ALEVELS P3

Integration

A11

1 **(i)** Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

$$\int_0^{\frac{1}{4}\pi} (\tan^{n+2} x + \tan^n x)\,dx = \frac{1}{n+1}.$$ [4]

(ii) Hence find the exact value of

(a) $\int_0^{\frac{1}{4}\pi} (\sec^4 x - \sec^2 x)\,dx$, [3]

(b) $\int_0^{\frac{1}{4}\pi} (\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x)\,dx$. [3]

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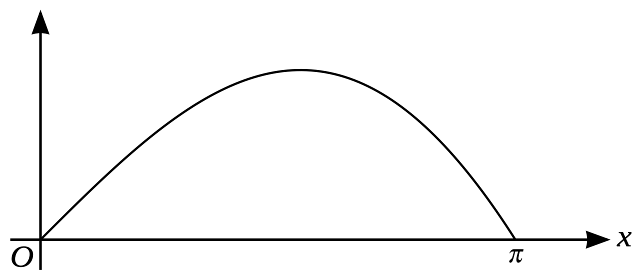
2



The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

(i) Find $\frac{dy}{dx}$ and show that $4\frac{d^2y}{dx^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x-axis. [5]

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3 **(i)** Show that $(x+1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6}\,dx$. [8]

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4 It is given that $x = \ln(1-y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \frac{e^{-x}}{1 + e^{-x}}$. [2]

(ii) Hence show that $\int_0^1 y\,dx = \ln\left(\frac{2e}{e+1}\right)$. [4]

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