

T3 TRIG IDENTITY QUESTIONS

- 1 (i) Prove the identity

$$\sin^2 \theta \cos^2 \theta \equiv \frac{1}{8}(1 - \cos 4\theta). \quad [3]$$

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- 2 (i) Prove the identity

$$\cos 4\theta + 4 \cos 2\theta \equiv 8 \cos^4 \theta - 3. \quad [4]$$

- (ii) Hence solve the equation

$$\cos 4\theta + 4 \cos 2\theta = 2,$$

$$\text{for } 0^\circ \leq \theta \leq 360^\circ. \quad [4]$$

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- 3 (i) Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot \theta.$ [3]

- (ii) Hence solve the equation $\operatorname{cosec} 2\theta + \cot 2\theta = 2$, for $0^\circ \leq \theta \leq 360^\circ.$ [2]

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- 4 (i) Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\frac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x. \quad [3]$$

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- 5 (i) Prove the identity $\cos 4\theta + 4 \cos 2\theta \equiv 8 \cos^4 \theta - 3.$ [4]

- (ii) Hence

(a) solve the equation $\cos 4\theta + 4 \cos 2\theta = 1$ for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi,$ [3]

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- 6 (i) By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \quad [4]$$

- (ii) Show that, after making the substitution $x = \frac{2 \sin \theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}.$ [1]

- (iii) Hence solve the equation

$$x^3 - x + \frac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

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7 (i) Prove the identity $\cos 4\theta - 4 \cos 2\theta \equiv 8 \sin^4 \theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4 \cos 2\theta + 3,$$

for $0^\circ \leq \theta \leq 360^\circ$.

[4]

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8 (i) Prove the identity $\tan 2\theta - \tan \theta \equiv \tan \theta \sec 2\theta$. [4]

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9 Prove the identity $\frac{1 + \sin \theta}{1 - \sin \theta} \equiv 2 \sec^2 \theta + 2 \sec \theta \tan \theta - 1$. [3]

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10 Prove the identity $\frac{\cot x - \tan x}{\cot x + \tan x} \equiv \cos 2x$. [3]

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11 Prove the identity $\tan(45^\circ + x) + \tan(45^\circ - x) \equiv 2 \sec 2x$. [4]

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12 Show that $\frac{2 \sin x - \sin 2x}{1 - \cos 2x} \equiv \frac{\sin x}{1 + \cos x}$. [4]

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13 (i) By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \frac{3}{4} \sin^2 2x. \quad [4]$$

(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \frac{2}{3},$$

for $0^\circ < x < 180^\circ$.

[4]

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14 By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [4]

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15 Let $f(\theta) = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

Show that $f(\theta) = \tan \theta$.

[3]

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16 By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$.

[4]

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