A7 Graphs
P3

1 Sketch the graph of $y = \sec x$, for $0 \leqslant x \leqslant 2\pi$. [3]

9709/03/M/J/04

2 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec} x = \tfrac{1}{2}x + 1,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

9709/3/M/J/05

3 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2\cot x = 1 + \mathrm{e}^{x},$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

9709/03/M/J/06

4 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

9709/03/O/N/07

5 **(i)** By sketching suitable graphs, show that the equation

$$4x^2 - 1 = \cot x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

9709/31/O/N/10

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

9709/33/M/J/11

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

9709/31/O/N/11

8 **(i)** By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation

$$\operatorname{cosec} x = x(\pi - x)$$

has exactly two real roots in the interval $0 < x < \pi$. [3]

9709/31/M/J/14

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

9709/33/O/N/14

10 Sketch the graph of $y = \mathrm{e}^{ax} - 1$ where a is a positive constant. [2]

9709/33/O/N/15

11 **(i)** By sketching a suitable pair of graphs, show that the equation

$$5\mathrm{e}^{-x} = \sqrt{x}$$

has one root. [2]

9709/31/M/J/16

12 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec} \tfrac{1}{2}x = \tfrac{1}{3}x + 1$$

has one root in the interval $0 < x \leqslant \pi$. [2]

9709/31/O/N/16

13 **(i)** Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2 \sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

9709/31/O/N/17

14 **(i)** By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

9709/31/O/N/18

15 **(i)** By sketching a suitable pair of graphs, show that the equation $\ln(x + 2) = 4e^{-x}$ has exactly one real root. [2]

9709/33/O/N/19