A LEVEL (P6) PERMUTATIONS AND COMBINATION

MARK SCHEME

1 **(a) (i)** $3 \times 5 \times 3 \times 2$ or ${}_3C_1 \times {}_5C_1 \times {}_3C_1 \times 2$ $= 90$	M1 A1	2	For multiplying $3 \times 5 \times 3$ For correct answer
(ii) $(3 \times 5 \times 2) + (3 \times 3) + (5 \times 2 \times 3)$ $= 69$	M1 M1 A1	3	For summing options that show S&M,S&D,M&D $3 \times 5 \times a + 3 \times 3 \times b + 5 \times 3 \times c$ seen for integers a,b,c For correct answer
(b) ${}_{14}C_5 \times {}_9C_5 \times {}_4C_4$ or equivalent $= 252252$	M1 M1 A1	3	For using combinations not all ${}_{14}C$… For multiplying choices for two or three groups For correct answer NB 14!/5!5!4! scores M2 and A1if correct answer

2 **(i)** $\frac{9!}{2!2!} = 90720$	**B1**		For dividing by 2! or 2 once or twice, or ${}_9P_7$ or ${}_9C_7$ seen, can be implied
	B1	**2**	For correct answer
(ii) $\frac{5!4!}{2!2!} = 720$ OR could do by probs and multiply by their **(i)**	**B1**		For 5! or 4! or ${}_4P_4$ or ${}_5P_5$ seen in num
	B1		For $5! \times 4! \times k$ in num of a term, $k = 1$ or 2 only
	B1	**3**	For correct final answer

3 **(a)(i)** ${}_3C_1 \times {}_5C_1$	**M1**		For multiplying two combinations together
$= 15$	**B1**	**[2**	For correct answer
(ii) ${}_5C_1 \times {}_6C_2$	**M1**		For seeing ${}_6C_2$, or separating it into three alternatives either added or multiplied
$= 75$	**A1**	**[2**	For correct answer
(b)(i) 9!/2!2! = 90720	**M1** **A1**	**[2]**	For dividing by 2! twice For correct answer
(ii) 5! Or ${}_5P_5$	**B1**		5! seen in a numerator
$= 120$	**B1**	**[2**	For correct final answer

4 (i) $_{13}P_9$ = 259,459,200 or 259,000,000	M1 A1 **2**	For using a permutation involving 13 For correct answer
(ii) 10! or $_{10}P_9$ = 3628800	M1 M1 A1 **3**	For using a 10 For using a 9! For correct answer
(iii) 1 – (ii) / (i) = 0.986	M1 A1ft **2**	For a subtraction of a suitable prob < 1, from 1 For correct answer, ft on their (i) and (ii)

5 (i) P(no orange) = $(2/3)^5$ or 0.132 or 32/243	B1 **1**	For correct final answer either as a decimal or a fraction
(ii) P(2 end in 6) = $(1/10)^2 \times (9/10)^3 \times {}_5C_2$ = 0.0729	B1 M1 A1 **3**	For using $(1/10)^k$ $k>1$ For using a binomial expression with their 1/10 or seeing some $p^2 * (1-p)^3$ For correct answer
(iii) P(2 orange end in 6) = $(1/30)^2 \times (29/30)^3 \times {}_5C_2$ = 0.0100 accept 0.01	M1 A1 **2**	For their (1/10)/3 seen For correct answer
(iv) $n = 5$, $p = 1/3$, mean = 5/3, variance = 10/9	B1 B1 ft **2**	For recognising $n=5$, $p = 1/3$ For correct mean and variance, ft their n and p, $p<1$

6 (i) $_{17}P_{11}$ $= 4.94 \times 10^{11}$	B1 B1 **2**	Or equivalent Or equivalent
(ii) $_{12}P_6 \times 5!$ = 79800000 (79833600)	B1 B1 B1 **3**	For 5! Multiplied by something For $_{12}P_6$ or $_{12}C_6$ multiplied by something Correct answer o.e.
(iii) $_3C_2 \times {}_7C_1$ =21	B1 M1 A1 **3**	3 or $_3C_2$ seen $_7C$ something seen correct answer

7 (i) 9! = 362880 (363000)	B1 B1 **2**	9! Or ${}_9P_9$ only correct answer
(ii) $6! \times {}_7P_3$ =151200	B1 M1 A1 A1 **4**	6! seen ${}_7P$ or ${}_7C$ something or 7 multiplied by something mult by ${}_7P_3$ correct answer
(iii) 1 woman: ${}_3C_1 \times {}_6C_2 = 45$ 2 women: ${}_3C_2 \times {}_6C_1 = 18$ 3 women: ${}_3C_3 = 1$ total = 64	M1 B1 A1	summing cases for 1, 2, 3 women one correct case correct answer
OR no restrictions 9C3 (84) Men only 84 – 20 = 64	B1 M1 A1 **3**	${}_9C_3$ or 84 or 3 times ${}_8C_2$ seen attempt at subt of their 'no women' case correct answer

8 (i) (a)	$\frac{12!}{4!2!} = 9979200\ (9980000)$	B1 B1 **2**	Dividing by 4! and 2! only Correct answer
(b)	$\frac{9!}{2!} = 181440\ (181000)$	B1 B1 **2**	9! or 9 × 8! seen not in denom correct answer
(ii)	${}_6C_2$ or ${}_4C_0 \times {}_2C_2 \times {}_6C_2$ or ${}_6C_4$ or ${}_6P_2/2!$ = 15	M1 M1 A1 **3**	for seeing ${}_6C_{\text{something}}$ or ${}_6P_{\text{something}}$ in a product (could be with 1) for seeing ${}_{\text{something}}C_2$ or ${}_6C_4$ correct answer 15 with no working scores full marks

9 (i) $\frac{6!}{3!} = 120$	M1 A1 **2**	For dividing by 3! Correct answer
(ii) 5 ….7 = $\frac{4!}{2!} = 12$ 7…. 5 = $\frac{4!}{2!} = 12$ 7…. 7 = 4! = 24 total = 48	M1 B1 B1 A1 **4**	For identifying different cases For 4!/2! seen For 4! alone seen or in a sum or product Correct final answer

10 (i) $\dfrac{12!}{2!2!3!4!} = 831600$	M1 A1 **[2]**	Dividing by 3! 4! and 2! once or twice o.e Correct final answer
(ii) $\dfrac{6!}{4!2!} \times \dfrac{6!}{2!3!}$	B1	$\dfrac{6!}{4!2!}$ and $\dfrac{6!}{2!3!}$ seen o.e
	M1	multiplying their numbers for group 1 with their numbers for group 2
$= 900$	A1 **[3]**	correct final answer
(iii) $2 \times 3 \times {}_7C_2$ or $2 \times 3 \times 21$	M1	${}_7C_2$ seen multiplied or 5 options added
$= 126$	A1 **[2]**	correct final answer

11 (i) ${}^{13}C_{10} \times {}^{12}C_9 \times {}^6C_4 \times {}^7C_4$	M1	Expression involving the product of 4 combinations
$= 33033000$ (33000000)	A1 **[2]**	Correct final answer allow 33×10^6 or 3.3×10^7
(ii) $5! \times 6!$	B1	6! or 5! or 4! oe seen no denom
$= 86400$	M1	a single product involving 6! and either 4! or 5! no denom
	A1 **[3]**	Correct final answer
(iii) $4! \times 3! \times 2$	B1 M1	4! or 3! or 4!/4 seen a single product involving 3! (or 4!/4) and 4!
$= 288$	A1 **[3]**	Correct final answer

12 (a) (i) $1 \times 5 \times 4 \times 3$ or ${}^5C_3 \times 3!$ or ${}^5P_3$ $= 60$	M1 A1 **[2]**	One of these oe Correct final answer
(ii) $1 \times 6^3 = 216$	M1 A1 **[2]**	Seeing 6^3 Correct answer
(b) (i) 5G 0B = ${}^8C_5 = 56$ $(\times\, {}^6C_0)$ 4G 1B = ${}^8C_4 \times {}^6C_1 = 420$ 3G 2B = ${}^8C_3 \times {}^6C_2 = 840$	M1 B1	Σ 2 or three 2-factor products, C or P Any correct option unsimplified
	A1	A second correct option unsimplified
total = 1316	A1 **[4]**	Correct answer
(ii) ${}^{11}C_2 + {}^{11}C_5$	M1	Adding two single perm or comb options ${}^{11}C_x + {}^{11}C_y$
$= 55 + 462$ $= 517$	B1 A1	One correct unsimplified option Correct answer
OR cousins in P(3B, 2G) + P(4B, 1G) + P(5B, 0G) + cousins out P(3B, 2G)	M1	Σ 5 or more 2-factor perm or comb terms
+ P(2B, 3G) + P(1B, 4G) + P(0B, 5G) = 28 + 24 + 3 + 28 + 168 + 210 + 56	B1	3 or more correct unsimplified options
= 517	A1 **[3]**	Correct answer

13 (a) (i) 24	B1 **[1]**	Correct final answer
(ii) 3 digit odd 500+ = 4 ways 3 digit odd 600+ = 3 × 2 = 6 ways 4 digit odd 1000+ = 4 ways 4 digit odd 3000+ = 4 ways 4 digit odd 5000+ = 4 ways 4 digit odd 6000+ = 6 ways OR 4 digit odd, last digit in 3 ways, 2nd to last in 3 ways, 2nd in 2 ways first in 1 way = 18 Total = 28 ways	M1 M1 M1 A1 **[4]**	Attempt for 3 digit odd numbers Attempt for 4 digit odd numbers For summing their number of ways with 3-digits and their number of ways with 4-digits Correct total
(b) no of ways 4 and 5 not next to each other = 6! − 5! × 2! = 720 − 240 = 480 Prob not next = 480/720 = 2/3	 M1 B1 A1 **[3]**	 Finding ways digits not next to each other 240 or 480 seen Correct answer

14 (i) $^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9$ = 512	M1 A1 A1 **[3]**	Summing some ^{10}C combinations with odd numbers, all different At least 3 correct unsimplified expressions Correct answer
(ii) 6! × 7 × 6 × 5 = 151200	B1 M1 A1 **[3]**	6! seen multiplying by $^7P_3$ o.e. correct answer
(iii) 12! / (4! × 7!) = 3960	B1 M1 A1 **[3]**	12! Seen dividing by 4!7! correct answer

15 (i) 362880 (363000)	B1 **[1]**	
(ii) PG or GP in 8! × 2 = 80640 or 7/9 of **(i)** 362880 – 80640 = 282240	M1 B1 A1ft **[3]**	Considering together and also subtracting from their **(i)** or using probabilities 8! × 2 or 80640 seen oe correct answer ft 40320 only
(iii) $^9P_3$ or $^9C_3 \times 3!$ or 9!/6! = 504	M1 A1 **[2]**	$^9P_3$ or $^9C_3$ oe seen allow extra multiplication correct final answer
(iv) $^8C_2 \times 3!$ or $504 - {}^8C_3 \times 3!$ or $^8P_2 \times 3$ = 168	M1 A1 **[2]**	^8C_x or ^8P_x seen allow extra mult, or **(iii)**/9 or **(iii)**/3 correct final answer
(v) PG and x in 7 × 2 × 2 ways = 28 Answer 504 – 28 = 476	M1 A1 **[2]**	$x \times 2 \times 2$ seen or their **(iii)** – 7 or $^7C_1$ or $^7C_2$ correct answer

16 **(i)** ends cola, 5!/2!2! = 30 ends green tea, 5!/3!2! = 10 ends orange juice, 5!/3!2! = 10 total = 50 ways	M1 A1 A1	Considering all three options Any one option correct Correct answer
OR P(ends same) $=\frac{3}{7}\times\frac{2}{6}+\frac{2}{7}\times\frac{1}{6}+\frac{2}{7}\times\frac{1}{6}$	M1	OR Considering all three options
$=\frac{5}{21}$	A1	Correct fraction
$\frac{5}{21}\times\frac{7!}{3!2!2!}=$ 50 ways	A1 [3]	Correct answer
(ii) colas together, no restrictions, 5!/2!2! = 30 ways colas together and green tea together, 4!/2! = 12 ways 30 – 12 = 18 ways.	M1 A1 M1 A1 A1	Considering all colas together, or 5! seen Correct answer Considering all colas tog and all green tea tog, or 4! seen Correct answer Correct final answer
OR$_1$ Attempt to list	M1A1 M1A1 A1	**OR$_1$** 10 or more, 12 or more correct 14 or more, 16 or more correct 18 correct
OR$_2$ $3\times\frac{4\times 3}{2}=18$	M1 A1 M1 A1 A1 [5]	**OR$_2$** Considering all colas together, or 3! seen 3 ways for colas and orange juice Considering green teas not together 4 × 3 or (4 × 3)/2 Correct final answer

17 **(i)** $^6P_4 = 6!/2!$ = 360	B1 [1]	Correct answer
(ii) 4!/2! = 12	B1 [1]	Correct answer
(iii) $4! \times {}^6C_4 = 360$ or $^6P_4$	B1 [1]	Correct final answer
(iv) e.g. 2R 1B 1G, 1R 2B 1G, 1R 1B 2G $=\frac{4!}{2!}+\frac{4!}{2!}+\frac{4!}{2!}=36$, mult by $^6C_3$ total = 720	M1 M1 A1 [3]	4!/2! seen Mult by $^6C_3$ Correct answer
(v) 2R 2B = 4!/2!2! = 6 Mult by $^6C_2$, total = 90 Answer = 360 + 720 + 90 = 1170	M1 A1 A1ft [3]	Considering 2 colours e.g. RRBB or RBBR or... mult by $^6C_2$ Ft their **(iii)** + **(iv)** + **(v)**

18	**(i)** 4M 2W or 5M 1W chosen in ${}^{10}C_4 \times {}^{9}C_2 + {}^{10}C_5 \times {}^{9}C_1$ = 9828	M1 A1 A1 [3]	At least 1 of ${}^{10}C_4 \times {}^{9}C_2$ *and* ${}^{10}C_5 \times {}^{9}C_1$ seen Correct unsimplified expression Correct answer
	(ii) ${}^{9}C_3 \times {}^{8}C_1 + {}^{9}C_4 = 798$ Prob = 798/9828 = 0.0812	M1 A1 [2]	One of ${}^{9}C_3 \times {}^{8}C_1$ *and* ${}^{9}C_4 \times ({}^{8}C_0)$ seen Correct answer
	(iii) Albert + not T... ${}^{9}C_3 \times {}^{8}C_2 + {}^{9}C_4 \times {}^{8}C_1$ = 3360 Tracey + not A... ${}^{9}C_4 \times {}^{8}C_1 + {}^{9}C_5$ = 1134 Number of ways = 4494	M1 A1 A1 [3]	One of ${}^{9}C_3 \times {}^{8}C_2$ or ${}^{9}C_4 \times {}^{8}C_1$ or ${}^{9}C_5 \times ({}^{8}C_0)$ seen Unsimplified 3360 or 1134 seen Correct final answer
	(iv) 6! – 4! × 5 × 2 or 6! – 5! × 2 (= 480) OR 4! × 5 × 4 or 4! × ${}^{5}P_2$ (= 480) prob = 480/6! = 2/3 (0.667) OR using probabilities…as above OR Women together 5!/4! (= 5) Women not together = 15 – 5 = 10 total ways MMMMWW = 6!/4!2! = 15 prob = 2/3	 B1 M1 A1 [3] B1 M1 A1	 6! – 4! × 5 × 2 or 6! – 5! × 2 or 4! × 5 × 4 or 4! × ${}^{5}P_2$ dividing by 6! correct answer 5 or 10 seen Dividing by 15 Correct answer

19	**(i)** ${}^{14}P_{12}$ = 4.36×10^{10}	M1 A1 [2]	${}^{14}P_{12}$ seen oe Correct answer
	(ii) business people 3! = 6 students 5! = 120 married couples ${}^{3}P_2 \times 2 \times 2 = 24$ total ways = 17280	B1 B1 B1 B1 [4]	3! oe seen, not in denominator 5! oe seen, not in denominator 24 oe seen, not in denominator correct final answer
	(iii) Mrs Brown 3 Mrs Lin 10 Student 5 Prob = 3 × 10 × 5 × ${}^{11}P_9$ / **(i)** = 0.0687 OR$_1$ 3/14 × 10/13 × 5/12 = 150/2184 (0.0687)	B1 B1 M1 A1 [4] B1 B1 M1 A1	any 2 of 3, 10, 5 oe seen, not in denominator ${}^{11}P_9$ seen multiplied dividing by their **(i)** correct answer any 2 of numerators 3, 10, 5 oe seen denominators 14, 13, 12 of 3 fractions multiplying 3 separate fractions correct answer

OR_2 1 − 3/14 = 11/14	B1	1 − 3/14 seen
1 − 11/14 × 5/13 = 127/182	B1	1 − 11/14 × 5/13 seen
8/14(4/13 × 12/12 + 9/13 × 7/12) + 3/14(3/13 × 12/12 + 10/13 × 7/12) = 1206/2184	M1	attempt to find P(Mrs Lin not behind a student and Mrs Brown not in front row), involving 8/14 × prob + 3/14 × prob
1 − (1524 + 1716 − 1206)/2184 = 150/2184	A1	correct answer

20 (i) Options 5 bat 5 bl 1 Wk in $^{10}C_5 \times {}^9C_5 \times {}^2C_1$ = 63504 ways	M1	Multiplying three combinations together
or 5 bat 4 bl 2 Wk in $^{10}C_5 \times {}^9C_4 \times {}^2C_2$ = 31752 ways	M1	Summing more than one sensible option
or 6 bat 4 bl 1 Wk in $^{10}C_6 \times {}^9C_4 \times {}^2C_1$ = 52920 ways	A1	Two options correct unsimplified
Total = 148176 (148000)	A1 **[4]**	Correct final answer
(ii) $\dfrac{11!}{5!4!2!} = 6930$	B1 **[1]**	Correct answer evaluated
(iii) Omit a pen $\dfrac{10!}{4!4!2!} = 3150$	M1	Summing three options
Omit a diary $\dfrac{10!}{5!3!2!} = 2520$	B1	One option correct
Omit a notebook $\dfrac{10!}{5!4!} = 1260$		
Total = 6930	A1 **[3]**	Correct final answer

21 (i) 90720	B1 **[1]**	Not 9!/2!2!
(ii) 3 vowels together	B1	3! oe seen multiplied by integer oe
= 3! × 7!/2!2! = 7560	B1	7 or 6! seen multiplied as a num
Prob(not together) = $\dfrac{90720-7560}{90720} = \dfrac{83160}{90720}$	M1	Subt from their **(i)** or dividing by their **(i)** or 1 – prob
= 0.917 (=11/12)	A1 **[4]**	Correct answer from correct working
(iii) One S in $^5C_3$ ways = 10	M1	$^5C_3$ seen added
SS in $^5C_2$ ways = 10	M1	$^5C_2$ seen added
Total = 20	A1 **[3]**	Correct answer
OR $^6C_3$	M1	$^6C_3$ × 2 or ÷ 2 or × 1 seen
	M1	$^6C_3$ only
= 20	A1	Correct answer

22 (i) $4\times3\times7$ $= 84$	B1	**[1]**	Correct answer
(ii) $10! - 9! \times 2$ $= 2903040$ (2900000)	B1 B1	 **[2]**	$10! - k \times 9!$ seen oe Correct answer
OR $8! \times 9 \times 8$ $= 2903040$ (2900000)	B1 B1		$8! \times 9 \times l$ seen oe Correct answer
(iii) ${}^9C_1 + {}^9C_2 + ... + {}^9C_9$ $= 511$	M1 M1 A1	 **[3]**	Using combinations Adding 9 combinations Correct answer
OR $2^9 - 1$ $= 511$	M1 M1 A1		2^9 seen Subtracting 1 Correct answer

23 (a) (i) $\dfrac{12!}{2!3!2!} = 19958400$ (20,000,000)	M1 A1	 [2]	Dividing by 2! 3! 2! Correct answer
(ii) $\dfrac{4!}{2!} \times \dfrac{9!}{2!3!} = 362880$	B1 B1 B1	 [3]	4! seen multiplied 9! or 9 × 8! seen multiplied Correct final answer
(b) (i) $3876 \times 4!$ $= 93024$	M1 A1	 [2]	Multiplying by 4! Correct answer
(ii) $(3!)^4 \times 4!$ $= 31104$	M1 A1	 [2]	3! or 6 or 4! seen Correct final answer

24 (i) each in 2 ways $= 2^{12}$ $= 4096$	M1 A1	 [2]	2^{12} seen Correct answer
(ii) $\dfrac{12!}{7!5!}$ $= 792$	B1	[1]	
25 (a) *G R L* 11 7 7 = 15C11 × 10C7 × 8C7 = 1310400 13 6 6 = 15C13 × 10C6 × 8C6 = 617400 15 5 5 = 15C15 × 10C5 × 8C5 = 14112 Total = 1941912 (1940000)	 M1 A1 M1 A1	 [4]	 Multiplying 3 combinations One of 15600, 617400, 14112 seen Adding 3 options Correct answer
(b) e.g. * E * R * E (GG) N * A * E * gives 6 ways for G $\dfrac{7!}{3!}\times 6$ or 8!/3! – 2 × 7!/3! = 5040 ways.	B1 B1 B1	 [3]	7! / 3! Or 7!/3!3! seen oe Multiplying by 6 (gaps) oe Correct final answer

26 **(i)** $3! \times 4! \times 8! \times 3!$	M1 M1	Multiplying 3 factorials together Multiplying by 3!
$= 34\,836\,480$ (34 800 000)	A1 [3]	Correct answer
(ii) ${}^3C_2 \times {}^4C_2 \times {}^8C_2$	M1	Multiplying (only) 3 combinations together
$= 504$	A1 [2]	Correct answer
(iii) Fr Fa H		
3 1 2 $= {}^8C_3 \times {}^3C_1 \times {}^4C_2 = 1008$	M1	Multiplying 3 combinations, only
3 2 1 $= {}^8C_3 \times {}^3C_2 \times {}^4C_1 = 672$	M1	Summing 3 options
4 1 1 $= {}^8C_4 \times {}^3C_1 \times {}^4C_1 = 840$	A1	3 correct combination answers
total ways = 2520	A1 [4]	Correct answer

27 **(a)** **(i)** 7 couples in 7! ways each couple in 2 ways so $7! \times 2^7$ = 645120	B1 M1 A1 **[3]**	7! seen multiplied mult by 2^7 correct final answer
OR $14 \times 12 \times 10 \times 8 \times 6 \times 4 \times 2 =$ 645120	B2 A1	correct unsimplified answer correct answer
(ii) $7! \times 7! \times 2$ = 50,803,200 (50,800,000)	B1 B1 **[2]**	$7! \times 7!$ seen Correct answer
OR $14 \times 6! \times 7!$	B1 B1	$14 \times 7!$ seen Correct answer
(b) **(i)** 7C2 = 21	B1 **[1]**	
(ii) all in: 1 all not in: 5C4 = 5 total 6	M1 A1 **[2]**	Considering both cases Correct answer
(iii) 2 girls in: 6C2 × 3C2 = 45 3 girls in: 6C1= 6 Total 51	M1 A1 **[2]**	Attempt at summing 2 and 3 girls in the team need not see 3C2 Correct answer

28 **(i)** ${}_{11}C_6 = 462$	B1	
OR A3 B3 or A4 B2 or A5 B1 or A6		
$= {}_8C_3 + {}_8C_4 \times {}_3C_2 + {}_8C_5 \times {}_3C_1 + {}_8C_6$	B1	
$= 56 + 210 + 168 + 28$		
$= 462$	[1]	
(ii) ${}_8C_4 \times {}_3C_2 + {}_8C_5 \times {}_3C_1 + {}_8C_6$	M1	$\sum$ 2 or more two-factor terms, P or C any numbers
$= 210 + 168 + 28$	B1	Any correct option unsimplified

$= 406$	A1	[3]	Correct answer
(iii) ${}_9C_4 + {}_9C_6 = 126 + 84$	M1		Summing ${}_9C_x + {}_9C_y$ can be mult by 2 no other terms
	B1		126 or 84 seen or unsimplified ${}_9C_4$, ${}_9C_6$
$= 210$	A1		Correct answer
OR			
1,2 in A tog with : A1B3 + A2B2 + A3B1 + A4B0 + 1,2 out of A : A3B3 + A4B2 + A5B1 + A6B0	M1		$\sum$ 5 or more 2-factor ${}_6P_x$ or ${}_6C_x$ with ${}_3C_x$ or ${}_3P_x$ only (can be mult by 2)
$= {}_6C_1 + {}_6C_2 \times {}_3C_2 + {}_6C_3 \times {}_3C_1 + {}_6C_4 + {}_6C_3 \times {}_3C_3 + {}_6C_4 \times {}_3C_2 + {}_6C_5 \times {}_3C_1 + {}_6C_6$	B1		3 or more correct unsimplified options
$= 6 + 45 + 60 + 15 + 20 + 45 + 18 + 1 = 210$	A1		Correct answer
OR			
$462 - {}_9C_5 - {}_9C_5$	M1 B1		subt two ${}_9C_x$ options from their **(i)** ${}_9C_5$ seen oe if using this method
$= 210$	A1	[3]	Correct answer

29 **(i)** $\frac{7!}{3!} \times 2$	B1		$\frac{7!}{3!}$ or 840 seen or implied
$= 1680$	B1	**[2]**	correct answer
(ii) ${}^6C_4 = 15$	B1	**[1]**	correct answer
(iii) 1E in ${}^6C_3$ ways	M1		k × ${}^6C_a$ or k × ${}^bC_3$ (k a constant) or ${}^6P_d$ or ${}^eP_3$ seen
$= 20$	A1	**[2]**	correct final answer
(iv) need 2Es in ${}^6C_2$ ways = 15 ways need 3Es in ${}^6C_1$ = 6 ways total = 15 + 20 + 15 + 6 = 56 ways	M1 A1 M1 A1ft	 **[4]**	attempt to find ways with 2Es or 3Es ${}^6C_2$ oe and ${}^6C_1$ oe seen summing ways for no Es, 1E, 2Es and 3Es correct final answer, ft on their four answers

30 **(a)** $\frac{10!}{5!4!} = 1260$	M1 A1	 [2]	10! or ${}_{10}P_{10}$ seen in num or alone or dividing by 5! 4! only Correct final answer
(b) **(i)** ${}_8P_4$ or ${}_8C_4 \times 4!$	M1		${}_8P_4$ or ${}_8C_4$ oe seen allow extra multiplication
$= 1680$	A1	[2]	Correct answer
(ii) 6C2 × 4!	M1		6C2 or 6P2 seen multiplied Mult by 4!
$= 360$	M1		Correct answer
OR 6P4 or 4 × 3 × 6 × 5 = 360	A1	[3]	Award full marks

(c) A B C			
1 1 7 = 9C1 × 8C1 × 7C7 (oe) × $_3C_1$ = 216	M1		Summing at least two options of 1, 1, 7 or 1, 3, 5 or 3, 3, 3
1 3 5 = 9C1 × 8C3 × 5C5(oe) × 3! = 3024	M1		Mult an option by 3C1 or 3! or 3C3
3 3 3 = 9C3 × 6C3 × 3C3 (oe) = 1680	M1		Any one of the 2nd term being xCy seen mult, fitting with the first (x could be 2, 4, 5, 6 or 8) and correspnding y
	A1		Any of unsimplified 72, 504 or 1680 seen
Total = 4920 ways	A1	[5]	Correct answer

31 (a)	Boys in:10C1 × 9C3 = 840 ways Boys out: 10C3 × 9C3 = 10080 ways Total = 10920 ways (10900)	M1 B1 A1	[3]	summing two 2-factor products, C or P Any correct option unsimplified Correct final answer
(b) (i)	$_{12}P_8$ = 19,958,400	B1	[1]	or 20,000,000
(ii)	together: $_{11}P_7$ = 1663200 × 2 = 3326400 Not tog: 19958400 – 3326400 =16,632,000 (16,600,000)	B1 M1 A1	[3]	$_{11}P_7$ seen 19958400 or their (i) – their together (must be >0) correct final answer
	OR M at end then not F in 10 × 10P6 × 2=3024000 ways not at end in 10 × 9 × 10P6 = 13608000 ways Total = 16,632,000 ways	 M1 B1 A1		 summing options for M at end and M not at end one correct option correct final answer
(iii)	8! × 5 = 201600 ways	B1 M1 A1	[3]	8! seen mult by equivalent of integer ≥ 1 Mult by 5 Correct answer SR 8! × 5!=4838400 B2

32 (a) twins in: $_6C_2$ twins out: $_5C_2 \times {_6C_2}$	B1		$_6C_2$ alone or $_5C_2$ multiplied seen or implied
Total = 15 + 150	M1		Summing two cases
= 165	A1	**3**	Correct final answer
OR all: $_7C_2 \times {_6C_2}$ one twin: $2 \times {_5C_1} \times {_6C_2}$	B1		$_7C_2 \times {_6C_2}$alone or $_5C_1$ multiplied seen or implied
Total = 315 − 150	M1		$2 \times {_5C_1} \times {_6C_2}$ seen, subtracted
= 165	A1		Correct final answer

(b)				
(i)	ends in 2, 6 or 8: 6!/2! (= 360) ways	B1		Correct option for ending with 2 or 6 or 8.6!/2! seen anywhere, not multiplied
	ends in 4: 6! (= 720) ways	B1		Correct option for ending in 4
	Total = 3 × 360 + 720	M1		Summing 3 or 4 even options
	= 1800 ways	A1	**4**	Correct final answer
				7!/2! seen anywhere, not multiplied
	OR_1 all: 7!/2! (= 2520) ways	B1		6!/2! seen, subtracted
	ends in 1 or 7: 6!/2! (= 360) ways	B1		Subtract 2 odd options from total options
	Total = 2520 − 2 × 360	M1		Correct final answer
	= 1800	A1		
				5 seen, multiplied
	OR_2 (4_A, 4_B) final digit: 5 ways	B1		6! seen and divide by 2! at some stage
	other digits: 6! ways and ÷ by 2!	B1		Multiplying their two numbers
	Total = 5 × 360	M1		Correct final answer
	= 1800	A1		
(ii)	$5\times4\times3\times2$ or ${}_5P_4$ or ${}_5C_4\times4!$ or 5! or ${}_5P_5$ or ${}_6P_5\div6$	M1		One of these oe
	= 120 ways	A1	**2**	Correct final answer
(c)	$\left(\frac{2}{3}\right)^7$	M1		2/3 seen multiplied
		M1		7 probabilities multiplied together
	$=\frac{128}{2187}$ (0.0585)	A1	**3**	Correct final answer

33 (i)	4! × 3! × 5! × 2! × 4! = 829440	B1		4!, 3!, 5!, 2 seen multiplied 1, not in denominator
		B1		Mult by 4!
		B1	**[3]**	Correct answer
(ii)	8! × 9 × 8 × 7 × 6 × 5 × 4	B1		8! seen multiplied 1
		B1		Mult by ${}_9P_6$
	= 2438553600 (2.44×10^9)	B1	**[3]**	Correct answer
(iii)	8C3 × 5C3 × 2C2	B1		8C3 seen mult
	= 560	B1		5C3 seen mult
		B1	**[3]**	Correct answer

34 (i)	H J O			Mult 3 combs, 2C2 may be implied
	1. 28 2 = 4C2×9C8×2C2 = 54	M1		4Cx×9Cy×2Cz
	3 7 2 = 4C3×9C7×2C2 = 144	M1		Summing 2 or 3 three-factor options
	4 6 2 = 4C4×9C6×2C2 = 84	A1		2 options correct unsimplified
	Total = 282 ways	A1	**[4]**	Correct answer
(ii)	4! × 6! × 2! × 3!	M1		4! × 6! × 2! oe seen multiplied by int ≥ 1
		M1		3! seen mult by int ≥ 1

	= 207360 (207000)	A1	**[3]**	Correct answer
(iii)	8 J and O trees in 8! = 40320 ways 9 gaps × 8 × 7 × 6	B1 M1		8! seen mult by int ≥ 1 no division 9P4 oe or 7P4 or 8P4 seen mult by int ≥ 1 no division
	= 121,927,680 (122,000,000)	A1	**[3]**	Correct answer
(i)	SR 4C2×9C2×2C2×9C6	M1		
(ii)	SR $\frac{4!\times 6!\times 2!}{4!\times 6!\times 2!}$ or 3! or both M1	M1		

35 (i) S(10) R(14) P(6) 1 2 4 = 10C1×14C2×6C4= 13650 1 3 3 = 10C1×14C3×6C3= 72800 2 2 3 = 10C2×14C2×6C3= 81900 Total = 168350 or 168000	M1 M1 B1 A1 **[4]**	Summing 2 or more 3-factor options perms or combs Mult 3 combs or 4 combs with Σr=7 2 options correct, unsimplified Correct answer
(ii) 2! × 2! × 5!	M1 M1	2! × 2! oe, seen mult by an integer ≥1, no division Mult by 5!, or 5! alone, seen mult by an integer ≥ 1 no division
= 480	A1 **[3]**	Correct answer
If M0 earned $\frac{2!\times 2!}{2!\times 2!}$ or $\frac{5!}{3!}$ or both, seen mult by an integer ≥ 1 Or 2!×2!×5! divided by a value	SCM1	
(iii) spaniels and retrievers in 4! ways gaps in 5P3 or 5 × 4 × 3 ways = 1440	M1 M1 A1 **[3]**	4! seen multiplied by an integer >1 Mult by 5P3 oe Correct answer
If M0 earned $\frac{4!}{2!\times 2!}$ or $\frac{{}_5P_3}{3!}$ or both, seen multiplied by an integer > 1	SCM1	${}_5C_3$ oe
or 7! – 5! × 3! – {(4! × 2 × 4 × 3!) + (4! × 3 × 4 × 3!)} = 1440	 M1 M1 A1	 oe oe, e.g. 6 × 5 × 4 × 4!
If M0 earned 3! × 2! × 2! used as a denominator in all 4 terms	SCM1	Marks cannot be earned from both methods.

36 **(i)** M R O 3 1 2 = 7C3 × 5C1 × 8C2 = 4900	M1		Summing more than one 3term option involving combs (can be added)
3 2 1 = 7C3 × 5C2 × 8C1 = 2800	M1		Mult 3 combs only (indep)
2 2 2 = 7C2 × 5C2 × 8C2 = 5880	A1		1 option correct unsimplified
Total = 13580	A1	**4**	Correct answer
(ii) 4 groups in 4! ways 3 mountain in 3! ways	M1		4! seen mult by something
2 ordinary in 2! ways	M1		Mult by 3! for racing or 2! for ordinary
4! × 3! × 2 = 288	A1	**3**	Correct answer
(iii) e.g. s O x x x x O s s s Ordinary in 2!	M1		2! or 4! seen mult
Rest of bikes in 4! Bikes and spaces 5 groups in 5 ways	M1		Mult by 5 (ssssb)
2! × 4! × 5 = 240	A1	**3**	Correct answer

37 **(i)** 1663200	B1	**[1]**	
(ii) M xxxxxxxxx M	M1		9! or 9P9 seen
Number of ways = $\frac{9!}{3!2!}$ = 30240	A1	**[2]**	Correct answer
(iii) 4 vowels together = 8! × 4/2!2!	M1		8!/2!2! seen mult by something
= 40320	M1		4 oe 4!/3! or 4C1 etc. seen mult by something
1663200 – 40320 = 1622880	B1	**[3]**	Correct answer SC 7!/2!2! × 8P4 or 7! × 8P4/3! Or 7!/2!2! × 8P4/3! M1
(iv) Exactly 2 Es 4C2 = 6	M1		Summing 2 options
Exactly 3 Es 4C1 = 4	B1		One option correct
Total = 10 ways	A1	**[3]**	Correct answer
OR 5C2	M2		M1 for k5C2
= 10	A1		Correct ans

38 **(i)** $\frac{8!}{3!2!2!}$	M1		8! Divided by at least one of 3!2!2! oe
= 1680	A1	**2**	Correct answer
(ii) 5!	M1		5! Seen (not added, may be divided/multipled)
= 120	A1	**2**	Correct answer

(iii) $\frac{5!4!}{3!2!2!}$	B1		5! Or 4! Seen in sum or product in numerator (denominator may by 1)
	M1		$\frac{k5!4!}{3!2!2!}$ in a numerical expression
= 120	A1	**3**	Correct final answer
(iv) GG with AA, AE, EE, RA, RE, RT, TA, TE, = 8 ways	M1		Summing 2 options (could be lists)
GGG with A, E, R, T = 4 ways	A1		1 correct option
Total = 12 ways	A1	**3**	Correct answer

39 (i) $\frac{6!}{2!} = 360$	B1 B1	**2**	6! Seen alone Dividing by 2! only
(ii) $\frac{4!}{2!} \times \frac{4!}{3!}$	B1 B1		4! seen mult Dividing by 2! or 3! (Mult by 4 implied B1B1)
= 48	B1	**3**	Correct answer
(iii) 1N and 1A: N A xx in ${}^3C_2$ = 3 ways	M1 A1	**2**	${}^3C_x$ or ${}^xC_2$ seen alone Correct answer
(iv) 0 A : Nxxx = 1 way 2 As: NAAx in ${}^3C_1$ = 3 ways 3 As: NAAA in 1 way	M1 M1		Finding ways with 0 or 2 or 3 As Summing 3 or 4 options
Total = 8 ways	A1	**3**	Correct answer

40 (i) 5! × 3! or 6!	B1		5! or 3! or 6! oe seen mult or alone
= 720	B1	**2**	Correct final answer
(ii) 3**4, 3**8, 4**8	M1		considering at least 2 types of 4-figure options ending with 4 or 8 and starting with 3 or 4
	B1		One option correct unsimplified can be implied
= 5 × 4 + 5 × 4 + 5 × 4 = 60	A1	**3**	Correct final answer
(iii) 5, *5, **5,	M1		Appreciating that the number must end in 5 (can be implied)
$= 1 + 7 + 7^2$	M1		summing numbers ending in 5 with at least 2 different numbers of digits
= 57	A1	**3**	Correct final answer

41 **(i)** **(a)**	6! (×) 4! **OR** (×) 4 × 3 ÷ 2!2!3! **OR** ÷ 2!3! Total 720 ways	M1 M1 M1 A1	 **4**	Seen in a single term expression as numerator Seen in a single term expression as numerator (denominator may be 1) Seen in a single term expression as denominator Correct ans
(i) **(b)**	1*******3 = $\frac{7!}{3!2!}$ = 420 3*******1= 420 3*******3= 420 Total = 1260 ways	B1 M1 A1	 **3**	$\frac{7!}{3!2!}$ seen oe Attempting to evaluate and sum at least 2 of 1***3, 3***1, 3***3 Correct ans
(ii) **(a)**	5 × 4 × 3 = 60 ways ($^5P_3$)	M1 A1	 **2**	$^5P_3$ or $^5C_3$ ×3! (can be implied) Correct ans
(ii) **(b)**	2** in 212, 213, 214, 216, 221, 223, 224, 226, 231, 232, 233, 234, 236, 241, 242, 243, 246 261, 262, 263, 264, 266 Total = 22 ways **Alternative Methods:** $3 \times {}^4C_1 + 2 \times {}^5C_1$ **OR** $^5P_2 + {}^2C_1$ **OR** $^4P_2 + 2 \times {}^4P_1 + {}^2C_1$	 M1 A1 M1 **OR** M1 **OR** M1	 **2**	 Listing attempt starting with 2, at least 10 correct entries Correct ans $p \times {}^4C_1 + q \times {}^5C_1$, oe $p + q > 2$ $^5P_2$ seen Any 2 terms added

42 **(i)**	W(8) M(5) 4 2 = $^8C_4 \times {}^5C_2$ = 700 5 1 = $^8C_5 \times {}^5C_1$ = 280 6 0 = $^8C_6 \times {}^5C_0$ = 28 Total = 1008	 M1 M1 A1 A1	 **4**	 Mult 2 combs, $^8C_x \times {}^5C_y$ Summing 2 or 3 options 2 correct options unsimplified Correct answer
(ii)	M1 and MMWWW = $^3C_2 \times {}^8C_3$ = 168 M2 and MMWWW = $^3C_2 \times {}^8C_3$ = 168 Neither and MMMWWW = $^3C_1 \times {}^8C_3$ = 56 Total = 392 OR total, no restrictions = $^5C_3 \times {}^8C_3$ = 560 M1M2 and MWWW = $^3C_1 \times {}^8C_3$ = 168 560 – 168 = 392	M1 B1 A1 M1 B1 A1	 **3**	Summing 3 options One correct option Correct answer Subt 2 men together from no restrictions One correct of 560 or 168 Correct answer

43 $^{48}C_{43}$ = 1712304 (1710000)	B1 B1 B1 **3**	48 seen in a single term combination oe 43 or 5 seen in a single term combination oe Both can be mult by integer $k \geqslant 1$ Correct final answer
44 **(i)** 6! ×5! = 86400 **(ii)** 6! × 7 × 6 × 5 × 4 = 604800	B1 B1 B1 **3** B1 B1 B1 **3**	6! oe seen multiplied by integer $k \geqslant 1$ 5! oe seen multiplied by integer $k \geqslant 1$ Correct final answer 6! seen mult by integer $k \geqslant 1$ Mult by $^7P_4$ oe Correct final answer

45 **(a)** 1*****3 or 3*****1 or 2*****2 $= 6^5 \times 3$ = 23328	M1 M1 A1 **3**	Mult by 6^5 (for middle 5 dice outcomes) Mult by 3 or summing 3 different combinations (for end dice outcomes) Correct answer accept 23 300
(b) W J H 1 1 7 = $^9C_1 \times {}^8C_1 \times 1 = 72$ 1 7 1 = $^9C_1 \times {}^8C_7 \times 1 = 72$ 7 1 1 = $^9C_7 \times {}^2C_1 \times 1 = 72$ 1 3 5 = $^9C_1 \times {}^8C_3 \times 1 = 504$ mult by 3! 3 3 3 = $^9C_3 \times {}^6C_3 \times 1 = 1680$ Total 4920 If no marks gained Listing all 10 different outcomes	 M1 A1 A1 M1 M1 A1 **6** SCM1	 Multiplying 3 combinations (may be implied) 1 unsimplified correct answer (72, 504, 1680, 216 or 3024) A 2nd unsimplified different correct answer Summing options for 1,1.7 or 1,3,5 oe (mult by 3 or 3!) Summing at least 2 different options of the 3 Correct ans If games replaced M1M1M1 max available If factorials used M0M1M1 max available

46 **(a)** **(i)**	$\frac{9!}{2!2!3!}$ = 15120 ways	B1 B1 **[2]**	Dividing by 2!2!3! Correct answer
(ii)	********3 in $\frac{8!}{2!2!3!}$ = 1680 ways ********7 in $\frac{8!}{2!3!}$ = 3360 ways Total even = 15120 – 1680 – 3360 = 10080 ways OR ********2 in 8!/2!3! = 3360 ways ********6 in 8!/2!2!3! = 1680 ways ********8 in 8!/2!2!2! = 5040ways Total = 10080 ways OR "15120" ×6/9 = 10080	B1 B1 M1 A1 **[4]** B1 B1 M1 A1 M2 A2	Correct ways end in 3 Correct ways end in 7 Finding odd and subt from 15120 or their **(i)** Correct answer One correct way end in even correct way end in another even Summing 2 or 3 ways Correct answer Mult their **(i)** by 2/3 oe Correct answer

(b)	T(3) S(6) G(14) 1 1 3 in $3\times6\times{}^{14}C_3 = 6552$ 1 3 1 in $3\times{}^{6}C_3\times14 = 840$ 3 1 1 in $1\times6\times14 = 84$ 2 2 1 in ${}^{3}C_2\times{}^{6}C_2\times14 = 630$ 2 1 2 in ${}^{3}C_2\times6\times{}^{14}C_2 = 1638$ 1 2 2 in $3\times{}^{6}C_2\times{}^{14}C_2 = 4095$ Total ways = 13839 (13800)	M1 M1 M1 B1 A1 **[5]**	Mult 3 (combinations) together assume $6 = {}^{6}C_1$etc Listing at least 4 different options Summing at least 4 different options At least 3 correct numerical options Correct answer

47 (a) (i)	N*****B Number of ways = $\frac{5!}{3!}$ = 20	B1 B1 B1 **3**	5! Seen in num oe or alone mult by $k \geqslant 1$ 3! Seen in denom can be mult by $k \geqslant 1$ Correct final answer
(ii)	B(AAA)NNS Number of ways = $\frac{5!}{2!}$ or ${}^{5}P_3$ = 60	M1 M1 A1 **3**	5! seen as a num can be mult by $k \geqslant 1$ Dividing by 2! Correct final answer
(b)	${}^{14}C_9$ total options = 2002 T and M both in ${}^{12}C_7 = 792$ Ans 2002 – 792 = 1210 OR Neither in ${}^{12}C_9 = 220$ One in ${}^{12}C_8 = 495$ Other in ${}^{12}C_8 = 495$	M1 B1 A1 **3** M1 B1	${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt ${}^{12}C_7$ (792) seen Correct final answer Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9 + {}^{12}P_8 + {}^{12}P_8$ here only Second correct option seen accept another 495 or if M1 not awarded, any correct option

48 (i)	W S D 1 1 3 = $6\times4\times{}^{3}C_3 = 24$ 1 3 1 = $6\times{}^{4}C_3\times3 = 72$ 3 1 1 = ${}^{6}C_3\times4\times3 = 240$ 1 2 2 = $6\times{}^{4}C_2\times{}^{3}C_2 = 108$ 2 1 2 = ${}^{6}C_2\times4\times{}^{3}C_2 = 180$ 2 2 1 = ${}^{6}C_2\times{}^{4}C_2\times3 = 270$ Total = 894	M1 M1 M1 B1 A1 **[5]**	Listing at least 4 different options Mult 3 (combs) together assume $6 = {}^{6}C_1$, $\Sigma r=5$ Summing at least 4 different evaluated/unsimplified options >1 At least 3 correct unsimplified options Correct answer
(ii)	${}^{3}P_2 \times {}^{10}P_8$ = 10886400	B1 B1 B1 **[3]**	${}^{3}P_2$ oe seen multiplied either here or in (iii) $k^{10}P_x$ seen or $k^{y}P_8$ with no addition, $k\geqslant1$, $y>8$, $x<10$ Correct answer, nfww
(iii)	DSWSWSWSWD or DWSWSWSWSD D in ${}^{3}P_2$ ways = 6 S in ${}^{4}P_4$ ways = 24 W in ${}^{6}P_4 = 360$ Swap SW in 2 ways Total = 103680 ways	 B1 B1 B1 **[3]**	If ${}^{3}P_2$ has not gained credit in (ii) may be awarded ${}^{4}P_4$ or ${}^{6}P_4$ oe seen multiplied or common in all terms (no division) Mult by 2 (condone 2!) Correct answer, 3sf or better, nfww

49 (i)	**5 (i)** eg **(EEEE)*** Number of ways = $\frac{6!}{2!2!}$ = 180	**M1** **M1** **A1** [3]	Mult by 6! oe Dividing by 2!2! oe Correct answer
(ii)	S*******T or T*******S Number of ways = $\frac{7!}{4!2!} \times 2$ = 210	**M1** **M1** **A1** [3]	Mult by 7! Or dividing by one of 2! or 4! Mult by 2 Correct answer
(iii)	exactly one E in $^6C_3$ ways = 20	**M1** **M1** **A1** [3]	^6C_x as a single answer $^xC_3$ as a single answer correct answer

50	P(no men) $\frac{^9C_6}{^{16}C_6} = \frac{84}{8008} = \frac{21}{2002} = \frac{3}{286}$ = 0.0105 OR $\frac{9}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \times \frac{5}{12} \times \frac{4}{11} = 0.0105$	**B1** **B1** **B1** 3 **B1** **B1** **B1**	$^9C_6$ seen anywhere $^{16}C_6$ seen as denom of fraction oe Correct final answer (9 × 8 × 7 × 6 × 5 × 4) seen anywhere Correct unsimplified denom Correct final answer
51 (i)	$\frac{1}{4}$	**B1** 1	
(ii)	$\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) = \frac{81}{1024} = 0.0791$	**M1** **A1** 2	Expression of form $p^4(1-p)$ only, p = 1/4 or 3/4 Correct answer
(iii)	P(all diff) = $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times 4!$ $= \frac{3}{32}$ (0.0938) OR $1 \times \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{3}{32}$	**M1** **M1** **A1** 3	4! on numerator seen mult by k ⩾ 1 or 3×2×1 on num oe, must be in a fraction. 4^4 on denom or 4^3 on denom with the 3× 2× 1 Correct answer
52 (i)	Two in same taxi: $^6C_2 \times {^4C_4} \times 2$ or $^6C_2 + {^6C_4}$ = 30	**M1** **M1** **A1** 3	$^6C_4$ or $^6C_2$ oe seen anywhere 'something' ×2 only or adding 2 equal terms Correct final answer
(ii)	MJS in taxi $(^5C_1 \times 2 \times 2) \times {^4P_4}$ = 480	**M1** **M1** **M1** **A1** 4	$^5P_1$, $^5C_1$ or 5 seen anywhere Mult by 2 or 4 oe Mult by $^4P_4$ oe eg 4! or $4 \times {^3P_3}$ or can be part of 5! Correct final answer

53 (a)	e.g. **(AAOOOI)***** $\frac{8!}{2!2!} \times \frac{6!}{2!3!} = 604800$	**B1** **M1** **A1** 3	8! (8 × 7!) or 6! seen anywhere, either alone or in numerator) Dividing by at least 3 of 2!2!2!3! (may be fractions added) Correct answer
(b)	C(7) E(6) A(4) 1 1 2 $= 7 \times 6 \times {}^4C_2 = 252$ 1 2 1 $= 7 \times {}^6C_2 \times 4 = 420$ 1 3 0 $= 7 \times {}^6C_3 \times 1 = 140$ 2 1 1 $= {}^7C_2 \times 6 \times 4 = 504$ 2 2 0 $= {}^7C_2 \times {}^6C_2 \times 1 = 315$ 3 1 0 $= {}^7C_3 \times 6 \times 1 = 210$ Total = 1841	**M1** **A1** **M1*** **DM1** **A1** 5	Mult 3 appropriate combinations together assume $6={}^6C_{1}$, $1={}^4C_0$ etc., $\sum r=4$, C&E both present At least 3 correct unsimplified products Listing at least 4 different correct options Summing at least 4 outcomes, involving 3 combs or perms, $\sum r=4$ Correct answer SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for ${}^7C_1 \times {}^6C_1 \times$(sum of at least 4 outcomes)