

Data Representation

Representation of real numbers

↳ decimal numbers e.g.: 14.525, -14.625

Mantissa: The significant digit of FPN (Floating Point Number)

Exponent: Power → Since studying binary, this is of no use.

0.11110000×2^4 ~ Exponent base

Mantissa

↳ since binary, base-2

Note: 128 64 32 16 8 4 2 1 . $\frac{1}{2^1}$ $\frac{1}{2^2}$ $\frac{1}{2^3}$ $\frac{1}{2^4}$...

OR

128 64 32 16 8 $\xleftarrow{\times 2}$ 4 $\xleftarrow{\times 2}$ 2 $\xleftarrow{\times 2}$ 1 . 0.5 $\xrightarrow{\div 2}$ 0.25 $\xrightarrow{\div 2}$ 0.125 $\xrightarrow{\div 2}$ 0.0625 $\xrightarrow{\div 2}$ 0.03125

CASE 1 : Decimal Number to Binary Conversion

- There will be certain things that would be given in the question
- No of bits used for Mantissa 12 bits
- No of bits used for exponent 4 bits

↓ below
question

Q- Represent +14.75 in floating-point Representation.

Step 1: Write down memorized list

128 64 32 16 8 4 2 1 . 0.5 0.25 0.125 0.0625 0.03125

Step 2: Put "1" on numbers which will be added to fulfill
question's requirement

* Include zeroes in the
middle

128 64 32 16 8 4 2 1 . 0.5 0.25 0.125 0.0625 0.03125

| | | | 0 . | | | → Fixed point Representation

Step 3: Move the decimal place all the way to the left and write the exponent value.

0 . 1 1 1 1 0 ; 1 1

4 3 2 1

$$\rightarrow 0.111011 \times 2^4$$

Step 4: Ignore decimal and convert the exponent into binary form

$$4 \Rightarrow \begin{array}{r} 1000 \\ 0100 \end{array}$$

0111011 0100

- To reduce memory consumption b/c "2" and ":" is repeating

Step 5: Store in given spaces

Mantissa

0 1 1 1 0 1 1 0 0 0 0 0

Exponent

0 1 0 0

- Always store mantissa from left side

↓ ↓
drop

• Always store exponent from right side (overflow, underflow)

CASE 2: Negative Decimal number to Binary Floating Point

Q- Convert -14.75 to Binary Floating Point representation

Step 1: Carry out steps 1 till 4 in CASE 1, ignoring negative sign

Step 2: Exponent would remain same and apply two's compliment method on mantissa only and store it

			Binary Addition		
Mantissa	Exponent	Sum	Carry		
0111011	0100	0	0		
		0	1		
0111011		1	0		
1000100 — One's compliment	— Two's compliment	1	1	0	1
<u>1</u> - Binary Addition		1	1	1	1
1000101					

Step 3: Store the new mantissa and the same exponent using the rules.

1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Mantissa

0	1	0	0
---	---	---	---

Exponent

- Apply one's compliment, then two's compliment

CASE 3 : Binary to Denary Conversion

Mantissa

0	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Exponent

0	1	0	0
---	---	---	---

Step 1 : Separate Mantissa and exponent

011101100000

0100

Step 2: Convert Exponent into denary format

8421

0100

Exponent = 4

Step 3: More decimal places number of times which equal the exponent

0 1 1 1 0 1 1 0 0 0 0 0
 \u203c\u203c 1 2 3 4

Step 4: Apply memorized List - zeroes at the start and end

8 4 2 1 . 0 8 0.25
1 1 1 0 . 1 1

can be removed since they represent no values

8 + 4 + 2 . 0.5 + 0.25
+ 14.75 Ans

CASE 4 : Negative Binary to Denary Conversion

- Identify +ve or -ve number by looking the left most bit

Mantissa

1 0 0 0 1 0 1 0 0 0 0 0

Exponent

0 1 0 0

Step 1 : Separate Mantissa , exponent and apply two's complement on Mantissa

$$\begin{array}{r} 1000101 \\ 0111010 \\ \hline 0111011 \end{array}$$

100

Step 2: Place decimal , apply memorized list and at the end, add -ve sign.

$$- \text{Exponent} = \frac{100}{100} = 4$$

- 0111011
1234

8+2+0.5+0.25
- 1110.11

$$8+4+2 \cdot 0.5 + 0.25 \rightarrow 14.75 \rightarrow -14.75 \quad \text{Ans}$$

Normalization

- It is a technique that is used to make your data more accurate/
precise

$0.1 \rightarrow$ Represents positive number

1.0 → Represents negative number

Q- How will we know/figure out that a binary representation is normalized?

- First and second bit should never be the same → You normalized
hai

Q- How to normalize?

Q- What problems could occur if the binary representation is not normalized?

- Multiple representation of a single number.
- Precision lost
- Redundant Leading zeroes in mantissa.

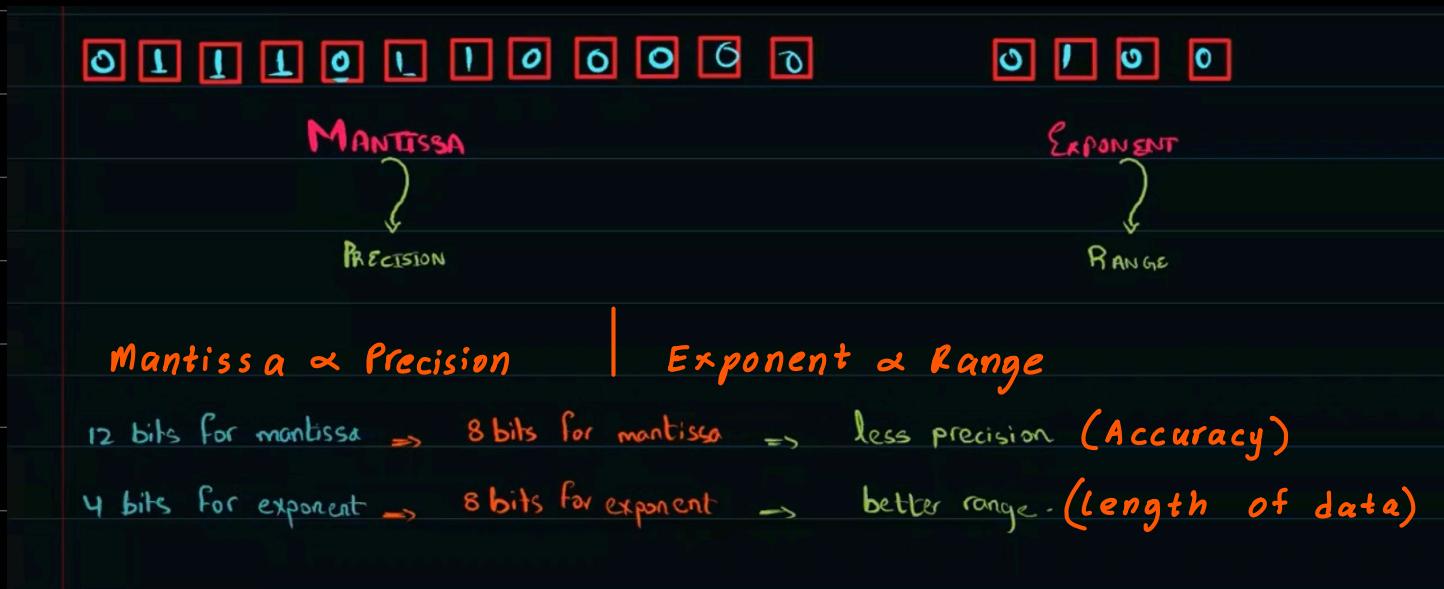
Q- What are the problems in floating Point Representation?

- $0.2 / 0.1 / 0.4 \rightarrow$ These numbers can not be exactly represented And the solution for this problem is rounding which would cause rounding error.
- 0.2 has been represented by value greater than 0.2
- 0.4 has been represented by value greater than 0.4
- So, after calculation with the rounded numbers the difference would increase
- The difference will be significant. (Compound Error)

- Q- Why rounding error occurs? → Sol. increase mantissa, normalize
- Because there is no exact representation of some binary numbers.

Trade off b/w Mantissa and Exponent

- Trade off means relationship



- Q- What is the trade off b/w Mantissa and exponent?
- The trade off is b/w precision and range
 - If more bits are used for Mantissa that means better precision
 - If more bits are used for exponent that means better range.

- More number of bits for mantissa mean less number of bits for exponent.

Largest Positive Number & Smallest Positive Number

LARGEST POSITIVE		SMALLEST POSITIVE	
0111	0111	0100	1000
Mantissa	Exponent	Mantissa	Exponent

OVERFLOW

less space more
bits to store

UNDERFLOW

More space less
bits to store

overflow: The result of carrying out a calculation which produces a value too large that can be stored.

Underflow: The result of carrying out a calculation which produces a value too small that can be represented.