I4 WITH TRIG IDENTITY MARKING SCHEME

1	(i)	Make relevant use of formula for sin 2θ or cos 2θ	M1	
		Make relevant use of formula for cos 4θ	M1	
		Complete proof of the given result	A1	**3**
	(ii)	Integrate and obtain $\frac{1}{8}(\theta - \frac{1}{4}\sin 4\theta)$ or equivalent	B1	
		Use limits correctly with an integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$	M1	
		Obtain answer $\frac{1}{8}(\frac{1}{3}\pi + \frac{\sqrt{3}}{8})$, or exact equivalent	A1	**3**
2	**(i)**	State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$	B1	
		Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent	M1	
		Simplify and obtain the given identity correctly	A1	[3]
	(ii)	Obtain integral $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x$	B1	
		Substitute limits correctly in an integral of the form $a\sin 2x + b\sin 4x$	M1	
		Obtain given answer following full, correct and exact working	A1	[3]
3	**(i)**	Express $\cos 4\theta$ as $2\cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2\sin^2 2\theta$	B1	
		Express $\cos 4\theta$ in terms of $\cos\theta$	M1	
		Obtain $8\cos^4\theta - 8\cos^2\theta + 1$	A1	
		Use $\cos 2\theta = 2\cos^2\theta - 1$ to obtain given answer $8\cos^4\theta - 3$ **AG**	A1	[4]
	(ii) (a)	State or imply $\cos^4\theta = \frac{1}{2}$	B1	
		Obtain 0.572	B1	
		Obtain –0.572	B1	[3]
	(b)	Integrate and obtain form $k_1\theta + k_2\sin 4\theta + k_3\sin 2\theta$	M1	
		Obtain $\frac{3}{8}\theta + \frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta$	A1	
		Obtain $\frac{3}{32}\pi + \frac{1}{4}$ following completely correct work	A1	[3]

4	**(i)**	Use correct quotient or chain rule	M1	
		Obtain the given answer correctly having shown sufficient working	A1	[2]
	(ii)	Use a valid method, e.g. multiply numerator and denominator by sec x + tan x, and a version of Pythagoras to justify the given identity	B1	[1]
	(iii)	Substitute, expand (sec x + tan x)^2 and use Pythagoras once	M1	
		Obtain given identity	A1	[2]
	(iv)	Obtain integral 2 tan $x - x$ + 2 sec x	B1	
		Use correct limits correctly in an expression of the form a tan x + bx + c sec x, or equivalent, where abc 0	M1	
		Obtain the given answer correctly	A1	[3]

5	**(i)**	Use correct quotient rule or equivalent	M1	
		Obtain $\dfrac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
		Substitute $x = 0$ and obtain $-\dfrac{1}{2}$ or equivalent	A1	[3]
	(ii)	Differentiate y^3 and obtain $3y^2\dfrac{dy}{dx}$	B1	
		Differentiate 5 y and obtain $5y+5x\dfrac{dy}{dx}$	B1	
		Obtain $6x^2+5y+5x\dfrac{dy}{dx}+3y^2\dfrac{dy}{dx}=0$	B1	
		Substitute $x=0, y=2$ to obtain $-\dfrac{5}{6}$ or equivalent following correct work	B1	[4]

6	**(i)**	*EITHER*: Use tan 2A formula to express LHS in terms of tan θ Express as a single fraction in any correct form Use Pythagoras or cos 2A formula Obtain the given result correctly	**M1** **A1** **M1** **A1**	
		OR: Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ Express as a single fraction in any correct form Use Pythagoras or cos 2A formula or sin($A - B$) formula Obtain the given result correctly	**M1** **A1** **M1** **A1**	[4]
	(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	**M1***	
		Obtain integral $-\frac{1}{2}\ln(\cos 2\theta)+\ln(\cos\theta)$, or equivalent	**A1**	
		Substitute limits correctly (expect to see use of both limits)	**DM1**	
		Obtain the given answer following full and correct working	**A1**	[4]

7(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
7(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and $\tan\theta$	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
	OR2: Express LHS in terms of secθ and tanθ	**(M1**
	Multiply numerator and denominator by secθ + tanθ and use Pythagoras	**M1**
	Complete the proof	**A1)**
	Total:	**3**
7(iii)	Use the identity and obtain integral $2\tan \quad 2\sec\theta-\theta$	**B2**
	Use correct limits correctly in an integral containing terms a tan and b sec	**M1**
	Obtain answer $2\sqrt{2}-\frac{1}{4}$	**A1**
	Total:	**4**

8(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	**M1**	$\frac{2\sin x - 2\sin x\cos x}{1-(2\cos^2 x - 1)}$
	Obtain a correct expression	**A1**	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos$	**M1**	
	Obtain the given RHS correctly *OR (working R to L)*:	**A1**	
	$\frac{\sin x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}=\frac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $=\frac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$ M1A1		Given answer so check working carefully
	$=\frac{2\sin x-\sin 2x}{1-\cos 2}$ M1A1		
		4	
8(ii)	State integral of the form $a\ln(1+\cos x)$	**M1***	If they use the substitution $u \quad 1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	**A1**	
	Substitute correct limits in correct order	**M1(dep)***	
	Obtain answer $\ln\left(\frac{3}{2}\right.$, or equivalent	**A1**	
		4	

9(i)	State correct expansion of $\sin(2x+x)$	**B1**	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	**M1**	
	Obtain a correct expression in any form	**A1**	e.g. $2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	**A1**	Accept $=$ for $\equiv$

Use identity, integrate and obtain $-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x$	**B1 B1**	One mark for each term correct
Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	**M1**	$\left(-\frac{3}{8}-\frac{1}{12}+\frac{3}{4}-\frac{1}{12}=-\frac{11}{24}+\frac{2}{3}\right)$
Obtain answer $\frac{5}{24}$	**A1**	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
	4	

10(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	**B1**	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	**M1**	
	Obtain $\sin 3\ \ \cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	**A1**	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4\ \ -\frac{1}{4}\cos 2x$	**B1 B1**	
	Substitute limits $x=0$ and $x=\frac{1}{3}$ correctly	**M1**	In their expression
	Obtain answer $\frac{9}{16}$	**A1**	From correct working seen.
		4	

10(iii)	State correct derivative $2\cos 4x + \cos 2x$	**B1**	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	**M1**	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	**A1**	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x = \dfrac{-1 \pm \sqrt{33}}{8}\right)$	**M1**	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = \ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer $x = 1.29$ only	**A1**	
		5	

11(i)	Use double angle formulae and express entire fraction in terms of sin and cos	**M1**	
	Obtain a correct expression	**A1**	
	Obtain the given answer	**A1**	
		3	
11(ii)	State integral of the form $\pm \ln \cos$	**M1***	
	Use correct limits correctly and insert exact values for the trig ratios	**DM1**	
	Obtain a correct expression, e.g. $-\ln \dfrac{1}{\sqrt{2}} + \ln \dfrac{\sqrt{3}}{2}$	**A1**	
	Obtain the given answer following full and exact working	**A1**	
		4	

12(i)	Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	**M1**	
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	**M1**	
	Obtain a correct expression in terms of $\cos x$ in any form	**A1**	
	Obtain $\cos 3 \quad \equiv 4\cos^3 x - 3\cos x$	**A1**	AG
		4	
12(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	**M1**	$\cos x = -0.6299...$
	Obtain answer 2.25 and no other in the interval	**A1**	Accept 0.717π M1A0 for 129.0°
		2	

12(iii)	Obtain indefinite integral $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$	**B1 + B1**	
	Substitute limits in an indefinite integral of the form $a\sin 3x + b\sin x$, where $ab \neq 0$	**M1**	$\frac{1}{4}\left[\frac{1}{3}\sin\pi + 3\sin\frac{\pi}{3} - \frac{1}{3}\sin\frac{\pi}{2} - 3\sin\frac{\pi}{6}\right]$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3} - 11\right)$, or exact equivalent	**A1**	
	Alternative method for question 12(iii)		
	$\int \cos x\left(1 - \sin^2 x\right)\mathrm{d}x = \sin x - \frac{1}{3}\sin^3 x\left(+C\right)$	**B1 + B1**	
	Substitute limits in an indefinite integral of the form $a\sin x + b\sin^3 x$ where $ab \neq 0$	**M1**	$\left(\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{1}{24}\right)$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3} - 11\right)$, or exact equivalent	**A1**	
		4	