A LEVEL (P6) NORMAL AND BINOMIAL DISTRIBUTION

MARK SCHEME

1 (i) $\bar{x} = 375.3$ $\sigma^2{}_{n-1} = 8.29$	B1 M1 A1 **3**	For correct mean (3.s.f) For legit method involving *n*-1, can be implied For correct answer
(ii) *p* = 0.19 or equiv. $0.19 \pm 2.055 \times \sqrt{\dfrac{0.19 \times 0.81}{200}}$ 0.133 < *p* < 0.247	B1 M1 B1 A1 **4**	For correct *p* For correct form $p \pm z \times \sqrt{\dfrac{pq}{n}}$ either/both sides For *z* = 2.054 or 2.055 For correct answer

2 (i) $c\int_0^5 t(25-t^2)\mathrm{d}t$ = 1 $c\left[\dfrac{25t^2}{2} - \dfrac{t^4}{4}\right]_0^5$ = 1 $c\left[\dfrac{625}{2} - \dfrac{625}{4}\right]$ = 1 $\Rightarrow c = \dfrac{4}{625}$	M1 A1 A1 **3**	For equating to 1 and a sensible attempt to integrate For correct integration and correct limits For given answer correctly obtained
(ii) $\int_2^4 ct(25-t^2)\mathrm{d}t = \left[\dfrac{25ct^2}{2} - \dfrac{ct^4}{4}\right]_2^4 = c[136] - c[46]$ = $\dfrac{72}{125}$ (0.576)	M1* M1*dep A1 **3**	For attempting to integrate f(*t*) between 2 and 4 (or attempt 2 and 4) For subtracting their value when t = 2 from their value when t = 4 For correct answer
(iii) $\int_0^5 ct^2(25-t^2)\mathrm{d}t$ = $\left[\dfrac{4}{625} \times \dfrac{25t^3}{3} - \dfrac{4}{625} \times \dfrac{t^5}{5}\right]_0^5$ = $\dfrac{8}{3}$	M1* A1 M1*dep A1 **4**	For attempting to integrate *t*f(*t*), no limits needed For correct integrand can have *c* (or their *c*) For subtracting their value when t=0 from their value when t=5 For correct answer

3 (i)	$z = 0.674$ or 0.675 allow 0.67 to 0.675	**B1**		For correct z, can be + or -
	$\frac{52-\mu}{5} = 0.674$	**M1**		For an equation relating 52, 5, μ and any $z \neq 0.5987$ or 0.7734 ish
	$\mu = 48.6$	**A1**		For correct answer
(ii)	$z_1 = \frac{40-48.63}{5} = -1.726$ $z_2 = \frac{46-48.63}{5} = 0.526$	**M1**		For standardising 40 or 46, 5 or $\sqrt{5}$ in denom or 5^2 with their mean, no cc
	prob = 0.9578 – 0.7005 = 0.2573	**M1**		For subtracting two probs consistent with their mean ie usually $\Phi_1 - \Phi_2$ or $(1 - \Phi_1) - (1 - \Phi_2)$ but could be of type $\Phi_1 - (1 - \Phi_2)$ if their mean is in between 40 and 46
	$(0.2573)^4$	**M1**		For raising their answer above to a power 4
	$= 0.00438$ or 4.38×10^{-3} accept 0.00449×10^{-3} NB 0.0045 gets A0 and RE #1	**A1 ft**	**4**	For correct answer

4 (i)	constant p, independent trials, fixed number of trials, only two outcomes	**B1**		For an option
		B1	**2**	For a second option
(ii)	$P(X<4) =$ $0.72^{14} + {}_{14}C_1 \times 0.28 \times 0.72^{13}$ $+ {}_{14}C_2 \times 0.28^2 \times 0.72^{12}$ $+ {}_{14}C_3 \times 0.28^3 \times 0.72^{11}$	**M1**		For adding with some C in P(0 + 1 + 2 + 3) or P(1 + 2 + 3) or P(0 + 1 + 2 + 3 + 4) or P(1 + 2 + 3 + 4)
		M1		For 0.28 and 0.72 to powers which sum to 14 Need 2 or more terms
	(= 0.0101 + 0.0548 + 0.1385 + 0.2154)	**A1**		For completely correct unsimplified form
	= 0.419	**A1**	**4**	For correct final answer **NB** 0.418 is A0 if PA # 1 or A1 if PA # 2

(iii) $\mu = 50 \times 0.28 (= 14)$	**B1**		For 14 and 10.08 seen, can be implied
$\sigma^2 = 50 \times 0.28 \times 0.72$ (=10.08)	**M1**		For standardising with or without cc, must have sq root
P(more than 18) = $1 - \Phi\left(\frac{18.5-14}{\sqrt{10.08}}\right)$	**M1**		For continuity correction 17.5 or 18.5 AND a final answer < 0.5
$= 1 - \Phi(1.417)$			
= 1 – 0.9218 or 0.9217			
= 0.0782 or 0.0783	**A1**	**4**	For correct answer
			NB 0.078 is A0 if RE # 1 or A1 if RE # 2

5 $\mu = 160$, $\sigma^2 = 96$	**B1**		For 160 and 96 seen or implied by 9.798
$P(\le 165) = \Phi\left(\frac{164.5-160}{\sqrt{96}}\right) = \Phi(0.4593)$	**M1**		For standardising, must have square root
	M1		For continuity correction, either 165.5 or 164.5
	M1		For using tables and finding correct area (i.e.> 0.5)
= 0.677	**A1**	**[5]**	For correct answer

6 (i) $z_1 = 0.02/0.15 = 0.1333$	**M1**		For standardising one value, no cc
$z_2 = -0.08/0.15 = -0.5333$	**M1**		For standardising the other value, no cc. SR ft on no sq rt
area= $\Phi(0.1333) - \Phi(-0.533)$ $= \Phi(0.1333) - [1 - \Phi(0.5333)]$ $= 0.5529 + 0.7029 - 1$ $= 0.256$	**M1** **A1**		For finding correct area (i.e. two Φ s - 1) For correct answer
Prob all 4 = $(0.256)^4$ (0.00428 to 0.00430)	**A1ft**	**[5]**	For correct answer, ft from their **(i)**, if $p<1$, allow 0.0043
(ii) $z = \pm 1.282$ or 1.28 or 1.281	**B1**		For correct z, + or - or both
$\pm 1.282 = \frac{b}{0.15}$	**M1**		For seeing an equation involving + or - of their z, b, 0.15 (their z can only be 0.842 or 0.84 or 0.841)
limits between 1.71 and 2.09	**A1ft**	**[3]**	both limits needed, ft 1.77 to 2.03 on 0.842 only

7 (i) $1.282 = (5130-\mu)/ 40.6$	B1		For $\pm$ 1.282 seen, or 1.28, 1.281, not 1.29 or 1.30
	M1		For standardising, with or without sq rt, squared, no cc
$\mu = 5080$ (5078) rounding to 5080	A1	**3**	For correct answer
(ii) $P(<5000) = \Phi[(5000-5078)/ 40.6]$	M1		For standardising, criteria as above, can include cc
$= \Phi(-1.921)$ $= 1 – 0.9727$	M1		For correct area found using tables ie < 0.5ft on wrong (i)
$= 0.0273$ or 2.73%	A1	**3**	For correct answer, accept 0.0274
(iii) $\mu = 60$, var = 54	B1		For 60 and 54 seen (could be sd or variance)
P(fewer than 65) $= \Phi\ (64.5 – 60) / \sqrt{54}$	M1		For using 64.5 or 65.5 in a standardising process
$= \Phi(0.6123)$	M1		For standardising, must have $\sqrt{(\text{ their } 54)}$ in denom
$= 0.730$ accept 0.73	A1	**4**	For correct answer

8 (i) $1.645 = \dfrac{50-38}{\sigma}$	B1		Using $z = +/-$ 1.645 or 1.65
	M1		Equation with 38, 50, σ and a recognisable z-value
$\sigma = 7.29$	A1	**3**	Correct answer
(ii) $z = \dfrac{30-38}{their\ \sigma}$ $(=-1.097)$	M1		Standardising, no cc
$P(z < 30) = 1 - \Phi\ (1.097)$ $= 1 - 0.8637$	M1		Finding correct area ie < 0.5
$= 0.136$	A1	**3**	Correct answer
(iii) $1-(0.95)^9$	B1		$(0.95)^9$ seen
$= 0.370$	B1	**2**	correct answer

9 (i) $1 - P(0, 1, 2)$	M1		For $1 - P(0, 1, 2)$
$= 1 - ((0.91)^{14} + (0.09)(0.91)^{13} \times {}_{14}C_1 +$	B1		Correct numerical expression for P(0) or P(1)
$(0.09)^2(0.91)^{12} \times {}_{14}C_2)$ $= 1 -(0.2670 + 0.3698 + 0.2377)$	B1		Correct numerical expression for P(2)
$= 0.126$	A1	**4**	Correct answer
(ii) $\mu = 200 \times 0.76 = 152$,	B1		For both mean and variance correct
$\sigma^2 = 200 \times 0.76 \times 0.24 = 36.48$ $P(X > 155)$	M1		For standardising , with or without cc, must have $\sqrt{\ }$ on denom
$= 1 - \Phi\left(\dfrac{155.5-152}{\sqrt{36.48}}\right) = 1 - \Phi(1.5795)$	M1		For use of continuity correction 154.5 or 155.5
	M1		For finding an area < 0.5 from their z
$= 1 - 0.7188 = 0.281$	A1	**5**	For answer rounding to 0.281

10 **(i)** heights, weights, times etc of something	B1	**1**	Any sensible set of data, must be qualified
(ii) $z = 0.64 = \dfrac{\mu - 10}{\sqrt{21}}$	B1 M1		z = ± 0.64 seen equation relating 10, $\sqrt{21}$, 21, μ and their z or 1 – their z, (must be a recognisable z value ie not 0.77)
μ = 12.9	A1	**3**	correct answer
(iii) $z = \dfrac{22 - 12.9}{\sqrt{21}}$ $= 1.986$	M1		standardising, with or without sq rt, no cc, must be their mean
$P(X > 22) = 1 - \Phi(1.986)$ $= 1 - 0.9765$ $= 0.0235$	M1ft		correct area ie < 0.5, ft on their mean >22
$300 \times 0.0235 = 7.05$	M1		mult by 300
answer = 7	A1	**4**	correct answer, accept 7 or 8 must be integer

11 **(i)** $(0.6)^{10} \times (0.4)^{10} \times {}_{20}C_{10}$	M1		3 term binomial expression involving ${}_{20}C_{\text{something}}$ and powers summing to 20
$= 0.117$	A1	**2**	Correct final answer
(ii) P(18, 19, 20)	M1		Summing three or 4 binomial expressions
$= (0.6)^{18}\,(0.4)^2\,{}_{20}C_2 + (0.6)^{19}(0.4)^1\,{}_{21}C_1 + (0.6)^{20}$	A1		One correct unsimplified expression allow 0.4 0.6 muddle
$= 0.003087 + 0.000487 + 0.00003635$ $= 0.00361$	A1		Correct answer
OR using normal approx N(12,4.8) $z = \dfrac{17.5 - 12}{\sqrt{4.8}}$	M1		Standardising, cc 16.5 or 17.5, their mean, $\sqrt{\ }$ (their var)
$= 2.51$	A1		2.51 seen
Prob = 1 – 0.9940 = 0.0060	A1	**3**	0.0060 seen must be 0.0060
(iii) $\mu = 150 \times 0.60 = 90$ $\sigma^2 = 150 \times 0.60 \times 0.40 = 36$	B1		For seeing 90 and 36
$P(88 < X < 97)$ $= \Phi\left(\dfrac{97.5 - 90}{6}\right) - \Phi\left(\dfrac{87.5 - 90}{6}\right)$	M1		For standardising , with or without cc, must have sq rt on denom
$= \Phi(1.25) - \Phi(-0.4166)$	M1		one continuity correction 97.5 or 96.5 or 87.5 or 88.5
$= 0.8944 - (1 - 0.6616)$	A1		0.8944 or 0.6616 or 0.3384 or 0.3944 or 0.1616 seen
$= 0.556$	M1		subtracting a probability from their standardised 97 prob
	A1	**6**	correct answer

12 (a)	$\frac{5.2-2s}{s}=-1.282$	M1	Equation with $\pm$correct LHS seen here or later, can be μ or s, no cc
		B1	±1.282 seen accept ±1.28 or anything in between
		M1	solving their equation with recognisable z-value and only 1 unknown occurring twice
	s = 7.24 or 7.23	A1 **4**	correct final answer
(b)	$\Phi\left(\frac{\mu+\sigma-\mu}{\sigma}\right)=0.8413$	B1	0.8413 (p) seen or implied (can use their own numbers)
	$P(\lvert z\rvert<1)=0.3413\times2=0.6826$	M1	finding the correct area i.e. $2p-1$
	$0.6826\times800=546$ (accept 547)	A1 **3**	correct answer, must be a positive integer
	OR *SR* $800\times2/3=533$ or 534	SR B1	for 2/3
		B1	for 533 or 534 or B2 if 533 or 534 and no working

13 (i)	$P(\geqslant3)=1-P(0, 1, 2)$	M1	For attempt at 1 – P(0, 1, 2) or 1 – P(0, 1, 2, 3) or P(3...15) or P(4...15)
	$=1-(6/7)^{15}-{}_{15}C_1\,(1/7)\,(6/7)^{14}-{}_{15}C_2\,(1/7)^2\,(6/7)^{13}$	M1	For 1 or more terms with 1/7 and 6/7 to powers which sum to 15 and ${}_{15}C_{\text{something}}$
	(= 1 – 0.0990 –0.2476 – 0.2889)	A1	Completely correct unsimplified form
	= 0.365 (accept 0.364)	A1 **4**	Correct final answer
(ii)	$\mu=56\times1/7$ (= 8) $\sigma^2=56\times1/7\times6/7$ (= 6.857)	B1	8 and 6.857 or 6.86 or 2.618 seen or implied
	P(more than 7) $=1-\Phi\left(\frac{7.5-8}{\sqrt{6.857}}\right)$	M1	Standardising attempt with or without cc, must have square root
	$=\Phi\left(\frac{8-7.5}{\sqrt{6.857}}\right)=\Phi(0.1909)$	M1	Continuity correction either 7.5 or 6.5
		M1	Final answer > 0.5 (award this if the long way is used and the final answer is > 0.5)
	= 0.576	A1 **5**	Correct final answer

14 (i) $z = \pm 1.68$	B1		Number rounding to 1.68 seen
$z = \dfrac{5.5-4.5}{\sigma}$	M1		Standardising and attempting to solve with their z, ; must be z value, no cc, no σ^2 ,no $\sqrt{\sigma}$
$\sigma = 0.595$ accept 25/42	A1	**3**	Correct answer
(ii) $z_1 = \dfrac{3.8-4.5}{0.5952}$ = -1.176	M1		For standardising 3.8 or 4.8, mean 4.5 not 5.5, their σ or $\sqrt{\sigma}$ or σ^2 in denom
$z_2 = \dfrac{4.8-4.5}{0.5952}$ = 0.504	A1ft		One correct z-value, ft on their σ
prob = $\Phi(0.504) - (1-\Phi(1.176))$ = 0.6929 – (1-0.8802)	M1		Correct area ie $\Phi_1 + \Phi_2 - 1$ or $\Phi_1 - \Phi_2$ if μ taken to be 5.5
= 0.573	A1	**4**	Correct answer only

15 (i) $P(X=5) = (0.65)^5 \times (0.35)^2 \times {}_7C_5$	M1		Expression with 3 terms, powers summing to 7 and a ${}_7C$ term
= 0.298 allow 0.2985	A1	**2**	Correct answer
(ii) $\mu = 50 \times 0.65$ (= 32.5), $\sigma^2 = 50 \times 0.65 \times 0.35$ (= 11.375)	B1		32.5 and 11.375 seen or implied
P(fewer than 29) = $\Phi\left(\dfrac{28.5-32.5}{\sqrt{11.375}}\right)$	M1		standardising, with or without cc, must have sq rt
	M1		for continuity correction 28.5 or 29.5
= 1 – Φ(1.186) = 1 – 0.8822	M1		correct area ie < 0.5 must be from a normal approx
= 0.118	A1	**5**	correct answer
(iii) $0.65\,n \geq 8$	M1		equality or inequality with *np* and 8
smallest n = 13	A1	**2**	correct answer

16 (i)	$-0.674 = \dfrac{7-\mu}{2.6}$	B1		± 0.674 seen only
		M1		Standardising must have a recognisable z-value, no cc and 2.6
		M1		For solving their equation with recognisable z-value, μ and 2.6 not 1 – 0.674 or 0.326, allow cc
	$\mu = 8.75$	A1	**4**	Correct answer
(ii)	$P(X > 6.2) = P\left(z > \dfrac{6.2-6.5}{2.6}\right)$	M1		Standardising, no cc on the 6.2
		M1		prob > 0.5
	= P(z > –0.1154) = 0.546	A1	**3**	Correct answer

17 (i)	$(0.05)(0.75)(0.15)$ $= 0.00563$ $(9 / 1600)$	M1 B1 A1 **3**	Multiplying 3 probs only, no Cs 0.05 or 0.15 or 1/5 × ¼ seen Correct answer
(ii)	P(at least 8) = P(8, 9, 10) $= {}_{10}C_8(0.75)^8(0.25)^2 + {}_{10}C_9(0.75)^9(0.25) + (0.75)^{10}$ $= 0.526$	B1 M1 A1 **3**	Binomial expression involving $(0.75)^r(0.25)^{10-r}$ and a C, $r \neq 0$ or 10 Correct unsimplified expression can be implied Correct answer
(iii)	$\mu = 90 \times 0.75 = 67.5$ $\sigma^2 = 90 \times 0.75 \times 0.25 = 16.875$ $P(X > 60)$ $= 1 - \Phi\left(\frac{60.5 - 67.5}{\sqrt{16.875}}\right) = \Phi(1.704)$ $= 0.956$	B1 M1 M1 M1 A1 **5**	90×0.75 (67.5) and $90 \times 0.75 \times 0.25$ (16.875 or 16.9) seen For standardising, with or without cc, must have $\sqrt{\ }$ on denom For use of continuity correction 60.5 or 59.5 For finding an area > 0.5 from their z For answer rounding to 0.956

18 mean $= 200 \times 0.08 = 16$ var = 14.72 $P(X \geq 15) = 1 - \Phi\left(\frac{14.5 - 16}{\sqrt{14.72}}\right)$ $= \Phi(0.391)$ $= 0.652$	B1 M1 M1 M1 A1 **[5]**	For both 16 and 14.7 seen For standardising, with or without cc, must have $\sqrt{\ }$ in denom For use of continuity correction 14.5 or 15.5 For finding a prob > 0.5 from their z, legit For answer rounding to 0.652 c.w.o
19 (i) $P(X > 0) = 1 - \Phi\left(\frac{0 - -15.1}{\sqrt{62}}\right)$ $= 1 - \Phi(1.918)$ $= 1 - 0.9724$ $= 0.0276$ or answer rounding to	M1 M1 A1 **[3]**	Standardising, sq rt, no cc Prob < 0.5 after use of normal tables Correct answer
(ii) $z = -1.22$ $-1.22 = \frac{0 - \mu}{\sqrt{40}}$ $\mu = 7.72$ c.a.o	B1 M1 A1 **[3]**	$z = \pm 1.22$ an equation in μ, recognisable z, $\sqrt{40}$, no cc correct answer c.w.o from same sign on both sides

20 **(i)**	$z = 0.674$ $\frac{1002-\mu}{8} = 0.674$ $\mu = 997$	B1 M1 A1 **[3]**	± 0.674 or rounding to, seen, e.g. 0.6743 Standardising and attempting to solve for μ, must use recognisable z-value, no cc, no sq rt, no sq Correct answer rounding to 997
(ii)	$P(2) = 3\times\frac{225}{900}\times\frac{224}{899}\times\frac{675}{898}$ $= 0.140$ OR $\frac{^{225}C_2\times{}^{675}C_1}{^{900}C_3}$	M1 A1 **[2]**	$900 \times 899 \times 898$ or $^{900}C_3$ seen in denom Correct answer not 0.141 or 0.14

21 **(i)**	$P(X < 3) = P(0) + P(1) + P(2)$ $= (0.84)^{11} + (0.16)(0.84)^{10} \times {}^{11}C_1 + (0.16)^2(0.84)^9 \times {}^{11}C_2$ $= 0.1469 + 0.30782 + 0.2931$ $= 0.748$	M1 M1 A1 **[3]**	Binomial term with $^{11}C_r p^r (1-p)^{11-r}$ seen Correct expression for P(0, 1, 2) or P(0, 1, 2, 3) Can have wrong p Correct final answer. Normal approx M0 M0 A0
(ii)	$\mu = 125 \times 0.64 = 80$ $\sigma^2 = 125 \times 0.64 \times 0.36 = 28.8$ $P(X > 73) = 1 - \Phi\left(\frac{73.5-80}{\sqrt{28.8}}\right)$ $= \Phi(1.211)$ $= 0.887$	B1 M1 M1 M1 A1 **[5]**	80 and 28.8 or 5.37 seen standardising, with or without cc, must have sq rt in denom continuity correction 73.5 or 72.5 only correct region (> 0.5 if mean > 73.5, vv if mean < 73.5 correct answer

22	$20p = 1.6 \quad p = 0.08$ $P(X > 2) = 1 - \{(0.92)^{20} + {}^{20}C_1(0.08)(0.92)^{19} + {}^{20}C_2(0.08)^2(0.92)^{18}\}$ $= 1 - (0.1887 + 0.3281 + 0.2711)$ $= 0.212$	M1 A1 M1 M1 A1 **[5]**	Equation relating $20p$ to the mean Correct p can be implied Bin expression involving $p^x(1-p)^{20-x}\,{}^{20}C_x$ any p Subtracting 2 or 3 binomial probs from 1, one of which is P(0) Correct answer

23 **(i)** $\dfrac{{}^4C_2 \times {}^7C_1}{{}^{11}C_3} = 0.255$	M1 M1 A1		Using 2 combs mult for numerator and 1 comb for denom Correct denom or num unsimplified Correct answer
OR $\dfrac{4}{11} \times \dfrac{3}{10} \times \dfrac{7}{9} \times 3$ $= 0.255$ (14/55) (42/165)	M1 M1 A1	**[3]**	Multiplying 3 correct probs Mult by 3 or Σ their 3 options Correct answer
(ii) P(3rd is orange) = P(*P, P, O*) + P(*P, O, O*) + P(*O, P, O*) + P(*O, O, O*)	M1		Summing four 3-factor options with or without replacement
$= \dfrac{4}{11} \times \dfrac{3}{10} \times \dfrac{7}{9} + \dfrac{4}{11} \times \dfrac{7}{10} \times \dfrac{6}{9} + \dfrac{7}{11} \times \dfrac{4}{10} \times \dfrac{6}{9} + \dfrac{7}{11} \times \dfrac{6}{10} \times \dfrac{5}{9}$	A1		At least 3 correct unsimplified options
$= \left[\dfrac{14}{165} + \dfrac{28}{165} + \dfrac{28}{165} + \dfrac{7}{33}\right]$ = 7/11 (0.636	A1		Correct answer. Award B3 if the correct answer is stated with no working.
OR using a tree diagram		**[3]**	
(iii) $P(P \mid O) = \dfrac{P(P \cap O)}{P(O)}$	M1		Substituting in cond prob formula with at least one 3-factor product in num, and denom their **(ii)** or 7/11
$= \dfrac{P(P,P,O) + P(P,O,O)}{P(O)}$	M1		Summing exactly 2 three-factor products in num
$= \dfrac{28/110}{7/11} = \dfrac{28}{70} = \dfrac{4}{10} = 0.4$	A1	**[3]**	Correct answer
(iv) $\mu = 121 \times \dfrac{4}{11} = 44$	B1		44 and 28 or 5.29 seen
$\sigma^2 = 121 \times \dfrac{4}{11} \times \dfrac{7}{11} = 28$	M1		Standardising, with or without cc, must have sq rt on denom
$P(X < 39) = \Phi\left(\dfrac{38.5 - 44}{\sqrt{28}}\right)$	M1		cc either 39.5 or 38.5
$= \Phi(-1.039)$ $= 1 - 0.8506$	M1		Correct area "$1 - \Phi$" seen
$= 0.149$	A1	**[5]**	Correct answer

24 **(i)** mean = 51	B1	**[1]**	
(ii) $z = \pm 0.674$	B1		Correct z
$\pm(63 - 51) / \sigma = 0.674$	M1		Standardising, no cc, no $\sqrt{\sigma}$, no σ^2
$\sigma = 17.8$	A1	**[3]**	Correct answer
25 P(total 7) = P(3,4 or 4,3) = 2/16 P(total 8) = P(4,4) = 1/16	M1		Attempt to find P(7) + P(8)
P(7 or more) = 3/16	A1		3/16 seen
Expected $200 \times \dfrac{3}{16} = 37.5$	M1 A1ft	**[4]**	Multiplying their prob by 200 Correct final answer ft their prob

26 (+/–) 1.045, (+/–) 0.313	B1, B1	1 correct z-value, the other correct z-value.
$20.9 - \mu = -0.313\,\sigma$ $30 - \mu = 1.045\,\sigma$	M1	Valid attempt to solve 2 equations relating to μ, σ, 30, 20.9. No $\sqrt{\sigma}$, σ^2
$\sigma = 6.70$ $\mu = 23.0$	A1 A1 **[5]**	correct answer correct answer

27 **(i)** $P(X = 2)) = (0.25)^2 \times (0.75)^6 \times {}^8C_2$	M1	3 term binomial expression involving ^8C something, powers summing to 8
$= 0.311$	A1 **[2]**	correct answer
(ii) $12 \times 0.25 = 3$, < 5 so not possible	B1 **[1]**	
(iii) mean $= 40 \times 0.25$ $(= 10)$ variance $= 40 \times 0.25 \times 0.75$ $(= 7.5)$	B1	40×0.25 and $40 \times 0.25 \times 0.75$ seen, o.e.
$P(X \text{ at least } 13) = P\left(z > \dfrac{12.5 - 10}{\sqrt{7.5}}\right)$	M1	standardising, ±, with or without cc, must have sq rt
$= P(z > 0.913)$	M1	continuity correction 12.5 or 13.5
$= 1 - \Phi(0.913)$	M1	correct area, i.e. < 0.5 legit
$= 1 - 0.8194$		
$= 0.181$	A1 **[5]**	correct answer

28 **(i)** $P(x > 10.9) = P(z > \dfrac{10.9 - 11}{0.095})$	M1	Standardising, no cc, no sq rt
$= P(z > -1.0526)$ $= 0.8538$ (0.854)	A1 **[2]**	Rounding to correct answer
(ii) P(at least 2 < 10.9) = 1 – P(0, 1)	M1	Bin expression with ∑ powers = 6, ^6C_x, $p + q = 1$.
$= 1 - (0.8538)^6 - {}^6C_1(0.1462)(0.8538)^5$	A1ft	Reasonably correct unsimplified expression ft their **(i)**
$= 0.215$	A1 **[3]**	Rounding to correct answer

29 **(i)** $P(X < 2\mu) = P\left(z < \dfrac{2\mu - \mu}{\sigma}\right)$	M1	Standardising, and attempt to get 1 variable, no cc, no $\sqrt{\ }$, no sq
$= P(z < \mu/\sigma) = P(z < 5/3)$	A1	±5/3 seen oe
$= 0.952$	A1 **[3]**	Rounding to correct answer

(ii) $P\left(X < \frac{\mu}{3}\right) = P\left(z < \frac{-2\mu}{3\sigma}\right)$	M1	standardising attempt resulting in $z \leqslant -$ some μ/σ allow $\pm\left(\frac{\mu/3-\mu}{\sigma}\right)$
$\frac{-2\mu}{3\sigma} = 1.047$	B1	± 1.047 seen
$\mu = -1.57\sigma$	A1 **[3]**	correct single number, answer must have a minus sign and $\mu = \ldots..\sigma$

30 $\frac{{}^{13}C_3 \times {}^{39}C_4}{{}^{52}C_7}$	M1	Using combinations with attempt to evaluate product of 2 in num and only 1 in denom
	M1	Correct numerator or denominator
$= 0.176$	A1	Correct answer
OR P(RRR) =	M1	**OR** Multiplying 3 unequal red probs with 4 unequal non-red probs
$\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{39}{49} \times \frac{38}{48} \times \frac{37}{47} \times \frac{36}{46} \times {}^7C_3$	M1	Multiplying a probability by ${}^7C_3$
$= 0.176$	A1 [3]	Correct answer

31 **(i)** $0.431 = \frac{135-\mu}{\sigma}$	B1	One $\pm z$-value correct, accept 0.430
	B1	A second $\pm z$-value correct
$-0.842 = \frac{127-\mu}{\sigma}$	M1	Solving two equations relating μ, σ, 135, 127 and their z-values (must be z-values)
$\sigma = 6.29$	A1	Correct answer accept 6.28
$\mu = 132$	A1 [5]	Correct answer
(ii) $P(X < 145) = P\left(z < \frac{145-132.3}{6.284}\right)$	M1	Standardising no sq rt no cc
$=P(z < 2.023)$	M1	Correct use of normal tables
$= 0.978$	A1 [3]	Answer rounding to 0.978 or 0.979
(iii) $p = 1/3$		
P(at least 2) = 1 – P(0, 1)	M1	Binomial expression with powers summing to 8 and ${}^8C_{\text{something}}$. (any p)
$= 1 - [\,(2/3)^8 + {}^8C_1 \times (1/3)^1 (2/3)^7\,]$	A1	Correct unsimplified expression
$= 0.805$	A1 [3]	Answer rounding to 0.805

32 mean = 200 × 2/15 (= 26.67) (80/3) variance = 200 × 2/15 × 13/15 (= 23.11)(208/9)	B1	mean and variance correct
$P(21 < X < 35) =$ $P\left(\frac{21.5-26.67}{\sqrt{23.11}}\right) < z < \frac{34.5-26.67}{\sqrt{23.11}}$	M1 M1	standardising, ±, with or without cc, must have sq rts continuity corrections 20.5 or 21.5, 34.5 or 35.5
$= P(-1.075 < z < 1.629)$ $= 0.8589 + 0.9483 - 1$ $= 0.807$	M1 A1 [5]	$\Phi_1 + \Phi_2 - 1$ answer rounding to 0.807
33 (i) $P(X > 20) = P(z > -6.4/3.7)$ $= P(z > -1.730)$ $= 0.9582$	M1 A1	Standardising no cc no sq rt Prob rounding to 0.958
Number of students = 335 or 336	A1ft [3]	Correct answer ft their prob, must be integer
(ii) P(very slow) = 0.05	B1	0.05 or 0.95 seen
$P(0, 1, 2) =$ $(0.95)^8 + {}^8C_1(0.05)^1(0.95)^7 + {}^8C_2(0.05)^2(0.95)^6$	M1 M1	Binomial term with ${}^8C_r p^r (1-p)^{8-r}$ seen any p Correct expression for P(0, 1, 2), p close to 0.05
$= 0.6634 + 0.2793 + 0.0515$ $= 0.994$	A1 [4]	Answer rounding to 0.994

34 (i) Zotoc: $z = \frac{367-320}{21.6} = 2.176$ Ganmor: $z = \frac{367-350}{7.5} = 2.267$	M1	Standardising either car's fuel, no cc, no sq, no √
P(Zotoc) = 0.985	A1	Correct answer
P(Ganmor) = 0.988	A1 [3]	Correct answer
(ii) $z = 0.23$	B1	± 0.23 seen
$0.23 = \frac{x-320}{21.6}$	M1	Standardising either car, no cc, no sq rt, no sq
$x = 324.968$	M1ind	320 + d – 320 i.e. just d on num
$d = 4.97$	A1 [4]	Correct answer, –4.97 gets A0

35	**(i)** constant/given prob, independent trials, fixed/given no. of trials, only two outcomes	B1 B1 [2]	One option correct Three options correct
	(ii) P(8, 9, 0, 1) = ${}^9C_8(0.3)^8(0.7) + (0.3)^9 + (0.7)^9 + {}^9C_1(0.3)(0.7)^8$ = 0.196	M1 A1 A1 [3]	One term seen involving $(0.3)^x(0.7)^{9-x}({}^9C_x)$ Correct unsimplified expression Correct answer
	(iii) mean = 90 × 0.3 = 27 var = 18.9 $P(X > 35) = 1 - \Phi\left(\frac{35.5 - 27}{\sqrt{18.9}}\right)$ = 1 – Φ(1.955) = 0.0253 $P(X < 27) = \Phi\left(\frac{26.5 - 27}{\sqrt{18.9}}\right) = 1 - \Phi(0.115)$ = 0.4542 Total prob = 0.480 accept 0.48	B1 M1 M1 M1 A1 [5]	Expressions for 27 and 18.9 (4.347) seen Standardising one expression, must have sq rt in denom, cc not necessary Continuity correction applied at least once $(1 - \Phi_1) + (1 - \Phi_2)$ accept (0.0329 + 0.5) if no cc Rounding to correct answer

36	Normal mean 60 kg, variance 90 kg^2	B1 B1 [2]	Any sensible values (mean 40–80 kg, variance 16–225 kg^2), could give s.d. 4–15 kg

37	**(i)** $z = 0.807$ $0.807 = \frac{10 - 8.2}{\sigma}$ s = 2.23	B1 M1 A1 [3]	0.807 seen standardising, must have σ, no sq rt, no cc and a z-value correct answer
	(ii) P(> 1 min from mean) = P(mod $z > \frac{1}{2.23}$) $= P(\lvert z\rvert > 0.4484)$ = (1 – 0.6729) × 2 = 0.654	M1 M1 A1 [3]	standardising, their sd, no cc and adding two areas using 1 – Φ(z) correct answer

(iii) P(> 2 longer) = 1 – P(0, 1, 2 longer)	M1	binomial term ${}^6C_x p^x(1-p)^{6-x}$
$= 1 - \{(0.79)^6 + {}^6C_1(0.21)(0.79)^5 + {}^6C_2(0.21)^2(0.79)^4\}$	A1	correct unsimplified answer
= 0.112	A1 [3]	correct answer
(iv) $\mu = 35 \times 0.5 = 17.5$ $\sigma^2 = 35 \times 0.5 \times 0.5 = 8.75$	B1	17.5 and 8.75 or $\sqrt{8.75}$ seen
$P(X < 16) = \Phi\left(\dfrac{15.5 - 17.5}{\sqrt{8.75}}\right)$	M1	standardising, with or without cc, must have sd in denom
	M1	continuity correction 15.5 or 16.5 only, seen
$= 1 - \Phi(0.676)$ $= 1 - 0.7505$	M1	using $1 - \Phi(z)$
= 0.2495 (0.249 or 0.250)	A1 [5]	correct answer
OR ${}^{35}C_0 0.5^0 0.5^{35} + {}^{35}C_1 0.5^1 0.5^{34} + {}^{35}C_2 0.5^2 0.5^{33} + \ldots$	M1	binomial term ${}^{35}C_x 0.5^x 0.5^{35-x}$
$= 8582372584/2^{35} = 0.250$	A1	at least 2 correct terms (x Þ 0) seen
	M1	summing 16 or 17 terms
	A1	correct expression
	A1	correct answer

38 $18p = 2.7$ $p = 0.15$	B1	Correct value for p
P(2, 3, 4) =	M1	Summing 3 binomial probs o.e
${}^{18}C_2 \times (0.15)^2(0.85)^{16} + {}^{18}C_3(0.15)^3(0.85)^{15} + {}^{18}C_4(0.15)^4(0.85)^{14}$	A1	Correct unsimplified answer
= 0.655	A1 **[4]**	Correct answer

39 (a) $z > \dfrac{2\mu - \mu}{\sigma} = \dfrac{\mu}{\sigma} = \dfrac{7\sigma^2}{3\sigma}$	M1	Standardising attempt resulting in $z >$ some μ/σ
$\dfrac{7\sigma}{3} = 1.272$	M1	Substituting to eliminate μ or σ
	B1	1.272 seen
$\sigma = 0.545$ $\mu = 0.693$	A1 **[4]**	Both answers correct
(b) $P(X < a + 33) = 0.75$	M1	Using 0.75 oe
$z = 0.674$	A1	± 0.674 seen
$\dfrac{a + 33 - 33}{\sqrt{21}} = 0.674$	M1	Standardising, no cc, must have sq rt
$a = 3.09$	A1 **[4]**	Correct answer

40 **(a)** **(i)** P(at least one 3) = 1 – P(no 3s) $= 1 - (5/6)^9$ = 0.806	 M1 A1 **[2]**	 Using 1 – none Correct answer
(ii) P(at least 1 three) = $1 - (5/6)^n$ $1 - (5/6)^n > 0.9$ $n > 12.6$ $n = 13$	B1 M1 M1 A1 **[4]**	 Equation or inequality involving n and 0.9 Solving attempt of sensible equation, can be trial Correct answer
(b) P(R wins his 1st ball) = P(GY) = 15/56 (0.268) P(R wins 2nd ball) = P(GGGY) = 3/28 P(R wins 3rd ball) = P(GGGGGY) $\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3} = 1/56$ P(R wins) = 11/28 (0.393)	M1 M1 M1 A1 **[4]**	Using P(GY) Attempt to find P(GGGY) or P(GGGGGY) Adding three options Correct answer

41 $20p = 4.8$ $p = 0.24$ or 4.8/20 $P(0, 1, 2) = (0.76)^{20} + {}^{20}C_1(0.24)^1(0.76)^{19}$ $+ {}^{20}C_2(0.25)^2(0.76)^{18}$ = 0.109 SR max 3 out of 4	B1 M1 A1 A1 **[4]** B1 M1 A1	Correct value for p Summing 2 or 3 binomial probs o.e., any p, n = 5 or 20 Correct unsimplified answer Correct answer As above Using N(4.8, 3.648) with cc 2.5 or 3.5 0.114 seen
42 **(i)** $np = 24$, $npq = 4.8$ $z = \pm\left(\frac{24.5-24}{\sqrt{4.8}}\right) = 0.228$ Prob = 0.590	B1 M1 M1 A1 **[4]**	24 and 4.8 or $\sqrt{4.8}$ seen can be unsimplified Standardising, need sq rt, cc not necessary Continuity correction 24.5 or 25.5 used Correct answer must be from 24.5
(ii) np and nq both > 5.	B1 **[1]**	Need both

43 **(i)** $z = -1.282$ $P(x < 20) = P\left(z < \frac{20-\mu}{0.8}\right)$ $-1.282 = \frac{20-\mu}{0.8}$ μ = 21.0 cm (21.0256)	B1 M1 A1 **[3]**	± 1.282 or ± 1.281 seen Standardising, no cc, must have 0.8, must be a z-value Correct answer

(ii) $P(21.5 < x < 22.5)$			
$= P\left(\frac{21.5-21.03}{0.8}\right) < z < \left(\frac{22.5-21.03}{0.8}\right)$	M1		2 attempts at standardising with their mean, must have 0.8 oe
$= \Phi(1.8375) - \Phi(0.5875)$	M1		Subtracting 2 Φs ft their mean
$= 0.9670 - 0.7217$			
$= 0.2453$	A1		Needn't be entirely accurate, rounding to 0.24 or 0.25
$P(< 2) = P(0) + P(1)$			
$= (0.7547)^4 + (0.2453)^1(0.7547)^3\,{}^4C_1$	M1		Binomial term with ${}^4C_r p^r (1-p)^{4-r}$ seen $r \neq 0$, any $p < 1$
	M1		Bin expression for $P(0) + P(1)$, any $p < 1$
$= 0.746$	A1	**[6]**	Accept 3sf rounding to 0.75

44 (i) $z = \pm 1.751$	B1		Correct z
$\pm \frac{20-\mu}{\mu/4} = 1.751$	M1		Standardising no cc, no sqrt, must be a z-value
$\mu = 13.9$	A1	**[3]**	Correct answer
(ii) $P(X < 10) = P(z < \pm\frac{10-13.91}{13.91/4})$	M1		Standardising attempt with 10, their μ and their $\mu/4$, no cc, no sqrt
$= P(z < -1.124)$	M1		"$\Phi_1 + \Phi_2 - 1$", ft their mean
$= 1 - 0.8694$			
$= 0.131$			
$P(10 < X < 20) = 0.96 - 0.131$			
$= 0.829$ or 0.830	A1	**[3]**	Correct answer
(iii) $\mu = 250 \times 0.96 = 240$	B1		240 and 9.6 or sq rt 9.6 seen unsimplified
$\sigma^2 = 250 \times 0.96 \times 0.04 = 9.6$			
$P(\geq 235) = 1 - \Phi\left(\pm\frac{234.5-240}{\sqrt{9.6}}\right)$	M1		Standardising, with or without cc, must have sq rt in denom
	M1		Continuity correction 234.5 or 235.5 only
$= \Phi(1.775)$	M1		Correct region > 0.5, ft their mean
$= 0.962$	A1	**[5]**	Correct answer

45 (i) $(0.75)^n < 0.06$	M1*		Equation or inequality with 0.75^n and 0.06 or 0.94 seen
$n > 9.78$	M1dep*		Attempt at solving by trial and error (can be implied) or using logarithms correctly
$n = 10$	A1	**[3]**	Correct answer
(ii) $E(X) = 14 \times 0.75$ or 10.5	M1		Evaluating binomial probability for an integer value directly above or below their mean
Try $P(10) = {}^{14}C_{10}(0.75)^{10}(0.25)^4 = 0.220$			
$P(11) = {}^{14}C_{11}(0.75)^{11}(0.25)^3 = 0.240$	M1		Evaluating the other binomial probability
(mode is) 11	A1	**[3]**	Correct answer
OR	M1		Evaluating binomial $P(n)$ and $P(n + 1)$
	M1		Evaluating binomial P(10), P(11) and P(12)
	A1		Correct answer

(iii) P(> 11) $= {}^{14}C_{12}(0.75)^{12}(0.25)^2 + {}^{14}C_{13}(0.75)^{13}(0.25)^1 + (0.75)^{14}$	M1 M1		A binomial term of the form ${}^{14}C_n p^n(1-p)^{14-n}$ seen, $n \neq 0$ or 14 Summing binomial P(12, 13, 14) or P(11, 12, 13, 14,)
= 0.281	A1		Correct answer 0.280 – 0.282
$P(3) = {}^5C_3\,(0.2811)^3(0.7189)^2$	M1		A binomial term of the form ${}^5C_3p^3(1-p)^2$ seen, any p
= 0.115	A1	**[5]**	Correct answer

46 $\mu = 250 \times 0.86 = 215$	B1		250 × 0.86 and 250 × 0.86 × 0.14 seen o.e
$\sigma^2 = 250 \times 0.86 \times 0.14 = 30.1$	M1		Standardising, with or without cc, must have sq rt in denom
$P(X > 210) = 1 - \Phi\left(\dfrac{210.5-215}{\sqrt{30.1}}\right)$	M1		Continuity correction 210.5 or 209.5 only
$= \Phi(0.820)$	M1		Correct region (> 0.5) ft their mean
= 0.794	A1	[5]	Correct answer

47 **(i)** $z = 1.882$ or 1.881	B1		±1.882 or ±1.881 seen
$1.882 = (32 - 20) / \sigma$	M1		Equation using their z (must be a z-value) 32, 20 and s
$\sigma = 6.38$	A1	[3]	Correct answer
(ii) $P(x > 13) = P\left(z > \dfrac{13-20}{6.376}\right)$	M1		Standardising
$= P(z > -1.0978)$	M1		Correct area > 0.5
= 0.864	A1	[3]	Correct answer
(iii) P(at least 2) = 1 – P(0, 1)	M1		Using 0.03 and 0.97 or 0.06 and 0.94 in a binomial expression powers summing to 7
$= 1 - (0.97)^7 - (0.03)(0.97)^6\,{}_7C_1$	M1		Correct unsimplified binomial expansion
= 0.0171	A1	[3]	Correct answer

48 **(i)** $4p + p + 3p = 1$ so P(blue) = 1/8 AG	B1	[1]	Must show something
(ii) $P(R) = ½, P(B) = 1/8, P(G) = 3/8$ P(all different) = ½ × 1/8 × 3/8 × 3! =9/64 (0.141)	M1 M1 A1	[3]	Multiplying P (R, B, G) together Mult by 3! Correct answer
(iii) mean = 136 × 1/8 = 17, var = 14.875 $P(<20) = P\left(z < \dfrac{19.5-17}{\sqrt{14.875}}\right)$ $= \Phi(0.648)$ = 0.742	B1 M1 M1 M1 A1	[5]	Unsimplified mean and variance correct Standardising, need sq rt Cont correction 19.5 or 20.5 Correct area, > 0.5 legit Correct answer

49 **(i)** $z_1 = \dfrac{12-8}{\sqrt{24}} = 0.816$ $\Phi_1(0.816) = 0.7926$	M1		Standardising any one, no sq rt no cc
$z_2 = \dfrac{7-8}{\sqrt{24}} = -0.204$ $\Phi_2(-0.204) = 1 - 0.5808$	M1		Correct area $\Phi_1 + \Phi_2 - 1$
Prob = 0.7926 – (1 – 0.5808) = 0.373	A1	[3]	Correct answer
(ii) $z = \dfrac{0-\mu}{2\mu} = -0.5$	M1		Standardising, no cc no sq rt, one variable
$P(z < -0.5) = 1 - 0.6915$ = 0.309 or 30.9%	A1	[2]	Correct answer oe
(iii) $z = \dfrac{3\mu-\mu}{2\mu} = 1$	M1		Standardising and eliminating μ
$P(z > 1) = 1 - 0.8413 = 0.1587$	M1		Subt from 1 and multiplying by 70
$70 \times 0.1587 = 11.1$	A1	[3]	Correct answer accept 11 or 12
(iv) $z = 1.45$	B1		± 1.45 seen
$1.45 = \dfrac{6-\mu}{2\mu}$	M1		Solving for μ with 6, 2μ, μ and their z
$\mu = 1.54$	A1	[3]	Correct answer

50 **(i)** $z = 0.38$	B1		± 0.38 (0) seen or implied
$\pm\dfrac{25-\mu}{\mu/3} = 0.38$	M1		Standardising attempt resulting in z = some μ/σ/both, no continuity correction
	M1		Substituting to eliminate μ or σ and attempt to solve linear equation
$\mu = 22.2$, $\sigma = 7.40$	A1	[4]	Both correct
(ii) $P(4) = {}^6C_4(0.352)^4(0.648)^2$	M1		${}^6C_r \times (p)^r \times (1-p)^{6-r}$, r = 2 or 4
= 0.0967	A1	[2]	Correct answer

51 **(i)** $P(O \text{ given } +) = \dfrac{0.37}{0.83}(0.4458)$	B1 A1		0.83 seen or implied Attempt to find P(O given +) using conditional probability fraction
	M1		Binomial term ${}^9C_rp^r(1-p)^{9-r}$, $r \neq 0$ or 9
$P(0, 1, 2) = (0.4458)^0(0.5542)^9 + {}^9C_1(0.4458)^1(0.5542)^8 + {}^9C_2(0.4458)^2(0.5542)^7$	M1		Binomial expression P(0, 1, 2) or P(0, 1, 2, 3) powers summing to 9 any $0 < p < 1$
	A1		Correct unsimplified expression
= 0.156	A1	[6]	Correct final answer
(ii) $\mu = 150 \times 0.35 = 52.5$,	B1		150 × 0.35 (52.5) and 150 × 0.35 × 0.65 (34.125) seen
$\sigma^2 = 150 \times 0.35 \times 0.65 = 34.125$	M1 M1		Standardising, using sd not variance Using continuity correction, 59.5 or 60.5
$P(> 60.5) = P\left(z > \pm\dfrac{60.5-52.5}{\sqrt{34.125}}\right)$	M1		correct area (< 0.5, for mean < their 60)
$= 1 - \Phi(1.369)$			
= 0.0854 or 0.0855	A1	[5]	correct value

52 $z_1 = \frac{30-28.3}{\sqrt{4.5}} = 0.8014$	M1	Standardising at least one value, sq rt.ess; no cc
$z_2 = \frac{25-28.3}{\sqrt{4.5}} = -1.5556$	M1	$\Phi_1 + \Phi_2 - 1$ oe
$\Phi_1 - (1 - \Phi_2) = 0.7884 + 0.9401 - 1$		
$= 0.729$	A1 **[3]**	Correct answer

53 **(i)** $-1.253 = \frac{6-\mu}{\sigma}$	B1	$Z = \pm\, 1.253$
	B1	$Z = \pm\, 0.648$
$0.648 = \frac{12-\mu}{\sigma}$	M1	Any equation with μ and σ and a reasonable z value not a prob. Allow cc or –, not √σ or σ^2
$\mu = 9.9$ $\sigma = 3.15$ or 3.16	M1 [Indpt] A1 **[5]**	Att. to solve by substitution or elimination
(ii) need $P(z < -1$ or $z > 1)$ $= 1 - \Phi 1) + \Phi(-1)$ $= 2 - 2 \times 0.8413$ $= 0.3174$ number = 317	B1 M1 M1 A1 **[4]**	$z = 1$ or -1 seen Correct area i.e. $2 - 2\mathbf{\Phi}$ Mult their prob if sensible, by 1000 Accept 317, 317.4, 318

54 **(i)** $P(2 < X < 12) = 1 - P(0, 1, 2, 12)$	M1	Using binomial with ${}_{12}C_{\text{something}}$ and powers summing to 12, $\Sigma p = 1$
$= 1 - (0.35)^{12} - (0.65)(0.35)^{11}\,{}_{12}C_1 - (0.65)^2(0.35)^{10}\,{}_{12}C_2 - (0.65)^{12}$	A1	Correct unsimplified answer
$= 1 - 0.0065359$		
$= 0.993$	A1 [3]	Accept 0.994 from correct working only
(ii) $1 - (0.87)^n > 0.95$	M1	Equality or inequality in (0.87 or 0.78 or 0.35), power n or $n - 1$, 0.95 or 0.05
$0.05 > (0.87)^n$	M1	Attempt to solve an equation with a power in (can be implied)
$n = 22$	A1 [3]	Correct answer

55 **(i)** $P(>42) = P\left(z > \frac{42 - 41.1}{3.4}\right)$	M1		Standardising no cc no sq rt no sq
$= P(z > 0.2647)$			
$= 1 - 0.6045$			
$= 0.3955$	A1		Correct prob rounding to 0.395 or 0.396
$\text{Prob} = (0.3955)(0.6045)^2\,{}_3C_1$	M1		Binomial ${}_3C_x$ powers summing to 3, any p, $\Sigma p = 1$
$= 0.433$ or 0.434	A1	[4]	Rounding to correct answer
(ii) $-1.282 = \frac{26.5 - \mu}{\sigma}$	B1 B1		± 1.282 seen ± 1.645 seen
$1.645 = \frac{34.6 - \mu}{\sigma}$	M1		An eqn with a z-value, μ and σ, no $\sqrt{\sigma}$ no σ^2
	M1		Sensible attempt to eliminate μ or σ by substitution or subtraction
$\mu = 30.0$ $\sigma = 2.77$	A1	[5]	Correct answers, accept 30.1, accept 30, rounding to 2.77
(iii) $P(B6 < 34.6) = P\left(z < \frac{34.6 - 41.1}{3.4}\right)$	M1		Standardising for B6 no cc no sq rt no sq
$= P(z < -1.912) = 1 - 0.9720$			
$= 0.0280$	A1		Correct answer rounding to
$P(B5 < 34.6) = 0.95$	M1		Mult by 0.95 or their regurgitated 0.95
$P(\text{both} < 34.6) = 0.028 \times 0.95$			
$= 0.0266$	A1	[4]	Correct answer rounding to 0.027, accept 0.027

56 **(i)** $\frac{34.1 - \mu}{\sigma} = 1.751$ $\frac{26.7 - \mu}{\sigma} = -0.524$ $\mu = 28.4$, $\sigma = 3.25$	B1 B1 M1 M1 A1	[5]	± 1.751 seen ± 0.524 seen a standardising equation with a z-value, μ and σ valid attempt to eliminate μ or σ correct answers
(ii) $\Phi\left(\frac{34.5 - 32.}{2.94}\right) - \Phi\left(\frac{33.5 - 32.}{2.94}\right)$	M1		one numerical standardising expression, no cc, no square root, can have 34
$= \Phi(0.667) - \Phi(0.25) = 0.7477 - 0.5987$ $= 0.149$	M1 A1	[3]	subtracting two areas correct answer
(iii) $\Phi\left(\frac{t - 32.4}{2.94}\right) - \Phi\left(\frac{31.8 - 32.4}{2.94}\right) = 0.5$	M1		using 2 standardising expressions to give an equation involving subtraction and 0.5, oe
$\Phi\left(\frac{t - 32.4}{2.94}\right) - (1 - 0.6765) = 0.5$			
$\Phi\left(\frac{t - 32.4}{2.94}\right) = 0.8235$	M1		adding their tail to 0.5 oe
$\frac{t - 32.4}{2.94} = 0.929$	M1		solving a standardised equation, must be a z-value from their 0.8235
$t = 35.1$	A1	[4]	correct final answer

57	**(i)**	$z = -1.036 = \dfrac{73-75}{\sigma}$ $\sigma = 1.93$	B1 M1 A1 [3]	$\pm$ correct z value accept ± 1.037 Equation with 73, 75, σ and a z value Rounding to correct answer
	(ii)	$P(>77) = 0.15$ $P(<3) = P(0, 1, 2)$ $= (0.85)^8 + {}_8C_1(0.15)(0.85)^7 + {}_8C_2(0.15)^2(0.85)^6$ $= 0.895$	M1 M1 A1 [3]	Prob rounding to 0.15 and 0.85 ${}_8C_x p^x(1-p)^{8-x}$ seen any p, $0<p<1$ Correct answer

58	**(i)**	$p = 0.2$ $\mu = 96 \times 0.2 = 19.2 \quad \sigma^2 = 96 \times 0.2 \times 0.8 = 15.36$ $P(<20) = P(z < \dfrac{19.5-19.2}{\sqrt{15.36}}) = P(z < 0.07654)$ $= 0.531$	B1 M1 M1 M1 A1 [5]	96×0.2 and $96 \times 0.2 \times 0.8$ seen standardising must have sq rt continuity correction either 19.5 or 20.5 correct area (> 0.5) correct value
	(ii)	$P(OT\mid B) = \dfrac{0.2 \times 0.6}{0.05 \times 0.3 + 0.2 \times 0.6 + 0.75}$ $= \dfrac{0.12}{0.885}$ $= 0.136$ (8/59)	B1 M1 A1 A1 [4]	their $0.2 \times (0.6$ or $0.4)$ as numerator of a fraction attempt at P(B) or P(NB) anywhere involving sum of 2 or 3 products correct unsimplified num or denom of a fraction correct answer

59	**(i)**	$z_1 = \dfrac{12-6.4}{5.2} = 1.077$ $z_2 = \dfrac{10-6.4}{5.2} = 0.692$ $\Phi(z_1) - \Phi(z_2) = 0.8593 - 0.7556$ $= 0.104$	M1 M1 A1	 [3]	Standardising, can be all in thousands, no mix, no cc no sq rt no sq $\Phi_2 - \Phi_1$, Φ_2 must be $> \Phi_1$ Correct answer
	(ii)	P(loss) $= P(z < \dfrac{0-6.4}{5.2}) = P(z < -1.231)$ $= 1 - 0.8909$ $= 0.109$ P(1) $= (0.1091)^1(0.8909)^3 \times 4C1$ $= 0.309$ or 0.308	M1 A1 M1 A1	 [4]	Standardising using $x = 0$, accept $\dfrac{0.5-6.4}{5.2}$ Correct prob Binomial term ${}_4C_x p^x(1-p)^{4-x}$ any p $x \neq 0$ Correct answer

60 **(i)** z = 1.036 or 1.037 $1.036 = \frac{5-4s}{s}$ $s = 0.993$ $\mu = 3.97$	B1 B1 M1 A1	[4]	± 1.036 or ± 1.037 seen $\frac{5-4\sigma}{\sigma}$ seen or $\frac{5-\mu}{\mu/4}$ oe One variable and sensible solving attempt z-value not nec Both answers correct
(ii) p = 0.85 $\mu = 200 \times 0.85 = 170$, var = $200 \times 0.85 \times 0.15 = 25.5$ P(at least 160) = $P\left(z > \frac{159.5-170}{\sqrt{25.5}}\right)$ = P(z > -2.079) = 0.981	B1 M1 M1 M1 A1	[5]	200×0.85 (170) and $200 \times 0.85 \times 0.15$ (25.5) seen Standardising, sq rt and must have used 200 continuity correction 159.5 or 160.5 correct area (> 0.5) must have used 200 correct value

61 $z = -1.036 = \frac{5.6-93}{\sigma}$ $\sigma = 3.57$	B1 M1 A1	**3**	± (1.036 to 1.037) seen Equation with 5.6 or 13.0, 9.3, σ and a *z* value, no cc Correct final answer

62 **(i)** $\Phi\left(\frac{84.5-82}{\sqrt{126}}\right) \Phi\left[\frac{83.5-82}{\sqrt{126}}\right]$ $= \Phi(0.2227) - \Phi(0.1336)$ $= 0.5883 - 0.5533$ $= 0.0350$	M1 M1 A1	**3**	Standardising using 83.5 or 84.5, must have square root Subtracting two probabilities, both > 0.5 or both < 0.5 Correct answer
(ii) $P(x > 87) = 1 - \Phi\left(\frac{87-82}{\sqrt{126}}\right) = 1 - \Phi(0.445)$ $= 1 - 0.6718 = 0.3282$ P(0, 1) $= (0.6718)^5 + {}_5C_1(0.3282)(0.6718)^4$ $= 0.471$	M1 A1 M1 A1	**4**	Standardising, no cc, must have square root Correct probability Any binomial term of form ${}_5C_x p^x(1-p)^{5-x}, x \neq 0$ Correct answer
(iii) $P(x < 87) = 0.6718$ $P(x < k) = 0.9718$ $z = 1.908$ or 1.909 $1.909 = \pm\frac{k-82}{\sqrt{126}}$ $k = 103$	M1 M1 A1 M1 A1	**5**	Finding $P(x < 87)$, value > 0.5 Adding 0.3 to their 0.6718 or equivalent Correct z Equation with *k*, 82 or 81.5 or 82.5, $\sqrt{126}$, and a *z*-value Correct answer rounding to 103

63	$np = 350 \times 1/7$ (= 50) $npq = 350 \times 1/7 \times 6/7$ (= 42.857)	B1 M1		Correct unsimplified *np* and *npq* standardising, with or without cc, must have sq rt
	$P(x \quad 47) = P\left(z > \frac{46.5-50}{\sqrt{42.857}}\right) =$	M1 M1		continuity correction 46.5 or 47.5 correct area ie > 0.5 must be a Φ
	$P(z > -0.5346)$ = 0.704	A1	**[5]**	correct answer

64 (a)	$P(y < 0) = P\left(z < \frac{0-\mu}{\mu/2}\right)$	M1		Standardising containing 0 (can be implied) and μ only
	$= P(z < -2)$	A1		$z < -2$ seen
	$= 1 - 0.9772 = 0.0228$	A1	**[3]**	Correct answer
(b)	$P(x > 2.1) = 253/8000 = 0.031625$ $P(x < 2.1) = 0.968375 = \Phi(z)$	M1		1 – their 253/8000 used to obtain a *z*-value
	$z = 1.857$ or 1.858 or $1.859 = \frac{2.1-2.04}{\sigma}$	A1		Rounded to 1.86 seen
	$\sigma = 0.0323$	M1		Solving for σ using their *z* val must be a *z* val
		A1	**[4]**	Correct answer

65 (i)	$X \sim \text{Bin}(12, 0.2)$	B1 B1 B1	**[3]**	Bin or B 12 0.2 or 1/5
(ii)	$P(X = 3, 4, 5) = 0.2^3 0.8^9 {}_{12}C_3 + 0.2^4 0.8^8 {}_{12}C_4 + 0.2^5 0.8^7 {}_{12}C_5$	M1		Bin exprerssion with any p
	$= 0.23622 + 0.13287 + 0.05315$ $= 0.422$	A1ft A1	**[3]**	Correct unsimplified expression, their p Correct answer
(iii)	$P(X = 0) < 0.01$	M1	**[3]**	Statement involving P(X = 0) and 0.01 can be implied
	$0.8^n < 0.01$ $n = 21$	M1 A1		Equn involving '0.8', 0.01 or 0.99 Correct answer

66	$z = 1.452$	B1		Rounding to ± 1.45
	$1.452 = \frac{20-\mu}{\mu/5}$	B1		$\frac{20-\mu}{\mu/5}$ or $\frac{20-5\sigma}{\sigma}$ seen oe
	$\mu = 15.5$	B1	**[3]**	rounding to correct answer

67 (i)	P($x < 440$) $= P\left(z < \dfrac{440-445}{3.6}\right) = 1 - \Phi(1.389)$ $= 1 - 0.9176$	M1 M1		Standardising no cc no sq or sq rt Correct area $(1 - \Phi)$ oe (indep)
	Ans = 0.0824	A1	**[3]**	Rounding to correct answer accept 0.0825
(ii)	$z = 1.881$	M1		±1.88 or 1.881 or 1.882 or 1.555 seen±
	$\dfrac{c}{3.6} = 1.881$	M1		Equation with ± $c/3.6$ or $2c/3.6$ only = z or prob (can be implied)
	$c = 6.77$	A1	**[3]**	Correct answer accept 6.78
68 (i)	$p = 4/9$ or $5/9$ P(at least 2) = 1 – P(0, 1) $= 1 - (5/9)^5 - (4/9)(5/9)^4\ {}_5C_1$	B1 M1		Binomial term ${}_5C_x p^x(1-p)^{5-x}$ seen
	= 0.735	A1	**[3]**	Correct answer
(ii)	$np = 96$ $npq = 32$ $p = P(\le k)$	M1		Using $np = 96$ $npq = 32$ to obtain eqn in 1 variable
	$p = 2/3$ $q = 1/3$ $n = 144$ $k = 6$	A1 A1ft		1/3 or 2/3 seen or implied Correct k ft $k = 9p$
	$n = 144$	A1	**[4]**	correct n

69 (i)	P(tall) $= P\left(z > \dfrac{70-50}{16}\right) = P(z > 1.25)$	M1		+ve/-ve Standardising no cc no sq rt no sq
	$= 1 - 0.8944$ $= 0.106$	A1	**[2]**	Correct answer
(ii)	P(short) = (1 – 0.1056)/3	M1		Subt their **(i)** from 1 or their **(i)** and multiplying by $\frac{1}{3}$ or $\frac{2}{3}$
	= 0.2981	A1 ft		Rounding to 0.298, only ft for $\dfrac{(1-\textbf{(i)})}{3}$
	$z = -0.53$	A1		± z-value rounding to 0.53, condone ±0.24
	$-0.53 = \dfrac{x-50}{16}$	M1		Standardising with their z value (not a probability), no cc sq rt etc.
	$x = 41.5$	A1	**[5]**	Correct answer

70 **(i)** $(0.8)^n < 0.001$	M1	Eqn or inequ involving 0.8^n or 0.2^n and 0.001 or 0.999
$n > 30.9$	M1	Trial and error or logs (can be implied)
$n = 31$	A1 **[3]**	Correct answer **MR** 0.01, max available M1M1A0
(ii) $\mu = 120 \times 0.2 = 24$	B1	24 and 19.2 or $\sqrt{19.2}$ seen
$\sigma^2 = 120 \times 0.2 \times 0.8 = 19.2$	M1	Standardising with or without cc, must have sq rt in denom
$P(x < 33) = P \times \left(z < \dfrac{32.5 - 24}{\sqrt{19.2}}\right)$	M1	Continuity correction 32.5 or 33.5
$= P(z < 1.9398)$ $= 0.974$	A1 **[4]**	Correct answer

71 **(i)** $z = -1.406$	B1	Rounding to ± 1.41 seen
$\dfrac{c - 14.2}{3.6} = -1.406$	M1	Standardising allow sq rt no cc
$c = 9.14$	A1 **3**	Correct answer
(ii) $P\left(\dfrac{15 - 14.2}{3.6}\right) < z < \left(\dfrac{16 - 14.2}{3.6}\right)$	M1	2 attempts at standardising no cc no sq rt
$= \Phi(0.5) - \Phi(0.222)$	M1	Subt two Φs (indep mark)
$= 0.6915 - 0.5879$ $= 0.1036$	A1	Needn't be entirely accurate, rounding to 0.10
P(at least 2) = 1 – P(0, 1) $= 1 - (0.8964)^7 - (0.8964)^6(0.1036)_7C_1$	M1	Binomial term with ${}_7C_r p^r(1-p)^{7-r}$ seen $r \neq 0$ any $p < 1$
$= 1 - 0.8413$	M1	1 – P(0), 1 – P(1), 1 – P(0, 1) seen their p
$= 0.159$	A1 **6**	Correct answer accept 3sf rounding to 0.16

72 $P(x < -2.4) = P\left(z < \dfrac{-2.4 - 1.5}{3.2}\right)$	M1		Standardising no cc can have sq
$= P(z < -1.219)$	M1		Correct area, i.e. < 0.5
$= 1 - 0.8886$ $= 0.111$	A1	**[3]**	Correct answer rounding to 0.111

73 **(i)** $P(4, 5, 6) = (0.22)^4(0.78)^4 8C4 + (0.22)^5(0.78)^3 8C5 + (0.22)^6(0.78)^2 8C6$	M1 M1		Bin term with ${}_8C_r\, p^r\,(1-p)^{8-r}$ seen $r \neq 0$ any $p < 1$ Summing 2 or 3 bin probs $p = 0.22$, $n = 8$
$= 0.0763$	A1	**[3]**	Correct answer
(ii) prob = 0.13 mean = 300 × 0.13 = 39 var = 300 × 0.13 × 0.87 = 33.93	B1 B1ft		Correct prob can be implied Correct unsimplified np and npq ft wrong 0.13
$P(30 < x < 50) = P$	M1		Standardising a value need sq rt
$\left(\frac{30.5-39}{\sqrt{33.93}} < z < \frac{49.5-39}{\sqrt{33.93}}\right)$	M1		Cont correction 30.5 / 31.5 or 48.5/49.5 only
$= P(-1.4592 < z < 1.8026)$ $= \Phi(1.8026) + \Phi(1.4592) - 1$ $= 0.9643 + 0.9278 - 1 = 0.892$	M1 A1	**[6]**	Correct area $\Phi_1 + \Phi_2 - 1$ oe Rounding to correct answer SC $P(31,..49)=300C31(0.13)^{31}(0.87)^{269} + \ldots +300C49$ etc.) B1B1

74 $P(13.6 < X < 14.8) = P\left(\frac{13.6-14}{0.52} < z < \frac{14.8-14}{0.52}\right)$	M1		Standardising 1 expression, no cc, no sq rt, no sq, ±, mean on num.
$= P(-0.7692 < z < 1.538)$ $= \Phi(1.538) - [1 - \Phi(0.7692)]$ $= 0.9380 - [1 - 0.7791]$ $= 0.7171$	M1 A1		$\Phi1 + \Phi2 - 1$ (indep) oe ($\Phi2 - \Phi1$ if cc used) Correct probability rounding to 0.72 here
$P(8) = (0.7171)^8(0.2829)^2\,{}_{10}C_8$ $= 0.252$	M1 A1	5	Binomial expression $10C8\ p^8q^2$, $\Sigma p + q = 1$, any p Correct answer (rounding to 0.252)
75 **(i** ($p =$)0.85 $P(< 12) = 1 - P(12, 13, 14)$ $= 1 - [(0.85)^{12}(0.15)^2\,{}_{14}C_{12} + (0.85)^{13}(0.15)\,{}_{14}C_{13} + (0.85)^{14}]$ $= 1 - 0.6479$ $= 0.352$	B1 M1 A1	3	($p =$)0.85 oe seen anywhere Summing 2 or 3 consistent bin probs, any $p < 1$, $n = 14$ (or summing 12 or 13 consistent bin probs) Correct answer
(ii) $(0.85)^n \geqslant 0.1$ $n \leqslant 14.2$ $n = 14$	M1 M1 A1	3	Eqn or inequality in 0.85(or 0.15), n, 0.1, n as a power Attempt to solve (can be implied) if n a power Correct answer – must be equals, not approx. MR allowed for 0.01, M1M1A0 max.

76 **(a)** $P(X < q + 82) = 0.72$ $z = 0.583$	M1		Rounding to ± 0.58 or ± 0.15 seen
$\frac{\pm q}{7.4}$ *or* $\frac{\pm 2q}{7.4} = z$ *or probabilty* (o.e.)	M1		Standardising, no cc, no sq, no sq rt
$q = 4.31$	A1	3	correct answer

(b) $\frac{0.5\mu-\mu}{\sigma}=\frac{\pm 0.5\mu}{\sigma}$	M1		Standardising attempt some μ/σ allow cc, sq rt, sq Can be implied
$\frac{0.2\sigma^2}{\sigma}=-0.2\sigma=-0.580$	B1 M1		± 0.580 seen (accept ±0.58) substituting to eliminate μ or σ , arriving at numerical solution, any z value or probability – not dependent
σ = 2.90 μ = 3.36	A1	**4**	both answers correct , accept 2.9

77 $P(21.6 < x < 28.7)$			
$= P\left(\left(\frac{21.6-24}{4.7}\right) < z < \left(\frac{28.7-24}{4.7}\right)\right)$	M1 A1		Standardising; no cc, no sq rt One rounding to Φ(0.841 or 0.695)
$= P(-0.5106 < z < 1) = \Phi(1) - \Phi(-0.5106)$	M1		$\Phi_1 + \Phi_2 - 1$
$= 0.8413 - (1 - 0.6953)$			
$= 0.537$ (0.5366)	A1	**4**	Correct answer
78 $1.751 = \frac{12-\mu}{\sigma}$	B1		Rounding to ±1.75 seen
$0.468 = \frac{9-\mu}{\sigma}$	B1		±0.468 seen
	M1		An eqn with a z-value, μ and σ no $\sqrt{\sigma}$, no σ^2
$\sigma = 2.34$	M1		Sensible attempt to eliminate μ or σ by substitution or subtraction, need a value
$\mu = 7.91$	A1	**5**	correct answers
79 **(i)** constant / given p, independent trials, fixed / given no. of trials, only two outcomes	B1 B1	 **2**	Any one correct Any 3 correct
(ii) $P(x \geqslant 3) = 1 - P(0, 1, 2)$	M1		Any binomial expression $p^r(1-p)^{18-r}\,{}^{18}C_r$ seen
$= 1 - [(0.85)^{18} + (0.85)^{17}(0.15) \times 18 + (0.85)^{16}(0.15)^2 \times {}^{18}C_2]$	M1		$1 - P(0, 1, 2)$, any n,p,q
$= 0.520$	A1	**3**	Correct answer

80 $X \sim B(19, 0.12)$	M1		Any binomial term ${}^{19}C_x\, p^x(1-p)^{19-x}$, $0<p<1$
$P(X < 4) = P(0, 1, 2, 3)$ $= (0.88)^{19} + {}^{19}C_1(0.12)^1(0.88)^{18} + {}^{19}C_2(0.12)^2(0.88)^{17} + {}^{19}C_3(0.12)^3(0.88)^{16}$	M1		Any binomial term ${}^{n}Cx(0.12 \text{ or } 0.88)^x(0.88 \text{ or } 0.12)^{n-x}$
	M1		P(0, 1, 2, 3) binomial expr with at least 2 consistent terms
$= 0.813$	A1	**4**	Correct answer

81 Y1(7) Y2(2)Y3(2)			
1 2 2 $= 7 \times 1 \times 1 = 7$	B1		One unsimplified correct 3-factor product of combinations
2 1 2 $= {}^7C_2 \times {}^2C_1 \times 1 = 42$	B1		A second unsimplified correct 3-factor product of combinations
2 2 1 $= {}^7C_2 \times 1 \times {}^2C_1 = 42$			
3 1 1 $= {}^7C_3 \times {}^2C_1 \times {}^2C_1 = 140$	M1		Summing 3 or 4 options allow perms, wrong combs but second numbers must sum to 5 etc.
Total = 231	A1	**4**	Correct answer

82 (i) $z = -0.842$	B1		± rounding to 0.84 seen
$P(x > 1.35) = P\left(z > \dfrac{1.35-1.9}{\sigma}\right)$	M1		$\pm\dfrac{1.35-1.9}{\sigma}$ = a prob or a z-value NOT 0.8 or 0.2 allow a 1–...
$-0.842 = -0.55/\sigma$			
$\sigma = 0.653$	A1	**3**	Correct answer from correct working
(ii) $P(x < 2) = P\left(z < \dfrac{2-1.9}{0.6532}\right)$ $= P(z < 0.1531)$	M1		± standardising no continuity correction their σ
$= 0.561$	A1	**2**	Correct answer
(iii) $X \sim N(160, 32)$	B1		Unsimplified 160 and 32 seen
$P(162.5 < x < 173.5) = P\left(\dfrac{162.5-160}{\sqrt{32}} < z < \dfrac{173.5-160}{\sqrt{32}}\right)$	M1		Standardising need sq rt
$P(0.442 < z < 2.386)$	M1		Any of 162.5, 163.5, 172.5, 173.5 seen
$= \Phi(2.386) - \Phi(0.442)$	M1		$\Phi_2 - \Phi_1$ oe
$= 0.9915 - 0.6707$	A1		One correct Φ to 3sf
$= 0.321$	A1	**6**	Correct answer accept 0.320

83 (i) $np = 252 \times 1/7 = 36$, $npq = 252 \times 1/7 \times 6/7 = 30.857$	B1		Unsimplified 36 and 30.857 seen, oe
$P\left(z < \left(\dfrac{29.5-36}{\sqrt{30.857}}\right)\right) + P\left(z > \left(\dfrac{44.5-36}{\sqrt{30.857}}\right)\right)$	M1 M1		any standardising, sq rt needed any continuity correction either 29.5, 30.5, 43.5, 44.5
$= P(z < -1.170) + P(z > 1.530)$			
$= 1 - 0.8790 + 1 - 0.9370$	M1		correct area $2 - (\Phi_1 + \Phi_2)$
$= 0.184$	A1	**5**	correct answer
(ii) np and nq are both > 5	B1	**1**	must have both

84 (i) $z = -1.282$	B1		Rounding to ± 1.28 seen
$-1.282 = \dfrac{t-6.5}{1.76}$	M1		Standardising, no cc, no sq or sq rt, $z \neq \pm 0.9, \pm 0.1$
$t = 4.24$	A1	3	Correct answer, accept 4.25
(ii) $P(z < 1) = 0.8413$	M1		$z = 1$ used to find a probability
P(within 1sd of mean) = $2\Phi - 1$ $= 0.6826$	B1		correct prob, accept answer rounding to 0.66, 0.67, 0.68, not from wrong working. If quoted, then implies first M1.
P(8, 9) $= {}^9C_8(0.6826)^8(0.3174) + (0.6826)^9$	M1 M1		Binomial term $p^r(1-p)^{9-r}\,{}^9C_r$, ${}^9C_r$ must be seen Binomial expression for P(8)+P(9), any p
$= 0.167$	A1	5	Correct ans

85 (i) $1.2 = 15p$ $p = 0.08$ Var = $npq = 15 \times 0.08 \times 0.92 = 1.104$ **AG**	M1 A1	2	Attempt to find p using $1.2 = 15p$ Correct answer
(ii) $P(0, 1, 2) = (0.92)^{15} + {}^{15}C_1(0.08)(0.92)^{14} + {}^{15}C_2(0.08)^2(0.92)^{13}$ $= 0.887$	M1 M1 A1	3	Binomial expression ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$ Correct unsimplified expression for P(0, 1, 2) Correct answer
(iii) P(at least 1 faulty screw) = 1 – P(0) = 1 – $(0.92)^{15}$ = 0.7137… P(at least 1 faulty screw in 7 packets) = ${}^8C_7(0.713\ldots)^7(0.2863\ldots)$ = 0.216	M1 A1 M1 A1	4	Attempt at P(0) or 1 – P(0) Rounding to 0.71 Binomial expression ${}^8C_7p^7(1-p)$ $0 < p < 1$ Correct answer
86 (i) $z_1 = \dfrac{70-66.4}{5.6} = 0.6429$	M1		Standardising one variable, no cc, no sq rt
$z_2 = \dfrac{72.5-66.4}{5.6} = 1.089$	M1		Correct area $\Phi_2 - \Phi_1$
$\Phi(1.089) - \Phi(0.643) = 0.8620 - 0.7399$ $= 0.1221$	A1		Correct answer rounding to 0.12
$0.1221 \times 250 = 30.5$ 30 or 31 sheep	M1 A1ft	5	Mult by 250 Correct answer ft their 0.1221
(ii) $66.4 - 59.2 = 7.2$ $66.4 + 7.2 = 73.6$	M1 A1	2	Subt from 66.4 Correct answer
(iii) $z = 0.674$	B1		± 0.674 or 0.675 seen
$\dfrac{67.5-\mu}{4.92} = 0.674$	M1		Standardising with a z-value no cc no sq rt
$\mu = 64.2$	A1	3	Correct answer

87 **(a)** **(i)** $P(x < 8) = P\left(z < \frac{8-7.15}{0.88}\right)$	M1		Standardising ± , no cc no sq rt no sq
$= \Phi\ (0.9659)$ $= 0.833$	A1	**2**	Correct answer
(ii) $z = 0.674$	B1		Accept ± 0.674 or 0.675 only
$\frac{q-7.15}{0.88} = 0.674$	M1		Standardised eqn = ± their z-value, allow sq or sq rt if already penalised in **(i)**
$q = 7.74$	A1	**3**	Correct answer
(b) $P(Y > 4\mu) = P\ (z > \left(\frac{4\mu - \mu}{(3\mu/2)}\right)) = P(z > 2)$	M1		Standardising no sq rt, no cc, no sq, one variable
$= 1 - 0.9772$	A1		$z = \pm 2$ seen
$= 0.0228$	A1	**3**	correct ans SR B1 if made-up values used and 0.0228 obtained

88 **(i)** $P\ (4, 5, 6) = (0.75)^4 (0.25)^4 \times {}^8C_4 + (0.75)^5 (0.25)^3 \times {}^8C_5 + (0.75)^6 (0.25)^2 \times {}^8C_6$	M1		Bin term $p^r(1-p)^{8-r} \times {}^8C_r$ seen any p
	M1		Correct unsimplified answer
$= 0.606$	A1	**3**	Correct ans
(ii) $np = 160 \times 0.75 = 120 \quad npq = 30$	B1		Unsimplified mean and var correct
$P(>114) = P\left(z > \left(\frac{114.5-120}{\sqrt{30}}\right)\right)$	M1 M1		Standardising, need sq rt Cont correction either 114.5 or 113.5
$= P\ (z > -1.004)$	M1		Correct area consistent with their working
$= \Phi(1.004) = 0.842$	A1	**5**	Correct ans
(iii) np and nq both > 5	B1	**1**	Need both

89 $z = -2.326$	B1		± 2.325 to ± 2.33 seen
$\frac{250-260}{\sigma} = -2.326$	M1		Standardising and = or < their z, no cc, sq, sq rt
$\sigma = 4.30$	A1	**3**	Correct ans

90 **(i)** max = 12	B1		(Implied by P(12) with power 12)
$P(12) = (0.7)^{12} = 0.0138$	B1	**2**	Accept 0.014
(ii) P(fewer than 10) = 1– P (10, 11, 12) $= 1 - {}^{12}C_{10} \times (0.7)^{10}(0.3)^2 - 12 \times (0.7)^{11}(0.3) - (0.7)^{12}$	M1		Binomial term ${}^{12}C_r(0.7)^r(0.3)^{12-r}$ or ${}^{12}C_r(p)^r(q)^{12-r}$, $0.99 \leqslant p + q \leqslant 1.00$
$= 1 - 0.2528$	A1		Correct unsimplified expression oe
$= 0.747$	A1	**3**	Correct answer

91 (i)	$P(<1.2)=P\left(z<\frac{1.2-1.9}{0.55}\right)=P(z<-1.2727)$ $= 1-\Phi(1.273)=1-0.8986$	M1	Standardising for wt 1.2 or 2.5, no cc, sq, sq rt May be awarded in (ii) if not attempted in (i) Accept 0.102
	$= 0.1014$	A1	First correct proportion seen
	$P(>2.5)=P\left(z<\frac{2.5-1.9}{0.55}\right)=P(z>1.0909)$ $= 1-\Phi(1.0909)=1-0.8623$	A1	Second correct proportion seen
	$= 0.138$ $P(1.2 < wt < 2.5) = 1 - 0.101 - 0.138$	M1	Third proportion 1 – their previous 2 proportions or correct attempt for remaining proportion
	$= 0.761$	A1√ **5**	Correct answer or 1 – *their* 2 previous correct proportions
(ii)	$P(x > k) = 0.8 + 0.1377 = 0.9377$	M1	Valid method to obtain $P(x > k)$ or $P(x < k)$
	$z = -1.536$	A1	$\pm$ 1.536 seen accept 3sf rounding to 1.53 or 1.54
	$-1.536=\frac{k-1.9}{0.55}$	M1	Attempt to solve equation with their 'correct' area z value, *k*, 1.9 and 0.55
	$k = 1.06$	A1 **4**	Correct answer or rounding to 1.05

92	$P(x < 3.273) = 0.5 - 0.475 = 0.025$	**M1**	Attempt to find *z*-value using tables in reverse
	$z = -1.96$	**A1**	± 1.96 seen
	$\frac{3.2-\mu}{0.714}=-1.96$	**M1**	Solving their standardised equation *z*-value not nec
	$\mu = 4.60$s	**A1 [4]**	Correct ans accept 4.6

93 (i)	$P(5, 6, 7) = {}^8C_5(0.68)^5(0.32)^3 + {}^8C_6(0.68)^6(0.32)^2 + {}^8C_7(0.68)^7(0.32)$	M1	Binomial term ${}^8C_x\, p^x(1-p)^{8-x}$ seen $0 < p < 1$
		M1	Summing 3 binomial terms
		A1	Correct unsimplified answer
	$= 0.722$	A1 **[4]**	Correct answer
(ii)	$np = 340$, $npq = 108.8$	B1	Correct (unsimplified) mean and var
	$P(x > 337) = P\left(z>\frac{337.5-340}{\sqrt{108.8}}\right)$	M1	standardising with sq rt must have used 500
		M1	cc either 337.5 or 336.5
	$= P(z > -0.2396)$	M1	correct area (> 0.5) must have used 500
	$= 0.595$	A1 **[5]**	correct answer
(iii)	np (340) > 5 and nq(160) > 5	B1 **[1]**	must have both or at least the smaller, need numerical justification

94	P(3, 4, 5) = ${}^{10}C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7 + {}^{10}C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6 + {}^{10}C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^5$ = 0.222	M1 A1 A1 **3**	Bin expression of form ${}^{10}C_x(p)^x(1-p)^{10-x}$ any x any p Correct unsimplified answer accept (0.17, 0.83), (0.16, 0.84), (0.16, 0.83), (0.17, 0.84) or more accurate Correct answer

95 (a) (i)	prob = $p\left(z < \frac{30-35.2}{4.7}\right)$ = P($z < -1.106$) = 1 – 0.8655 = 0.1345 0.1345×52 = 6.99	M1 M1 A1 A1 **4**	Standardising no sq rt no cc no sq $1-\Phi$ Correct ans rounding to 0.13 Correct final answer accept 6 or 7 if 6.99 not seen but previous prob 0,1345 correct
(ii)	$\Phi(t) = 0.648$ $z = 0.380$ $0.380 = \frac{t-35.2}{4.7}$ $t = 37.0$	B1 M1 A1 **3**	0.648 seen standardising allow cc, sq rt,sq, need use of tables not 0.148, 0.648, 0.352, 0.852 correct answer rounding to 37.0
(b)	$\frac{7-\mu = -0.8\sigma}{\sigma}$ so $7-\mu = -0.8\sigma$ $\frac{10-\mu}{\sigma} = 0.44$ so $10-\mu = 0.44\sigma$ $\mu = 8.94$ $\sigma = 2.42$	B1 B1 M1 M1 A1 **5**	± 0.8 seen ± 0.44 seen An eqn with z-value, μ and σ no sq rt no cc no sq Sensible attempt to eliminate μ or σ by subst or subtraction, need at least one value Correct answers

96	z = 1.136 $1.136 = \frac{195-\mu}{22}$ $\mu = 170$	B1 M1 A1 **[3]**	± 1.136 seen, not ±1.14, Standardising, no cc no sq rt, equated to their z not 0.128 or 0.872 Correct answer, nfww

97	$\mu = 300\times0.072 = 21.6$, $\sigma^2 = 20.0448$ P($x < 18$) = $P\left(z < \frac{17.5-21.6}{\sqrt{20.0448}}\right)$ =P (z < -0.9157) = 1 – 0.8201 = 0.180	B1 M1 M1 M1 A1 **[5]**	300×0.072 seen and 300×0.072×0.928 seen or implied ($\sigma = 4.4771, \sigma^2 = 20(.0)$) oe ±Standardising, their mean/var, with sq root Cont corr 17.5 or 18.5 Correct area 1 - Φ Answer wrt 0.180, nfww

98 **(i)**	P(large) = $1 - \Phi\left(\frac{29-21.7}{6.5}\right)$ $= 1 - \Phi(1.123) = 1 - 0.8692$ $= 0.1308$ $P(0,1) = (0.8692)^{8} + {}^8C_1(0.1308)(0.8692)^7$ $= 0.718$	M1 M1 A1 M1 M1 A1 **[6]**	Standardising no cc no sq rt Correct area 1 – Φ Rounding to 0.13 Any bin term with ${}^8C_xp^x(1-p)^{8-x}$ $0 < p < 1$ Summing bin P(0) + P(1) only with $n = 8$, oe Correct ans
(ii)	$= 1 - (0.8692)^n > 0.98$ $(0.8692)^n < 0.02$ Least number = 28	M1 M1 A1 **[3]**	eq/ineq involving their $(0.8692)^n$ or $(0.1308)^n$, 0.02 or 0.98 oe with or without a 1 solving attempt (could be trial and error) – may be implied by their answer correct answer

99	$p = 0.76$ P(fewer than 10) = 1 – P(10, 11) $= 1 - (0.76)^{10}(0.24)^{11}C_{10} - (0.76)^{11}$ $= 1 - 0.219$ $= 0.781$	**M1** **M1** **M1** **A1** [4]	Any binomial term ${}^{11}C_xp^x(1\text{-}p)^{11-x}$, $0 < p < 1$ Any binomial term ${}^nC_x(0.76)^x(0.24)^{n-x}$ 1 – P(10, 11) oe binomial expression Correct answer
100	$\mu = 54.1$ $z = -1.11$ $-1.11 = \frac{50.9-54.1}{\sigma}$ $\sigma = 2.88$	**B1** **B1** **M1** **A1** [4]	Stated or evaluated Accept rounding to ± 1.1 Standardising no cc no sq rt Correct answer

101 (i)	$\bar{x} = 80 - 147/30 = 80 - 4.9$ $= 75.1$ sd $= \sqrt{\left(\frac{952}{30} - \left(\frac{147}{30}\right)^2\right)} = \sqrt{7.72\ldots}$ sd = 2.78	**M1** **A1** **M1** **A1** [4]	For –147/30 oe seen Correct answer 952/30 – (± their coded mean)^2 Correct answer
(ii)	$P(x > 160) = P\left(z > \frac{160-148.6}{18.5}\right)$ $= P(z > 0.616)$ $= 1 - 0.7310$ $= 0.269$	**M1** **M1** **A1** [3]	Standardising no cc no sq rt 1 – Φ Correct answer

102 (i)	let P(2, 4, 6) all = p then P(1, 3, 5) all = 2p $3p + 6p = 1$ $p = 1/9$ so prob (3) = 2/9 (0.222)	**M1** **M1** **A1** [3]	Using P(even) = 2P(odd) or vice versa oe Summing P(odd+ even) or P(1, 2, 3, 4, 5, 6) = 1 Correct answer
(ii)	$P(5, 5, 6) = 2/9 \times 2/9 \times 1/9 \times {}^3C_2$ $= 4/243\ (0.0165)$	**M1** **M1** **A1** [3]	Mult three probs together Mult by 3 oe ie summing 3 options Correct answer
(iii)	$\mu = 100 \times 1/3 = 33.3,\ \sigma = 100 \times 1/3 \times 2/3 = 22.2$ $P(x \leqslant 37) = P\left(z \leqslant \dfrac{37.5 - \frac{100}{3}}{\sqrt{\frac{200}{9}}}\right) = P(z \leqslant 0.8839)$ $= 0.812$	**B1** **M1** **M1** **M1** **A1** [5]	Unsimplified 100/3 and 200/9 seen Standardising need sq rt 36.5 or 37.5 seen correct area using their mean Correct answer

103 **(a)** **(i)**	$P(x > 3900) = P\left(z > \dfrac{3900 - 4520}{560}\right)$ $= P(z > -1.107) = \Phi(1.107)$ $= 0.8657$ Number of days = 365 × 0.0.8657 = 315 or 316 (315.98)	**M1** **M1** **A1** **B1**√ 4	Standardising no cc no sq rt no sq Correct area Φ ie > 0.5 Prob rounding to 0.866 Correct answer ft their wrong prob if previous A0, $p < 1$, ft must be accurate to 3sf
(ii)	$z = 1.165$ $1.165 = \dfrac{8000 - m}{560}$ $m = 7350\ (7347.6)$	**B1** **M1** **A1** 3	± 1.165 seen Standardising eqn allow sq, sq rt, cc, must have z-value eg not 0.122, 0.878, 0.549, 0.810. Correct answer rounding to 7350
(iii)	$P(0, 1) = (0.878)^6 + {}^6C_1(0.122)^1(0.878)^5$ = 0.840 accept 0.84 Normal approx. to Binomial. M0, M0, A0	**M1** **M1** **A1** 3	Binomial term ${}^6C_x\, p^x(1-p)^{6-x}$ $0 < p < 1$ seen Correct unsimplified expression Correct answer
(b)	$P(< 2\mu) = P\left(z > \dfrac{2\mu - \mu}{\sigma}\right) = P(z < 1.5)$ $= 0.933$	**M1** **M1** **A1** 3	Standardising with μ and σ Attempt at one variable and cancel Correct answer

104 (i	$z = 1.127$ $1.127 = \dfrac{136-125}{\sigma}$ $\sigma = 9.76$	B1 M1 A1 3	± 1.127 seen accept rounding to ±1.13 Standardising no cc no sq rt, with attempt at z (not ±0.8078, ±0.5517, ±0.13, ±0.87) Correct ans

105 (i)	P(0, 1, 2) = $(0.92)^{19}+{}^{19}C_1(0.08)(0.92)^{18}+{}^{19}C_2(0.08)^2(0.92)^{17}$ = 0.809	**M1** **M1** **A1** 3	Binomial term ${}^{19}C_x p^x(1-p)^{19-x}$ seen $0<p<1$ Correct unsimplified expression Correct answer (no working SC B2)
(ii)	P(at least 1) = 1 – P(0) $= 1 - P(0.92)^n > 0.90$ $0.1 > (0.92)^n$ $n > 27.6$ Ans 28	 **M1** **M1** **A1** 3	 Eqn with their 0.92^n, 0.9 or 0.1, 1 not nec Solving attempt by logs or trial and error, power eqn with one unknown power Correct answer, not approx., ≈, ⩾, >, ⩽, <
(iii)	np = 1800 × 0.08 = 144 npq = 132.48 P(at least 152) = $P\left(z > \left(\dfrac{151.5-144}{\sqrt{132.48}}\right)\right)$ $= P(z > 0.6516)$ $= 1 - 0.7429$ $= 0.257$	**B1** **M1** **M1** **M1** **A1** 5	correct unsimplified np and npq seen accept 132.5, 132, 11.5, awrt 11.51 standardising, with √ cont correction 151.5 or 152.5 seen correct area 1 – Φ (probability) correct answer
(iv)	Use because 1800 ×0.08 (and 1800 × 0.92 are both) > 5	**B1** 1	1800 ×0.08 > 5 is sufficient np>5 is sufficient if clearly evaluated in (iii) If npq>5 stated then award B0