D2 PARAMETRIC MARKING SCHEME

1 State that $\frac{dx}{d\theta} = 2 + 2\cos 2\theta$ or $\frac{dy}{d\theta} = 2\sin 2\theta$ B1

Use $\frac{dy}{dx} = \frac{dy}{d\theta} + \frac{dx}{d\theta}$ M1

Obtain answer in any correct form, e.g. $\frac{2\sin 2\theta}{2 + 2\cos 2\theta}$ A1

Make relevant use of $\sin 2A$ and $\cos 2A$ formulae M1

Obtain given answer correctly A1 **5**

2 State or imply $\frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$ or $\frac{dy}{d\theta} = 2a\sin 2\theta$ B1

Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ M1

Obtain $\frac{dy}{dx} = \frac{\sin 2\theta}{(1 - \cos 2\theta)}$, or equivalent A1

Make use of correct $\sin 2A$ and $\cos 2A$ formulae M1

Obtain the given result following sufficient working A1 [5]

[SR: An attempt which assumes a is the parameter and θ a constant can only earn the two M marks. One that assumes θ is the parameter and a is a function of θ can earn B1M1A0M1A0.]

[SR: For an attempt that gives a a value, e.g. 1, or ignores a, give B0 but allow the remaining marks.]

3 **(i) EITHER** State $\frac{dx}{dt} = -3a\cos^2 t\sin t$ or $\frac{dy}{dt} = 3a\sin^2 t\cos t$, or equivalent B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

OR State $\frac{2}{3}x^{-\frac{1}{3}}dx$ or $\frac{2}{3}y^{-\frac{1}{3}}dy$ as differentials of $x^{\frac{2}{3}}$ or $y^{\frac{2}{3}}$ respectively, or equivalent B1

Obtain $\frac{dy}{dx}$ in terms of t, having taken the differential of a constant to be zero M1

Obtain $\frac{dy}{dx}$ in any correct form A1 **3**

(ii) Form the equation of the tangent M1

Obtain the equation in any correct form A1

Obtain the given answer A1 **3**

(iii) State the x-coordinate of X or the y-coordinate of Y in any correct form B1

Obtain the given answer with no errors seen B1 **2**

4 Use of correct quotient or product rule to differentiate x or t — M1

Obtain correct $\frac{3}{(2t+3)^2}$ or unsimplified equivalent — A1

Obtain $-2e^{-2t}$ for derivative of y — B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$ or equivalent — M1

Obtain –6 — cwo A1 [5]

Alternative:

Eliminate parameter and attempt differentiation $\left(y = e^{\frac{-6x}{1\ 2x}} \right)$ — B1

Use correct quotient or product rule — M1

Use chain rule — M1

Obtain $\frac{dy}{dx} = \frac{-6}{(1-2x)^2} e^{\frac{-6x}{1-2x}}$ — A1

Obtain –6 — cwo A1

5 **(i)** *EITHER*: State $\frac{dx}{dt} = \sec^2 t / \tan t$, or equivalent — B1

State $\frac{dy}{dt} = 2\sin t \cos t$, or equivalent — B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ — M1

Obtain correct answer in any form, e.g. $2\sin^2 t \cos^2 t$ — A1

OR: Obtain $y = e^{2x} / (1 + e^{2x})$, or equivalent — B1

Use correct quotient or product rule — M1

Obtain correct derivative in any form, e.g. $2e^{2x} / (1 + e^{2x})^2$ — A1

Obtain correct derivative in terms of t in any form, e.g. $(2\tan^2 t) / (1 + \tan^2 t)^2$ — A1 [4]

(ii) State or imply $t = \frac{1}{4}\pi$ when $x = 0$ — B1

Form the equation of the tangent at $x = 0$ — M1

Obtain correct answer in any horizontal form, e.g. $y = \frac{1}{2}x + \frac{1}{2}$ — A1 [3]

[SR: If the *OR* method is used in part **(i)**, give B1 for stating or implying $y = \frac{1}{2}$ or $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.]

6	*EITHER*: Use chain rule	M1	
	obtain $\frac{dx}{dt} = 6\sin t\cos t$, or equivalent	A1	
	obtain $\frac{dy}{dt} = -6\cos^2 t\sin t$, or equivalent	A1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain final answer $\frac{dy}{dx} = -\cos t$	A1	
	OR: Express y in terms of x and use chain rule	M1	
	Obtain $\frac{dy}{dx} = k(2 - \frac{x}{3})^{\frac{1}{2}}$, or equivalent	A1	
	Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$, or equivalent	A1	
	Express derivative in terms of t	M1	
	Obtain final answer $\frac{dy}{dx} = -\cos t$	A1	[5]

7	**(i)**	Differentiate y to obtain $3\sin^2 t\cos t - 3\cos^2 t\sin t$ o.e.	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dt}{dx}$	M1	
		Obtain given result $-3\sin t\cos t$	A1cwo	[3]
	(ii)	Identify parameter at origin as $t = \frac{3}{4}\pi$	B1	
		Use $t = \frac{3}{4}\pi$ to obtain $\frac{3}{2}$	B1	[2]
	(iii)	Rewrite equation as equation in one trig variable e.g. $\sin 2t = -\frac{2}{3}$, $9\sin^4 x - 9\sin^2 x + 1 = 0$, $\tan^2 x + 3\tan x + 1 = 0$	B1	
		Find at least one value of t from equation of form $\sin 2t = k$ o.e.	M1	
		Obtain 1.9	A1	
		Obtain 2.8 and no others	A1	[4]

8 Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos 2\theta - 1$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta} = -2\sin 2\theta + 2\cos\theta$, or equivalent — B1

Use $\frac{\mathrm{d}y}{\mathrm{d}} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$ — M1

Obtain $\frac{\mathrm{d}y}{\mathrm{d}} = \frac{-2\sin 2\theta + 2\cos\theta}{2\cos 2\theta - 1}$, or equivalent — A1

At any stage use correct double angle formulae throughout — M1

Obtain the given answer following full and correct working — A1 [5]

9 **(i)** Either Use correct quotient rule or equivalent to obtain

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4(2t+3) - 8t}{(2t+3)^2}$ or equivalent — B1

Obtain $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{4}{2t+3}$ or equivalent — B1

Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \Big/ \frac{\mathrm{d}x}{\mathrm{d}t}$ or equivalent — M1

Obtain $\frac{1}{3}(2t+3)$ or similarly simplified equivalent — A1

Or Express t in terms of x or y e.g. $t = \frac{3x}{4-2x}$ — B1

Obtain Cartesian equation e.g. $y = 2\ln\left(\frac{6}{2-x}\right)$ — B1

Differentiate and obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{2-x}$ — M1

Obtain $\frac{1}{3}(2t+3)$ or similarly simplified equivalent — A1 [4]

(ii) Obtain $2t = 3$ or $t = \frac{3}{2}$ — B1

Substitute in expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and obtain 2 — B1 [2]

10 Use correct product or quotient rule at least once — M1*

Obtain $\frac{dx}{dt} = e^{t}\sin t - e^{t}\cos t$ or $\frac{dy}{dt} = e^{t}\cos t - e^{t}\sin t$, or equivalent — A1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ — M1

Obtain $\frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent — A1

EITHER: Express $\frac{dy}{dx}$ in terms of tan t only — M1(dep*)

Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$ — A1

OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t — M1

Show expression is identical to $\frac{dy}{dx}$ — A1 **[6]**

11 Obtain $\frac{2}{2t+3}$ for derivative of x — B1

Use quotient of product rule, or equivalent, for derivative of y — M1

Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent — A1

Obtain $t = -1$ — B1

Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form — M1

Obtain gradient $\frac{5}{2}$ — A1 [6]

12 (**i**) State $\frac{dx}{dt} = 1 - \sec^2 t$, or equivalent — B1

Use chain rule — M1

Obtain $\frac{dy}{dt} = -\frac{\sin t}{\cos t}$, or equivalent — A1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ — M1

Obtain the given answer correctly. — A1 **5**

(**ii**) State or imply $t = \tan^{-1}(\frac{1}{2})$ — B1

Obtain answer $x = -0.0364$ — B1 **2**

13	(i)	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
		Use $\frac{d}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(ii)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**

14	Use correct product rule or correct chain rule to differentiate y	M1	
	Use $\frac{dy}{dx} = \frac{\frac{d}{d\theta}}{\frac{d}{d\theta}}$	M*1	
	Obtain $\frac{-4\cos\theta\sin^2\theta + 2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
	Express $\frac{d}{dx}$ in terms of $\cos\theta$	DM*1	
	Confirm given answer $6\cos^5\theta - 4\cos^3\theta$ legitimately	A1	[5]

15	**(i)**	State $\frac{dx}{dt} = -4a\cos^3 t\sin t$, or $\frac{dy}{dt} = 4a\sin^3 t\cos t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	**3**
	(ii)	Form the equation of the tangent	M1	
		Obtain a correct equation in any form	A1	
		Obtain the given answer	A1	**3**
	(iii)	State the x-coordinate of P or the y-coordinate of Q in any form	B1	
		Obtain the given result correctly	B1	**2**

16	**(i)**	State $\frac{dx}{dt} = 1 - \sin t$	**B1**
		Use chain rule to find the derivative of y	**M1**
		Obtain $\frac{d}{dt} = \frac{\cos t}{1 + \sin t}$, or equivalent	**A1**
		Use $\frac{d}{dx} = \frac{d}{dt} \div \frac{dx}{dt}$	**M1**
		Obtain the given answer correctly	**A1**
			[5]
	(ii)	State or imply $t = \cos^{-1}(\frac{1}{3})$	**B1**
		Obtain answers $x = 1.56$ and $x = -0.898$	**B1 + B1**
			[3]

17(i)	Use chain rule to differentiate x $\left(\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\frac{\sin\theta}{\cos\theta}\right)$	**M1**
	State $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 3 - \sec^2\theta$	**B1**
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$	**M1**
	Obtain correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form e.g. $\frac{3 - \sec^2\theta}{-\tan\theta}$	**A1**
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\tan^2\theta - 2}{\tan\theta}$, or equivalent	**A1**
	Total:	**5**
17(ii)	Equate gradient to −1 and obtain an equation in $\tan\theta$	**M1**
	Solve a 3 term quadratic $\left(\tan^2\theta + \tan\theta - 2 = 0\right)$ in $\tan\theta$	**M1**
	Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only	**A1**
	Total:	**3**

18(i)	State $\frac{\mathrm{d}y}{\mathrm{d}t} = 4 + \frac{2}{2t-1}$	**B1**
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**
	Obtain answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2 - 2t}$	**A1**
	Total:	**3**
18(ii)	Use correct method to find the gradient of the normal at $t = 1$	**M1**
	Use a correct method to form an equation for the normal at $t = 1$	**M1**
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	**A1**
	Total:	**3**

19(i)	Obtain $\frac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**
	Use $dy/dx = dy/d\theta \div dx/d\theta$	**M1**
	Obtain correct $\frac{dy}{dx}$ in any form, e.g. $-\frac{2\sin \quad + 2\sin 2\theta}{2\cos \quad + 2\cos 2\theta}$	**A1**
		3
19(ii)	Equate denominator to zero and use any correct double angle formula	**M1***
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	**A1**
	Solve for θ	**depM1***
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	**A1**
		4

20	State $\frac{dx}{dt} = 2 + 2\cos 2t$	**B1**	
	Use the chain rule to find the derivative of y	**M1**	
	Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1 - \cos 2t}$	**A1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain $\frac{dy}{dx} = \operatorname{cosec} 2t$ correctly	**A1**	AG
		5	