C5 With Polynomial Answers
P3

1	**(i)**	Attempt to solve for m the equation $p(-2) = 0$ or equivalent	M1	
		Obtain $m = 6$	A1	[2]
		Alternative:		
		Attempt $p(z) \div (z + 2)$, equate a constant remainder to zero and solve for m.	M1	
		Obtain $m = 6$	A1	

(ii)	**(a)**	State $z = -2$	B1	
		Attempt to find quadratic factor by inspection, division, identity, …	M1	
		Obtain $z^2 + 4z + 16$	A1	
		Use correct method to solve a 3-term quadratic equation	M1	
		Obtain $-2 \pm 2\sqrt{3}i$ or equivalent	A1	[5]
	(b)	State or imply that square roots of answers from part **(ii)(a)** needed	M1	
		Obtain $\pm i\sqrt{2}$	A1	
		Attempt to find square root of a further root in the form $x + iy$ or in polar form	M1	
		Obtain $a^2 - b^2 = -2$ and $ab = (\pm)\sqrt{3}$ following their answer to part **(ii)(a)**	A1√	
		Solve for a and b	M1	
		Obtain $\pm(1 + i\sqrt{3})$ and $\pm(1 - i\sqrt{3})$	A1	[6]

2	**(i)**	Substitute $x = 2$ and equate to zero, or divide by $x - 2$ and equate constant remainder to zero, or equivalent	M1	
		Obtain $a = 4$	A1	[2]

(ii)	**(a)**	Find further (quadratic or linear) factor by division, inspection or factor theorem or equivalent	M1	
		Obtain $x^2 + 2x - 8$ or $x + 4$	A1	
		State $(x - 2)^2(x + 4)$ or equivalent	A1	[3]
	(b)	State any two of the four (or six) roots	B1√	
		State all roots ($\pm\sqrt{2}$, $\pm 2i$), provided two are purely imaginary	B1√	[2]

3 (i)	*EITHER*	Substitute $x = 1 + \sqrt{2}\,i$ and attempt the expansions of the x^2 and x^4 terms	M1	
		Use $i^2 = -1$ correctly at least once	B1	
		Complete the verification	A1	
		State second root $1 - \sqrt{2}\,i$	B1	
	OR 1	State second root $1 - \sqrt{2}\,i$	B1	
		Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,i$	M1	
		Obtain $x^2 - 2x + 3$, or equivalent	A1	
		Show that the division of p(x) by $x^2 - 2x + 3$ gives zero remainder and complete the verification	A1	
	OR 2	Substitute $x = 1 + \sqrt{2}\,i$ and use correct method to express x^2 and x^4 in polar form	M1	
		Obtain x^2 and x^4 in any correct polar form (allow decimals here)	B1	
		Complete an exact verification	A1	
		State second root $1 - \sqrt{2}\,i$, or its polar equivalent (allow decimals here)	B1	[4]

(ii)	Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,i$	M1*	
	Obtain $x^2 - 2x + 3$, or equivalent	A1	
	Attempt division of p(x) by $x^2 - 2x + 3$ reaching a partial quotient $x^2 + kx$, or equivalent	M1 (dep*)	
	Obtain quadratic factor $x^2 - 2x + 2$	A1	
	Find the zeros of the second quadratic factor, using $i^2 = -1$	M1 (dep*)	
	Obtain roots $-1 + i$ and $-1 - i$	A1	[6]

[The second M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + (6/3)$ and an equation in A and/or B]
[If part **(i)** is attempted by the *OR 1* method, then an attempt at part **(ii)** which uses or quotes relevant working or results obtained in part **(i)** should be marked using the scheme for part **(ii)**]

(i)	State answer $-1-\sqrt{3}i$	B1	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

4 (i)	Substitute -2 and equate to zero or divide by $x + 2$ and equate remainder to zero or use -2 in synthetic division	M1	
	Obtain $a = -1$	A1	[2]
(ii)	Attempt to find quadratic factor by division reaching $x^2 + kx$, or inspection as far as $(x+2)(x^2+Bx+c)$ and equations for one or both of B and C, or $(x+2)(Ax^2+Bx+7)$ and equations for one or both of A and B.	M1	
	Obtain $x^2 - 3x + 7$	A1	
	Use discriminant to obtain -19, or equivalent, and **confirm one root** cwo	A1	[3]

5 (i)	Commence division by $x^2 - x + 2$ and reach a partial quotient $4x^2 + kx$	**M1**	
	Obtain quotient $4x^2 + 4x + a - 4$ or $4x^2 + 4x + b/2$	**A1**	
	Equate x or constant term to zero and solve for a or b	**M1**	
	Obtain $a = 1$	**A1**	
	Obtain $b = -6$	**A1**	[5]
(ii)	Show that $x^2 - x + 2 = 0$ has no real roots	**B1**	
	Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2 + 4x - 3 = 0$	**B1**	[2]

Question	Answer	Marks	Guidance
6 (i)	State answer $-1-\sqrt{3}\mathrm{i}$	B1	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

Question	Answer	Marks	Guidance
(ii)	Substitute $x=-1+\sqrt{3}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2=-1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $k=2$	A1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$	M1	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain x^2+2x+4	A1	Using factor theorem, obtain $\mathrm{f}\left(-\frac{1}{2}\right)=0$
	Obtain root $x=-\frac{1}{2}$, or equivalent, *via* division or inspection	A1	Final answer

Question	Answer	Marks	Guidance
(ii)	**Alternative method 1**		
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$ (multiplying two linear factors or using sum and product of roots)	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2=-1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain x^2+2x+4	A1	Allow M1A0 for x^2+2x+3
	Obtain linear factor $kx+1$ and compare coefficients of x or x^2 and solve for k	M1	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k=2$	A1	
	Obtain root $x=-\frac{1}{2}$	A1	Final answer
			Note: Verification that $x=-\frac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained

Question	Answer	Marks	Guidance
(ii)	**Alternative method 2**		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	M1	
	Use $\mathrm{i}^2=-1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k}=-2+\gamma$, $\quad -\frac{4}{k}=4\gamma$	A1	
	Solve simultaneous equations for k and γ	M1	
	Obtain $k=2$	A1	
	Obtain root $\gamma=-\frac{1}{2}$	A1	Final answer
		6	