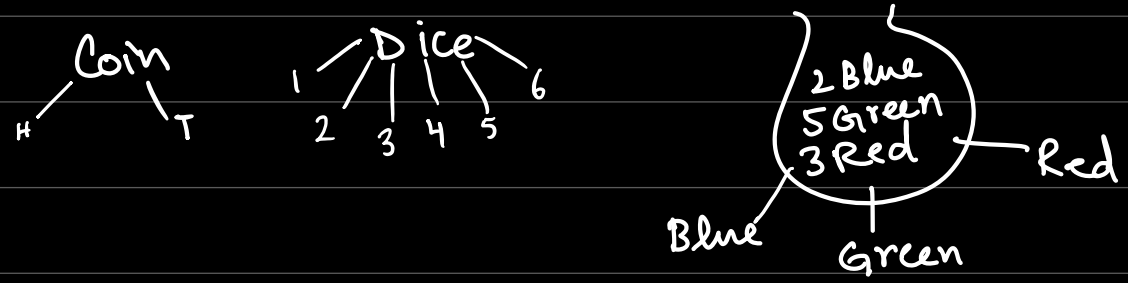
Binomial Distribution

(without replacement cannot be Binomial ever)

Conditions

1- Repeated experiment (n)

2- Discrete (fixed) outcomes



3- Success (p) and failure (q) $q = 1 - p$

4. n, p, and q are constants.

$$P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

- r: Desired number of times success happens
- Success

P1

${}^{n}C_{r}\, a^{n-r} b^{r}$

Be careful.

Q. A Fair dice is thrown 20 times. → Repeated experiment $n=20$

X (success) denotes the random variable for number of times that dice lands on a multiple of 3. Find the probability that: 1, 2, (3), 4, 5, (6)

X
0
1
2
⋮
20

$n = 20$ $\quad p = \frac{2}{6} = \frac{1}{3}$ $\quad q = 1 - \frac{1}{3} = \frac{2}{3}$

(a) Dice lands on multiple of 3 (Success (X)) exactly twice. $X = 2$

$$P(X=2) = {}^{20}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{18}$$

This means that we threw dice 20 times, out of which two times it landed on multiple of 3 and 18 times it did not land on a multiple of 3.

(b) Dice lands on multiple of 3 (Success (X)) at least 18 times. $X \geq 18$

$$P(X \geq 18) = P(18, 19, 20)$$
$$= P(X=18) + P(X=19) + P(X=20)$$
$$= {}^{20}C_{18} \left(\frac{1}{3}\right)^{18} \left(\frac{2}{3}\right)^2 + {}^{20}C_{19} \left(\frac{1}{3}\right)^{19} \left(\frac{2}{3}\right)^1 + {}^{20}C_{20} \left(\frac{1}{3}\right)^{20} \left(\frac{2}{3}\right)^0$$
$$= \boxed{}$$

(c) Dice lands on multiple of 3 (Success (X)) at most twice. $X \leq 2$

$$P(X \leq 2) = P(0, 1, 2)$$
$$= P(X=0) + P(X=1) + P(X=2)$$

$$= {}^{20}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{20} + {}^{20}C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{19} + {}^{20}C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{18}$$

(d) Dice lands on multiple of 3 at least once.

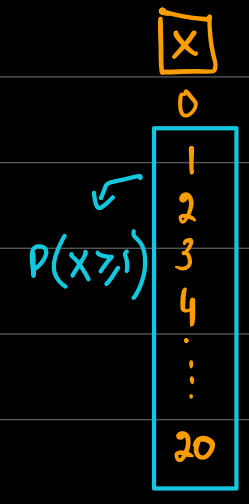
Success (X) $\quad X \geq 1$

$$P(X \geq 1) = P(1, 2, 3, 4 \ldots\ldots 20)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^{20}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{20}$$

$$= \boxed{}$$



(e) Dice lands on a multiple of 3 at least twice.

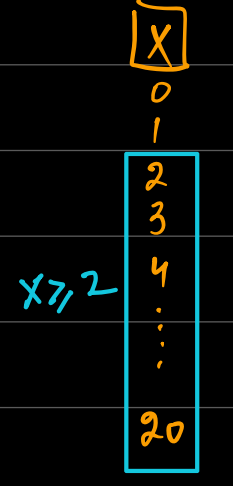
Success (X) $\quad X \geq 2$

$$P(X \geq 2) = P(2, 3, 4 \ldots\ldots 20)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - {}^{20}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{20} - {}^{20}C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{19}$$

$$= \boxed{}$$



ADVANCED

THIS QUESTION IS ALL ABOUT YOUR GRADE 7 ENGLISH TEACHER!

Q. A Fair dice is thrown n times.

(X) denotes the random variable for number of times that dice lands on a multiple of 3. Find the probability that:

Repeat = n , $p = \frac{1}{3}$, $q = \frac{2}{3}$ (Binomial)

(i) Find smallest value of n for which the probability that dice lands on a multiple of three at least once is at least 0.95.

$X \geq 1$

Probability of Success is at least once is at least 0.95

$$P(\text{at least once}) \geq 0.95$$

$$P(X \geq 1) \geq 0.95$$

$$P(1, 2, 3 \ldots\ldots n) \geq 0.95$$

$$1 - P(X = 0) \geq 0.95$$

$$1 - {}^{n}C_{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{n} \geq 0.95$$

$$1 - (1)(1)\left(\frac{2}{3}\right)^{n} \geq 0.95$$

$$-\left(\frac{2}{3}\right)^{n} \geq 0.95 - 1$$

X
0
1
2
3
⋮
n

$P(X \geq 1)$

${}^{2}C_{0} = 1$, ${}^{5}C_{0} = 1$, ${}^{n}C_{0} = 1$
${}^{2}C_{1} = 2$, ${}^{5}C_{1} = 5$, ${}^{8}C_{1} = 8$ ${}^{n}C_{1} = n$
${}^{2}C_{2} = 1$, ${}^{5}C_{5} = 1$, ${}^{8}C_{8} = 1$ ${}^{n}C_{n} = 1$

$$-\left(\frac{2}{3}\right)^n \geqslant -0.05$$

IF we divide/cancel -ve sign, inequality flips

$$\left(\frac{2}{3}\right)^n \leqslant 0.05$$

$$\ln\left(\frac{2}{3}\right)^n \leqslant \ln 0.05$$

$$n \ln\left(\frac{2}{3}\right) \leqslant \ln 0.05$$

Those who have not studied logs in P3, Please memorize these.

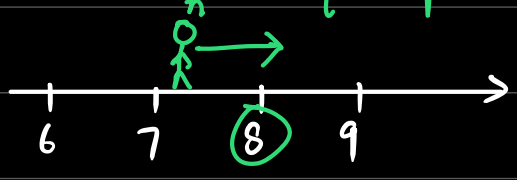
Dont make n subject before finding ln values on calculator

$$n(-0.405465) \leqslant -2.99573$$

$$n \geqslant \frac{-2.99573}{-0.405465}$$

$$n \geqslant 7.38847$$

Now, Since n is number of repeats, it must be INTEGER



Smallest value of $n = 8$

(i) Find Largest value of n for which the probability that dice lands on a multiple of three at least once is less than 0.10.

$n = ?$ $\quad p = \frac{1}{3}$ $\quad q = \frac{2}{3}$

$$P(x \geq 1) < 0.1$$

$$1 - P(x=0) < 0.1$$

$$1 - {}^{n}C_{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{n} < 0.1$$

$$1 - (1)(1)\left(\frac{2}{3}\right)^{n} < 0.1$$

$$-\left(\frac{2}{3}\right)^{n} < -0.9$$

$$\ln\left(\frac{2}{3}\right)^{n} > \ln 0.9$$

$$n\ln\left(\frac{2}{3}\right) > \ln 0.9$$

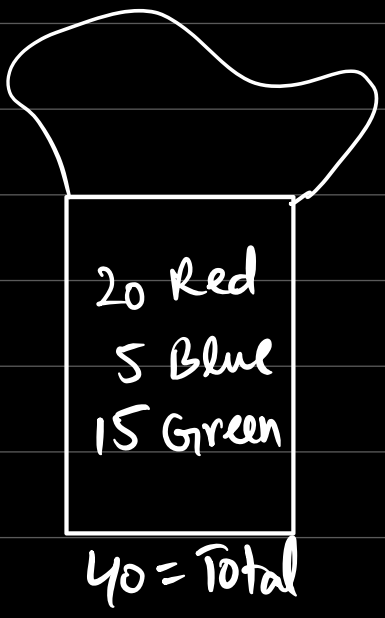
$$n(-0.40546) > -0.1053$$

$$n < \frac{0.1053}{0.40546}$$

$$n < 0.25985$$

Since n is integer.

$$n = 0$$



Three sweets are taken out of this bag without replacement. X denotes the number of Green Sweets.

Conditions:

1) Repeat ✓
2) Discrete ✓
3) Sucess (Green) ✓
4) n, P, q must be constant X

V.V.IMP Without replacement Cannot be BINOMIAL.