I6 WITH SUBSTITUTION QUESTIONS

1 **(i)** Use the substitution $x = \tan\theta$ to show that

$$\int \frac{1 - x^2}{(1 + x^2)^2}\,dx = \int \cos 2\theta\,d\theta.$$ [4]

(ii) Hence find the value of

$$\int_0^1 \frac{1 - x^2}{(1 + x^2)^2}\,dx.$$ [3]

9709/3/M/J/05

2 **(i)** Use the substitution $x = \sin^2\theta$ to show that

$$\int \sqrt{\left(\frac{x}{1 - x}\right)}\,dx = \int 2\sin^2\theta\,d\theta.$$ [4]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}} \sqrt{\left(\frac{x}{1 - x}\right)}\,dx.$$ [4]

9709/03/O/N/05

3 Let $I = \displaystyle\int_1^4 \frac{1}{x(4 - \sqrt{x})}\,dx.$

(i) Use the substitution $u = \sqrt{x}$ to show that $I = \displaystyle\int_1^2 \frac{2}{u(4 - u)}\,du.$ [3]

(ii) Hence show that $I = \frac{1}{2}\ln 3$. [6]

9709/03/M/J/07

4 **(i)** Use the substitution $x = 2\tan\theta$ to show that

$$\int_0^2 \frac{8}{(4 + x^2)^2}\,dx = \int_0^{\frac{1}{4}\pi} \cos^2\theta\,d\theta.$$ [4]

(ii) Hence find the exact value of

$$\int_0^2 \frac{8}{(4 + x^2)^2}\,dx.$$ [4]

9709/32/O/N/09

5 Let $I = \displaystyle\int_0^1 \frac{x^2}{\sqrt{(4-x^2)}}\,dx$.

(i) Using the substitution $x = 2\sin\theta$, show that

$$I = \int_0^{\frac{1}{6}\pi} 4\sin^2\theta\,d\theta. \qquad [3]$$

(ii) Hence find the exact value of I. [4]

9709/31/O/N/10

6 The integral I is defined by $I = \displaystyle\int_0^2 4t^3\ln(t^2+1)\,dt$.

(i) Use the substitution $x = t^2 + 1$ to show that $I = \displaystyle\int_1^5 (2x-2)\ln x\,dx$. [3]

(ii) Hence find the exact value of I. [5]

9709/31/M/J/11

7 Let $I = \displaystyle\int_2^5 \frac{5}{x+\sqrt{(6-x)}}\,dx$.

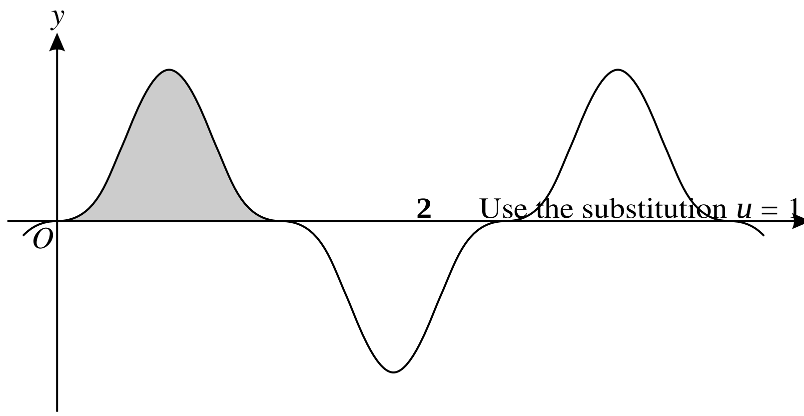
(i) Using the substitution $u = \sqrt{(6-x)}$, show that

$$I = \int_1^2 \frac{10u}{(3-u)(2+u)}\,du. \qquad [4]$$

(ii) Hence show that $I = 2\ln\left(\frac{9}{2}\right)$. [6]

9709/32/M/J/12

8



2 Use the substitution $u = 1 + 3\tan x$ to find the exact valu

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3\tan x)}}{\cos^2 x}$$

The diagram shows part of the curve $y = \sin^3 2x \cos^3 2x$. The shaded region shown is bounded by the curve and the x-axis and its exact area is denoted by A.

(i) Use the substitution $u = \sin 2x$ in a suitable integral to find the value of A. [6]

(ii) Given that $\int_0^{k\pi} |\sin^3 2x \cos^3 2x| \, dx = 40A$, find the value of the constant k. [2]

9709/33/O/N/12

9 **(a)** Show that $\int_2^4 4x \ln x \, dx = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\int_0^{\frac{1}{24}\pi} \cos^3 4x \, dx$. [5]

9709/31/M/J/13

10 **(i)** By differentiating $\dfrac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\dfrac{dy}{dx} = \sec x$. [4]

(ii) Using the substitution $x = (\sqrt{3}) \tan \theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{(3 + x^2)}} \, dx,$$

expressing your answer as a single logarithm. [4]

9709/32/M/J/13

11 Use the substitution $u = 3x + 1$ to find $\displaystyle\int \frac{3x}{3x + 1} \, dx$. [4]

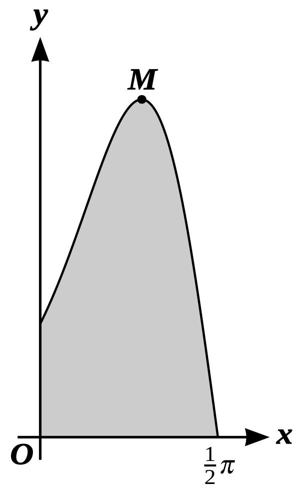
9709/33/O/N/13

12 Use the substitution $u = 1 + 3\tan x$ to find the exact value o

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3\tan x)}}{\cos^2 x}\,dx. \qquad [5]$$

9709/31/M/J/14

13



The diagram shows the curve $y = e^{2\sin x}\cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

(ii) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

9709/33/M/J/14

14 By first using the substitution $u = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\,dx = \ln\left(\frac{8}{5}\right). \qquad [10]$$

9709/33/O/N/14

15 Let $I = \displaystyle\int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}}\,dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \displaystyle\int_1^2 \frac{2(2-u)^2}{u}\,du$. [4]

(ii) Hence show that $I = 8\ln 2 - 5$. [4]

9709/32/M/J/15

16 Use the substitution $u = 4 - 3\cos x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{9\sin 2x}{\sqrt{(4 - 3\cos x)}}\,dx$. [8]

9709/33/O/N/15

17 Let $I = \displaystyle\int_0^1 \frac{x^5}{(1 + x^2)^3}\,dx$.

(i) Using the substitution $u = 1 + x^2$, show that $I = \displaystyle\int_1^2 \frac{(u-1)^2}{2u^3}\,du$. [3]

(ii) Hence find the exact value of I. [5]

9709/33/M/J/16

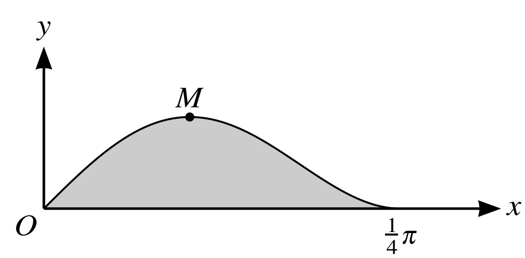
18 Let $I = \displaystyle\int_1^4 \frac{(\sqrt{x}) - 1}{2(x + \sqrt{x})}\,dx$.

(i) Using the substitution $u = \sqrt{x}$, show that $I = \displaystyle\int_1^2 \frac{u-1}{u+1}\,du$. [3]

(ii) Hence show that $I = 1 + \ln\frac{4}{9}$. [6]

9709/33/O/N/16

19



The diagram shows the curve $y = \sin x \cos^2 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$ and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Find the x-coordinate of M. Give your answer correct to 2 decimal places. [6]

9709/31/M/J/17

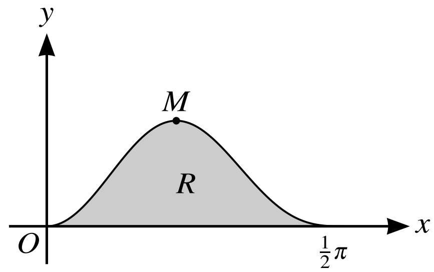
20 Let $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,dx$.

(i) Using the substitution $x = \cos^2\theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2\cos^2\theta\,d\theta$. [4]

(ii) Hence find the exact value of I. [4]

9709/31/M/J/18

21



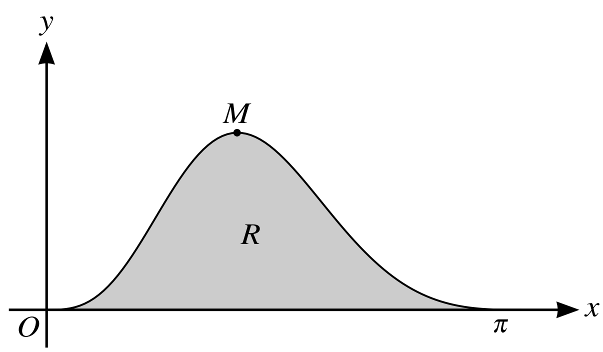
The diagram shows the curve $y = 5\sin^2 x\cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

9709/31/O/N/18

22



The diagram shows the graph of $y = e^{\cos x}\sin^3 x$ for $0 \leqslant x \leqslant \pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M. Show all necessary working and give your answer correct to 2 decimal places. [5]

(ii) By first using the substitution $u = \cos x$, find the exact value of the area of R. [7]

9709/33/O/N/19