A6 Inequalities and Modulus Answers
P3

1 *EITHER*: State or imply non-modular inequality $(2x+1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x + 1) = \pm x$ B1

Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1

Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only A1

State answer $-1 < x < -\frac{1}{3}$ A1

OR: Obtain the critical value $x = -1$ from a graphical method , or by inspection, or by solving a linear inequality or equation B1

Obtain the critical value $x = -\frac{1}{3}$ (deduct B1 from B3 if extra values are obtained) B2

State answer $-1 < x < -\frac{1}{3}$ B1 **4**

[Condone $\leq$ for <; accept –0.33 for $-\frac{1}{3}$.]

2 *EITHER*: State or imply non-modular inequality $(x-3a)^2 > (x-a)^2$, or corresponding equation B1

Expand and solve the inequality, or equivalent M1

Obtain critical value $2a$ A1

State correct answer $x < 2a$ only A1

OR: State a correct linear equation for the critical value, e.g. $x - 3a = -(x - a)$, or corresponding inequality B1

Solve the linear equation for x, or equivalent M1

Obtain critical value $2a$ A1

State correct answer $x < 2a$ only A1

OR: Make recognizable sketches of both $y = |x - 3a|$ and $y = |x - a|$ on a single diagram B1

Obtain a critical value from the intersection of the graphs M1

Obtain critical value $2a$ A1

Obtain correct answer $x < 2a$ only A1 **[4]**

3 *EITHER*: State or imply non-modular inequality $(2x)^2 > (x-1)^2$, or corresponding equation B1

Expand and make a reasonable solution attempt at a 2- or 3-term quadratic M1

Obtain critical value $x = \frac{1}{3}$ A1

State answer $x > \frac{1}{3}$ only A1

OR: State the relevant critical linear equation, i.e. $2x = 1 - x$ B1

Obtain critical value $x = \frac{1}{3}$ B1

State answer $x > \frac{1}{3}$ B1

State or imply by omission that no other answer exists B1

OR: Obtain the critical value $x = \frac{1}{3}$ from a graphical method, or by inspection, or by solving a linear inequality B2

State answer $x > \frac{1}{3}$ B1

State or imply by omission that no other answer exists B1 **4**

4

EITHER: State or imply non-modular inequality $-0.5 < 3^x - 8 < 0.5$, or $(3^x - 8)^2 < (0.5)^2$, or corresponding pair of linear equations or quadratic equation — B1

Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ — M1

Obtain critical values 1.83 and 1.95, or exact equivalents — A1

State correct answer $1.83 < x < 1.95$ — A1

OR: Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ — M1

Obtain one critical value, e.g. 1.95, or exact equivalent — A1

Obtain the other critical value 1.83, or exact equivalent — A1

State correct answer $1.83 < x < 1.95$ — A1 — 4

[Do not condone ≤ for <. Allow final answer given in the form $1.83 < x$, (and) $x < 1.95$.]

[Exact equivalents must be in terms of ln or logarithms to base 10.]

[SR: Solutions given as logarithms to base 3 can only earn M1 and B1 of the first scheme.]

5 **EITHER** State or imply non-modular inequality $(x-2)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(x-2) = \pm 3(2x+1)$ — B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations — M1

Obtain critical values $x = -1$ and $x = -\frac{1}{7}$ — A1

State answer $-1 < x < -\frac{1}{7}$ — A1

OR Obtain the critical value $x = -1$ from a graphical method, or by inspection, or by solving a linear equation or inequality — B1

Obtain the critical value $x = -\frac{1}{7}$ similarly — B2

State answer $-1 < x < -\frac{1}{7}$ — B1 — [4]

[Do not condone ≤ for <; accept $-\frac{5}{35}$ and − 0.14 for $-\frac{1}{7}$.]

6 *EITHER:* State or imply non-modular inequality $(x+3a)^2 > (2(x-2a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+3a) = \pm 2(x-2a)$ — B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations — M1

Obtain critical values $x = \frac{1}{3}a$ and $x = 7a$ — A1

State answer $\frac{1}{3}a < x < 7a$ — A1

OR: Obtain the critical value $x = 7a$ from a graphical method, or by inspection, or by solving a linear equation or inequality — B1

Obtain the critical value $x = \frac{1}{3}a$ similarly — B2

State answer $\frac{1}{3}a < x < 7a$ — B1 — [4]

[Do not condone ⩽ for <; accept 0.33 for $\frac{1}{3}$.]

7 EITHER: State or imply non-modular inequality $(2(x-3))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-3) = \pm(3x+1)$ B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1

Obtain critical values $x = -7$ and $x = 1$ A1

State answer $-7 < x < 1$ A1

OR: Obtain critical value $x = -7$ or $x = 1$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1

Obtain critical values $x = -7$ and $x = 1$ B2

State answer $-7 < x < 1$ B1 [4]

[Do not condone: ‹ for <.]

8 *EITHER*: State or imply non-modular inequality $x^2 < (5+2x)^2$, or corresponding equation, or pair of linear equations $x = \pm(5+2x)$ M1

Obtain critical values –5 and $-\frac{5}{3}$ only A1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$ A1

OR: State one critical value e.g. –5, by solving a linear equation or inequality, or from a graphical method, or by inspection B1

State the other critical value, e.g. $-\frac{5}{3}$, and no other B1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$ B1 [3]

[Do not condone ≤ or ≥.]

9 *EITHER* State or imply non-modular inequality $(3(x-1))^2 < (2x+1)^2$

or corresponding quadratic equation, or pair of linear equations $3(x-1) = \pm(2x+1)$ B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1

Obtain critical values $x = \frac{2}{5}$ and $x = 4$ A1

State answer $\frac{2}{5} < x < 4$ A1

OR by Obtain critical value $x = \frac{2}{5}$ or $x = 4$ from a graphical method, or by inspection, or

solving a linear equation or inequality B1

Obtain critical values $x = \frac{2}{5}$ and $x = 4$ B2

State answer $\frac{2}{5} < x < 4$ B1 [4]

[Do not condone ≤ for < .]

10 *EITHER*: State or imply non-modular equation $(x-2)^2 = \left(\frac{1}{3}x\right)^2$,

or pair of equations $x - 2 = \pm\frac{1}{3}x$ M1

Obtain answer $x = 3$ A1

Obtain answer $x = \frac{3}{2}$, or equivalent A1

OR: Obtain answer $x = 3$ by solving an equation or by inspection B1

State or imply the equation $x - 2 = -\frac{1}{3}$, or equivalent M1

Obtain answer $x = \frac{3}{2}$, or equivalent A1 [3]

11 *EITHER*: State or imply non-modular inequality $(4x + 3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$ M1

Obtain a critical value, e.g. –1 A1

Obtain a second critical value, e.g. $-\frac{3}{5}$ A1

State final answer $x < -1, x > -\frac{3}{5}$ A1

OR: Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection B1

Obtain the critical value $-\frac{3}{5}$ similarly B2

State final answer $x < -1, x > -\frac{3}{5}$ B1 [4]

[Do not condone ≤ or ≥.]

12 *EITHER:* State or imply non-modular inequality $(x + 2a)^2 > (3(x - a))^2$, or corresponding quadratic equation, or pair of linear equations $(x + 2a) = \pm 3(x - a)$ B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x M1

Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$ A1

State answer $\frac{1}{4}a < x < \frac{5}{2}a$ A1

OR: Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1

Obtain critical value $x = \frac{1}{4}a$ similarly B2

State answer $\frac{1}{4}a < x < \frac{5}{2}a$ B1 **4**

[Do not condone ⩽ for <.]

13 Either State or imply non-modular inequality $(3x-1)^2 < (2x+5)^2$ or corresponding quadratic equation or pair of linear equations $3x-1 = \pm(2x+5)$ B1

Solve a three-term quadratic or two linear equations $5x^2 - 26x - 24 < 0$ M1

Obtain $-\frac{4}{5}$ and 6 A1

State $-\frac{4}{5} < x < 6$ A1

Or Obtain value 6 from graph, inspection or solving linear equation B1

Obtain value $-\frac{4}{5}$ similarly B2

State $-\frac{4}{5} < x < 6$ B1 [4]

14 *EITHER*: State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation B1

Solve a 3-term quadratic, as in Q1. M1

Obtain critical value $x = \frac{5}{3}$ A1

State final answer $x < \frac{5}{3}$ only A1

*OR*1: State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation B1

Solve inequality or equation for x M1

Obtain critical value $x = \frac{5}{3}$ A1

State final answer $x < \frac{5}{3}$ only A1

*OR*2: Make recognisable sketches of $y = 2x - 3$ and $y = |x-2|$ on a single diagram B1

Find x-coordinate of the intersection M1

Obtain $x = \frac{5}{3}$ A1

State final answer $x < \frac{5}{3}$ only A1 **4**

15 *EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**

Obtain critical values -2 and $\frac{1}{4}$ **A1**

State final answer $-2 < x < \frac{1}{4}$ **A1**

OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality **B1**

Obtain critical value $x = \frac{1}{4}$ similarly **B2**

State final answer $-2 < x < \frac{1}{4}$ **B1** [4]

[Do not condone $\leqslant$ for $<$]

16 *EITHER*: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**

Obtain critical values $x = -5$ and $x = \frac{3}{5}$ **A1**

State final answer $-5 < x < \frac{3}{5}$ **A1**

OR: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality **(B1**

Obtain critical value $x = \frac{3}{5}$ similarly **B2**

State final answer $-5 < x < \frac{3}{5}$ **B1)**

[Do not condone ≤ for <.] [4]

Question		
17	**EITHER:** State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	**M1**
	Obtain critical values $x = 1$ and $x = 7$	**A1**
	State final answer $x < 1$ and $x > 7$	**A1)**
	OR: Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = 1$ similarly	**B2**
	State final answer $x < 1$ and $x > 7$	**B1)**
	Total:	**4**

18	*EITHER*: State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$, or corresponding equation	**(B1**
	Make reasonable attempt at solving a three term quadratic	**M1**
	Obtain critical value $x = \frac{7}{4}$	**A1**
	State final answer $x > \frac{7}{4}$ only	**A1)**
	*OR*1: State the relevant critical inequality $3 - x < 3x - 4$, or corresponding equation	**(B1**
	Solve for x	**M1**
	Obtain critical value $x = \frac{7}{4}$	**A1**
	State final answer $x > \frac{7}{4}$ only	**A1)**
	*OR*2: Make recognizable sketches of $y = \lvert x-3 \rvert$ and $y = 3x - 4$ on a single diagram	**(B1**
	Find x-coordinate of the intersection	**M1**
	Obtain $x = \frac{7}{4}$	**A1**
	State final answer $x > \frac{7}{4}$ only	**A1)**

Question		
19	*EITHER*: State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a) = \pm(x+3a)$	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	
	Obtain critical values $x = \frac{5}{3}a$ and $x = -\frac{1}{5}a$	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$	**A1**
	OR: Obtain critical value $x = \frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	
	Obtain critical value $x = -\frac{1}{5}a$ similarly	**B2**
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	
		4

Question	Answer	Marks	Guidance
20	State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	**B1**	$35x^2 - 44x - 7 = 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Allow for reasonable attempt at factorising e.g. $(5x-7)(7x+1)$
	Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	**A1**	Accept 1.4 and -0.143 or better for penultimate A mark
	State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	**A1**	'and' is A0, $\frac{7}{5} < x < -\frac{1}{7}$ is A0. Must be exact values. Must be strict inequalities in final answer
	Alternative		
	Obtain critical value $x = \frac{7}{5}$ from a graphical method	**B1**	or by inspection, or by solving a linear equation or an inequality
	Obtain critical value $x = -\frac{1}{7}$ similarly	**B2**	
	State final answer $x > \frac{7}{5}$ or $x < -\frac{1}{7}$ or equivalent	**B1**	[Do not condone $\geqslant$ for $>$, or $\leqslant$ for $<$.]
		4	

Question	Answer	Marks	Guidance
21	State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3) = \pm 4(x+1)$	B1	$12x^2 + 44x + 7 < 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Correct method seen, or implied by correct answers
	Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$	A1	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	A1	
	Alternative method for question 2		
	Obtain critical value $x = -\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{1}{6}$ similarly	B2	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	B1	
		4	

Question			
22	State or imply non-modular inequality $(x \quad 2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x + 2) = \pm (3x - 1)$	B1	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = -\frac{3}{5}$ and $x = 5$	A1	
	State final answer $-\frac{3}{5} < \quad < 5$	A1	
	Alternative method for question 1		
	Obtain critical value $x = 5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{3}{5}$ similarly	B2	
	State final answer $-\frac{3}{5} < x < 5$	B1	
		4	