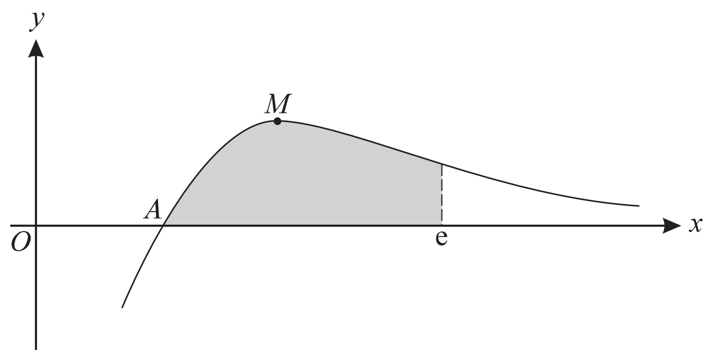
I1 WITH DIFFERENTIATION QUESTIONS

1



The diagram shows the curve $y = \dfrac{\ln x}{x^2}$ and its maximum point M. The curve cuts the x-axis at A.

(i) Write down the x-coordinate of A. [1]

(ii) Find the exact coordinates of M. [5]

(iii) Use integration by parts to find the exact area of the shaded region enclosed by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

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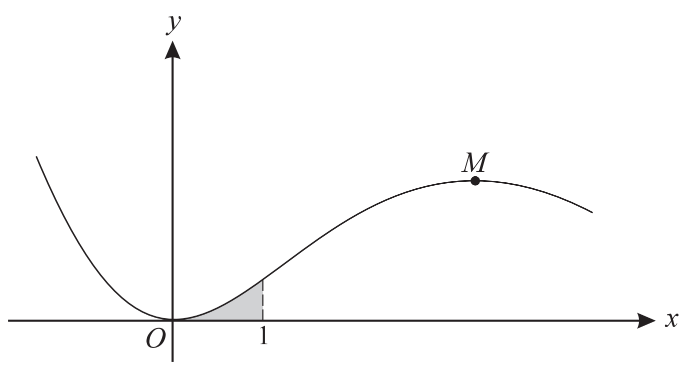
2



The diagram shows the curve $y = x^2 \mathrm{e}^{-\frac{1}{2}x}$.

(i) Find the x-coordinate of M, the maximum point of the curve. [4]

(ii) Find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 1$, giving your answer in terms of e. [5]

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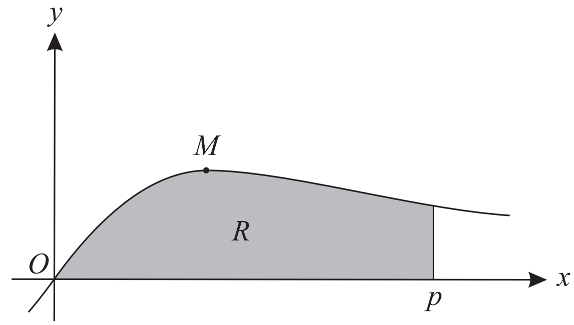

The diagram shows part of the curve $y = \dfrac{x}{x^2 + 1}$ and its maximum point M. The shaded region R is bounded by the curve and by the lines $y = 0$ and $x = p$.

(i) Calculate the x-coordinate of M. [4]

(ii) Find the area of R in terms of p. [3]

(iii) Hence calculate the value of p for which the area of R is 1, giving your answer correct to 3 significant figures. [2]

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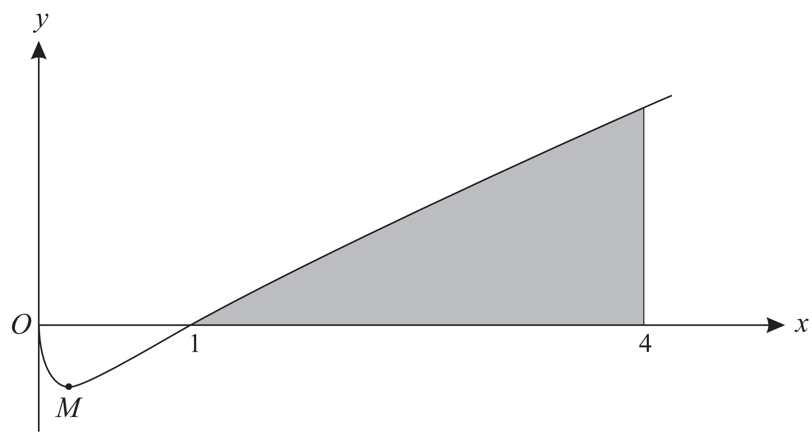
4



The diagram shows a sketch of the curve $y = x^{\frac{1}{2}} \ln x$ and its minimum point M. The curve cuts the x-axis at the point $(1, 0)$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Use integration by parts to find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 4$. Give your answer correct to 2 decimal places. [5]

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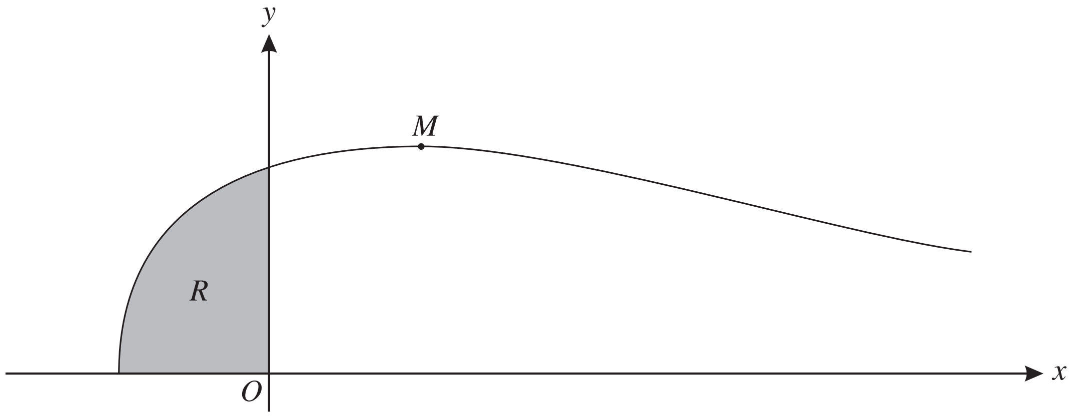
5



The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.

(i) Find the x-coordinate of M. [4]

(ii) Find by integration the volume of the solid obtained when R is rotated completely about the x-axis. Give your answer in terms of π and e. [6]

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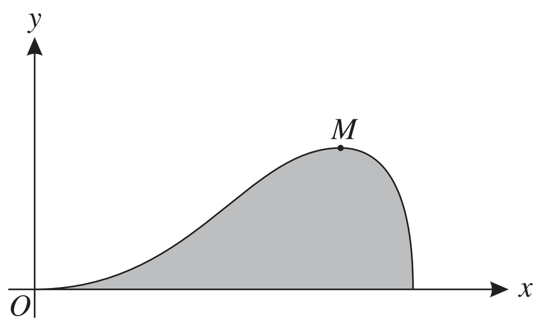

The diagram shows the curve $y = x^2\sqrt{(1 - x^2)}$ for $x \geqslant 0$ and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Show, by means of the substitution $x = \sin\theta$, that the area A of the shaded region between the curve and the x-axis is given by

$$A = \frac{1}{4}\int_0^{\frac{1}{2}\pi} \sin^2 2\theta \, d\theta. \quad [3]$$

(iii) Hence obtain the exact value of A. [4]

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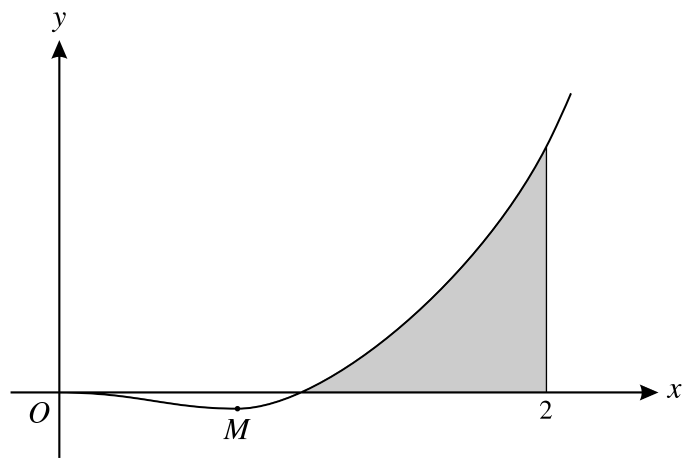
7



The diagram shows the curve $y = x^3 \ln x$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

(ii) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 2$. [5]

9709/31/O/N/10

8 It is given that $\mathrm{f}(x) = 4\cos^2 3x$.

(i) Find the exact value of $\mathrm{f}'\left(\frac{1}{9}\pi\right)$. [3]

(ii) Find $\int \mathrm{f}(x)\,\mathrm{d}x$. [3]

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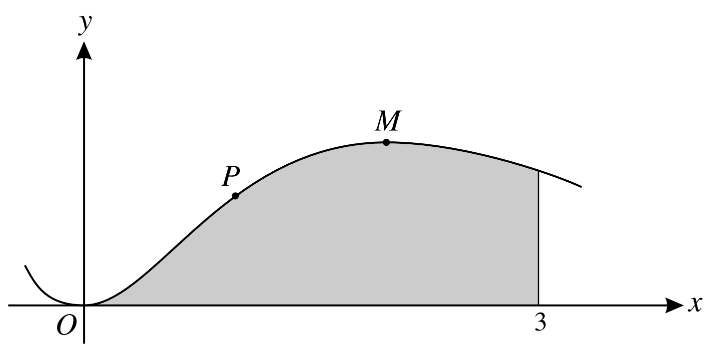
9



The diagram shows the curve $y = x^2 \mathrm{e}^{-x}$.

(i) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 3$ is equal to $2 - \dfrac{17}{\mathrm{e}^3}$. [5]

(ii) Find the x-coordinate of the maximum point M on the curve. [4]

(iii) Find the x-coordinate of the point P at which the tangent to the curve passes through the origin. [2]

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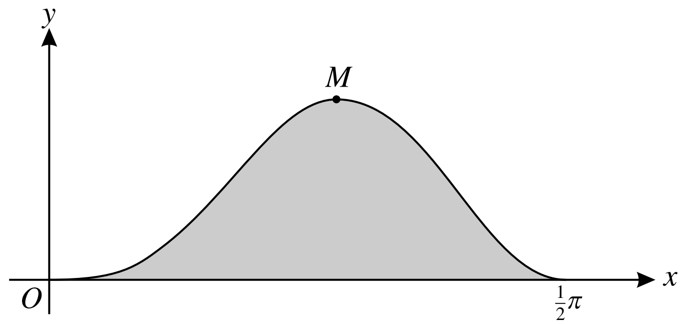
10



The diagram shows the curve $y = 5\sin^3 x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [5]

(ii) Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [5]

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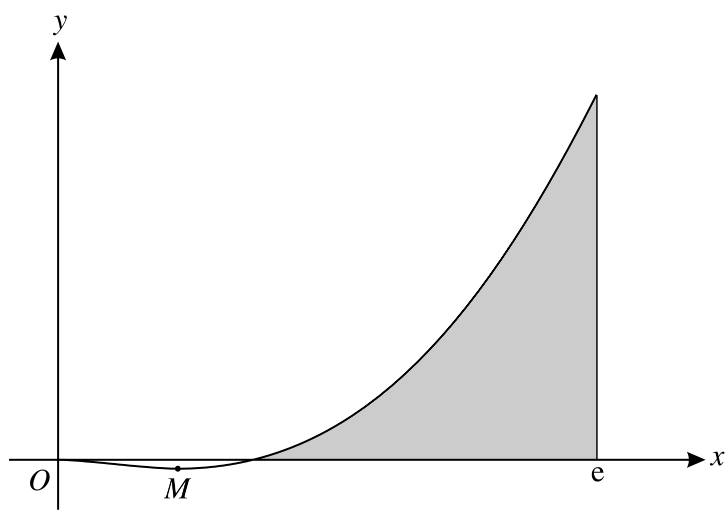
11



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = e$. [5]

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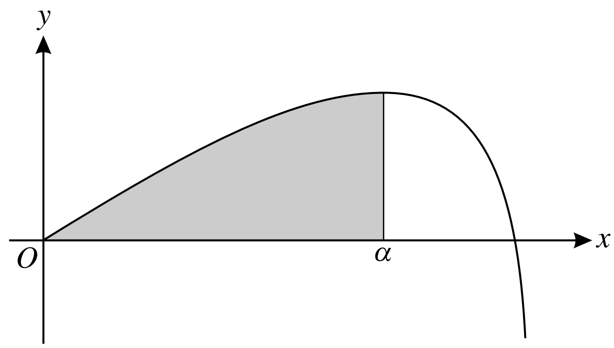
12



The diagram shows the curve

$$y = 8\sin\tfrac{1}{2}x - \tan\tfrac{1}{2}x$$

for $0 \leqslant x < \pi$. The x-coordinate of the maximum point is α and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$.

(i) Show that $\alpha = \frac{2}{3}\pi$. [3]

(ii) Find the exact value of the area of the shaded region. [4]

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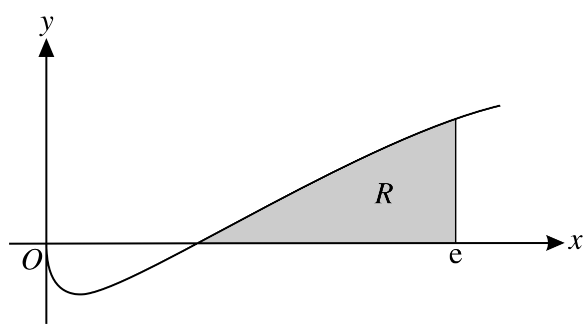
13



The diagram shows the curve $y = x^{\frac{1}{2}}\ln x$. The shaded region between the curve, the x-axis and the line $x = \mathrm{e}$ is denoted by R.

(i) Find the equation of the tangent to the curve at the point where $x = 1$, giving your answer in the form $y = mx + c$. [4]

(ii) Find by integration the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π and e. [7]

9709/32/M/J/12

14 The expression $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 3x\mathrm{e}^{-2x}$.

(i) Find the exact value of $\mathrm{f}'\left(-\frac{1}{2}\right)$. [3]

(ii) Find the exact value of $\displaystyle\int_{-\frac{1}{2}}^{0} \mathrm{f}(x)\,\mathrm{d}x$. [5]

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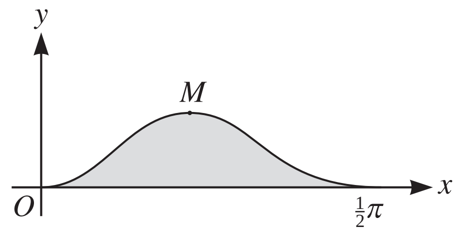

The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [6]

(ii) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [4]

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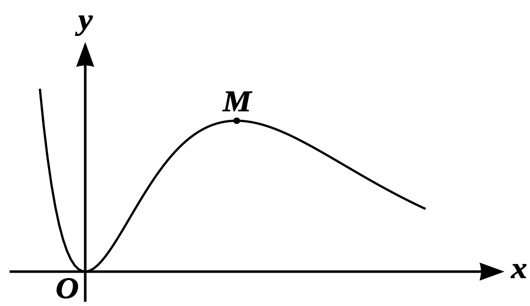

The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x}\,dx$. [6]

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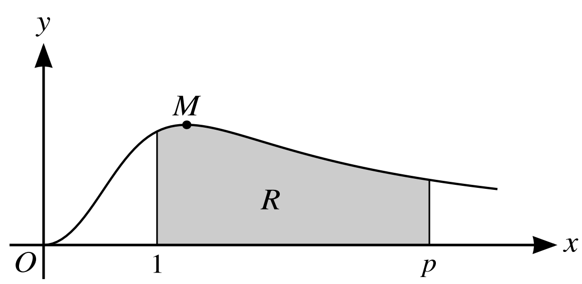
17



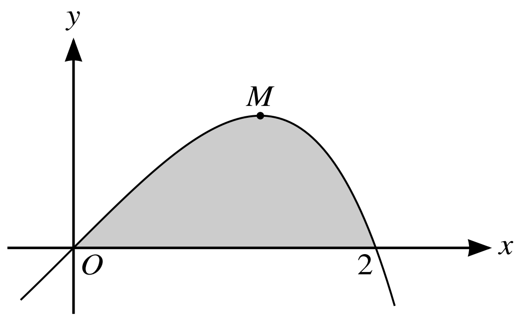
The diagram shows the curve $y = \dfrac{x^2}{1 + x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

9709/31/O/N/15

18



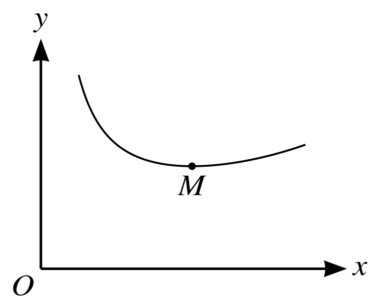
The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M.

(i) Find the exact x-coordinate of M. [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x-axis. [5]

9709/31/O/N/16

19



The diagram shows a sketch of the curve $y = \dfrac{e^{\frac{1}{2}x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) The estimate found in part **(ii)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E. [1]

9709/33/M/J/17

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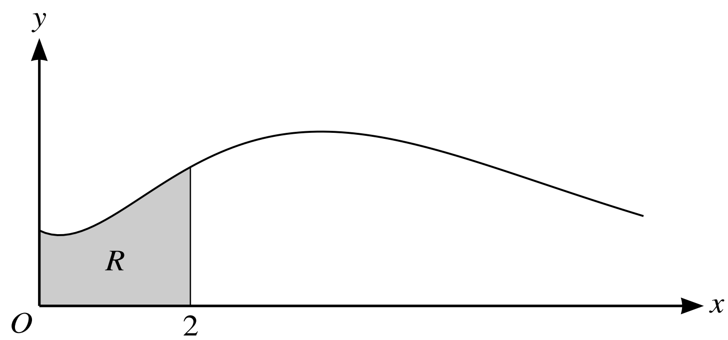

The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geqslant 0$. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x-coordinates of the stationary points of the curve. [4]

(ii) Show that the exact value of the area of R is $18 - \dfrac{42}{e}$. [5]

9709/31/O/N/17

21 The positive constant a is such that $\displaystyle\int_0^a xe^{-\frac{1}{2}x}\,dx = 2$.

(i) Show that a satisfies the equation $a = 2\ln(a + 2)$. [5]

(ii) Verify by calculation that a lies between 3 and 3.5. [2]

(iii) Use an iteration based on the equation in part **(i)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9709/31/M/J/18

22 A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) The constant a is such that $\displaystyle\int_0^a \frac{3\cos x}{2 + \sin x}\,dx = 1$. Find the value of a, giving your answer correct to 3 significant figures. [4]

9709/32/O/N/18

23 **(i)** By differentiating $\dfrac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

(ii) Show that $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x\operatorname{cosec}^2 x\,dx = \frac{1}{4}(\pi + \ln 4)$. [6]

9709/31/O/N/19