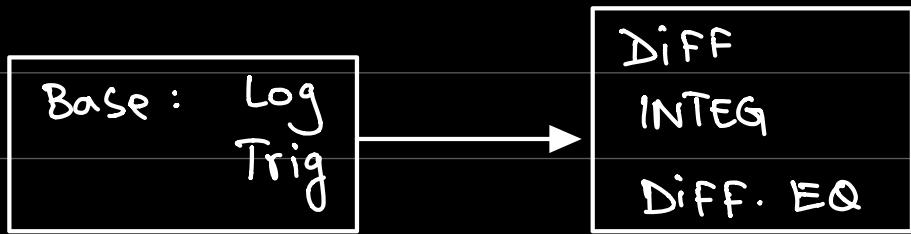


(P3) DIFFERENTIATION



OPERATORS

1 POWER $(\square)^n \rightarrow n(\square)^{n-1} \times \square'$

2 TRIG $\sin \square \rightarrow \cos \square \times \square'$
 $\cos \square \rightarrow -\sin \square \times \square'$
 $\tan \square \rightarrow \sec^2 \square \times \square'$

3 EXP $e^\square \rightarrow e^\square \times \square'$

4 Logs $\ln \square \rightarrow \frac{1}{\square} \times \square'$

5 Inverse Tan: $\tan^{-1} \square \rightarrow \frac{1}{1 + \square^2} \times \square'$

NOTATION : $y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2}$

$$y \longrightarrow y' \longrightarrow y''$$

$$f(x) \longrightarrow f'(x) \longrightarrow f''(x)$$

SINGLE OPERATORS

1) $y = (2x - 3)^6$

$$y = (2x - 3)^6$$

$$\frac{dy}{dx} = 6(2x - 3)^5 \times (2) = 12(2x - 3)^5$$

2) $y = \sin(2x)$

$$y = \underline{\sin}(2x)$$

$$\frac{dy}{dx} = \cos(2x) \times 2$$

$$\frac{dy}{dx} = 2\cos 2x$$

3) $y = 2 \cos\left(2x + \frac{\pi}{6}\right)$

$$\frac{dy}{dx} = 2 \left(-\sin\left(2x + \frac{\pi}{6}\right) \right) \times (2)$$

$$\frac{dy}{dx} = -4 \sin\left(2x + \frac{\pi}{6}\right)$$

$$3) \quad y = \tan 3x$$

$$y = \underline{\tan} [3x]$$

$$\frac{dy}{dx} = \sec^2 3x \times 3 = 3 \sec^2 3x$$

$$4) \quad y = e^{2x+1}$$

$$y = \underline{e} [2x+1]$$

$$\frac{dy}{dx} = e^{2x+1} \times 2 = 2e^{2x+1}$$

$$5) \quad y = \ln(2x+5)$$

$$y = \underline{\ln} (2x+5)$$

$$\frac{dy}{dx} = \frac{1}{2x+5} \times 2 = \frac{2}{2x+5}$$

$$6) \quad y = \tan'(3x)$$

$$\tan' \square \rightarrow \frac{1}{1+\square^2} \times \square'$$

$$y = \underline{\tan}' (3x)$$

$$\frac{dy}{dx} = \frac{1}{1 + [\boxed{3x}]^2} \times \boxed{3}'$$

$$= \frac{1}{1 + 9x^2} \times 3$$

$$\frac{dy}{dx} = \frac{3}{1 + 9x^2}$$

MULTIPLE OPERATORS

OUTSIDE OPERATOR IS DIFF FIRST.

INSIDE OPERATOR = WHICH LIES INSIDE BOX OF ANOTHER OPERATOR

$$y = \sin^3 x$$

$$y = (\underline{\sin x})^3$$

$\underbrace{\hspace{10em}}$

Power = outside operator
 sin = inside operator.

$$\frac{dy}{dx} = 3(\sin x)^2 \times (\cos x \times 1)$$

$$\frac{dy}{dx} = 3 \sin^2 x \cos x$$

$$(ii) \quad y = e^{\tan 3x}$$

e = outside
 \tan = inside

$$\frac{dy}{dx} = e^{\tan 3x} \times [\sec^2 3x \times 3]$$

$\tan \square \rightarrow \sec^2 \square \times \square'$

$$(iii) \quad y = \ln |\cos 2x| \quad \ln u \rightarrow \frac{1}{u} \times u'$$

$$\frac{dy}{dx} = \frac{1}{\cos 2x} \times [-\sin 2x \times 2]$$

$$(iv) \quad y = \cos^3 4x$$

$$y = (\underline{\cos 4x})^3$$

$$\frac{dy}{dx} = 3(\cos 4x)^2 [-\sin 4x \times 4]$$

$$\frac{dy}{dx} = -12 \cos^2 4x \sin 4x$$

RULE: Use Product & Quotient rule only when two VARIABLE terms MULTIPLY / DIVIDE.

PRODUCT RULE	QUOTIENT RULE
$(uv)' = uv' + vu'$	$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

$$y = \frac{x \sin 3x}{u v}$$

$$\frac{dy}{dx} = x [\cos 3x \times 3] + \sin 3x [1]$$

$$y = \frac{2x+1}{5x+4} \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{(5x+4)[2] - (2x+1)[5]}{(5x+4)^2}$$

$$\frac{dy}{dx} = 3x \cos 3x + \sin 3x \quad \left| \frac{dy}{dx} = \frac{10x+8-10x-5}{(5x+4)^2} = \frac{3}{(5x+4)^2} \right.$$

$$y = \frac{1}{\sin^2 x} = (\sin x)^{-2}$$

It's not good idea
to use quotient rule.

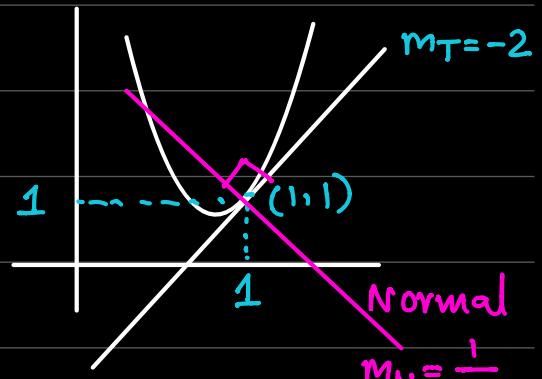
$$\frac{dy}{dx} = -2(\sin x)^{-3} \times [\cos x \times 1]$$

$$\frac{dy}{dx} = -\frac{2 \cos x}{\sin^3 x}.$$

BASIC OUTCOMES

$$y = 2x^2 - 6x + 5$$

(i) Find $\frac{dy}{dx} = 4x - 6$



(ii) Find gradient of tangent at $x=1$

$$m_T = 4(1) - 6 = -2$$

(iii) Find equation of tangent at $x=1$

$$y = 2x^2 - 6x + 5$$

$$y = 2(1)^2 - 6(1) + 5 = 1$$

$$m_T = -2, (1, 1)$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -2(x - 1)$$
$$y = -2x + 2 + 1$$

TANGENT.

$$y = -2x + 3$$

iv) Find equation of Normal at $x = 1$

$$m_T = -2 \longrightarrow m_N = \frac{1}{2} \quad \text{use same point } (1, 1)$$

$$y - 1 = \frac{1}{2}(x - 1)$$

$$2y - 2 = x - 1$$

$$\boxed{\text{NORMAL}} \quad 2y = x + 1$$

(v) Find x coordinate of stationary point.

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 4x - 6$$

$$0 = 4x - 6$$

$$x = 1.5$$

(vi) Find nature of stationary point.

Method 1:

$$y = 2x^2 - 6x + 5$$

$$\frac{dy}{dx} = 4x - 6$$

$$\frac{d^2y}{dx^2} = 4 \quad (\text{tve}) \\ (\text{Min}).$$

Just for practice lets see method 2.

$$\frac{dy}{dx} = 4x - 6$$

STATIONARY

$$x = 1.5$$

0.1 smaller

$$x = 1.4$$

0.1 bigger.

$$x = 1.6$$

$$\begin{aligned}\frac{dy}{dx} &= 4(1.4) - 6 \\ &= -0.4\end{aligned}$$

$$\frac{dy}{dx} = 4(1.6) - 6$$

$$0.4$$

⊖



⊕ (Min).

STATIONARY POINTS:

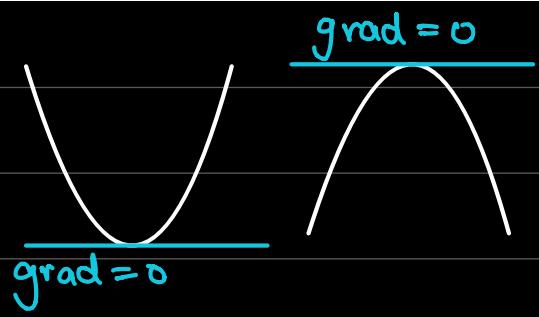
Turning points

Max. Points

Min. Points

Critical Points

Vertex.



$$\frac{dy}{dx} = 0$$

NATURE OF A STATIONARY POINT

Method 1 is used for P1.

Both Method 1 and Method 2 is used for P3.

Method 1

$$y \xrightarrow{\text{DIFF}} \frac{dy}{dx} \xrightarrow{\text{DIFF}} \frac{d^2y}{dx^2}$$

+ve (Min)
-ve (Max)

Method 2: when double derivative is not feasible. When you have to use product or quotient rule for $\frac{d^2y}{dx^2}$
(P3 only)

NOW YOU ONLY HAVE $\frac{dy}{dx}$ AND

x Coordinate of stationary point.

STATIONARY $x = \boxed{\quad}$

0.1 smaller
 ↓
 $x = \boxed{}$

0.1 bigger
 ↓
 $x = \boxed{}$

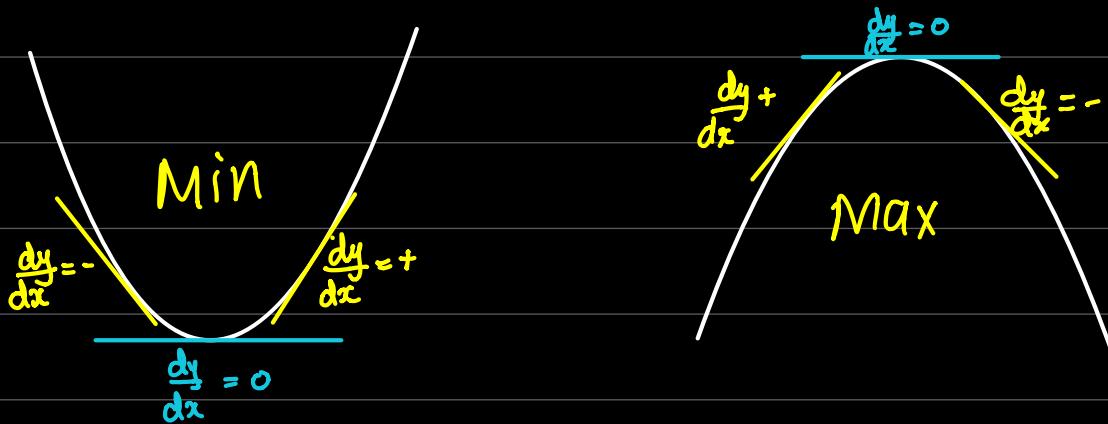
Find $\frac{dy}{dx}$ on both values.

$$\frac{dy}{dx} = \boxed{}$$

$$\frac{dy}{dx} = \boxed{}$$

If $- \rightarrow +$ Min

$+ \rightarrow -$ Max.



DIFFERENTIATION
 INTEGRATION
 ITERATION
 DIFF. EQUATIONS

RADIANS
 MODE

- 1 The equation of a curve is $y = x + \cos 2x$. Find the x -coordinates of the stationary points of the curve for which $0 \leq x \leq \pi$, and determine the nature of each of these stationary points. [7]

$$y = x + \cos 2x$$

$$0 \leq x \leq \pi$$

$$\frac{dy}{dx} = 1 + (-\sin 2x)(2)$$

$$\frac{dy}{dx} = 1 - 2 \sin 2x$$

$$0 = 1 - 2 \sin 2x$$

$$\sin 2x = \frac{1}{2}$$

$$0 \leq x \leq \pi$$

$$2x = A$$

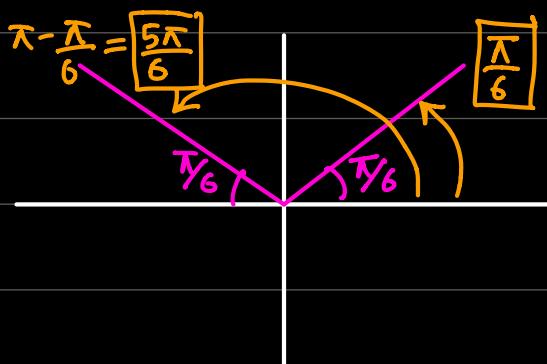
$$\times 2$$

$$0 \leq 2x \leq 2\pi$$

$$\sin A = \frac{1}{2}$$

$$0 \leq A \leq 2\pi$$

$$A = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$



$$2x = A = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

$$\frac{dy}{dx} = 1 - 2 \sin 2x$$

$$\frac{d^2y}{dx^2} = 0 - 2(\cos 2x)(2)$$

$$\frac{d^2y}{dx^2} = -4 \cos 2x$$

$$x = \frac{\pi}{12}, \frac{d^2y}{dx^2} = -4 \cos \left[2 \times \frac{\pi}{12} \right] = -2\sqrt{3} \quad (\text{max})$$

$$x = \frac{5\pi}{12}, \frac{d^2y}{dx^2} = -4 \cos \left[2 \times \frac{5\pi}{12} \right] = 2\sqrt{3} \quad (\text{min})$$

2 The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

(i) Find the x -coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

$$(i) y = 6e^x - e^{3x}$$

$$\frac{dy}{dx} = 6e^x(1) - e^{3x}(3)$$

$$\frac{dy}{dx} = 6e^x - 3e^{3x}$$

$$0 = 6e^x - 3e^{3x}$$

$$3e^{3x} = 6e^x$$

$$\frac{e^{3x}}{e^x} = \frac{6}{3}$$

$$\ln e^{2x} = \ln 2$$

$$2x \ln e = \ln 2$$

$$2x(1) = \ln 2$$

$$x = \frac{\ln 2}{2} = 0.347$$

$$(ii) \frac{dy}{dx} = 6e^x - 3e^{3x}$$

$$\frac{d^2y}{dx^2} = 6e^x(1) - 3e^{3x}(3)$$

$$\frac{d^2y}{dx^2} = 6e^x - 9e^{3x}$$

$$x = 0.347$$

$$\frac{d^2y}{dx^2} = 6e^{0.347} - 9e^{3(0.347)}$$

$$\frac{d^2y}{dx^2} = -16.995 \quad \text{Max.}$$

3 The curve with equation $y = e^{-x} \sin x$ has one stationary point for which $0 \leq x \leq \pi$.

(i) Find the x -coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

$$y = \frac{e^{-x}}{u} \sin x$$

$$\frac{dy}{dx} = e^{-x} [\cos x \times 1] + \sin x [e^{-x} \times -1]$$

$$\frac{dy}{dx} = e^{-x} (\cos x - \sin x)$$

$$0 = e^{-x} (\cos x - \sin x)$$

$$e^{-x} = 0$$

No solution

$$\cos x - \sin x = 0$$

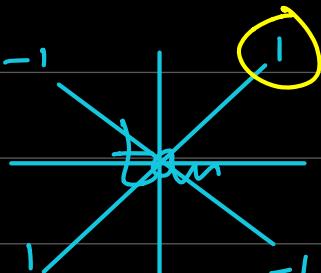
$$\sin x = \cos x$$

$$\frac{\sin x}{\cos x} = 1$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}$$

$$x = 0.785$$



(ii) $\frac{dy}{dx} = e^{-x} (\cos x - \sin x)$

Here we use method 2.

$$x = 0.785$$

$$x = 0 \cdot 7$$

$$x = 0 \cdot 8$$

$$\frac{dy}{dx} = e^{-0.7} (\cos 0.7 - \sin 0.7)$$

$$\frac{dy}{dx} = e^{-0.8} (\cos 0.8 - \sin 0.8)$$

$$\frac{dy}{dx} = 0.0599$$

$$\frac{dy}{dx} = -9.27 \times 10^{-3}$$

(+)



(-)

Max.

TYPE 2

PARAMETRIC EQUATIONS

You will have separate equations for x & y .
Third variable is called parameter.

Find $\frac{dy}{dx}$

$$x = 3 - 2 \underline{\cos 2\theta}$$

$$y = 5 \underline{\sin 2\theta} - 3$$

$$\frac{dx}{d\theta} = 0 - 2 [-\underline{\sin 2\theta} \times 2]$$

$$\frac{dy}{d\theta} = 5 \cos 2\theta \times 2 - 0$$

$$\frac{dx}{d\theta} = 4 \sin 2\theta$$

$$\frac{dy}{d\theta} = 10 \cos 2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\frac{dy}{dx} = \frac{10 \cos 2\theta}{4 \sin 2\theta} = \frac{5}{2} \cot 2\theta$$

9709/33/O/N/11

8 The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2 \sin \theta.$$

Show that $\frac{dy}{dx} = \frac{2 \cos \theta}{1 + 2 \sin \theta}$.

[5]

9709/33/M/J/12

$$x = \sin 2\theta - \theta$$

$$y = \cos 2\theta + 2 \sin \theta$$

$$\frac{dx}{d\theta} = (\cos 2\theta)(2) - 1$$

$$\frac{dy}{d\theta} = (-\sin 2\theta)(2) + 2(\cos \theta)(1)$$

Differentiation

$$\frac{dx}{d\theta} = 2 \cos 2\theta - 1$$

$$\frac{dy}{d\theta} = 2 \cos \theta - 2 \sin 2\theta$$

$$\frac{dy}{dx} = \frac{2 \cos \theta - 2 \sin 2\theta}{2 \cos 2\theta - 1}$$

$$= \frac{2 \cos \theta - 2(2 \sin \theta \cos \theta)}{2(1 - 2 \sin^2 \theta) - 1}$$

$$= \frac{2 \cos \theta - 4 \sin \theta \cos \theta}{2 - 4 \sin^2 \theta - 1}$$

$$= \frac{2 \cos \theta (1 - 2 \sin \theta)}{1 - 4 \sin^2 \theta}$$

$$\rightarrow (1)^2 - (2 \sin \theta)^2$$

$$= \frac{2 \cos \theta (1 - 2 \sin \theta)}{(1 + 2 \sin \theta)(1 - 2 \sin \theta)}$$

$$(1 + 2 \sin \theta)(1 - 2 \sin \theta)$$

$$= \frac{2 \cos \theta}{1 + 2 \sin \theta}$$

9 The parametric equations of a curve are

$$x = \frac{4t}{2t+3}, \quad y = 2 \ln(2t+3).$$

(i) Express $\frac{dy}{dx}$ in terms of t , simplifying your answer. [4]

(ii) Find the gradient of the curve at the point for which $x = 1$. [2]

9709/33/O/N/12

$$x = \frac{4t}{2t+3}$$

$$\frac{dx}{dt} = \frac{(2t+3)(4) - (4t)(2)}{(2t+3)^2}$$

$$\frac{dx}{dt} = \frac{8t+12 - 8t}{(2t+3)^2}$$

$$\frac{dx}{dt} = \frac{12}{(2t+3)^2}$$

$$y = 2 \ln(2t+3)$$

$$\frac{dy}{dt} = 2 \cdot \frac{1}{2t+3} \times 2$$

$$\frac{dy}{dt} = \frac{4}{2t+3}$$

$$\frac{dy}{dx} = \frac{\frac{4}{2t+3}}{\frac{12}{(2t+3)^2}} \div \frac{12}{(2t+3)^2}$$

$$= \frac{4}{2t+3} \times \frac{(2t+3)^2}{12}$$

$$\boxed{\frac{dy}{dx} = \frac{2t+3}{3}}$$

put $x = 1$???

(ii) put $x = 1$ and find t

$$x = \frac{4t}{2t+3}$$

$$1 = \underline{\underline{\frac{4t}{2t+3}}}$$

$$2t+3$$

$$2t+3 = 4t$$

$$3 = 2t$$

$$t = 1.5$$

$$\frac{dy}{dx} = \frac{2t+3}{3} = \frac{2(1.5)+3}{3} = \boxed{2}$$

9709/03/O/N08

- 3 The parametric equations of a curve are

$$x = a \cos^3 t, \quad y = a \sin^3 t,$$

where a is a positive constant and $0 < t < \frac{1}{2}\pi$.

- (i) Express $\frac{dy}{dx}$ in terms of t . [3]

- (ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x \sin t + y \cos t = a \sin t \cos t. \quad [3]$$

- (iii) Hence show that, if this tangent meets the x -axis at X and the y -axis at Y , then the length of XY is always equal to a . [2]

9709/03/M/J/09

$$(i) x = a \cos^3 t$$

$$x = a (\cos t)^3$$

$$\frac{dx}{dt} = a(3)(\cos t)^2(-\sin t)$$

$$\frac{dx}{dt} = -3a \sin t \cos^2 t$$

$$\frac{dy}{dx} = \frac{\cancel{3a} \sin^2 t \cos t}{\cancel{-3a} \sin t \cos^2 t} = -\frac{\sin t}{\cos t} = -\tan t.$$

$$y = a \sin^3 t$$

$$y = a (\sin t)^3$$

$$\frac{dy}{dt} = a(3)(\sin t)^2(\cos t \times 1)$$

$$\frac{dy}{dt} = 3a \sin^2 t \cos t$$

(ii) Tangent: $\frac{dy}{dx} = -\tan t = -\frac{\sin t}{\cos t}$, $x = a \cos^3 t$
 $y = a \sin^3 t$

$$y - a \sin^3 t = -\frac{\sin t}{\cos t} (x - a \cos^3 t)$$

$$ycost - a \sin^3 t \cos t = -x \sin t + a \sin t \cos^3 t$$

$$x \sin t + y \cos t = a \sin^3 t \cos t + a \sin t \cos^3 t$$

$$x \sin t + y \cos t = a \sin t \cos t (\sin^2 t + \cos^2 t)$$

$$x \sin t + y \cos t = a \sin t \cos t (1)$$

$$x \sin t + y \cos t = a \sin t \cos t$$

(iii) Tangent: $x \sin t + y \cos t = a \sin t \cos t$

x -axis at x

$$y=0$$

$$x \sin t + 0 \cos t = a \sin t \cos t$$

$$x \sin t = a \sin t \cos t$$

$$x = a \cos t$$

$$x (a \cos t, 0)$$

y -axis at y

$$x=0$$

$$0 \sin t + y \cos t = a \sin t \cos t$$

$$y \cos t = a \sin t \cos t$$

$$y = a \sin t$$

$$y (0, a \sin t)$$

$$xy = \sqrt{(a \sin t - 0)^2 + (0 - a \cos t)^2}$$

$$= \sqrt{a^2 \sin^2 t + a^2 \cos^2 t}$$

$$= \sqrt{a^2 (\sin^2 t + \cos^2 t)}$$

$$= \sqrt{a^2 (1)}$$

$$= \sqrt{a^2}$$

$$xy = a$$

TYPE 3 : IMPLICIT DIFFERENTIATION

y will not be subject of our equation

Do not even try to make y subject.

RULE 1 : $x \xrightarrow{\text{diff}} 1 \quad \frac{d \boxed{x}}{dx} = 1$

RULE 2 : $y \xrightarrow{} \frac{dy}{dx} \quad \frac{d \boxed{y}}{dx}$

RULE 3 : STRICT DIFFERENTIATION WITH PROPER ATTENTION TO DIFF OF BOX.

Q: Find $\frac{dy}{dx}$: $x^3 + 4x^2y + y^3 = 10$

$$x^3 + 4 \underbrace{x^2 y}_{u v} + y^3 = 10$$

Diff

$$3(x^2)(1) + 4 \left[x^2 \left(\frac{dy}{dx} \right) + y [2x(1)] \right] + 3(y^2) \left(\frac{dy}{dx} \right) = 0$$

$$3x^2 + 4x^2 \frac{dy}{dx} + 8xy + 3y^2 \frac{dy}{dx} = 0$$

$$4x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -8xy - 3x^2$$

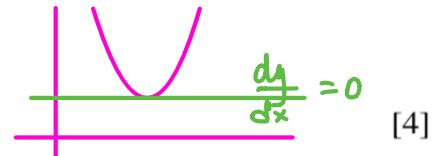
$$(4x^2 + 3y^2) \frac{dy}{dx} = -8xy - 3x^2$$

$$\frac{dy}{dx} = \frac{-8xy - 3x^2}{4x^2 + 3y^2}$$

2 The equation of a curve is $x^3 + 2y^3 = 3xy$.

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{2y^2 - x}$.

$x \neq 0, y \neq 0$



(ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

grad of tangent = 0
 $\frac{dy}{dx} = 0$

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9709/03/O/N/06

$$x^3 + 2y^3 = 3xy$$

$$3x^2(1) + 6y^2 \frac{dy}{dx} = 3 \left[x \frac{dy}{dx} + y(1) \right]$$

$$3x^2 + 6y^2 \frac{dy}{dx} = 3x \frac{dy}{dx} + 3y$$

$$6y^2 \frac{dy}{dx} - 3x \frac{dy}{dx} = 3y - 3x^2$$

$$(6y^2 - 3x) \frac{dy}{dx} = 3y - 3x^2$$

$$\frac{dy}{dx} = \frac{3y - 3x^2}{6y^2 - 3x}$$

$$\frac{dy}{dx} = \frac{3(y - x^2)}{3(2y^2 - x)}$$

$$\frac{dy}{dx} = \frac{y-x^2}{2y^2-x}$$

(ii) $\frac{dy}{dx} = 0$

$$\frac{y-x^2}{2y^2-x} = 0$$

$$y - x^2 = 0$$

$$y = x^2$$

IN IMPLICIT WHEN WE PUT $\frac{dy}{dx} = 0$, we do not get a value for x . we get another equation.

Solve this simultaneously with original equation.

$$y = x^2 \quad x^3 + 2y^3 = 3xy$$

$$x^3 + 2(x^2)^3 = 3x(x^2)$$

$$x^3 + 2x^6 = 3x^3$$

$$2x^6 = 3x^3 - x^3$$

$$2x^6 = 2x^3$$

$$x^3 = 1$$

$$x = 1$$

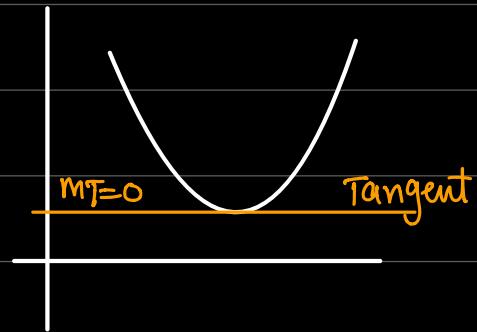
$$y = (1)^2 = 1$$

$$(1, 1)$$

IMP

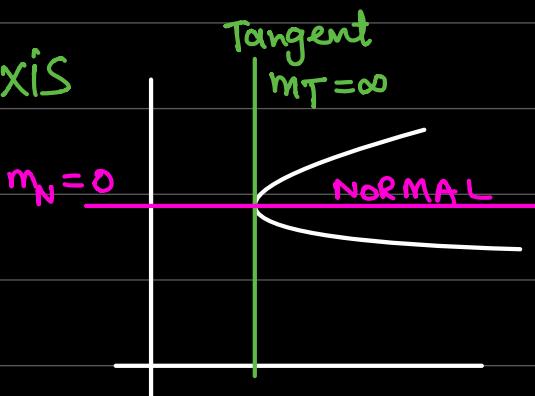
1) TANGENT PARALLEL TO X-AXIS

$$\frac{dy}{dx} = 0$$



2) TANGENT IS PARALLEL TO Y-AXIS

$$\frac{dy}{dx} = \infty \text{ (you cannot write this in an equation)}$$



$$\frac{dy}{dx} = m_T \xrightarrow{\text{Negative reciprocal.}} m_N$$

Now put $m_N = 0$

7 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\frac{dy}{dx} = \frac{y}{x(3y^2 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures.

$$m_T \longrightarrow m_N = 0$$

$$\ln(xy) - y^3 = 1$$

$$\frac{1}{xy} \left[x \frac{dy}{dx} + y(1) \right] - 3y^2 \left(\frac{dy}{dx} \right) = 0$$

$$\frac{1}{y} \frac{dy}{dx} + \frac{1}{x} - 3y^2 \frac{dy}{dx} = 0$$

$$\frac{1}{x} = 3y^2 \frac{dy}{dx} - \frac{1}{y} \frac{dy}{dx}$$

$$\frac{1}{x} = \frac{dy}{dx} \left(3y^2 - \frac{1}{y} \right)$$

$$\frac{1}{x} = \frac{dy}{dx} \left(\frac{3y^3 - 1}{y} \right)$$

$$\frac{y}{x} = \frac{dy}{dx} (3y^3 - 1)$$

$$\frac{dy}{dx} = \frac{y}{x(3y^3 - 1)}$$

(ii) $m_T = \frac{dy}{dx} = \frac{y}{x(3y^3 - 1)}$

$$m_N = -\frac{x(3y^3 - 1)}{y} \quad m_N = 0$$

$$-\frac{x(3y^3 - 1)}{y} = 0$$

$$-x(3y^3 - 1) = 0$$

$$-x = 0$$

$$x = 0$$

rejected.

$$3y^3 - 1 = 0$$

$$y^3 = \frac{1}{3}$$

$$y = \sqrt[3]{\frac{1}{3}} = 0.693$$

$$\ln(xy) - y^3 = 1$$

$$\ln(0.693x) - 0.693^3 = 1$$

$$\ln(0.693x) = 1.3328$$

$$0.693x = e^{1.3328}$$

$$x =$$

~~Ans~~ $y - 5$? ? <