ALEVELS P3 MARKING SCHEME

DIFFERENTIAL EQUATIONS WITH PROOF (SIMPLE)

DE2

1	**(i)**	Separate variables correctly and attempt to integrate both sides	M1	
		Obtain term $\ln x$, or equivalent	A1	
		Obtain term $-\frac{1}{2}kt^2$, or equivalent	A1	
		Use $t = 0$, $x = 100$ to evaluate a constant, or as limits	M1	
		Obtain solution in any correct form, e.g. $\ln x = -\frac{1}{2}kt^2 + \ln 100$	A1	**[5]**
	(ii)	Use $t = 20$, $x = 90$ to obtain an equation in k	M1*	
		Substitute $x = 50$ and attempt to obtain an unsimplified numerical expression for t^2, such as $t^2 = 400(\ln 100 - \ln 50)/(\ln 100 - \ln 90)$	M1(dep*)	
		Obtain answer $t = 51.3$	A1	**[3]**
2	(i)	State or imply that $\frac{dx}{dt} = kx - 25$	B1	
		Show that $k = 0.1$ and justify the given statement	B1	2
	(ii)	Separate variables and attempt integration	M1	
		Obtain $\ln(x - 250)$, or equivalent	A1	
		Obtain $0.1t$, or equivalent	A1	
		Evaluate a constant or use limits $t = 0$, $x = 1000$ with a solution containing terms $a\ln(x - 250)$ and bt	M1	
		Obtain any correct form of solution, e.g. $\ln(x - 250) = 0.1t + \ln 750$	A1	
		Rearrange and obtain $x = 250(3e^{0.1t} + 1)$, or equivalent	A1	6
3	(i)	State $\frac{dh}{dt} = k(9-h)^{\frac{1}{3}}$	B1	
		Show that $k = 0.1$	B1	2
	(ii)	Separate variables correctly and attempt integration of at least one side	M1	
		Obtain terms $-\frac{3}{2}(9-h)^{\frac{2}{3}}$ and $0.1t$, or equivalent	A1 + A1	
		Evaluate a constant, or use limits $t = 0$, $h = 1$ with a solution containing terms of the form $a(9-h)^p$ and bt, where $p > 0$	M1*	
		Obtain solution in any form, e.g. $-\frac{3}{2}(9-h)^{\frac{2}{3}} = 0.1t - 6$	A1	
		Rearrange and make h the subject	M1(dep*)	
		Obtain answer $h = 9 - (4 - \frac{1}{15}t)^{\frac{3}{2}}$, or equivalent	A1	7
	(iii)	State that the maximum height is $h = 9$	B1	
		State that the time taken is 60 years	B1	2
	(iv)	Substitute $h = 9/2$ and obtain $t = 19.1$ (accept 19, 19.0 and 19.2)	B1	1

4	(i)	State $\dfrac{y}{TN} = \dfrac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent	B1	
		Express area of PTN in terms of y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, and equate to $\tan x$	M1	
		Obtain given relation correctly	A1	[3]
	(ii)	Separate variables correctly	B1	
		Integrate and obtain term $-\dfrac{2}{y}$, or equivalent	B1	
		Integrate and obtain term $\ln(\sin x)$, or equivalent	B1	
		Evaluate a constant or use limits $y = 2,\ x = \frac{1}{6}\pi$ in a solution containing a term of the form a/y or $b\ln(\sin x)$	M1	
		Obtain correct solution in any form, e.g. $-\dfrac{2}{y} = \ln(2\sin x) - 1$	A1	
		Rearrange as $y = 2/(1 - \ln(2\sin x))$, or equivalent	A1	[6]
		[Allow decimals, e.g. as in a solution $y = 2/(0.3 - \ln(\sin x))$.]		
5	(i)	State or imply $\dfrac{\mathrm{d}x}{\mathrm{d}t} = k(20 - x)$	B1	
		Show that $k = 0.05$	B1	[2]
	(ii)	Separate variables correctly and integrate both sides	B1	
		Obtain term $-\ln(20 - x)$, or equivalent	B1	
		Obtain term $\frac{1}{20}t$, or equivalent	B1	
		Evaluate a constant or use limits $t = 0, x = 0$ in a solution containing terms $a\ln(20 - x)$ and bt	M1*	
		Obtain correct answer in any form, e.g. $\ln 20 - \ln(20 - x) = \frac{1}{20}t$	A1	[5]
	(iii)	Substitute $t = 10$ and calculate x	M1(dep*)	
		Obtain answer $x = 7.9$	A1	[2]
	(iv)	State that x approaches 20	B1	[1]
6	(i)	State $\dfrac{\mathrm{d}A}{\mathrm{d}t} = k\sqrt{2A - 5}$	B1	[1]
	(ii)	Separate variables correctly and attempt integration of each side	M1	
		Obtain $(2A - 5)^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Obtain $= kt$ or equivalent	A1	
		Use $t = 0$ and $A = 7$ to find value of arbitrary constant	M1	
		Obtain $C = 3$ or equivalent	A1	
		Use $t = 10$ and $A = 27$ to find k	M1	
		Obtain $k = 0.4$ or equivalent	A1	
		Substitute $t = 20$ and values for C and k to find value of A	M1	
		Obtain 63	cwo A1	[9]

7 **(i)** Show that the differential equation is $\frac{dy}{dx} = 2xy$ B1

Separate variables correctly and attempt integration of both sides M1

Obtain term $\ln y$, or equivalent A1

Obtain term x^2, or equivalent A1

Evaluate a constant, or use limits $x = 1$, $y = 2$, in a solution containing terms $a\ln y$ and bx^2 M1

Obtain correct solution in any form A1

Obtain the given answer correctly A1 [7]

(ii) State that the gradient at $(-1, 2)$ is -4 B1

Show the sketch of curve with correct concavity, positive y-intercept and axis of symmetry $x = 0$ B1 [2]

[SR: A solution with $k \neq 2$, or not evaluated, can earn B0M1A1A1M1A1A0 in part **(i)**.]

[SR: If given answer is assumed valid, give B1 if $\frac{dy}{dx}$ is shown correctly to be equal to $2xy$, is stated to be proportional to xy, and shown to be equal to 4 at (1, 2).]

8 **(i)** State or imply $\frac{dx}{dt} = k(10 - x)(20 - x)$ and show $k = 0.01$ B1 [1]

(ii) Separate variables correctly and attempt integration of at least one side M1

Carry out an attempt to find A and B such that $\frac{1}{(10-x)(20-x)} \equiv \frac{A}{10-x} + \frac{B}{20-x}$, or equivalent M1

Obtain $A = \frac{1}{10}$ and $B = -\frac{1}{10}$, or equivalent A1

Integrate and obtain $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x)$, or equivalent A1√

Integrate and obtain term $0.01t$, or equivalent A1

Evaluate a constant, or use limits $t = 0$, $x = 0$, in a solution containing terms of the form $a\ln(10-x)$, $b\ln(20-x)$ and ct M1

Obtain answer in any form, e.g. $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x) = 0.01t + \frac{1}{10}\ln 2$ A1√

Use laws of logarithms to correctly remove logarithms M1

Rearrange and obtain $x = 20(\exp(0.1t) - 1)/(2\exp(0.1t) - 1)$, or equivalent A1 [9]

(iii) State that x approaches 10 B1 [1]

9 **(i)** Substitute for x, separate variables correctly and attempt integration of both sides M1

Obtain term $\ln y$, or equivalent A1

Obtain term e^{-3t}, or equivalent A1

Evaluate a constant, or use $t = 0$, $y = 70$ as limits in a solution containing terms $a\ln y$ and be^{-3t} M1

Obtain correct solution in any form, e.g. $\ln y - \ln 70 = e^{-3t} - 1$ A1

Rearrange and obtain $y = 70\exp(e^{-3t} - 1)$, or equivalent A1 [6]

(ii) Using answer to part (**i**), either express p in terms of t or use $e^{-3t} \to 0$ to find the limiting value of y M1

Obtain answer $\frac{100}{e}$ from correct exact work A1 [2]

10	**(i)**	State $\frac{dV}{dt} = 80 - kV$	B1	
		Correctly separate variables and attempt integration of one side	M1	
		Obtain $a\ln(80 - kV) = t$ or equivalent	M1*	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent	A1	
		Use $t = 0$ and $V = 0$ to find constant of integration or as limits	M1 (dep*)	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent	A1	
		Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly	A1	[7]
	(ii)	Use iterative formula correctly at least once	M1	
		Obtain final answer 0.14	A1	
		Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]
	(iii)	State a value between 530 and 540 cm^3 inclusive	B1	
		State or imply that volume approaches 569 cm^3 (allowing any value between 567 and 571 inclusive)	B1	[2]
11	**(i)**	State or imply $\frac{dN}{dt} = kN(1 - 0.01N)$ and obtain the given answer $k = 0.02$	B1	**1**
	(ii)	Separate variables and attempt integration of at least one side	M1	
		Integrate and obtain term $0.02t$, or equivalent	A1	
		Carry out a relevant method to obtain A or B such that $\frac{1}{N(1-0.01N)} \equiv \frac{A}{N} + \frac{B}{1-0.01N}$, or equivalent	M1*	
		Obtain $A = 1$ and $B = 0.01$, or equivalent	A1	
		Integrate and obtain terms $\ln N - \ln(1 - 0.01N)$, or equivalent	A1√	
		Evaluate a constant or use limits $t = 0$, $N = 20$ in a solution with terms $a\ln N$ and $b\ln(1 - 0.01N)$, $ab \neq 0$	M1(dep*)	
		Obtain correct answer in any form, e.g. $\ln N - \ln(1 - 0.01N) = 0.02t + \ln 25$	A1	
		Rearrange and obtain $t = 50\ln(4N/(100 - N))$, or equivalent	A1	**8**
	(iii)	Substitute $N = 40$ and obtain $t = 49.0$	B1	**1**
12	**(i)**	State $\frac{dN}{dt} = k(N - 150)$	**B1**	[1]
	(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k	**M1**	
		Obtain $k = 0.08$	**A1**	
		Separate variables and obtain general solution involving $\ln(N - 150)$	**M1***	
		Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$	**A1√**	
		Substitute $t = 0$ and $N = 650$ to find c	**dep M1***	
		Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent	**A1**	
		Obtain $N = 500e^{0.08t} + 150$	**A1**	[7]
	(iii)	Either Substitute $t = 15$ to find N or solve for t with $N = 2000$	**M1**	
		Obtain Either $N = 1810$ or $t = 16.4$ and conclude target not met	**A1**	[2]

13 (i)	State equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}xy$	**B1**	[1]
(ii)	Separate variables correctly and attempts to integrate one side of equation	**M1**	
	Obtain terms of the form $a \ln y$ and bx^2	**A1**	
	Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing a ln y or bx^2	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$	**A1**	
	Obtain correct expression for y, e.g. $y = 2e^{\frac{1}{4}x^2}$	**A1**	[5]
(iii)	Show correct sketch for $x \geqslant 0$. Needs through (0, 2) and rapidly increasing positive gradient.	**B1**	[1]

14(i)	State $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2y}{(1+t)^2}$, or equivalent	**B1**
	Separate variables correctly and attempt integration of one side	**M1**
	Obtain term $\ln y$, or equivalent	**A1**
	Obtain term $\frac{2}{(1+t)}$, or equivalent	**A1**
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a \ln y$ and $\frac{b}{1+t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{2}{1+t} - 2 + \ln 100$	**A1**
	Total:	**6**
(ii)	State that the mass of B approaches $\frac{100}{e^2}$, or **exact** equivalent	**B1**
	State or imply that the mass of A tends to zero	**B1**
	Total:	**2**

15(i)	Justify the given differential equation	**B1**
	Total:	**1**
(ii)	Separate variables correctly and attempt to integrate one side	**B1**
	Obtain term kt, or equivalent	**B1**
	Obtain term $-\ln(50-x)$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 0$, $t = 0$ in a solution containing terms $a\ln(50-x)$ and bt	**M1***
	Obtain solution $-\ln(50-x) = kt - \ln 50$, or equivalent	**A1**
	Use $x = 25$, $t = 10$ to determine k	**DM1**
	Obtain correct solution in any form, e.g. $\ln 50 - \ln(50-x) = \frac{1}{10}(\ln 2)t$	**A1**
	Obtain answer $x = 50(1 - \exp(-0.0693t))$, or equivalent	**A1**
	Total:	**8**

16(i)	Fully justify the given statement	**B1**
		1
(ii)	Separate variables and attempt integration of at least one side Obtain terms $\ln y$ and $\frac{1}{2}x$	**B1 B1**
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx, where $ab \neq 0$	**M1**
	Obtain correct solution in any form	**A1**
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	**A1**
		5

1

17	State equation $\frac{dy}{dx} = k\ \frac{y^2}{x}$, , or equivalent	**B1**
	Separate variables correctly and integrate at least one side	**B1**
	Obtain terms $-\frac{1}{y}$ and $k \ln x$	**B1 + B1**
	Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b\ln x$ and C	**M1**
	Obtain $k = \frac{1}{2}$ and $c = -1$, or equivalent	**A1 + A1**
	Obtain answer $y = \frac{2}{2 - \ln x}$, or equivalent, and ISW	**A1**
		8 .

18(i)	State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$	**B1**
		1
(ii)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln N$	**B1**
	Obtain term $0.5e^{-0.02t}$	**B1**
	Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a\ln N$ and $be^{-0.02t}$, where $ab \neq 0$	**M1**
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t} - 1\right)$	**A1**
	Substitute $N = 800$ and obtain $t = 29.6$	**A1**
		6
(iii)	State that N approaches $\frac{1000}{\sqrt{e}}$	**B1**
		1