A14 Vectors Answers
P3

1 **(i)** *EITHER*: Express general point of l or m in component form

e.g. $(2 + s, -1 + s, 4 - s)$ or $(-2 - 2t, 2 + t, 1 + t)$ — B1

Equate at least two pairs of components and solve for s or for t — M1

Obtain correct answer for s or t (possible answers are $\frac{2}{3}$, 10, or 3 for s and $-\frac{7}{3}$, -7, or 0 for t) — A1

Verify that all three component equations are not satisfied — A1

OR: State a Cartesian equation for l or for m, e.g. $\frac{x-2}{1} = \frac{y-(-1)}{1} = \frac{z-4}{-1}$ for l — B1

Solve a pair of equations for a pair of values, e.g. x and y — M1

Obtain a pair of correct answers, e.g. $x = \frac{8}{3}$ and $y = -\frac{1}{3}$ — A1

Find corresponding remaining values, e.g. of z, and show lines do not intersect — A1

OR: Form a relevant triple scalar product, e.g.

$(4\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}).((\mathbf{i} + \mathbf{j} - \mathbf{k})\times(-2\mathbf{i} + \mathbf{j} + \mathbf{k}))$ — B1

Attempt to use correct method of evaluation — M1

Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant — A1

Obtain correct non-zero value, e.g.14, and state that the lines cannot intersect — A1 — **4**

(ii) *EITHER*: Express $\overrightarrow{PQ}$ or $\left(\overrightarrow{QP}\right)$ in terms of s in any correct form e.g.

$-s\mathbf{i} + (1 - s)\mathbf{j} + (-5 + s)\mathbf{k}$ — B1

Equate its scalar product with a direction vector for l to zero, obtaining a linear equation in s — M1

Solve for s — M1

Obtain $s = 2$ and $\overrightarrow{OP}$ is $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ — A1

OR: Take a point A on l, e.g. $(2, -1, 4)$, and use scalar product to calculate AP, the length of the projection of AQ onto l — M1

Obtain answer $AP = 2\sqrt{3}$, or equivalent — A1

Carry out method for finding $\overrightarrow{OP}$ — M1

Obtain answer $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ — A1 — **4**

(iii) Show that Q is the point on m with parameter $t = -2$, or that $(2, 0, -1)$ satisfies the Cartesian equation of m — B1

Show that PQ is perpendicular to m e.g. by verifying fully that $(-2\mathbf{i} - \mathbf{j} - 3\mathbf{k}).(-2\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$ — B1 — **2**

2 **(i)** State or imply a direction vector for AB is $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent **B1**

EITHER: State equation of AB is $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent **B1√**

Equate at least two pairs of components of AB and l and solve for s or for t **M1**

Obtain correct answer for s or for t, e.g. $s = 0$ or $t = -2$; $s = -\frac{5}{3}$ or $t = -\frac{1}{3}$ or $s = 5$ or $t = 3$ **A1**

Verify that all three pairs of equations are not satisfied and that the lines fail to intersect **A1**

OR: State a Cartesian equation for AB, e.g. $\frac{x-2}{-1} = \frac{y-2}{2} = \frac{z-1}{2}$, and for l,

e.g. $\frac{x-4}{1} = \frac{y+2}{2} = \frac{z-2}{1}$ **B1√**

Solve a pair of equations, e.g. in x and y, for one unknown **M1**

Obtain one unknown, e.g. $x = 4$ or $y = -2$ **A1**

Obtain corresponding remaining values, e.g. of z, and show lines do not intersect **A1**

OR: Form a relevant triple scalar product,
e.g. $(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}).((-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})\times(\mathbf{i} + 2\mathbf{j} + \mathbf{k}))$ **B1√**

Attempt to use correct method of evaluation **M1**

Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant **A1**

Obtain correct non-zero value, e.g. – 20, and state that the lines do not intersect **A1** **5**

3 (i) State $\mathbf{r} = \begin{pmatrix}-1\\3\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\-4\end{pmatrix}$, or equivalent B1 1

(ii) Express $\overrightarrow{BN}$ in terms of λ, e.g. $\begin{pmatrix}-1+3\lambda\\3-\lambda\\5-4\lambda\end{pmatrix} - \begin{pmatrix}3\\-1\\-4\end{pmatrix}$, or equivalent B1

Equate its scalar product with $\begin{pmatrix}3\\-1\\-4\end{pmatrix}$ to zero and solve for λ M1

Obtain $\lambda = 2$ A1

Obtain $\overrightarrow{ON} = \begin{pmatrix}5\\1\\-3\end{pmatrix}$, or equivalent A1√

Carry out method for calculating BN, i.e. $|2\mathbf{i} + 2\mathbf{j} + \mathbf{k}|$ M1

Obtain the given answer $BN = 3$ correctly A1 6

4 **(i)** State a vector equation for the line through A and B, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - \mathbf{j})$ B1

Equate at least two pairs of components of general points on AB and l, and solve for s or for t M1

Obtain correct answer for s or t, e.g. $s = -6, 2, -2$ when $t = 3, -1, -1$ respectively A1

Verify that all three component equations are not satisfied A1 [4]

(ii) State or imply a direction vector for AP has components $(-2t, 3 + t, -1-t)$, or equivalent B1

State or imply cos 60° equals $\dfrac{\overrightarrow{AP}.\overrightarrow{AB}}{\left|\overrightarrow{AP}\right|.\left|\overrightarrow{AB}\right|}$ M1*

Carry out correct processes for expanding the scalar product and expressing the product of the moduli in terms of t, in order to obtain an equation in t in any form M1(dep*)

Obtain the given equation $3t^2 + 7t + 2 = 0$ correctly A1

Solve the quadratic and use a root to find a position vector for P M1

Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{1}{3}$ for a valid reason A1 [6]

5 **(i)** Express general point of l or m in component form, e.g. $(1 + s, 1 - s, 1 + 2s)$ or $(4 + 2t, 6 + 2t, 1 + t)$ B1

Equate at least two corresponding pairs of components and solve for s or t M1

Obtain $s = -1$ or $t = -2$ A1

Verify that all three component equations are satisfied A1 [4]

(ii) Carry out correct process for evaluating the scalar product of the direction vectors of l and m M1

Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result M1

Obtain answer 74.2° (or 1.30 radians) A1 [3]

6 **(i)** State correct equation in any form, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ B1 [1]

(ii) *EITHER*: Equate a relevant scalar product to zero and form an equation in λ M1

OR 1: Equate derivative of OP^2 (or OP) to zero and form an equation in λ M1

OR 2: Use Pythagoras in OAP or OBP and form an equation in λ M1

State a correct equation in any form A1

Solve and obtain $\lambda = -\frac{1}{6}$ or equivalent A1

Obtain final answer $\overrightarrow{OP} = \frac{2}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} + \frac{7}{3}\mathbf{k}$, or equivalent A1 [4]

7 (i)	*EITHER*:	Express general point of l or m in component form, e.g. $(2 + \lambda, -\lambda, 1 + 2\lambda)$ or $(\mu, 2 + 2\mu, 6 - 2\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ (possible answers for λ are -2, $\frac{1}{4}$, 7 and for μ are 0, $2\frac{1}{4}$, $-4\frac{1}{2}$)	A1	
		Verify that all three component equations are not satisfied	A1	
	OR:	State a relevant scalar triple product, e.g. $(2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}) \,.\, ((\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}))$	B1	
		Attempt to use the correct method of evaluation	M1	
		Obtain at least two correct simplified terms of the three terms of the expansion of the triple product or of the corresponding determinant, e.g. $-4, -8, -15$	A1	
		Obtain correct non-zero value, e.g. -27, and state that the lines do not intersect	A1	[4]
(ii)		Carry out the correct process for evaluating scalar product of direction vectors for l and m	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 47.1° or 0.822 radians	A1	[3]

8 (i)		Use a correct method to express $\overrightarrow{OP}$ in terms of λ	M1	
		Obtain the given answer	A1	[2]
(ii)	*EITHER*:	Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$ in terms of λ	M1	
		Using the correct method for the moduli, divide scalar products by products of moduli and express cos *AOP* = cos *BOP* in terms of λ, or in terms of λ and *OP*	M1*	
	OR1:	Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of λ	M1	
		Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express cos *AOP* = cos *BOP* in terms of λ, or λ and *OP*	M1*	
		Obtain a correct equation in any form, e.g. $\dfrac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}} = \dfrac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$	A1	
		Solve for λ	M1(dep*)	
		Obtain $\lambda = \frac{3}{8}$	A1	[5]
		[SR: The M1* can also be earned by equating cos *AOP* or cos *BOP* to a sound attempt at cos $\frac{1}{2}$ *AOB* and obtaining an equation in λ. The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.]		
		[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for *OP* to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]		
(iii)		Verify the given statement correctly	B1	[1]

9	**(i)**	Either	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line) or equivalent	B1	
			Use scalar product to find cosine of angle between PA and line	M1	
			Obtain $\dfrac{42}{\sqrt{14\times 230}}$ or equivalent	A1	
			Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 1	Obtain $\pm\begin{pmatrix}2n+2\\n-1\\3n-15\end{pmatrix}$ for PN (where N is foot of perpendicular)	B1	
			Equate scalar product of PN and line direction to zero Or equate derivative of PN^2 to zero Or use Pythagoras' theorem in triangle PNA to form equation in n	M1	
			Solve equation and obtain $n = 3$	A1	
			Obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 2	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line)	B1	
			Evaluate vector product of PA and line direction	M1	
			Obtain $\pm\begin{pmatrix}12\\-36\\-4\end{pmatrix}$	A1	
			Divide modulus of this by modulus of line direction and obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 3	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line)	B1	
			Evaluate scalar product of PA and line direction to obtain distance AN	M1	
			Obtain $3\sqrt{14}$ or equivalent	A1	
			Use Pythagoras' theorem in triangle PNA and obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 4	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line)	B1	
			Use a second point B on line and use cosine rule in triangle ABP to find angle A or angle B or use vector product to find area of triangle	M1	
			Obtain correct answer (angle A = 42.25…)	A1	
			Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent	A1	[4]

10 or (i) Express general point of l or m in component form, i.e. $(3-\lambda, -2+2\lambda, 1+\lambda)$ $(4+a\mu, 4+b\mu, 2-\mu)$ B1

Equate components and eliminate either λ or μ from a pair of equations M1
Eliminate the other parameter and obtain an equation in a and b M1
Obtain the given answer A1 [4]

(ii) Using the correct process equate the scalar product of the direction vectors to zero M1*
Obtain $-a+2b-1=0$, or equivalent A1
Solve simultaneous equations for a or for b M1(dep*)
Obtain $a = 3$, $b = 2$ A1 [4]

(iii) Substitute found values in component equations and solve for λ or for μ M1
Obtain answer $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ from either $\lambda = 2$ or from $\mu = -1$ A1 [2]

11 **(ii)** *EITHER* Find $\overrightarrow{CP}$ for a point P on AB with a parameter t, e.g. $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + t(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ B1√

Either: Equate scalar product $\overrightarrow{CP}.\overrightarrow{AB}$ to zero and form an equation in t

Or 1: Equate derivative for CP^2 (or CP) to zero and form an equation in t
Or 2: Use Pythagoras in triangle CPA (or CPB) and form an equation in t M1
Solve and obtain correct value of t, e.g. $t = -2$ A1
Carry out a complete method for finding the length of CP M1
Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1

OR 1 State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form B1√
Using a relevant scalar product find the cosine of CAB (or CBA) M1
Obtain cost $CAB = -\dfrac{22}{\sqrt{11}.\sqrt{62}}$, or cos $CBA = \dfrac{33}{\sqrt{11}.\sqrt{117}}$, or equivalent A1
Use trig to find the length of the perpendicular M1
Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1

OR 2 State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form B1√
Using a relevant scalar product find the length of the projection AC (or BC) on AB M1
Obtain answer $2\sqrt{11}$ (or), $3\sqrt{11}$ or equivalent A1
Use Pythagoras to find the length of the perpendicular M1
Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1

OR 3 State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form B1√
Calculate their vector product, e.g. $(-2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ M1
Obtain correct product, e.g. $-2\mathbf{i} + 13\mathbf{j} - 5\mathbf{k}$ A1
Divide modulus of the product by the modulus of $\overrightarrow{AB}$ M1
Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1

OR 4 State two of $\overrightarrow{AB}$, $\overrightarrow{BC}$) and $\overrightarrow{AC}$ in component form B1√
Use cosine formula in triangle ABC to find cos A or cos B M1
Obtain cos $A = -\dfrac{44}{2\sqrt{11}.\sqrt{62}}$, or cos $B = \dfrac{66}{2\sqrt{11}.\sqrt{117}}$ A1
Use trig to find the length of the perpendicular M1
Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1 [5]
[The f.t is on $\overrightarrow{AB}$]

12 **(i)** State or imply general point of either line has coordinates $(5 + s, 1 - s, -4 + 3s)$ or $(p + 2t, 4 + 5t, -2 - 4t)$ B1

Solve simultaneous equations and find s and t M1

Obtain $s = 2$ and $t = -1$ or equivalent in terms of p A1

Substitute in third equation to find $p = 9$ A1

State point of intersection is $(7, -1, 2)$ A1 [5]

13 (**i**) *EITHER:* State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form B1

Using the correct processes evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, or equivalent M1

Using the correct process for the moduli divide the scalar product by the product of the moduli M1

Obtain answer $\frac{20}{21}$ A1

OR: Use correct method to find lengths of all sides of triangle ABC M1

Apply cosine rule correctly to find the cosine of angle BAC M1

Obtain answer $\frac{20}{21}$ A1 **4**

(**ii**) State an exact value for the sine of angle BAC, e.g. $\sqrt{41}/21$ B1√

Use correct area formula to find the area of triangle ABC M1

Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent A1 **3**

[SR: Allow use of a vector product, e.g. $\overrightarrow{AB} \times \overrightarrow{AC} = -6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ B1√. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]

14 (i) *EITHER*:	Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i} - 17\mathbf{j} + 4\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	B1	
	Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1	
	Solve and obtain $\lambda = 3$	A1	
	Carry out a complete method for finding the length of AP	M1	
	Obtain the given answer 15 correctly	A1	
*OR*1:	Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}$	B1	
	Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	M1	
	Obtain correct answer, e.g. $-30\mathbf{i} + 6\mathbf{j} + 33\mathbf{k}$	A1	
	Divide the modulus of the product by that of the direction vector	M1	
	Obtain the given answer correctly	A1	
*OR*2:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the projection of BA (or AB) on l	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}}$	A1	
	Use Pythagoras to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the cosine of ABP	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}.\sqrt{306}}$	A1	
	Use trig. to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Find a second point C on l and use the cosine rule in triangle ABC to find the cosine of angle A, B, or C, or use a vector product to find the area of ABC	M1	
	Obtain correct answer in any form	A1	
	Use trig. or area formula to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ in any form	B1	
	Use correct method to express AP^2 (or AP) in terms of λ	M1	
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2 + (-17+\lambda)^2 + (4-2\lambda)^2$	A1	
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
	Obtain the given answer correctly	A1	**[5]**

15	(i)	State at least two of the equations $1+\lambda = a+\mu$, $4 = 2+2\mu$, $-2+3\lambda = -2+3a\mu$	B1	
		Solve for λ or for μ	M1	
		Obtain $\lambda = a$ (or $\lambda = a+\mu-1$) and $\mu = 1$	A1	
		Confirm values satisfy third equation	A1	[4]
	(ii)	State or imply point of intersection is $(a+1, 4, 3a-2)$	B1	
		Use correct method for the modulus of the position vector and equate to 9, following their point of intersection	M*1	
		Solve a three-term quadratic equation in a $\quad (a^2 - a - 6 = 0)$	DM*1	
		Obtain –2 and 3	A1	[4]

16	(i)	Obtain $\pm\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ as direction vector of l_1	B1	
		State that two direction vectors are not parallel	B1	
		Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answers for λ and μ	A1	
		Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
	(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1	
		Use correct process for finding modulus and evaluating inverse cosine	M1	
		Obtain $79.5°$ or 1.39 radians	A1	[3]

17	(i)	Carry out a correct method for finding a vector equation for *AB*	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		Equate at least two pairs of components of general points on *AB* and *l* and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$; or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$	A1	
		Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1	**[5]**
	(ii)	*EITHER*: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	

18	**(i)**	Use correct method to form a vector equation for AB	**M1**	
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**A1**	[2]

19	**(i)**	*Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$	**B1**	
		Use a correct method for finding the position vector of D	**M1**	
		Obtain answer $3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, or equivalent	**A1**	
		EITHER: Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and BC	**M1**	
		Show that $ABCD$ has a pair of adjacent sides that are equal	**A1**	
		OR: Calculate scalar product $\overrightarrow{AC}.\overrightarrow{BD}$ or equivalent	**M1**	
		Show that $ABCD$ has perpendicular diagonals	**A1**	[5]

20	(i)	State a correct equation for AB in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent	**B1**
		Equate at least two pairs of components of AB and l and solve for λ or for μ	**M1**
		Obtain correct answer for λ or for μ, e.g. $\lambda = -1$ or $\mu = 2$	**A1**
		Show that not all three equations are not satisfied and that the lines do not intersect	**A1**
			[4]
	(ii)	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l, e.g. $(1-\mu)\mathbf{i} + (-3+2\mu)\mathbf{j} + (-2+\mu)\mathbf{k}$	**B1**
		Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	**M1**
		Solve and obtain $\mu = \frac{3}{2}$	**A1**
		Carry out a method to calculate AP when $\mu = \frac{3}{2}$	**M1**
		Obtain the given answer $\dfrac{1}{\sqrt{2}}$ correctly	**A1**
		OR 1:Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l	**(B1**
		Use correct method to express AP^2 (or AP) in terms of μ	**M1**
		Obtain a correct expression in any form, e.g. $(1-\mu)^2 + (-3+2\mu)^2 + (-2+\mu)^2$	**A1**
		Carry out a complete method for finding its minimum	**M1**
		Obtain the given answer correctly	**A1)**
		OR 2:Calling (2, –2, –1) C, state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	**(B1**
		Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on l	**M1**
		Obtain correct answer in any form, e.g. $\dfrac{9}{\sqrt{6}}$	**A1**
		Use Pythagoras to find the perpendicular	**M1**
		Obtain the given answer correctly	**A1)**
		OR 3:State $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form	**(B1**
		Calculate vector product of $\overrightarrow{AC}$ and a direction vector for l, e.g. $(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**
		Obtain correct answer in any form, e.g. $\mathbf{i} + \mathbf{j} - \mathbf{k}$	**A1**
		Divide modulus of the product by that of the direction vector	**M1**
		Obtain the given answer correctly	**A1)**
			[5]

21	**(i)**	Express general point of l in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$	**B1**	
		Using the correct process for the modulus form an equation in λ	**M1***	
		Reduce the equation to a quadratic, e.g. $6\lambda^2 + 2\lambda - 4 = 0$	**A1**	
		Solve for λ (usual requirements for solution of a quadratic)	**DM1**	
		Obtain final answers $-\mathbf{i} + 3\mathbf{j}$ and $\frac{7}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$	**A1**	[5]

22(i)	*EITHER*: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	**(B1**
	Equate scalar product of $\overrightarrow{AP}$ and direction vector of l to zero and solve for λ	**M1**
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	**A1)**
	OR: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	**(B1**
	Differentiate $\lvert AP \rvert^2$ and solve for λ at minimum	**M1**
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	**A1)**
	Total:	**5**

23(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$, or equivalent	**A1**
	Equate two pairs of components of general points on AB and l and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda = \frac{5}{7}$ or $\mu = \frac{3}{7}$	**A1**
	Obtain $m = 3$	**A1**
	Total:	**5**

24(i)	Equate at least two pairs of components of general points on l and m and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	**A1**
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	**A1**
		3

25(i)	*EITHER*: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l, e.g. $(1+\mu)\,\mathbf{i} + (4+2\mu)\mathbf{j} + (4+3\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain correct solution e.g. $\mu = -\dfrac{3}{2}$	**A1**
	Carry out method to calculate PQ	**M1**
	Obtain answer 1.22	**A1**
	*OR*1: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	**B1**
	Use a correct method to express PQ^2 (or PQ) in terms of μ	**M1**
	Obtain a correct expression in any form	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain answer 1.22	**A1**
	*OR*2: Calling (4, 2, 5) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$	**B1**
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on l	**M1**
	Obtain correct answer $21/\sqrt{14}$, or equivalent	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain answer 1.22	**A1**
	*OR*3: State $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form	**B1**
	Calculate vector product of $\overrightarrow{PA}$ and a direction vector for l	**M1**
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**A1**
	Divide modulus of the product by that of the direction vector	**M1**
	Obtain answer 1.22	**A1**

26(i)	Equate at least two pairs of components and solve for s or for t	**M1**	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-\ldots}{5\ldots} \\ t = \frac{-\ldots}{5\ldots} \\ \frac{6}{5} \neq \ldots \end{cases}$
	Obtain correct answer for s or t, e.g. $s = -6$, $t = -11$	**A1**	
	Verify that all three equations are not satisfied and the lines fail to intersect	**A1**	
	State that the lines are not parallel	**B1**	
		4	

27(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	**A1**
	Equate pair(s) of components AB and l and solve for λ or μ	**M1(dep*)**
	Obtain correct answer for λ or μ	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Total:	**5**
(ii)	State or imply a direction vector for AP has components $(2 + t, 5 + 2t, -3 - 2t)$	**B1**
	State or imply that cos 120° equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	**M1**
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of t, and obtain an equation in terms of t	**M1**
	Obtain the given equation correctly	**A1**
	Solve the quadratic and use a root to find a position vector for P	**M1**
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	**A1**
	Total:	**6**

28(i)	Carry out correct method for finding a vector equation for *AB*	**M1**	
	Obtain $(\mathbf{r} =)\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+2\mathbf{k})$, or equivalent	**A1**	
	Equate two pairs of components of general points on *their AB* and l and solve for λ or for μ	**M1**	
	Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$	**A1**	
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$) Conclusion needs to follow correct values	**A1**	Alternatives ***A*** λ μ
		5	

29(i)	Find $\overrightarrow{PQ}$ for a general point Q on l, e.g. $-3\mathbf{i}+6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	**B1**
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate the result to zero	**M1**
	Solve for μ and obtain $\mu = 2$	**A1**
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	**M1**
	Obtain answer 3	**A1**
	Alternative method for question 10(i)	
	Calling the point (1, 2, 3) A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. $3\mathbf{i} - 6\mathbf{k}$	**B1**
	Use a scalar product with a direction vector for l to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l	**M1**
	Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain answer 3	**A1**

30(i)	Express general point of l or m in component form e.g. $(a + \lambda, 2 - 2\lambda, 3 + 3\lambda)$ or $(2 + 2\mu, 1 - \mu, 2 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain **either** $\lambda = -2$ or $\mu = -5$ **or** $\lambda = \frac{1}{3}a$ or $\mu \quad \frac{2}{3}a - 1$ **or** $\lambda = \frac{1}{5}(a-4)$ or $\mu \quad \frac{1}{5}(3a-7)$	**A1**	
	Obtain $a = -6$	**A1**	
		4	