



ZAIÑEMATICS

FOR THE LOVE OF MATHS

A - Level

M1 Notes

By

Zain Afaq

Compiled By Rafay Mushtaq



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ZAIÑEMATICS

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**3| Forces &
Equilibrium**

8| Tension

17| Friction

**27| Work, Power
& Energy**

38| Momentum

44| Kinematics



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Forces & Equilibrium M1

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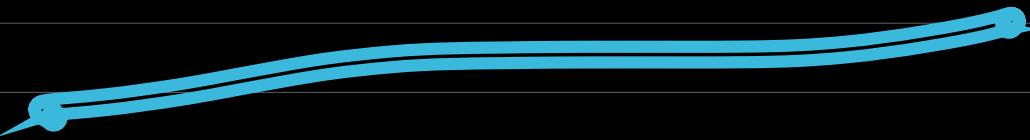
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M1

Welcome!



M1 contains diff and integration of P1 only.

M1 P4 (50 Marks) (1h 15m) (7 to 8 Questions)

(VERY EASY PAPER EVEN FOR NON-SCIENCE PPL.)

HIGH PERCENTILE : 42 - 46 out of 50.

(TOTAL 12 WORKINGS)

FORCES

PUSH/PULL → FORCE → UNITS (NEWTON)



SPECIAL FORCE : **WEIGHT**

~~75kg~~ → Mass

Pull of gravity on an object .

$$w = 75(10)$$

$$w = 750 \text{ N}$$

$$W = m g$$

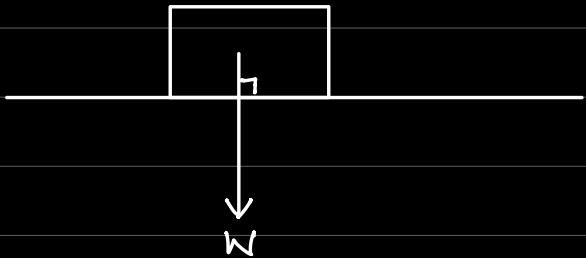
$$\text{mass in (kg)} \quad g = 10.$$

g = acceleration due to gravity = 10 N/kg
 10 m/s^2

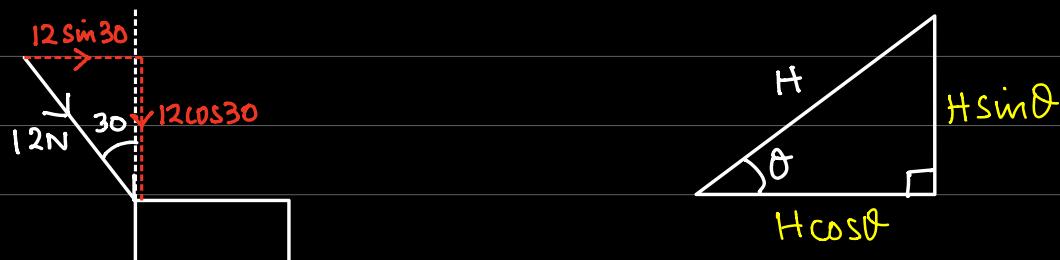
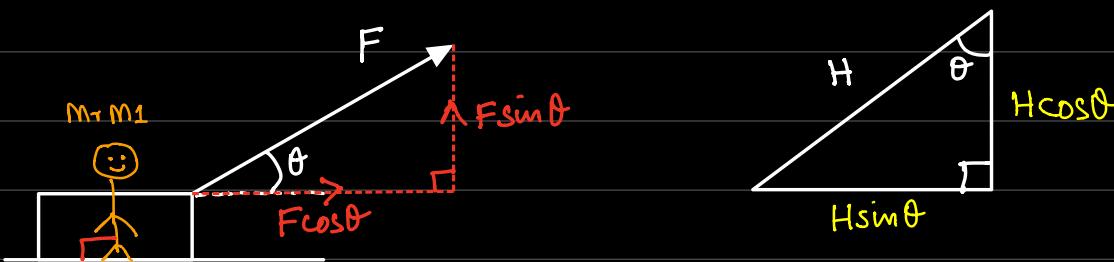
(In whole M1, $g=10$) (In Physics $g=9.81$)

DIRECTION: TOWARDS GROUND MAKING 90° WITH GROUND (HORIZONTAL).

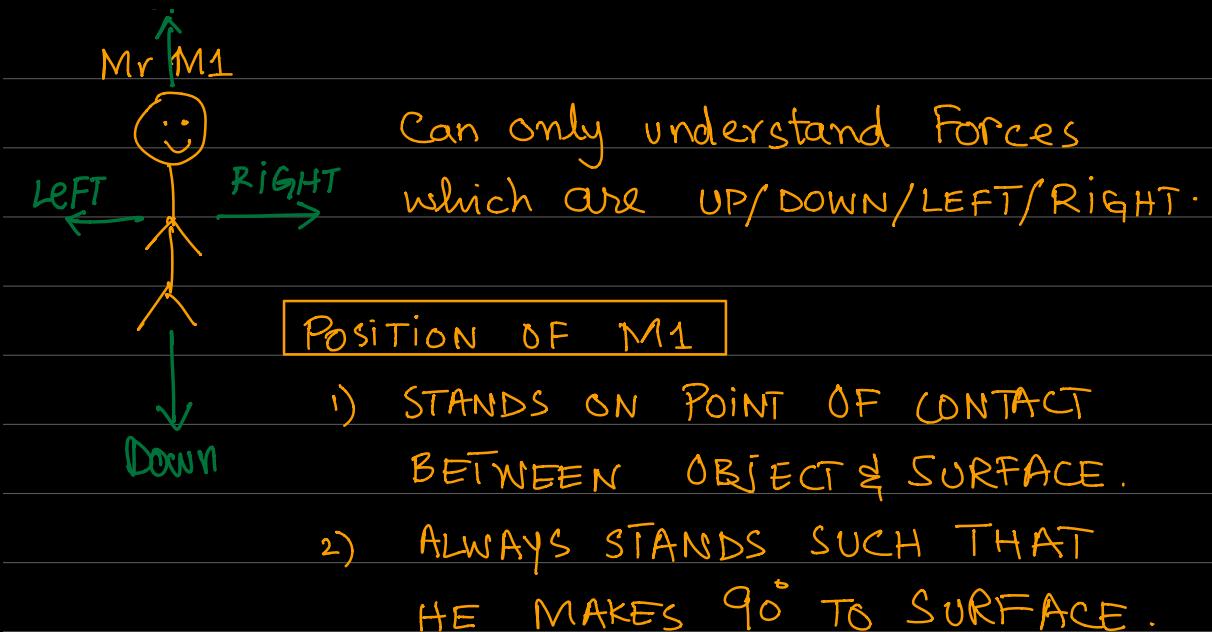
Line of weight starts from centre of object.



COMPONENTS OF A FORCE:-

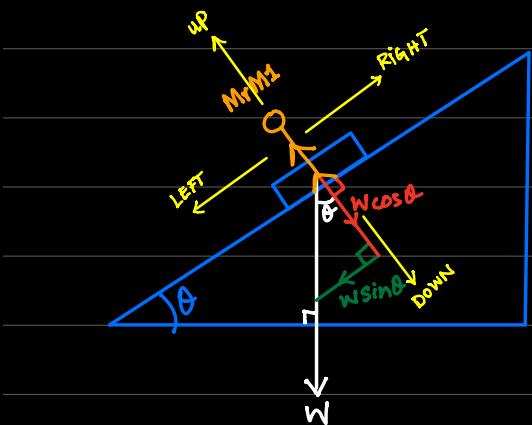


UP



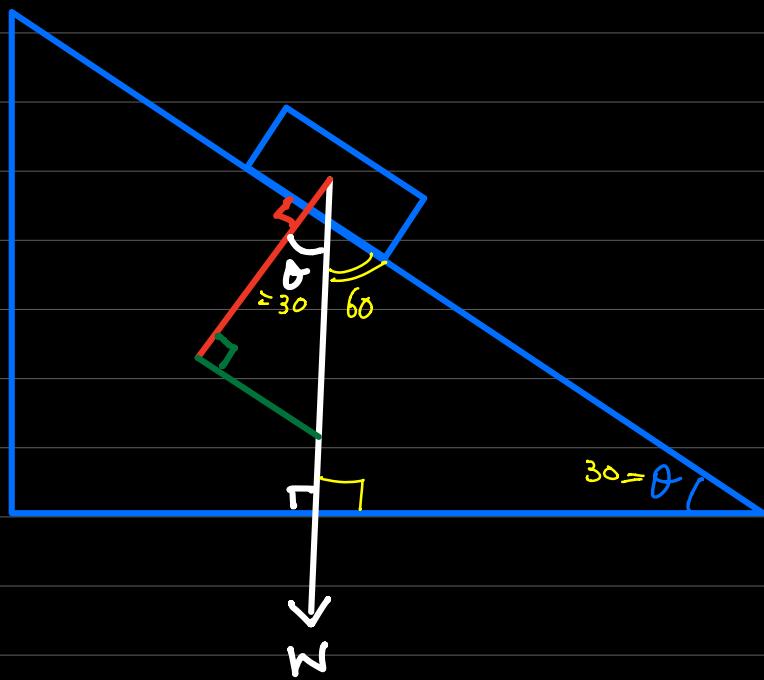
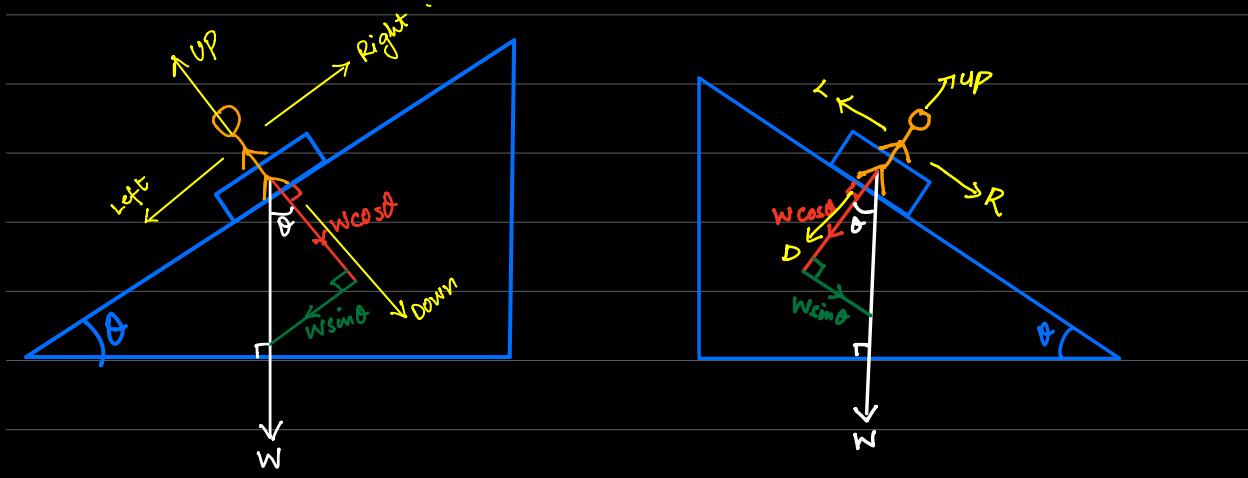
IN COMPLETE M1 THE WORDS UP/ DOWN / LEFT/ RIGHT ARE USED FROM MR M1's PERSPECTIVE.

INCLINED PLANES:



THREE LINES.

- 1) Weight (90° to ground)
- 2) 90° TO PLANE
- 3) PARALLEL TO PLANE.





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Tension M1

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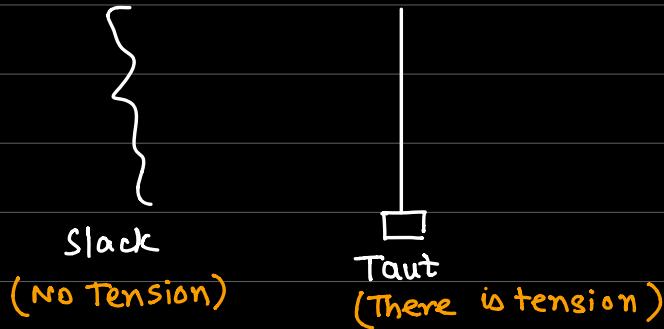
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TENSION

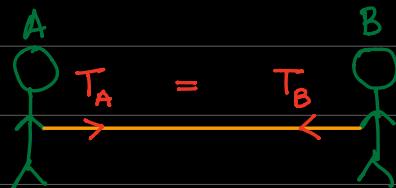
FORCE APPLIED BY THE ROPE/STRING ON AN OBJECT.



DIRECTION OF TENSION:

ACTS AWAY FROM POINT OF OBSERVATION

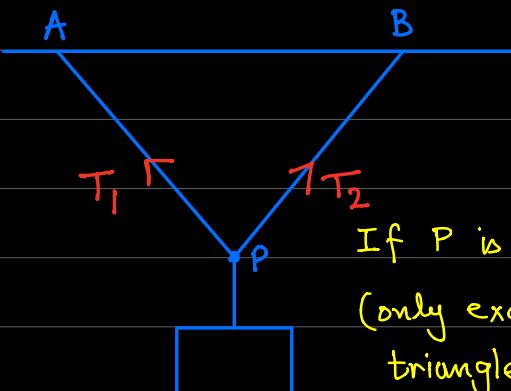
POINT OF OBSERVATION IS MENTIONED IN QUESTION.



Rope is Taut.



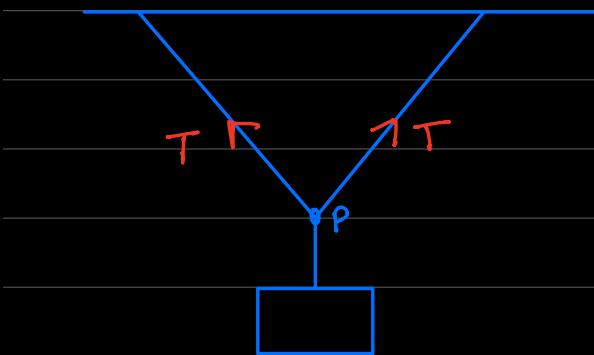
Box is AT REST.



P is a fixed point.

P is point of observation.

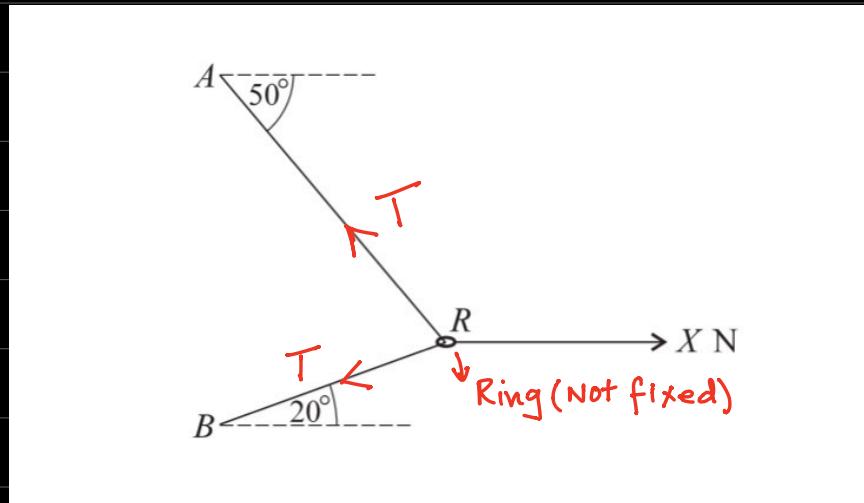
If P is fixed T_1 and T_2 are different
(only exception is if there is an isosceles triangle. In that case only $T_1 = T_2$).



P is a smooth ring.
String is threaded through the ring.
(Ring can slip).

P = Point of observation.

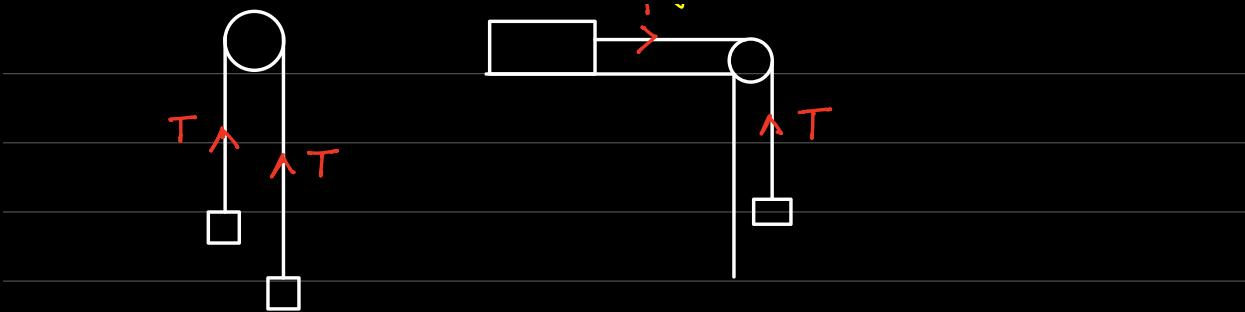
IF POINT P can slip, BOTH TENSIONS ARE
ALWAYS SAME REGARDLESS OF SHAPE OF
DIAGRAM.



PULLEYS / PEG :-

IN ANY SHAPE, TENSION IN A PULLEY
SYSTEM IS SAME AND ACTS TOWARDS
THE PULLEY.

Even if weights are different, objects are at
rest or are moving.



Objects are points of observation here.

WORKING #1 EQUILIBRIUM

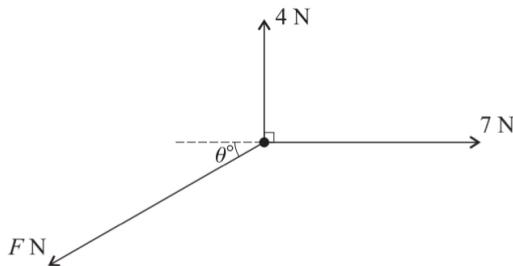
(BALANCED) (STATIONARY) (REST)

UP = down

Left = Right.

(UP, DOWN, LEFT, RIGHT is FROM Mr M1's PERSPECTIVE)

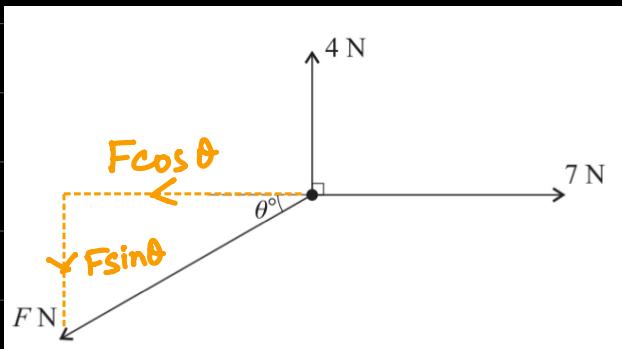
8



A particle is in **equilibrium** on a **smooth** horizontal table when acted on by the three horizontal forces shown in the diagram.
→ NO FRICTION

- (i) Find the values of F and θ .

[4]



up = down

Left = Right.

$$4 = F \sin \theta$$

$$F \sin \theta = 4$$

$$F \cos \theta = 7$$

THIS TYPE OF SIMULTANEOUS IS SPECIAL.

$\left\{ \begin{array}{l} F \sin \theta = \square \\ F \cos \theta = \square \end{array} \right\}$ THIS WORKING COMES
 IN P3 AND M1

STEP 1

$$\frac{F \sin \theta}{F \cos \theta} = \frac{4}{7}$$

$$\tan \theta = \frac{4}{7}$$

STEP 2 SQUARE BOTH EQUATIONS AND ADD.

$$\begin{aligned}
 F^2 \sin^2 \theta &= 16 \\
 + F^2 \cos^2 \theta &= 49 \\
 \hline
 F^2 \sin^2 \theta + F^2 \cos^2 \theta &= 16 + 49
 \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{4}{7}\right)$$

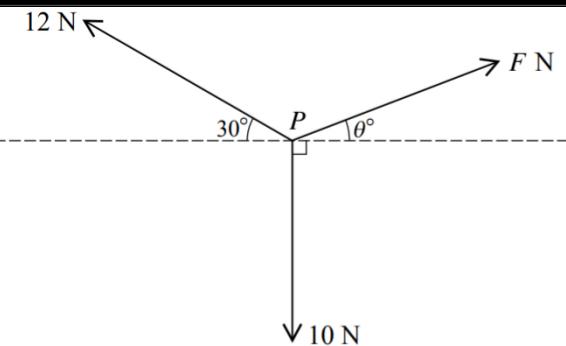
$$\theta = 29.74$$

$$F^2 (\sin^2 \theta + \cos^2 \theta) = 65$$

$$F^2 (1) = 65$$

$$F^2 = 65$$

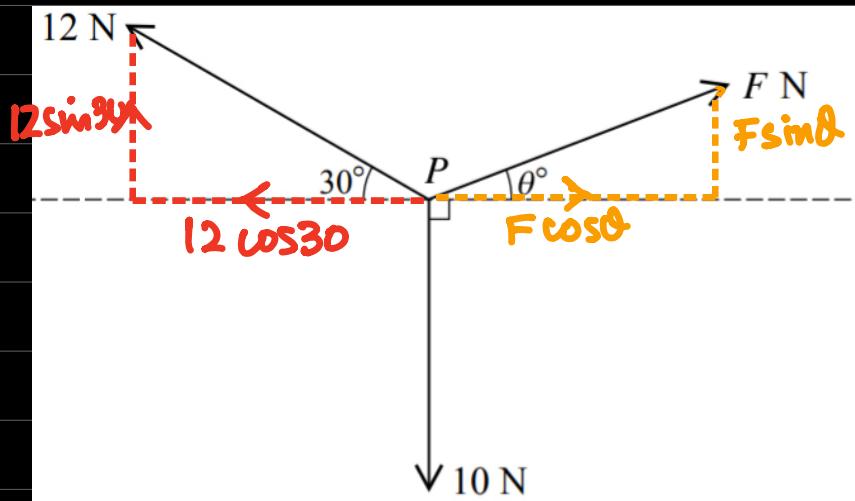
$$F = \sqrt{65} = 8.0622 .$$



The three **coplanar** forces shown in the diagram act at a point P and are in **equilibrium**.

- (i) Find the values of F and θ .

[6]



up = down

$$F \sin \theta + 12 \sin 30 = 10$$

$$F \sin \theta = 4$$

Left = Right

$$12 \cos 30 = F \cos \theta$$

$$F \cos \theta = 12 / \sqrt{3}$$

(2)

STEP1

$$\frac{F \sin \theta}{F \cos \theta} = \frac{4}{6\sqrt{3}}$$

$$\tan \theta = \frac{2}{3\sqrt{3}}$$

$$\theta = \tan^{-1} \left(\frac{2}{3\sqrt{3}} \right)$$

$$\theta = 21.05^\circ$$

STEP2

$$\begin{aligned} F^2 \sin^2 \theta &= 16 \\ + F^2 \cos^2 \theta &= 108 \end{aligned}$$

$$F^2 \sin^2 \theta + F^2 \cos^2 \theta = 124$$

$$F^2 (\sin^2 \theta + \cos^2 \theta) = 124$$

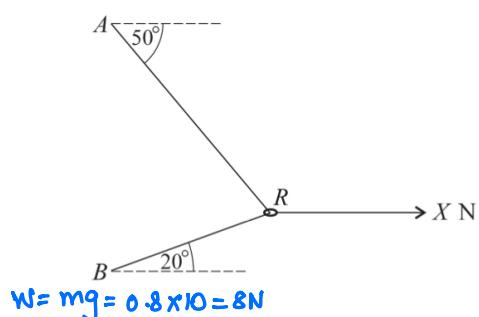
$$F^2 (1) = 124$$

$$F^2 = 124$$

$$F = \sqrt{124}$$

$$F = 11.1355$$

1



A light inextensible string has its ends attached to two fixed points A and B, with A vertically above B. A smooth ring R, of mass 0.8 kg, is threaded on the string and is pulled by a horizontal force of magnitude X newtons. The sections AR and BR of the string make angles of 50° and 20° respectively with the horizontal, as shown in the diagram. The ring rests in equilibrium with the string taut. Find

(i) the tension in the string,

↓
point of observation.

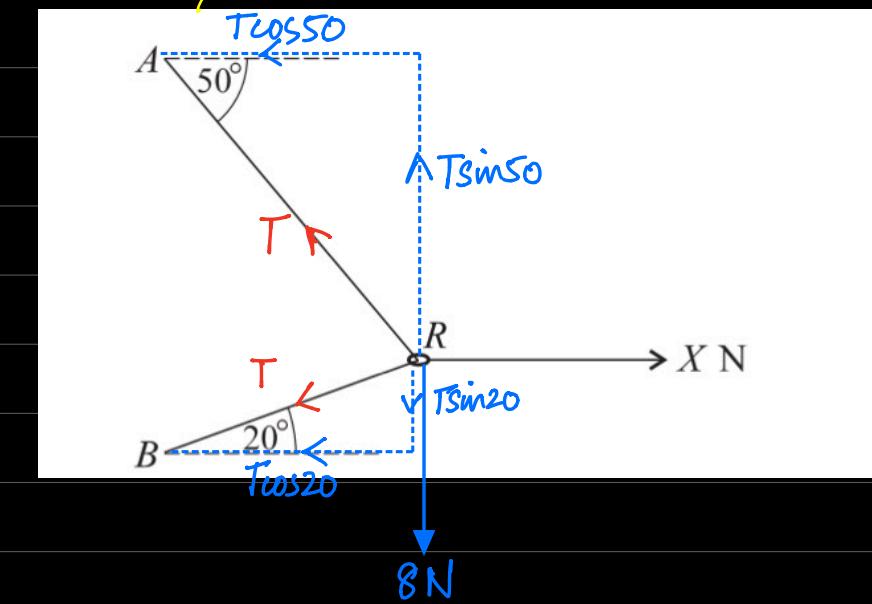
[3]

(ii) the value of X.

[3]

NEVER MARK ANY FORCES ON THE
DIAGRAM OF QUESTION.

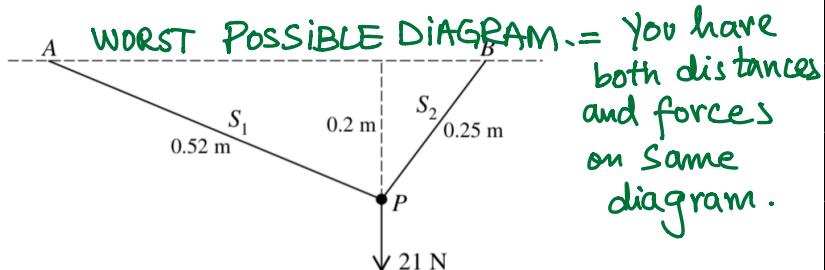
DRAW A HUGE DIAGRAM FOR
YOURSELF.



$$\begin{aligned}
 \text{up} &= \text{down} \\
 Ts\sin 50 &= 8 + Ts\sin 20 \\
 Ts\sin 50 - Ts\sin 20 &= 8 \\
 T(\sin 50 - \sin 20) &= 8 \\
 T &= \frac{8}{\sin 50 - \sin 20} \\
 T &= 18.87.
 \end{aligned}$$

$$\begin{aligned}
 \text{Left} &= \text{Right} \\
 T\cos 50 + T\cos 20 &= X \\
 T(\cos 50 + \cos 20) &= X \\
 18.87(\cos 50 + \cos 20) &= X \\
 X &= 29.86.
 \end{aligned}$$

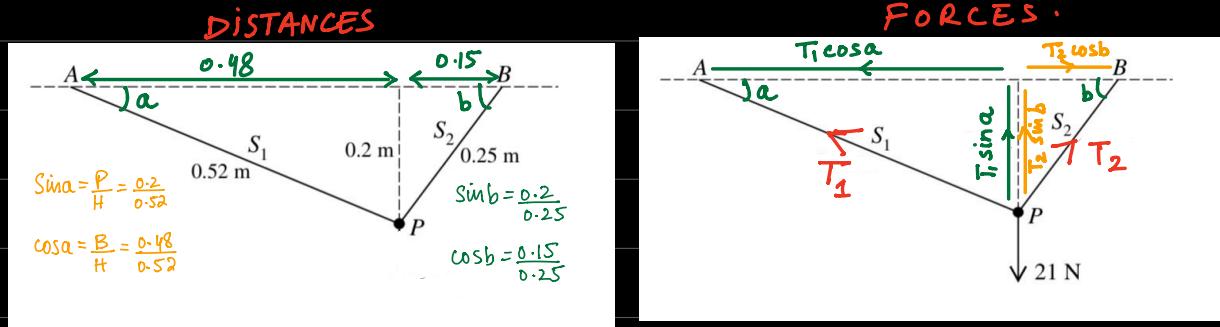
26



A particle P of weight 21 N is attached to one end of each of two light inextensible strings, S_1 and S_2 , of lengths 0.52 m and 0.25 m respectively. The other end of S_1 is attached to a fixed point A , and the other end of S_2 is attached to a fixed point B at the same horizontal level as A . The particle P hangs in equilibrium at a point 0.2 m below the level of AB with both strings taut (see diagram). Find the tension in S_1 and the tension in S_2 .

[6]

EXAM TIP : DRAW SEPERATE DIAGRAMS FOR DISTANCES AND FORCES.



STAY IN FRACTIONS THROUGHOUT QUESTION.

up = down

Left = Right.

$$T_1 \sin \alpha + T_2 \sin \beta = 21$$

$$T_1 \cos \alpha = T_2 \cos \beta$$

$$T_1 \left(\frac{0.2}{0.52} \right) + T_2 \left(\frac{0.2}{0.25} \right) = 21$$

$$T_1 \left(\frac{0.48}{0.52} \right) = T_2 \left(\frac{0.15}{0.25} \right)$$

$$\boxed{\frac{5}{13} T_1 + \frac{4}{5} T_2 = 21}$$

$$\frac{12}{13} T_1 = \frac{3}{5} T_2$$

Solving Simultaneously.

$$\boxed{T_1 = \frac{13}{20} T_2}$$

$$\frac{5}{13} \left(\frac{13}{20} T_2 \right) + \frac{4}{5} T_2 = 21$$

$$T_1 = \frac{13}{20} \times 20$$

$$\frac{1}{4} T_2 + \frac{4}{5} T_2 = 21$$

$$\frac{21}{20} T_2 = 21$$

$$\boxed{T_1 = 13}$$

$$\boxed{T_2 = 20}$$



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Friction

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FRICITION

$$F = \mu R$$

Maximum Friction. Coefficient of friction. Normal Reaction force

This formula can only be applied if body is **ABOUT TO MOVE** or **MOVING**
 This formula cannot be used if a body is at **REST**

TYPE OF SURFACE

Coefficient of Friction (μ)

Slippery surface ($\mu \downarrow$) (Friction \downarrow)

Rough surface ($\mu \uparrow$) (Friction \uparrow)

CONTACT FORCE

NORMAL FORCE

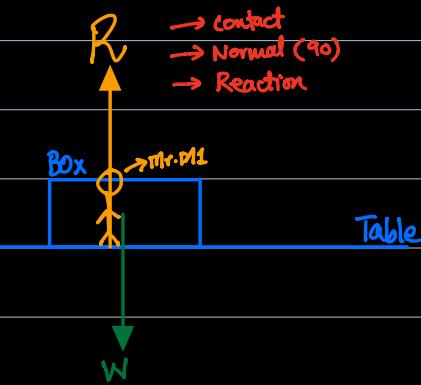
REACTION FORCE

R

Always Find R using

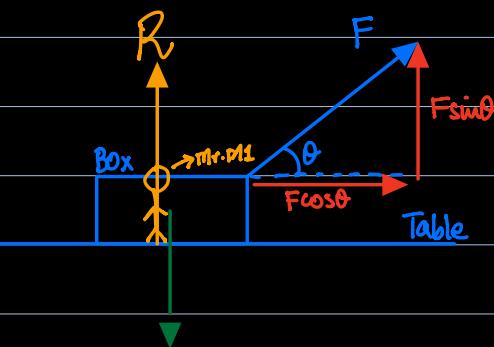
UP = DOWN

R acts towards head of Mr. D1.



up = down

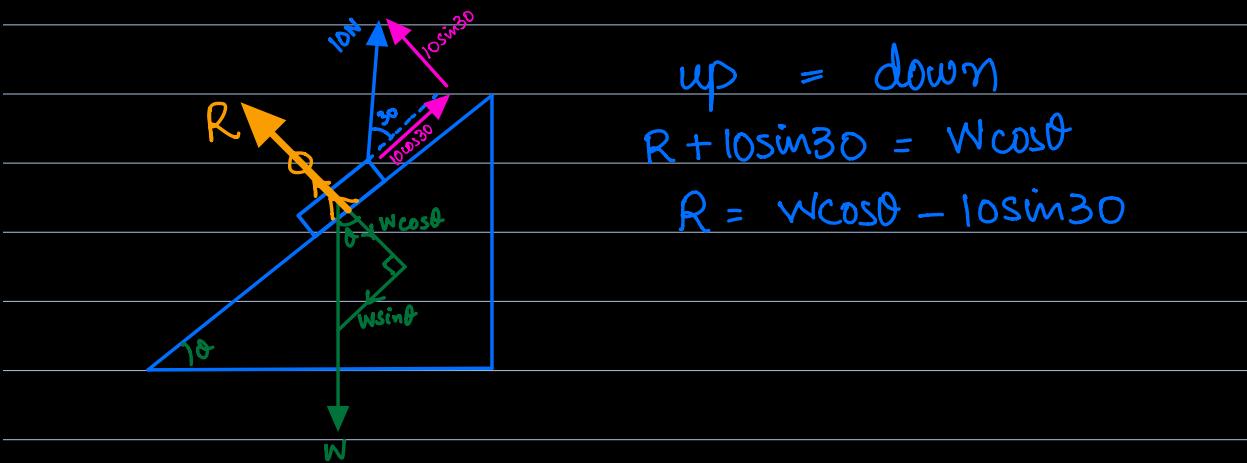
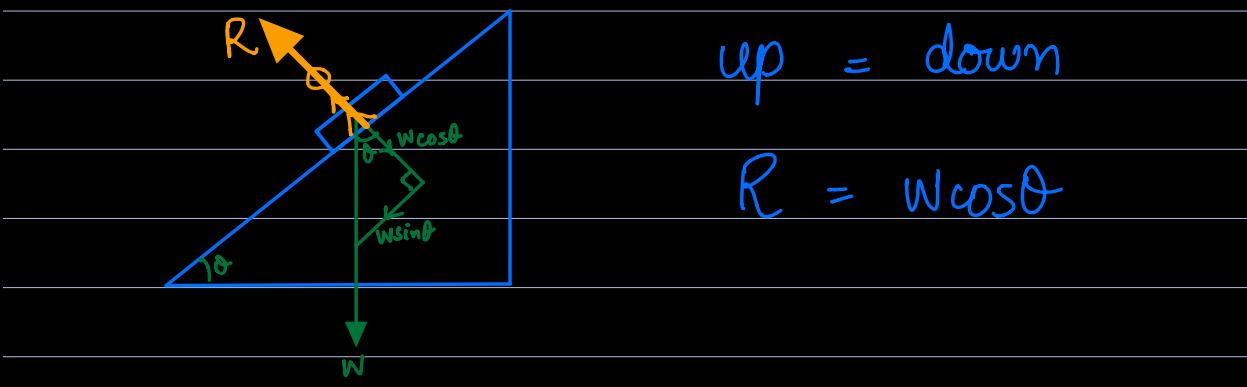
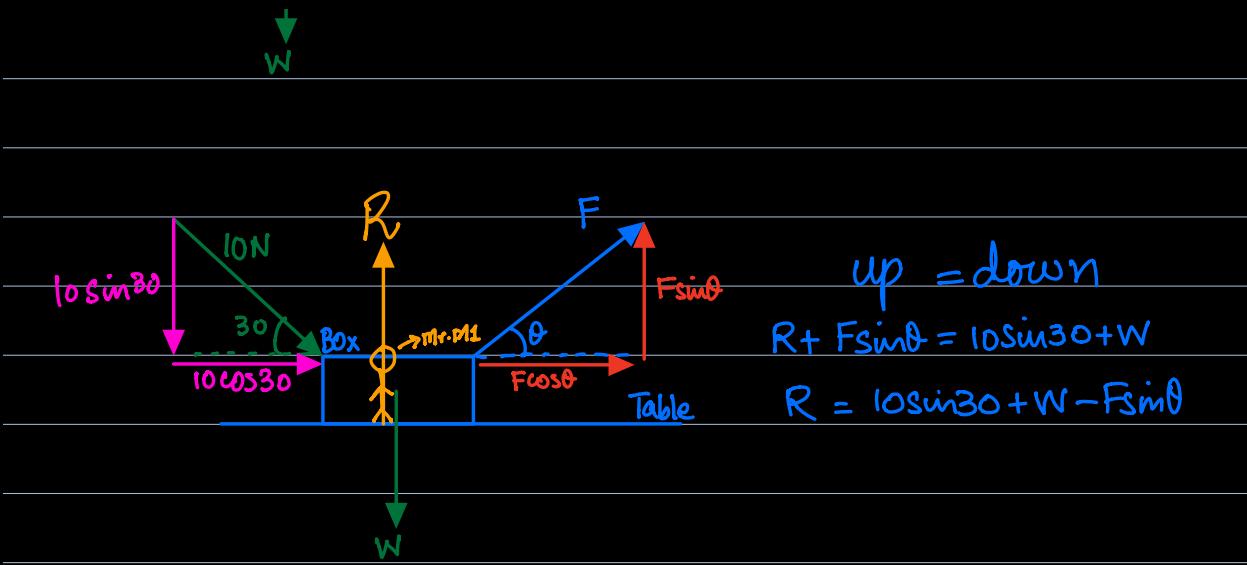
$$R = W$$



up = down

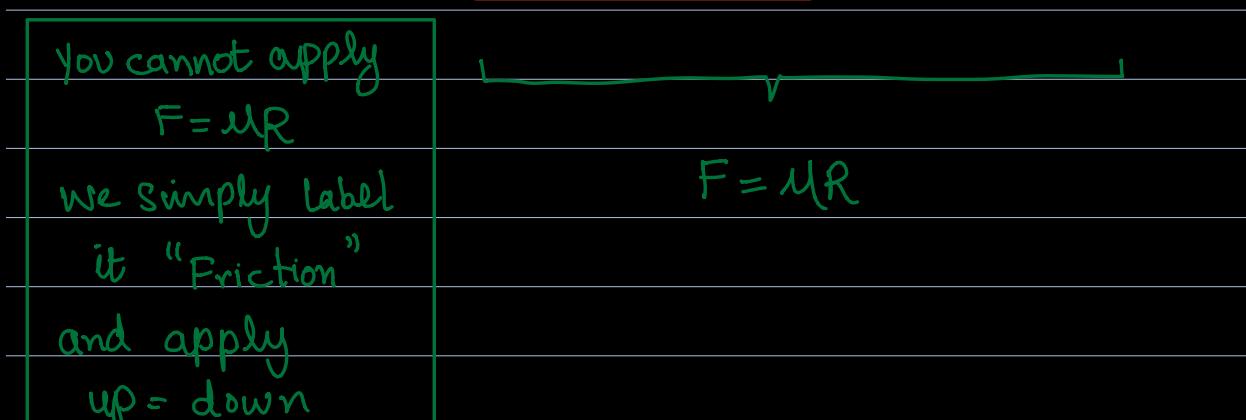
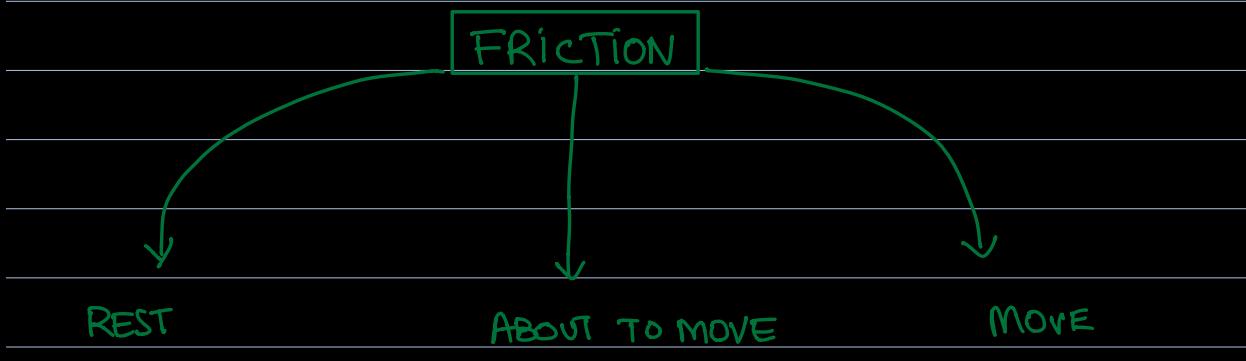
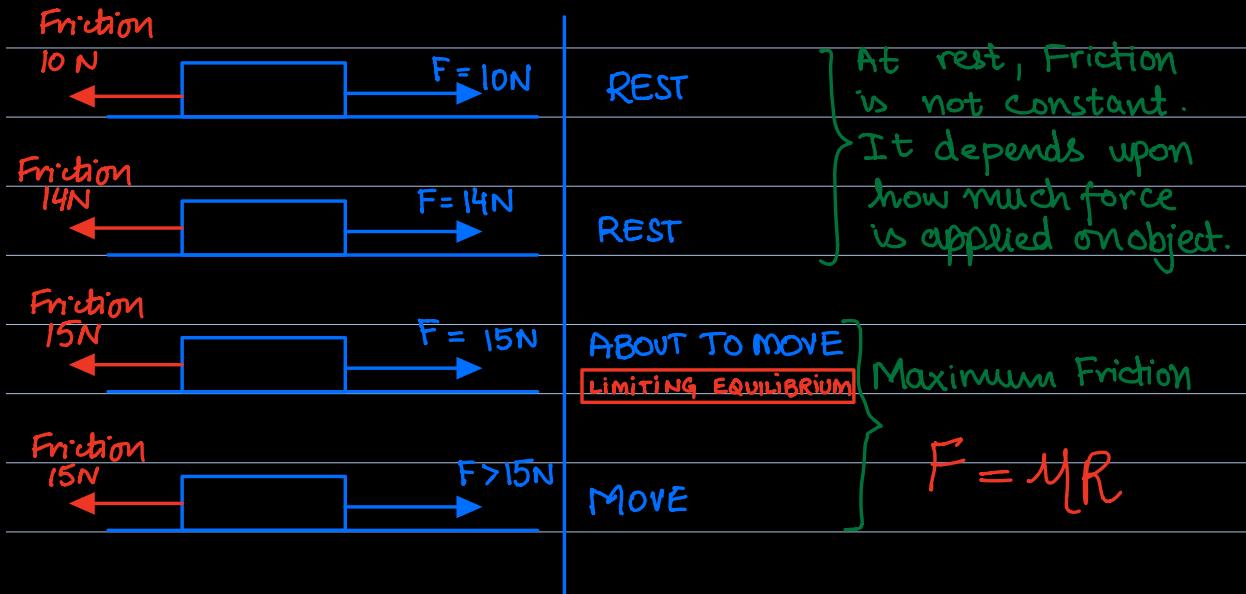
$$R + F\sin\theta = W$$

$$R = W - F\sin\theta$$



THOUGHT EXPERIMENT

Lets suppose , Max friction between box and surface is 15N.



left = Right.

LIMITING EQUILIBRIUM

ABOUT TO MOVE /SLIP.

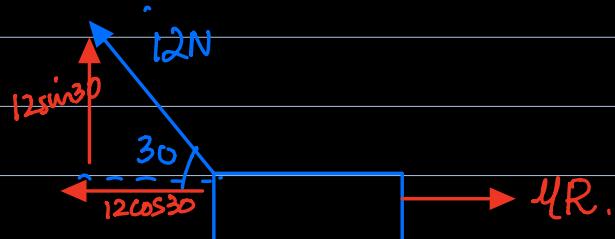
STEP1

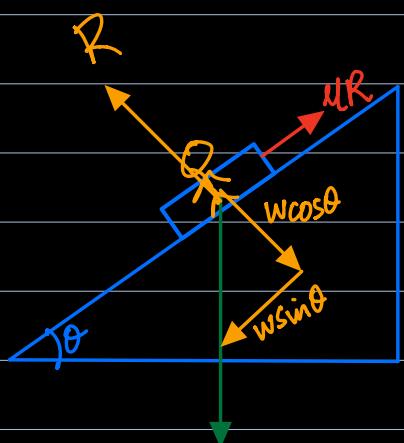
MARK THE DIRECTION
OF FRICTION AGAINST DIRECTION
OF MOTION AND LABEL IT
 $F = \mu R$

STEP2

Apply up = down
left = Right.

Box is ABOUT TO MOVE.





Force P pulls the box upwards.
Box is about to move.

In this case we cannot determine the direction of motion of box.
Hence we cannot label friction.

In this case there are two options:

- 1) Question will tell you direction of motion (4marks)

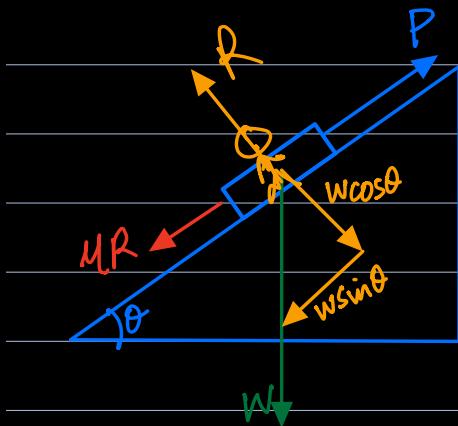
2) Question will ask for two values
of unknown.

(8marks)

You will assume two cases.

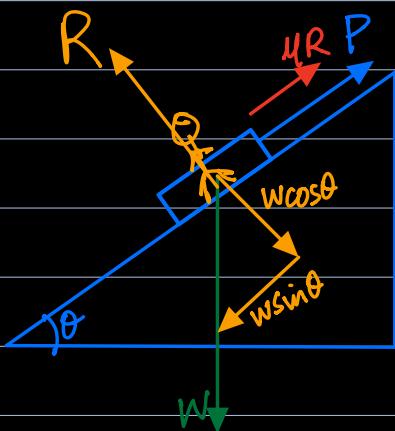
Case1

box moves up

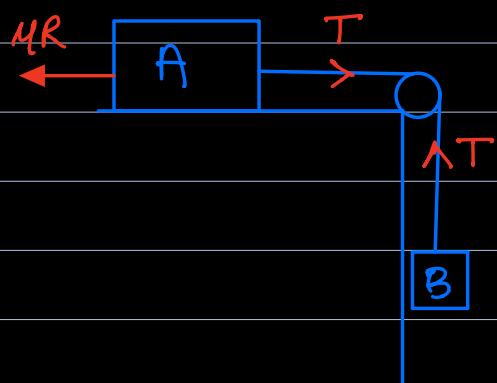


Case2

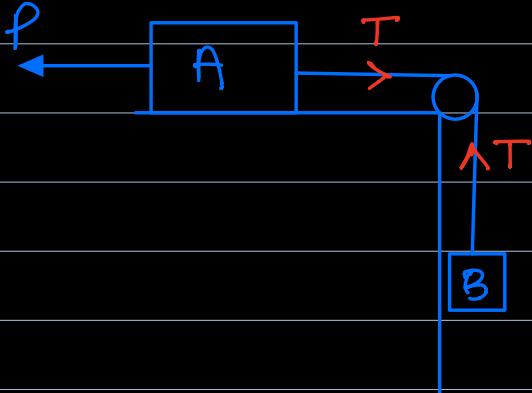
box moves down



moves



System is released
from rest and
Box A is about
to move.



System is released from rest and Box A is about to move.

We cannot determine direction of motion.

5

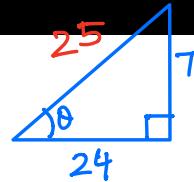
$$W = 3200 \text{ N}$$

we cannot use $F = \mu R$.

A stone slab of mass 320 kg rests in equilibrium on rough horizontal ground. A force of magnitude XN acts upwards on the slab at an angle of θ to the vertical, where $\tan \theta = \frac{7}{24}$ (see diagram).

$$\sin \theta = \frac{7}{25}$$

$$\cos \theta = \frac{24}{25}$$



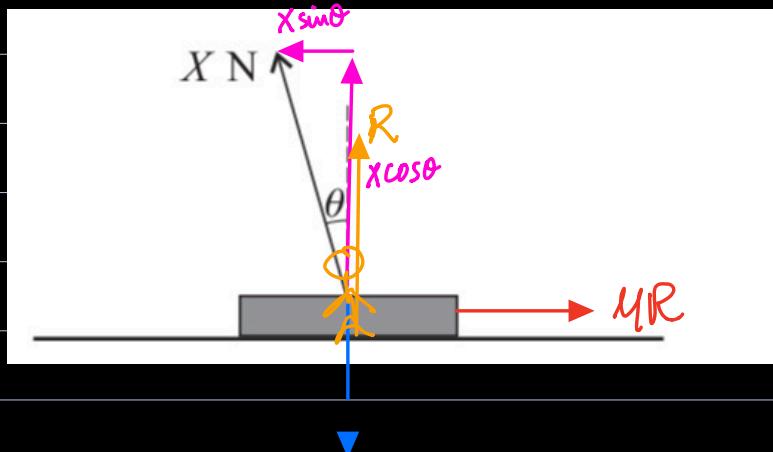
(i) Find, in terms of X , the normal component of the force exerted on the slab by the ground. [3]

(ii) Given that the coefficient of friction between the slab and the ground is $\frac{3}{8}$, find the value of X for which the slab is about to slip. (limiting equilibrium) [3]

Now Label friction

$$\mu = \frac{3}{8}$$

$$\text{as } F = \mu R$$



↓
3200N

(i) up = down

$$R + X \cos \theta = 3200$$

$$R = 3200 - X \cos \theta$$

$$R = 3200 - \frac{24}{25} X$$

① Left = Right

$$X \sin \theta = \mu R .$$

$$X \left(\frac{1}{25} \right) = \frac{3}{8} \left(\frac{3200 - 24}{25} X \right)$$

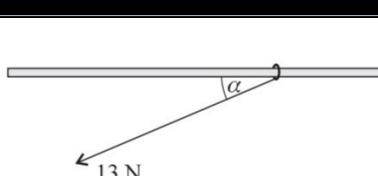
$$\frac{1}{25} X = 1200 - \frac{9}{25} X$$

$$\frac{16}{25} X = 1200$$

$$X = 1875 N .$$

1

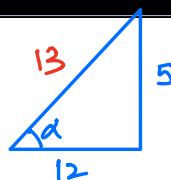
$$W = 11 N$$



Friction

$$\sin \alpha = \frac{5}{13}$$

$$\cos \alpha = \frac{12}{13}$$



A ring of mass 1.1 kg is threaded on a fixed rough horizontal rod. A light string is attached to the ring and the string is pulled with a force of magnitude 13 N at an angle α below the horizontal, where $\tan \alpha = \frac{5}{12}$ (see diagram). The ring is in equilibrium.

(i) Find the frictional component of the contact force on the ring. [2]

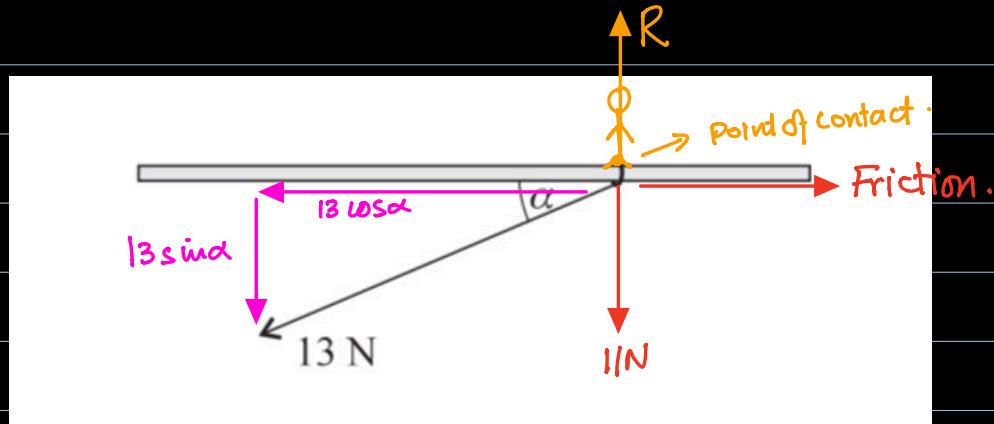
(ii) Find the normal component of the contact force on the ring. [2]

(iii) Given that the equilibrium of the ring is limiting, find the coefficient of friction between the ring and the rod. [1]

\downarrow
Friction = μR

$$\mu$$

1



① Left = Right
 $13 \cos \alpha = \text{Friction}$
 $13 \left(\frac{12}{13}\right) = \text{Friction}$

Friction = 12 N.

② up = down
 $R = 11 + 13 \sin \alpha$
 $R = 11 + 13 \left(\frac{5}{13}\right)$

$R = 16 \text{ N}$

③ $F = \mu R$

$12 = \mu (16)$

$\mu = \frac{3}{4}$



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Work, Power & Energy

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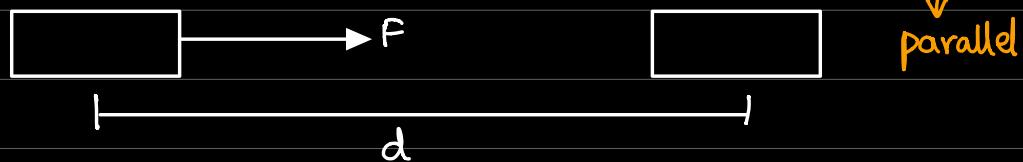
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WORK - POWER - ENERGY.

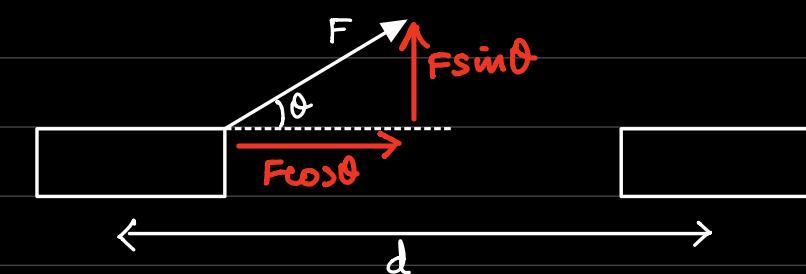
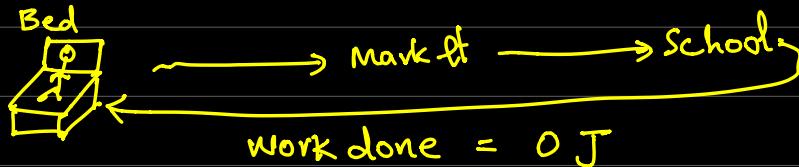
WORK Work is said to be done when a **force** **displaces** an object **in its direction**



$$\text{WORK DONE} = \underline{F \times d}$$

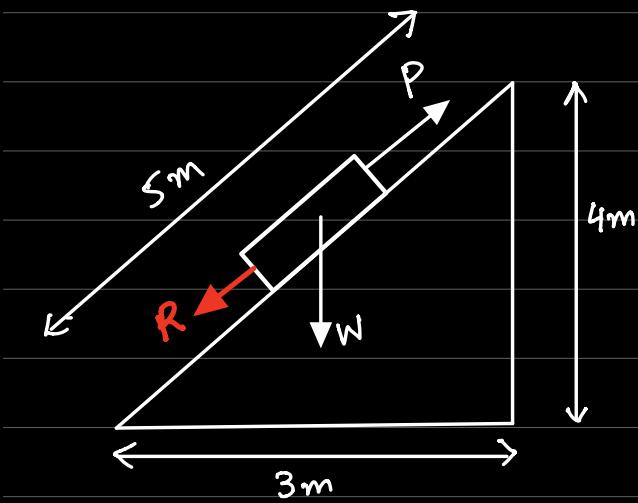
must be parallel to each other

UNIT = Joules (J)



$$W = (\underline{F \cos \theta}) \times (d)$$

FORCE AND DISPLACEMENT MUST BE PARALLEL.
ARROWS CAN BE POINTING AWAY.



(i) Work done by Force P
in pulling object from
bottom to top.

$$W = F \times d$$

$$W = (P) \times (5)$$

(ii) Work done against gravity.
gravity
weight

$$W = F \times d$$

$$W = (W) \times (4)$$

(iii) Work done against resistive
forces.

$$W = F \times d$$

$$W = (R) \times (5)$$

ENERGY

GRAVITATIONAL POTENTIAL ENERGY

Due to height of object

$$PE = m g h$$

Units = Joules (J)

KINETIC ENERGY.

Due to speed of object.

$$KE = \frac{1}{2} m v^2$$

Units = Joules (J)

POWER

$$\text{Power} = \frac{\text{Work done/Energy}}{\text{time}} = \frac{J}{s}$$

$$P = \frac{W}{t} = \frac{F \times d}{t} = F \times v$$

$$P = F v$$

UNITS: WATTS (W) OR J/S

TRICKS: 1) KW means 1000W

5 KW means 5000W

P KW means 1000P W

2) They use units

J/S or $J s^{-1}$

instead of Watts.

F is different in all of these four formulas.

$$F = ma$$

↓

Fwd - Bwd

$$W = F \times d$$

↓

Desired force
whose work done
is to be found.

$$P = F \times v$$

↓

Driving
force

$$F = \mu R$$

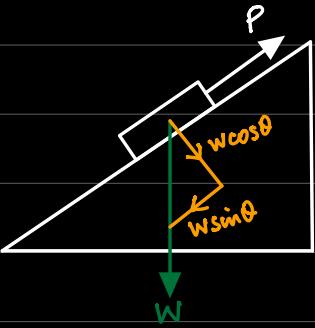
↓

Maximum
Friction

FORWARD FORCE

Any force acting in
the direction of motion

Box is moving upwards



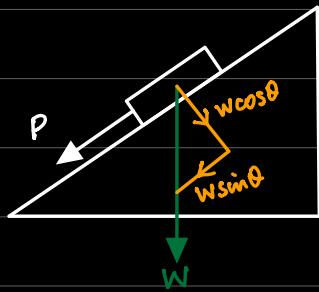
$$\text{Fwd force} = P$$

$$\text{Bwd force} = ws\sin\theta$$

DRIVING FORCE

- Has to be an external force
- Acts in same direction as direction of motion
- Cannot include weight (w) or components ($ws\sin\theta, ws\cos\theta$)

Box is moving downwards.



$$\text{Fwd Force} = P + ws\sin\theta$$

$$\text{Driving Force} = P$$

Box is moving down.



$$F_{wd} = W \sin \theta$$

$$\text{Driving force} = 0$$



Ball is falling freely under gravity.

$$F_{wd} \text{ force} = w$$

$$\text{Driving force} = 0$$



TYPE 1

QUESTION WILL DISCUSS

FORCE

, ACCELERATION ,

POWER

$$F_{wd} - B_{wd} = ma$$

$$P = F v$$

\nearrow Driving

$$P = (DF)v$$

Apply both of these at same time and find unknown.

- 46 A car of mass 880 kg travels along a straight horizontal road with its engine working at a constant rate of P W. The resistance to motion is 700 N. At an instant when the car's speed is 16 m s^{-1} its acceleration is 0.625 m s^{-2} . Find the value of P . [4]



$$\text{Power} = P$$

$$\text{acc} = 0.625$$

$$v = 16$$

$$\text{Fwd} - \text{Bwd} = ma$$

$$P = (DF)(v)$$

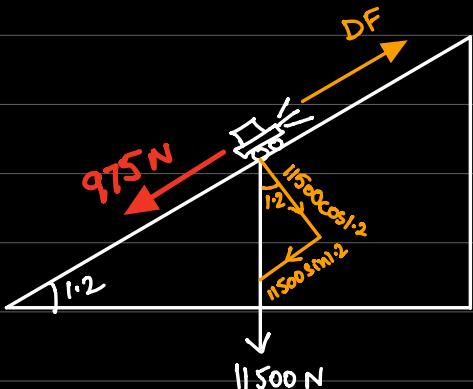
$$DF - 700 = 880(0.625)$$

$$P = (1250)(16)$$

$$DF = 1250$$

$$P = 20,000 \text{ W}$$

- 25 A car of mass 1150 kg travels up a straight hill inclined at 1.2° to the horizontal. The resistance to motion of the car is 975 N. Find the acceleration of the car at an instant when it is moving with speed 16 m s^{-1} and the engine is working at a power of 35 kW. [4]



$$a = ?$$

$$v = 16$$

$$P = 35 \text{ kW} = 35000 \text{ W}$$

$$\text{Fwd} - \text{Bwd} = ma$$

$$P = (DF) v$$

$$DF - 975 - 11500 \sin 1.2 = 1150a$$

$$35000 = (DF)(16)$$

$$DF = 2187.5$$

$$2187.5 - 975 - 11500 \sin 1.2 = 1150a$$

$$a = 0.8449.$$

$$\text{Power} = 1000 P$$

- 67 A car has mass 800 kg. The engine of the car generates constant power P kW as the car moves along a straight horizontal road. The resistance to motion is constant and equal to R N. When the car's speed is 14 m s^{-1} its acceleration is 1.4 m s^{-2} , and when the car's speed is 25 m s^{-1} its acceleration is 0.33 m s^{-2} . Find the values of P and R . [6]



$$v = 14, a = 1.4$$

$$\text{Power} = 1000P$$

$$\text{Fwd} - \text{Bwd} = ma$$

$$DF - R = 800(1.4)$$

$$\frac{1000P}{14} - R = 1120$$

$$P = (DF)v$$

$$1000P = DF(14)$$

$$DF = \frac{1000P}{14}$$



$$v = 25, a = 0.33$$

$$\text{Power} = 1000P$$

$$\text{Fwd} - \text{Bwd} = ma$$

$$DF - R = 800(0.33)$$

$$40P - R = 264$$

$$P = (DF)v$$

$$1000P = DF(25)$$

$$DF = \frac{1000P}{25}$$

$$DF = 40P$$

$$R = \frac{1000P}{14} - 1120$$

$$R = 40P - 264$$

NOW SOLVE SIMULTANEOUS.

$$\frac{1000P}{14} - 1120 = 40P - 264$$

$$\frac{1000P}{14} - 40P = 1120 - 264$$

$$\frac{220}{7} P = 856$$

$$P = 27.236, R = 40(27.236) - 264$$

$$R = 825.45$$

TYPE 2

WORK DONE

or

ENERGY

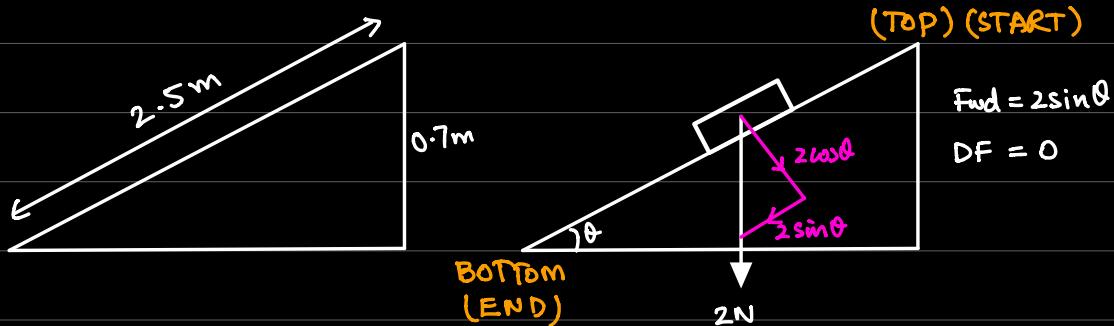
MARKS: Range from 1 mark to 9 marks.

BEFORE

AFTER

$$\left(\begin{array}{l} PE = \\ KE = \\ \text{Workdone by} \\ \text{Driving force} = \end{array} \right) = \left(\begin{array}{l} PE = \\ KE = \\ \text{Workdone against} \\ \text{friction} = \end{array} \right)$$

- 1 The top of an inclined plane is at a height of 0.7 m above the bottom. A block of mass 0.2 kg is released from rest at the top of the plane and slides a distance of 2.5 m to the bottom. Find the kinetic energy of the block when it reaches the bottom of the plane in each of the following cases:
- (i) the plane is smooth, (**No friction**) [2]
- (ii) the coefficient of friction between the plane and the block is 0.15. [5]



BEFORE

TOP

$$\left(\begin{array}{l} PE = mgh = (0.2)(10)(0.7) = 1.4 \\ KE = 0 \\ \text{Workdone by} \\ \text{Driving force} = 0 \end{array} \right) = \left(\begin{array}{l} PE = 0 \\ KE = ? \\ \text{Workdone against} \\ \text{friction} = 0 \end{array} \right)$$

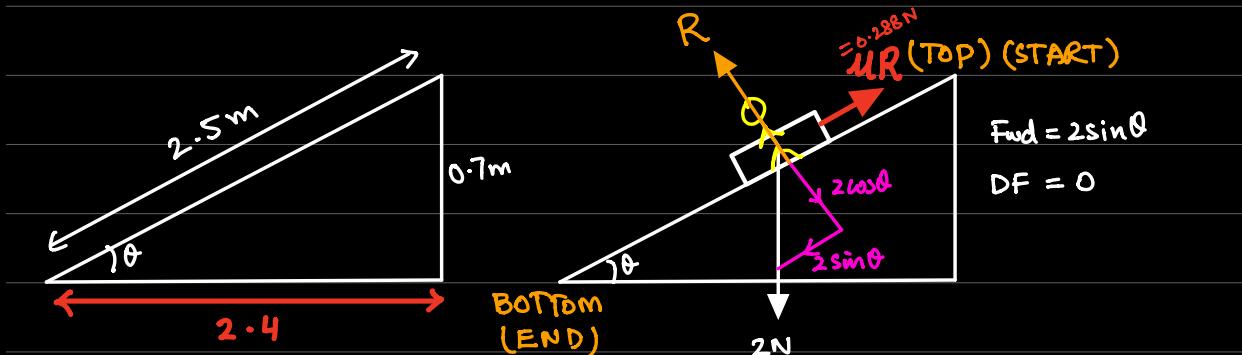
AFTER

BOTTOM

$$1.4 + 0 + 0 = 0 + KE_B + 0$$

$$KE_B = 1.4 \text{ J}$$

(ii) PLANE IS ROUGH: $\mu = 0.15$



$$\sin \theta = \frac{0.7}{2.5} = \frac{7}{25}$$

$$\cos \theta = \frac{2.4}{2.5} = \frac{24}{25}$$

Apply up = down to find R

$$R = 2 \cos \theta$$

$$R = 2 \left(\frac{24}{25} \right) = 1.92$$

Apply $F = \mu R$ to find friction.

$$F = (0.15)(1.92)$$

$$F = 0.288.$$

BEFORE

AFTER

TOP

Bottom

$$PE = (0.2)(10)(0.7) = 1.4$$

$$PE = 0$$

$$KE = 0$$

$$KE = ?$$

Workdone by
Driving force = 0

Workdone against
friction = $(0.288) \times (2.5)$

$$1.4 + 0 + 0 = 0 + KE_B + (0.288)(2.5)$$

$$KE_B = 0.68 \text{ J}$$



ZAIÑEMATICS

FOR THE LOVE OF MATHS

Momentum M1

Compiled by Rafay Mushtaq



zainematics



zainematics



zainematics

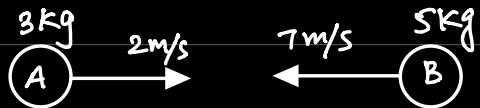
38

MOMENTUM = PRODUCT OF MASS AND VELOCITY.

$$P = m v$$

$$\text{Units} = \text{kg}\cdot\text{m/s} \quad \text{or} \quad \text{kg m s}^{-1}$$

IMP: DIRECTION WITH VELOCITY.



$$\rho_A = (3)(2) = 6 \text{ kg ms}^{-1}$$

$$\rho_B = (5)(-7) \\ -35 \text{ kg m}^{-1}$$

COLLISIONS

TOTAL MOMENTUM

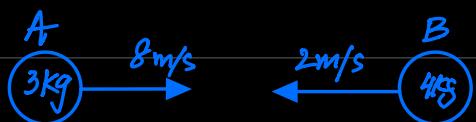
1

TOTAL MOMENTUM

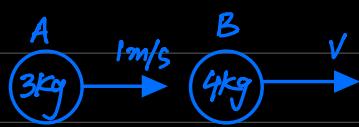
BEFORE COLLISION

AFTER COLLISION

Q: Before



After.



, Find v.

TOTAL MOMENTUM BEFORE = TOTAL MOMENTUM AFTER

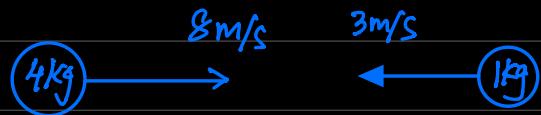
$$(3)(8) + (4)(-2) = (3)(1) + (4)(r)$$

$$24 - 8 = 3 + 4v$$

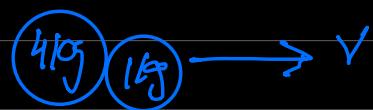
$$16 - 3 = 4 \vee$$

$$v = 3.25$$

Q:



After the collision, Both boxes coalesce,



Coalesce = Join together

TOTAL MOMENTUM BEFORE = TOTAL MOMENTUM AFTER.

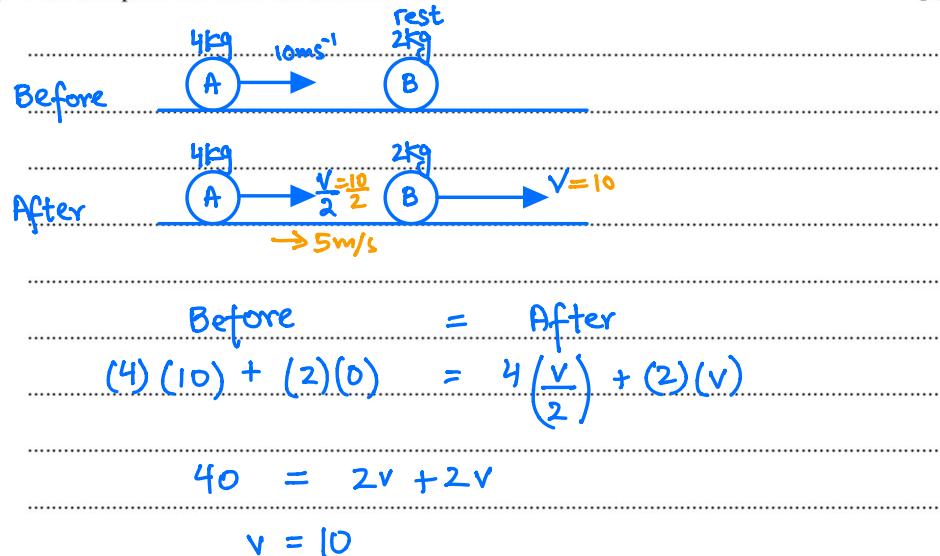
$$(4)(8) + (1)(-3) = (4+1)(v)$$

$$32 - 3 = 5v$$

$$v = 5.8$$

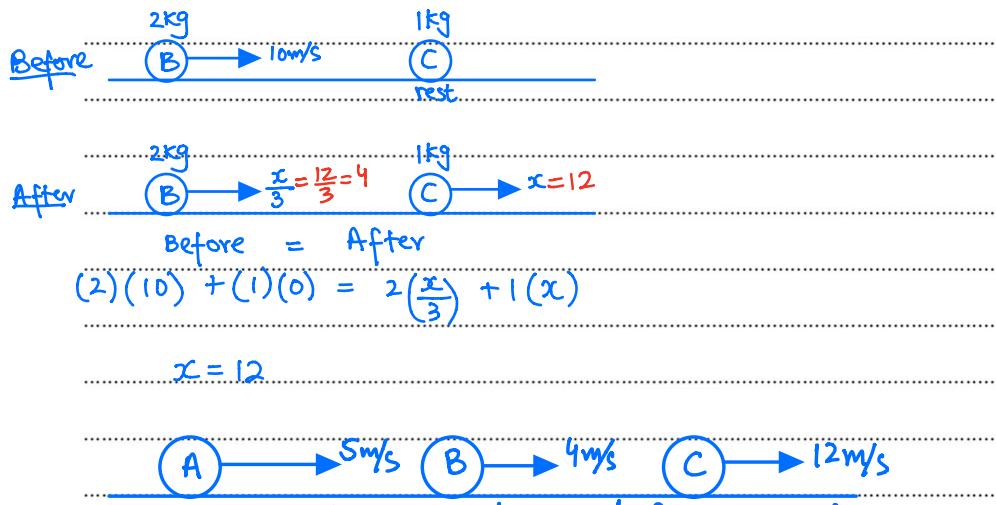
- 4 Small smooth spheres A and B, of equal radii and of masses 4 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed 10 m s^{-1} . After the spheres collide A continues to move in the same direction but with half the speed of B.

- (a) Find the speed of B after the collision. [2]



A third small smooth sphere C, of mass 1 kg and with the same radius as A and B, is at rest on the plane. B now collides directly with C. After this collision B continues to move in the same direction but with one third the speed of C.

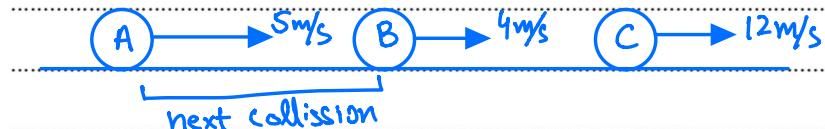
- (b) Show that there is another collision between A and B. [3]



Since Speed of A = 5 and speed of B = 4, A will catch up and there will be another collision.

(c) A and B coalesce during this collision.

Find the total loss of kinetic energy in the system due to the three collisions. [5]



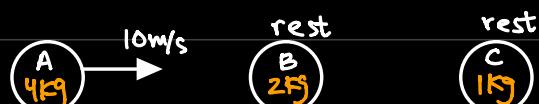
$$\text{Before} = \text{After}$$

$$(4)(5) + (2)(4) = (4+2)v$$

$$28 = 6v$$

$$v = 4.667$$

START



$$\text{TOTAL KE} = \frac{1}{2}(4)(10)^2 + \frac{1}{2}(2)(0)^2 + \frac{1}{2}(1)(0)^2 = 200 \text{ J}$$

END



$$\text{TOTAL KE} = \frac{1}{2}(4+2)(4.667)^2 + \frac{1}{2}(1)(12)^2 = 137.15 \text{ J}$$

$$\text{Loss in KE} = 200 - 137.15 = 62.85 \text{ J}.$$



ZAIİNEMATICS

FOR THE LOVE OF MATHS

Kinematics

M1

Compiled by Rafay Mushtaq



zainematics



zainematics



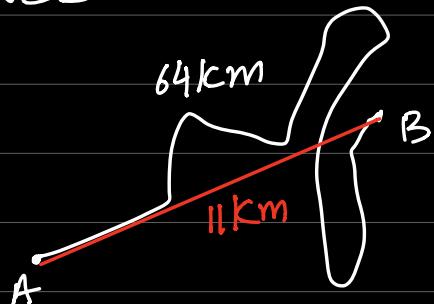
zainematics

44

KINEMATICS

(M1) (20 + Marks)

DISTANCE
(mm, m, Km)



DISPLACEMENT
(mm, m, Km)

SHORTEST DISTANCE
BETWEEN START
AND END POINT.

SPEED
(m/s or km/h)

VELOCITY.
(m/s or km/h)

IF QUESTION USES TERM "DISPLACEMENT"
OR "VELOCITY" BODY IS ALWAYS
MOVING IN STRAIGHT LINE.

ACCELERATION Rate of change in speed.

$$a = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{time}}$$

$$a = \frac{v - u}{t}$$

u = initial velocity
 v = final velocity .

a = acceleration .
 t = time .

UNITS , m/s^2 or ms^{-2}

(A) $u=0$ $t=4 \text{ sec}$ $v=100$ $a = \frac{100-0}{4} = 25 \text{ ms}^{-2}$

(B) $u=0$ $t=10 \text{ sec}$ $v=100$ $a = \frac{100-0}{10} = 10 \text{ ms}^{-2}$

Acc $\rightarrow +ve \rightarrow$ BODY IS \rightarrow ACCELERATION
SPEEDING UP

Acc $\rightarrow -ve \rightarrow$ BODY IS \rightarrow RETARDATION
SLOWING DOWN \rightarrow DECELERATION.

+/- SIGN OF ACCELERATION CANNOT
COMMENT ON DIRECTION OF MOTION
OF BODY.

TYPE 1: CONSTANT ACCELERATION

CONSTANT SPEED

$$(a = 0)$$

$$d = s \times t$$

↑ distance ↑ speed ↑ time

$$s = v \times t$$

↑ displacement ↑ velocity ↑ time

CONSTANT (UNIFORM) ACCELERATION

1) INCLINED PLANE

2) PULLEY (in any shape)

3) FREEFALL:

(a) body is in air

(b) only force acting on body is weight.

$$a = 10 \text{ (speedup)} \quad \text{or} \quad a = -10 \text{ (slowdown)}$$

$$v = u + at$$

$$2as = v^2 - u^2$$

$$s = ut + \frac{1}{2}at^2$$

v = final velocity

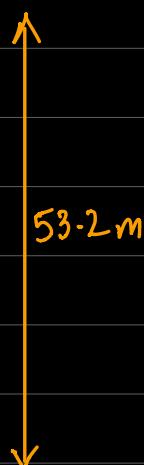
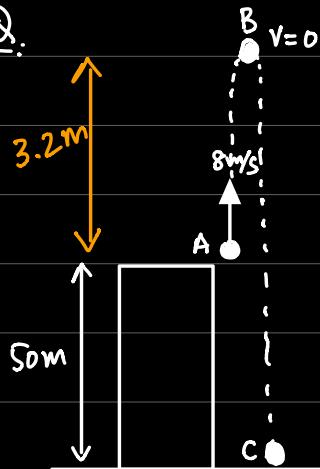
u = initial velocity

t = time

s = displacement.

a = acceleration.

Q:



A ball is thrown upwards from top of 50m tall building with 8 m/s.

Find:

(i) Max height reached by ball.

A $\xrightarrow{\text{freefall}}$ B

$$u = 8, \quad a = -10, \quad v = 0$$

$$2as = v^2 - u^2$$

$$2(-10)(s) = 0^2 - 8^2$$

$$s = 3.2 \text{ m}$$

$$\text{Max height} = 50 + 3.2 = 53.2 \text{ m.}$$

(ii) Velocity with which ball hits ground.

$$B \xrightarrow{\text{free fall}} C$$
$$u=0, a=+10, v=? , s=53.2$$

$$2as = v^2 - u^2$$

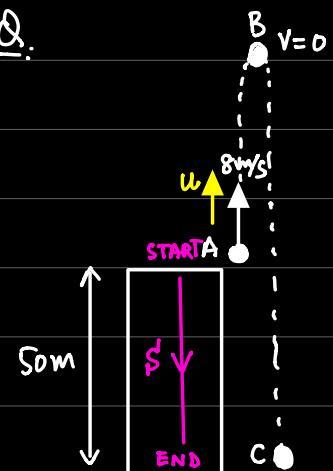
$$2(10)(53.2) = v^2 - 0^2$$

$$v^2 = 1064$$

$$v = \sqrt{1064} = 32.6 \text{ m/s.}$$

IN Q1, examiner requires to calculate from complete Journey from A \rightarrow C.

Q:



A ball is thrown upwards from top of 50m tall building with 8m/s.

Find velocity of ball just before hitting ground.

$$A \xrightarrow{\text{FREEFALL}} C$$

$$u=8, a=-10, s=-50, v=?$$

$$2as = v^2 - u^2$$

$$2(-10)(-50) = v^2 - 8^2$$

$$1000 = v^2 - 64$$

$$v^2 = 1064, v=32.6.$$

+/- SIGNS

ACCELERATION

STAND AT initial velocity (u)

IF BODY IS ABOUT TO
Speed up $\rightarrow a=+ve$
Slow down $\rightarrow a=-ve$

DISPLACEMENT

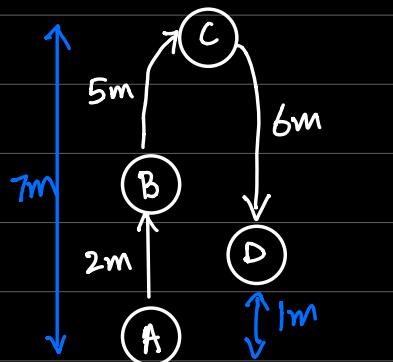
Mark an arrow from
 $START \rightarrow END$ and label
it (s).

Now compare this arrow
with initial velocity (u)

If both arrows are
in:

same direction $\rightarrow s=+ve$
opp direction $\rightarrow s=-ve$.

IF A BODY STARTS FROM GROUND
AND MOVES VERTICALLY, ITS
DISPLACEMENT AND HEIGHT IS ALWAYS
SAME



	height	Displacement
A	0	0
B	2	2
C	7	7
D	1	1

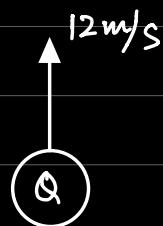
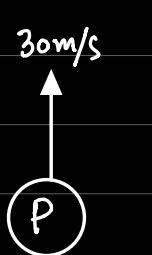
TIME DELAY BETWEEN TWO OBJECTS

In equations of motion, time of motion is needed.

$$t_p = 5$$

$$t_q = 5 - 2 = 3$$

$t = 5 \text{ sec (Pause)}$.



Q is released
2 seconds after P.

$$\text{time} = t \rightarrow \text{time} = t - 2$$

$$\text{time} = T + 2 \leftarrow \text{time} = T$$

HEIGHT = DISPLACEMENT.

- 27 Particles P and Q are projected vertically upwards, from different points on horizontal ground, with velocities of 20 m s^{-1} and 25 m s^{-1} respectively. Q is projected 0.4 s later than P. Find
- the time for which P's height above the ground is greater than 15 m, [3]
 - the velocities of P and Q at the instant when the particles are at the same height [5]

Same displacement.



(i)

$u = 20, a = -10$

$s = 15, t = ?$

$s = ut + \frac{1}{2}at^2$

$$15 = 20t + \frac{1}{2}(-10)t^2$$

$$15 = 20t - 5t^2$$

$$5t^2 - 20t + 15 = 0$$

$$t^2 - 4t + 3 = 0$$

$$t^2 - 3t - t + 3 = 0$$

$$t(t - 3) - 1(t - 3) = 0$$

$$t = 1, t = 3$$

Time for which body stayed above 15m height = $3 - 1 = 2 \text{ sec}$

(ii) DISPLACEMENTS (HEIGHTS)

h_P = height of P

h_Q = height of Q.

$$\boxed{P} \quad u = 20, a = -10 \\ s = h_P, \text{ time} = t$$

$$s = ut + \frac{1}{2} at^2$$

$$h_P = 20t + \frac{1}{2}(-10)t^2$$

$$\boxed{h_P = 20t - 5t^2}$$

$$\boxed{Q} \quad u = 25, a = -10 \\ s = h_Q, \text{ time} = t - 0.4$$

$$s = ut + \frac{1}{2} at^2$$

$$h_Q = 25(t - 0.4) + \frac{1}{2}(-10)(t - 0.4)^2$$

$$\boxed{h_Q = 25(t - 0.4) - 5(t - 0.4)^2}$$

$$h_P = h_Q$$

$$20t - 5t^2 = 25(t - 0.4) - 5(t - 0.4)^2$$

$$20t - 5t^2 = 25t - 10 - 5(t^2 - 0.8t + 0.16)$$

$$20t - 5t^2 = 25t - 10 - 5t^2 + 4t - 0.8$$

$$0 = 9t - 10.8$$

$$t = 1.2$$

VELOCITIES

$$\boxed{P} \quad v = u + at$$

$$v_P = 20 + (-10)(1.2)$$

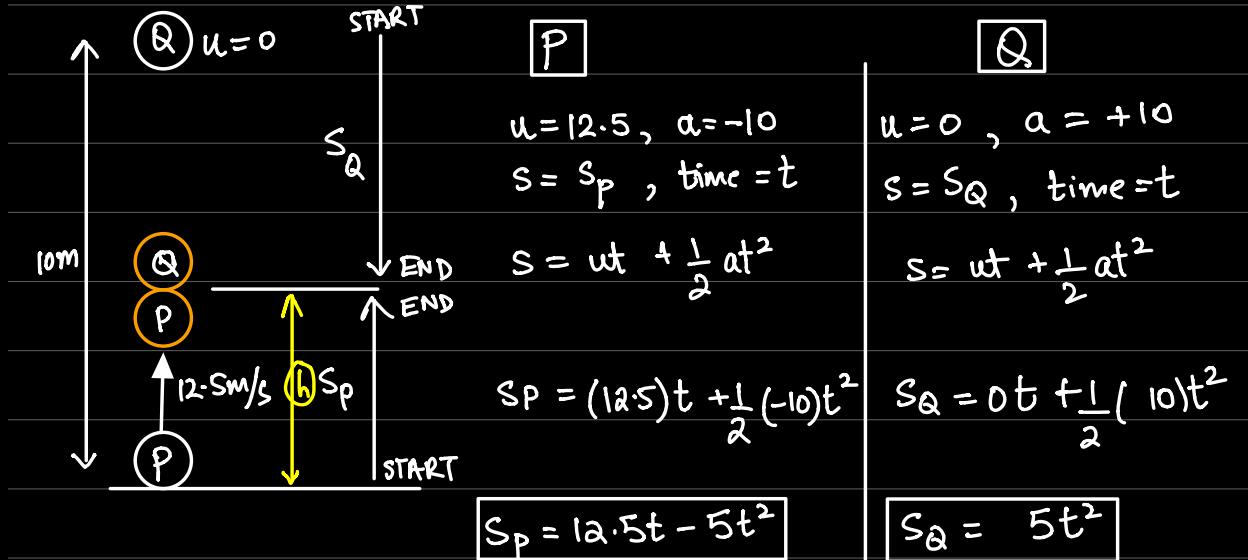
$$\boxed{Q} \quad v = u + a(t - 0.4)$$

$$v_Q = 25 + (-10)(1.2 - 0.4)$$

$$v_P = 8 \text{ m/s}$$

$$v_Q = 17 \text{ m/s}$$

- 21 A particle is projected vertically upwards from a point O with initial speed 12.5 m s^{-1} . At the same instant another particle is released from rest at a point 10 m vertically above O . Find the height above O at which the particles meet. [5]



$$S_p + S_q = 10$$

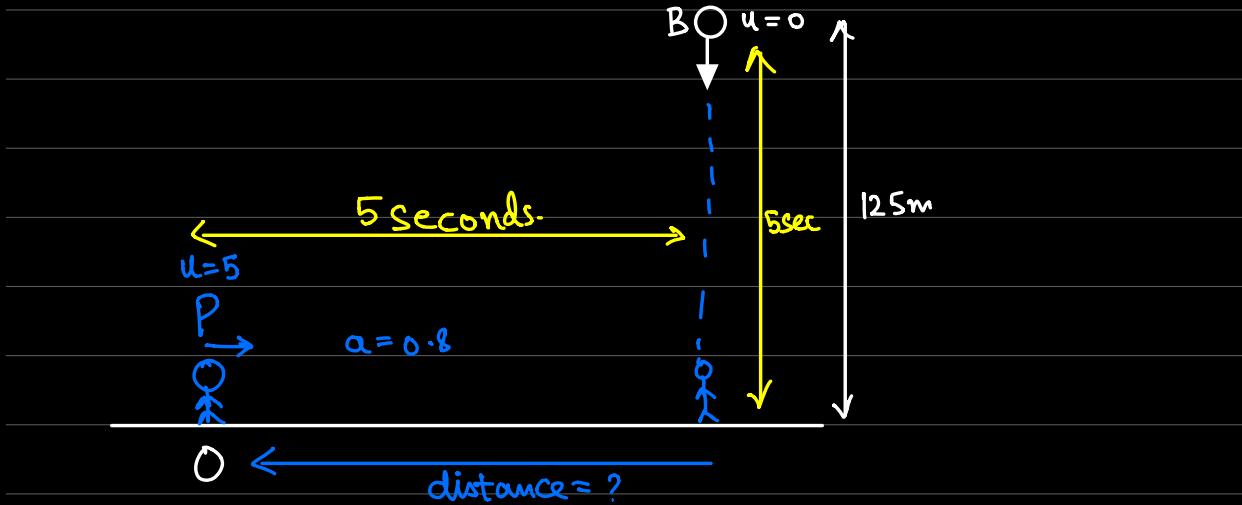
$$12.5t - 5t^2 + 5t^2 = 10$$

$$12.5t = 10$$

$$t = 0.8$$

$$\begin{aligned} h &= S_p = 12.5t - 5t^2 \\ &= 12.5(0.8) - 5(0.8)^2 \\ &= 6.8 \text{ m.} \end{aligned}$$

- 51 An object is released from rest at a height of 125 m above horizontal ground and falls freely under gravity, hitting a moving target P. The target P is moving on the ground in a straight line, with constant acceleration 0.8 m s^{-2} . At the instant the object is released P passes through a point O with speed 5 m s^{-1} . Find the distance from O to the point where P is hit by the object. [4]



$$B \quad u=0, s=125, t=? , a=+10$$

$$s = ut + \frac{1}{2}at^2$$

$$125 = 0t + \frac{1}{2}(10)t^2$$

$$125 = 5t^2$$

$$t=5$$

$$P \quad u=5$$

$$a=0.8$$

$$t=5$$

$$s=?$$

$$s = ut + \frac{1}{2}at^2$$

$$= 5(5) + \frac{1}{2}(0.8)(5)^2$$

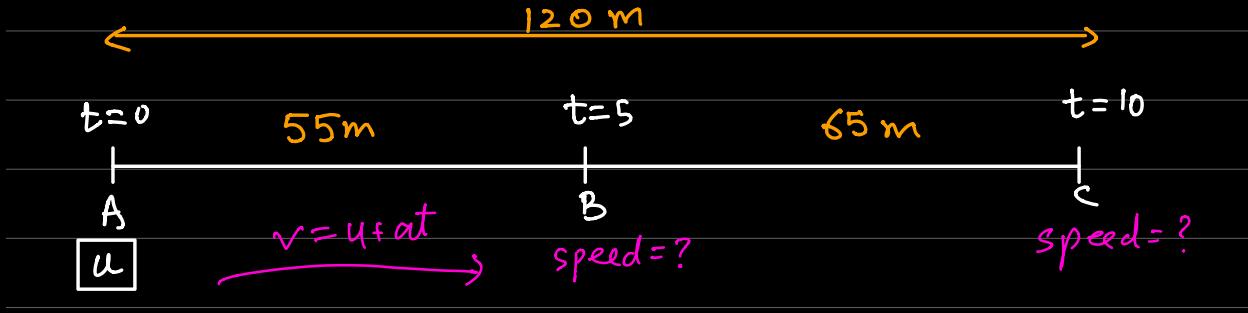
$$s = 35 \text{ m.}$$

$$v = u + at$$

$$2as = v^2 - u^2$$

$$s = ut + \frac{1}{2}at^2$$

- 54 A car travels along a straight road with constant acceleration $a \text{ m s}^{-2}$. It passes through points A, B and C; the time taken from A to B and from B to C is 5 s in each case. The speed of the car at A is $u \text{ m s}^{-1}$ and the distances AB and BC are 55 m and 65 m respectively. Find the values of a and u . [6]



$$A \longrightarrow B$$

$$u = u, a = a \\ t = 5, s = 55$$

$$s = ut + \frac{1}{2}at^2$$

$$A \longrightarrow C$$

$$u = u, a = a \\ t = 10, s = 120$$

$$s = ut + \frac{1}{2}at^2$$

$$55 = u(5) + \frac{1}{2}a(5)^2$$

$$120 = u(10) + \frac{1}{2}(a)(10)^2$$

$$55 = 5u + 12.5a \quad \leftrightarrow \quad 120 = 10u + 50a$$

$$u = \\ a =$$

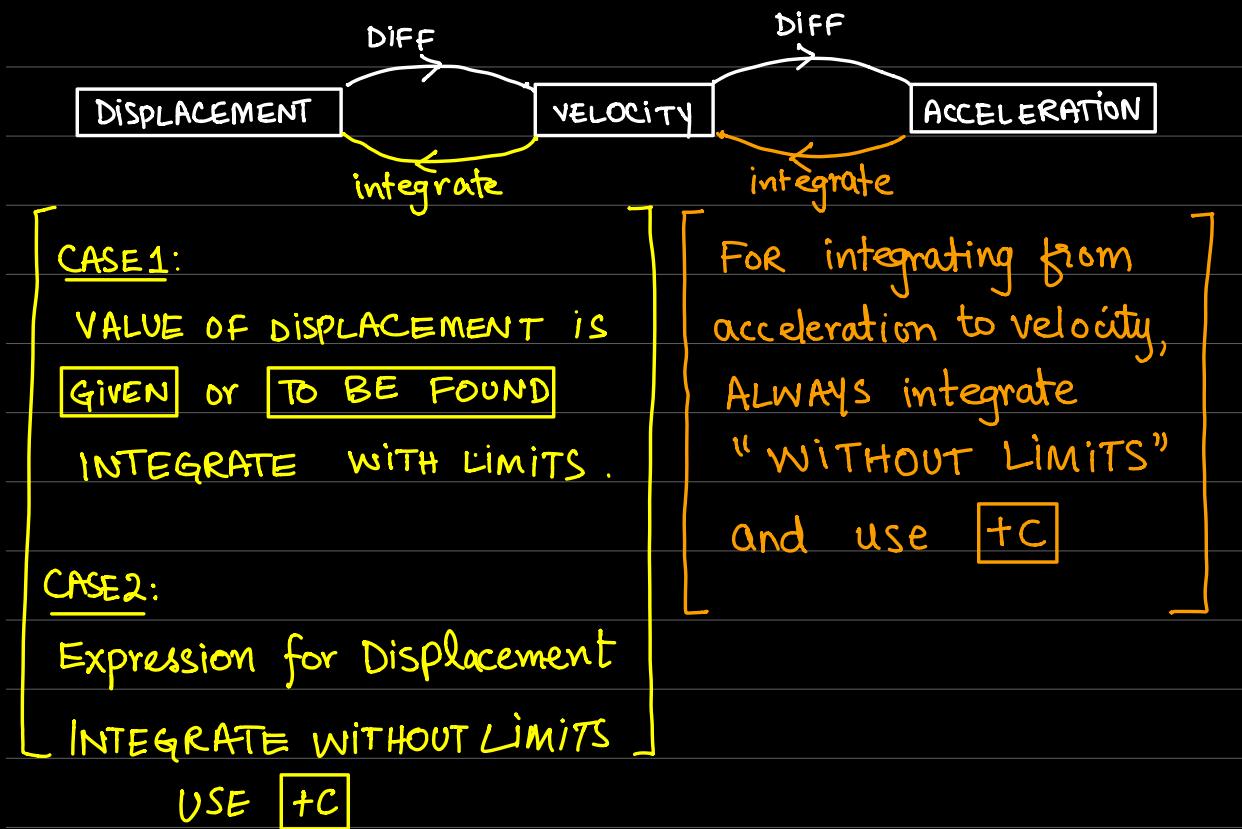
- 71 A particle is projected vertically upwards with speed 9 m s^{-1} from a point 3.15 m above horizontal ground. The particle moves freely under gravity until it hits the ground. For the particle's motion from the instant of projection until the particle hits the ground, find the total distance travelled and the total time taken.

[6]

<p>4.05 9m/s 3.15m 4.05 A → B → C</p>	<p>$A \rightarrow B$</p> <p>$u = 9, a = -10$</p> <p>$s = ?, v = 0$</p> <p>$2as = v^2 - u^2$</p> <p>$2(-10)s = 0^2 - 9^2$</p> <p>$s = 4.05$</p> <p>$v = u + at$</p> <p>$0 = 9 + (-10)t$</p> <p>$t = 0.9$</p>	<p><u>Total Distance</u></p> <p>$4.05 + 4.05 + 3.15$</p> <p>$= 11.25 \text{ m}$</p>
<p><u>Direct: A → C</u></p> <p>$u = 9, a = -10$</p> <p>$s = -3.15, t = ?$</p> <p>$s = ut + \frac{1}{2}at^2$</p> <p>$-3.15 = 9t + \frac{1}{2}(-10)t^2$</p>	<p>$B \rightarrow C$</p> <p>$u = 0, t = ?, a = 10$</p> <p>$s = 4.05 + 3.15 = 7.2$</p> <p>$s = ut + \frac{1}{2}at^2$</p> <p>$7.2 = 0t + \frac{1}{2}(10)t^2$</p> <p>$t = 1.2$</p>	<p>$\boxed{t = 1.2}$</p> <p>Total time = $1.2 + 0.9 = 2.1$</p>

Solve Quadratic.

TYPE 2: NON CONSTANT ACCELERATION



$$s = 6t^3 - 7t^2 + 12$$

$\left(\begin{matrix} \text{Diff} \\ \downarrow \end{matrix} \right) \text{integrate.}$

$$v = 18t^2 - 14t$$

$\left(\begin{matrix} \text{Diff} \\ \downarrow \end{matrix} \right) \text{integrate}$

$$a = 36t - 14$$

TURNING POINT (INSTANTANEOUS REST)		$v=0$
MAX DISPLACEMENT	disp $\xrightarrow{\text{diff}}$ velocity	$v=0$
MAX VELOCITY	velocity $\xrightarrow{\text{diff}}$ acc	$a=0$
MAX ACCELERATION	acc $\xrightarrow{\text{diff}}$ $\frac{da}{dt}$	$\frac{da}{dt} = 0$

$t=0 \quad v=0$

2 A particle starts from rest at the point A and travels in a straight line until it reaches the point B . The velocity of the particle t seconds after leaving A is $v \text{ ms}^{-1}$, where $v = 0.009t^2 - 0.0001t^3$. Given that the velocity of the particle when it reaches B is zero, find

- (i) the time taken for the particle to travel from A to B , [2]
- (ii) the distance AB , (value) (with limits integration). [4]
- (iii) the maximum velocity of the particle. ($acc=0$) [4]

$$v = 0.009t^2 - 0.0001t^3$$

(i) Put $v=0$

$$0 = 0.009t^2 - 0.0001t^3$$

$$0 = t^2(0.009 - 0.0001t)$$

$$t^2 = 0, \quad 0.009 - 0.0001t = 0$$

$$t=0 \quad t=90$$

A

B

(ii) Integration . (with limits)

$$s = \int_0^{90} (0.009t^2 - 0.0001t^3) dt$$

$$s = \left| \frac{0.009t^3}{3} - \frac{0.0001t^4}{4} \right|_0^{90}$$

$$s = \left| \left(\frac{0.009(90)^3}{3} - \frac{0.0001(90)^4}{4} \right) - \left(\frac{0.009(0)^3}{3} - \frac{0.0001(0)^4}{4} \right) \right|$$

$$s = 546.75.$$

(iii) Max velocity

$$a = 0$$

$v = 0.009t^2 - 0.0001t^3$

↓ diff

$$a = 0.018t - 0.0003t^2$$
$$0 = 0.018t - 0.0003t^2$$
$$0.0003t^2 = 0.018t$$
$$0.0003t = 0.018$$
$$t = 60$$

$$v = 0.009(60)^2 - 0.0001(60)^3$$

$$v = 10.8 \text{ m/s}$$

- 40 A particle travels in a straight line from A to B in 20 s. Its acceleration t seconds after leaving A is $a \text{ m s}^{-2}$, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B .

(i) Show that the initial speed of the particle is zero. [4]

(ii) Find the maximum speed of the particle. (put acc = 0) [2]

(iii) Find the distance AB . (integrate velocity) (with limits) [4]

$$t=0$$

$$\begin{array}{c} | \\ A \end{array}$$

$$v=?$$

$$a = \frac{3}{160}t^2 - \frac{1}{800}t^3$$

$$t=20$$

$$\begin{array}{c} | \\ B \end{array}$$

$$v=0$$

(ii) integrate without limits (+C)

$$v = \int \left(\frac{3}{160}t^2 - \frac{1}{800}t^3 \right) dt .$$

$$v = \frac{3}{160} \frac{t^3}{3} - \frac{1}{800} \frac{t^4}{4} + C$$

$$v = \frac{1}{160}t^3 - \frac{1}{3200}t^4 + C$$

$$\begin{array}{l} v=0 \text{ at } B \\ t=20 \end{array}$$

$$0 = \frac{1}{160}(20)^3 - \frac{1}{3200}(20)^4 + C$$

$$C=0$$

$$v = \frac{1}{160}t^3 - \frac{1}{3200}t^4$$

For initial velocity, put $t=0$

$$v = \frac{1}{160}(0)^3 - \frac{1}{3200}(0)^4 = 0 .$$

(ii) Max Speed ($acc = 0$)

$$a = \frac{3}{160} t^2 - \frac{1}{800} t^3$$

Max velocity, put
 $t = 15$

$$0 = \frac{3}{160} t^2 - \frac{1}{800} t^3$$

$$v = \frac{1}{160} t^3 - \frac{1}{3200} t^4$$

$$0 = t^2 \left(\frac{3}{160} - \frac{1}{800} t \right)$$

$$v = \frac{1}{160} (15)^3 - \frac{1}{3200} (15)^4$$

$$\frac{3}{160} - \frac{1}{800} t = 0$$

$$v = 5.27 .$$

$$t = 15$$

$$s = \int_0^{20} \left(\frac{1}{160} t^3 - \frac{1}{3200} t^4 \right) dt$$

$$\left| \frac{1}{160} \frac{t^4}{4} - \frac{1}{3200} \frac{t^5}{5} \right|_0^{20}$$

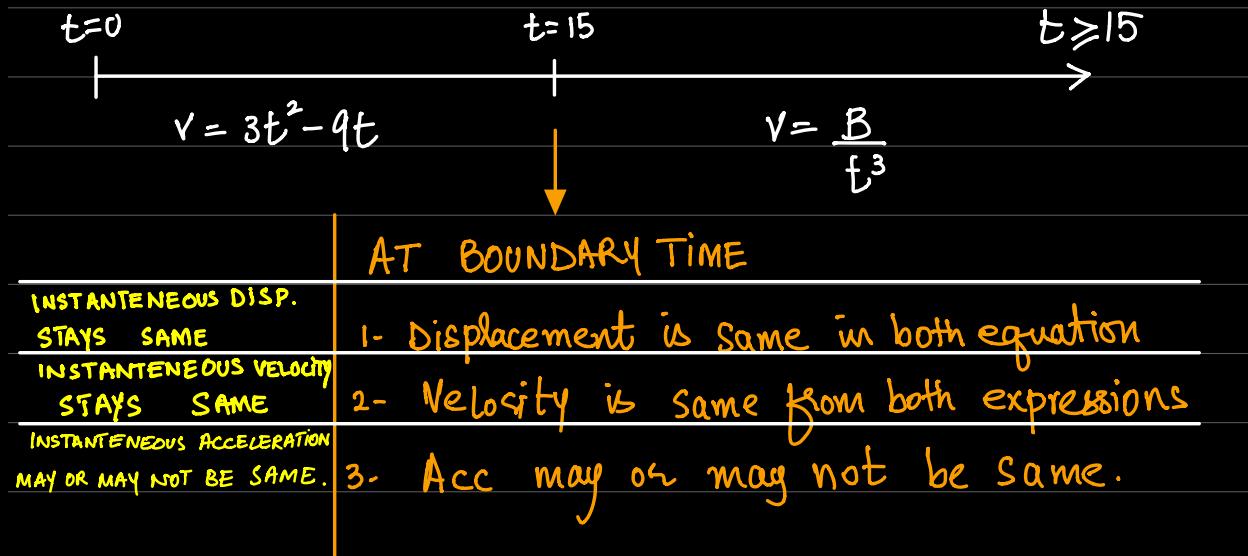
$$\left| \frac{t^4}{640} - \frac{t^5}{16000} \right|_0^{20}$$

$$\left| \left(\frac{20^4}{640} - \frac{20^5}{16000} \right) - \left(\frac{0^4}{640} - \frac{0^5}{16000} \right) \right|$$

$$= 50 .$$

TYPE 2 ADVANCED

SPLIT JOURNEY.



Find value of B.

put $t=15$ in both velocity expressions
and equate them.

$$v = 3t^2 - 9t \quad \boxed{t=15} \quad v = \frac{B}{t^3}$$

$$3(15)^2 - 9(15) = \frac{B}{15^3}$$

$$B = 1822500 .$$

- 21 A vehicle is moving in a straight line. The velocity v m s⁻¹ at time t s after the vehicle starts is given by

$$v = A(t - 0.05t^2) \quad \text{for } 0 \leq t \leq 15,$$

$$v = \frac{B}{t^2} \quad \text{for } t \geq 15,$$

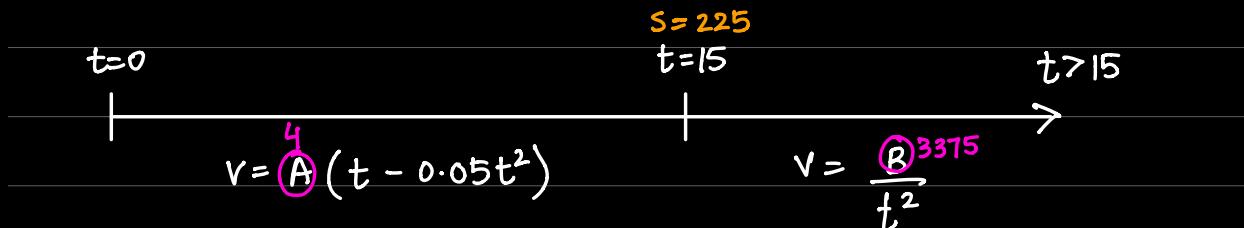
where A and B are constants. The distance travelled by the vehicle between $t = 0$ and $t = 15$ is 225 m.

- (i) Find the value of A and show that $B = 3375$.

$$v = \frac{3375}{t^2} \quad [5]$$

- (ii) Find an expression in terms of t for the total distance travelled by the vehicle when $t \geq 15$. [3]

- (iii) Find the speed of the vehicle when it has travelled a total distance of 315 m. [3]



$$(i) \quad S = \int A(t - 0.05t^2) dt$$

$$225 = \int_0^{15} A(t - 0.05t^2) dt$$

$$225 = A \int_0^{15} (t - 0.05t^2) dt$$

$$225 = A \left| \frac{t^2}{2} - \frac{0.05t^3}{3} \right|_0^{15}$$

$$225 = A \left| \left(\frac{15^2}{2} - \frac{0.05(15)^3}{3} \right) - \left(\frac{0^2}{2} - \frac{0.05(0)^3}{3} \right) \right|$$

$$225 = A \left| \frac{225}{4} \right|$$

$$225 = A \left(\frac{225}{4} \right)$$

$$A = 4$$

FOR B WE USE BOUNDARY TIME

$$t = 15$$

$$v = 4(t - 0.05t^2) \quad \rightarrow \quad v = \frac{B}{t^2}$$

$$4(15 - 0.05(15)^2) = \frac{B}{15^2}$$

$$15 = \frac{B}{225}$$

$$B = 3375$$

(ii) $s = \int \frac{3375}{t^2} dt$

$$s = \int 3375t^{-2} dt$$

$$s = 3375 \frac{t^{-1}}{-1} + c$$

$$s = -\frac{3375}{t} + c$$

$$s = 225, t = 15$$

This c will always be calculated from the displacement and time of BOUNDARY TIME.

$$225 = -\frac{3375}{15} + c$$

$$c = 450$$

$$s = -\frac{3375}{t} + 450$$

Expression for total distance travelled.

(iii) Total distance = 315

$$S = -\frac{3375}{t} + 450$$

$$315 = -\frac{3375}{t} + 450$$

$$\frac{3375}{t} = 450 - 315$$

$$t = 25$$

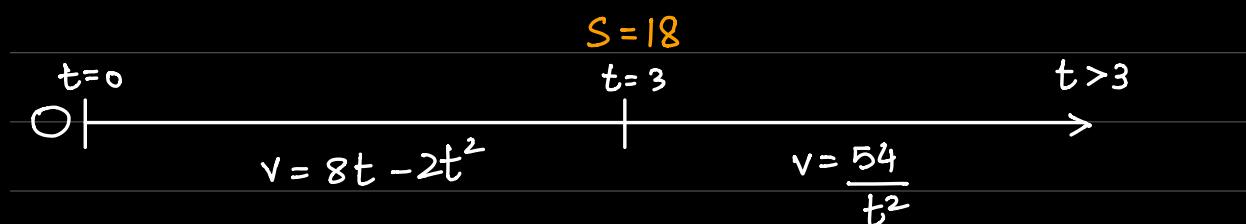
For Speed put $t=25$
in:

$$v = \frac{3375}{t^2}$$

$$v = \frac{3375}{25^2}$$

$$v = 5.4.$$

- 5 A particle P starts from rest at O and travels in a straight line. Its velocity $v \text{ m s}^{-1}$ at time $t \text{ s}$ is given by $v = 8t - 2t^2$ for $0 \leq t \leq 3$, and $v = \frac{54}{t^2}$ for $t > 3$. Find
- (i) the distance travelled by P in the first 3 seconds, [4]
 - (ii) an expression in terms of t for the displacement of P from O , valid for $t > 3$, [3]
 - (iii) the value of v when the displacement of P from O is 27 m. [3]



(i) $S = \int_0^3 (8t - 2t^2) dt$

$$S = \left| \frac{8t^2}{2} - \frac{2t^3}{3} \right|_0^3$$

$$S = \left| 4t^2 - \frac{2t^3}{3} \right|_0^3$$

$$s = \left| \left(\frac{4(3)^2 - 2(3)^3}{3} \right) - \left(\frac{4(0)^2 - 2(0)^3}{3} \right) \right|$$

$$s = 18m.$$

$$(ii) \quad s = \int \frac{54}{t^2} dt$$

$$s = \int 54t^{-2} dt$$

$$s = \frac{54t^{-1}}{-1} + C$$

$$\boxed{s = -\frac{54}{t} + C} \quad \begin{aligned} t &= 3 \\ s &= 18 \end{aligned} \quad \begin{aligned} &\text{BOUNDARY} \\ &\text{TIME.} \end{aligned}$$

$$18 = -\frac{54}{3} + C$$

$$C = 36$$

$$\boxed{s = -\frac{54}{t} + 36}$$

$$(iii) \quad s = 27$$

$$s = -\frac{54}{t} + 36$$

$$27 = -\frac{54}{t} + 36$$

$$\frac{54}{t} = 36 - 27$$

$$\boxed{t = 6}$$

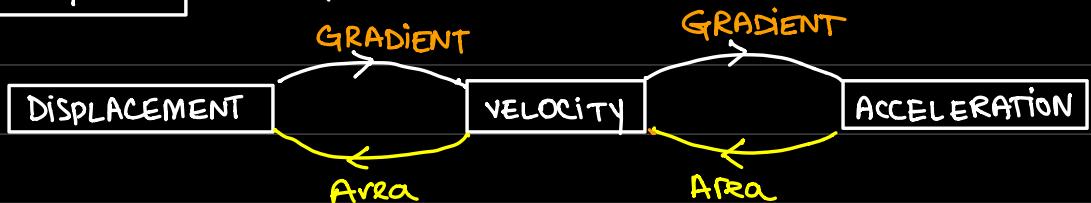
$$v = \frac{54}{t^2}$$

$$v = \frac{54}{6^2}$$

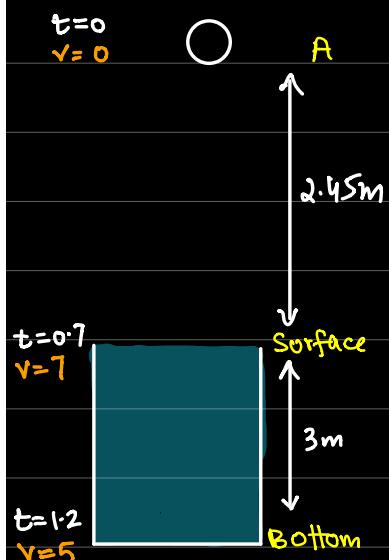
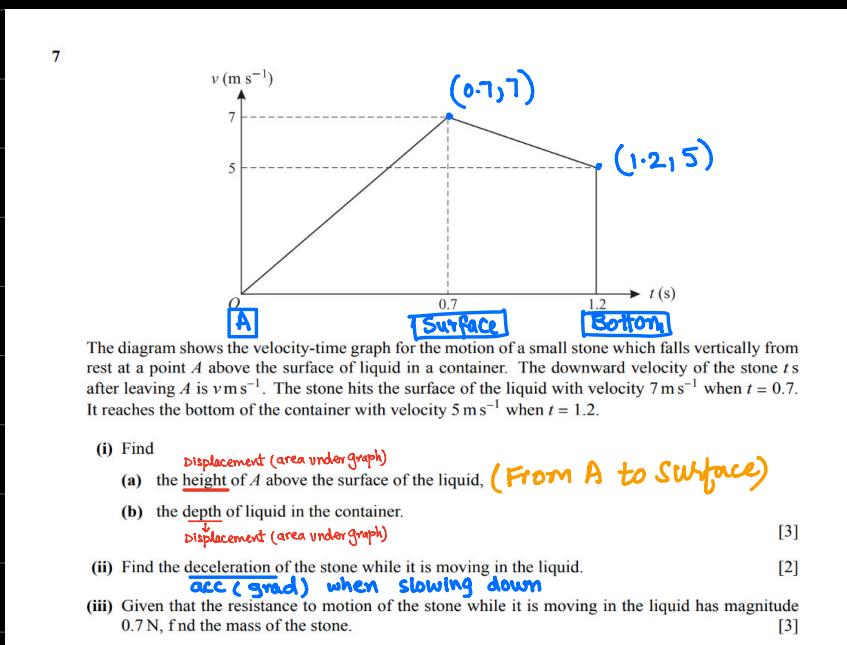
$$v = 1.5$$

TYPE 3

GRAPHS



IMP: ALWAYS DRAW REAL LIFE DIAGRAM OF SCENARIO AND RELATE KEY TIMES TO GRAPH.



(i) height of *A* to Surface

$$\text{Area} = \frac{1}{2}(0.7)(7) = 2.45\text{m}$$

(ii) Depth = Area from Surface to Bottom

$$\text{Area of trapezium} = \frac{1}{2} \times h \times (a+b)$$

$$= \frac{1}{2} \times 0.5 \times (7+5)$$

$$= 3 \text{ m}$$

(iii) Deceleration: $(0.7, 7) (1.2, 5)$

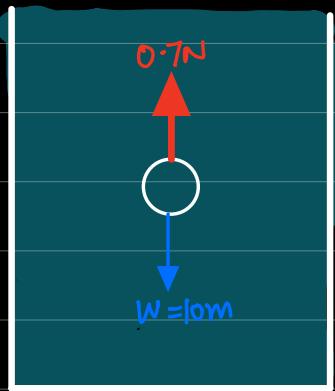
$$\text{grad} = \text{acc} = \frac{5-7}{1.2 - 0.7} = \frac{-2}{0.5} = -4 \text{ m/s}^2$$

$$\text{acc} = -4$$

$$\text{Deceleration} = 4 \text{ m/s}^2$$

(iii)

- Given that the resistance to motion of the stone while it is moving in the liquid has magnitude 0.7 N, find the mass of the stone. [3]



$$\text{fwd} - \text{Bwd} = ma \rightarrow \text{acc}$$

$$10m - 0.7 = m(-4)$$

$$10m + 4m = 0.7$$

$$14m = 0.7$$

$$m = 0.05$$

$$W = mg = m(10) = 10m$$

Acc

KINEMATICS

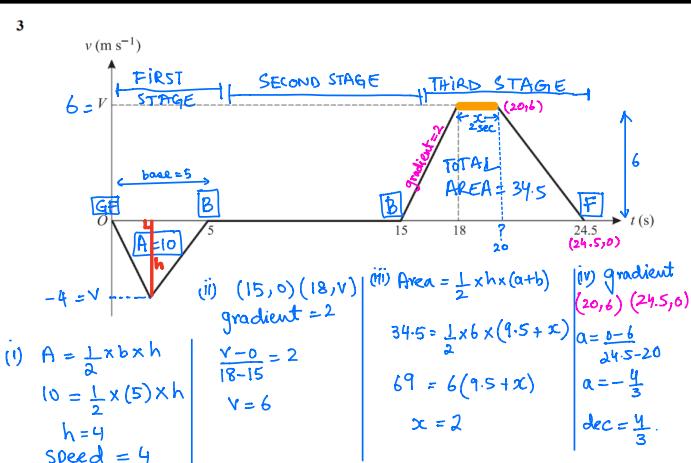
FORCES

$$v = u + at$$

$$2as = v^2 - u^2$$

$$s = ut + \frac{1}{2}at^2$$

$$F_{\text{wd}} - F_{\text{bd}} = ma$$



The diagram shows the velocity-time graph for a lift moving between floors in a building. The graph consists of straight line segments. In the first stage the lift travels downwards from the ground floor for 5 s, coming to rest at the basement after travelling 10 m. (area = 10)

(i) Find the greatest speed reached during this stage. [2]

The second stage consists of a 10 s wait at the basement. In the third stage, the lift travels upwards until it comes to rest at a floor 34.5 m above the basement, arriving 24.5 s after the start of the first stage. The lift accelerates at 2 m s^{-2} for the first 3 s of the third stage, reaching a speed of $V \text{ m s}^{-1}$. Find gradient = 2

(ii) the value of V . [2]

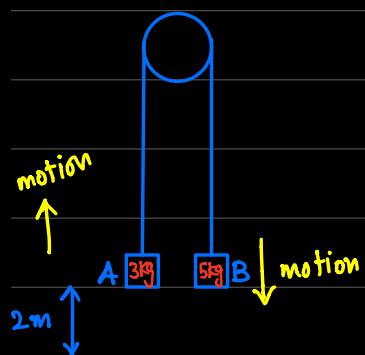
(iii) the time during the third stage for which the lift is moving at constant speed. [3]

(iv) the deceleration of the lift in the final part of the third stage. [2]

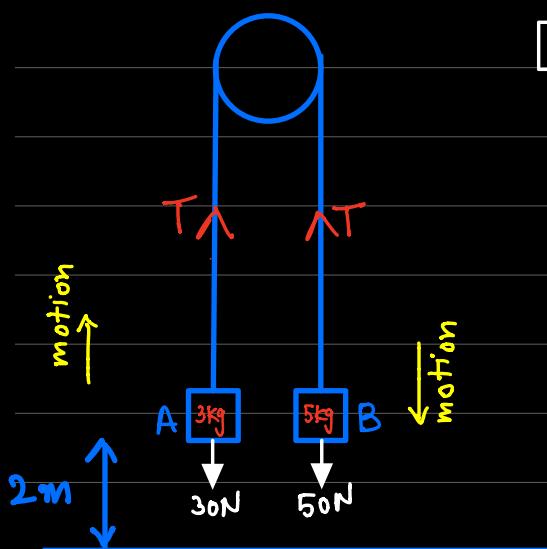
PULLEYS

Mass of 3kg and 5kg are held at rest

The system is released and objects start to move.



(i) Find the acceleration of both particles and tension in string.



$$[A] F_{\text{wd}} - B_{\text{wd}} = ma$$

$$T - 30 = 3a$$

$$T = 30 + 3a$$

$$30 + 3a = 50 - 5a$$

$$8a = 20$$

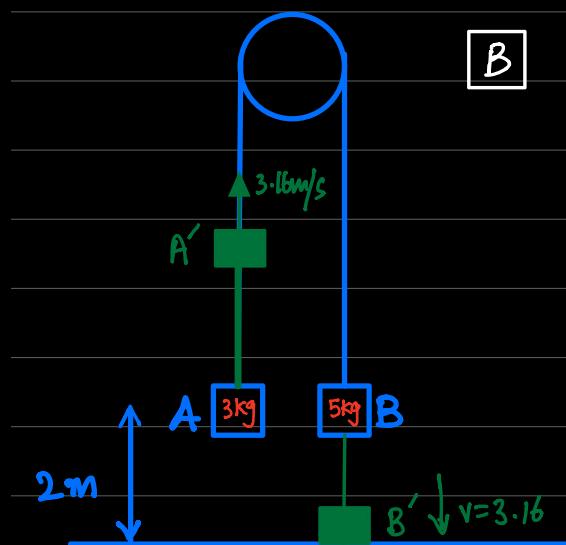
$$a = 2.5 \text{ m/s}^2$$

$$T = 30 + 3(2.5) = 37.5 \text{ N.}$$

$$[B] F_{\text{wd}} - B_{\text{wd}} = ma$$

$$50 - T = 5a$$

$$T = 50 - 5a$$



(ii) Find speed with which B hits ground.

$$[B] u=0, v=? , s=2, a=2.5$$

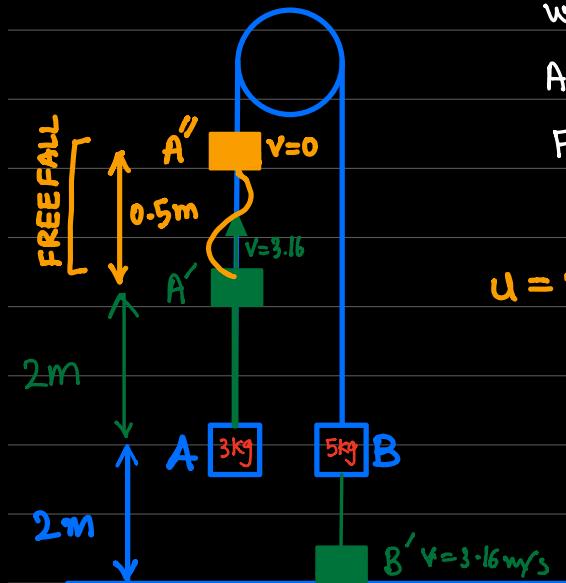
$$2as = v^2 - u^2$$

$$2(2.5)(2) = v^2 - 0^2$$

$$10 = v^2$$

$$v = \sqrt{10} = 3.16$$

(iii) Find greatest height above ground reached by A (5)



when B hits ground it comes to rest

A keeps moving up, under gravity

Freefall because string is not taut

$$A' \longrightarrow A''$$

$$u = 3.16, s = ?, a = -10, v = 0$$

$$2as = v^2 - u^2$$

$$2(-10)s = 0^2 - 3.16^2$$

$$s = 0.5$$

VARIATIONS

Find:

(a) Max height above ground reached by A.

$$2 + 2 + 0.5 = 4.5 \text{ m}$$

(b) Distance travelled by A to reach max height.

$$2 + 0.5 = 2.5 \text{ m}$$

(c) Distance travelled by A from start

of journey till it comes to rest again.

$$2 + 0.5 + 0.5 = 3 \text{ m}$$