PREVIOUSLY (2004 - 2019):

P1
Vectors
V. Easy

P3
VECTORS
Line — Easy
~~Plane~~ — ~~Difficult~~

NOW:

P1
No Vectors
all content shifted to P3

P3 (v. v. easy)
P1 old vectors
Lines

$\boxed{\text{VECTORS}}$ 3-D COORDINATE GEOMETRY.

$$\overrightarrow{AB} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \overrightarrow{AB} = xi + yj + zk$$

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}, \quad \overrightarrow{AB} = 5i + 3j - 2K$$

$\boxed{\text{MAGNITUDE OF A VECTOR (LENGTH)}}$

$$\vec{AB} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad |\vec{AB}| = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{AB} = \begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix} \qquad |\vec{AB}| = \sqrt{(-6)^2 + (8)^2 + 0^2}$$

$$= 10$$

$\boxed{\text{UNIT VECTORS}}$

Length of a unit vector is 1

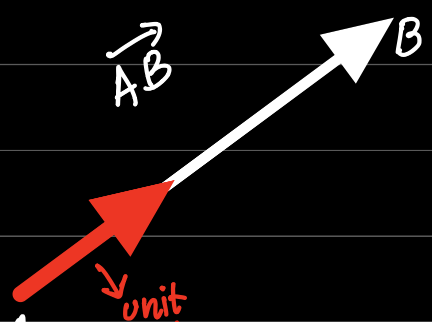
(MAGNITUDE) of a unit vector is 1

$$\text{UNIT VECTOR} = \frac{\text{VECTOR}}{\text{ITS OWN MAGNITUDE}}$$

$$\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

Q: $\vec{AB} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ Find the unit vector in the direction of $\vec{AB}$.

$\vec{AB}$



$$|\vec{AB}| = \sqrt{3^2 + 4^2 + 0^2} = 5$$

$$\text{Unit vector} = \frac{\text{vector}}{\text{magnitude}}$$

$$= 1 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3i + 4j + 0k$$

A vector

$\frac{}{5}\begin{pmatrix} \\ 0 \end{pmatrix}$ $\frac{}{5}$

$$= \begin{pmatrix} 3/5 \\ 4/5 \\ 0 \end{pmatrix} = \frac{3}{5}i + \frac{4}{5}j$$

Q. $\overrightarrow{PQ} = \begin{pmatrix} 0.2 \\ 0.1 \\ p \end{pmatrix}$ Given that $\overrightarrow{PQ}$ is a unit vector, find values of p.

$$|\overrightarrow{PQ}| = 1$$

$$\sqrt{0.2^2 + 0.1^2 + p^2} = 1$$

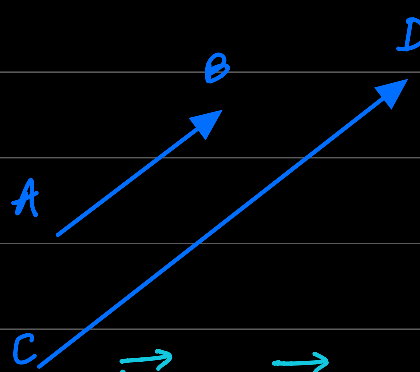
$$0.04 + 0.01 + p^2 = 1$$

$$p^2 = 0.95$$

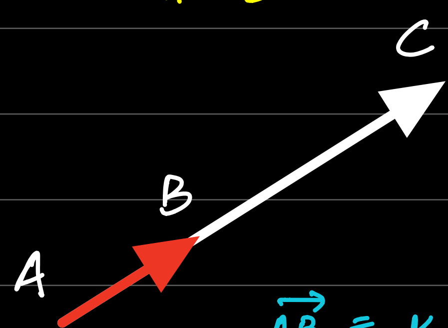
$$p = \pm\sqrt{0.95} = \pm 0.975$$

PARALLEL & COLLINEAR

ON THE SAME LINE



$\overrightarrow{AB} = K\,\overrightarrow{CD}$



$\overrightarrow{AB} = K\,\overrightarrow{AC}$

Collinear will have one alphabet repeated in names of vector.

$$A = K\,B$$

First vector constant second vector

NOTES: 1) PARALLEL/COLLINEAR VECTORS ARE MULTIPLES OF EACH OTHER.

$$\vec{AB} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad \vec{CD} = \begin{pmatrix} 10 \\ 5 \\ 0 \end{pmatrix}$$

$$\vec{CD} = 5\,\vec{AB}$$

(parallel.)

2) If there is K already present in question, change the formula.

eg: $A = \cancel{K}B$

$A = tB$

$A = xB$

Q: $a = \begin{pmatrix} 3 \\ 12 \\ 27 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ x \\ K-1 \end{pmatrix}$

Given that a and b are $\boxed{\text{parallel}}$, find values of K and x

$A = t\,B$

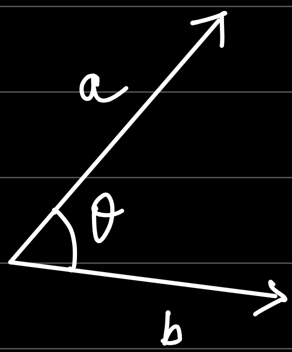
$$\begin{pmatrix} 1 \\ x \\ K-1 \end{pmatrix} = t\begin{pmatrix} 3 \\ 12 \\ 27 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ x \\ K-1 \end{pmatrix} = \begin{pmatrix} 3t \\ 12t \\ 27t \end{pmatrix}$$

$1 = 3t$	$x = 12t$	$K-1 = 27t$
$t = \frac{1}{3}$	$x = 12\left(\frac{1}{3}\right)$	$K-1 = 27\left(\frac{1}{3}\right)$
	$x = 4$	$K-1 = 9$
		$K = 10$

DOT PRODUCT (SCALAR PRODUCT)

USAGE: ANGLE BETWEEN TWO VECTORS

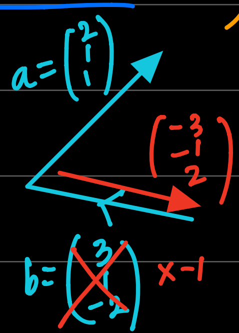


$$\boxed{a \cdot b = |a|\,|b| \cos\theta}$$

DOT PRODUCT special way to multiply two vectors

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$$

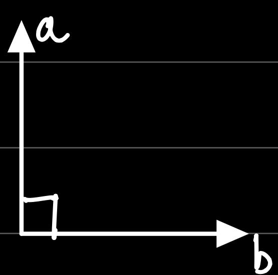
NOTES : 1) TO APPLY THIS FORMULA, BOTH VECTORS MUST:



1) EITHER DIVERGE FROM A POINT
2) CONVERGE TO A POINT.

IF THIS IS NOT THE CASE, MULTIPLY ONE OF VECTORS WITH −1 TO CHANGE ITS DIRECTION.

2) **PERPENDICULAR VECTORS**



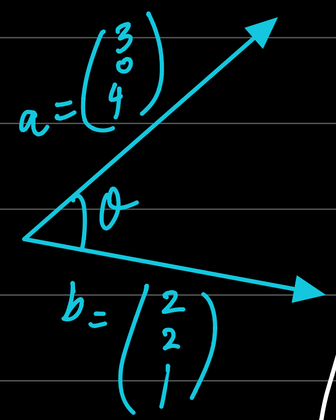
$$a \cdot b = |a|\,|b| \cos\theta$$

$$a \cdot b = |a|\,|b| \cos 90$$

$$\boxed{a \cdot b = 0}$$

DOT PRODUCT OF PERPENDICULAR VECTOR IS ALWAYS ZERO

Q:



Find angle θ. (4 marks)

$$a \cdot b = |a|\,|b| \cos\theta$$

$$\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \sqrt{3^2+0^2+4^2}\sqrt{2^2+2^2+1^2} \cos\theta$$

$$(3)(2) + (0)(2) + (4)(1) = \sqrt{25}\sqrt{9} \cos\theta$$

$$10 = 15 \cos\theta$$

$$\cos\theta = \frac{10}{15}$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right) = 48.18^\circ$$

Q:

$$a = \begin{pmatrix} 5 \\ 4 \\ p \end{pmatrix} \qquad c = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$$

Given that a and c are perpendicular, find the value of p.

$$a \cdot c = 0$$

$$\begin{pmatrix} 5 \\ 4 \\ \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ \end{pmatrix} = 0$$

$p \quad 5$

$(5)(-3) + (4)(2) + (5)(p) = 0$

$p = \frac{7}{5}$

THERE ARE TWO WAYS IN WHICH WE CAN MULTIPLY VECTORS

DOT/ SCALAR PRODUCT	CROSS/ VECTOR PRODUCT
$W = F \cdot d$ Scalar = vector · vector	$\tau = F \times d$ vector = vector × vector

Not in syllabus now.

ANUSHA AND EMAN WERE HERE

VECTORS (P3) (3-D COORDINATE)

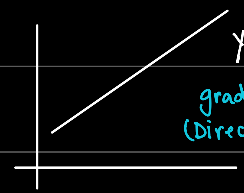
LINES

PLANES
↓
out of syllabus.

LINE

EQUATION OF A LINE

gradient (2D) (m)
↓
DIRECTION VECTOR (m)



$y - y_1 = m(x - x_1)$

$y = mx + c$

gradient (Direction)

y-int (a point on line)

EQUATION OF LINE:

VECTOR FORM

$$r = a + \lambda m$$

r: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

a: a point on line

λ: parameter

m: Direction vector

PARAMETRIC FORM

$$x = a + d\lambda$$
$$y = b + e\lambda$$
$$z = c + f\lambda$$

a, b, c: point

d, e, f: Direction vector.

CARTESIAN FORM

Point

$$\underbrace{\frac{x - \triangle}{\square} = \frac{y - \triangle}{\square} = \frac{z - \triangle}{\square}}_{\text{Direction vector.}}$$

SOMETIMES CARTESIAN IS NOT IN CORRECT FORM:

$$\frac{x+3}{5} = \frac{2y-8}{12} = \frac{3z+5}{7}$$

This is not a cartesian form Right now

$$\boxed{\frac{x-(-3)}{5}} \checkmark$$

$$\frac{2(y-4)}{12} \qquad \boxed{\frac{y-4}{6}} \checkmark$$

$$\frac{3 \div 3\left(z+\frac{5}{3}\right)}{7 \div 3} \qquad \frac{z+\frac{5}{3}}{\frac{7}{3}} \qquad \boxed{\frac{z-\left(-\frac{5}{3}\right)}{\frac{7}{3}}} \checkmark$$

$$\frac{x - \triangle(-3)}{\boxed{5}} = \frac{y - \triangle 4}{\boxed{6}} = \frac{z - \triangle\left(-\frac{5}{3}\right)}{\boxed{\frac{7}{3}}}$$

Point $= \left(-3, 4, -\frac{5}{3}\right)$

Direction vector $= \begin{pmatrix} 5 \\ 6 \\ 7/3 \end{pmatrix}$

Q: Find equation of line passing through $A(1,3,7)$ and has direction vector of $\begin{pmatrix}2\\5\\8\end{pmatrix}$.

$a = \begin{pmatrix}1\\3\\7\end{pmatrix}$ $\quad m = \begin{pmatrix}2\\5\\8\end{pmatrix}$

$r = a + \lambda m$

VECTOR FORM $\quad \boxed{r = \begin{pmatrix}1\\3\\7\end{pmatrix} + \lambda \begin{pmatrix}2\\5\\8\end{pmatrix}}$

$\begin{pmatrix}1\\3\\7\end{pmatrix} \downarrow$ Point $\quad \begin{pmatrix}2\\5\\8\end{pmatrix} \downarrow$ Direction vector.

Parameter

$y = 2x + 1$

$x = 2, \; y = 2(2)+1 = 5$
$(2,5)$

$x = 1, \; y = 2(1)+1 = 3$
$(1,3)$

IF WE WANT A POINT ON THIS LINE, PUT RANDOM VALUES FOR λ.

$\lambda = 1, \quad r = \begin{pmatrix}1\\3\\7\end{pmatrix} + (1)\begin{pmatrix}2\\5\\8\end{pmatrix} = \begin{pmatrix}3\\8\\15\end{pmatrix}$

$\lambda = 5, \quad r = \begin{pmatrix}1\\3\\7\end{pmatrix} + (5)\begin{pmatrix}2\\5\\8\end{pmatrix} = \begin{pmatrix}11\\28\\47\end{pmatrix}$

VECTOR FORM TO PARAMETRIC FORM

Substitute r with $\begin{pmatrix}x\\y\\z\end{pmatrix}$ and form 3 separate equations

VECTOR FORM $\quad \boxed{r = \begin{pmatrix}1\\3\\7\end{pmatrix} + \lambda \begin{pmatrix}2\\5\\8\end{pmatrix}}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 2\lambda \\ 5\lambda \\ 8\lambda \end{pmatrix}$$

PARAMETRIC FORM

$$\boxed{\begin{aligned} x &= \underbrace{1}_{\text{point}} \underbrace{+ 2}_{\text{direction vector}}\lambda \\ y &= 3 + 5\lambda \\ z &= 7 + 8\lambda \end{aligned}}$$

PARAMETRIC FORM TO **CARTESIAN FORM**

make λ subject from all equations and equate.

$$\lambda = \frac{x-1}{2}, \quad \lambda = \frac{y-3}{5}, \quad \lambda = \frac{z-7}{8}$$

Point

CARTESIAN FORM

$$\boxed{\frac{x-1}{\boxed{2}} = \frac{y-3}{\boxed{5}} = \frac{z-7}{\boxed{8}}}$$

Direction vector.

EQUATION OF A LINE IS ASKED IN TWO WAYS	
Given: 1) Direction Vector (m) 2) A point on line (a)	Given: Two points on the line.

EQUATION OF A LINE:-

(1) Point on line(a) and direction vector (m) is given.

(2) Two points are given.

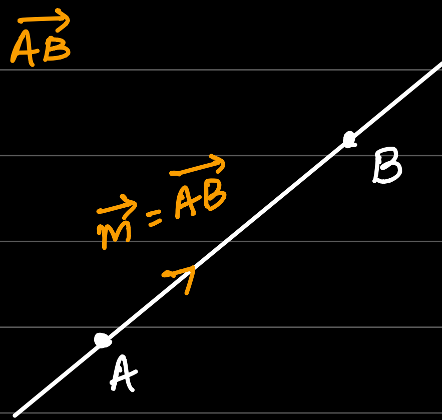
Q. Find equation of line that passes through A(1, 0, 5) and B(3, 7, 11).

STEP 1: Find the direction vector $\overrightarrow{AB}$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= \begin{pmatrix} 3 \\ 7 \\ 11 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$$

$$\vec{m} = \overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \\ 6 \end{pmatrix}$$



EQUATION:

$$r = a + \lambda m$$

Vector form

$$\begin{array}{cccc} & A & & \overrightarrow{AB} \\ r = & \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} & + \lambda & \begin{pmatrix} 2 \\ 7 \\ 6 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 2\lambda \\ 7\lambda \\ 6\lambda \end{pmatrix}$$

NOTE:

if $m = \overrightarrow{AB}$ then take point A.

if $m = \overrightarrow{BA}$, then take point B.

$$\boxed{\begin{aligned} x &= 1 + 2\lambda \\ y &= 7\lambda \\ z &= 5 + 6\lambda \end{aligned}}$$

Parametric form

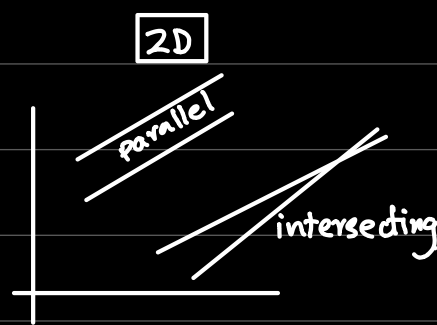
Make λ subject in all three equations.

$$\frac{x-1}{2} = \frac{y}{7} = \frac{z-5}{6}$$

Point

$$\frac{x-1}{2} = \frac{y-0}{7} = \frac{z-5}{6}$$

direction.



TWO LINES

PARALLEL

1) Their direction vectors are parallel

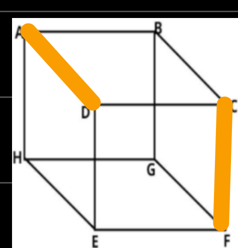
$m_1 = k\, m_2$

2) m_1 and m_2 are

NON-PARALLEL

INTERSECTING

SKEW.



multiples of each other.

1) Point of intersection
2) Angle of intersection.

not multiples so not parallel

Q: $r_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ (m_1) $\quad r_2 = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ (m_2)

STEP 1: BRING BOTH LINES TO PARAMETRIC FORM, EQUATE AND MAKE 3 EQUATIONS.

$x = 1 + \lambda \qquad x = 4 + 2t$

$y = 1 - \lambda \qquad y = 6 + 2t$

$z = 1 + 2\lambda \qquad z = 1 + t$

$1 + \lambda = 4 + 2t \qquad 1 - \lambda = 6 + 2t \qquad 1 + 2\lambda = 1 + t$

$\boxed{\lambda = 3 + 2t} \qquad \boxed{\lambda = -5 - 2t} \qquad \boxed{2\lambda = t}$

STEP 2: SOLVE ANY TWO OF THEM AND FIND VALUES OF λ AND t. IF THESE VALUES SATISFY THE THIRD EQUATION, THESE ARE INTERSECTING LINES. OTHERWISE, SKEW.

$\boxed{\lambda = 3 + 2t} \qquad \boxed{\lambda = -5 - 2t}$

$3 + 2t = -5 - 2t \qquad \lambda = -5 - 2(-2)$

CHECK

$\boxed{2\lambda = t}$

$2(-1) = (-2)$

$4t = -8$ $\qquad$ $\lambda = -5+4$ $\qquad$ $-2 = -2$

$\boxed{t = -2}$ $\qquad$ $\boxed{\lambda = -1}$ $\qquad$ $\boxed{\text{INTERSECTING}}$

POINT OF INTERSECTION:

$$r_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + (-1)\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \longrightarrow (0,2,-1)$$

$$r_2 = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + (-2)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$$

ANGLE OF INTERSECTION:

APPLY DOT PRODUCT ON DIRECTION VECTORS (m_1 & m_2)
(SCALAR)

$$r_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \qquad r_2 = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$m_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \qquad m_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$m_1 \cdot m_2 = |m_1||m_2| \cos\theta$$

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \sqrt{(1)^2+(-1)^2+2^2}\sqrt{2^2+2^2+1^2} \cos\theta$$

$$(1)(2) + (-1)(2) + (2)(1) = \sqrt{6}\sqrt{9} \cos\theta$$

$$2 = \sqrt{6}\sqrt{9} \cos\theta$$

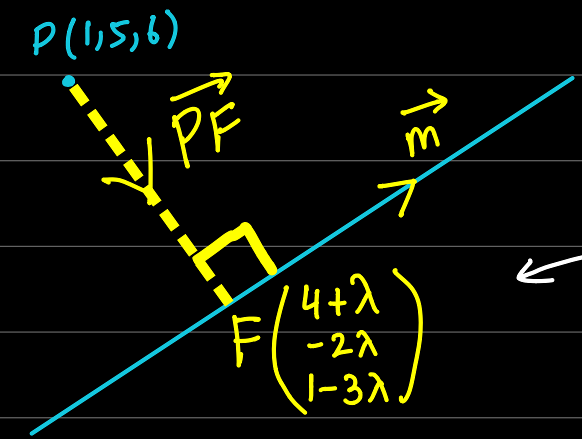
$$\cos\theta = \frac{2}{}$$

$3\sqrt{6}$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{6}}\right) = 74.21^\circ$$

FOOT OF PERPENDICULAR (5 Marks)

Find the shortest distance from point P to the line.



$$r = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$$

$$x = 4 + \lambda$$
$$y = 0 - 2\lambda$$
$$z = 1 - 3\lambda$$

STEP 1: Bring line to parametric form and use those as coordinates of F.

STEP 2: Find vector $\vec{PF}$

$$\vec{PF} = \vec{OF} - \vec{OP}$$

$$= \begin{pmatrix} 4+\lambda \\ -2\lambda \\ 1-3\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}$$

$$\vec{PF} = \begin{pmatrix} 3+\lambda \\ -5-2\lambda \\ -5-3\lambda \end{pmatrix}$$

STEP 3: $\vec{PF}$ and $\vec{m}$ are now perpendicular.

$$\vec{PF} \perp \vec{m}$$

$$\vec{PF} \cdot \vec{m} = 0$$

$$\begin{pmatrix} 3+\lambda \\ -5-2\lambda \\ -5-3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = 0$$

$$(1)(3+\lambda) + (-2)(-5-2\lambda) + (-3)(-5-3\lambda) = 0$$

$$3+\lambda + 10 + 4\lambda + 15 + 9\lambda = 0$$

$$28 + 14\lambda = 0$$

$$\boxed{\lambda = -2}$$

$\vec{OF}$, F

Coordinates of F $= \begin{pmatrix} 4+\lambda \\ -2\lambda \\ 1-3\lambda \end{pmatrix} = \begin{pmatrix} 4+(-2) \\ -2(-2) \\ 1-3(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$

P(1, 5, 6) F(2, 4, 7)

TO FIND DISTANCE WE HAVE TWO APPROACHES.

1) COORDINATE GEOMETRY

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(2-1)^2 + (4-5)^2 + (7-6)^2}$$

$$= \sqrt{3}$$

2) VECTORS

$$\vec{PF} = \begin{pmatrix} 3+\lambda \\ -5-2\lambda \\ -5-3\lambda \end{pmatrix} = \begin{pmatrix} 3+(-2) \\ -5-2(-2) \\ -5-3(-2) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Distance $= |\vec{PF}| = \sqrt{1^2 + (-1)^2 + 1^2}$

$= \sqrt{3}$