C3 With Diagram Easy Answers P3

1 **(i)**	State $u - v$ is $-3 + i$	B1	
	EITHER: Carry out multiplication of numerator and denominator of u/v by $4 - 2i$, or equivalent	M1	
	Obtain answer $\frac{1}{2} + \frac{1}{2}i$, or any equivalent	A1	
	OR: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain answer $\frac{1}{2} + \frac{1}{2}i$, or any equivalent	A1	3
(ii)	State argument is $\frac{1}{4}\pi$ (or 0.785 radians or 45°)	A1√	1
(iii)	State that OC and BA are equal (in length)	B1	
	State that OC and BA are parallel or have the same direction	B1	2
(iv)	*EITHER*: Use fact that angle $AOB = \arg u - \arg v = \arg(u/v)$	M1	
	Obtain given answer (or 45°)	A1	
	OR: Obtain tan AOB from gradients of OA and OB and the $\tan(A \pm B)$ formula	M1	
	Obtain given answer (or 45°)	A1	
	OR: Obtain cos AOB by using the cosine rule or a scalar product	M1	
	Obtain given answer (or 45°)	A1	
	OR: Prove angle $OAB = 90°$ and $OA = AB$	M1	
	Derive the given answer (or 45°)	A1	2
	[SR: Obtaining a value for angle AOB by calculating $\arctan(3) - \arctan\left(\frac{1}{2}\right)$ earns a maximum of B1.]		

2 **(i)**	Use quadratic formula, or the method of completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$	**M1**	
	Obtain a root, e.g. $2 + i$	**A1**	
	Obtain the other root $-2 + i$	**A1**	3
	[Roots given as $\pm 2 + i$ earn **A1** + **A1**.]		
(ii)	Obtain modulus $\sqrt{5}$ (or 2.24) of both roots	**B1**√	
	Obtain argument of $2 + i$ as 26.6° or 0.464 radians (allow ±1 in final figure)	**B1**√	
	Obtain argument of $-2 + i$ as 153.4° or 2.68 radians (allow ±1 in final figure)	**B1**√	3
	[SR: in applying the follow through to the roots obtained in **(i)**, if both roots are real or pure imaginary, the mark for the moduli is not available and only **B1**√ is given if both arguments are correct; also if one of the two roots is real or pure imaginary and the other is neither then **B1**√ is given if both moduli are correct and **B1**√ if both arguments are correct.]		
(iii)	Show both roots on an Argand diagram in relatively correct positions	**B1**√	1
	[This follow through is only available if at least one of the two roots is of the form $x + iy$ where $xy \neq 0$.]		

3	(i)	Substitute $x = 1 + 2i$ and attempt expansions	M1	
		Use $i^2 = -1$ correctly at least once	M1	
		Complete the verification correctly	A1	[3]
	(ii)	State that the other complex root is $1 - 2i$	B1	[1]
	(iii)	Show $1 + 2i$ in relatively correct position	B1	
		Sketch a locus which		
		(a) is a straight line	B1	
		(b) relative to the point representing $1 + 2i$ (call it A), passes through the mid-point of OA	B1	
		(c) intersects OA at right angles	B1	[4]

4	(i)	Show u and u^* in relatively correct positions	B1	
		Show $u + u^*$ in relatively correct position	B1√	
		State or imply that $OACB$ is a parallelogram	B1√	
		State or imply that $OACB$ has a pair of adjacent equal sides	B1√	4
		[The statement that $OACB$ is a rhombus, or equivalent, earns B2√.]		
	(ii)	*EITHER*: Multiply numerator and denominator of $\frac{u}{u^*}$ by $2 + i$	M1	
		Simplify numerator to $3 + 4i$ or denominator to 5	A1√	
		Obtain answer $\frac{3}{5} + \frac{4}{5}i$, or equivalent	A1√	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{5}$ or $y = \frac{4}{5}$	A1√	
		Obtain answer $\frac{3}{5} + \frac{4}{5}i$	A1√	3
	(iii)	*EITHER*: State or imply $\arg\left(\frac{u}{u^*}\right) = 2\arg u$	M1	
		Justify the given statement correctly	A1	
		OR: Use tan 2A formula with $\tan A = \frac{1}{2}$	M1	
		Justify the given statement correctly	A1	2
		[The f.t. is on $-2 + i$ as complex conjugate.]		

5	(i)	*EITHER*: Carry out multiplication of numerator and denominator by $-1 - i$, or solve for x or y	M1	
		Obtain $u = -1 - i$, or any equivalent of the form $(a + ib)/c$	A1	
		State modulus of u is $\sqrt{2}$ or 1.41	A1	
		State argument of u is $-\frac{3}{4}\pi$ (−2.36) or −135°, or $\frac{5}{4}\pi$ (3.93) or 225°	A1	
		OR: Divide the modulus of the numerator by that of the denominator	M1	
		State modulus of u is $\sqrt{2}$ or 1.41	A1	
		Subtract the argument of the denominator from that of the numerator, or equivalent	M1	
		State argument of u is $-\frac{3}{4}\pi$ (−2.36) or −135°, or $\frac{5}{4}\pi$ (3.93) or 225°	A1	
		Carry out method for finding the modulus or the argument of u^2	M1	
		State modulus of u is 2 and argument of u^2 is $\frac{1}{2}\pi$ (1.57) or 90°	A1	6
	(ii)	Show u and u^2 in relatively correct positions	B1√	
		Show a circle with centre at the origin and radius 2	B1	
		Show the line which is the perpendicular bisector of the line joining u and u^2	B1√	
		Shade the correct region, having obtained u and u^2 correctly	B1	4

6	(i)	Use quadratic formula, or completing the square, or the substitution $z = x + \mathrm{i}y$ to find a root, using $\mathrm{i}^2 = -1$	M1	
		Obtain a root, e.g. $1 - \sqrt{3}\mathrm{i}$	A1	
		Obtain the other root, e.g. $-1 - \sqrt{3}\mathrm{i}$	A1	**3**
	(ii)	Represent both roots on an Argand diagram in relatively correct positions	B1√	**1**
	(iii)	State modulus of both roots is 2	B1√	
		State argument of $1 - \sqrt{3}\mathrm{i}$ is $-60°$ (or $300°$, $-\frac{1}{3}\pi$, $-\frac{5}{3}\pi$)	B1√	
		State argument of $-1 - \sqrt{3}\mathrm{i}$ is $-120°$ (or $240°$, $-\frac{2}{3}\pi$, $-\frac{4}{3}\pi$)	B1√	**3**
	(iv)	Give a complete justification of the statement	B1	**1**
		[The A marks in **(i)** are for the final versions of the roots. Allow $(\pm 2 - 2\sqrt{3}\mathrm{i})/2$ as final answer. The remaining marks are only available for roots such that $xy \neq 0$.]		
		[Treat answers to **(iii)** in polar form as a misread]		

7	**(i)**	State modulus is 2	B1	
		State argument is $\frac{1}{6}\pi$, or $30°$, or 0.524 radians	B1	[2]
	(ii) (a)	State answer $3\sqrt{3} + \mathrm{i}$	B1	
	(b)	*EITHER*: Multiply numerator and denominator by $\sqrt{3} - \mathrm{i}$, or equivalent	M1	
		Simplify denominator to 4 or numerator to $2\sqrt{3} + 2\mathrm{i}$	A1	
		Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR 1: Obtain two equations in x and y and solve for x or for y	M1	
		Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$	A1	
		Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR 2: Using the correct processes express $\mathrm{i}z^*/z$ in polar form	M1	
		Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$	A1	
		Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	[4]
	(iii)	Plot A and B in relatively correct positions	B1	
		EITHER: Use fact that angle $AOB = \arg(\mathrm{i}z^*) - \arg z$	M1	
		Obtain the given answer	A1	
		OR 1: Obtain $\tan A\hat{O}B$ from gradients of OA and OB and the correct $\tan(A - B)$ formula	M1	
		Obtain the given answer	A1	
		OR 2: Obtain $\cos A\hat{O}B$ by using correct cosine formula or scalar product	M1	
		Obtain the given answer	A1	[3]

8	(i)	Attempt multiplication and use $i^2 = -1$	M1	
		Obtain $3 + 4i$	A1	
		Obtain 5 for <u>modulus</u>	B1	[3]
	(ii)	Draw complete circle with centre corresponding to their w^2 …	B1√	
		… and radius corresponding to their $\|w^2\|$	B1√	
		Shade the correct region	cwo B1	[3]

9	(i)	Either	Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent	B1	
			Multiply numerator and denominator by $2 - i$	M1	
			Obtain correct numerator $-2 + 11i$ or correct denominator 5	A1	
			Obtain $-\frac{2}{5}+\frac{11}{5}i$ or equivalent	A1	
		Or	Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent	B1	
			Obtain two equations in x and y and solve for x or y	M1	
			Obtain final answer $x = -\frac{2}{5}$	A1	
			Obtain final answer $y = \frac{11}{5}$	A1	[4]
	(ii)		Draw a circle	M1	
			Show centre at relatively correct position, following their u	A1√	
			Draw circle passing through the origin	A1	[3]

10	**(a)**	EITHER: Use quadratic formula to solve for w Use $\mathrm{i}^2 = -1$	**M1** **M1**	
		Obtain one of the answers $w = \frac{1}{2\mathrm{i}+1}$ and $w = -\frac{5}{2\mathrm{i}+1}$	**A1**	
		Multiply numerator and denominator of an answer by –2i + 1, or equivalent	**M1**	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}\mathrm{i}$ and $-1 + 2\mathrm{i}$	**A1**	
		OR1: Multiply the equation by $1 - 2\mathrm{i}$	**M1**	
		Use $\mathrm{i}^2 = -1$	**M1**	
		Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	**A1**	
		Use quadratic formula or factorise to solve for w	**M1**	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}\mathrm{i}$ and $-1 + 2\mathrm{i}$	**A1**	
		OR2: Substitute $w = x + \mathrm{i}y$ and form equations for real and imaginary parts	**M1**	
		Use $\mathrm{i}^2 = -1$	**M1**	
		Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	**A1**	
		Form equation in x only or y only and solve	**M1**	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}\mathrm{i}$ and $-1 + 2\mathrm{i}$	**A1**	[5]
	(b)	Show a circle with centre $1 + \mathrm{i}$	**B1**	
		Show a circle with radius 2	**B1**	
		Show half-line $\arg z = \frac{1}{4}\pi$	**B1**	
		Show half-line $\arg z = -\frac{1}{4}\pi$	**B1**	
		Shade the correct region	**B1**	[5]

11(a)	Square $x + \mathrm{i}y$ and equate real and imaginary parts to 8 and –15	**M1**
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	**A1**
	Obtain answers $\pm\frac{1}{\sqrt{2}}(5 - 3\mathrm{i})$ or equivalent	**A1**
		5
11(b)	Show a circle with centre $2 + \mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

12(i)	State modulus 2	**B1**
	State argument $-\frac{1}{3}\pi$ or $-60°$ ($\frac{5}{3}\pi$ or $300°$)	**B1**
		2
12(ii)	*EITHER:* Expand $(1-(\sqrt{3})\text{i})^3$ completely and process i^2 and i^3	**(M1**
	Verify that the given relation is satisfied	**A1)**
	OR: $u^3 = 2^3\left(\cos\left(-\pi\right)+i\sin\left(-\pi\right)\right)$ or equivalent: follow their answers to **(i)**	**(M1**
	Verify that the given relation is satisfied	**A1)**
		2

12(iii)	Show a circle with centre $1-(\sqrt{3})\text{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 passing through the origin	**B1**
	Show the line Re $z = 2$	**B1**
	Shade the correct region	**B1**
		4

13(i)	Use quadratic formula, or completing the square, or the substitution $z = x + \text{i}y$ to find a root, using $\text{i}^2 = -1$	**M1**
	Obtain a root, e.g. $-\sqrt{6}-\sqrt{2}\text{i}$	**A1**
	Obtain the other root, e.g. $-\sqrt{6}-\sqrt{2}\text{i}$	**A1**
		3
13(ii)	Represent both roots in relatively correct positions	**B1ft**
		1
13(iii)	State or imply correct value of a relevant length or angle, e.g. *OA*, *OB*, *AB*, angle between *OA* or *OB* and the real axis	**B1ft**
	Carry out a complete method for finding angle *OAB*	**M1**
	Obtain *AOB* = 60° correctly	**A1**
		3
13(iv)	Give a complete justification of the given statement	**B1**
		1

Substitute in uv, expand the product and use $i^2 = -1$		M1	
Obtain answer $uv = -11 - 5\sqrt{3}i$		A1	
EITHER:	Substitute in u/v and multiply numerator and denominator by the conjugate of v, or equivalent	M1	
	Obtain numerator $-7 + 7\sqrt{3}i$ or denominator 7	A1	
	Obtain final answer $-1 + \sqrt{3}i$	A1	
OR:	Substitute in u/v, equate to $x + iy$ and solve for x or for y	M1	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
Obtain $x = -1$ or $y = \sqrt{3}$		A1	
Obtain final answer $-1 + \sqrt{3}\,i$		A1	
		5	

(ii)	Show the points A and B representing u and v in relatively correct positions	B1	
	Carry out a complete method for finding angle AOB, e.g. calculate $\arg(u/v)$ If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	M1	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan(a-b) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1 - \frac{2}{9}} = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix} -3\sqrt{3} \\ 1 \end{pmatrix}\begin{pmatrix} \sqrt{3} \\ 2 \end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	A1	Given answer so check working carefully
		3	

15(i)	Multiply numerator and denominator by $1+\sqrt{3}\mathrm{i}$, or equivalent	**M1**	
	$4\mathrm{i}-4\sqrt{3}$ and $3+1$	**A1**	
	Obtain final answer $-\sqrt{3}+\mathrm{i}$	**A1**	
		3	

15(ii)	State that the modulus of u is 2	**B1**	
	State that the argument of u is $\frac{5}{6}\pi$ (or 150°)	**B1**	
		2	
15(iii)	Show a circle with centre the origin and radius 2	**B1**	
	Show u in a relatively correct position	**B1**	**FT**
	Show the perpendicular bisector of the line joining u and the origin	**B1**	**FT**
	Shade the correct region	**B1**	
		4	