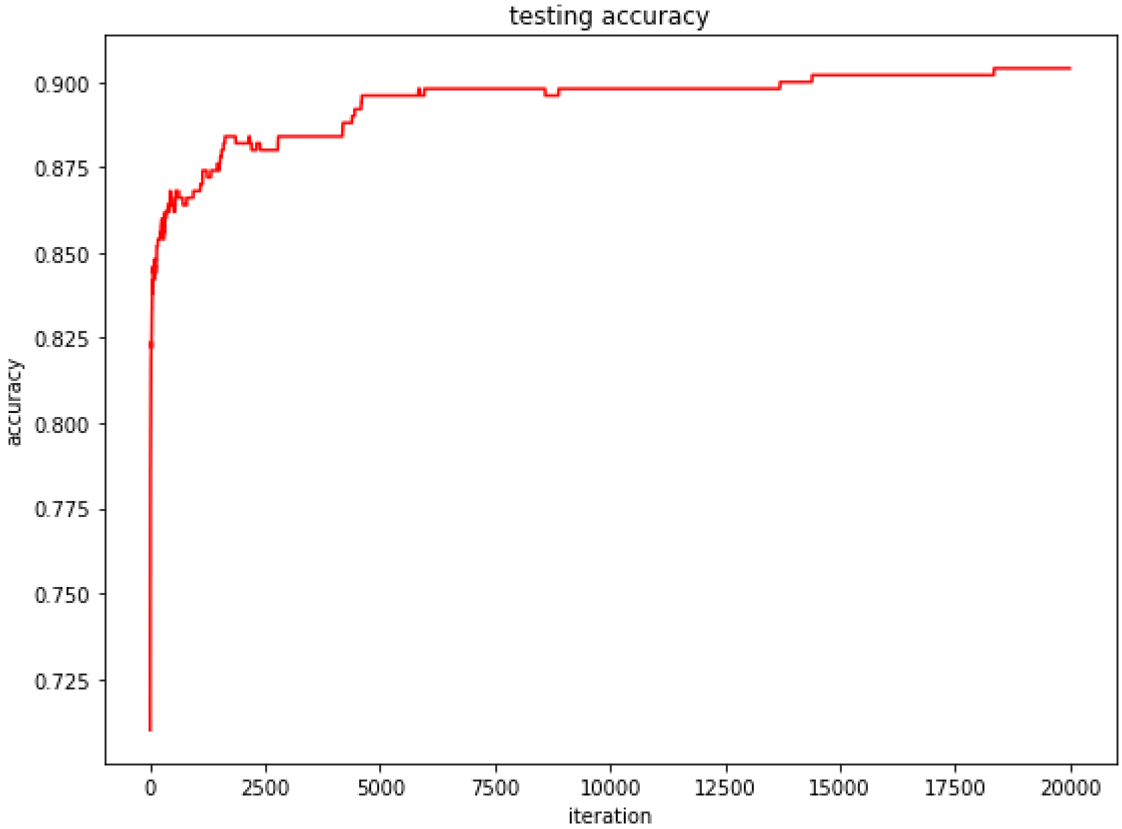

```
**************************************************
## [RESULT 04]
**************************************************
```


testing loss
loss
iteration
0.7
0.6
0.5
0.4
0.3
0
2500
5000
7500
10000
12500
15000
17500
20000

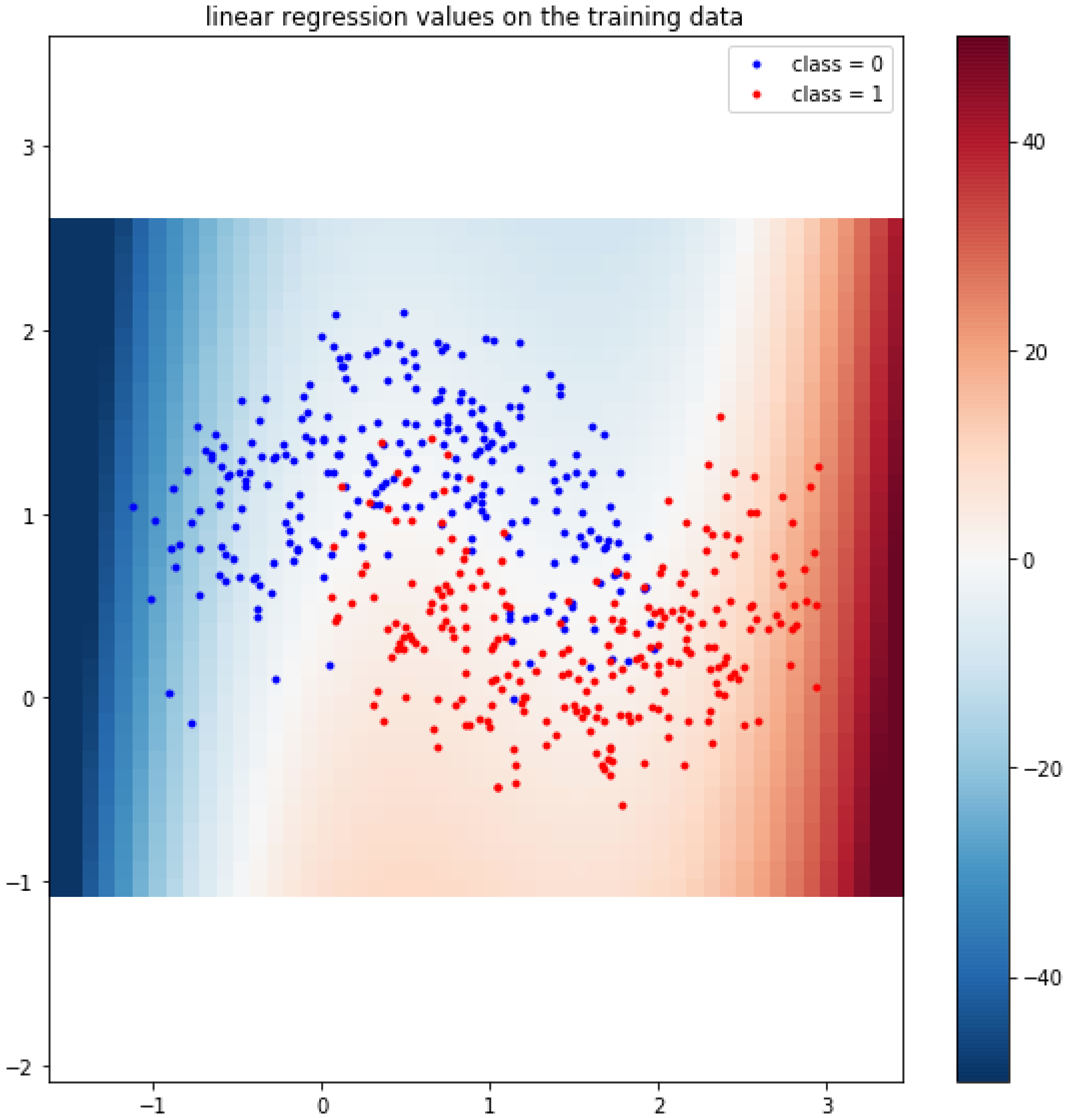
```
****************************************************
## [RESULT 05]
****************************************************
```


training accuracy
accuracy
iteration
0.90
0.85
0.80
0.75
0.70
0
2500
5000
7500
10000
12500
15000
17500
20000

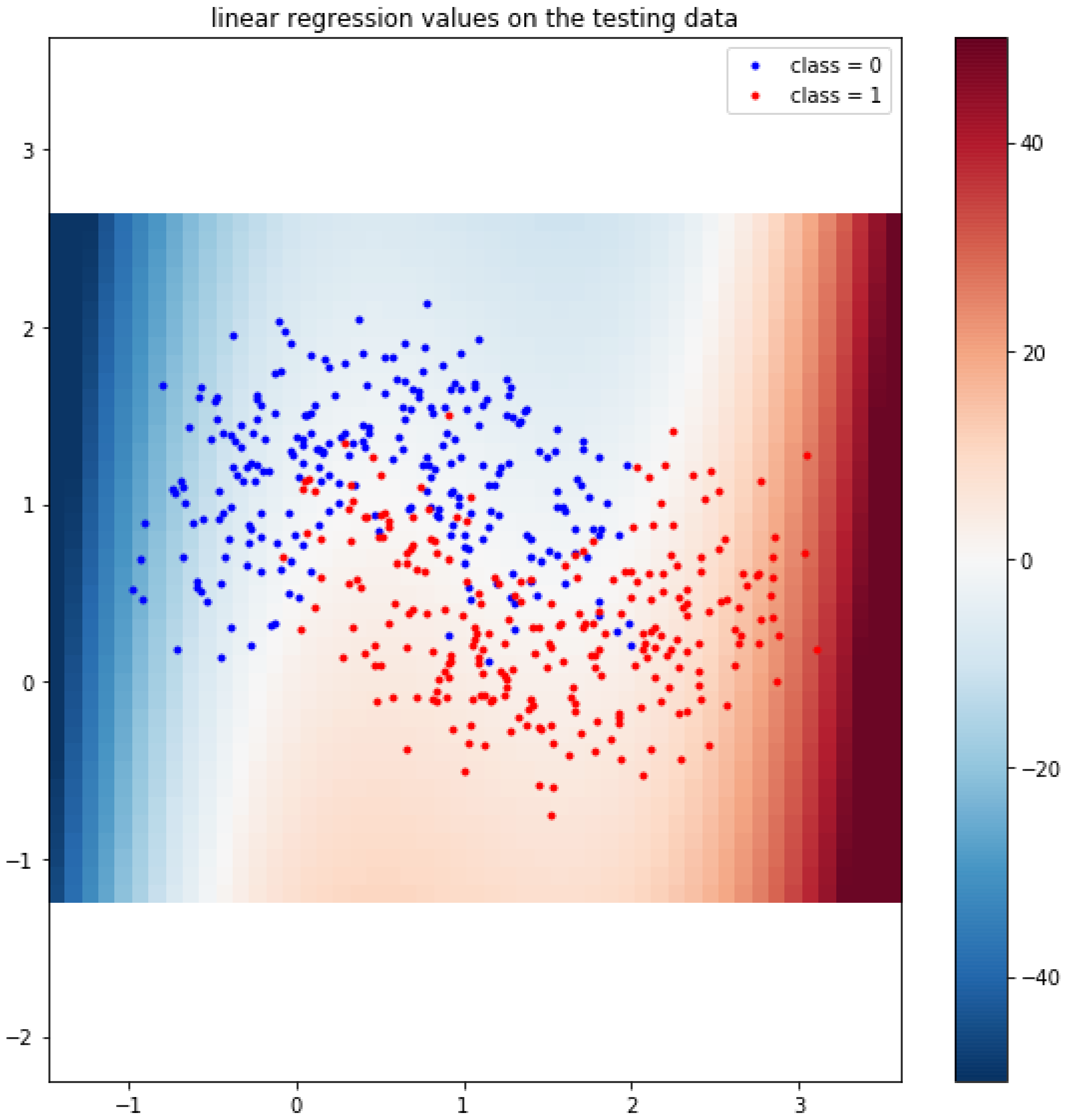
```
**************************************************
## [RESULT 06]
**************************************************
```



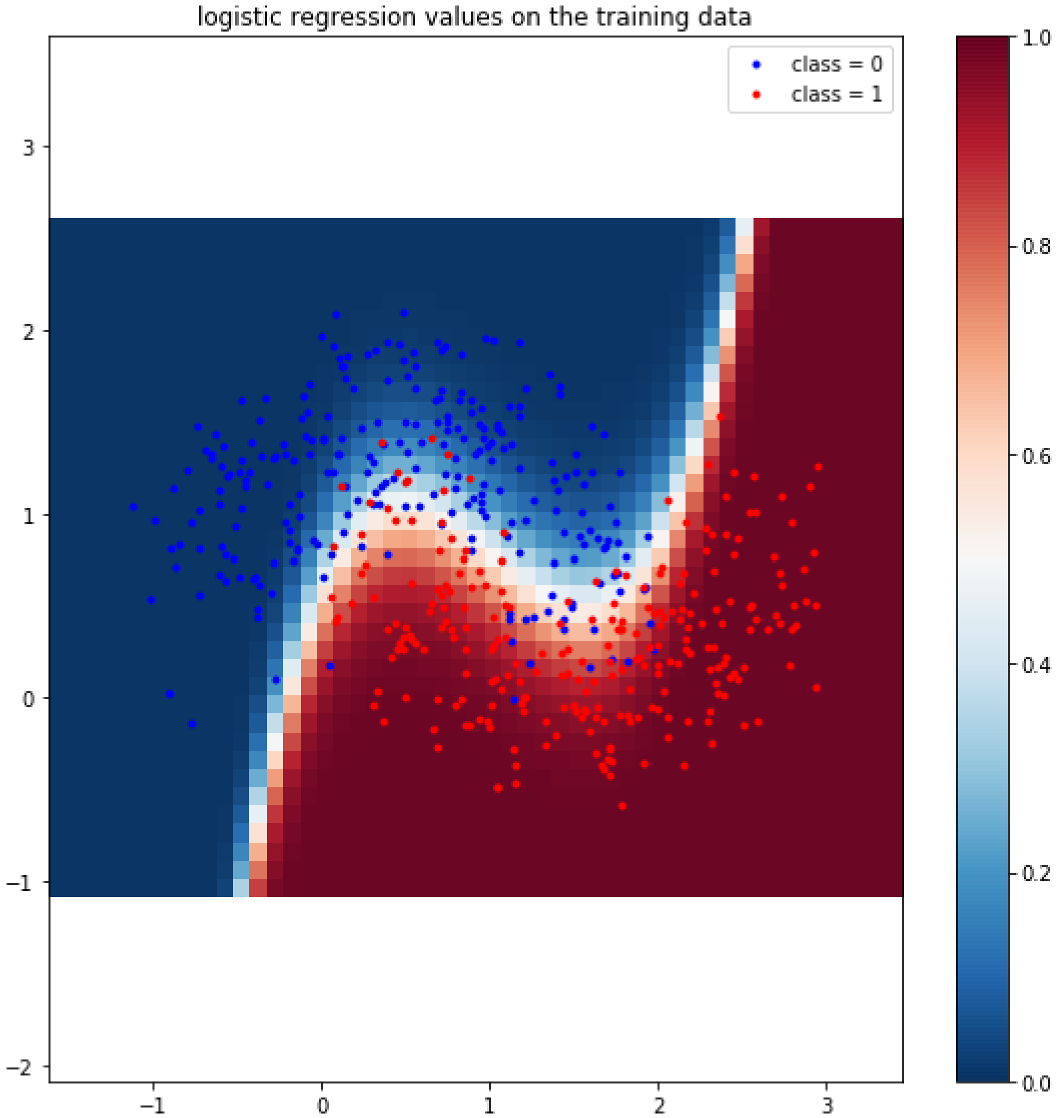

```
**************************************************
## [RESULT 07]
**************************************************
```


linear regression values on the training data
class = 0
class = 1
3
2
1
0
−1
−2
−1
0
1
2
3
40
20
0
−20
−40

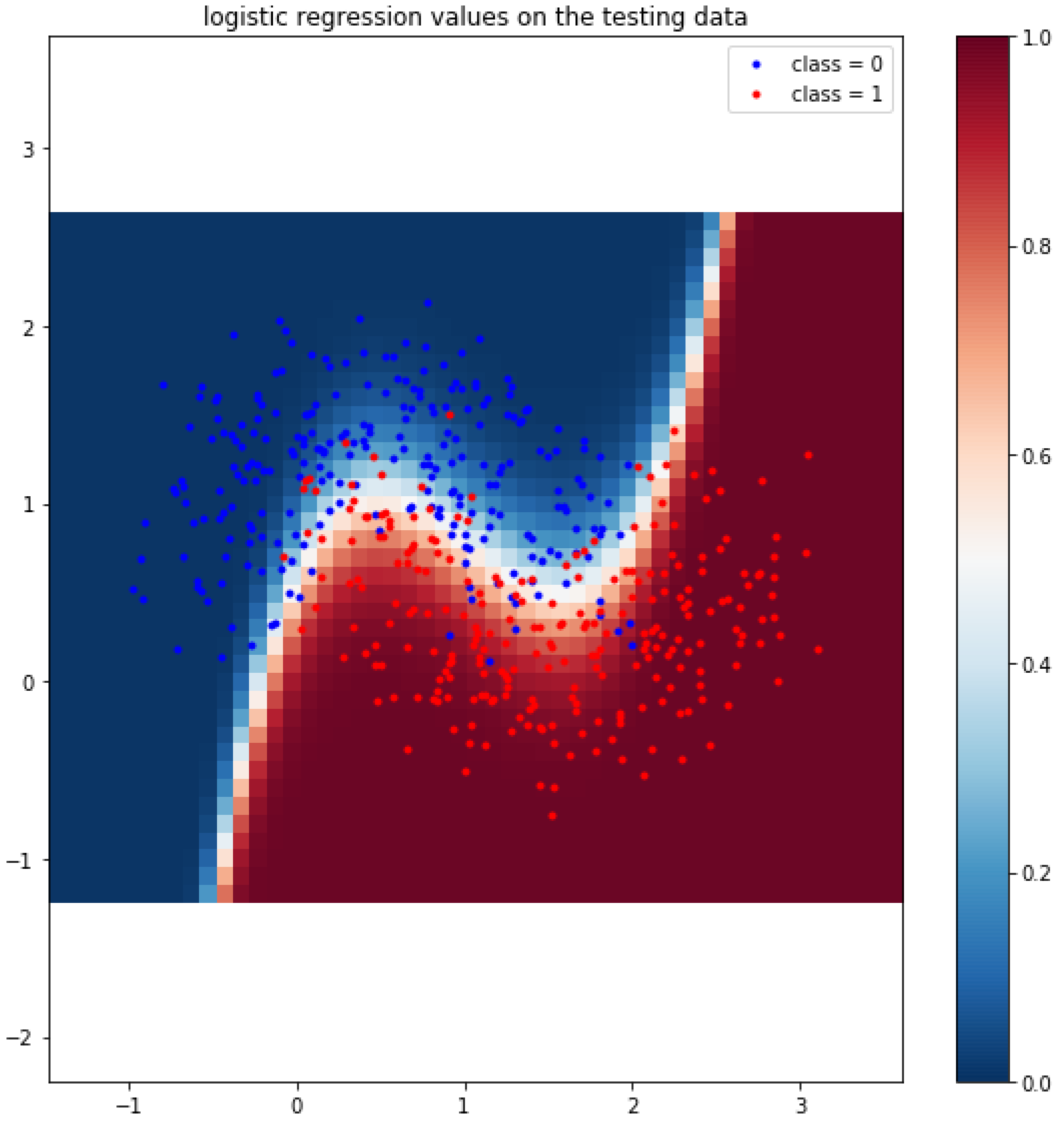
```
*****************************************************
## [RESULT 08]
*****************************************************
```




```
*****************************************************
## [RESULT 09]
*****************************************************
```


logistic regression values on the training data
class = 0
class = 1
3
2
1
0
−1
−2
−1
0
1
2
3
1.0
0.8
0.6
0.4
0.2
0.0

```
**************************************************
## [RESULT 10]
**************************************************
```


logistic regression values on the testing data
class = 0
class = 1
3
2
1
0
−1
−2
−1
0
1
2
3
1.0
0.8
0.6
0.4
0.2
0.0

In [ ]:

```

```