POLYNOMIALS

P3

(5-6 MARK)

BASIC

ADVANCED WITH COMPLEX NUMBERS

$\underbrace{\text{POLY}}_{\text{MANY}}\ \underbrace{\text{NOM}}_{\text{TERM}}\ \text{IALS.}$

$p(x) = 2x^2 + 6x + 1$

$p(1) = 2(1)^2 + 6(1) + 1 =$ ______

LONG DIVISION

Q. Find remainder when $p(x) = 2x^4 - 6x^2 + 5$ is divided by $x - 3$.

$$
\begin{array}{r|l}
 & 2x^3 + 6x^2 + 12x + 36 \\
\hline
x-3 & \cancel{2x^4} + 0x^3 - 6x^2 + 0x + 5 \\
 & -\cancel{2x^4} \mp 6x^3 \\
\hline
 & \cancel{6x^3} - 6x^2 + 0x + 5 \\
 & -\cancel{6x^3} \mp 18x^2
\end{array}
$$

Q = inner first / outer first.

$\frac{2x^4}{x} = 2x^3$

$\frac{6x^3}{x} = 6x^2$

$$\begin{array}{r} 12\cancel{x^2} + 0x + 5 \\ -\cancel{12x^2} \mp 36x \\ \hline 36\cancel{x} + 5 \\ -\cancel{36x} \mp 108 \\ \hline 113 \\ \hline \end{array}$$

x

$\frac{12x^2}{x} = 12x$

$\frac{36x}{x} = 36$

$\boxed{\text{REMAINDER THEOREM}}$

If $p(x)$ is divided by $(x-a)$ then the remainder is $p(a)$.

$x - a = 0$

$\boxed{x = a}$

Q. Find remainder when $p(x) = 2x^4 - 6x^2 + 5$ is divided by $x-3$.

$p(x) = 2x^4 - 6x^2 + 5 \quad \div \quad x-3$

$p(3) = 2(3)^4 - 6(3)^2 + 5$

$x - 3 = 0$

$x = 3$

$p(3) = 113$ (Remainder).

FACTOR THEOREM

If $(x-a)$ is a factor of $p(x)$,

$x-a=0$

$x=a$

then $\underbrace{p(a)}_{\text{Remainder}=0} = 0$

4 is factor 12

Y / N

$$\begin{array}{r} 3 \\ 4\overline{)12} \\ -12 \\ \hline 0 \end{array}$$

For a factor

Rem = 0

Q. $p(x) = 2x^3 - ax + 3$ has a factor $(x-2)$. Find a.

$p(2) = 0$ (remainder)

$x-2=0$

$x=2$

$2(2)^3 - a(2) + 3 = 0$

$16 - 2a + 3 = 0$

$2a = 19$

$a = 9.5$

DIVISOR / FACTOR

- LINEAR
  - REMAINDER AND FACTOR THEOREM
- NON-LINEAR
  - LONG DIVISION

3 The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

$p(x) = x^3 - 2x + a$

factor: $x + 2$

$x + 2 = 0$

$x = -2$

$p(-2) = 0$

$0 = (-2)^3 - 2(-2) + a$

$0 = -8 + 4 + a$

$\boxed{a = 4}$

4 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $x^2 + x + 2$ is a factor of $\mathrm{p}(x)$. Find the value of a and the other quadratic factor of $\mathrm{p}(x)$. [4]

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$p(x) = x^4 + 3x^2 + a$

Factor: $x^2 + x + 2$

Long Division

$$
\begin{array}{r|l}
 & x^2 - x + 2 \\
\hline
x^2+x+2 & \cancel{x^4} + 0x^3 + 3x^2 + 0x + a \\
 & -\cancel{x^4} \pm x^3 \pm 2x^2 \\
\hline
 & -\cancel{x^3} + x^2 + 0x + a \\
 & \mp \cancel{x^3} \mp x^2 \mp 2x \\
\hline
 & \cancel{2x^2} + \cancel{2x} + a \\
 & -\cancel{2x^2} \pm \cancel{2x} \pm 4 \\
\hline
 & a - 4
\end{array}
$$

$Q = \dfrac{x^4}{x^2} = x^2$

$Q = \dfrac{-x^3}{x^2} = -x$

$Q = \dfrac{2x^2}{x^2} = 2$

Since x^2+x+2 is factor, Remainder should be zero

$a - 4 = 0$

$\boxed{a = 4}$

Other factor $= x^2 - x + 2$

9 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

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$p(x) = ax^3 + bx^2 + 5x - 2$

factor: $2x - 1$

$2x - 1 = 0$

$x = \frac{1}{2}$

$p\left(\frac{1}{2}\right) = 0$

$0 = a\left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 2$

$\frac{a}{8} + \frac{b}{4} + \frac{5}{2} - 2 = 0$

$a + 2b + 20 - 16 = 0$

$\boxed{a + 2b = -4}$

Divisor: $x - 2$

Remainder: 12

$x - 2 = 0$

$x = 2$

$p(2) = 12$

$12 = a(2)^3 + b(2)^2 + 5(2) - 2$

$12 = 8a + 4b + 10 - 2$

$4 = 8a + 4b$

$\boxed{2a + b = 1}$

$a = -2b - 4$

$2(-2b - 4) + b = 1$

$-4b - 8 + b = 1$

$-3b = 9$

$\boxed{b = -3}$

$a = -2(-3) - 4$

$\boxed{a = 2}$

(ii) $p(x) = 2x^3 - 3x^2 + 5x - 2$ Factor: $2x-1$

Max $= 3$ Lin $= 1$ Rem $= 2$

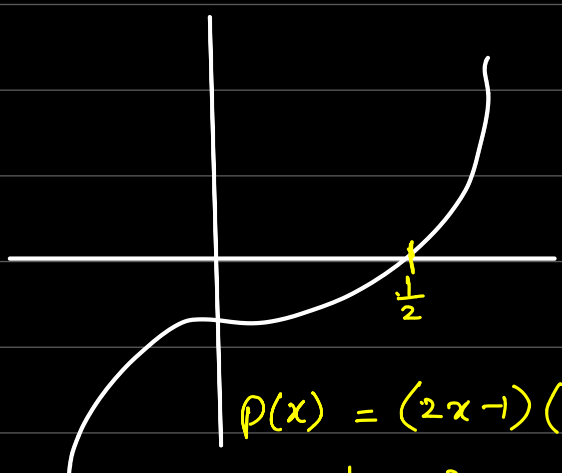
$$2x^3 - 3x^2 + 5x - 2 \equiv (2x-1)(ax^2+bx+c)$$

max power	Constants	x term or x^2 term Middle Term
$2x^3 = 2ax^3$	$-2 = -1c$	$-3x^2 = 2bx^2 - ax^2$
$2 = 2a$		$-3 = 2b - a$
$\boxed{a=1}$	$\boxed{c=2}$	$-3 = 2b - 1$
		$-2 = 2b$
		$\boxed{b=-1}$

$(2x-1)(ax^2+bx+c)$

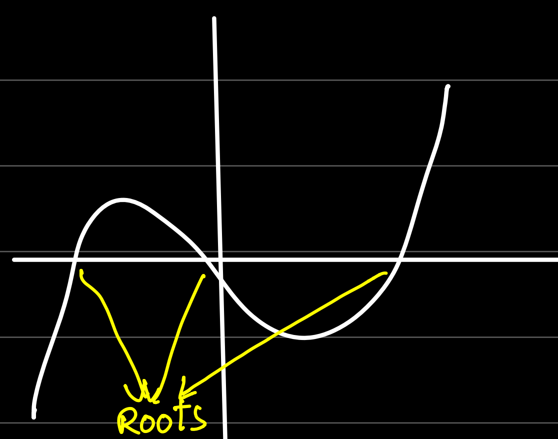
$a=1,\ b=-1,\ c=2$

$(2x-1)(x^2-x+2)$



$p(x) = (2x-1)(x^2-x+2)$

$2x-1=0$	$x^2-x+2=0$
$x=\frac{1}{2}$	No real Roots.



$p(x) = (x-1)(x-2)(x-5)$

$0 = (x-1)(x-2)(x-5)$

$x=1,\ x=2,\ x=5$

Q $p(x) = 8x^3 + 6x^2 - 3x - 1$ has factor $(x+1)$
Factorize $p(x)$ completely.

$$8x^3 + 6x^2 - 3x - 1 \equiv (x+1)(ax^2 + bx + c)$$

Max	Constant	Mid Term (x-term)
$8x^3 = ax^3$	$-1 = 1c$	$-3x = cx + bx$
		$-3 = c + b$
$\boxed{a = 8}$	$\boxed{c = -1}$	$-3 = -1 + b$
		$b = -2$

$$p(x) = (x+1)(8x^2 - 2x - 1)$$
$$(x+1)[8x^2 - 4x + 2x - 1]$$
$$(x+1)[4x(2x-1) + 1(2x-1)]$$
$$p(x) = (x+1)(4x+1)(2x-1)$$