# ALEVELS P3

## COMPLEX NUMBERS WITHOUT DIAGRAM (HARD)

## C2

1 The variable complex number $z$ is given by

$$z = 1 + \cos 2\theta + \mathrm{i}\sin 2\theta,$$

where $\theta$ takes all values in the interval $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

**(i)** Show that the modulus of $z$ is $2\cos\theta$ and the argument of $z$ is $\theta$. [6]

**(ii)** Prove that the real part of $\dfrac{1}{z}$ is constant. [3]

9709/32/M/J/10/Q8

2 **(a)** The complex number $u$ is defined by $u = \dfrac{5}{a + 2\mathrm{i}}$, where the constant $a$ is real.

**(i)** Express $u$ in the form $x + \mathrm{i}y$, where $x$ and $y$ are real. [2]

**(ii)** Find the value of $a$ for which $\arg(u^*) = \frac{3}{4}\pi$, where $u^*$ denotes the complex conjugate of $u$. [3]

**(b)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers $z$ which satisfy both the inequalities $|z| < 2$ and $|z| < |z - 2 - 2\mathrm{i}|$. [4]

9709/32/M/J/11/Q7

3 **(i)** Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

giving your answers in the form $x + \mathrm{i}y$, where $x$ and $y$ are real. [2]

**(ii)** State the modulus and argument of each root. [3]

**(iii)** Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64.$$ [3]

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4 **Throughout this question the use of a calculator is not permitted.**

The complex number $u$ is defined by

$$u = \frac{1 + 2\mathrm{i}}{1 - 3\mathrm{i}}.$$

**(i)** Express $u$ in the form $x + \mathrm{i}y$, where $x$ and $y$ are real. [3]

**(ii)** Show on a sketch of an Argand diagram the points $A$, $B$ and $C$ representing the complex numbers $u$, $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$ respectively. [2]

**(iii)** By considering the arguments of $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$, show that

$$\tan^{-1}2 + \tan^{-1}3 = \frac{3}{4}\pi.$$ [3]

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5 The complex number $z$ is defined by $z = a + ib$, where $a$ and $b$ are real. The complex conjugate of $z$ is denoted by $z^*$.

**(i)** Show that $|z|^2 = zz^*$ and that $(z - ki)^* = z^* + ki$, where $k$ is real. [2]

In an Argand diagram a set of points representing complex numbers $z$ is defined by the equation $|z - 10i| = 2|z - 4i|$.

**(ii)** Show, by squaring both sides, that

$$zz^* - 2iz^* + 2iz - 12 = 0.$$

Hence show that $|z - 2i| = 4$. [5]

**(iii)** Describe the set of points geometrically. [1]

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6 The complex number $z$ is defined by $z = \dfrac{9\sqrt{3} + 9i}{\sqrt{3} - i}$. Find, showing all your working,

**(i)** an expression for $z$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, [5]

**(ii)** the two square roots of $z$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

9709/31/M/J/14/Q5

7 **(a)** It is given that $-1 + (\sqrt{5})i$ is a root of the equation $z^3 + 2z + a = 0$, where $a$ is real. Showing your working, find the value of $a$, and write down the other complex root of this equation. [4]

**(b)** The complex number $w$ has modulus 1 and argument $2\theta$ radians. Show that $\dfrac{w-1}{w+1} = i\tan\theta$. [4]

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8 **Throughout this question the use of a calculator is not permitted.**

The complex numbers $w$ and $z$ satisfy the relation

$$w = \frac{z + i}{iz + 2}.$$

**(i)** Given that $z = 1 + i$, find $w$, giving your answer in the form $x + iy$, where $x$ and $y$ are real. [4]

**(ii)** Given instead that $w = z$ and the real part of $z$ is negative, find $z$, giving your answer in the form $x + iy$, where $x$ and $y$ are real. [4]

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9 The complex numbers $w$ and $z$ are defined by $w = 5 + 3i$ and $z = 4 + i$.

**(i)** Express $\dfrac{iw}{z}$ in the form $x + iy$, showing all your working and giving the exact values of $x$ and $y$. [3]

**(ii)** Find $wz$ and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]

9709/33/O/N/14/Q5

**10 Throughout this question the use of a calculator is not permitted.**

The complex number $(\sqrt{3}) + \mathrm{i}$ is denoted by $u$.

**(i)** Express $u$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of $r$ and $\theta$. Hence or otherwise state the exact values of the modulus and argument of $u^4$. [5]

**(ii)** Verify that $u$ is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

**(iii)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers $z$ satisfying the inequalities $|z - u| \leqslant 2$ and $\mathrm{Im}\, z \geqslant 2$, where $\mathrm{Im}\, z$ denotes the imaginary part of $z$. [5]

9709/31/M/J/19/Q10