A12 POLYNOMIALS
ANSWERS
P3

1	Substitute 2 for x and equate to zero, or divide by $x - 2$ and equate remainder to zero	M1	
	Obtain answer $a = -3$	A1	2
(ii)	Attempt to find quadratic factor by division or inspection	M1	
	State quadratic factor $2x^2 + x + 2$	A1	2
	[The M1 is earned if division reaches a partial quotient of $2x^2 + kx$, or if inspection has an unknown factor of $2x^2 + bx + c$ and an equation in b and/or c, or if two coefficients with the correct moduli are stated without working.]		
(iii)	State answer $x > 2$ (and nothing else)	B1*	
	Make a correct justification e.g. $2x^2 + x + 2$ (has no zeros and) is always positive	B1(dep*)	2
	[SR: The answer $x \geq 2$ gets B0, but in this case allow the second B mark if the remaining work is correct.]		

9709/03/O/N/04

2 **(i)**

EITHER:	Attempt division by $x^2 - x + 3$ reaching a partial quotient $x^2 + x$	**B1**	
	Complete division and equate constant remainder to zero	**M1**	
	Obtain answer $a = -6$	**A1**	
OR:	Commence inspection and reach unknown factor of $x^2 + x + c$	**B1**	
	Obtain $3c = a$ and an equation in c	**M1**	
	Obtain answer $a = -6$	**A1**	
	State or obtain factor $x^2 + x - 2$	**B1**	
	State or obtain factors $x + 2$ and $x - 1$	**B1 + B1**	**6**
(ii)	State that $x^2 + x - 2 = 0$, has two (real) roots	**B1**	
	Show that $x^2 - x + 3 = 0$, has no (real) roots	**B1**	**2**

3 **(i)**	Substitute $x = -2$ and equate to zero, or divide by $x + 2$ and equate constant remainder to zero, or use a factor $Ax^2 + Bx + C$ and reach an equation in a	M1	
	Obtain answer $a = 4$	A1	2
(ii)	Attempt to find quadratic factor by division or inspection	M1	
	State or exhibit quadratic factor $x^2 - 2x + 2$	A1	2
	[The M1 is earned if division reaches a partial quotient $x^2 + kx$, or if inspection has an unknown factor $x^2 + bx + c$ and an equation in b and/or c, or if inspection without working states two coefficients with the correct moduli.]		

4	*EITHER*: Attempt division by x^2+x+2 reaching a partial quotient of x^2+kx	M1	
	Complete the division and obtain quotient x^2-x+2	A1	
	Equate constant remainder to zero and solve for a	M1	
	Obtain answer $a = 4$	A1	
	OR: Calling the unknown factor x^2+bx+c, obtain an equation in b and/or c, or state without working two coefficients with the correct moduli	M1	
	Obtain factor x^2-x+2	A1	
	Use $a = 2c$ to find a	M1	
	Obtain answer $a = 4$	A1	**[4]**

5	(**i**) *EITHER*: Attempt division by $2x^2-3x+3$ and state partial quotient $2x$	B1	
	Complete division and form an equation for a	M1	
	Obtain $a = 3$	A1	
	*OR*1: By inspection or using an unknown factor $bx + c$, obtain $b = 2$	B1	
	Complete the factorisation and obtain a	M1	
	Obtain $a = 3$	A1	
	*OR*2: Find a complex root of $2x^2-3x+3=0$ and substitute it in p(x)	M1	
	Equate a correct expression to zero	A1	
	Obtain $a = 3$	A1	
	*OR*3: Use $2x^2 \equiv 3x-3$ in p(x) at least once	B1	
	Reduce the expression to the form $a + c = 0$, or equivalent	M1	
	Obtain $a = 3$	A1	[3]

6	**(i)** Substitute $x = -2$, equate to zero and state a correct equation, e.g. $-16 + 4a - 2b - 4 = 0$	B1	
	Differentiate p(x), substitute $x = -2$ and equate to zero	M1	
	Obtain a correct equation, e.g. $24 - 4a + b = 0$	A1	
	Solve for a or for b	M1	
	Obtian $a = 7$ and $b = 4$	A1	[5]
	(ii) *EITHER*: State or imply $(x + 2)^2$ is a factor	B1	
	Attempt division by $(x + 2)^2$ reaching a quotient $2x + k$ or use inspection with unknown factor $cx + d$ reaching $c = 2$ or $d = -1$	M1	
	Obtain factorisation $(x + 2)^2 (2x - 1)$	A1	
	OR: Attempt division by $(x + 2)$	M1	
	Obtain quadratic factor $2x^2 + 3x - 2$	A1	
	Obtain factorisation $(x + 2)(x + 2)(2x - 1)$	A1	[3]
	[The M1 is earned if division reaches a partial quotient of $2x^2 + kx$, or if inspection has an unknown factor of $2x^2 + ex + f$ and an equation in e and/or f, or if two coefficients with the correct moduli are stated without working.]		

7 **(i)** Substitute $x = -\frac{1}{2}$, equate to zero and obtain a correct equation, e.g.

$-\frac{1}{4} + \frac{5}{4} - \frac{1}{2}a + b = 0$ B1

Substitute $x = -2$ and equate to 9 M1

Obtain a correct equation, e.g. $-16 + 20 - 2a + b = 9$ A1

Solve for a or for b M1

Obtain $a = -4$ and $b = -3$ A1 [5]

(ii) Attempt division by $2x + 1$ reaching a partial quotient of $x^2 + kx$ M1

Obtain quadratic factor $x^2 + 2x - 3$ A1

Obtain factorisation $(2x+1)(x+3)(x-1)$ A1 [3]

[The M1 is earned if inspection has an unknown factor of $x^2 + ex + f$ and an equation in e and/or f, or if two coefficients with the correct moduli are stated without working.]
[If linear factors are found by the factor theorem, give B1 + B1 for $(x - 1)$ and $(x + 3)$, and then B1 for the complete factorisation.]

 9709/32/M/J/10

8 **(i)** Verify that $-96 + 100 + 8 - 12 = 0$ B1

Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity M1

Obtain $12x^2 + x - 6$ A1

State $(x + 2)(4x + 3)(3x - 2)$ A1 [4]

[The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C.]

(ii) State $3^y = \frac{2}{3}$ and no other value B1

Use correct method for finding y from equation of form $3^y = k$, where $k > 0$ M1

Obtain –0.369 and no other value A1 [3]

 9709/31/M/J/11

9 **(i)** Substitute $x = \dfrac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g.

$\dfrac{1}{8}a + \dfrac{1}{4}b + \dfrac{5}{2} - 2 = 0$ B1

Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12 M1

Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$ A1

Solve for a or for b M1

Obtain $a = 2$ and $b = -3$ A1 [5]

(ii) Attempt division by $2x - 1$ reaching a partial quotient $\dfrac{1}{2}ax^2 + kx$ M1

Obtain quadratic factor $x^2 - x + 2$ A1 [2]

[The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B, or an unknown factor of $\dfrac{1}{2}ax^2 + Bx + C$ and an equation in B and/or C.]

10 **(i)** *EITHER*: Attempt division $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$		M1	
	Obtain quotient $x^2 + 4x + 3$	A1	
	Equate remainder of form lx to zero and solve for a, or equivalent	M1	
	Obtain answer $a = 1$	A1	
OR:	Substitute a complex zero of $x^2 - x + 1$ in p(x) and equate to zero	M1	
	Obtain a correct equation in a in any unsimplified form	A1	
	Expand terms, use $i^2 = -1$ and solve for a	M1	
	Obtain answer $a = 1$	A1	[4]

[SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B. The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.]

(ii)	State answer, e.g. $x = -3$	B1	
	State answer, e.g. $x = -1$ and no others	B1	[2]

11	**(i)**	Substitute $x = \frac{1}{2}$ and equate to zero or divide by $(2x - 1)$, reach $\frac{a}{2}x^2 + kx + ...$and equate remainder to zero or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B	M1	
		Obtain $a = 2$	A1	
		Attempt to find quadratic factor by division or inspection or equivalent	M1	
		Obtain $(2x - 1)(x^2 + 2)$	A1cwo	[4]
	(ii)	State or imply form $\frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$, following factors from part **(i)**	B1√	
		Use relevant method to find a constant	M1	
		Obtain $A = -4$, following factors from part **(i)**	A1√	
		Obtain $B = 2$	A1	
		Obtain C = 5	A1	

12 Carry out division or equivalent at least as far as two terms of quotient M1
Obtain quotient $2x-4$ A1
Obtain remainder 8 A1 [3]

13 **(i)** Substitute $x=-\frac{1}{3}$, or divide by $3x+1$, and obtain a correct equation,

e.g. $-\frac{1}{27}a-\frac{20}{9}-\frac{1}{3}+3=0$ B1
Solve for a an equation obtained by a valid method M1
Obtain $a = 12$ A1 [3]

(ii) Commence division by $3x+1$ reaching a partial quotient $\frac{1}{3}ax^2+kx$ M1

Obtain quadratic factor $4x^2-8x+3$ A1
Obtain factorisation $(3x+1)(2x-1)(2x-3)$ A1 [3]

[The M1 is earned if inspection reaches an unknown factor $\frac{1}{3}ax^2+Bx+C$ and an equation in B and/or C, or an unknown factor Ax^2+Bx+3 and an equation in A and/or B, or if two coefficients with the correct moduli are stated without working.]
[If linear factors are found by the factor theorem, give B1B1 for $(2x-1)$ and $(2x-3)$, and B1 for the complete factorisation.]
[Synthetic division giving $12x^2-24x+9$ as quadratic factor earns M1A1, but the final factorisation needs $(x+\frac{1}{3})$, or equivalent, in order to earn the second A1.]

[SR: If $x=\frac{1}{3}$ is used in substitution or synthetic division, give the M1 in part **(i)** but give M0 in part **(ii)**.]

14 **(i)** Substitute $x=-\frac{1}{2}$, or divide by $(2x+1)$, and obtain a correct equation, e.g. $a-2b+8=0$ B1

Substitute $x=\frac{1}{2}$ and equate to 1, or divide by $(2x-1)$ and equate constant remainder to 1 M1

Obtain a correct equation, e.g. $a+2b+12=0$ A1
Solve for a or for b M1
Obtain $a=-10$ and $b=-1$ A1 [5]

(ii) Divide by $2x^2-1$ and reach a quotient of the form $4x+k$ M1
Obtain quotient $4x-5$ A1
Obtain remainder $3x-2$ A1 [3]

15 **(i)** Use law for the logarithm for a product or quotient or exponentiation AND for a power — M1

Obtain $(4x-5)^2(x+1) = 27$ — B1

Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$ — A1 [3]

(ii) Obtain $x = 2$ is root or $(x-2)$ is a factor, or likewise with $x = -\frac{1}{4}$ — B1

Divide by $(x-2)$ to reach a quotient of the form $16x^2 + kx$ — M1

Obtain quotient $16x^2 + 8x + 1$ — A1

Obtain $(x-2)(4x+1)^2$ or $(x-2)$, $(4x+1)$, $(4x+1)$ — A1 [4]

(iii) State $x = 2$ only — A1 [1]

16 **(i)** Differentiate f(x) and obtain $f'(x) = (x-2)^2 g'(x) + 2(x-2)g(x)$ — B1

Conclude that $(x-2)$ is a factor of $f'(x)$ — B1 **2**

(ii) *EITHER:* Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$ — B1

Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero — M1*

Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$ — A1

OR1: Identify given polynomial with $(x-2)^2(x^3 + Ax^2 + Bx + C)$ and obtain an equation in a and/or b — M1*

Obtain a correct equation, e.g. $\frac{1}{4}a - 4(4+a) + 4 = 3$ — A1

Obtain a second correct equation, e.g. $-\frac{3}{4}a + 4(4+a) = b$ — A1

OR2: Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b — M1*

Obtain a correct equation, e.g. $29 + 8a + b + 0$ — A1

Obtain a second correct equation, e.g. $176 + 47a + 4b = 0$ — A1

Solve for a or for b — M1(dep*)

Obtain $a = -4$ and $b = 3$ — A1 **5**

17 Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$ — B1

Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21 — M1

Obtain a correct equation, e.g. $8a + 4b + 5 = 21$ — A1

Solve for a or for b — M1

Obtain $a = 12$ and $b = -20$ — A1 **[5]**

18	**(i)**	Either	Equate p(–1) or p(–2) to zero or divide by $(x+1)$ or $(x+2)$ and equate constant remainder to zero.	M*1	
			Obtain two equations $a-b=6$ and $4a-2b=34$ or equivalents	A1	
			Solve pair of equations for a or b	DM*1	
			Obtain $a=11$ and $b=5$	A1	
		Or	State or imply third factor is $4x-1$	B1	
			Carry out complete expansion of $(x+1)(x+2)(4x-1)$ or $(x+1)(x+2)(Cx+D)$	M1	
			Obtain $a=11$	A1	
			Obtain $b=5$	A1	[4]
	(ii)		Use division or equivalent and obtaining linear remainder	M1	
			Obtain quotient $4x+a$, following their value of a	A1√	
			Indicate remainder $x-13$	A1	[3]

19	**(i)**	Substitute $x=-1$, equate to zero and simplify at least as far as $-8+a-b-1=0$	**B1**	
		Substitute $x=-\frac{1}{2}$ and equate the result to 1	**M1**	
		Obtain a correct equation in any form, e.g. $-1+\frac{1}{4}a-\frac{1}{2}b-1=1$	**A1**	
		Solve for a or for b	**M1**	
		Obtain $a = 6$ and $b = -3$	**A1**	[5]
	(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2+kx$	**M1**	
		Obtain quadratic factor $8x^2-2x-1$	**A1**	
		Obtain factorisation $(x+1)(4x+1)(2x-1)$	**A1**	[3]

[The M1 is earned if inspection reaches an unknown factor $8x^2+Bx+C$ and an equation in B and/or C, or an unknown factor Ax^2+Bx-1 and an equation in A and/or B.]
[If linear factors are found by the factor theorem, give B1B1 for $(2x-1)$ and $(4x + 1)$, and B1 for the complete factorisation.]

20	**(i)**	Either	Substitute $x=-1$ and evaluate	**M1**	
			Obtain 0 and conclude $x+1$ is a factor	**A1**	
		Or	Divide by $x+1$ and obtain a constant remainder	**M1**	
			Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]

Question		
21	Commence division and reach a partial quotient $x^2 + kx$	**M1**
	Obtain quotient $x^2 - 2x + 5$	**A1**
	Obtain remainder $-12x + 5$	**A1**
		3

Question		
22	*EITHER:* Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	**M1**
	Obtain quotient $x^2 + 3x + 2$	**A1**
	Either Set remainder identically equal to zero and solve for a or for b, *or* multiply given divisor and found quotient and obtain a or b	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
	OR: Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in B and/or C	**M1**
	Obtain $B = 3$ and $A = 2$	**A1**
	Either Use equations to obtain a or b *or* multiply given divisor and found factor to obtain a or b	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
		5

uestion			
23	Commence division and reach partial quotient ${}^{2}\quad kx$	**M1**	
	Obtain correct quotient ${}^{2}\quad 2x-1$	**A1**	
	Set their linear remainder equal to $2x + 3$ and solve for a or for b	**M1**	Remainder = $a \quad 3\ x+\ b-1$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
	Alternative method for question 3		
	State ${}^{4}\quad 3x^3+ax+b=\left(x^2+x-1 \quad x^2+Ax+B \quad +2x+3\right.$ and form and solve two equations in A and B	**M1**	e.g. $3 \quad 1+ \quad$ and $0 \quad -1+ \quad +B$
	Obtain $A=2,\ B=-1$	**A1**	
	Form and solve equations for a or b	**M1**	e.g. $a \quad B-A+2, \quad b=-B+3$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
		5	

Question			
24	Substitute $x=-\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $\frac{6}{8}+\frac{1}{4}a-\frac{1}{2}b-2=0$	**B1**	
	Substitute $x = -2$ and equate result to -24	***M1**	
	Obtain a correct equation, e.g. $48+4a-2b-2=-24$	**A1**	
	Solve for a or for b	**DM1**	
	Obtain $a = 5$ and $b = -3$	**A1**	
		5	