

A8 TRIGONOMETRY MARKING SCHEME

1	<i>EITHER:</i> Use tan 2A formula and obtain a horizontal equation in tan x Simplify the equation to the form $3\tan^2 x = 1$, or equivalent Obtain answer 30° <i>OR:</i> Use sin 2A and cos 2A formulae and obtain a horizontal equation in sin x or cos x Simplify the equation to $4\sin^2 x = 1$, $4\cos^2 x = 3$, or equivalent Obtain answer 30° Obtain second answer 150° and no others in the range [Ignore answers outside the given range.] [Treat answers in radians as a MR and deduct one mark from the marks for the angles.] [Methods leading to an equation in $\cos 3x$ or $\cos 2x$, or to the equality of two tangents can also earn M1 A1, and then A1 + A1 for 30° and 150° only.] [SR: If the answer 30° is found by inspection or from a graph, and is exactly verified, award B2. If a second answer 150° is found and verified, and no others stated, award B2.]	M1 A1 A1 A1 M1 A1 A1 A1 4
2	(i) State or imply $\sin a = 4/5$ Use $\sin(A - B)$ formula and substitute for $\cos a$ and $\sin a$ Obtain answer $\frac{1}{10}(4\sqrt{3} - 3)$, or exact equivalent	B1 M1 A1 [3]
	(ii) Use tan 2A formula and substitute for $\tan a$, or use sin 2A and cos 2A formulae, substitute $\sin a$ and $\cos a$, and divide Obtain $\tan 2a = -\frac{24}{7}$, or equivalent Use $\tan(A + B)$ formula with $A = 2a$, $B = a$ and substitute for $\tan 2a$ and $\tan a$ Obtain $\tan 3a = -\frac{44}{117}$	M1 A1 M1 A1 [4]
3	Use correct trig formula (or formulae) and obtain an equation in $\cos\theta$ Obtain $8\cos^2\theta + \cos\theta - 7 = 0$, or equivalent Solve a 3-term quadratic in $\cos\theta$ and reach $\theta = \cos^{-1}(a)$ Obtain answer 29.0° Obtain answer 180° and no others [Ignore answers outside the given interval. Treat answers in radians (0.505 and 3.14 or π) as a misread.] [SR: The answer 180° found by inspection can earn B1.]	M1 A1 M1 A1 A1 5
4	Use correct tan 2A formula and $\cot x = 1/\tan x$ to form an equation in $\tan x$ Obtain a correct horizontal equation in any form Solve an equation in $\tan^2 x$ for x Obtain answer, e.g. 40.2° Obtain second answer, e.g. 139.8° , and no other in the given interval [Ignore answers outside the given interval.] [Treat answers in radians as a misread and deduct A1 from the marks for the angles.] [SR: For the answer $x = 90^\circ$ give B1 and A1 for one of the other angles.]	M1 A1 M1 A1 A1 [5]

5	(i) State $\sin 2\alpha = 2\sin \alpha \cos \alpha$ and $\sec \alpha = 1/\cos \alpha$ Obtain $2\sin \alpha$	B1 B1	[2]
	(ii) Use $\cos 2\beta = 2\cos^2 \beta - 1$ or equivalent to produce correct equation in $\cos \beta$ Solve three-term quadratic equation for $\cos \beta$	B1 M1	
	Obtain $\cos \beta = \frac{1}{3}$ only	A1	[3]
6	(i) Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin \theta$ Obtain a correct expression in terms of $\sin \theta$ in any form Obtain the given identity [SR: Give M1 for using correct formulae to express RHS in terms of $\sin \theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin \theta$ and $\sin 3\theta$ only, or in terms of $\cos \theta$, $\sin \theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]	M1 M1 A1 A1	[4]
	(ii) Substitute for x and obtain the given answer	B1	[1]
	(iii) Carry out a correct method to find a value of x Obtain answers $0.322, 0.799, -1.12$ [Solutions with more than 3 answers can only earn a maximum of A1 + A1.]	M1 A1 + A1 + A1	[4]
7	Use correct $\tan 2A$ and $\cot A$ formulae to form an equation in $\tan x$ Obtain a correct equation in any form Reduce equation to the form $\tan^2 x + 6\tan x - 3 = 0$, or equivalent Solve a three term quadratic in $\tan x$ for x , as in Q1. Obtain answer, e.g. 24.9° (24.896) Obtain second answer, e.g. 98.8° (98.794) and no others in the given interval [Ignore outside the given interval. Treat answers in radians as a misread.] Radian answers $0.43452, 1.7243$	M1 A1 A1 M1 A1 A1	6
8	Correctly restate the equation in terms of $\sin \theta$ and $\cos \theta$ Using Pythagoras obtain a horizontal equation in $\cos \theta$ Reduce the equation to a correct quadratic in $\cos \theta$, e.g. $3\cos^2 \theta - \cos \theta - 2 = 0$ Solve a 3-term quadratic for $\cos \theta$ Obtain answer $\theta = 131.8^\circ$ only [Ignore answers outside the given interval.]	B1 M1 A1 M1 A1 [5]	

9	<p><i>EITHER:</i> Correctly restate the equation in terms of $\sin \theta$ and $\cos \theta$ Correct method to obtain a horizontal equation in $\sin \theta$ Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2 \theta - \sin \theta - 2 = 0$ Solve a three-term quadratic for $\sin \theta$ Obtain final answer $\theta = -41.8^\circ$ only [Ignore answers outside the given interval.]</p> <p><i>OR 1:</i> Square both sides of the equation and use $1 + \tan^2 \theta = \sec^2 \theta$ Correct method to obtain a horizontal equation in $\sin \theta$ Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2 \theta - 6\sin \theta - 8 = 0$ Solve a three-term quadratic for $\sin \theta$ Obtain final answer $\theta = -41.8^\circ$ only</p> <p><i>OR 2:</i> Multiply through by $(\sec \theta + \tan \theta)$ Use $\sec^2 \theta - \tan^2 \theta = 1$ Obtain $1 = 3 + 3\sin \theta$ Solve for $\sin \theta$ Obtain final answer $\theta = -41.8^\circ$ only</p>	B1 M1 A1 M1 A1 B1 M1 A1 M1 A1 M1 B1 A1 M1 A1	[5]
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10	Use the $\tan 2A$ formula to obtain an equation in $\tan \theta$ only Obtain a correct horizontal equation Rearrange equation as a quadratic in $\tan \theta$, e.g. $3\tan^2 \theta + 2\tan \theta - 1 = 0$ Solve for θ (usual requirements for solution of quadratic) Obtain answer, e.g. 18.4° Obtain second answer, e.g. 135° , and no others in the given interval	M1 A1 A1 M1 A1 A1	[6]
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11(i)	Use correct formulae to express the equation in terms of $\cos \theta$ and $\sin \theta$	M1	
	Use Pythagoras and express the equation in terms of $\cos \theta$ only	M1	
	Obtain correct 3-term equation, e.g. $2\cos^4 \theta + \cos^2 \theta - 2 = 0$	A1	
	Total:	3	
11(ii)	Solve a 3-term quadratic in $\cos^2 \theta$ for $\cos \theta$	M1	
	Obtain answer $\theta = 152.1^\circ$ only	A1	
	Total:	2	

12	Use correct trig formulae to obtain an equation in $\tan \theta$ or equivalent (e.g all in $\sin \theta$ or all in $\cos \theta$)	*M1	$\frac{1-\tan^2 \theta}{2\tan \theta} = 2\tan \theta . \text{Allow } \frac{\cot^2 \theta - 1}{2\cot \theta} = \frac{2}{\cot \theta}$
	Obtain a correct simplified equation	A1	$5\tan^2 \theta = 1$ or $\sin^2 \theta = \frac{1}{6}$ or $\cos^2 \theta = \frac{5}{6}$
	Solve for θ	DM1	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	A1	One correct in range to at least 3 sf
	Obtain second answer	A1	FT $180^\circ - \text{their } 24.1^\circ$ and no others in range. Correct to at least 3 sf. Accept 156° but not 156.0 Ignore values outside range If working in $\tan \theta$ or $\cos \theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	