## 5.4 Discrete random variables

| Candidates should be able to: | Notes and examples |
|---|---|
| • draw up a probability distribution table relating to a given situation involving a discrete random variable $X$, and calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ | |
| • use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models | Including the notations $\mathrm{B}(n, p)$ and $\mathrm{Geo}(p)$. $\mathrm{Geo}(p)$ denotes the distribution in which $p_r = p(1-p)^{r-1}$ for $r = 1, 2, 3, \ldots$ . |
| • use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution. | Proofs of formulae are not required. |

# GEOMETRIC DISTRIBUTION

**BINOMIAL:**

**Coin:**

$n = 20$

$p = \text{Head} = 0.5$

no. of times success occur.

$P(X=8) = {}^{20}C_{8}(0.5)^{8}(0.5)^{12}$

Binomial looks at a repeated experiment where n = total repeats and r = no. of time success occur.

$X = 0, 1, 2, 3 \ldots\ldots n$

**Geometric**

1) Repeated
2) Success/failure
3) Discrete outcomes.

Repeat an experiment till success happens for the first time.

There is no n = no. of repeats involved.

eg: Toss a coin till you get first head.
Roll a dice till it lands on a 6.

Q: Roll a $\underbrace{\text{dice}}_{1,2,3,4,5,6}$ till it $\underbrace{\text{lands on } 6}_{\text{sucess } = \; p = \frac{1}{6}, \; q = \frac{5}{6}}$.

Geometric : Repeat until first success.

(i) Find probability that 6 lands on 8<sup></sup>th time we rolled the dice.

$$\begin{array}{ccccccccc} & F & F & F & F & F & F & F & S \end{array}$$

$$P(X=8) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) = \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

This means that it took 8 times to repeat experiment to get first success.

$$P(X=8) = q^{r-1}p = q^{8-1}p = q^7 p = \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

$$\boxed{\text{GEOMETRIC DISTRIBUTION}}$$

Repeat experiment until success occurs for first time.

$$P(X=r) = q^{r-1}\cdot p$$

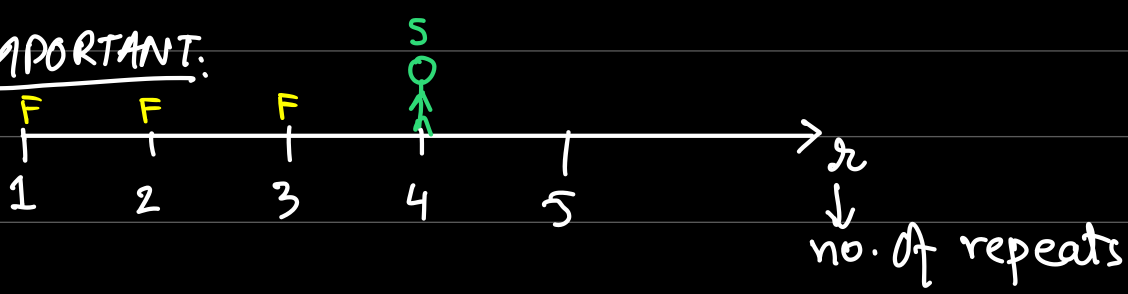
$r$ : number of repeatiton on which we want success for first time.

$X = 1, 2, 3 \ldots\ldots \infty$

$$P(X=6) = q^5 p$$

$$P(X=4) = q^3 p$$

IMPORTANT:



# $\boxed{\text{LETS CRAM}}$

$\boxed{\text{GEOMETRIC}}$ Repeat experiment until first success.

$\boxed{1}$ $P(X=r) = q^{r-1} \cdot p$

$(1-p)^{r-1} \cdot p$

$\boxed{2}$ Mean = Expected Value $= \dfrac{1}{p}$

$\boxed{3}$ $P(X \leq r) = 1 - q^{r}$

$\boxed{4}$ $P(X > r) = q^{r}$

Be careful of $\leq$ and $>$ sign.

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**Q** Roll a dice till it lands on 6.

Geometric : $p = \frac{1}{6}$ , $q = \frac{5}{6}$ Mean $= \frac{1}{p} = \frac{1}{1/6} = 6$

(i) Find probability that dice lands on 6 on fourth turn.

$P(X=4) = q^{r-1} \cdot p = q^{3} p = \left(\frac{5}{6}\right)^{3}\left(\frac{1}{6}\right) = \dfrac{125}{1296}$

(ii) $P(X \leq 6) =$

$P(X \leq r) = 1 - q^{r}$

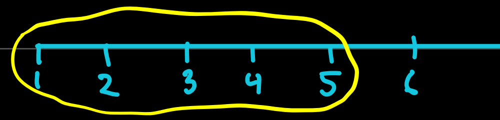
$= 1 - \left(\frac{5}{6}\right)^{6} = \boxed{\phantom{00}}$

$P(X \leq r) = 1 - q^{r}$

(iii) $P(X<6) =$

$P(X \leq 5)$

$P(X \leq r) = 1 - q^r = 1 - \left(\frac{5}{6}\right)^5$

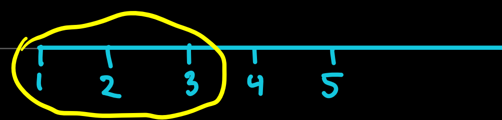



$X < 6$
$X \leq 5$

(iv) $P(X < 4) =$

$P(X \leq 3)$

$P(X \leq r) = 1 - q^r = 1 - \left(\frac{5}{6}\right)^3$

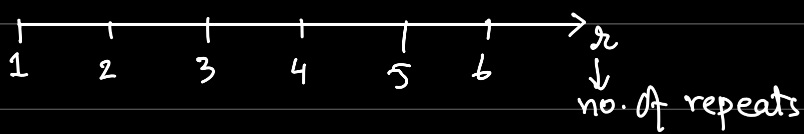



$X < 4$
$X \leq 3$

(v) $P(X > 7) =$

$P(X > r) = q^r$

$= \left(\frac{5}{6}\right)^7$

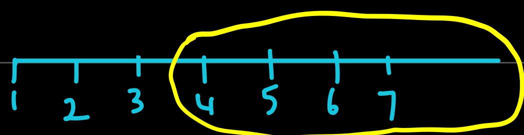



(vi) $P(X \geq 4)$

$P(X > 3)$

$P(X > r) = q^r$

$= \left(\frac{5}{6}\right)^3$




$X \geq 4$
$X > 3$

$$\boxed{\begin{array}{l} P(X \leq r) = 1 - q^r \\ P(X > r) = q^r \end{array}}$$

| BINOMIAL | GEOMETRIC |
|---|---|
| Toss a coin 20 times<br>Success = head<br>failure = Tail.<br>$P(X=r) = {}^nC_r\, p^r q^{n-r}$<br>out of n repeats,<br>r = no of times success happened | Toss a coin till it lands on Head<br>Success = head<br>failure = Tail.<br>$P(X=r) = q^{r-1} \cdot p$<br>The number of repetition on which success happened for the first time. |

**1** The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

Total = 36

**(a)** Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

$(1,3)\ (3,1)\ (2,2)$ } Favorable outcomes

$P(X=4) = \frac{3}{36} = \frac{1}{12}$

Geometric

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable $X$.

sucess $= p = \frac{1}{12}$

$q = \frac{11}{12}$

**(b)** Find the mean of $X$. [1]

$\text{Mean}(X) = \frac{1}{p} = \frac{1}{1/12} = 12$

success X

**(c)** Find the probability that a score of 4 is first obtained on the 6th throw. [1]

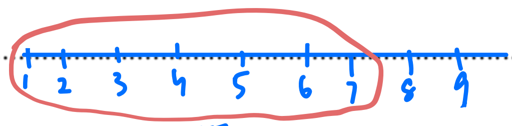
$P(X=6) = q^{6-1}p$

$= q^5 p = \left(\frac{11}{12}\right)^5 \left(\frac{1}{12}\right) = \boxed{0.053935}$

**(d)** Find $P(X < 8)$. [2]

$X<8$

$P(X \leq r) = 1 - q^r$




$X \leq 7$

$P(X \leq 7) = 1 - \left(\frac{11}{12}\right)^7$

$= \boxed{\phantom{0}}$

**7** On any given day, the probability that Moena messages her friend Pasha is 0.72.

**(a)** Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

Binomial: $n = 12$ $p = 0.72$ $q = 0.28$

$P(X \leqslant 9) = 1 - P(10, 11, 12)$

$= 1 - {}^{12}C_{10}(0.72)^{10}(0.28)^{2} - {}^{12}C_{11}(0.72)^{11}(0.28)^{1} - {}^{12}C_{12}(0.72)^{12}(0.28)^{0}$

$= \boxed{\phantom{0000}}$

$\boxed{X}$

0
1
2
3
4
5
6
7
8
9
10
11
12

$X \leqslant 9$

**(b)** Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

2 jan 3 Jan 4 jan 5 jan

F F F S

$(0.28)(0.28)(0.28)(0.72) = \boxed{\phantom{0000}}$

**5** geometric

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable $X$ denotes the number of throws required to obtain a pair of tails.

**(a)** Find the expected value of $X$. [1]

HH, HT, TH $q = \frac{3}{4}$

TT $p = \frac{1}{4}$

Expected Value of $X = E(X) = \text{mean}(X) = \frac{1}{p} = \frac{1}{1/4} = 4$

**(b)** Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

$P(X=3) = q^{r-1}p$

$= q^{3-1}p$

$= q^2 p = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \boxed{\phantom{00}}$

**(c)** Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

$P(X < 6)$

$P(X \leqslant r) = 1 - q^r$

$P(X \leqslant 5) = 1 - \left(\frac{3}{4}\right)^5$

$= \boxed{\phantom{00}}$

$X < 6$

1 2 3 4 5 6 7

$X \leqslant 5$

**3** Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

**(a)** Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

**(b)** Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

**1** A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

**(a)** Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

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On another occasion, the die is thrown 10 times.

**(b)** Find the probability that a 4 is obtained at least 3 times. [3]

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**2** An ordinary fair die is thrown until a 6 is obtained.

**(a)** Find the probability that obtaining a 6 takes more than 8 throws. [2]

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable $X$.

**(b)** Find the expected value of $X$. [1]

**(c)** Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

7 Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable $X$ is the number of tins that she needs to open.

**(a)** Show that $P(X = 3) = \frac{6}{35}$. [2]

$P(\text{carrot}) = \frac{3}{7}$

Not a geometric

**(b)** Draw up the probability distribution table for $X$. [4]

FOR BOTH GEOMETRIC & BINOMIAL,

$p$ and $q$ are constant.

So, for without replacement, it can never be binomial or geometric.