ALEVEL P3

T2 TRIG COMPOUND ANGLE

1 **(i)** Show that the equation

$$\tan(45^\circ + x) = 2\tan(45^\circ - x)$$

can be written in the form

$$\tan^2 x - 6\tan x + 1 = 0.$$ [4]

(ii) Hence solve the equation $\tan(45^\circ + x) = 2\tan(45^\circ - x)$, for $0^\circ < x < 90^\circ$. [3]

9709/03/O/N/04

2 **(i)** Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

 9709/03/O/N/07

3 **(i)** Show that the equation $\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0.$$ [4]

(ii) Hence, or otherwise, solve the equation

$$\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta),$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

 9709/03/M/J/08

4 The angles α and β lie in the interval $0^\circ < x < 180^\circ$, and are such that

$$\tan\alpha = 2\tan\beta \quad \text{and} \quad \tan(\alpha + \beta) = 3.$$

Find the possible values of α and β. [6]

 9709/32/O/N/09

5 Solve the equation

$$\cos(\theta + 60^\circ) = 2\sin\theta,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

 9709/31/O/N/10

6 **(i)** Show that the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k$$

can be written in the form

$$(2\sqrt{3})(1 + \tan^2\theta) = k(1 - 3\tan^2\theta). \qquad [4]$$

(ii) Hence solve the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

 9709/33/M/J/11

7 It is given that $\tan 3x = k \tan x$, where k is a constant and $\tan x \neq 0$.

(i) By first expanding $\tan(2x + x)$, show that

$$(3k - 1)\tan^2 x = k - 3. \qquad [4]$$

(ii) Hence solve the equation $\tan 3x = k \tan x$ when $k = 4$, giving all solutions in the interval $0^\circ < x < 180^\circ$. [3]

(iii) Show that the equation $\tan 3x = k \tan x$ has no root in the interval $0^\circ < x < 180^\circ$ when $k = 2$. [1]

 9709/33/M/J/12

8 Solve the equation

$$\sin(\theta + 45^\circ) = 2\cos(\theta - 30^\circ),$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

 9709/31/O/N/12

9 **(i)** By first expanding $\cos(\boldsymbol{x} + 45^\circ)$, express $\cos(\boldsymbol{x} + 45^\circ) - (\sqrt{2})\sin \boldsymbol{x}$ in the form $\boldsymbol{R}\cos(\boldsymbol{x} + \alpha)$, where $\boldsymbol{R} > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of $\boldsymbol{R}$ correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$\cos(\boldsymbol{x} + 45^\circ) - (\sqrt{2})\sin \boldsymbol{x} = 2,$$

for $0^\circ < \boldsymbol{x} < 360^\circ$. [4]

 9709/32/M/J/13

10 Solve the equation

$$\cos(x + 30^\circ) = 2\cos x,$$

giving all solutions in the interval $-180^\circ < x < 180^\circ$. [5]

 9709/32/M/J/14

11 **(i)** Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0. \quad [3]$$

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

 9709/33/M/J/14

12 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

 9709/33/O/N/14

13 The angles θ and ϕ lie between $0°$ and $180°$, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

 9709/31/O/N/15

14 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

 9709/33/O/N/15

15 By expressing the equation $\tan(\theta + 60°) + \tan(\theta - 60°) = \cot\theta$ in terms of $\tan\theta$ only, solve the equation for $0° < \theta < 90°$. [5]

 9709/32/O/N/17

16 **(i)** Given that $\sin(x - 60°) = 3\cos(x - 45°)$, find the exact value of $\tan x$. [4]

(ii) Hence solve the equation $\sin(x - 60°) = 3\cos(x - 45°)$, for $0° < x < 360°$. [2]

 9709/31/M/J/18

17 Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45°) = 2$, for $0° < \theta < 180°$. [5]

 9709/32/M/J/18

18 Showing all necessary working, solve the equation $\sin(\theta - 30°) + \cos\theta = 2\sin\theta$, for $0° < \theta < 180°$. [4]

 9709/32/O/N/18

19 By first expressing the equation $\cot\theta - \cot(\theta + 45°) = 3$ as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 180°$. [6]

20 **(i)** By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3\cot x$ can be written in the form $\tan^4 x - 12\tan^2 x + 3 = 0$. [4]

(ii) Hence solve the equation $\tan 3x = 3\cot x$ for $0° < x < 90°$. [3]

