ALEVELS P3
MARKING SCHEME

INTEGRATION WITH PARTIAL
I2

1	**(b)**	Stating or implying $f(x) \equiv \frac{A}{x+1} + \frac{B}{x-2}$, use a relevant method to determine A or B	M1	
		Obtain A = 1 and B = 2	A1	
		[SR: If A = 1 and B = 2 stated without working, award B1 + B1.]		
		Integrate and obtain terms $\ln(x + 1) + 2\ln(x - 2)$	A1√ + A1√	
		Use correct limits correctly in the complete integral	M1	
		Obtain given answer ln 5 following full and exact working	A1	**6**

2	(i)	*EITHER*: State or imply $f(x) \equiv \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A = 2$, $B = -1$, $C = 3$	A1	
		Obtain the remaining two values	A1 + A1	
		[A correct solution starting with third term $\frac{Cx}{(x+1)^2}$ or $\frac{Cx+D}{(x+1)^2}$ is also possible.]		
		OR: State or imply $f(x) \equiv \frac{A}{2x+1} + \frac{Dx+E}{(x+1)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A = 2$, $D = -1$, $E = 2$	A1	
		Obtain the remaining two values	A1 + A1	**5**
	(ii)	Integrate and obtain terms $\frac{1}{2}.2\ln(2x+1) - \ln(x+1) - \frac{3}{x+1}$, or equivalent	B1√ + B1√ + B1√	
		Use limits correctly, having integrated all the partial fractions	M1	
		Obtain given answer following full and exact working	A1	**5**
		[The f.t. is on A, B, C etc.]		
		[SR: If B, C, or E are omitted, give B1M1in part (i) and B1√B1√M1 in part (ii): max 5/10.]		

3	**(i)**	State or imply the form $A + \frac{B}{x+1} + \frac{C}{x+3}$	B1	
		State or obtain $A = 1$	B1	
		Use correct method for finding B or C	M1	
		Obtain $B = \frac{1}{2}$	A1	
		Obtain $C = -\frac{3}{2}$	A1	[5]
	(ii)	Obtain integral $x + \frac{1}{2}\ln(x+1) - \frac{3}{2}\ln(x+3)$	B2√	
		[Award B1√ if only one error. The f.t. is on A, B, C.]		
		Substitute limits correctly	M1	
		Obtain given answer following full and exact working	A1	[4]
		[SR: if A omitted, only M1 in part **(i)** is available, then in part **(ii)** B1√ for each correct integral and M1.]		

4	**(i)**	State or imply the form $\dfrac{A}{x+1}+\dfrac{B}{x+3}$ and use a relevant method to find A or B	M1	
		Obtain $A = 1$, $B = -1$	A1	[2]
	(ii)	Square the result of part **(i)** and substitute the fractions of part **(i)**	M1	
		Obtain the given answer correctly	A1	[2]
	(iii)	Integrate and obtain $-\dfrac{1}{x+1}-\ln(x+1)+\ln(x+3)-\dfrac{1}{x+3}$	B3	
		Substitute limits correctly in an integral containing at least two terms of the correct form	M1	
		Obtain given answer following full and exact working	A1	[5]
5	**(i)**	*EITHER*: Divide by denominator and obtain quadratic remainder	M1	
		Obtain $A = 1$	A1	
		Use any relevant method to obtain B, C or D	M1	
		Obtain one correct answer	A1	
		Obtain $B = 2$, $C = 1$ and $D = -3$	A1	
		OR: Reduce RHS to a single fraction and equate numerators, or equivalent	M1	
		Obtain $A = 1$	A1	
		Use any relevant method to obtain B, C or D	M1	
		Obtain one correct answer	A1	
		Obtain $B = 2$, $C = 1$ and $D = -3$	A1	[5]
		[SR: If $A = 1$ stated without working give B1.]		
	(ii)	Integrate and obtain $x+2\ln x-\dfrac{1}{x}-\dfrac{3}{2}\ln(2x-1)$, or equivalent	B3√	
		(The f.t. is on A, B, C, D. Give B2√ if only one error in integration; B1√ if two.)		
		Substitute limits correctly in the complete integral	M1	
		Obtain given answer correctly following full and exact working	A1	[5]
6		State or imply form $\dfrac{A}{2x+1}+\dfrac{B}{x+2}$	B1	
		Use relevant method to find A or B	M1	
		Obtain $\dfrac{4}{2x+1}-\dfrac{1}{x+2}$	A1	
		Integrate and obtain $2\ln(2x+1)-\ln(x+2)$ (ft on their A, B)	B1√B1√	
		Apply limits to integral containing terms $a\ln(2x+1)$ and $b\ln(x+2)$ and apply a law of logarithms correctly.	M1	
		Obtain given answer $\ln 50$ correctly	A1	[7]

7	**(i)**	Use any relevant method to determine a constant	M1	
		Obtain one of the values $A = 3$, $B = 4$, $C = 0$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[4]
	(ii)	Integrate and obtain term $-3\ln(2-x)$	B1√	
		Integrate and obtain term $k\ln(4+x^2)$	M1	
		Obtain term $2\ln(4+x^2)$	A1√	
		Substitute correct limits correctly in a complete integral of the form $a\ln(2-x)+b\ln(4+x^2)$, $ab \neq 0$	M1	
		Obtain given answer following full and correct working	A1	[5]
8		State or imply form $A+\dfrac{B}{2x+1}+\dfrac{C}{x+2}$	B1	
		State or obtain $A = 2$	B1	
		Use correct method for finding B or C	M1	
		Obtain $B = 1$	A1	
		Obtain $C = -3$	A1	
		Obtain $2x+\dfrac{1}{2}\ln(2x+1)-3\ln(x+2)$ [Deduct B1√ for each error or omission]	B3√	
		Substitute limits in expression containing $a\ln(2x+1)+b\ln(x+2)$	M1	
		Show full and exact working to confirm that $8+\dfrac{1}{2}\ln 9-3\ln 6+3\ln 2$, or an equivalent expression, simplifies to given result $8-\ln 9$	A1	[10]

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B0√B1√B1√M1A0 in (**ii**).]

[SR: For a solution starting with $\dfrac{M}{2x+1}+\dfrac{Nx}{x+2}$ or $\dfrac{Px}{2x+1}+\dfrac{Q}{x+2}$, give B0B0M1A0A0 in (**i**); B1√B1√B1√, if recover correct form, M1A0 in (**ii**).]

[SR: For a solution starting with $\dfrac{B}{2x+1}+\dfrac{Dx+E}{x+2}$, give M1A1 for one of $B = 1$, $D = 2$, $E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -3$.]

[SR: For a solution starting with $\dfrac{Fx+G}{2x+1}+\dfrac{C}{x+2}$, give M1A1 for one of $C = -3$, $F = 4$, $G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = 1$.]

9	(i)	State or imply the form $A+\dfrac{B}{x+1}+\dfrac{C}{2x-3}$	B1	
		State or obtain $A = 2$	B1	
		Use a correct method for finding a constant	M1	
		Obtain $B = -2$	A1	
		Obtain $C = -1$	A1	[5]
	(ii)	Obtain integral $2x - 2\ln(x+1) - \dfrac{1}{2}\ln(2x-3)$	B3$\sqrt{}$	
		(Deduct B1$\sqrt{}$ for each error or omission. The f.t. is on A, B, C.)		
		Substitute limits correctly in an expression containing terms $a\ln(x+1)$ and $b\ln(2x-3)$	M1	
		Obtain the given answer following full and exact working	A1	[5]
		[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B1$\sqrt{}$B1$\sqrt{}$M1A0 in (**ii**).]		
		[SR: For a solution starting with $\dfrac{B}{x+1}+\dfrac{Dx+E}{2x-3}$, give M1A1 for one of $B = -2$, $D = 4$, $E = -7$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -1$.]		
		[SR: For a solution starting with $\dfrac{Fx+G}{x+1}+\dfrac{C}{2x-3}$ or with $\dfrac{Fx}{x+1}+\dfrac{C}{2x-3}$, give M1A1 for one of $C = -1$, $F = 2$, $G = 0$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = -2$.]		
10	(i)	Use a correct method for finding a constant	M1	
		Obtain one of $A = 3$, $B = 3$, $C = 0$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	**4**
	(ii)	Integrate and obtain term $-3\ln(2-x)$	B1$\sqrt{}$	
		Integrate and obtain term of the form $k\ln(2+x^2)$	M1	
		Obtain term $\frac{3}{2}\ln(2+x^2)$	A1$\sqrt{}$	
		Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab \neq 0$	M1	
		Obtain given answer after full and correct working	A1	**5**
11	(i)	State or imply $f(x) \equiv \dfrac{A}{2x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A = 2$, $B = -1$, $C = 3$	A1	
		Obtain the remaining values A1 +	A1	**5**
		[Apply an analogous scheme to the form $\dfrac{A}{2x-1}+\dfrac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1$, $E = 1$.]		
	(ii)	Integrate and obtain terms $\dfrac{1}{2}\cdot 2\ln(2x-1) - \ln(x+2) - \dfrac{3}{x+2}$	B1$\sqrt{}$ + B1$\sqrt{}$ + B1$\sqrt{}$	
		Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG.	M1	
		Obtain the given answer following full and exact working	A1	**5**
		[The t marks are dependent on A, B, C etc.]		
		[SR: If B, C or E omitted, give B1M1 in part (**i**) and B1$\sqrt{}$B1$\sqrt{}$M1 in part (**ii**).]		
		[NB: Candidates who follow the A, D, E scheme in part (**i**) and then integrate $\dfrac{-x+1}{(x+2)^2}$ by parts should obtain $\dfrac{1}{2}\cdot 2\ln(2x-1) - \ln(x+2) + \dfrac{x-1}{x+2}$ (the third term is equivalent to $-\dfrac{3}{x+2}+1$).]		

12 **(i)** State or imply the form $\dfrac{A}{x+1}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$ **B1**

Use a correct method to determine a constant **M1**

Obtain one of the values $A = 1$, $B = 3$, $C = 12$ **A1**

Obtain a second value **A1**

Obtain a third value **A1**

[5]

[Mark the form $\dfrac{A}{x+1}+\dfrac{Dx+E}{(x-3)^2}$, where $A = 1$, $D = 3$, $E = 3$, B1M1A1A1A1 as above.]

(ii) Use correct method to find the first two terms of the expansion of $(x+1)^{-1}, (x-3)^{-1}, (1-\frac{1}{3}x)^{-1}$,

$(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$ **M1**

Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction **A1√ + A1√ + A1√**

Obtain final answer $\frac{4}{3}-\frac{4}{9}x+\frac{4}{3}x^2$, or equivalent **A1**

[5]

13 **(i)** State or imply the form $A+\dfrac{B}{2x+1}+\dfrac{C}{x+2}$ **B1**

State or obtain $A = 2$ **B1**

Use a correct method for finding a constant **M1**

Obtain one of $B = 1$, $C = -2$ **A1**

Obtain the other value **A1** [5]

(ii) Integrate and obtain terms $2x+\frac{1}{2}\ln(2x+1)-2\ln(x+2)$ **B3√**

Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$ **M1**

Obtain the given answer after full and correct working **A1** [5]

[2]

14 **(i)** State or imply the form $\dfrac{A}{x+3}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ **B1**

Use a correct method to determine a constant **M1**

Obtain one of the values $A = -3$, $B = 1$, $C = 2$ **A1**

Obtain a second value **A1**

Obtain the third value **A1**

[Mark the form $\dfrac{A}{x+3}+\dfrac{Dx+E}{(x-1)^2}$, where $A = -3$, $D = 1$, $E = 1$, B1M1A1A1A1 as above.] [5]

(ii) Use a correct method to find the first two terms of the expansion of $(x+3)^{\ 1}, (1+\frac{1}{3}x)^{-1}$,

$(x-1)^{-1}$, $(1-x)^{-1}$, $(x-1)^{-2}$, or $(1-x)^{-2}$ **M1**

Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction **A1√ + A1√ + A1√**

Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2$, or equivalent **A1**

[5]

15	(i)	State or imply the form $\dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+4}$	B1	
		Use a correct method to determine a constant	M1	
		Obtain one of $A = 2$, $B = 1$, $C = -1$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	[5]
	(ii)	Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√ + A1√	
		Multiply out fully by $Bx + C$, where $BC \neq 0$	M1	
		Obtain final answer $\frac{3}{4}-\frac{1}{4}x+\frac{5}{16}x^2$, or equivalent	A1	[5]
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on A, B, C.]		
		[In the case of an attempt to expand $(3x^2+x+6)(x+2)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		

Question		
16(i)	State or imply the form $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{3x+2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 3$, $B = -2$, $C = -6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\dfrac{Ax+B}{x^2}+\dfrac{C}{3x+2}$ using same pattern of marks.]	**A1**
	Total:	**5**

16(ii)	Integrate and obtain terms $3\ln x = \frac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x + \frac{2}{x} - 3$ or equivalent	**B3 FT**
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x + \frac{b}{x} + c\ln(3x+2)$	**M1**
	Obtain the given answer following full and exact working	**A1**
	Total:	**5**

Question		
17(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
17(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question	Answer	Marks
18(i)	State or imply the form $\dfrac{A}{2-x} + \dfrac{B}{3\quad 2} + \dfrac{C}{(3+2\quad)^2}$	**B1**
	Use a correct method to find a constant	**M1**
	Obtain one of $A = 1$, $B = -1$, $C = 3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\dfrac{\quad}{2-\quad} + \dfrac{Dx+E}{(3+2\quad)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	**A1**
5		
18(ii)	Integrate and obtain terms $-\ln(2-\quad) - \dfrac{1}{2}\ln(3+2x) - \dfrac{3}{2(3+2\quad)}$	**B3ft**
	Substitute correctly in an integral with terms $a \ln(2-x)$, $b \ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	**M1**
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	**A1**
		5

Question	Answer	Marks
19(i)	State or imply the form $\dfrac{A}{2x+1}$ + $\dfrac{B}{2x+3}$ + $\dfrac{C}{(2x+3)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain the values $A = 1$, $B = -1$, $C = 3$	A1 A1 A1
	[Mark the form $\dfrac{A}{2x+1}$ + $\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	
		5
19(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1)-\frac{1}{2}\ln(2x+3)-\dfrac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1)+\dfrac{x}{2x+3}-\frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	B1 B1 B1
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	M1
	Obtain the **given answer** following full and correct working	A1
		5

Question	Answer	Marks
20(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^{2}}+\frac{C}{x-2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A = -1$, $B = 3$, $C = 2$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5

Question	Answer	Marks
20(ii)	Integrate and obtain terms $\ln x \quad \frac{3}{x} + 2\ln(\ x+2)$	**B1FT + B1FT + B1FT**
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{\ }$ and $c\ln \ \ x \quad 2$, where $abc \neq 0$	**M1**
	Obtain $\frac{9}{4}$ following full and exact working	**A1**
		5

Question	Answer	Marks
21(i)	State or imply the form $\frac{\ }{2 \quad 1} + \frac{x \quad C}{x^2 \quad 2}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A = 4$, $B = -1$, $C = 0$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
21(ii)	Integrate and obtain term $2\ln \ \ 2x \quad 1)$	**B1FT**
	Integrate and obtain term of the form $k\ln \ \ x^2 \quad 2)$	***M1**
	Obtain term $-\frac{1}{2}\ln\left(x^2+2\right)$	**A1FT**
	Substitute limits correctly in an integral of the form $a\ln \ \ 2x \quad 1) + b\ln(x^2+2)$, where $ab \neq 0$	**DM1**
	Obtain answer ln 27 after full and correct exact working	**A1**
		5