A LEVEL (P6) RANDOM VARIABLE

MARK SCHEME

1 **(i)**			
x | 1 | 2 | 3 | 4 | 5 | 6	M1		For 36 in the uncancelled denominator somewhere, accept decimals eg 0.305 recurring or 0.306 etc
P($X = x$) | $11/36$ | $9/36$ | $7/36$ | $5/36$ | $3/36$ | $1/36$	A1		For 3 correct probabilities
	A1		All correct
		3	
(ii) $E(X) = 1\times 11/36 + 2\times 9/36 + 3\times 7/36 + 4\times 5/36 + 5\times 3/36 + 6\times 1/36 = 91/36$	M1		For calculation of $\sum xp$ where all probs < 1
	A1		
		2	

2 **(i)**	Options 123, 124, 125, 134, 135, 145, 234, 235, 245, 345	**M1**	For listings options, at least 4 different ones
	P (odd) = 0.4	**B1**	For correct answer, legit obtained
(ii)	P(largest is 4) = 0.3	**B1**	For correct answer
	OR $\frac{1\times {}_3C_2}{{}_5C_3}$		**SR** if 9 options in **(i)** give B1 for 3/9 or 2/9 depending on their missing option
(iii)	l | 3 | 4 | 5 P($L = l$) | 0.1 | 0.3 | 0.6	**M1**	For 3, 4, 5, in table or 1, 2 as well, no need for any probs but need to see an (uncompleted) second line
		M1	For evaluating another probability based on their list
		A1	For correct answer
(iv)	E(L) = $\sum lp = 3\times 0.1 + 4\times 0.4 + 5\times 0.6 = 4.5$	**B1 ft**	For correct answer, ft if their $\sum p = 1$
	Var (L) = $3^2\times 0.1 + 4^2\times 0.3 + 5^2\times 0.6 -$ (their 4.5^2)	**M1**	For evaluating their $\sum l^2 p -$ (their 4.5^2) (must see – their 4. ^{2}) each $p < 1$, in first numerical instance, ie can forget the sq rt subsequently
	= 0.45	**A1**	For correct answer

3 (i) P(G, G, G, G, NG) = $(0.25)^4 \times (0.75)^1 \times {}_5C_4$ = 0.0146 AG	**M1** **A1**	**[2**	For relevant binomial calculation, need ${}_5C_r$ or 5 or all 5 options For correct answer. AG
(ii) X 0 1 2 P($X = x$) 0.2373 0.3955 0.2637	**B1**		For all correct X values
	B1		For one correct prob excluding P($X = 4$)
(cont) X 3 4 5 P($X = x$) 0.0879 0.0146 0.0010	**B1** **B1**		For 2 correct probs excluding P($X = 4$) For 3 correct probs excluding P($X = 4$)
	B1	**[5**	All correct and in decimals

4 **(i)** \$2	B1	**1**	For correct answer
(ii) P(MMMH) +P(MMMMH) $= 0.8^3 \times 0.2 + 0.8^4 \times 0.2 = 0.184$ **AG**	M1 A1	**2**	For attempting to sum P(MMMH) and P(MMMMH) For correct answer
(iii) x 4 2 0 -1 P($X = x$) 0.2 0.288 0.184 0.328	B1 B1ft B1	**3**	For one correct prob other than 0.184 For another correct prob other than 0.184, ft only if the –1 ignored and their 3rd prob is 1 – Σ the other 2 For correct table, can have separate 2s
(iv) E(X) = 0.8 + 0.576 – 0.328 = \$1.05	M1 A1	**2**	For attempt at Σ xp from their table, at least 2 non-zero terms For correct answer

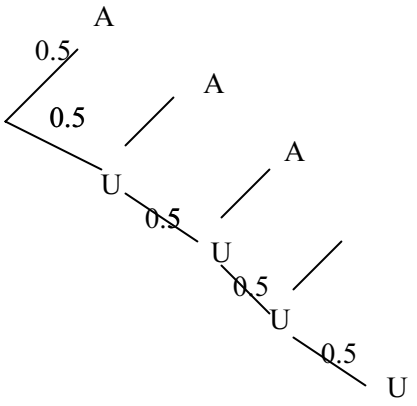
5 **(i)** 16	B1	**1**	
(ii) 8	B1	**1**	
(iii) Matches 1 2 3 4 5 freq 16 8 4 2 2	M1 A1 A1	**3**	Matches 1,2,3,4,5 3 correct frequencies All correct
(iv) mean = 62/32 =1.9375 (= 1.94) var = 166/32 – (62/32)² = 1.43	M1 A1 M1 A1	**4**	Using their $\Sigma fx/\Sigma f$ Correct answer Subst in $\Sigma fx^2 - (\Sigma fx/n)^2$ formula Correct answer, or B2 if used calculator

6 **(i)** $q + 3q + 0.26 + 0.05 + 0.09 = 1$	M1		Equation with q in summing probs to 1must be probs
$q = 0.15$	A1	**2**	Correct answer
(ii) E(X) = 1.56	B1ft		Correct final answer, ft on wrong q
Var (X) = 0.15 + 1.8 + 0.45 + 1.44 – mean^2	M1		Subst in Σpx^2 – mean^2 formula
= 1.41	A1	**3**	Correct final answer

7 **(i)**	P (all different) $= \frac{{}_3C_1 \times {}_4C_1 \times {}_5C_1}{{}_{12}C_3} =$	M1		Attempt using combinations, with ${}_{12}C_3$ denom, or P(RGY) in any order, i.e. 12 x 11 x 10 in denom
		M1		Correct numerator, or multiplying by 6
	= 3/11 (= 0.273)	A1	**3**	Correct answer
(ii)	P(exactly 2 G) $= \frac{{}_4C_2 \times {}_8C_1}{{}_{12}C_3}$	M1		Attempt using combinations, or mult any P $\left(GG\overline{G}\right)$ x 3 Or P(GGY) x 3+P(GGR) x 3
	= 12/55 AG	A1	**2**	Correct answer AG
(iii)		M1		For seeing P(0, 1, 2, 3) only and 1 or more probs
		M1		For reasonable attempt at P(X = 0 or 1 or 3)
		A1		For one correct probability seen other than P(X=2)
		A1		For a second probability correct other than P(X=2)
		A1	**5**	All correct

x	0	1	2	3
P(X=x)	14/55	28/55	12/55	1/55
decimal	0.255	0.509	0.218	0.018

8 **(i)** $2p + p + 3p =$ 1 $p = 1/6$ (= 0.167)	M1		Equation involving ps and summing to 1
	A1	**2**	Correct answer
(ii) E(X) = -2× 2/6 + 0 + 4 × 3/6 = 4/3 (=1.33)	M1		Using correct formula for E(X), in terms of p or their p< 1
	A1ft		Correct expectation ft on their p if $p \leq 1/3$
Var (X) = 4 × 2/6 + 0 + 16 × 3/6 – (4/3)^2 = 7.56 (68/9)	M1		Substitution in their Σpx^2 – their E^2(X) need 2 terms
	A1	**4**	Correct answer

9	**(i)**		M1 A1 A1 **3**	4 or 5 pairs A and U seen no extra bits but condone (0, 1) branches after any or all As. Exactly 4 pairs of A and U, must be labelled Correct diagram with all probs correct, allow A1ft for 4 correct pairs and (0,1) branch(es) or A1ft for 5 correct pairs and no (0, 1) branch(es)
	(ii)	x **0** 1 2 3 4 P(X=x) ½ ¼ 1/8 1/16 1/16	B1 B1 B1 B1 **4**	P(0) correct P(2) correct P(3) correct P(4) correct
	(iii)	E(X) = 15/16 (0.938 or 0.9375)	M1 A1 **2**	attempt at $\Sigma(xp)$ only with no other numbers correct answer

10	**(i)**	P(odd) = 2/3 or 0.667 P(7) = $_8C_7 \times (2/3)^7 (1/3)$ = 0.156 P(8) = $(2/3)^8$ = 0.0390 P(7 or 8) = 0.195 (1280/6561)	B1 M1 M1 A1 **[4]**	Can be implied if normal approx used with $\mu = 5.333 (= 8 \times 2/3)$ Binomial expression with C in and 2/3 and 1/3 in powers summing to 8 Summing P(7) + P(8) binomial expressions Correct answer
	(ii)	<table><tr><td>x</td><td>2</td><td>4</td><td>6</td><td>7</td><td>8</td></tr><tr><td>P(X=x)</td><td>1/36</td><td>2/36</td><td>5/36</td><td>4/36</td><td>4/36</td></tr></table> <table><tr><td>x</td><td>9</td><td>10</td><td>11</td><td>12</td></tr><tr><td>P(X=x)</td><td>4/36</td><td>4/36</td><td>8/36</td><td>4/36</td></tr></table>	B1 B2 **[3]**	Values of x all correct in table of probabilities All probs correct and not duplicated, –1 ee
	(iii)	E(X) = $\sum p_i x_i$ = 2×1/36 + 4×2/36 + ... = 312/36 (26/3) (8.67)	M1 A1 **[2]**	attempt to find $\sum p_i x_i$, all $p < 1$ and no further division of any sort correct answer
	(iv)	P(X > E(X)) = P(X = 9, 10, 11, 12) = 20/36 (5/9) (0.556)	M1 A1 **[2]**	attempt to add their relevant probs correct answer

11 **(i)**	$P(X = 2) = 1/4 \times 1/4 + 1/4 = 5/16$ AG OR can use a table | | 1 | 2 | 3 | 4 | | 1 | 2 | 2 | 4 | 4 | | 2 | 3 | 2 | 5 | 4 | | 3 | 4 | 2 | 6 | 4 | | 4 | 5 | 2 | 7 | 4 |	M1 A1 **[2]**	Considering cases (1, 1) and (2) Correct given answer legitimately obtained (1/16 + 4/16 needs some justification but 1/16 + 1/4 is acceptable)
(ii)	$E(X) = \Sigma xp$ $= 15/4$ (3.75) $Var(X) = 2^2 \times 5/16 + 3^2 \times 1/16 + 4^2 \times 3/8 + ... - (15/4)^2$ $= 260/16 - 225/16 = 35/16$ (2.19)	M1 A1 M1 A1 **[4]**	Using correct formula for $E(X)$, no extra division Correct answer Using a variance formula correctly with mean^2 subtracted numerically, no extra division Correct final answer

12 **(i)**	$-0.16 - p + 0.16 + 2q + 0.66 = 1.05$ $-p + 2q = 0.39$ $p + q = 0.42$ $q = 0.27$ $p = 0.15$	M1 A1 B1 A1 **[4]**	Attempt at $\Sigma px = 1.05$ no dividing Correct simplified equation Accept $p = 0.42 - q$ oe Both answers correct
(ii)	Var $(X) = 4 \times 0.08 + p + 0.16 + 4q + 1.98 - (1.05)^2$ $= 2.59$	M1 A1 **[2]**	Subst in Σpx^2 – mean^2 formula, mean^2 subt numerically, p +ve and < 1 Correct answer

13 **(i)**	$40 = 120 / 3$ so $r = 3$ $P(40) = 3/45 = 1/15$ AG	M1 A1 **[2]**	$r = 3$ seen or obtained from table Given answer legit obtained
(ii)	| x | 120 | 60 | 40 | 30 | | $P(X = x)$ | 1/45 | 2/45 | 3/45 | 4/45 | | 24 | 20 | 17.14 | 15 | 13.3 | | 5/45 | 6/45 | 7/45 | 8/45 | 9/45 |	B1 B1 B1 **[3]**	8 or 9 values for x, correct to nearest integer One correct probability apart from 1/15 Correct table
(iii)	40/3 oe (13.3)	B1ft **[1]**	ft their table
(iv)	$P(18 < X < 100) = (2 + 3 + 4 + 5 + 6)/45$ $= 20/45$ (4/9) (0.444)	M1 A1 **[2]**	Adding 5 probabilities o.e. Correct answer

14 $a + b = 0.45$ $-3a - b + 1.6 = 0.75$ $a = 0.2$ $b = 0.25$	B1 M1 A1 A1 **[4]**	Correct sum probs = 1 o.e. Attempt at $\Sigma xp = 0.75$ Correct a Correct b

15 **(i)**		B1 B1 B1 **[3]**	0, 1, 2 only in table or listed with some prob 3, 4… if in table must have blank or 0 for prob One correct probability All correct
(ii)	E(X) = 8/7 (1.14) AG Var(X) = 12/7 – $(8/7)^2$ = 20/49 (0.408)	B1 M1 A1 **[3]**	Legitimate correct given answer rounding to 1.14 Correct method with mean^2 subt numerically no dividing by anything Correct final answer
(iii)	$P(G \mid NA) = \dfrac{P(G \cap NA)}{P(NA)}$ $= \dfrac{2/5 \times 1/4}{2/5 \times 1/4 + 3/5 \times 9/10}$ $= \dfrac{5}{32}$ (0.156)	M1 M1 A1 A1 **[4]**	Attempt at $P(G \cap NA)$ or $P(G \cap A)$ as numerator of a fraction Attempt at P(NA) or P(A) in form of summing two 2-factor products, seen anywhere Correct unsimplified denominator of a fraction Correct answer

16 **(i**	P(2) = P(0,2) + P(2,0) = 6/10 × 3/7 + 3/10 × 4/7 = 30/70 = 3/7 AG	M1 A1 [2]	Summing two 2-factor probabilities Correct answer legit obtained
(ii)	<table><tr><td>x</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>P($X = x$)</td><td>24/70</td><td>30/70</td><td>13/70</td><td>3/70</td></tr></table>	B1 B1 [2]	Correct values for rv X Correct probs
(iii)	E(X) = 13/7 Var(X) = 120/70 + 208/70 + 108/70 – $(13/7)^2$ = 2.78	B1ft M1 A1 [3]	 Using variance formula correctly with mean^2 subtracted numerically, no extra division Correct final answer
(iv)	P(A2 | Sum 2) = $\dfrac{3/10 \times 4/7}{30/70}$ = 0.4	M1 A1 [2]	Correct numerator with a 0 < denom < 1 Correct answer

17 **(i)**	If y = P(odd number) then P(even number) = $2y$ $3y + 6y = 1$ so $y = 1/9$ oe. OR prob = 1/3	M1 A1 [2]	2P(Odd) shown = P(Even) and summed to 1 correct answer accept either
(ii)	Score of 8 means throwing a 6 6 is even so P(8) = 2/9 (AG)	B1 B1 [2]	 legit justification of use of 2/9

(iii) Var(X) = (48 + 36 + 98 + 128 + 100)/9 – (58/9)^2 = 4.02 accept 4.025 (326/81)	M1 A1 [2]	Correct method no dividings, 6.44 squared subt numerically Correct answer
(iv) P(score 6,10) + P(score 10,6) + P(score 8,8) = 1/81 + 1/81 + 4/81 = 6/81 (2/27) (0.0741)	M1 A1 [2]	Summing two different 2-factor probabilities Correct answer
(v) P(score 6, 10) = 1/81 P(1st score 6 given total 16) = (1/81) ÷ (6/81) = 1/6	B1 M1 A1 [3]	1/81 seen in numerator Dividing by their **(iv)** Correct answer

18 $4p + 5p^2 + 1.5p + 2.5p + 1.5p = 1$ $10p^2 + 19p – 2 = 0$ $p = 0.1$ or –2 $p = 0.1$	M1 A1 A1 [3]	Summing 5 probs to = 1 can be implied For 0.1 seen with or without –2 Choosing 0.1 must be by rejecting –2

19 **(i)** x | 1 | 2 | 3 | 4 | 5 Prob | k | $2k$ | $3k$ | $4k$ | $5k$ $15k = 1$ $k = 1/15$ (0.0667)	M1 M1 A1 [3]	1, 2, 3, 4, 5 seen, together with some probabilities involving k but not x summing probs involving k to 1 correct answer
(ii) E(X) $= k + 4k + 9k + 16k + 25k$ $= 55k = 11/3$ (3.67)	M1 A1ft [2]	using Σpx no dividing correct answer, ft on $55k$, $0 < k < 1$

20 **(i)** P(any other number) = 9/70 P($X < 2$) = 27/70 + 1/10 = 34/70 (17/35) (0.486)	B1 B1ft **[2]**	9/70 Seen Ft their probs if < 1
(ii) E(X) = 108/70 (54/35) (1.543) Var(X) = ((–2)^2 + … + 5^2) × 9 / 70 – (54/35)^2 = 5.33	M1 M1 A1 **[3]**	Valid attempt at E(X) (needn't be accurate) Using a variance formula correctly with mean^2 subtracted numerically, no extra division Correct final answer
(iii) $a = 1$	B1 **[1]**	

21 **(i)** P(6) = P(3, 9) + P(9, 3) = 2/25 = 0.08 AG	B1 **[1]**	Accept 2/25 seen
(ii) <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Prob</td><td>0.2</td><td>0.24</td><td>0.08</td><td>0.08</td><td>0.16</td><td>0.16</td><td>0.08</td></tr></table>	M1 A1 **[2]**	Values 0 – 6 seen could be in list All correct
(iii) Mean = Σxp = 2.56 (64/25)	B1 **[1]**	
(iv) P(4, 5, 6) = 0.4(10/25) or 0.16 + 0.16 + 0.08 = P(draw) × 0.4 = 0.2 × 0.4 = 0.08 (2/25)	B1 ft M1 A1ft **[3]**	ft their P(4, 5, 6) providing $p < 1$ Multiplying by their P(draw) providing $p < 1$ Correct answer
(v) P(J wins on nth go) = $(0.2)^{n-1} \times 0.4$ oe	M1 A1ft **[2]**	Mult by any p^n or p^{n-1}, $p < 1$ ft their probs

22 **(i)** $(0.75)^n < 0.06$	M1*	Equation or inequality with 0.75^n and 0.06 or 0.94 seen
$n > 9.78$	M1dep*	Attempt at solving by trial and error (can be implied) or using logarithms correctly
$n = 10$	A1 **[3]**	Correct answer
(ii) E(X) = 14 × 0.75 or 10.5 Try P(10) = $^{14}C_{10}(0.75)^{10}(0.25)^4 = 0.220$	M1	Evaluating binomial probability for an integer value directly above or below their mean
P(11) = $^{14}C_{11}(0.75)^{11}(0.25)^3 = 0.240$ (mode is) 11	M1 A1 **[3]**	Evaluating the other binomial probability Correct answer
OR	M1 M1 A1	Evaluating binomial P(n) and P(n + 1) Evaluating binomial P(10), P(11) and P(12) Correct answer
(iii) P(> 11) = $^{14}C_{12}(0.75)^{12}(0.25)^2 + {}^{14}C_{13}(0.75)^{13}(0.25)^1 + (0.75)^{14}$	M1 M1	A binomial term of the form $^{14}C_n p^n(1-p)^{14-n}$ seen, $n \neq 0$ or 14 Summing binomial P(12, 13, 14) or P(11, 12, 13, 14,)
= 0.281	A1	Correct answer 0.280 – 0.282
P(3) = $^5C_3\,(0.2811)^3(0.7189)^2$	M1	A binomial term of the form $^5C_3 p^3(1-p)^2$ seen, any p
= 0.115	A1 **[5]**	Correct answer

23 **(i** P(X = 1) = P(GBBB) 4 × C_1	M1	Considering values of X of 1, 2, 3, 4
= 5/8 × 3/7 × 2/6 × 1/5 × 4 = 1/14	M1	Attempting to find the probability of at least 2 values of X
P(X = 2) = P(GGBB) × $_4C_2$ = 3/7		
P(X = 3) = P(GGGB) × $_4C_3$ = 3/7	A1	One correct probability
P(X = 4) = P(GGGG) × $_4C_4$ = 1/14	A1	All correct
OR		
P(1) = $_5C_1$ / $_8C_4$ = 1/14	M1	Considering values of X of 1, 2, 3, 4
P(2) = $_3C_2$ × $_5C_2$ / $_8C_4$ = 3/7	M1	Dividing by $_8C_4$
P(3) = $_3C_1$ × $_5C_3$ / $_8C_4$ = 3/7	A1	One correct probability
P(4) = $_5C_4$ / $_8C_4$ = 1/14	A1 [4]	All correct

(ii)	Var(X) = 1/14 + 12/7 + 27/7 + 16/14 – $(5/2)^2$	M1		Using a variance formula correctly with mean^2 subtracted numerically, no extra division
	= 15/28 (0.536)	A1	[2]	Correct final answer

24 **(i)**	P(2, N2, 2) = 1/4 × 1 × 1/7 = 1/28	M1		Considering at least two options of 2s and 8s
	P(8, 8, N8) = 1/4 × 2/5 × 3/7 = 3/70	M1		Considering three options for the 8s
	P(8, N8, 8) = 1/4 × 3/5 × 4/7 = 3/35	M1		Summing their options if more than 3 in total
	P(N8, 8, 8) = 3/4 × 2/5 × 4/7 = 6/35	B1		One option correct
	$\sum$ = 47/140 (0.336)	A1	[5]	Correct answer
(ii)	P(2, 2 given same) = $\dfrac{1/28}{47/140}$	M1		1/28 in numerator of a fraction
	= 5/47 (0.106)	A1	[2]	Correct answer
(iii)	P(X) = 47/140	M1		Attempt to compare P(A and B) with P(A) × P(B) or using conditional probabilities
	P(Y) = 1/4			
	P(X and Y) = 1/28 ≠ 47/140 × 1/4	A1		Legitimate correct answer
	Not independent		[2]	

25 **(i)**	P(3m) = 4/5 (0.8) P(5m) = 1/5 (0.2)	B1		P(3m) = 4/5 or P(5m) = 1/5 seen or implied
	E(X) = 17/5 (3.4)	B1		Correct E(X)
		M1		Subtract their mean^2 numerically from $\sum x^2$p, no extra dividing
	Var(X) = 16/25 (0.64)	A1	[4]	Correct answer
(ii)	P(3, 5) + P(5, 3) = 0.8 × 0.2 +0.2 × 0.8	M1		Summing two 2-factor terms
	= 8/25 (0.32)	A1√	[2]	Correct answer, ft on 2 × p × (1 − p), their p
(iii)	P(11) = P(3, 3, 5) + P(3, 5, 3) + P(5, 3, 3)	M1		Mult 2 probs for 3 with 1 prob for 5
	= (4/5 × 4/5 × 1/5) × 3	M1		Multiplying probs for 11 by 3 or summing 3 options
	= 48/125 (0.384)	A1	[3]	Correct final answer

26 **(i)**	p = 0.1	B1	**[1]**	
(ii) **(a)**	P(X = 1, Y = 3) = 0.3 × 0.2 = 0.06	M1		Summing 2 or 3 options
	P(X = 2, Y = 2) = 0.15 × 0.5 = 0.075	B1		One option correct unsimplified
	P(X = 3, Y = 1) = 0.3 × 0.3 = 0.09			
	P(sum is 4) = 0.225	A1	**[3]**	correct final answer

(b)	P($X = 1$, Y = anything) = 0.3	M1		Σ 3 or more two-factor options
	P($X = 2$, Y = anything) = 0.15	B1		Two correct options
	P($X = 3$, $Y = 1, 2$) = $0.3 \times 0.8 = 0.24$			
	P($X = 4$, $Y = 1$) = $0.2 \times 0.3 = 0.06$			
	P($X = 5$, $Y = 1$) = $0.05 \times 0.3 = 0.015$			
	P(product < 8) = 0.765	A1	**[3]**	Correct answer
OR	P($Y = 1$, $X =$ anything) = 0.3	M1		
	P($Y = 2$, $X = 1, 2, 3$) = 0.5×0.75 = 0.375			
	P($Y = 3$, $X = 1, 2$) = $0.2 \times 0.45 = 0.09$	B1		
	P(product < 8) = 0.765	A1		

27 **(i)**

y	0	2	4
P($Y = y$)	0.42	0.48	0.1

B1		0, 2, 4 only seen for Y no probs needed. Accept other vals if P(value) = 0 seen in table, allow 0002244 with probs
M1		Summing two or more 2-factor probs (can be implied)
A1		One correct prob
A1	[4]	Correct table or list

(ii) 0.96 + 0.4 = 1.36

B1ft	[1]	Ft their table for Y or X $\Sigma p = 1$

28**(i)** mean = 11/6 (1 $\frac{5}{6}$, 1.83)

sd = $\sqrt{(1+1+1+4+9+9)/6-(11/6)^2}$

= $\sqrt{29}/6$ (0.898)

B1		correct answer
M1		numerical use of a correct sd/variance formula
A1	**[3]**	correct answer

(ii)

x	2	3	4	5	6
Pr	9/3	6/36	13/36	4/36	4/36

B1		all correct x values
B1		P(2) and P(6) correct
M1		considering more than 1 case for a sum of 3 or 4 or 5
A1	**[4]**	P(3), P(4) and P(5) correct

(iii) $p = 1/3$

$np = 8$ $n = 24$

Var = $24 \times 1/3 \times 2/3 = 16/3$ (5.33)

B1		correct p
M1		using $np = 8$ to find n or $8(1 - p)$ to find var, $0<p<1$
A1ft	**[3]**	correct answer, ft their p

29 P(0) = $7/10 \times 6/9 \times 5/8 = 210/720$

P(1) = $3/10 \times 7/9 \times 6/8 \times 3C1 = 378/720$

P(2) = $3/10 \times 2/9 \times 7/8 \times 3C2 = 126/720$

P(3) = $3/10 \times 2/9 \times 1/8 = 6/720$ (1/120)

B1		Finding P(0, 1, 2, 3)
B1		1 or 2 correct
B1		3 correct
B1	[4]	All correct

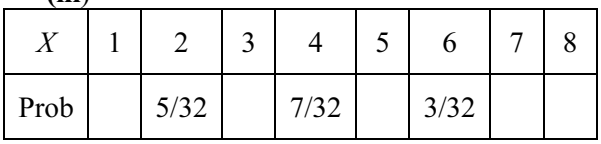
30	**(i** P(9) = P(1,4,4) × 3 + P(2,3,4) × 6 + P(3,3,3)	M1 M1		Listing at least 2 different options Multiplying P(4,3,2) by 6 or P(1,4,4) by 3
	= 10/64 (5/32) (0.156) AG	A1	[3]	Correct answer must see numerical justification
	(ii) probs 1/64, 3/64, 6/64, 10/64, 12/64, 12/64, 10/64, 6/64, 3/64, 1/64.	B1 B1 B1	[3]	3 or more additional correct probs 5 or more correct All correct
	(iii) P(S) = 6/64(3/32)	M1 A1		An attempt at P(S) 4,4,1 or 4,2,2 Correct P(S)
	P($R \cap S$) = 3/64, ≠ 15/1024 ie P(R) ×P(S)	B1		Correct P($R \cap S$) in either intersection or cond prob cases
	OR $P(R\|S) = \frac{3/64}{6/64} = 1/2, \neq 10/64$ ie P(R)	M1		comparing their P($R \cap S$) with their P(R) × P(S) or their $P(R\|S)$ with their P(R) need numerical vals
	Not independent	A1ft	[5]	correct conclusion ft wrong P(S) or P($R \cap S$) only

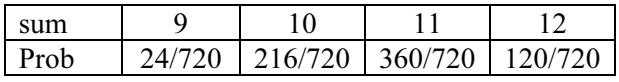
31	$-3p + 2r + 4 \times 0.4 = 2.3$	B1		Correct unsimplified equation, oe
	$(-3)^2p + 2^2r + 4^2 \times 0.4 - 2.3^2 = 3.01$	B1		Correct unsimplified equation, oe
	$p + q + r + 0.4 = 1$	B1		Correct equation, oe
	$-3p + 2r = 0.7$			
	$9p + 4r = 1.9$			
	so $-9p + 6r = 2.1$ or $-6p + 4r = 1.4$			
	$4r + 6r = 1.9 + 2.1$ or $9p + 6p = 1.9 - 1.4$	M1		Obtain an equation in 1 unknown
	$r = \frac{2}{5}$ (0.4), $p = \frac{1}{30}$ (0.0333)	A1		One correct answer
	$q = 0.6 - 0.4 - 0.0333 = \frac{1}{6}$ (0.167)	A1	**6**	Remaining two answers correct

32	**(i)** $X \sim$ Bin (12, 0.2)	B1 B1 B1	**[3]**	Bin or B 12 0.2 or 1/5
	(ii) P (X = 3, 4, 5) = $0.2^3 0.8^9 {}_{12}C_3 + 0.2^4 0.8^8 {}_{12}C_4 + 0.2^5 0.8^7 {}_{12}C_5$	M1		Bin exprerssion with any p
	= 0.23622 + 0.13287 + 0.05315	A1ft		Correct unsimplified expression, their p
	= 0.422	A1	**[3]**	Correct answer
	(iii) P (X = 0) < 0.01	M1	**[3]**	Statement involving P(X = 0) and 0.01 can be implied
	$0.8^n < 0.01$	M1		Equn involving '0.8', 0.01 or 0.99
	$n = 21$	A1		Correct answer

33	**(i)**	$P(T,B) = \frac{5}{12} \times \frac{2}{10} = \frac{1}{12}$ (0.0833)	M1 A1	**[2]**	Mult their P(T) by 2/9 or 2/10 only Correct answer
	(ii)	$P(C_S \cap C_A) = \frac{7}{12} \times \frac{4}{10} = \frac{28}{120}$ (0.2333)	M1		Mult their P(C_S) by 3/9 or 4/10 seen as num or denom of a fraction
		$P(C_A) = \frac{7}{12} \times \frac{4}{10} + \frac{5}{12} \times \frac{3}{10} = \frac{43}{120}$ (0.3583)	M1		Summing 2 two-factor products to find P(C_A) seen anywhere
		$P(C_S \mid C_A) = \frac{P(C \cap C)}{P(C_A)} = \frac{28/120}{43/120}$	A1		Correct unsimplified P(C_A) seen as num or denom of a fraction
		$= \frac{28}{43}$ (0.651)	A1	**[4]**	Correct answer
	(iii)				
		x | 0 | 1 | 2 Prob | 7 | 19/40 | 7/30	B1		x = 0, 1, 2, can be implied from table or working
		$P(X = 0) = P(T, B) + P(T, T)$	M1		1 or 2 two-factor products, denoms 12 and 10 or 12 and 9, implied if ans is correct
		$= \frac{5}{12} \times \frac{2}{10} + \frac{5}{12} \times \frac{5}{10} = \frac{7}{24}$ (0.292)	A1		One correct unsimplified
		$P(X = 2) = P(C, C) = \frac{7}{12} \times \frac{4}{10} = \frac{28}{120}$ (0.233)	B1		One other correct unsimplified
		$P(X = 1) = 1 - 7/24 - 28/120 = \frac{19}{40}$ (0.475)	B1ft	**[5]**	Third correct ft 1 – P(2 of their probs))

34	P(at least 2) = P(2, 3) or 1 – P(0, 1)	M1		Summing, or 1–, two different three-factor prob expressions, ${}_3C_2$ not needed
	$= \frac{5}{12} \times \frac{4}{11} \times \frac{7}{10} \times {}_3C_2 + \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$	M1		12, 11, 10 seen or implied in denominator
		M1		Mult a prob by ${}_3C_2$ or ${}_3C_1$ oe
	$= \frac{4}{11}$ (0.364)	A1	**[4]**	Correct answer
	OR $\frac{({}_5C_3) + ({}_5C_2 \times {}_7C_1)}{{}_{12}C_3}$	M1		${}_5C_3$ seen added in numerator
		M1		${}_5C_2$ seen mult alone or in numerator
		M1		${}_{12}C_3$ seen in denom
		A1		Correct answer

35	**(i)**	if throw H then smallest score is 2 P(T, 1) = 1/2 × 1/4 = 1/8 **AG**	B1 B1 **2**	Or equivalent
	(ii)	P(3) from two dice = 2/16 seen P(H, 3) = 1/2 × 2/16 = 2/32 P(T, 3) = 1/2 × 1/4 = 1/8 So P(3) = 6/32 = 3/16 **AG**	B1 M1 A1 A1 **4**	From (1, 2) and (2, 1) Summing P(H, 3) and P(T, 3) One correct Correct answer must see clear reasoning
	(iii)	 *X* 1 2 3 4 5 6 7 8 Prob 5/32 7/32 3/32	B1 B1 B1 **3**	One correct prob A second correct prob A third correct prob
	(iv)	P($Q \cap R$) = 0 or 'if you throw a tail you can't get a 7' Yes they are exclusive	M1 A1dep **2**	Stating P($Q \cap R$) = 0 or implying by words Dep on previous M

36	**(i)**	options (3, 4, 4,) or (4, 3, 4) or (4, 4, 3) Probs (4/10 × 6/9 × 5/8) ×3C1 = 360/720 = ½ AG	M1 M1 A1	**[3]**	Summing three 3-factor options oe 10 × 9 × 8 seen in denom Correct answer
		OR $\frac{{}_6C_2 \times {}_4C_1}{{}_{10}C_3} = \frac{1}{2}$ AG	M1 M1 A1		One of 6C2 or 4C1 seen in num 10C3 in denom Correct answer
	(ii)	 sum 9 10 11 12 Prob 24/720 216/720 360/720 120/720	B1 B1	**[4]**	9, 10, 11, 12 only seen One correct prob other than P(11), with or without replacement
		P(3, 3, 3) = 4/10 × 3/9 × 2/8 = 24/720 (1/30) P(3, 3, 4) = 4/10 × 3/9 × 6/8 × 3C1 = 216/720 (3/10) P(4, 4, 4) = 6/10 × 5/9 × 4/8 = 120/720(1/6)	B1 B1		Another correct prob Σ all 4 probs = 1
	(iii)	P(R) = 0.5 P(S) = 0.4 P($R \cap S$) = 120/720 P($R \cap S$) = 120/720 ≠ P(R) × P(S) Not indep	B1 M1 A1ft	**[3]**	P($R \cap S$) = 120/720 (1/6) Numerical attempt to compare P(R and S) with P(R) × P(S) provided P($R \cap S$) ≠ 1/5 Correct conclusion ft wrong P($R \cap S$) ≠ 1/5, P(S) correct
	(iv)	P($R \cap S$) ≠ 0 or there is an overlap between R and S (34,4) Not exclusive Σxf/Σf	B1ft	**[1]**	Correct answer following correct reasoning ft wrong non zero P($R \cap S$)

37 **(i**	P(same) = P(1, 1)+ P(3, 3) + P(5, 5) $= \frac{2}{9}\times\frac{1}{8}+\frac{4}{9}\times\frac{3}{8}+\frac{3}{9}\times\frac{2}{8}$ = 5/18 (0.278) Alt. method: $\frac{2C2+4C2+3C2}{9C2}$ or $\frac{2\times1+3\times4+2\times3}{9C2\times2}$ *oe*	M1 M1 A1 **3**	Summing 3 two-factor options Multiplying terms by one less in the numerator or denominator Correct answer M1 for numerator, M1 for denominator, A1 correct answer
(ii)	$P(5,\overline{5}) + P(\overline{5},5)$ $= \frac{3}{9}\times\frac{6}{8}+\frac{6}{9}\times\frac{3}{8}=\frac{36}{72}$ = ½ or 0.5 Alt. method: $\frac{6C1\times3C1(\times2)}{9C2(\times2)}$ *oe*	M1 M1 A1 **3**	Mult 2 probs whose numerators sum to 9 o.e. Summing 2 options or mult by 2 (may be 4 options) Correct answer M1 for numerator, M1 for denominator, A1 correct answer
(iii)	$P(5\cap\overline{5}) = \frac{3}{9}\times\frac{6}{8}=\frac{1}{4}$ $P(\overline{5}) = \frac{1}{4}+\frac{6}{9}\times\frac{5}{8}$ = 48/72 = 0.6666 $P(5_1 \mid \overline{5}_2) = \frac{1/4}{48/72}$ = 3/8 = 0.375	M1 M1 A1 A1 **4**	Attempt at P(5 and not 5) seen as numerator or denominator of a fraction Attempt at P(not 5) sum of 2 two-factor terms seen anywhere Correct $P(\overline{5})$ as numerator or denominator in fraction Correct answer
(iv)	<table><tr><td>*x*</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(*X* = *x*)</td><td>5/12</td><td>1/2</td><td>1/12</td></tr></table> $P(0) = P(\overline{5},\overline{5}) = \frac{6}{9}\times\frac{5}{8}$ = 30/72 (5/12) (0.4166) P(1) = 0.5 from part (ii) P(2) = 6/72 (1/12) (0.0833) from part (i)	B1 B1 B1ft **3**	Values 0, 1, 2 seen in table with at least 1 prob Correct P(0) unsimplified If *x*=0,1,2(,3) ft Σp = 1, no –ve values, all probabilities <1

38	**(i)**	$P(\text{exactly } 2) = \frac{{}^6C_2}{{}^8C_4} = \frac{15}{70} = \frac{3}{14}$ AG OR $P(2) = \frac{6}{8}\times\frac{5}{7}\times\frac{2}{6}\times\frac{1}{5}\times{}^4C_2 = \frac{3}{14}$ AG	M1 A1 **2**	${}^6Cx$ / ${}^8Cx$ seen or ${}^4C_2$ mult by 4 fractions (last 2 can be implied) Answer legit obtained
	(ii)	x: 2, 3, 4 Prob: 3/14, 8/14, 3/14	B1 B1 B1√ **3**	2, 3, 4 only in top line one correct prob other than $P(2)$ third correct prob ft $\Sigma = 1$
	(iii)	$\text{Var}(X) = \frac{12}{14}+\frac{72}{14}+\frac{48}{14} - 3^2$ $= \frac{3}{7}$ (0.429)	M1 A1 **2**	using $\Sigma x^2p - 3^2$ (or their $\{E(X)\}^2$) must be evaluated correct answer

39	**(i)**	A:P(H) = 2/3, P(T) = 1/3 B: P(H) = ¼, P(T) = 3/4 P(1H) = P(HTT) + P(THT) + P(TTH) = (2/3 × 1/3 × 3/4) + (1/3 × 2/3 × 3/4) + (1/3 × 1/3 × 1/4) = 13/36 **AG**	M1 M1 A1 **3**	Using some of 2/3, 1/3, ¼ or 3/4 in a calculation involving prod of 3 probs Summing 3 options not all the same Correct answer
	(ii)	x: 0, 1, 2, 3 P: 3/36, 13/36, 16/36, 4/36 P(0H) = P(TTT) = 1/3 × 1/3 × 3/4 = 1/12 P(2H) = P(HHT) + P(HTH) + P(THH) = (2/3 × 2/3 × 3/4) + (2/3 × 1/3 × 1/4) + (1/3 × 2/3 × 1/4) = 4/9 not 2/3 × 2/3 P(3H) = P(HHH) = 2/3 × 2/3 × 1/4 = 1/9	B1 B1 B1 B1√ **4**	0, 1, 2, 3 seen for table no probs needed, table not absolutely necessary if calcs shown One prob correct other than (i) condone 0.083 for 0.0833 A second prob correct need 3 factors can be implied A third prob correct ft 23/36 – Σ their 2 probs
	(iii)	E(X) = 13/36 + 32/36 + 12/36 = 57/36 (19/12) (1.58)	M1 A1 **2**	Attempt to evaluate Σxp at least 3 vals of x in table Correct answer

40	**(i)**	$P(2) = {}^6C_3 \times {}^3C_2/{}^9C_5$ **OR** $\frac{{}^6C_3\times{}^3C_2}{{}^6C_5+{}^6C_4\times{}^3C_1+{}^6C_3\times{}^3C_2+{}^6C_2\times{}^3C_3}$ **OR** 3/9 × 2/8 × 6/7 × 5/6 × 4/5 × ${}^5C_2$ = 10/21 = 60/126 **AG**	M1 **OR** M1 **OR** M1 A1 **2**	Using combinations ${}^aC_b \times {}^cC_d/{}^eC_f$ Mult 5 probs with a ${}^pC_q$ If ${}^5C_2$ replace by 10, oe must be justified Legit method, as answer given

(ii)

x	0	1	2	3
Prob	2/42	15/42	20/42	5/42

B1 — 0, 1, 2, 3 only seen in table. Condone x = 4,5 in table if P(x) = 0 or blank and values in table for x = 0,1,2,3

$P(0) = {}^6C_5/{}^9C_5 = 6/126$
$P(1) = {}^6C_4 \times {}^3C_1/{}^9C_5 = 45/126$
$P(3) = {}^6C_2 \times {}^3C_3/126 = 15/126$

B1 — Any correct prob other than P(2)
B1 — Any other correct prob
B1√ **4** — $\Sigma P(x) = 1$, $3<n(x)<6$

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41 **(i** $0.24 + 0.35 + 2k + k + 0.05 = 1$
$k = 0.12$

M1 — Summing probs = 1
A1 **2** — Correct answer

(ii) model number is 1

B1 **1**

(iii) mean = $1 \times 0.35 + 2 \times 0.24 + 3 \times 0.12 + 4 \times 0.05$
P(>1.39) = P(2, 3, 4) = 0.41

B1 — 1.39 seen
M1 — Finding P(X > their mean)
B1 **3** — Correct ans following mean or mode only

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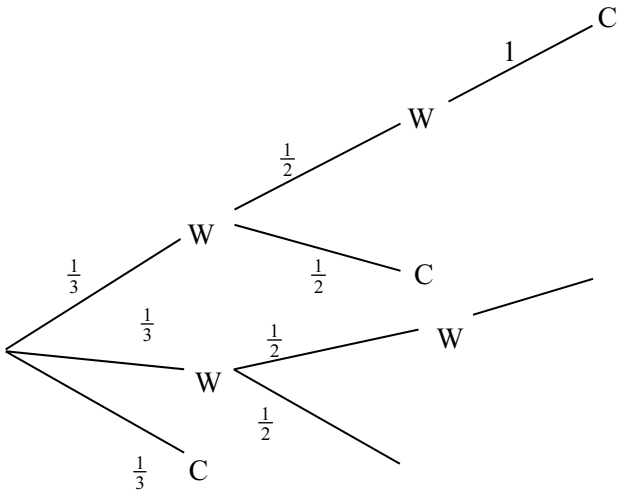
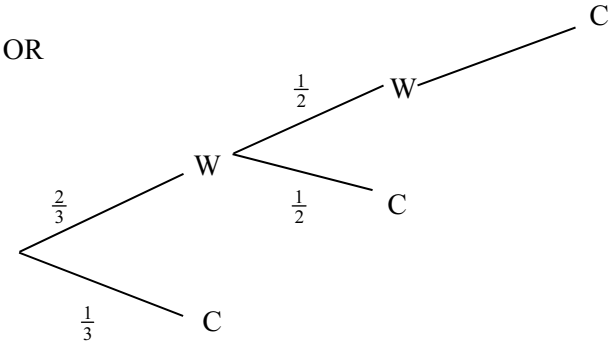
42 **(i** W = wrong, C = correct



M1 — 3 branches first qn and 2 by 2 for second qn only
M1 — One branch twice for third qn or two branches twice with 0 and 1 seen on branches
B1 — Any two of $\frac{1}{3}$, $\frac{1}{2}$ and 1 seen as probs
A1 **4** — Probs all correct and sensible labels NB SR for 4 outcomes instead of 3, M1 B1 only

OR



M1 — 2 branches first qn and 1 by 2 for second qn only
M1 — One branch once for third qn or two branches with 0 and 1 seen on branches
B1 — Any two of $\frac{1}{3}$ or $\frac{2}{3}$, $\frac{1}{2}$ and 1 seen as probs
A1 — Probs all correct and sensible labels

(ii)

x	1	2	3
Prob	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

B1 1, 2, 3 seen only oe

B1 2 correct probs

P(1) = P(C) say $= \frac{1}{3}$

P(2) = P(WC) = $\frac{1}{6}$ P(WC) = $\frac{1}{6}$ total P (2) $= \frac{1}{3}$

P(3) = P(WWC) = $\frac{1}{6}$ P(WWC) = $\frac{1}{6}$ total P(3) = $\frac{1}{3}$

B1 3 correct probs

$E(X) = 1 \times \frac{1}{3} + 2 \times \frac{1}{3} + 3 \times \frac{1}{3} = 2$

B1√ **4** Correct answer ft their probs provided $0.999 \leqslant \Sigma p \leqslant 1$

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43 **(i** max = 12 — B1 (Implied by P(12) with power 12)

$P(12) = (0.7)^{12} = 0.0138$ — B1 **2** Accept 0.014

(ii) P(fewer than 10) = 1– P (10, 11, 12) — M1 Binomial term ${}^{12}C_r(0.7)^r(0.3)^{12-r}$ or ${}^{12}C_r(p)^r(q)^{12-r}$, $0.99 \leqslant p + q \leqslant 1.00$

$= 1 - {}^{12}C_{10} \times (0.7)^{10}(0.3)^2 - 12 \times (0.7)^{11}(0.3) - (0.7^{12}$ — A1 Correct unsimplified expression oe

= 1 – 0.2528

= 0.747 — A1 **3** Correct answer

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44 **(a) (i)** $\frac{9!}{2!2!3!}$ = 15120 ways — B1 Dividing by 2!2!3!; B1 **[2]** Correct answer

(ii)

********3 in $\frac{8!}{2!2!3!}$ = 1680 ways — B1 Correct ways end in 3

********7 in $\frac{8!}{2!3!}$ = 3360 ways — B1 Correct ways end in 7

Total even

= 15120 – 1680 – 3360 — M1 Finding odd and subt from 15120 or their **(i)**

= 10080 ways — A1 **[4]** Correct answer

OR

********2 in 8!/2!3! = 3360 ways — B1 One correct way end in even

********6 in 8!/2!2!3! = 1680 ways — B1 correct way end in another even

********8 in 8!/2!2!2! = 5040ways — M1 Summing 2 or 3 ways

Total = 10080 ways — A1 Correct answer

OR

"15120" ×6/9 = 10080 — M2 Mult their **(i)** by 2/3 oe; A2 Correct answer

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(b) T(3) S(6) G(14)

1 1 3 in $3 \times 6 \times {}^{14}C_3 = 6552$ — M1 Mult 3 (combinations) together assume $6 = {}^6C_1$etc

1 3 1 in $3 \times {}^6C_3 \times 14 = 840$

3 1 1 in $1 \times 6 \times 14 = 84$ — M1 Listing at least 4 different options

2 2 1 in ${}^3C_2 \times {}^6C_2 \times 14 = 630$ — M1 Summing at least 4 different options

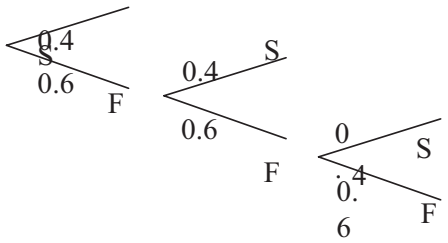
2 1 2 in ${}^3C_2 \times 6 \times {}^{14}C_2 = 1638$

1 2 2 in $3 \times {}^6C_2 \times {}^{14}C_2 = 4095$ — B1 At least 3 correct numerical options

Total ways = 13839 (13800) — A1 **[5]** Correct answer

45 **(i)**	P(2Es 1O) = $\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times{}^3C_2 = \frac{3}{5}$ (0.6) OR P(2Es 1O) = $\frac{{}^3C_2\times{}^2C_1}{{}^5C_3} = \frac{6}{10}$ = 0.6 OR 241, 247, 261, 267, 461, 467 = 6 options 124 126 127 146 147 167 246 247 267 467 Prob = 6/10	M1 M1 A1 **3** M1 M1 A1 M1 M1 A1	5×4×3 seen in denom Mult a prob by ${}^3C_2$ oe Correct answer ${}^3C_x$ or ${}^yC_2$ or ${}^2C_1$ oe seen mult by $k \geqslant 1$ in num ${}^5C_3$ seen in denom Correct answer List at least 3 of 241, 247, 261, 267, 461, 467 ${}^5C_3$ or list to get all 10 options in denom see below Correct answer
(ii)	**124** 12 **246** 247 s | 1 | 2 | 4 P(S =s) | 6/10 | 3/10 | 1/10	M1 A1 B1 B1 B1 **5**	Attempt at listing with at least 7 correct All correct and no others or all 60 1, 2, 4 only seen in top row Any two correct All correct

46 **(i)**	P(1 W) = 6/9×3/8 + 3/9×6/8 = ½ AG OR $\frac{{}^6C_1\times{}^3C_1}{{}^9C_2}$ = ½ AG	M1 A1 **[2]** M1 A1	summing 2 two-factor probs (condone replacement) not ½×½ + ½×½ Correct answer, fully justified Using combinations consistent, correct format Correct answer, fully justified
(ii)	P$(\overline{W},\overline{W})$= 3/9×2/8 = 6/72 (1/12) P(W,W)= 6/9×5/8 = 30/72 (5/12) x | 0 | 1 | 2 Prob | 1/12 | 1/2 | 5/12	B1 B1 B1√ **[3]**	Distribution table with 0,1,2 only P(W,W) or P$(\overline{W},\overline{W})$ correct P(W,W) + P$(\overline{W},\overline{W})$= 0.5
(iii)	E(X) = 16/12 (4/3) (1.33) isw	B1 **[1]**	Condone 1(.3) if correct working seen, nfww

47 **(i)**		**M1** **A1** **A1** [3]	3 pairs S (bank, log in, success oe) and F oe seen no extra bits. Exactly 3 pairs, must be labelled Correct diagram with all probs correct
(ii)	x | 0 | 1 | 2 | 3 Prob | 0.4 | | 0.144 | 0.216	**B1** **M1** **A1** **B1** [4]	P(0) correct Multiplying two of more factors of 0.4 and 0.6 One more correct prob One more correct prob
(iii)	E(X) = 0.24+ 2× 0.144 + 3× 0.216 = 1.176 (1.18)	**M1** **A1** [2]	Using $\Sigma p_i x_i$ Correct answer

48 **(i)**

Spinner *A*

Spinner *B* \ Spinner *A*	1	2	3	3
–3	(–2)	–1	0	0
–2	–1	0	(1)	1
–1	0	1	2	2
1	2	3	4	4

B1 1

(ii)

x	–2	–1	0	1	2	3	4
prob	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{2}{16}$

M1 Their values in (i) as the top line, seen listed in (ii) or used in part (iii)

M1 Attempt at probs seen evaluated, need at least 4 correct from their table

A1 3 Correct table seen

(iii)

$E(X) = 1$

$\text{Var}(X) = ((-2)^2 + 2 + 3 + 12 + 9 + 32)/16 - 1^2$

$= \frac{62}{16} - 1$

$= \left(\frac{23}{8}\right)(2.875)$

OR using $\Sigma p(x - \bar{x})^2 = (9 + 8 + 4 + 0 + 3 + 4 + 18)/16$

$= \frac{46}{16} = 2.875$

M1 Attempt at $E(X)$ from their table if $\Sigma p = 1$

M1 Evaluating $\Sigma x^2 p$ – [their $E(X)$]^2 allow $\Sigma p \neq 1$ but all p's <1

A1 3 Correct answer

M1

M1

A1

(iv)

P(even given +ve)

$= \frac{5}{9}$

OR P(even given +ve) $= \frac{\left(\frac{5}{16}\right)}{\left(\frac{9}{16}\right)}$

$= \frac{5}{9}(0.556)$

M1 Counting their even numbers and dividing by their positive numbers

A1 2 Correct answer

M1 Using cond prob formula not P(E) × P(+ve) need fraction over fraction accept any of $\frac{5/16 \text{ or } 6/16 \text{ or } 9/16}{9/16 \text{ or } 10/16 \text{ or } 13/16}$

A1 Correct answer