TYPE 1
FOWARD

12

    (b) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$?

19   **(i)** The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above $0\,°C$. [3]

22   The mean number of defective batteries in packs of 20 is 1.6. Use a binomial distribution to calculate the probability that a randomly chosen pack of 20 will have more than 2 defective batteries. [5]

28   The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

    (i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]

    (ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm. [3]

33   The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.

    (i) For a random sample of 350 students, f nd the number who would be expected to take longer than 20 minutes to get up in the morning. [3]

    (ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

39

  (b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

52 It is given that $X \sim N(28.3, 4.5)$. Find the probability that a randomly chosen value of X lies between 25 and 30. [3]

62 The random variable X is such that $X \sim N(82, 126)$.

    (i) A value of X is chosen at random and rounded to the nearest whole number. Find the probability that this whole number is 84. [3]

    (ii) Five independent observations of X are taken. Find the probability that at most one of them is greater than 87. [4]

    (iii) Find the value of k such that $P(87 < X < k) = 0.3$. [5]

TYPE 2
RESERVE

12 (a)The random variable X is normally distributed. The mean is twice the standard deviation. It is given that P(X > 5.2)=0.9. Find the standard deviation. [4]

19 **(ii)** In another city the daily minimum temperature in $^{\circ}$C in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above $0\,^{\circ}$C is 0.8888. Find the value of μ. [3]

20 The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

  (i) Calculate the value of μ. [3]

  (ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres. [2]

24

39   63   wind speed (km h^{-1})
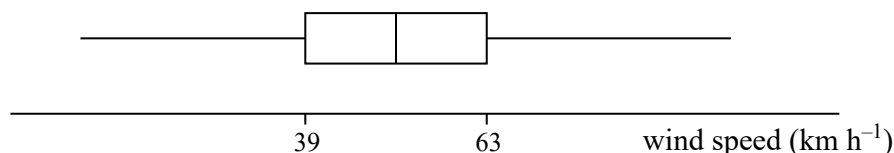
Measurements of wind speed on a certain island were taken over a period of one year. A box-and-whisker plot of the data obtained is displayed above, and the values of the quartiles are as shown. It is suggested that wind speed can be modelled approximately by a normal distribution with mean $\mu\,$km h^{-1} and standard deviation $\sigma\,$km h^{-1}.

  (i) Estimate the value of μ. [1]

  (ii) Estimate the value of σ. [3]

26 The random variable X is the length of time in minutes that Jannon takes to mend a bicycle puncture. X has a normal distribution with mean μ and variance σ^2. It is given that $P(X > 30.0) = 0.1480$ and $P(X > 20.9) = 0.6228$. Find μ and σ. [5]

29 The random variable X is normally distributed with mean μ and standard deviation σ.

    (i) Given that $5\sigma = 3\mu$, f nd $P(X < 2\mu)$.   [3]

    (ii) With a different relationship between μ and σ, it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ.   [3]

39   **(a)** The random variable X is normally distributed with mean μ and standard deviation σ. It is given

that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

61   In a normal distribution with mean 9.3, the probability of a randomly chosen value being greater than 5.6 is 0.85. Find the standard deviation.

                                                              [3]

TYPE 3
Binomial----> Normal

2 A shop sells sold videotapes, of which 1 in 5 on average are known to be damaged.

  (i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged.   [3]

  (ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape.   [3]

  (iii) A random sample of 1600 tapes is taken. Use a suitable approximation to f nd the probability that there are at least 290 damaged tapes.   [5]

4   **7i)** State two conditions which must be satisf ed for a situation to be modelled by a binomial distribution.   [2]

In a certain village 28% of all cars are made by Ford.

  (ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford.   [4]

  (iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford.   [4]

5 It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, fin the probability that fewer than 165 people in the sample are overweight.   [5]

9 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

  (i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch.   [4]

  (ii) A random sample of 200 adults was taken. Using a suitable approximation, fin the probability that more than 155 wore a watch on their left wrist.   [5]

11   A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

   (i) equal numbers of large and small bands,   [2]

   (ii) more than 17 small bands.   [3]

   An off ce pack contains 150 elastic bands.

   (iii) Using a suitable approximation, calculate the probability that the number of small bands in the off ce pack is between 88 and 97 inclusive.   [6]

13   The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

   (i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday.   [4]

   (ii) 56 years are chosen randomly. Use a suitable approximation to f nd the probability that more than 7 of these years have New Year's Day on a Saturday.   [5]

15   On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

   (i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7.
                                                                                                       [2]

   (ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions.   [5]

   (iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8.   [2]

17   A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

   (i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number.   [3]

   (ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5.   [3]

   (iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times.   [5]

18   On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to f nd the probability that there are at least 15 faulty toys.   [5]

21 On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

    (i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses.   [3]

    (ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, f nd the probability that more than 73 are cars.   [5]

23 A box contains 4 pears and 7 oranges. Three fruits are taken out at random and eaten. Find the probability that

    (i) 2 pears and 1 orange are eaten, in any order,   [3]

    (ii) the third fruit eaten is an orange,   [3]

    (iii) the f rst fruit eaten was a pear, given that the third fruit eaten is an orange.   [3]

There are 121 similar boxes in a warehouse. One fruit is taken at random from each box.

    (iv) Using a suitable approximation, f nd the probability that fewer than 39 are pears.   [5]

25 Two unbiased tetrahedral dice each have four faces numbered 1, 2, 3 and 4. The two dice are thrown together and the sum of the numbers on the faces on which they land is noted. Find the expected number of occasions on which this sum is 7 or more when the dice are thrown together 200 times.

    [4]

32 On average, 2 apples out of 15 are classif ed as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35.   [5]

35   **(i)** State three conditions that must be satisf ed for a situation to be modelled by a binomial distribution.   [2]

On any day, there is a probability of 0.3 that Julie's train is late.

    (ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days.   [3]

    (iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days.   [5]

38   Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits.   [4]

42   In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

  (i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

  (ii) Give a reason why the use of a normal approximation is justified. [1]

46   When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to find the probability that more than 210 germinate. [5]

54   In Restaurant Bijoux 13% of customers rated the food as 'poor', 22% of customers rated the food as 'satisfactory' and 65% rated it as 'good'. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

  (i) Find the probability that more than 2 and fewer than 12 of them rated the food as 'good'. [3]

  On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

  (ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as 'poor' is greater than 0.95. [3]

TYPE 4
Advanced Mix

1   Melons are sold in three sizes: small, medium and large. The weights follow a normal distribution with mean 450 grams and standard deviation 120 grams. Melons weighing less than 350 grams are classifi d as small.

   (i) Find the proportion of melons which are classifi d as small. [3]

   (ii) The rest of the melons are divided in equal proportions between medium and large. Find the weight above which melons are classif ed as large. [5]

3   The length of Paulo's lunch break follows a normal distribution with mean μ minutes and standard deviation 5 minutes. On one day in four, on average, his lunch break lasts for more than 52 minutes.

   (i) Find the value of μ. [3]

   (ii) Find the probability that Paulo's lunch break lasts for between 40 and 46 minutes on every one of the next four days. [4]

6   Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

   (i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars. [5]

   (ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits. [3]

7   In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

   (i) Find the mean lifetime, giving your answer to the nearest hour. [3]

   (ii) Find the probability that a light bulb fails to last for 5000 hours. [3]

   (iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, fin the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

8   The lengths of fis of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the f sh are longer than 50 cm.

   (i) Find the standard deviation. [3]

   (ii) When fis are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fis rejected. [3]

   (iii) 9 f sh are chosen at random. Find the probability that at least one of them is longer than 50 cm. [2]

10 **(i)** Give an example of a variable in real life which could be modelled by a normal distribution. [1]

(ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, f nd the value of μ. [3]

(iii) If 300 observations are taken at random from the distribution in part **(ii)**, estimate how many of these would be greater than 22.0. [4]

14
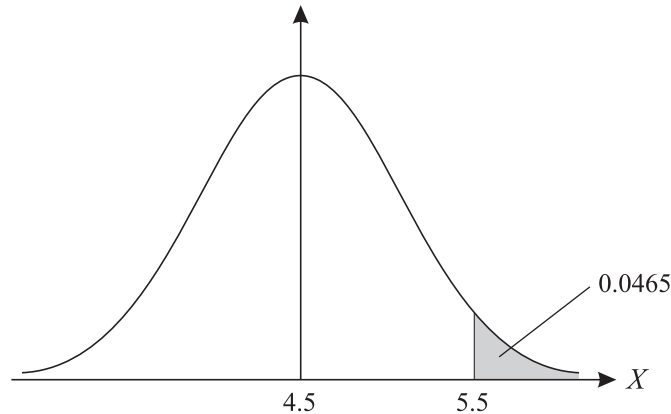
The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

(i) Find the standard deviation of X. [3]

(ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

16 In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]

27 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

(ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to f nd the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]

(iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

31   The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

  (i) Find the mean and standard deviation of the heights the children can jump. [5]

  (ii) Find the probability that a randomly chosen child will **not** be able to jump a height of 145 cm. [3]

  (iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]

34   The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are f lled with 20 litres of fuel and are driven towards a place 367 km away.

  (i) For each car, f nd the probability that it runs out of fuel before it has travelled 367 km. [3]

  (ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d. [4]

37   The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.

  (i) Find the standard deviation of the times spent by people visiting this dentist. [3]

  (ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute. [3]

  (iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes. [3]

  (iv) Find the probability that, of 35 randomly chosen people, fewer than 16 have visits lasting less than 8.2 minutes. [5]

43   The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

  (i) Find the value of μ. [3]

  (ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

44   The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $P(X > 20) = 0.04$.

  (i) Find μ. [3]

  (ii) Find $P(10 < X < 20)$. [3]

  (iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20. [5]

45   The probability that Sue completes a Sudoku puzzle correctly is 0.75.

(i) Sue attempts n Sudoku puzzles. Find the least value of n for which the probability that she completes all n puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by X.

(ii) Find the value of X that has the highest probability. You may assume that this value is one of the two values closest to the mean of X. [3]

(iii) Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]