ALEVELS P3
MARKING SCHEME
TRIG COMPOUND ANGLE
T2

1**(i)**	*EITHER*:	Use tan($A \pm B$) formula correctly to obtain an equation in tan x	M1	
		State or imply the equation $\frac{1+\tan x}{1-\tan x} = \frac{2(1-\tan x)}{1+\tan x}$ or equivalent	A1	
		Transform to an expanded horizontal quadratic equation in tan x	M1	
		Obtain given answer correctly	A1	
	OR:	Use sin($A \pm B$) and cos($A \pm B$) formulae correctly to obtain an equation in sin x and cos x	M1	
		Using values of sin 45°and cos 45°, or their equality, obtain an expanded horizontal equation in sin x and cos x	A1	
		Transform to a quadratic equation in tan x	M1	
		Obtain given answer correctly	A1	**4**
(ii)		Solve the given quadratic and calculate an angle in degrees or radians	M1	
		Obtain one answer e.g. 80.3°	A1	
		Obtain second answer 9.7° and no others in the range [Ignore answers outside the given range.]	A1	**3**

2	**(i)**	Use correct tan($A + B$) formula to obtain an equation in tan x	M1*	
		Use tan 45° = 1	M1(dep*)	
		Obtain the given answer	A1	**[3]**
	(ii)	Make reasonable attempt to solve the given quadratic for one value of tan x	M1	
		Obtain tan $x = -1 \pm \sqrt{2}$, or equivalent in the form $(a \pm \sqrt{b})/c$ (accept 0.4, –2.4)	A1	
		Obtain answer x = 22.5°	A1	
		Obtain second answer x = 112.5 and no others in the range [Ignore answers outside the range.] [Treat answers in radians as a MR and deduct one mark from the marks for the angles.]	A1	**[4]**

3	**(i)**	Use tan($A \pm B$) formula correctly at least once to obtain an equation in tan θ	M1	
		Obtain a correct horizontal equation in any form	A1	
		Use correct exact values of tan 30° and tan 60° throughout	M1	
		Obtain the given equation correctly	A1	[4]
	(ii)	Make reasonable attempt to solve the given quadratic in tan θ	M1	
		Obtain answer $\theta = 24.7°$	A1	
		Obtain answer $\theta = 95.3°$ and no others in the given range [Ignore answers outside the given range.] [Treat answers in radians as MR and deduct one mark from the marks for the angles.]	A1	[3]

4 Use tan($A \pm B$) formula and obtain an equation in tan α and tan β — M1*
Substitute throughout for tan α or for tan β — M1(dep*)
Obtain $2\tan^2\beta + \tan\beta - 1 = 0$ or $\tan^2\alpha + \tan\alpha - 2 = 0$, or equivalent — A1
Solve a 3-term quadratic and find an angle — M1
Obtain answer $\alpha = 45°$, $\beta = 26.6°$ — A1
Obtain answer $\alpha = 116.6°$, $\beta = 135°$ — A1 [6]
[Treat answers given in radians as a misread. Ignore answers outside the given range.]
[SR: Two correct values of α (or β) score A1; then A1 for both correct α, β pairs]

5 Attempt use of cos($A + B$) formula to obtain an equation in cos θ and sin θ — M1
Use trig formula to obtain an equation in tan θ (or cos θ, sin θ or cot θ) — M1
Obtain $\tan\theta = 1/(4 + \sqrt{3})$ or equivalent (or find cos θ, sin θ or cot θ) — A1
Obtain answer $\theta = 9.9°$ — A1
Obtain $\theta = 189.9°$, and no others in the given interval — A1 [5]
[Ignore answers outside the given interval. Treat answers in radians as a misread (0.173, 3.31).]
[The other solution methods are *via* $\cos\theta = \pm(4 + \sqrt{3})/\sqrt{\left(1 + \left(4 + \sqrt{3}\right)^2\right)}$ or
$\sin\theta = \pm 1/\sqrt{\left(1 + \left(4 + \sqrt{3}\right)^2\right)}$.]

6 **(i)** Use tan($A \pm B$) formula correctly at least once and obtain an equation in tanθ — M1
Obtain a correct horizontal equation in any form — A1
Use $\tan 60° = \sqrt{3}$ throughout — M1
Obtain the given equation correctly — A1 [4]

(ii) Set $k = 3\sqrt{3}$ and obtain $\tan^2\theta = \frac{1}{11}$ — B1
Obtain answer 16.8° — B1√
Obtain answer 163.2° — B1√ [3]
[Ignore answers outside the given interval. Treat answers in radians (0.293 and 2.85) as a misread.]

7 **(i)** Use tan ($A + B$) and tan $2A$ formulae to obtain an equation in tan x — M1
Obtain a correct equation in tan x in any form — A1
Obtain an expression of the form $a\tan^2 x = b$ — M1
Obtain the given answer — A1 [4]

(ii) Substitute $k = 4$ in the given expression and solve for x — M1
Obtain answer, e.g. $x = 16.8°$ — A1
Obtain second answer, e.g. $x = 163.2°$, and no others in the given interval — A1 [3]
[Ignore answers outside the given interval. Treat answers in radians as a misread and deduct A1 from the marks for the angles.]

(iii) Substitute $k = 2$, show $\tan^2 x < 0$ and justify given statement correctly — B1 [1]

8 Attempt use of sin $(A + B)$ and cos $(A - B)$ formulate to obtain an equation in cos θ and sin θ M1

Obtain a correct equation in any form A1

Use trig. formula to obtain an equation in tan θ (or cos θ, sin θ or cot θ) M1

Obtain tan θ = , or equivalent (or find cost θ, sin θ or cot θ) A1

Obtain answer θ = 105.9°, and no others in the given interval A1 [5]

[Ignore answers outside the given material]

9 **(i)** Use cos $(A + B)$ formula to express the given expression in terms of cos x and sin x M1

Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}} \sin x$, or equivalent A1

Obtain $R = 2.236$ A1

Use trig formula to find α M1

Obtain $\alpha = 71.57°$ with no errors seen A1 [5]

(ii) Evaluate $\cos^{-1}$ (2/2.236) to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°) B1√

Carry out an appropriate method to find a value of x in the interval $0° < x < 360°$ M1

Obtain answer, e.g. $x = 315°$ (315.0°) A1

Obtain second answer, e.g. 261.9° and no others in the given interval A1 [4]

[Ignore answers outside the given range.]

[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]

[SR: Conversion of the equation to a correct quadratic in sin x, cos x, or tan x earns B1, then M1 for solving a 3-term quadratic and obtaining a value of x in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]

10 Use cos$(A + B)$ formula to obtain an equation in cos x and sin x M1

Use trig formula to obtain an equation in tan x (or cos x or sin x) M1

Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find cos x or sin x) A1

Obtain answer $x = -66.2°$ A1

Obtain answer $x = 113.8°$ and no others in the given interval A1 **5**

[Ignore answers outside the given interval. Treat answers in radians as a misread (−1.16, 1.99).]

[The other solution methods are *via* $\cos x = \pm 1/\sqrt{(1 + (\sqrt{3} - 4)^2)}$ and $\sin x = \pm(\sqrt{3} - 4)/\sqrt{(1 + (\sqrt{3} - 4)^2)}$.]

11 **(i)** Use $\tan(A \pm B)$ formula and obtain an equation in tan x M1

Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan x in any correct form A1

Reduce the equation to the given form A1 **3**

(ii) Solve the given quadratic for tan x M1

Obtain a correct answer, e.g. $x = 21.6°$ A1

Obtain a second answer, e.g. $x = 128.4°$, and no others A1 **3**

[Ignore answers outside the given interval. Treat answers in radians as a misread (0.377, 2.24).]

12 **(i)** Either Use $\cos(A \pm B)$ correctly at least once — M1

State correct complete expansion — A1

Confirm given answer $\cos\theta$ with explicit use of $\cos 60^o = \frac{1}{2}$ — A1

SR: "correct" answer from sign errors in both expansions is B1 only

Or Use correct $\cos A + \cos B$ formula — M1

State correct result e.g. $2\cos\left(\frac{2\theta}{2}\right)\cos\left(\frac{-120}{2}\right)$ — A1

Confirm given answer $\cos\theta$ with explicit use of $\cos(\pm 60^o) = \frac{1}{2}$ — A1 [3]

(ii) State or imply $\frac{\cos 2x}{\cos x} = 3$ — B1

Obtain equation $2\cos^2 x - 3\cos x - 1 = 0$ — B1

Solve a three-term quadratic equation for $\cos x$ — M1

Obtain $\frac{1}{4}\left(3 - \sqrt{17}\right)$ or exact equivalent and, finally, no other — A1 [4]

13 Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$ — **M1***

Substitute throughout for $\tan\theta$ or for $\tan\phi$ — **dep M1***

Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent — **A1**

Solve a 3-term quadratic and find an angle — **M1**

Obtain answer $\theta = 135°$, $\phi = 63.4°$ — **A1**

Obtain answer $\theta = 53.1°$, $\phi = 161.6°$ — **A1** [6]

[Treat answers in radians as a misread. Ignore answers outside the given interval.]

[SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]

14 State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$ — **B1**

Divide by $\cos A$ to find value of $\tan A$ — **M1**

Obtain $\tan A = 3$ — **A1**

Use identity $\sec^2 B = 1 + \tan^2 B$ — **B1**

Solve three-term quadratic equation and find $\tan B$ — **M1**

Obtain $\tan B = \frac{3}{2}$ only — **A1**

Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$ — **M1**

Obtain $\frac{3}{11}$ — **A1** [8]

Question	Answer	Marks
15	Use correct $\tan(A \pm B)$ formula and express LHS in terms of $\tan\theta$	M1
	Using $\tan 60° = \sqrt{3}$ and $\cot\theta = 1/\tan\theta$, obtain a correct equation in $\tan\theta$ in any	A1
	Reduce the equation to one in $\tan^2\theta$ only	M1
	Obtain $11\tan^2\theta = 1$, or equivalent	A1
	Obtain answer 16.8°	A1
		5

Question	Answer	Marks
16(i)	Use trig formulae and obtain an equation in sin x and cos x	M1*
	Obtain a correct equation in any form	A1
	Substitute exact trig ratios and obtain an expression for $\tan x$	M1(dep*)
	Obtain answer $\tan x = \dfrac{-\left(6+\sqrt{6}\right)}{\left(6-\sqrt{2}\right)}$ or equivalent	A1
		4
16(ii)	State answer, e.g. 118.5°	B1
	State second answer, e.g. 298.5°	B1ft
		2

Question	Answer	Marks
17	Use correct tan ($A \pm B$) formula and obtain an equation in tan θ	**M1**
	Obtain a correct equation in any form	**A1**
	Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**
	Obtain answer $x = 30°$	**A1**
	Obtain answer $x = 150°$	**A1**
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1	
	Reduce to $\tan^2\theta = \frac{1}{3}$, $\sin^2\theta = \frac{1}{4}$, $\cos^2\theta = \frac{3}{4}$ or $\cot^2\theta = 3$ A1 etc.	
		5

Question	Answer	Marks
18	Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$	**M1***
	Obtain an equation in $\tan\theta$	**M1(dep*)**
	Obtain $\tan\theta = 1/\left(4-\sqrt{3}\right)$, or equivalent	**A1**
	Obtain final answer $\theta = 23.8°$ and no others in range	**A1**
		4

Question	Answer	Marks
19	Use correct trig formula and obtain an equation in tan θ	**M1**
	Obtain a correct horizontal equation in any form	**A1**
	Reduce to $2\tan^2\theta + 3\tan\theta - 1 = 0$	**A1**
	Solve 3-term quadratic and find a value of θ	**M1**
	Obtain answer $15.7°$	**A1**
	Obtain answer $119.(3)°$	**A1**
		6

Question	Answer	Marks
20(i)	Use tan (A + B) formula to express the LHS in terms of tan $2x$ and tan x	**M1**
	Using the tan $2A$ formula, express the entire equation in terms of tan x	**M1**
	Obtain a correct equation in tan x in any form	**A1**
	Obtain the given form correctly	**A1**
		4
20(ii)	Use correct method to solve the given equation for x	**M1**
	Obtain answer, e.g. $x = 26.8°$	**A1**
	Obtain second answer, e.g. $x = 73.7°$ and no other	**A1**
		3