C2 Without Diagram Hard
Answers
P3

1	**(i)** *EITHER*:	State a correct expression for $\lvert z \rvert$ or $\lvert z \rvert^2$, e.g. $(1+\cos 2\theta)^2 + (\sin 2\theta)^2$	B1	
		Use double angle formulae throughout or Pythagoras	M1	
		Obtain given answer $2\cos\theta$ correctly	A1	
		State a correct expression for tangent of argument, e.g. $(\sin 2\theta/(1+\cos 2\theta)$	B1	
		Use double angle formulae to express it in terms of $\cos\theta$ and $\sin\theta$	M1	
		Obtain $\tan\theta$ and state that the argument is θ	A1	
	OR:	Use double angle formulae to express z in terms of $\cos\theta$ and $\sin\theta$	M1	
		Obtain a correct expression, e.g. $1+\cos^2\theta - \sin^2\theta + 2\mathrm{i}\sin\theta\cos\theta$	A1	
		Convert the expression to polar form	M1	
		Obtain $2\cos\theta(\cos\theta + \mathrm{i}\sin\theta)$	A1	
		State that the modulus is $2\cos\theta$	A1	
		State that the argument is θ	A1	[6]
	(ii)	Substitute for z and multiply numerator and denominator by the conjugate of z, or equivalent	M1	
		Obtain correct real denominator in any form	A1	
		Identify and obtain real part equal to $\frac{1}{2}$	A1	[3]

2	**(a) (i)** *EITHER*:	Multiply numerator and denominator by $a - 2\mathrm{i}$, or equivalent	M1	
		Obtain final answer $\dfrac{5a}{a^2+4} - \dfrac{10\mathrm{i}}{a^2+4}$, or equivalent	A1	
	OR:	Obtain two equations in x and y, solve for x or for y	M1	
		Obtain final answer $x = \dfrac{5a}{a^2+4}$ and $y = \dfrac{10}{a^2+4}$, or equivalent	A1	[2]
	(ii)	Either state $\arg(u) = -\dfrac{3}{4}\pi$, or express u^* in terms of a (f.t. on u)	B1√	
		Use correct method to form an equation in a, e.g. $5a = -10$	M1	
		Obtain $a = -2$ correctly	A1	[3]
	(b)	Show a point representing $2 + 2\mathrm{i}$ in relatively correct position in an Argand diagram	B1	
		Show the circle with centre at the origin and radius 2	B1	
		Show the perpendicular bisector of the line segment from the origin to the point representing $2 + 2\mathrm{i}$	B1√	
		Shade the correct region	B1	[4]
		[SR: Give the first B1 and the B1√ for obtaining $y = 2 - x$, or equivalent, and sketching the attempt.]		

3 **(i)** Use the quadratic formula, completing the square, or the substitution $z = x + iy$ to find a root and use $i^2 = -1$ M1

Obtain final answers $-\sqrt{3} \pm i$, or equivalent A1 [2]

(ii) State that the modulus of both roots is 2 B1√

State that the argument of $-\sqrt{3} + i$ is 150° or $\frac{5}{6}\pi$ (2.62) radians B1√

State that the argument of $-\sqrt{3} - i$ is –150° (or 210°) or $-\frac{5}{6}\pi$ (–2.62) radians or $\frac{7}{6}\pi$ (3.67) radians B1√ [3]

(iii) Carry out an attempt to find the sixth power of a root M1

Verify that one of the roots satisfies $z^6 = -64$ A1

Verify that the other root satisfies the equation A1 [3]

4 **(i)** *EITHER*: Multiply numerator and denominator by 1 + 3i, or equivalent M1

Simplify numerator to –5 + 5i, or denominator to 10, or equivalent A1

Obtain final answer $-\frac{1}{2} + \frac{1}{2}i$, or equivalent A1

OR: Obtain two equations in x and y, and solve for x or for y M1

Obtain $x = -\frac{1}{2}$ or $y = \frac{1}{2}$, or equivalent A1

Obtain final answer $-\frac{1}{2} + \frac{1}{2}i$, or equivalent A1 [3]

(ii) Show B and C in relatively correct positions in an Argand diagram B1

Show u in a relatively correct position B1√ [2]

(iii) Substitute exact arguments in the LHS $\arg(1 + 2i) - \arg(1 - 3i) = \arg u$, or equivalent M1

Obtain and use $\arg u = \frac{3}{4}\pi$ A1

Obtain the given result correctly A1 [3]

5	**(i)**	Show that $a^2 + b^2 = (a + \mathrm{i}b)(a - \mathrm{i}b)$	B1	
		Show that $(a + \mathrm{i}b - k\mathrm{i})^* = a - \mathrm{i}b + k\mathrm{i}$	B1	[2]
	(ii)	Square both sides and express the given equation in terms of z and z^*	M1	
		Obtain a correct equation in any form, e.g. $(z - 10\mathrm{i})(z^* + 10\mathrm{i}) = 4(z - 4\mathrm{i})(z^* + 4\mathrm{i})$	A1	
		Obtain the given equation	A1	
		Either express $\lvert z - 2i \rvert = 4$ in terms of z and z^* or reduce the given equation to the form $\lvert z - u \rvert = r$	M1	
		Obtain the given answer correctly	A1	[5]
	(iii)	State that the locus is a circle with centre 2i and radius 5	B1	[1]

6	**(i)**	Either	Multiply numerator and denominator by $\sqrt{3} + \mathrm{i}$ and use $\mathrm{i}^2 = -1$	M1	
			Obtain correct numerator $18 + 18\sqrt{3}\mathrm{i}$ or correct denominator 4	B1	
			Obtain $\frac{9}{2} + \frac{9}{2}\sqrt{3}\mathrm{i}$ or $\left(18 + 18\sqrt{3}\mathrm{i}\right)/4$	A1	
			Obtain modulus or argument	M1	
			Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1	[5]
		OR	Obtain modulus and argument of numerator or denominator, or both moduli or both arguments	M1	
			Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$ or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees)	B1	
			Obtain $18\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} \div 2\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}$ or equivalent	A1	
			Divide moduli and subtract arguments	M1	
			Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1	[5]
	(ii)		State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$, following through their answer to part (i)	B1$\sqrt{}$	
			State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i} \pm \frac{1}{2}\pi\mathrm{i}}$, following through their answer to part (i)	B1$\sqrt{}$	
			Obtain $3\mathrm{e}^{-\frac{5}{6}\pi\mathrm{i}}$	B1	[3]

7	(a)	*EITHER*:	Substitute and expand $(-1+\sqrt{5}\,\mathrm{i})^3$ completely	M1	
			Use $\mathrm{i}^2 = -1$ correctly at least once	M1	
			Obtain $a = -12$	A1	
			State that the other complex root is $-1-\sqrt{5}\,\mathrm{i}$	B1	
		OR1:	State that the other complex root is $-1-\sqrt{5}\,\mathrm{i}$	B1	
			State the quadratic factor z^2+2z+6	B1	
			Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for *a* or, using a 3-term quadratic, factorise the cubic and determine *a*	M1	
			Obtain $a = -12$	A1	
		OR2:	State that the other complex root is $-1-\sqrt{5}\mathrm{i}$	B1	
			State or show the third root is 2	B1	
			Use a valid method to determine *a*	M1	
			Obtain $a = -12$	A1	
		OR3:	Substitute and use De Moivre to cube $\sqrt{6}\mathrm{cis}(114.1^\circ)$, or equivalent	M1	
			Find the real and imaginary parts of the expression	M1	
			Obtain $a = -12$	A1	
			State that the other complex root is $-1-\sqrt{5}\mathrm{i}$	B1	**4**
	(b)	*EITHER:*	Substitute $w = \cos 2\theta + \mathrm{i}\sin 2\theta$ in the given expression	B1	
			Use double angle formulae throughout	M1	
			Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
			Obtain given answer correctly	A1	
		OR:	Substitute $w = \mathrm{e}^{2\mathrm{i}\theta}$ in the given expression	B1	
			Divide numerator and denominator by $\mathrm{e}^{\mathrm{i}\theta}$, or equivalent	M1	
			Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
			Obtain the given answer correctly	A1	**4**

8 **(i)** Substitute $z = 1 + \text{i}$ and obtain $w = \dfrac{1+2\text{i}}{1+\text{i}}$ B1

EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent M1

Simplify numerator to $3 + \text{i}$ or denominator to 2 A1

Obtain final answer $\frac{3}{2} + \frac{1}{2}\text{i}$, or equivalent A1

OR: Obtain two equations in x and y, and solve for x or for y M1

Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent A1

Obtain final answer $\frac{3}{2} + \frac{1}{2}\text{i}$, or equivalent A1 **[4]**

(ii) *EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\text{i}z^2 + z - \text{i} = 0$ B1

Solve a 3-term quadratic for z or substitute $z = x + \text{i}y$ and use a correct method to solve for x and y M1

OR: Substitute $w = x + \text{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts B1

Solve for x and y M1

Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\text{i}}{2\text{i}}$ A1

Obtain final answer $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\text{i}$ A1 **[4]**

9 **(i)** State or imply $\text{i}w = -3 + 5\text{i}$ B1

Carry out multiplication by $\dfrac{4-\text{i}}{4-\text{i}}$ M1

Obtain final answer $-\dfrac{7}{17} + \dfrac{23}{17}\text{i}$ or equivalent A1 [3]

(ii) Multiply w by z to obtain $17 + 17\text{i}$ B1

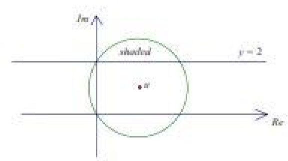
State $\arg w = \tan^{-1}\dfrac{3}{5}$ or $\arg z = \tan^{-1}\dfrac{1}{4}$ B1

State $\arg wz = \arg w + \arg z$ M1

Confirm given result $\tan^{-1}\dfrac{3}{5} + \tan^{-1}\dfrac{1}{4} = \dfrac{1}{4}\pi$ legitimately A1 [4]

Question	Answer	Marks	Guidance
10(i)	State or imply $r = 2$	B1	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	B1	
	Use a correct method for finding the modulus or the argument of u^4	M1	Allow correct answers from correct u with minimal working shown
	Obtain modulus 16	A1	
	Obtain argument $\frac{2}{3}\pi$	A1	Accept $16e^{i\frac{2\pi}{3}}$
		5	
10(ii)	Substitute u and carry out a correct method for finding u^3	M1	$\left(u^3 = 8i\right)$ Follow *their* u^3 if found in part **(i)**
	Verify u is a root of the given equation	A1	
	State that the other root is $\sqrt{3} - \mathrm{i}$	B1	
	Alternative method		
	State that the other root is $\sqrt{3} - \mathrm{i}$	B1	
	Form quadratic factor and divide cubic by quadratic	M1	$\left(z - \sqrt{3} - i\right)\left(z - \sqrt{3} + i\right)\left(= z^2 - 2\sqrt{3}z + 4\right)$
	Verify that remainder is zero and hence that u is a root of the given equation	A1	
		3	

PUBLISHED

Question	Answer	Marks	Guidance
10(iii)	Show the point representing u in a relatively correct position	B1	
	Show a circle with centre u and radius 2	B1	**FT** on the point representing u. Condone near miss of origin
	Show the line $y = 2$	B1	
	Shade the correct region	B1	
	Show that the line and circle intersect on $x = 0$	B1	Condone near miss
		5	