# A LEVEL (P6) NORMAL AND BINOMIAL DISTRIBUTION

QUESTION'S

1 Melons are sold in three sizes: small, medium and large. The weights follow a normal distribution with mean 450 grams and standard deviation 120 grams. Melons weighing less than 350 grams are classifi d as small.

   (i) Find the proportion of melons which are classifi d as small.   [3]

   (ii) The rest of the melons are divided in equal proportions between medium and large. Find the weight above which melons are classif ed as large.   [5]

2 Ashopsellsoldvideotapes, of which1in 5onaverageare known tobedamaged.

   (i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged.   [3]

   (ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape.   [3]

   (iii) A random sample of 1600 tapes is taken. Use a suitable approximation to f nd the probability that there are at least 290 damaged tapes.   [5]

3 The length of Paulo's lunch break follows a normal distribution with mean μ minutes and standard deviation 5 minutes. On one day in four, on average, his lunch break lasts for more than 52 minutes.

   (i) Find the value of μ.   [3]

   (ii) Find the probability that Paulo's lunch break lasts for between 40 and 46 minutes on every one of the next four days.   [4]

4   **7i)** State two conditions which must be satisf ed for a situation to be modelled by a binomial distribution.   [2]

In a certain village 28% of all cars are made by Ford.

   (ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford.   [4]

   (iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford.   [4]

5 It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, fin the probability that fewer than 165 people in the sample are overweight.   [5]

6 Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

   (i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars.   [5]

   (ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits.   [3]

7 In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

  (i) Find the mean lifetime, giving your answer to the nearest hour. [3]

  (ii) Find the probability that a light bulb fails to last for 5000 hours. [3]

  (iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, fin the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

8 The lengths of fis of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the f sh are longer than 50 cm.

  (i) Find the standard deviation. [3]

  (ii) When fis are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fis rejected. [3]

  (iii) 9 f sh are chosen at random. Find the probability that at least one of them is longer than 50 cm. [2]

9 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

  (i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

  (ii) A random sample of 200 adults was taken. Using a suitable approximation, fin the probability that more than 155 wore a watch on their left wrist. [5]

10 **(i)** Give an example of a variable in real life which could be modelled by a normal distribution. [1]

  (ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, f nd the value of μ. [3]

  (iii) If 300 observations are taken at random from the distribution in part **(ii)**, estimate how many of these would be greater than 22.0. [4]

11 A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

  (i) equal numbers of large and small bands, [2]

  (ii) more than 17 small bands. [3]

  An off ce pack contains 150 elastic bands.

  (iii) Using a suitable approximation, calculate the probability that the number of small bands in the off ce pack is between 88 and 97 inclusive. [6]

12 (a) The random variable X is normally distributed. The mean is twice the standard deviation. It is given that P(X > 5.2)=0.9. Find the standard deviation. [4]

    (b) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$? [3]

13 The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

    (i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]

    (ii) 56 years are chosen randomly. Use a suitable approximation to f nd the probability that more than 7 of these years have New Year's Day on a Saturday. [5]

14
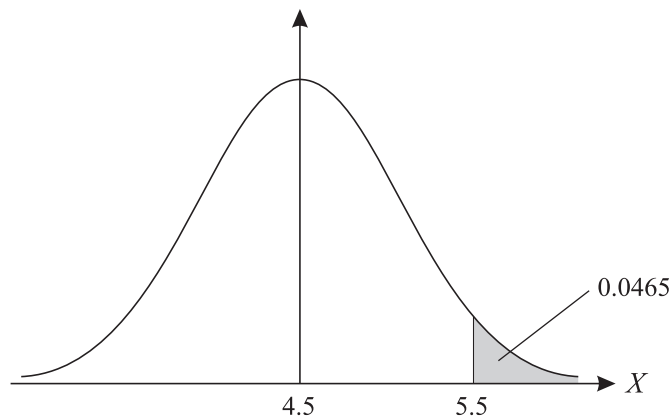
The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

    (i) Find the standard deviation of X. [3]

    (ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

15 On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

    (i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. [2]

    (ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions. [5]

    (iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8. [2]

16   In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

   (i) Find the value of μ.   [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

   (ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up.   [3]

17   A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

   **(i)** The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number.   [3]

   (ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5.   [3]

   (iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times.   [5]

18   On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to f nd the probability that there are at least 15 faulty toys.   [5]

19   **(i)** The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution N(−15.1, 62.0). Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above $0\,°C$.   [3]

(ii) In another city the daily minimum temperature in °C in January is a random variable with distribution N(μ, 40.0). In this city the probability that a randomly chosen day in January has a minimum temperature above $0\,°C$ is 0.8888. Find the value of μ.   [3]

20   The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

(i) Calculate the value of μ.   [3]

(ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres.   [2]

21   On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

(i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses.   [3]

(ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, f nd the probability that more than 73 are cars.   [5]

22   The mean number of defective batteries in packs of 20 is 1.6. Use a binomial distribution to calculate the probability that a randomly chosen pack of 20 will have more than 2 defective batteries.
                                                                                                                       [5]

23   A box contains 4 pears and 7 oranges. Three fruits are taken out at random and eaten. Find the probability that

(i) 2 pears and 1 orange are eaten, in any order,   [3]

(ii) the third fruit eaten is an orange,   [3]

(iii) the f rst fruit eaten was a pear, given that the third fruit eaten is an orange.   [3]

There are 121 similar boxes in a warehouse. One fruit is taken at random from each box.

(iv) Using a suitable approximation, f nd the probability that fewer than 39 are pears.   [5]

24

$$\text{39} \qquad\qquad \text{63} \qquad\qquad \text{wind speed (km h}^{-1})$$
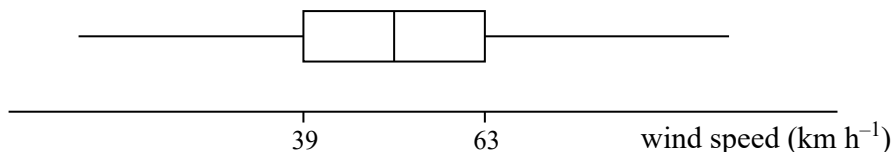
Measurements of wind speed on a certain island were taken over a period of one year. A box-and-whisker plot of the data obtained is displayed above, and the values of the quartiles are as shown. It is suggested that wind speed can be modelled approximately by a normal distribution with mean μ km h^{-1} and standard deviation σ km h^{-1}.

   (i) Estimate the value of μ.   [1]

   (ii) Estimate the value of σ.   [3]

25   Two unbiased tetrahedral dice each have four faces numbered 1, 2, 3 and 4. The two dice are thrown together and the sum of the numbers on the faces on which they land is noted. Find the expected number of occasions on which this sum is 7 or more when the dice are thrown together 200 times.   [4]

26   The random variable X is the length of time in minutes that Jannon takes to mend a bicycle puncture. X has a normal distribution with mean μ and variance σ^2. It is given that $P(X > 30.0) = 0.1480$ and $P(X > 20.9) = 0.6228$. Find μ and σ. [5]

27   In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

   (i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones.   [2]

   (ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to f nd the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones.   [1]

   (iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones.   [5]

28   The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

   (i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm.   [2]

   (ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm.   [3]

29 The random variable X is normally distributed with mean μ and standard deviation σ.

   (i) Given that $5\sigma = 3\mu$, f nd $P(X < 2\mu)$.   [3]

   (ii) With a different relationship between μ and σ, it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ.   [3]

30   A bottle of sweets contains 13 red sweets, 13 blue sweets, 13 green sweets and 13 yellow sweets. 7 sweets are selected at random. Find the probability that exactly 3 of them are red. [3]

31   The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

   (i) Find the mean and standard deviation of the heights the children can jump. [5]

   (ii) Find the probability that a randomly chosen child will **not** be able to jump a height of 145 cm. [3]

   (iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]

32   On average, 2 apples out of 15 are classif ed as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35. [5]

33   The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.

   (i) For a random sample of 350 students, f nd the number who would be expected to take longer than 20 minutes to get up in the morning. [3]

   (ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

34   The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are f lled with 20 litres of fuel and are driven towards a place 367 km away.

   (i) For each car, f nd the probability that it runs out of fuel before it has travelled 367 km. [3]

   (ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d. [4]

35   **(i)** State three conditions that must be satisf ed for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

   (ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

   (iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

36   Name the distribution and suggest suitable numerical parameters that you could use to model the weights in kilograms of female 18-year-old students.   [2]

37   The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.

   (i) Find the standard deviation of the times spent by people visiting this dentist.   [3]

   (ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute.   [3]

   (iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes.   [3]

   (iv) Find the probability that, of 35 randomly chosen people, fewer than 16 have visits lasting less than 8.2 minutes.   [5]

38   Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits.   [4]

39   **(a)** The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

   (b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

40   **(a)**   **(i)** Find the probability of getting at least one 3 when 9 fair dice are thrown.   [2]

   (ii) When n fair dice are thrown, the probability of getting at least one 3 is greater than 0.9. Find the smallest possible value of n.   [4]

   (b) A bag contains 5 green balls and 3 yellow balls. Ronnie and Julie play a game in which they take turns to draw a ball from the bag at random without replacement. The winner of the game is the f rst person to draw a yellow ball. Julie draws the f rst ball. Find the probability that Ronnie wins the game.   [4]

41   A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three.   [4]

42   In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

   (i) Use a normal approximation to f nd the probability of there being fewer than 25 cloudy days in Scotland in November (30 days).   [4]

   (ii) Give a reason why the use of a normal approximation is justif ed.   [1]

43 The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

(i) Find the value of μ. [3]

(ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

44 The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $P(X > 20) = 0.04$.

(i) Find μ. [3]

(ii) Find $P(10 < X < 20)$. [3]

(iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20. [5]

45 The probability that Sue completes a Sudoku puzzle correctly is 0.75.

(i) Sue attempts n Sudoku puzzles. Find the least value of n for which the probability that she completes all n puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by X.

(ii) Find the value of X that has the highest probability. You may assume that this value is one of the two values closest to the mean of X. [3]

(iii) Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]

46 When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to f nd the probability that more than 210 germinate. [5]

47 The weights of letters posted by a certain business are normally distributed with mean 20 g. It is found that the weights of 94% of the letters are within 12 g of the mean.

(i) Find the standard deviation of the weights of the letters. [3]

(ii) Find the probability that a randomly chosen letter weighs more than 13 g. [3]

(iii) Find the probability that at least 2 of a random sample of 7 letters have weights which are more than 12 g above the mean. [3]

48   A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

    (i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$.   [1]

    (ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time.   [3]

    (iii) The spinner is spun 136 times. Use a suitable approximation to f nd the probability that it lands on the blue side fewer than 20 times.   [5]

49   **(i)** In a certain country, the daily minimum temperature, in °C, in winter has the distribution N(8, 24). Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7 °C and 12 °C.   [3]

The daily minimum temperature, in °C, in another country in winter has a normal distribution with mean μ and standard deviation 2μ.

    (ii) Find the proportion of winter days on which the minimum temperature is below zero.   [2]

    (iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean.   [3]

    (iv) The probability of the minimum temperature being above 6 °C on any winter day is 0.0735. Find the value of μ.   [3]

50   The random variable X is normally distributed and is such that the mean μ is three times the standard deviation σ. It is given that $P(X < 25) = 0.648$.

    (i) Find the values of μ and σ.   [4]

    (ii) Find the probability that, from 6 random values of X, exactly 4 are greater than 25.   [2]

51   Human blood groups are identif ed by two parts. The f rst part is A, B, AB or O and the second part (the Rhesus part) is + or −. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A−, 2% are B−, 1% are AB− and 7% are O−.

    (i) A random sample of 9 people in the UK who are Rhesus + is taken. Find the probability that fewer than 3 are group O+.   [6]

    (ii) A random sample of 150 people in the UK is taken. Find the probability that more than 60 people are group A+.   [5]

52   It is given that $X \sim N(28.3, 4.5)$. Find the probability that a randomly chosen value of X lies between 25 and 30. [3]

53 The lengths of body feathers of a particular species of bird are modelled by a normal distribution. A researcher measures the lengths of a random sample of 600 body feathers from birds of this species and f nds that 63 are less than 6 cm long and 155 are more than 12 cm long.

   (i) Find estimates of the mean and standard deviation of the lengths of body feathers of birds of this species. [5]

   (ii) In a random sample of 1000 body feathers from birds of this species, how many would the researcher expect to f nd with lengths more than 1 standard deviation from the mean? [4]

54 In Restaurant Bijoux 13% of customers rated the food as 'poor', 22% of customers rated the food as 'satisfactory' and 65% rated it as 'good'. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

   (i) Find the probability that more than 2 and fewer than 12 of them rated the food as 'good'. [3]

On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

   (ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as 'poor' is greater than 0.95. [3]

55 The times taken to play Beethoven's Sixth Symphony can be assumed to have a normal distribution with mean 41.1 minutes and standard deviation 3.4 minutes. Three occasions on which this symphony is played are chosen at random.

   (i) Find the probability that the symphony takes longer than 42 minutes to play on exactly 1 of these occasions. [4]

The times taken to play Beethoven's Fifth Symphony can also be assumed to have a normal distribution. The probability that the time is less than 26.5 minutes is 0.1, and the probability that the time is more than 34.6 minutes is 0.05.

   (ii) Find the mean and standard deviation of the times to play this symphony. [5]

   (iii) Assuming that the times to play the two symphonies are independent of each other, f nd the probability that, when both symphonies are played, both of the times are less than 34.6 minutes. [4]

56 The lengths, in cm, of trout in a f sh farm are normally distributed. 96% of the lengths are less than 34.1 cm and 70% of the lengths are more than 26.7 cm.

   (i) Find the mean and the standard deviation of the lengths of the trout. [5]

In another f sh farm, the lengths of salmon, X cm, are normally distributed with mean 32.9 cm and standard deviation 2.4 cm.

   (ii) Find the probability that a randomly chosen salmon is 34 cm long, correct to the nearest centimetre. [3]

   (iii) Find the value of t such that $P(31.8 < X < t) = 0.5$. [4]

57 Lengths of rolls of parcel tape have a normal distribution with mean 75 m, and 15% of the rolls have lengths less than 73 m.

    (i) Find the standard deviation of the lengths.   [3]

Alison buys 8 rolls of parcel tape.

    (ii) Find the probability that fewer than 3 of these rolls have lengths more than 77 m.   [3]

58 Ana meets her friends once every day. For each day the probability that she is early is 0.05 and the probability that she is late is 0.75. Otherwise she is on time.

    (i) Find the probability that she is on time on fewer than 20 of the next 96 days.   [5]

    (ii) If she is early there is a probability of 0.7 that she will eat a banana. If she is late she does not eat a banana. If she is on time there is a probability of 0.4 that she will eat a banana. Given that for one particular meeting with friends she does not eat a banana, f nd the probability that she is on time.   [4]

59 The random variable X is the daily prof t, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

    (i) Find the probability that, on a randomly chosen day, the company makes a prof t between $10 000 and $12 000.   [3]

    (ii) Find the probability that the company makes a loss on exactly 1 of the next 4 consecutive days.   [4]

60 The mean of a certain normally distributed variable is four times the standard deviation. The probability that a randomly chosen value is greater than 5 is 0.15.

    (i) Find the mean and standard deviation.   [4]

    (ii) 200 values of the variable are chosen at random. Find the probability that at least 160 of these values are less than 5.   [5]

61 In a normal distribution with mean 9.3, the probability of a randomly chosen value being greater than 5.6 is 0.85. Find the standard deviation.   [3]

62 The random variable X is such that $X \sim N(82, 126)$.

    (i) A value of X is chosen at random and rounded to the nearest whole number. Find the probability that this whole number is 84.   [3]

    (ii) Five independent observations of X are taken. Find the probability that at most one of them is greater than 87.   [4]

    (iii) Find the value of k such that $P(87 < X < k) = 0.3$.   [5]

63   Assume that, for a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday. Find the probability that, out of 350 randomly chosen people, at least 47 will have their next birthday on a Monday.   [5]

64   **(a)** The random variable Y is normally distributed with positive mean μ and standard deviation $\frac{1}{2}\mu$. Find the probability that a randomly chosen value of Y is negative. [3]

     (b)   The weights of bags of rice are normally distributed with mean $2.04\,\text{kg}$ and standard deviation $\sigma\,\text{kg}$. In a random sample of 8000 such bags, 253 weighed over $2.1\,\text{kg}$. Find the value of σ.   [4]

65   Fiona uses her calculator to produce 12 random integers between 7 and 21 inclusive. The random variable X is the number of these 12 integers which are multiples of 5.

     (i) State the distribution of X and give its parameters.   [3]

     (ii) Calculate the probability that X is between 3 and 5 inclusive.   [3]

     Fiona now produces n random integers between 7 and 21 inclusive.

     (iii) Find the least possible value of n if the probability that none of these integers is a multiple of 5 is less than 0.01.   [3]

66   The random variable Y is normally distributed with mean equal to five times the standard deviation. It is given that $\text{P}(Y > 20) = 0.0732$. Find the mean. [3]

67   Cans of lemon juice are supposed to contain $440\,\text{ml}$ of juice. It is found that the actual volume of juice in a can is normally distributed with mean $445\,\text{ml}$ and standard deviation $3.6\,\text{ml}$.

     (i) Find the probability that a randomly chosen can contains less than $440\,\text{ml}$ of juice.   [3]

     It is found that 94% of the cans contain between $(445 - c)\,\text{ml}$ and $(445 + c)\,\text{ml}$ of juice.

     (ii) Find the value of c.   [3]

68   Robert uses his calculator to generate 5 random integers between 1 and 9 inclusive.

     (i) Find the probability that at least 2 of the 5 integers are less than or equal to 4.   [3]

     Robert now generates n random integers between 1 and 9 inclusive. The random variable X is the number of these n integers which are less than or equal to a certain integer k between 1 and 9 inclusive. It is given that the mean of X is 96 and the variance of X is 32.

     (ii) Find the values of n and k.   [4]

69   Buildings in a certain city centre are classifi d by height as tall, medium or short. The heights can be modelled by a normal distribution with mean 50 metres and standard deviation 16 metres. Buildings with a height of more than 70 metres are classifi d as tall.

    (i) Find the probability that a building chosen at random is classifi d as tall.   [2]

    (ii) The rest of the buildings are classifi d as medium and short in such a way that there are twice as many medium buildings as there are short ones. Find the height below which buildings are classifi d as short.   [5]

70   In a certain country, on average one student in fi e has blue eyes.

    (i) For a random selection of n students, the probability that none of the students has blue eyes is less than 0.001. Find the least possible value of n.   [3]

    (ii) For a random selection of 120 students, fi d the probability that fewer than 33 have blue eyes.   [4]

71   Lengths of a certain type of carrot have a normal distribution with mean 14.2 cm and standard deviation 3.6 cm.

    (i) 8% of carrots are shorter than c cm. Find the value of c.   [3]

    (ii) Rebekah picks 7 carrots at random. Find the probability that at least 2 of them have lengths between 15 and 16 cm.   [6]

72   It is given that $X \sim N(1.5, 3.2^2)$. Find the probability that a randomly chosen value of X is less than -2.4. [3]

73   On trains in the morning rush hour, each person is either a student with probability 0.36, or an off ce worker with probability 0.22, or a shop assistant with probability 0.29 or none of these.

    (i) 8 people on a morning rush hour train are chosen at random. Find the probability that between 4 and 6 inclusive are off ce workers.   [3]

    (ii) 300 people on a morning rush hour train are chosen at random. Find the probability that between 31 and 49 inclusive are neither students nor off ce workers nor shop assistants.   [6]

74   A factory produces fl wer pots. The base diameters have a normal distribution with mean 14 cm and standard deviation 0.52 cm. Find the probability that the base diameters of exactly 8 out of 10 randomly chosen fl wer pots are between 13.6 cm and 14.8 cm.   [5]

75   In a large consignment of mangoes, 15% of mangoes are classifi d as small, 70% as medium and 15% as large.

    (i) Yue-chen picks 14 mangoes at random. Find the probability that fewer than 12 of them are medium or large.   [3]

    (ii) Yue-chen picks n mangoes at random. The probability that none of these n mangoes is small is at least 0.1. Find the largest possible value of n.   [3]

76 **(a)** The random variable X is normally distributed with mean 82 and standard deviation 7.4. Find the value of q such that $P(82 - q < X < 82 + q) = 0.44$. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. It is given that $5\mu = 2\sigma^2$ and that $P(Y < \frac{1}{2}\mu) = 0.281$. Find the values of μ and σ. [4]

77 The petrol consumption of a certain type of car has a normal distribution with mean 24 kilometres per litre and standard deviation 4.7 kilometres per litre. Find the probability that the petrol consumption of a randomly chosen car of this type is between 21.6 kilometres per litre and 28.7 kilometres per litre. [4]

78 Lengths of a certain type of white radish are normally distributed with mean μ cm and standard deviation σ cm. 4% of these radishes are longer than 12 cm and 32% are longer than 9 cm. Find μ and σ. [5]

79 **(i)** State three conditions which must be satisf ed for a situation to be modelled by a binomial distribution. [2]

George wants to invest some of his monthly salary. He invests a certain amount of this every month for 18 months. For each month there is a probability of 0.25 that he will buy shares in a large company, there is a probability of 0.15 that he will buy shares in a small company and there is a probability of 0.6 that he will invest in a savings account.

(ii) Find the probability that George will buy shares in a small company in at least 3 of these 18 months. [3]

80 In a certain country 12% of houses have solar heating. 19 houses are chosen at random. Find the probability that fewer than 4 houses have solar heating. [4]

81 A school club has members from 3 different year-groups: Year 1, Year 2 and Year 3. There are 7 members from Year 1, 2 members from Year 2 and 2 members from Year 3. Five members of the club are selected. Find the number of possible selections that include at least one member from each year-group. [4]

82 The time Rafa spends on his homework each day in term-time has a normal distribution with mean 1.9 hours and standard deviation σ hours. On 80% of these days he spends more than 1.35 hours on his homework.

(i) Find the value of σ. [3]

(ii) Find the probability that, on a randomly chosen day in term-time, Rafa spends less than 2 hours on his homework. [2]

(iii) A random sample of 200 days in term-time is taken. Use an approximation to f nd the probability that the number of days on which Rafa spends more than 1.35 hours on his homework is between 163 and 173 inclusive. [6]

83   There is a probability of $\frac{1}{7}$ that Wenjie goes out with her friends on any particular day. 252 days are chosen at random.

  (i) Use a normal approximation to f nd the probability that the number of days on which Wenjie goes out with her friends is less than than 30 or more than 44.   [5]

  (ii) Give a reason why the use of a normal approximation is justif ed.   [1]

84   When Moses makes a phone call, the amount of time that the call takes has a normal distribution with mean 6.5 minutes and standard deviation 1.76 minutes.

  (i) 90% of Moses's phone calls take longer than t minutes. Find the value of t.   [3]

  (ii) Find the probability that, in a random sample of 9 phone calls made by Moses, more than 7 take a time which is within 1 standard deviation of the mean.   [5]

85   Screws are sold in packets of 15. Faulty screws occur randomly. A large number of packets are tested for faulty screws and the mean number of faulty screws per packet is found to be 1.2.

  (i) Show that the variance of the number of faulty screws in a packet is 1.104.   [2]

  (ii) Find the probability that a packet contains at most 2 faulty screws.   [3]

  Damien buys 8 packets of screws at random.

  (iii) Find the probability that there are exactly 7 packets in which there is at least 1 faulty screw. [4]

86 A farmer finds that the weights of sheep on his farm have a normal distribution with mean 66.4 kg and standard deviation 5.6 kg.

  (i) 250 sheep are chosen at random. Estimate the number of sheep which have a weight of between 70 kg and 72.5 kg.   [5]

  (ii) The proportion of sheep weighing less than 59.2 kg is equal to the proportion weighing more than y kg. Find the value of y.   [2]

  Another farmer finds that the weights of sheep on his farm have a normal distribution with mean μ kg and standard deviation 4.92 kg. 25% of these sheep weigh more than 67.5 kg.

  (iii) Find the value of μ.   [3]

87   **(a)** The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.

    (i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night.   [2]

    (ii) Find the value of q such that $P(X < q) = 0.75$.   [3]

  (b) The random variable Y has the distribution $N(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $P(Y > 4\mu)$.   [3]

88   In Marumbo, three quarters of the adults own a cell phone.

   (i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive.   [3]

   (ii) A random sample of 160 adults from Marumbo is taken. Use an approximation to f nd the probability that more than 114 of them own a cell phone.   [5]

   (iii) Justify the use of your approximation in part **(ii)**.   [1]

89   Packets of tea are labelled as containing 250 g. The actual weight of tea in a packet has a normal distribution with mean 260 g and standard deviation σ g. Any packet with a weight less than 250 g is classed as 'underweight'. Given that 1% of packets of tea are underweight, f nd the value of σ. [3]

90   The number of books read by members of a book club each year has the binomial distribution B(12, 0.7).

   (i) State the greatest number of books that could be read by a member of the book club in a particular year and f nd the probability that a member reads this number of books.   [2]

   (ii) Find the probability that a member reads fewer than 10 books in a particular year.   [3]

91   Gem stones from a certain mine have weights, X grams, which are normally distributed with mean 1.9 g and standard deviation 0.55 g. These gem stones are sorted into three categories for sale depending on their weights, as follows.

   Small: under 1.2 g   Medium: between 1.2 g and 2.5 g   Large: over 2.5 g

   (i) Find the proportion of gem stones in each of these three categories.   [5]

   (ii) Find the value of k such that $P(k < X < 2.5) = 0.8$.   [4]

92   The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ.   [4]

93   **(i)** In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer.   [4]

   (ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer.   [5]

   (iii) Justify your use of the approximation in part **(ii)**.   [1]

94   A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive.   [3]

95 **(a)** Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

    (i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]

    (ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t. [3]

  (b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ. [5]

96 The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ. [3]

97 On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard. [5]

98 The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than $29\,\text{cm}$ is classified as 'large'.

    (i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large. [6]

    (ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n. [3]

99 In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

100 The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ. [4]

101 **(a)** Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

    Find the mean and standard deviation of Amy's pulse rate. [4]

  (b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute. [3]

102 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

   (i) Find the probability of throwing a 3. [3]

   (ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

   (iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

103 **(a)** A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

     (i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

     The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

     (ii) Find the value of m. [3]

     (iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

   (b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

104 The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

   (i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

   (ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

105 A factory makes water pistols, 8% of which do not work properly.

   (i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

   (ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

   (iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

   (iv) Justify the use of your approximation in part **(iii)**. [1]