ALEVELS P3

BINOMIAL A3

1 Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = -3 + 2x - \tfrac{3}{2}x^2 + \tfrac{11}{4}x^3.$$ [5]

9709/03/M/J/04

2 Expand $\dfrac{1}{(2+x)^3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

9709/03/O/N/04

3 Expand $(1 + 4x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

9709/3/M/J/05

4 **(i)** Express $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in partial fractions. (partial fraction) [5]

(ii) Hence obtain the expansion of $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in ascending powers of x, up to and including the term in x^3. [5]

9709/03/O/N/05

5 **(i)** Express $\dfrac{10}{(2-x)(1+x^2)}$ in partial fractions. (partial fraction) [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\dfrac{10}{(2-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

9709/03/M/J/06

6 **(i)** Simplify $\left(\sqrt{(1+x)} + \sqrt{(1-x)}\right)\left(\sqrt{(1+x)} - \sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}} = \frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{2x}.$$ [2]

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}}$$

in ascending powers of x, up to and including the term in x^2. [4]

9709/03/O/N/06

7 Expand $(2 + 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

9709/03/M/J/07

8 **(i)** Express $\frac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in partial fractions.(partial fraction) [5]

(ii) Hence obtain the expansion of $\frac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in ascending powers of x, up to and including the term in x^2. [5]

9709/03/O/N/07

9 Expand $(1 + x)\sqrt{(1 - 2x)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

9709/03/O/N/08

10 When $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

(i) Find the value of a. [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

9709/03/M/J/09

11 **(i)** Express $\frac{5x + 3}{(x + 1)^2(3x + 2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{5x + 3}{(x + 1)^2(3x + 2)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

9709/31/O/N/09

12 **(i)** Express $\frac{1 + x}{(1 - x)(2 + x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1 + x}{(1 - x)(2 + x^2)}$ in ascending powers of x, up to and including the term in x^2. [5]

9709/32/O/N/09

13 Let $f(x) = \dfrac{3x}{(1+x)(1+2x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

9709/31/O/N/10

14 Expand $(1 + 2x)^{-3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

9709/33/O/N/10

15 Expand $\sqrt[3]{(1 - 6x)}$ in ascending powers of x up to and including the term in x^3, simplifying the coefficients. [4]

9709/31/M/J/11

16 **(i)** Express $\dfrac{5x - x^2}{(1+x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{5x - x^2}{(1+x)(2+x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

9709/32/M/J/11

17 Expand $\dfrac{16}{(2+x)^2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

9709/33/O/N/11

18 **(i)** Expand $\dfrac{1}{\sqrt{(1 - 4x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\dfrac{1 + 2x}{\sqrt{(4 - 16x)}}$. [2]

9709/31/M/J/12

19 Expand $\sqrt{\left(\dfrac{1 - x}{1 + x}\right)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

9709/32/M/J/12

20 Expand $\dfrac{1}{\sqrt{(4 + 3x)}}$ in ascending powers of x, up to and including the term in x^2 simplifying the coefficients. [4]

9709/33/M/J/12

21 When $(1 + \boldsymbol{a}x)^{-2}$, where $\boldsymbol{a}$ is a positive constant, is expanded in ascending powers of $\boldsymbol{x}$, the coefficients of $\boldsymbol{x}$ and $\boldsymbol{x}^3$ are equal.

(i) Find the exact value of $\boldsymbol{a}$. [4]

(ii) When $\boldsymbol{a}$ has this value, obtain the expansion up to and including the term in $\boldsymbol{x}^2$, simplifying the coefficients. [3]

9709/31/O/N/12

22 **(i)** Express $\dfrac{9-7x+8x^2}{(3-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{9-7x+8x^2}{(3-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

9709/33/O/N/12

23 Expand $\dfrac{1+3x}{\sqrt{(1+2x)}}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

9709/31/M/J/13

24 Let $\mathrm{f}(x) = \dfrac{2x^2-7x-1}{(x-2)(x^2+3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/31/O/N/13

25 **(i)** Express $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in partial fractions. [5]

(ii) Hence expand $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [5]

9709/33/O/N/13

26 **(i)** Express $\dfrac{4+12x+x^2}{(3-x)(1+2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4+12x+x^2}{(3-x)(1+2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

9709/31/M/J/14

27 Expand $(1+3\boldsymbol{x})^{-\frac{1}{3}}$ in ascending powers of $\boldsymbol{x}$, up to and including the term in $\boldsymbol{x}^3$, simplifying the coefficients. [4]

9709/33/M/J/14

28 Let $\mathrm{f}(x) = \dfrac{x^2-8x+9}{(1-x)(2-x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/31/O/N/14

29 Show that, for small values of $\boldsymbol{x}^2$,

$$\left(1-2\boldsymbol{x}^2\right)^{-2}-\left(1+6\boldsymbol{x}^2\right)^{\frac{2}{3}} \approx \boldsymbol{k}\boldsymbol{x}^4,$$

where the value of the constant $\boldsymbol{k}$ is to be determined. [6]

9709/31/M/J/15

30 Let $\mathrm{f}(x)=\dfrac{5x^2+x+6}{(3-2x)(x^2+4)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/32/M/J/15

31 Given that $\sqrt[3]{(1+9x)} \approx 1+3x+ax^2+bx^3$ for small values of x, find the values of the coefficients a and b. [3]

9709/33/O/N/15

32 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

9709/32/M/J/16

33 Expand $(2-x)(1+2x)^{-\frac{3}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

9709/31/O/N/16

34 Expand $\dfrac{1}{\sqrt[3]{(1+6x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

9709/31/M/J/17

35 Let $\mathrm{f}(x)=\dfrac{5x^2-7x+4}{(3x+2)(x^2+5)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/32/M/J/17

36 Expand $(3+2x)^{-3}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

9709/33/M/J/17

37 Let $\mathrm{f}(x)=\dfrac{8x^2+9x+8}{(1-x)(2x+3)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/32/O/N/17

38 Let $f(x) = \dfrac{12x^2 + 4x - 1}{(x-1)(3x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/31/M/J/18

39 Let $f(x) = \dfrac{x - 4x^2}{(3-x)(2+x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{3-x} + \dfrac{Bx+C}{2+x^2}$. [4]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

9709/32/M/J/18

40 Expand $\dfrac{4}{\sqrt{(4-3x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

9709/33/M/J/18

41 Let $f(x) = \dfrac{7x^2 - 15x + 8}{(1-2x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/32/O/N/18

42 Let $f(x) = \dfrac{16 - 17x}{(2+x)(3-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/31/M/J/19

43 Find the coefficient of x^3 in the expansion of $(3-x)(1+3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

9709/32/M/J/19

44 Let $f(x) = \dfrac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3. [5]

9709/33/M/J/19