PARTIAL FRACTION (5 MARK)

10 MARKS
PARTIAL (5)
BINOMIAL (5)

10 MARKS
PARTIAL (5)
INTEGRATION (5)

O- LEVELS

$$\frac{3}{x+1} + \frac{5}{x+5}$$ ANSWER

PARTIAL FRACTION

$$\frac{3(x+5) + 5(x+1)}{(x+1)(x+5)}$$

$$\frac{3x + 15 + 5x + 5}{(x+1)(x+5)}$$

$$\frac{8x+20}{(x+1)(x+5)}$$ QUESTION

FORMS

LINEAR

power of x is 1.

$$\frac{3}{(x+1)(x+2)} \equiv \frac{A}{x+1} + \frac{B}{x+2}$$

REPEATED

one bracket with whole squared power.

$$\frac{3}{(x+5)(x-3)^2} \equiv \frac{A}{x+5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

Memorize this case as variation of REPEATED

$$\frac{3}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$

QUADRATIC

x^2 term inside the bracket

$$\frac{3}{(x+5)(x^2+1)} \equiv \frac{A}{x+5} + \frac{Bx+C}{x^2+1}$$

PARTIAL FRACTION

check max power of x in expanded form in both the numerator and denominator

PROPER	**IMPROPER**
$N < D$	$N \geqslant D$
$\dfrac{3x^2+5x+1}{\underbrace{(x+1)(x^2+5)}_{x^3}}$ $\quad \dfrac{2}{3}$	$\dfrac{2x^3-7x^2+1}{(x+1)(x+2)}$ $\quad \dfrac{3}{2}$
$\dfrac{x^2+3}{(x+1)(x+2)^2}$ $\quad \dfrac{2}{3}$	$\dfrac{5x^2+6x+8}{(x+1)(x+2)}$ $\quad \dfrac{2}{2}$
$\dfrac{2x+5}{(x+1)(x+2)}$ $\quad \dfrac{1}{2}$	
FOR PROPER FRACTIONS YOU CAN USE FORMS GIVEN ABOVE DIRECTLY.	FOR IMPROPER YOU CANNOT USE PARTIAL FRACTION FORMS DIRECTLY.
	STEP 1: USE LONG DIVISION

TO BRING $Q + \frac{R}{D}$

proper

STEP2: Apply partial fraction form on the proper fraction only.

Improper

Divisor

Quotient

$$\frac{11}{9} \rightarrow 9\overline{)11} \rightarrow 1\frac{2}{9}$$

$$\begin{array}{r} 1 \\ 9\overline{)11} \\ -9 \\ \hline 2 \end{array}$$

Remainder.

$$1 + \frac{2}{9}$$ proper

$$Q + \frac{R}{D}$$

IDENTITY: $(\equiv)$

(1) You cannot switch sides of terms.

(2) you are allowed to put any value of x.

1 Let $f(x) = \frac{x^2 + 7x - 6}{\underbrace{(x-1)(x-2)(x+1)}_{x^3}}$. $\frac{2}{3}$ (proper) (form applied directly)

(i) Express $f(x)$ in partial fractions. [4]

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$$\frac{x^2+7x-6}{(x-1)(x-2)(x+1)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\frac{x^2+7x-6}{(x-1)(x-2)(x+1)} \equiv \frac{A(x-2)(x+1)+B(x-1)(x+1)+C(x-1)(x-2)}{(x-1)(x-2)(x+1)}$$

$$\boxed{x^2+7x-6 \equiv A(x-2)(x+1)+B(x-1)(x+1)+C(x-1)(x-2)}$$

$x-1=0$	$x-2=0$	$x+1=0$
$x=1$	$x=2$	$x=-1$
$(1)^2+7(1)-6 = A(1-2)(1+1)$	$(2)^2+7(2)-6 = B(2-1)(2+1)$	$(-1)^2+7(-1)-6 = C(-1-1)(-1-2)$
$2 = A(-1)(2)$	$12 = B(1)(3)$	$-12 = C(-2)(-3)$
$A=-1$	$B=4$	$C=-2$

$$\frac{x^2+7x-6}{(x-1)(x-2)(x+1)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\boxed{\frac{-1}{x-1} + \frac{4}{x-2} - \frac{2}{x+1}}$$

5 Let $f(x) = \frac{7x+4}{\underbrace{(2x+1)(x+1)^2}_{x^3}}$. $\frac{1}{3}$ (proper) (apply form).

(i) Express $f(x)$ in partial fractions. [5]

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$$\frac{7x+4}{(2x+1)(x+1)^2} \equiv \frac{A}{2x+1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$\frac{7x+4}{(2x+1)(x+1)^2} \equiv \frac{A(x+1)^2+B(2x+1)(x+1)+C(2x+1)}{(2x+1)(x+1)^2}$$

$$\boxed{7x+4 \equiv A(x+1)^2+B(2x+1)(x+1)+C(2x+1)}$$

$2x+1=0$

$x=-\frac{1}{2}$

$7\left(-\frac{1}{2}\right)+4 = A\left(-\frac{1}{2}+1\right)^2$

$\frac{1}{2} = A\left(\frac{1}{4}\right)$

$A = 2$

$x+1=0$

$x=-1$

$7(-1)+4 = C(2(-1)+1)$

$-3 = C(-1)$

$C=3$

$(x+1)^2=0$

$x+1=0$

$x=-1$

we need a new value each time.

↓

you are allowed any value of x.

Now Put $\boxed{x=0}$

$7(0)+4 = 2(0+1)^2+B(2(0)+1)(0+1)+3(2(0)+1)$

$4 = 2(1)+B(1)(1)+3(1)$

$4 = 2+B+3$

$B=-1$

$$\frac{7x+4}{(2x+1)(x+1)^2} \equiv \frac{A}{2x+1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$$

$$\boxed{\frac{2}{2x+1}-\frac{1}{x+1}+\frac{3}{(x+1)^2}}$$

3 (i) Express $\dfrac{3x^2+x}{(x+2)(x^2+1)}$ in partial fractions. $\frac{2}{3}$ (proper) (apply form). [5]

$$\frac{3x^2+x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

V.IMP

$$\frac{3x^2+x}{(x+2)(x^2+1)} \equiv \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$\boxed{3x^2+x \equiv A(x^2+1)+(Bx+C)(x+2)}$$

$x+2=0$	$x^2+1=0$	Now you have to
$x=-2$	$x^2=-1$	put two new values
$3(-2)^2+(-2)=A((-2)^2+1)$	No solutions	of x.
$10 = A(5)$		First value $x=0$
$\boxed{A=2}$		Second value $x=$ any.

$\boxed{x=0}$ $3(0)^2+0 = 2(0^2+1)+(B(0)+C)(0+2)$

$0 = 2(1)+(C)(2)$

$-2 = 2C$

$C=-1$

$\boxed{x=1}$ $3(1)^2+1 = 2(1^2+1)+(B(1)-1)(1+2)$

$4 = 4 + (B-1)(3)$

$0 = 3(B-1)$

$B=1$

$$\frac{3x^2+x}{(x+2)(x^2+1)} = \frac{\overset{2}{A}}{x+2} + \frac{\overset{1}{B}x+\overset{-1}{C}}{x^2+1}$$

$$= \boxed{\frac{2}{x+2} + \frac{x-1}{x^2+1}}$$

$\boxed{\text{New values}}$

- **Linear** → No new values are needed
- **Repeated** → one new value is needed. $\boxed{x=0}$
- **QUADRATIC** → Two new values needed.
  First : $x = 0$
  Second : $x = \text{any}$.

$$\frac{3x^2+x}{(x+2)(x^2+1)} = \frac{\overset{3}{A}}{x+2} + \frac{\overset{-2}{B}x+\overset{5}{C}}{x^2+1}$$

$$\boxed{\frac{3}{x+2} + \frac{-2x+5}{x^2+1}} \checkmark$$

$$\frac{3}{x+2} + \frac{-(2x-5)}{x^2+1}$$

$$\boxed{\frac{3}{x+2} - \frac{2x-5}{x^2+1}} \checkmark$$

Guys please practice these Questions for today's class can't take live class Today.

10 (i) Express $\dfrac{1+x}{(1-x)(2+x^2)}$ in partial fractions. [5]

 9709/32/O/N/09

8 (i) Express $\dfrac{100}{x^2(10-x)}$ in partial fractions. [4]

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6 (i) Express $\dfrac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

 9709/03/O/N/07

21 (i) Express $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in partial fractions. [5]

 9709/33/O/N/13

New google class room code . g7zctrr

IMPROPER FRACTIONS

Long Division $\rightarrow$ $Q + \dfrac{R}{D}$ $\rightarrow$ Apply partial fraction form only on the proper part

16 Let $f(x) = \dfrac{4x^2 - 7x - 1}{(x+1)(2x-3)}$. $\frac{2}{2}$ (improper) (Long Division)

(i) Express $f(x)$ in partial fractions. [5]

$(x+1)(2x-3) = 2x^2 + 2x - 3x - 3 = \boxed{2x^2 - x - 3}$

$$\begin{array}{r} \xrightarrow{\times} 3 \to \text{Quotient} \\ \text{Divisor} \swarrow 4 \overline{)\,13} \\ -12 \\ \hline 1 \to \text{Remainder.} \end{array}$$

$$\begin{array}{r|l} & \quad 2 \\ \hline 2x^2 - x - 3 & 4x^2 - 7x - 1 \\ & \cancel{4x^2} - 2x - 6 \\ & - \quad + \quad + \\ \hline & \quad -5x + 5 \end{array}$$

Quotient = $\dfrac{\text{inner first term}}{\text{outer first term}}$

$Q = \dfrac{4x^2}{2x^2} = 2$

$$Q + \frac{R}{D} = 2 + \boxed{\frac{-5x+5}{(x+1)(2x-3)}}$$

(W1) proper fraction

(W1) $\dfrac{-5x+5}{(x+1)(2x-3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{2x-3}$

$\dfrac{-5x+5}{(x+1)(2x-3)} \equiv \dfrac{A(2x-3) + B(x+1)}{(x+1)(2x-3)}$

$\boxed{-5x+5 \equiv A(2x-3) + B(x+1)}$

$x + 1 = 0$

$x = -1$

$-5(-1) + 5 = A(2(-1) - 3)$

$2x - 3 = 0$

$x = \frac{3}{2}$

$-5\left(\frac{3}{2}\right) + 5 = B\left(\frac{3}{2} + 1\right)$

$10 = -5a$ | $\frac{-5}{2} = \frac{5}{2}B$

$A = -2$

$B = -1$

$$\frac{-5x+5}{(x+1)(2x-3)} \equiv \frac{\overset{-2}{A}}{x+1} + \frac{\overset{-1}{B}}{2x-3}$$

$$Q + \frac{R}{D} = 2 + \boxed{\frac{-5x+5}{(x+1)(2x-3)}}$$

$$2 + \left[\frac{-2}{x+1} - \frac{1}{2x-3}\right]$$

$$\boxed{\frac{4x^2-7x-1}{(x+1)(2x-3)} = 2 - \frac{2}{x+1} - \frac{1}{2x-3}}$$

EXAM QUESTIONS : { (a) Partial (5) / (b) Binomial (5) } (10 marks) (15 mins)

41 Let $f(x) = \dfrac{7x^2 - 15x + 8}{(1-2x)(2-x)^2}$. $\equiv \dfrac{1}{1-2x} + \dfrac{3}{2-x} - \dfrac{2}{(2-x)^2}$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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(ii) $f(x) = \underbrace{1(1-2x)^{-1}}_{W1} + \underbrace{3(2-x)^{-1}}_{W2} - \underbrace{2(2-x)^{-2}}_{W3}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$$

(W1) $$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-1-1)(-2x)^2}{2!} = \boxed{1+2x+4x^2}$$

(W2) $$(2-x)^{-1}$$

$$\left[2\left(1-\frac{x}{2}\right)\right]^{-1}$$

$$2^{-1}\left(1-\frac{x}{2}\right)^{-1}$$

$$\frac{1}{2}\left[1+(-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2\right]$$

$$\boxed{\frac{1}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}\right)}$$

(W3) $$(2-x)^{-2}$$

$$\left[2\left(1-\frac{x}{2}\right)\right]^{-2}$$

$$2^{-2}\left(1-\frac{x}{2}\right)^{-2}$$

$$\frac{1}{4}\left[1+(-2)\left(-\frac{x}{2}\right) + \frac{(-2)(-2-1)}{2!}\left(-\frac{x}{2}\right)^2\right]$$

$$\boxed{\frac{1}{4}\left(1+x+\frac{3}{4}x^2\right)}$$

$$f(x) = 1(1-2x)^{-1} + 3(2-x)^{-1} - 2(2-x)^{-2}$$

W1 W2 W3

$$= 1\left[1 + 2x + 4x^2\right] + 3\left[\frac{1}{2}\left(1 + \frac{x}{2} + \frac{x^2}{4}\right)\right] - \cancel{2}^{1}\left[\frac{1}{\cancel{4}_{2}}\left(1 + x + \frac{3x^2}{4}\right)\right]$$

$$= 1 + 2x + 4x^2 + \frac{3}{2} + \frac{3}{4}x + \frac{3}{8}x^2 - \frac{1}{2} - \frac{x}{2} - \frac{3}{8}x^2$$

$$= 2 + \frac{9}{4}x + 4x^2$$

PARTIAL DEADLINE : MONDAY MIDNIGHT

BINOMIAL : FRIDAY MIDNIGHT

$$\frac{2x+3}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$

$$\frac{A(x)(2x+1) + B(2x+1) + C(x^2)}{x^2(2x+1)}$$