ALEVEL P3
A3 BINOMIAL
MARK SCHEME

1 Use correct method to obtain the first two terms of the expansion of $(x-1)^{-1}$ or $(x-2)^{-1}$ or $(x+1)^{-1}$ M1

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3

(deduct A1 for each incorrect expansion) A1√ + A1√ + A1√

Obtain the given answer correctly A1 **5**

[Binomial coefficients involving –1, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the M1 mark. The f.t. is on *A*, *B*, *C*.]

[Apply a similar scheme to the alternative form of fractions in (**i**), awarding M1*A1√A1√ for the expansions, M1(dep*) for multiplying by *Bx* + *C*, and A1 for obtaining the given answer correctly.]

[In the case of an attempt to expand $(x^2+7x-6)(x-1)^{-1}(x-2)^{-1}(x+1)^{-1}$, give M1A1A1A1 for the expansions and A1 for multiplying out and obtaining the given answer correctly.]

[Allow attempts to multiply out $(x-1)(x-2)(x+1)(-3+2x-\frac{3}{2}x^2+\frac{11}{4}x^3)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^3, M1 for attempting to multiply out at least as far as terms in x^2, A1 for a correct expansion up to terms in x^3, and A1 for correctly obtaining the answer x^2+7x-6 and also showing there is no term in x^3.]

[Allow the use of Maclaurin, giving M1A1√ for f(0) = –3 and f ′(0) = 2, A1√ for f ″(0) = –3, A1√ for f ‴(0) = $\frac{33}{2}$, and A1 for obtaining the given answer correctly (f.t. is on *A*, *B*,*C* if used).]

2 *EITHER*: Obtain correct unsimplified version of the *x* or x^2 term in the expansion of $(2+x)^{-3}$ or $\left(1+\frac{1}{2}x\right)^{-3}$ M1

State correct first term $\frac{1}{8}$ B1

Obtain next two terms $-\frac{3}{16}x+\frac{3}{16}x^2$ A1 + A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as $\begin{pmatrix}-3\\1\end{pmatrix}$.]

[Accept exact decimal equivalents of fractions.]

[SR: Answers given as $\frac{1}{8}\left(1-\frac{3}{2}x+\frac{3}{2}x^2\right)$ can earn M1B1A1.]

[SR: Solutions involving $k\left(1+\frac{1}{2}x\right)^{-3}$, where *k* = 2, 8 or $\frac{1}{2}$, can earn M1 and A1√ for correctly simplifying both the terms in *x* and x^2.]

OR: Differentiate expression and evaluate f(0) and f′(0), where f′(*x*) = *k*(2 + *x*)$^{-4}$ M1

State correct first term $\frac{1}{8}$ B1

Obtain next two terms $-\frac{3}{16}x+\frac{3}{16}x^2$ A1 + A1 **4**

[Accept exact decimal equivalents of fractions.]

3 **EITHER:** Obtain correct unsimplified version of the x or x^2 *or* x^3 term **M1**

State correct first two terms $1 - 2x$ **A1**

Obtain next two terms $6x^2 - 20x^3$ **A1 + A1**

[The M mark is not earned by versions with unexpanded binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{2} \\ 2 \end{pmatrix}$.]

OR: Differentiate expression and evaluate f(0) and f′(0),

where $f'(x) = k(1+4x)^{-\frac{3}{2}}$ **M1**

State correct first two terms $1 - 2x$ **A1**

Obtain next two terms $6x^2 - 20x^3$ **A1 + A1** **4**

4 Use correct method to obtain the first two terms of the expansion of $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$, or $(1+x^2)^{-1}$ M1*

Obtain complete unsimplified expansions of the fractions, e.g. $2.\frac{1}{2}(1-\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^3)$;

$(x-1)(1-x^2)$ A1√ + A1√

Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(x-1)$ M1(dep*)

Obtain answer $\frac{1}{2}x+\frac{5}{4}x^2-\frac{9}{8}x^3$ A1 **[5]**

[Binomial coefficients involving –1, such as $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, are not sufficient for the first M1.]

[f.t. is on *A*, *B*, *C*.]

[Apply this scheme to attempts to expand $(3x^2+x)(x+2)^{-1}(1+x^2)^{-1}$, giving M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

5 Use correct method to obtain the first two terms of the expansion of $(2-x)^{-1}$ or $(1-\frac{1}{2}x)^{-1}$ or $(1+x^2)^{-1}$ M1

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3, e.g. $(2x+4)(1+(-1)x^2)$ (deduct A1 for each incorrect expansion) A1√ + A1√

Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(2x+4)$ M1

Obtain answer $5+\frac{5}{2}x-\frac{15}{4}x^2-\frac{15}{8}x^3$ A1 5

[Binomial coefficients involving −1, e.g. $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, are not sufficient for the M1 mark. The f.t. is on *A*, *B*, *C*.]

[In the case of an attempt to expand $10(2-x)^{-1}(1+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

[Allow the use of Maclaurin, giving M1A1√ for f(0) = 5 and $f'(0) = \frac{5}{2}$, A1√ for $f''(0) = -\frac{15}{2}$, A1√ for $f'''(0) = -\frac{45}{4}$, and A1 for obtaining the correct final answer (f.t. is on *A*, *B*,*C* if used).]

6 (i) Simplify product and obtain $(1+x)-(1-x)$ B1

Complete the proof of the given result with no errors seen B1 **2**

(ii) Use correct method to obtain the first two terms of the expansion of $\sqrt{1+x}$ or $\sqrt{1-x}$ M1

EITHER: Obtain any correct unsimplified expansion of the numerator of the RHS of the identity up to the terms in x^3 A1

Obtain final answer with constant term $\frac{1}{2}$ A1

Obtain term $\frac{1}{16}x^2$ and no term in x A1

OR: Obtain any correct unsimplified expansion of the denominator of the LHS of the identity up to the terms in x^2 A1

Obtain final answer with constant term $\frac{1}{2}$ A1

Obtain term $\frac{1}{16}x^2$ and no term in x A1 **4**

[Symbolic binomial coefficients are not sufficient for the M1. Allow two correct separate expansions to earn the first A1 if the context is clear and appropriate.]

[Allow the use of Maclaurin, giving M1A1 for $f(0)=\frac{1}{2}$ and $f'(0)=0$, A1 for $f''(0)=\frac{1}{8}$, and A1 for obtaining the correct final answer.]

7 *EITHER*: Obtain correct unsimplified version of the x or x^2 term in the expansion of $(2+3x)^{-2}$ or $(1+\frac{3}{2}x)^{-2}$ M1

State correct first term $\frac{1}{4}$ B1

Obtain the next two terms $-\frac{3}{4}x+\frac{27}{16}x^2$ A1 + A1

[The M mark is not earned by versions with symbolic binomial coefficients such as $\binom{-2}{1}$.]

[The M mark is earned if division of 1 by the expansion of $(2+3x)^2$, with a correct unsimplified x or x^2 term, reaches a partial quotient of $a+bx$.]

[Accept exact decimal equivalents of fractions.]

[SR: Answer given as $\frac{1}{4}(1-3x+\frac{27}{4}x^2)$ can earn B1M1A1 (if $\frac{1}{4}$ seen but then omitted, give M1A1).]

[SR: Solutions involving $k(1+\frac{3}{2}x)^{-2}$, where $k=2$, 4 or $\frac{1}{2}$, can earn M1and A1✓ for correctly simplifying both the terms in x and x^2.]

OR: Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x)=k(2+3x)^{-3}$ M1

State correct first term $\frac{1}{4}$ B1

Obtain the next two terms $-\frac{3}{4}x+\frac{27}{16}x^2$ A1 + A1 **4**

8 Use correct method to obtain the first two terms of the expansion of $(1-x)^{-1}$, $(1+2x)^{-1}$, $(2+x)^{-1}$, $(1+\frac{1}{2}x)^{-1}$ M1

Obtain complete unsimplified expansions up to x^2 of each partial fraction A1√ + A1√ + A1√

Combine expansions and obtain answer $1-2x+\frac{17}{2}x^2$ A1 **[5]**

[Binomial coefficients such as $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A*, *B*, *C*.]

[Apply this scheme to attempts to expand $(2-x+8x^2)(1-x)^{-1}(1+2x)^{-1}(2+x)^{-1}$, giving M1A1A1A1 for the expansions, and A1 for the final answer.]

[Allow Maclaurin, giving M1A1√A1√ for f(0) = 1 and f ′(0) = –2, A1√ for f ″(0) = 17 and A1 for the final answer (f.t. is on *A*, *B*,*C*).]

9 *EITHER*: State correct unsimplified first two terms of the expansion of $\sqrt{(1-2x)}$, e.g. $1+\frac{1}{2}(-2x)$ B1

State correct unsimplified term in x^2, e.g. $\frac{1}{2}.(\frac{1}{2}-1).(-2x)^2/2!$ B1

Obtain sufficient terms of the product of $(1+x)$ and the expansion up to the term in x^2 of $\sqrt{(1-2x)}$ M1

Obtain final answer $1-\frac{3}{2}x^2$ A1

[The B marks are not earned by versions with symbolic binomial coefficients such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}$.]

[SR: An attempt to rewrite $(1+x)\sqrt{(1-2x)}$ as $\sqrt{(1-3x^2)}$ earns M1 A1 and the subsequent expansion $1-\frac{3}{2}x^2$ gets M1 A1.]

OR: Differentiate expression and evaluate f(0) and f ′(0), having used the product rule M1

Obtain f(0) = 1 and f ′(0) = 0 correctly A1

Obtain f ″(0) = –3 correctly A1

Obtain final answer $1-\frac{3}{2}x^2$, with no errors seen A1 [4]

10 **(i)** State correct first two terms of the expansion of $(1+ax)^{\frac{2}{3}}$, i.e. $1+\frac{2}{3}ax$ B1

Form an expression for the coefficient of x in the expansion of $(1+2x)(1+ax)^{\frac{2}{3}}$ and equate it to zero M1

Obtain $a=-3$ A1 **3**

(ii) Obtain correct unsimplified terms in x^2 and x^3 in the expansion of $(1-3x)^{\frac{2}{3}}$ or $(1+ax)^{\frac{2}{3}}$ B1√ + B1√

Carry out multiplication by $1+2x$ obtaining two terms in x^3 M1

Obtain final answer $-\frac{10}{3}x^3$, or equivalent A1 **4**

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}\frac{2}{3}\\1\end{pmatrix}$, are not acceptable for the B marks in **(i)** or **(ii)**]

11 Use correct method to obtain the first two terms of the expansion of $(x+1)^{-1}$, $(x+1)^{-2}$, $(3x+2)^{-1}$ or $(1+\frac{3}{2}x)^{-1}$ M1

Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction A1√ + A1√ + A1√

Obtain answer $\frac{3}{2}-\frac{11}{4}x+\frac{29}{8}x^2$, or equivalent A1 [5]

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the first M1. The f.t. is on A, B, C.]

[The form $\dfrac{Dx+E}{(x+1)^2}+\dfrac{C}{3x+2}$, where $D=1$, $E=3$, $C=-3$, is acceptable. In part **(i)** give B1M1A1A1A1.
In part **(ii)** give M1A1√A1√ for the expansions, and, if $DE\neq 0$, M1 for multiplying out fully and A1 for the final answer.]
[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10]
[If D or E omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10]
[In the case of an attempt to expand $(5x+3)(x+1)^{-2}(3x+2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]
[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining $f(0)=\frac{3}{2}$ and $f'(0)=-\frac{11}{4}$, A1√ for $f''(0)=\frac{29}{4}$, and A1 for the final answer (the f.t. is on A, B, C if used).]

12 **(i)** State or imply the form $\dfrac{A}{1+x}+\dfrac{Bx+C}{1+2x^2}$ B1

Use any relevant method to evaluate a constant M1

Obtain one of $A=-1$, $B=2$, $C=1$ A1

Obtain a second value A1

Obtain the third value A1 [5]

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$ or $(1+2x^2)^{-1}$ M1

Obtain correct expansion of each partial fraction as far as necessary A1√ + A1√

Multiply out fully by $Bx+C$, where BC Þ 0 M1

Obtain answer $3x-3x^2-3x^3$ A1 [5]

[Symbolic binomial coefficients, e.g., $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the first M1. The f.t. is on A, B, C.]
[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]
[If a constant D is added to the correct form, give M1A1A1A1 and B1 if and only if $D=0$ is stated.]
[If an extra term $D/(1+2x^2)$ is added, give B1M1A1A1, and A1 if $C+D=1$ is resolved to $1/(1+2x^2)$.]
[In the case of an attempt to expand $3x(1+x)^{-1}(1+2x^2)^{-1}$, give M1A1A1 for the expansions up to the term in x^2, M1 for multiplying out fully, and A1 for the final answer.]
[For the identity $3x\equiv(1+x+2x^2+2x^3)(a+bx+cx^2+dx^3)$ give M1A1; then M1A1 for using a relevant method to find two of $a=0$, $b=3$, $c=-3$ and $d=-3$; and then A1 for the final answer in series form.]

13	**(i)**	State or imply the form $\dfrac{A}{1+x}+\dfrac{Bx+C}{1+2x^2}$	B1	
		Use any relevant method to evaluate a constant	M1	
		Obtain one of $A = -1$, $B = 2$, $C = 1$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[5]
	(ii)	Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$ or $(1+2x^2)^{-1}$	M1	
		Obtain correct expansion of each partial fraction as far as necessary	A1√ + A1√	
		Multiply out fully by $Bx + C$, where BC Þ 0	M1	
		Obtain answer $3x - 3x^2 - 3x^3$	A1	[5]

[Symbolic binomial coefficients, e.g., $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the first M1. The f.t. is on A, B, C.]

[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]

[If a constant D is added to the correct form, give M1A1A1A1 and B1 if and only if $D = 0$ is stated.]

[If an extra term $D/(1 + 2x^2)$ is added, give B1M1A1A1, and A1 if $C + D = 1$ is resolved to $1/(1 + 2x^2)$.]

[In the case of an attempt to expand $3x(1 + x)^{-1}(1 + 2x^2)^{-1}$, give M1A1A1 for the expansions up to the term in x^2, M1 for multiplying out fully, and A1 for the final answer.]

[For the identity $3x \equiv (1 + x + 2x^2 + 2x^3)(a + bx + cx^2 + dx^3)$ give M1A1; then M1A1 for using a relevant method to find two of $a = 0$, $b = 3$, $c = -3$ and $d = -3$; and then A1 for the final answer in series form.]

14 Obtain $1 - 6x$ B1

State correct unsimplified x^2 term. Binomial coefficients must be expanded. M1

Obtain … $+ 24x^2$ A1 [3]

15 Either: Obtain $1+\frac{1}{3}kx$, where $k = \pm 6$ or ± 1 M1

Obtain $1 - 2x$ A1

Obtain $-4x^2$ A1

Obtain $-\frac{40}{3}x^3$ or equivalent A1

Or: Differentiate expression to obtain form $k(1-6x)^{-\frac{2}{3}}$ and evaluate f(0) and f′(0) M1

Obtain $f'(x) = -2(1 - 6x)^{-\frac{2}{3}}$ and hence the correct first two terms $1 - 2x$ A1

Obtain $f''(x) = -8(1 - 6x)^{-\frac{5}{3}}$ and hence $-4x^2$ A1

Obtain $f'''(x) = -80(1 - 6x)^{-\frac{8}{3}}$ and hence $-\frac{40}{3}x^3$ or equivalent A1 [4]

16 **(i)** State or imply partial fractions are of the form $\frac{A}{1+x}+\frac{Bx+C}{2+x^2}$ B1

Use a relevant method to determine a constant M1

Obtain one of the values $A = -2$, $B = 1$, $C = 4$ A1

Obtain a second value A1

Obtain the third value A1 [5]

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$, $\left(1+\frac{1}{2}x^2\right)^{-1}$ or $(2+x^2)^{-1}$ in ascending powers of x M1

Obtain correct unsimplified expansion up to the term in x^3 of each partial fraction A1√ + A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ M1

Obtain final answer $\frac{5}{2}x - 3x^2 + \frac{7}{4}x^3$, or equivalent A1 [5]

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the first M1. The f.t. is on A, B, C.]

[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]

[In the case of an attempt to expand $(5x - x^2)(1 + x)^{-1}(2 + x^2)^{-1}$, give M1A1A1 for the expansions, M1 for the multiplying out fully, and A1 for the final answer.]

[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining f(0) = 0 and $f'(0) = \frac{5}{2}$, A1√ for $f''(0) = -6$, and A1 for $f'''(0) = \frac{21}{2}$ and the final answer (the f.t. is on A, B, C if used).]

[For the identity $5x - x^2 \equiv (2 + 2x + x^2 + x^3)(a + bx + cx^2 + dx^3)$ give M1A1; then M1A1 for using a relevant method to obtain two of $a = 0$, $b = \frac{5}{2}$, $c = -3$ and $d = \frac{7}{4}$; then A1 for the final answer in series form.]

17	**Either** Obtain correct unsimplified version of x or x^2 term in expansion of $(2+x)^{-2}$ or $(1+\frac{1}{2}x)^{-2}$	M1	
	Correct first term 4 from correct work	B1	
	Obtain $-4x$	A1	
	Obtain $+ 3x^2$	A1	
	Or Differentiate and evaluate f(0) and f′ (0) where $f'(x) = k(2+x)^{-3}$	M1	
	State correct first term 4	B1	
	Obtain $-4x$	A1	
	Obtain $+ 3x^2$	A1	[4]

18 **(i)** <u>Either</u> Obtain correct (unsimplified) version of x or x^2 term from $(1-4x)^{\frac{1}{2}}$ M1

Obtain $1 + 2x$ A1

Obtain $+ 6x^2$ A1

<u>Or</u> Differentiate and evaluate f(0) and f′(0) where $f'(x) = k(1-4x)^{-\frac{3}{2}}$ M1

Obtain $1 + 2x$ A1

Obtain $+ 6x^2$ A1 [3]

(ii) Combine both x^2 terms from product of $1 + 2x$ and answer from part **(i)** M1

Obtain 5 A1 [2]

19 *EITHER*: State a correct unsimplified term in x or x^2 of $(1-x)^{\frac{1}{2}}$ or $(1+x)^{-\frac{1}{2}}$ B1

State correct unsimplified expansion of $(1-x)^{\frac{1}{2}}$ up to the term in x^2 B1

State correct unsimplified expansion of $(1+x)^{-\frac{1}{2}}$ up to the term in x^2 B1

Obtain sufficient terms of the product of the expansions of $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$ M1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1

*OR*1: State that the given expression equals $(1-x)(1-x^2)^{-\frac{1}{2}}$ and state that the first term of the expansion of $(1-x^2)^{-\frac{1}{2}}$ is 1 B1

State correct unsimplified term in x^2 of $(1-x^2)^{-\frac{1}{2}}$ B1

State correct unsimplified expansion of $(1-x^2)^{-\frac{1}{2}}$ up to the term in x^2 B1

Obtain sufficient terms of the product of $(1-x)$ and the expansion M1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1

*OR*2: State correct unsimplified expansion of $(1+x)^{\frac{1}{2}}$ up to the term in x^2 B1

Multiply expansion by $(1-x)$ and obtain $1-2x+2x^2$ B1

Carry out correct method to obtain one non-constant term of the expansion of $\left(1-2x+2x^2\right)^{\frac{1}{2}}$ M1

Obtain a correct unsimplified expansion with sufficient terms A1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1 [5]

[Treat $(1+x)^{-1}(1-x^2)^{\frac{1}{2}}$ by the *EITHER* scheme.]

[Symbolic coefficients, e.g. $\binom{\frac{1}{2}}{2}$, are not sufficient for the B marks.]

20 *EITHER*: Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4+3x)^{-\frac{1}{2}}$ or $(1+\frac{3}{4}x)^{-\frac{1}{2}}$ M1

State correct first term $\frac{1}{2}$ B1

Obtain the next two terms $-\frac{3}{16}x+\frac{27}{256}x^2$ A1 + A1

OR: Differentiate and evaluate f(0) and f ′(0), where $f'(x)=k(4+3x)^{-\frac{3}{2}}$ M1

State correct first term $\frac{1}{2}$ B1

Obtain the next two terms $-\frac{3}{16}x+\frac{27}{256}x^2$ A1 + A1 [4]

[Symbolic coefficients, e.g. $\binom{-\frac{1}{2}}{2}$ are not sufficient for the M or B mark.]

21 **(i)** Obtain correct unsimplified terms in x and x^3 B1 + B1

Equate coefficients and solve for a M1

Obtain final answer $a = \frac{1}{\sqrt{2}}$, or exact equivalent A1 [4]

(ii) Use correct method and value of a to find the first two terms of the expansion $(1 + ax)^{-2}$ M1

Obtain $1 - \sqrt{2}x$, or equivalent A1 $\sqrt{}$

Obtain term $\frac{3}{2}x^2$ A1 $\sqrt{}$ [3]

[Symbolic coefficients, e.g. a, are not sufficient for the first B marks]

[The f.t. is solely on the value of a.]

22 **(i)** State or imply form $\frac{A}{3-x} + \frac{Bx+C}{1+x^2}$ B1

Use relevant method to determine a constant M1

Obtain $A = 6$ A1

Obtain $B = -2$ A1

Obtain $C = 1$ A1 [5]

(ii) Either Use correct method to obtain first two terms of expansion of $(3-x)^{-1}$ or $\left(1-\frac{1}{3}x\right)^{-1}$ or $\left(1+x^2\right)^{-1}$ M1

Obtain $\frac{A}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2+\frac{1}{27}x^3\right)$ A1

Obtain $(Bx + C)(1 - x^2)$ A1

Obtain sufficient terms of the product $(Bx + C)(1 - x^2)$, $B, C \neq 0$ and add the two expansions M1

Obtain final answer $3-\frac{4}{3}x-\frac{7}{9}x^2+\frac{56}{27}x^3$ A1

Or Use correct method to obtain first two terms of expansion of $(3-x)^{-1}$ or $\left(1-\frac{1}{3}x\right)^{-1}$ or $\left(1+x^2\right)^{-1}$ M1

Obtain $\frac{1}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2+\frac{1}{27}x^3\right)$ A1

Obtain $(1 - x^2)$ A1

Obtain sufficient terms of the product of the three factors M1

Obtain final answer $3-\frac{4}{3}x-\frac{7}{9}x^2+\frac{56}{27}x^3$ A1 [5]

23 Obtain $1-x$ as first two terms of $(1+2x)^{-\frac{1}{2}}$ B1

Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1+2x)^{\frac{1}{2}}$ B1

Multiply $1+3x$ by attempt at $(1+2x)^{-\frac{1}{2}}$, obtaining sufficient terms M1

Obtain final answer $1+2x-\frac{3}{2}x^2$ A1 [4]

24	**(i)** State or imply partial fractions are of the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+3}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A=-1$, $B=3$, $C=-1$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	**[5]**
	(ii) Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\dfrac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\dfrac{1}{3}x^2\right)^{-1}$	M1	
	Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction	A1√+A1√	
	Multiply out fully by $Bx+C$, where $BC\neq 0$	M1	
	Obtain final answer $\dfrac{1}{6}+\dfrac{5}{4}x+\dfrac{17}{72}x^2$, or equivalent	A1	**[5]**
	[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A, B, C.*]		
	[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
	[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]		
25	**(i)** <u>Either</u> State or imply form $\dfrac{A}{1+x}+\dfrac{B}{(1+x)^2}+\dfrac{C}{2-3x}$	B1	
	Use any relevant method to find at least one constant	M1	
	Obtain $A=-1$	A1	
	Obtain $B=3$	A1	
	Obtain $C=4$	A1	
	<u>Or</u> State or imply form $\dfrac{A}{1+x}+\dfrac{Bx}{(1+x)^2}+\dfrac{C}{2-3x}$	B1	
	Use any relevant method to find at least one constant	M1	
	Obtain $A=2$	A1	
	Obtain $B=-3$	A1	
	Obtain $C=4$	A1	
	<u>Or</u> State or imply form $\dfrac{Dx+E}{(1+x)^2}+\dfrac{F}{2-3x}$	B1	
	Use any relevant method to find at least one constant	M1	
	Obtain $D=-1$	A1	
	Obtain $E=2$	A1	
	Obtain $F=4$	A1	[5]

(ii)	Either	Use correct method to find first two terms of expansion of $(1+x)^{-1}$ or $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$	M1	
		Obtain correct unsimplified expansion of first partial fraction up to x^2 term	A1$\sqrt{}$	
		Obtain correct unsimplified expansion of second partial fraction up to x^2 term	A1$\sqrt{}$	
		Obtain correct unsimplified expansion of third partial fraction up to x^2 term	A1$\sqrt{}$	
		Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	
	Or 1	Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$	M1	
		Obtain correct unsimplified expansion of first partial fraction up to x^2 term	A1$\sqrt{}$	
		Obtain correct unsimplified expansion of second partial fraction up to x^2 term	A1$\sqrt{}$	
		Expand and obtain sufficient terms to obtain three terms	M1	
		Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	
	Or 2	(expanding original expression)		
		Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$	M1	
		Obtain correct expansion $1-2x+3x^2$ or unsimplified equivalent	A1	
		Obtain correct expansion $\frac{1}{2}\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$ or unsimplified equivalent	A1	
		Expand and obtain sufficient terms to obtain three terms	M1	
		Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	
	Or 3	(McLaurin expansion)		
		Obtain first derivative $f'(x)=(1+x)^{-2}-6(1+x)^{-3}+12(2-3x)^{-2}$	M1	
		Obtain $f'(0)=1-6+3$ or equivalent	A1	
		Obtain $f''(0)=-2+18+9$ or equivalent	A1	
		Use correct form for McLaurin expansion	M1	
		Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	[5]

26	**(i)** Either	State or imply partial fractions are of form $\dfrac{A}{3-x}+\dfrac{B}{1+2x}+\dfrac{C}{(1+2x)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain $A = 1$	A1	
		Obtain $B = \dfrac{3}{2}$	A1	
		Obtain $C = -\dfrac{1}{2}$	A1	[5]
	Or	State or imply partial fractions are of form $\dfrac{A}{3-x}+\dfrac{Dx+E}{(1+2x)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain $A = 1$	A1	
		Obtain $D = 3$	A1	
		Obtain $E = 1$	A1	[5]

(ii) Obtain the first two terms of one of the expansion of $(3-x)^{-1}$, $\left(1-\dfrac{1}{3}x\right)^{-1}$

$(1+2x)^{-1}$ and $(1+2x)^{-2}$ — M1

Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C — A1√ A1√ A1√

Obtain answer $\dfrac{4}{3}-\dfrac{8}{9}x+\dfrac{1}{27}x^2$ — A1 [5]

[If A, D, E approach used in part (i), give M1A1√A1√ for the expansions, M1 for multiplying out fully and A1 for final answer]

27	State a correct unsimplified version of the x or x^2 or x^3 term	M1	
	State correct first two terms $1 - x$	A1	
	Obtain the next two terms $2x^2 - \frac{14}{3}x^3$	A1 + A1	**4**

[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{3} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]

28 (**i**) State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$ B1

Use a correct method to determine a constant M1

Obtain one of $A = 2$, $B = -1$, $C = 3$ A1

Obtain a second value A1

Obtain a third value A1 **[5]**

[The alternative form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2-x)^2}$, where $A = 2$, $D = 1$, $E = 1$ is marked B1M1A1A1A1 as above.]

(**ii**) Use correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2-x)^{-1}$, $(2-x)^{-2}$, $(1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$ M1

Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction A1✓ + A1✓ + A1✓

Obtain final answer $\dfrac{9}{4}+\dfrac{5}{2}x+\dfrac{39}{16}x^2$, or equivalent A1 **[5]**

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for M1. The ✓ is on A,B,C.]

[For the A,D,E form of partial fractions, give M1 A1✓ A1✓ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]

[In the case of an attempt to expand $(x^2-8x+9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

29 <u>Either</u> Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$ M1

Obtain $1+4x^2$ A1

Obtain … $+12x^4$ A1

Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$ M1

Obtain $1+4x^2-4x^4$ A1

Combine expansions to obtain $k = 16$ with no error seen A1

<u>Or</u> Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$ M1

Obtain $1+4x^2$ A1

Obtain … $-4x^4$ A1

Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$ M1

Obtain $1+4x^2+12x^4$ A1

Combine expansions to obtain $k = 16$ with no error seen A1 [6]

30	**(i)** State or imply the form $\dfrac{A}{3-2x}+\dfrac{Bx+C}{x^2+4}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A=3$, $B=-1$, $C=-2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	**[5]**
	(ii) Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√+A1√	
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	M1	
	Obtain final answer $\frac{1}{2}+\frac{5}{12}x+\frac{41}{72}x^2$, or equivalent	A1	**[5]**
	[Symbolic coefficients, e.g. $\binom{-1}{2}$ are not sufficient for the first M1. The f.t. is on A, B, C.]		
	[In the case of an attempt to expand $(5x^2+x+6)(3-2x)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		

31	Either: State correct unsimplified x^2 or x^3 term	**M1**	
	Obtain $a=-9$	**A1**	
	Obtain $b=45$	**A1**	
	Or: Use chain rule to differentiate twice to obtain form $k(1+9x)^{-\frac{5}{3}}$	**M1**	
	Obtain $f''(x)=-18(1+9x)^{-\frac{5}{3}}$ and hence $a=-9$	**A1**	
	Obtain $f'''(x)=270(1+9x)^{-\frac{8}{3}}$ and hence $b=45$	**A1**	[3]

32	State a correct un-simplified version of the x or x^2 or x^3 term	**M1**	
	State correct first two terms $1+x$	**A1**	
	Obtain the next two terms $\frac{3}{2}x^2+\frac{5}{2}x^3$	**A1 A1**	[4]
	[Symbolic binomial coefficients, e.g. $\binom{-\frac{1}{2}}{3}$ are not sufficient for the M mark.]		

33	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$	**B1**	
	State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$	**B1**	
	Obtain sufficient terms of the product of $(2-x)$ and the expansion up to the term in x^2	**M1**	
	Obtain **final** answer $2-7x+18x^2$ Do not ISW	**A1**	[4]

34	**EITHER:** State a correct unsimplified version of the x or x^2 or x^3 term in the expansion of $(1+6x)^{-\frac{1}{3}}$	**(M1**
	State correct first two terms $1-2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ $\left(37\frac{1}{3}x^3\right)$ in final answer	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x)=k(1+6x)^{-\frac{4}{3}}$	**(M1**
	Obtain correct first two terms $1-2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ in final answer	**A1)**
	Total:	**4**

35(i)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+5}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 1$, $C = –3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**
35(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}$, $(1+\frac{3}{2}x)^{-1}$, $(5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient]	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C. from part (i)	**A1FT + A1FT**
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	**M1**
	Obtain **final answer** $\frac{2}{5}-\frac{13}{10}x+\frac{237}{100}x^2$, or equivalent	**A1**
	Total:	**5**

36	*EITHER:* State a correct unsimplified version of the x or x^2 term in the expansion of $\left(1+\frac{2}{3}x\right)^{-3}$ or $\left(3+2x\right)^{-3}$ [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-3\\2\end{pmatrix}$, are not sufficient for **M1**.]	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x)=k(3+2x)^{-4}$	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	Total:	**4**

37(i)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 1$, $B = -2$, $C = 5$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
	[Mark the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -4$, $E = -1$, **B1M1A1A1A1** as above.]	
37(ii)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(1+\frac{2}{3}x)^{-1}$, $(2x+3)^{-1}$, $(1+\frac{2}{3}x)^{-2}$ or $(2x+3)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**
	Obtain final answer $\frac{8}{9}+\frac{19}{27}x+\frac{13}{9}x^2$, or equivalent	**A1**
		5

38(i)	State or imply the form $A+\frac{B}{x-1}+\frac{C}{3x+2}$	**B1**
	State or obtain $A=4$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $B=3$, $C=-1$	**A1**
	Obtain the other value	**A1**
		5
38(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft + A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $\frac{1}{2}-\frac{9}{4}x-\frac{33}{8}x^2$	**A1**
		5

39(i)	Use a correct method to find a constant	**M1**
	Obtain one of the values $A=-3$, $B=1$, $C=2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
39(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(2+x^2\right)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**
	Multiply out their expansion, up to the terms in x^3, by $Bx+C$, where $BC\neq 0$	**M1**
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**
		5

40	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4-3x)^{-\frac{1}{2}}$ or $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	**M1**
	State correct first term 2	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{64}x^2$	**A1 + A1**
	Total:	**4**

41(i)	State or imply the form $\frac{A}{1-2x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	**B1**
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is "correct")	**M1**
	Obtain one of $A = 1$, $B = 3$, $C = -2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	[Mark the form $\frac{A}{1-2x}+\frac{Dx+E}{(2-x)^2}$, where $A = 1$, $D = -3$ and $E = 4$, B1M1A1A1A1 as above.]	
		5

41(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}\right)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3ft**
	Obtain **final** answer $2+\frac{9}{4}x+4x^2$	**A1**
	[For the A, D, E form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]	
		5

42(i)	State or imply the form $\dfrac{A}{2+x}+\dfrac{B}{3-x}+\dfrac{C}{(3-x)^2}$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $A=2$, $B=2$, $C=-7$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
42(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\dfrac{1}{2}x\right)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1** **A1** **A1**
	Obtain final answer $\dfrac{8}{9}-\dfrac{43}{54}x+\dfrac{7}{108}x^2$	**A1**
		5

43	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	**B1**
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	**B1**
	Multiply by $(3-x)$ to give 2 terms in x^3, or their coefficients	**M1**
	Obtain answer 6	**A1**
		4

44(i)	State or imply the form $\frac{A}{3+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A = -3$, $B = -1$, $C = 2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
44(ii)	Use a correct method to find the first two terms of the expansion of $(3+x)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(1-x)^{-1}$ or $(1-x)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1**
		A1
		A1
	Obtain final answer $\frac{10}{3}x+\frac{44}{9}\,{}^{2}+\frac{190}{27}\,{}^{3}$	**A1**
		5