D1 Stationary Point Answers P3

A1 **[5]**

1 State correct derivative $1 - 2\sin 2x$ B1

Equate derivative to zero and solve for x M1

Obtain answer $x = \frac{1}{12}\pi$ A1

Carry out an appropriate method for determining the nature of a stationary point M1

Show that $x = \frac{1}{12}\pi$ is a maximum with no errors seen A1

Obtain second answer $x = \frac{5}{12}\pi$ in range A1√

Show this is a minimum point A1 **[7]**

2 (i) State derivative is $6e^{x} - 3e^{3x}$ B1

EITHER: Equate derivative to zero and simplify to an equation of the form $e^{2x} = a$ M1*

Carry out method for calculating x, where $a > 0$ M1(dep*)

Obtain answer $x = \frac{1}{2}\ln 2$, or equivalent (0.347, or 0.346, or 0.35) A1

OR: Equate terms of the derivative and obtain a linear equation in x by taking logs correctly M1*

Solve the linear equation for x M1(dep*)

Obtain answer $x = \frac{1}{2}\ln 2$, or equivalent (0.347, or 0.346, or 0.35) A1 4

(ii) Carry out a method for determining the nature of a stationary point M1

Show that the point is a maximum with no errors seen A1 2

A1 **[4]**

3 **(i)** Use correct product or quotient rule M1

Obtain derivative in any correct form A1

Equate derivative to zero and solve for x M1

Obtain answer $x = \frac{1}{4}\pi$ or 0.785 with no errors seen A1 **[4]**

(ii) Use an appropriate method for determining the nature of a stationary point M1

Show the point is a maximum point with no errors seen A1 **[2]**

[SR: for the answer 45° deduct final A1 in part (i), and deduct A1 in part (ii) if this value in degrees is used in the exponential.]

4 Use correct quotient or product rule M1

Obtain correctly the derivative in any form, e.g. $\dfrac{e^{x}\cos x + e^{x}\sin x}{\cos^{2} x}$ A1

Equate derivative to zero and reach $\tan x = k$ M1*

Solve for x M1(dep*)

Obtain $x = -\frac{1}{4}\pi$ (or -0.785) only (accept x in [-0.79, -0.78] but not in degrees) A1 [5]

[The last three marks are independent. Fallacious log work forfeits the M1*. For the M1(dep*) the solution can lie outside the given range and be in degrees, but the mark is not available if $k = 0$. The final A1 is only given for an entirely correct answer to the whole question.]

5	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form, e.g. $-\dfrac{3\ln x}{x^4}+\dfrac{1}{x^4}$	A1	
	Equate derivative to zero and solve for x an equation of the form $\ln x = a$, where $a > 0$	M1	
	Obtain answer $\exp(\frac{1}{3})$, or 1.40, from correct work	A1	[4]

6 **(i)**	State derivative in any correct form, e.g. $3\cos x - 12\cos^2 x \sin x$	B1 + B1	
	Equate derivative to zero and solve for sin 2x, or sin x or cos x	M1	
	Obtain answer $x = \dfrac{1}{12}\pi$	A1	
	Obtain answer $x = \dfrac{5}{12}\pi$	A1	
	Obtain answer $x = \dfrac{1}{2}\pi$ and no others in the given interval	A1√	[6]
(ii)	Carry out a method for determining the nature of the relevant stationary point	M1	
	Obtain a maximum at $\dfrac{1}{12}\pi$ correctly	A1	[2]
	[Treat answers in degrees as a misread and deduct A1 from the marks for the angles.]		

7 **(i)**	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form, e.g. $\dfrac{2e^{2x}}{x^3}-\dfrac{3e^{2x}}{x^4}$	A1	
	Equate derivative to zero and solve a 2-term equation for non-zero x	M1	
	Obtain $x = \dfrac{3}{2}$ correctly	A1	[4]
(ii)	Carry out a method for determining the nature of a stationary point, e.g. test derivative either side	M1	
	Show point is a minimum with no errors seen	A1	[2]

8	Differentiate to obtain form $a\sin 2x + b\cos x$	M1	
	Obtain correct $-6\sin 2x + 7\cos x$	A1	
	Use identity $\sin 2x = 2\sin x\cos x$	B1	
	Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x	M1	
	Obtain 0.623	A1	
	Obtain 2.52	A1	
	Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$	A1	
	Treat answers in degrees as MR – 1 situation		[7]

9	EITHER:	Use correct product rule	M1	
		Obtain correct derivative in any form, e.g. $-\sin x\cos 2x - 2\cos x\sin 2x$	A1	
		Use the correct double angle formulae to express derivative in cos x and sin x, or cos $2x$ and sin x	M1	
	*OR*1:	Use correct double angle formula to express y in terms of cos x and attempt differentiation	M1	
		Use chain rule correctly	M1	
		Obtain correct derivative in any form, e.g. $-6\cos^2 x\sin x + \sin x$	A1	
	*OR*2:	Use correct factor formula and attempt differentiation	M1	
		Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
		Use correct trig formulae to express derivative in terms of cos x and sin x, or sin x	M1	
	Equate derivative to zero and obtain an equation in one trig function		M1	
	Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$		A1	
	Obtain answer $x = 1.15$ (or 65.9°) and no other in the given interval		A1	**[6]**
	[Ignore answers outside the given interval.]			
	[SR: Solution attempts following the *EITHER* scheme for the first two marks can earn the second and third method marks as follows:			
	Equate derivative to zero and obtain an equation in tan $2x$ and tan x		M1	
	Use correct double angle formula to obtain an equation in tan x		M1]	

10	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and obtain a horizontal equation	M1	
	Carry out complete method for solving an equation of the form $ae^{3x} = b$, or $ae^{5x} = be^{2x}$	M1	
	Obtain $x = \ln 2$, or exact equivalent	A1	
	Obtain $y = \frac{1}{3}$, or exact equivalent	A1	**6**

11 Use product rule **M1**

Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2\sin x \sin 2x$ **A1**

Equate derivative to zero and use double angle formulae **M1**

Remove factor of $\cos x$ and reduce equation to one in a single trig function **M1**

Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$ **A1**

Solve and obtain $x = 0.421$ **A1**

[6]

[Alternative: Use double angle formula M1.Use chain rule to differentiate M1. Obtain correct derivative
e.g. $\cos\theta - 6\sin^2\theta\cos\theta$ A1, then as above.]

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12 State or imply derivative of $(\ln x)^2$ is $\dfrac{2\ln x}{x}$ **B1**

Use correct quotient or product rule **M1**

Obtain correct derivative in any form, e.g. $\dfrac{2\ln x}{x^2} - \dfrac{(\ln x)^2}{x^2}$ **A1**

Equate derivative (or its numerator) to zero and solve for $\ln x$ **M1**

Obtain the point (1, 0) with no errors seen **A1**

Obtain the point $(e^2,\ 4e^{\ 2})$ **A1** [6]

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Answer	Marks
Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	M1
Obtain correct derivative in any form	A1
Equate the derivative to zero and solve for x	M1
Obtain $x = \frac{1}{6}\pi$	A1
Obtain $y = \sqrt{3}$	A1
	5
Carry out an appropriate method for determining the nature of a stationary point	M1
Show the point is a minimum point with no errors seen	A1
	2

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Question		
14	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a\sin x = b$	**M1 ***
	Solve for x	**M1(dep*)**
	Obtain answer 0.340	**A1**
	Obtain second answer 2.802 and no other in the given interval	**A1**
		6

Question			
15	Use product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
		2	
(ii)	Equate derivative to zero and use correct $\cos(A + B)$ formula	**M1**	
	Obtain the given equation	**A1**	
		2	
(iii)	Use correct method to solve for x	**M1**	
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	**A1**	
	Obtain second answer, e.g. $\frac{7}{12}\pi$, and no other	**A1**	
		3	

Use correct quotient rule or correct product rule	**M1**	
Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\mathrm{e}^{-2x} \quad 1-x^2\big) + 2x\mathrm{e}^{-2x}}{\left(1-x^2\right)^2}$
Equate derivative to zero and obtain a 3 term quadratic in x	**M1**	
Obtain a correct 3-term equation e.g. $2x^2 \quad 2x - 2 = 0$ or $^2 \quad x = 1$	**A1**	From correct work only
Solve and obtain x = 0.618 only	**A1**	From correct work only
	5	