A LEVEL (P6) PROBABLITY

QUESTION'S

1 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]

2 When Andrea needs a taxi, she rings one of three taxi companies, *A*, *B* or *C*. 50% of her calls are to taxi company *A*, 30% to *B* and 20% to *C*. A taxi from company *A* arrives late 4% of the time, a taxi from company *B* arrives late 6% of the time and a taxi from company *C* arrives late 17% of the time.

(i) Find the probability that, when Andrea rings for a taxi, it arrives late. [3]

(ii) Given that Andrea's taxi arrives late, find the conditional probability that she rang company *B*. [3]

3 Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let *M* be the event that the person is male and let *E* be the event that the person is employed.

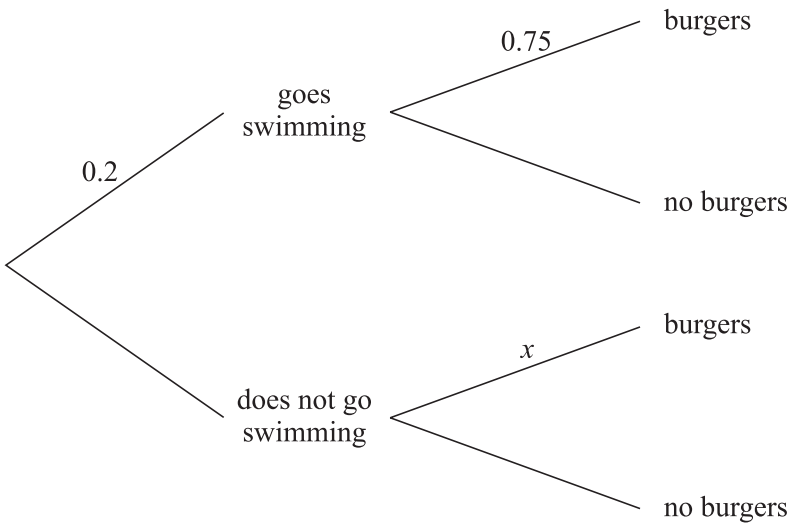
(i) Find P(*M*). [2]

(ii) Find P(*M* and *E*). [1]

(iii) Are *M* and *E* independent events? Justify your answer. [3]

(iv) Given that the person chosen is unemployed, find the probability that the person is female. [2]

4 The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.



The probability that Henk has burgers for supper on any day is 0.5.

(i) Find x. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

5 Two fair dice are thrown.

(i) Event A is 'the scores differ by 3 or more'. Find the probability of event A. [3]

(ii) Event B is 'the product of the scores is greater than 8'. Find the probability of event B. [2]

(iii) State with a reason whether events A and B are mutually exclusive. [2]

6 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]

7 In country A 30% of people who drink tea have sugar in it. In country B 65% of people who drink tea have sugar in it. There are 3 million people in country A who drink tea and 12 million people in country B who drink tea. A person is chosen at random from these 15 million people.

(i) Find the probability that the person chosen is from country A. [1]

(ii) Find the probability that the person chosen does not have sugar in their tea. [2]

(iii) Given that the person chosen does not have sugar in their tea, find the probability that the person is from country B. [2]

8 There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information. [2]

(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set. [2]

(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights. [3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights. [3]

9 At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he goes on each of the three rides. [2]

The probabilities that Ravi is frightened on each of the rides are as follows:

$$\text{elephant ride } \frac{6}{10}, \quad \text{camel ride } \frac{7}{10}, \quad \text{jungle tractor ride } \frac{8}{10}.$$

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened. [2]

Ravi goes on a ride.

(iii) Find the probability that he is frightened. [2]

(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride. [3]

10 Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

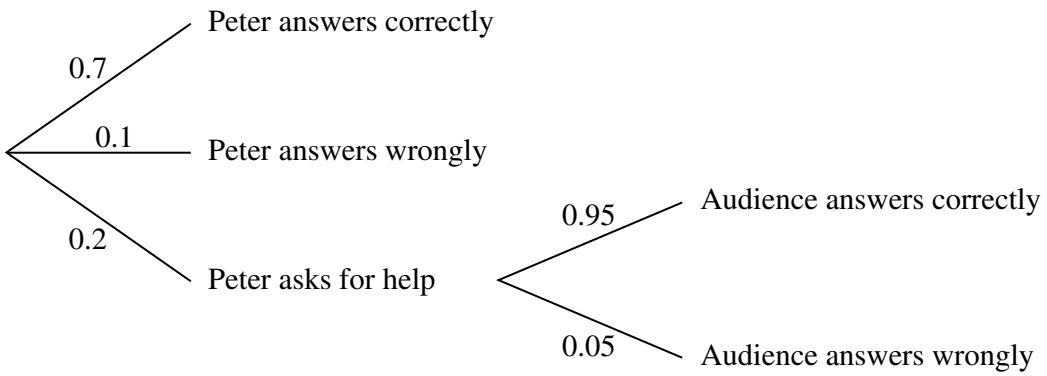
(i) Draw a fully labelled tree diagram to show this information. [2]

(ii) Given that Maria did **not** have jam for breakfast, find the probability that she had toast. [4]

11 In a television quiz show Peter answers questions one after another, stopping as soon as a question is answered wrongly.

- The probability that Peter gives the correct answer himself to any question is 0.7.
- The probability that Peter gives a wrong answer himself to any question is 0.1.
- The probability that Peter decides to ask for help for any question is 0.2.

On the first occasion that Peter decides to ask for help he asks the audience. The probability that the audience gives the correct answer to any question is 0.95. This information is shown in the tree diagram below.



(i) Show that the probability that the first question is answered correctly is 0.89. [1]

On the second occasion that Peter decides to ask for help he phones a friend. The probability that his friend gives the correct answer to any question is 0.65.

(ii) Find the probability that the first two questions are both answered correctly. [6]

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience. [3]

12 Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24.

R : both of the numbers are greater than 8.

(i) Find P(Q). [2]

(ii) Find P(R). [2]

(iii) Are events Q and R exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

13 A bottle of sweets contains 13 red sweets, 13 blue sweets, 13 green sweets and 13 yellow sweets. 7 sweets are selected at random. Find the probability that exactly 3 of them are red. [3]

14 Three friends, Rick, Brenda and Ali, go to a football match but forget to say which entrance to the ground they will meet at. There are four entrances, A, B, C and D. Each friend chooses an entrance independently.

- The probability that Rick chooses entrance A is $\frac{1}{3}$. The probabilities that he chooses entrances B, C or D are all equal.
- Brenda is equally likely to choose any of the four entrances.
- The probability that Ali chooses entrance C is $\frac{2}{7}$ and the probability that he chooses entrance D is $\frac{3}{5}$. The probabilities that he chooses the other two entrances are equal.

(i) Find the probability that at least 2 friends will choose entrance B. [4]

(ii) Find the probability that the three friends will all choose the same entrance. [4]

15 A fair five-sided spinner has sides numbered 1, 2, 3, 4, 5. Raj spins the spinner and throws two fair dice. He calculates his score as follows.

- If the spinner lands on an **even-numbered** side, Raj **multiplies** the two numbers showing on the dice to get his score.
- If the spinner lands on an **odd-numbered** side, Raj **adds** the numbers showing on the dice to get his score.

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side. [6]

16 It was found that 68% of the passengers on a train used a cell phone during their train journey. Of those using a cell phone, 70% were under 30 years old, 25% were between 30 and 65 years old and the rest were over 65 years old. Of those not using a cell phone, 26% were under 30 years old and 64% were over 65 years old.

(i) Draw a tree diagram to represent this information, giving all probabilities as decimals. [2]

(ii) Given that one of the passengers is 45 years old, find the probability of this passenger using a cell phone during the journey. [3]

17 When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]

18 A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

(i) One rope is chosen at random. Find the expectation and variance of its length. [4]

(ii) Two ropes are chosen at random. Find the probability that they have different lengths. [2]

(iii) Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

19 Maria has 3 pre-set stations on her radio. When she switches her radio on, there is a probability of 0.3 that it will be set to station 1, a probability of 0.45 that it will be set to station 2 and a probability of 0.25 that it will be set to station 3. On station 1 the probability that the presenter is male is 0.1, on station 2 the probability that the presenter is male is 0.85 and on station 3 the probability that the presenter is male is p. When Maria switches on the radio, the probability that it is set to station 3 and the presenter is male is 0.075.

(i) Show that the value of p is 0.3. [1]

(ii) Given that Maria switches on and hears a male presenter, find the probability that the radio was set to station 2. [4]

20 Suzanne has 20 pairs of shoes, some of which have designer labels. She has 6 pairs of high-heeled shoes, of which 2 pairs have designer labels. She has 4 pairs of low-heeled shoes, of which 1 pair has designer labels. The rest of her shoes are pairs of sports shoes. Suzanne has 8 pairs of shoes with designer labels in total.

(i) Copy and complete the table below to show the number of pairs in each category. [2]

	Designer labels	No designer labels	Total
High-heeled shoes			
Low-heeled shoes			
Sports shoes			
Total			20

Suzanne chooses 1 pair of shoes at random to wear.

(ii) Find the probability that she wears the pair of low-heeled shoes with designer labels. [1]

(iii) Find the probability that she wears a pair of sports shoes. [1]

(iv) Find the probability that she wears a pair of high-heeled shoes, given that she wears a pair of shoes with designer labels. [1]

(v) State with a reason whether the events 'Suzanne wears a pair of shoes with designer labels' and 'Suzanne wears a pair of sports shoes' are independent. [2]

Suzanne chooses 1 pair of shoes at random each day.

(vi) Find the probability that Suzanne wears a pair of shoes with designer labels on at most 4 days out of the next 7 days. [3]

21 Fabio drinks coffee each morning. He chooses Americano, Cappucino or Latte with probabilities 0.5, 0.3 and 0.2 respectively. If he chooses Americano he either drinks it immediately with probability 0.8, or leaves it to drink later. If he chooses Cappucino he either drinks it immediately with probability 0.6, or leaves it to drink later. If he chooses Latte he either drinks it immediately with probability 0.1, or leaves it to drink later.

(i) Find the probability that Fabio chooses Americano and leaves it to drink later. [1]

(ii) Fabio drinks his coffee immediately. Find the probability that he chose Latte. [4]

22 Ronnie obtained data about the gross domestic product (GDP) and the birth rate for 170 countries. He classified each GDP and each birth rate as either 'low', 'medium' or 'high'. The table shows the number of countries in each category.

		Birth rate		
		Low	Medium	High
GDP	Low	3	5	45
	Medium	20	42	12
	High	35	8	0

One of these countries is chosen at random.

(i) Find the probability that the country chosen has a medium GDP. [1]

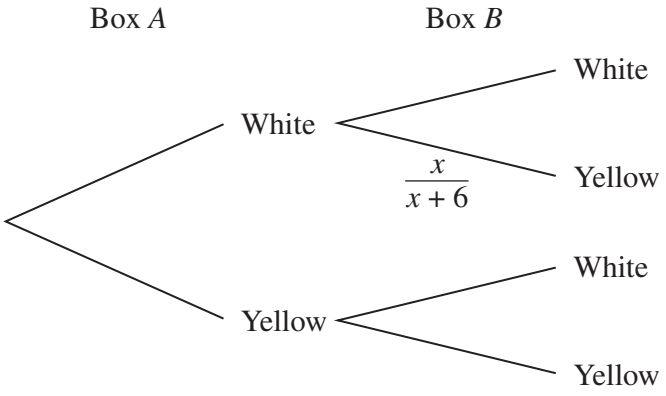
(ii) Find the probability that the country chosen has a low birth rate, given that it does not have a medium GDP. [2]

(iii) State with a reason whether or not the events 'the country chosen has a high GDP' and 'the country chosen has a high birth rate' are exclusive. [2]

One country is chosen at random from those countries which have a medium GDP and then a different country is chosen at random from those which have a medium birth rate.

(iv) Find the probability that both countries chosen have a medium GDP and a medium birth rate. [3]

23 Box A contains 8 white balls and 2 yellow balls. Box B contains 5 white balls and x yellow balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.



(i) Justify the probability $\frac{x}{x+6}$ on the tree diagram. [1]

(ii) Copy and complete the tree diagram. [4]

(iii) If the ball chosen from box A is white then the probability that the ball chosen from box B is also white is $\frac{1}{3}$. Show that the value of x is 12. [2]

(iv) Given that the ball chosen from box B is yellow, find the conditional probability that the ball chosen from box A was yellow. [4]

24 The people living in two towns, Mumbok and Bagville, are classified by age. The numbers in thousands living in each town are shown in the table below.

	Mumbok	Bagville
Under 18 years	15	35
18 to 60 years	55	95
Over 60 years	20	30

One of the towns is chosen. The probability of choosing Mumbok is 0.6 and the probability of choosing Bagville is 0.4. Then a person is chosen at random from that town. Given that the person chosen is between 18 and 60 years old, find the probability that the town chosen was Mumbok. [5]

25 On Saturday afternoons Mohit goes shopping with probability 0.25, or goes to the cinema with probability 0.35 or stays at home. If he goes shopping the probability that he spends more than \$50 is 0.7. If he goes to the cinema the probability that he spends more than \$50 is 0.8. If he stays at home he spends \$10 on a pizza.

(i) Find the probability that Mohit will go to the cinema and spend less than \$50. [1]

(ii) Given that he spends less than \$50, find the probability that he went to the cinema. [4]

26 Playground equipment consists of swings (S), roundabouts (R), climbing frames (C) and play-houses (P). The numbers of pieces of equipment in each of 3 playgrounds are as follows.

Playground X	Playground Y	Playground Z
$3S$, $2R$, $4P$	$6S$, $3R$, $1C$, $2P$	$8S$, $3R$, $4C$, $1P$

Each day Nur takes her child to one of the playgrounds. The probability that she chooses playground X is $\frac{1}{4}$. The probability that she chooses playground Y is $\frac{1}{4}$. The probability that she chooses playground Z is $\frac{1}{2}$. When she arrives at the playground, she chooses one piece of equipment at random.

(i) Find the probability that Nur chooses a play-house. [4]

(ii) Given that Nur chooses a climbing frame, find the probability that she chose playground Y. [4]

27 Roger and Andy play a tennis match in which the first person to win two sets wins the match. The probability that Roger wins the first set is 0.6. For sets after the first, the probability that Roger wins the set is 0.7 if he won the previous set, and is 0.25 if he lost the previous set. No set is drawn.

(i) Find the probability that there is a winner of the match after exactly two sets. [3]

(ii) Find the probability that Andy wins the match given that there is a winner of the match after exactly two sets. [2]

28 The following back-to-back stem-and-leaf diagram shows the times to load an application on 61 smartphones of type A and 43 smartphones of type B.

	Type A		Type B	
(7)	9 7 6 6 4 3 3	2	1 3 5 8	(4)
(7)	5 5 4 4 2 2 2	3	0 4 4 5 6 6 6 6 7 8 8 9	(12)
(13)	9 9 8 8 8 7 6 6 4 3 2 2 0	4	0 1 1 2 3 6 8 8 9 9	(10)
(9)	6 5 5 4 3 2 1 1 0	5	2 5 6 6 9	(5)
(4)	9 7 3 0	6	1 3 8 9	(4)
(6)	8 7 4 4 1 0	7	5 7	(2)
(10)	7 6 6 6 5 3 3 2 1 0	8	1 2 4 4	(4)
(5)	8 6 5 5 5	9	0 6	(2)

Key: 3 | 2 | 1 means 0.23 seconds for type A and 0.21 seconds for type B.

(i) Find the median and quartiles for smartphones of type A. [3]

You are given that the median, lower quartile and upper quartile for smartphones of type B are 0.46 seconds, 0.36 seconds and 0.63 seconds respectively.

(ii) Represent the data by drawing a pair of box-and-whisker plots in a single diagram on graph paper. [3]

(iii) Compare the loading times for these two types of smartphone. [1]

29 Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

(ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find $\mathrm{E}(X)$. [4]

30 In country X, 25% of people have fair hair. In country Y, 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y. A person is chosen at random from these 28 million people.

(i) Find the probability that the person chosen is from country X. [1]

(ii) Find the probability that the person chosen has fair hair. [2]

(iii) Find the probability that the person chosen is from country X, given that the person has fair hair. [2]