A1 ITERATION MARKING SCHEME

1 (i) Evaluate cubic when $x = -1$ and $x = 0$ M1

Justify given statement correctly A1 **2**

[If calculations are not given but justification uses correct statements about signs, award B1.]

(ii) State $x = \dfrac{2x^3 - 1}{3x^2 + 1}$, or equivalent B1

Rearrange this in the form $x^3 + x + 1 = 0$ (or *vice versa*) B1 **2**

2 **(i)** Obtain area of *ONB* in terms of *r* and α e.g. $\frac{1}{2}r^2 \cos\alpha \sin\alpha$ B1

Equate area of triangle in terms of *r* and α to $\frac{1}{2}\left(\frac{1}{2}r^2\alpha\right)$ or equivalent M1

Obtain given form, sin $2\alpha = \alpha$, correctly A1 **3**

[Allow use of *OA* and/or *OB* for *r*.]

(ii) Make recognisable sketch in one diagram over the given range of two suitable graphs, e.g. $y = \sin 2x$ and $y = x$ B1

State or imply link between intersections and roots and justify the given answer B1 **2**

[Allow a single graph and its intersection with $y = 0$ to earn full marks.]

(iii) Use the iterative formula correctly at least once M1

Obtain final answer 0.95 A1

Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in (0.945, 0.955) A1 **3**

[SR: Allow the M mark if calculations are attempted in degree mode.]

3 **(i)** Make recognisable sketch of a relevant graph over the given range,

e.g. $y = \text{cosec } x$ **B1**

Sketch the other relevant graph, e.g. $y = \frac{1}{2}x + 1$, and justify the given statement **B1** **2**

(ii) Consider sign of cosec $x - \frac{1}{2}x - 1$ at $x = 0.5$ and $x = 1$, or equivalent **M1**

Complete the argument correctly with appropriate calculations **A1** **2**

(iii) Rearrange $\text{cosec } x = \frac{1}{2}x + 1$ in the given form, or *vice versa* **B1** **1**

(iv) Use the iterative formula correctly at least once **M1**

Obtain final answer $x = 0.80$ **A1**

Show sufficient iterations to at least 3 d.p. to justify its accuracy to 2 d.p., **or** show there is a sign change in the interval (0.795, 0.805) **A1** **3**

4	**(i)**	Consider sign of $x^3 - x - 3$, or equivalent	M1	
		Justify the given statement	A1	**[2]**
	(ii)	Apply an iterative formula correctly at least once, with initial value $x_1 = 1.5$	M1	
		Show that (*A*) fails to converge	A1	
		Show that (*B*) converges	A1	
		Obtain final answer 1.67	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.665, 1.675)	A1	**[5]**

5	**(i)**	Make recognizable sketch of a relevant graph, e.g. $y = 2\cot x$	B1	
		Sketch an appropriate second graph, e.g. $y = 1 + e^x$ correctly and justify the given statement	B1	**2**
	(ii)	Consider sign of $2\cot x - 1 - e^x$ at $x = 0.5$ and $x = 1$, or equivalent	M1	
		Complete the argument with appropriate calculations	A1	**2**
	(iii)	Show that the given equation is equivalent to $x = \tan^{-1}\left(\frac{2}{1+e^x}\right)$, or *vice versa*.	B1	**1**
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.61	A1	
		Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (0.605, 0.615)	A1	**3**

6	**(i)**	Use product rule	M1	
		Obtain correct derivative $\cos 2x - 2x \sin 2x$	A1	
		Equate derivative to zero and obtain given answer correctly	A1	3
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.43	A1	
		Show sufficient iterations to at least 3d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.425, 0.435)	A1	3
	(iii)	Attempt integration by parts and obtain $\pm kx \sin 2x \pm \int l \sin 2x \, dx$, where $k, l = \frac{1}{2}, 1$, or 2	M1*	
		Obtain $\frac{1}{2}x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$	A1	
		Obtain indefinite integral $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x$	A1	
		Use limits $x = 0$ and $x = \frac{1}{4}\pi$ having integrated twice	M1(dep)*	
		Obtain answer $\frac{1}{8}\pi - \frac{1}{4}$, or exact equivalent	A1	5

7	**(i)**	Using the formulae $\frac{1}{2}r^2\alpha$ and $\frac{1}{2}r^2 \sin\alpha$, or equivalent, form an equation	M1	
		Obtain given equation correctly [Allow the use of *OA* and/or *OB* for *r*.]	A1	**2**
	(ii)	Consider sign of $x - 2\sin x$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	**2**
	(iii)	State or imply the equation $x = \frac{1}{3}(x + 4\sin x)$	B1	
		Rearrange this as $x = 2\sin x$, or work *vice versa*	B1	**2**
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.90	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.895, 1.905) [The final answer 1.9 scores A0.]	A1	**3**

8 **(i)** Make a recognisable sketch of an appropriate graph, e.g. $y = \ln x$ — B1
Sketch an appropriate second graph, e.g. $y = 2 - x$, correctly and justify the given statement — B1 **[2]**

(ii) Consider sign of $2 - x - \ln x$ when $x = 1.4$ and $x = 1.7$, or equivalent — M1
Complete the argument with correct calculations — A1 **[2]**

(iii) Rearrange the equation $x = \frac{1}{3}(4 + x - 2\ln x)$ as $2 - x = \ln x$, or *vice versa* — B1 **[1]**

(iv) Use the iterative formula correctly at least once — M1
Obtain final answer 1.56 — A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.555, 1.565) — A1 **[3]**

9 **(i)** State or imply $r = a$ cosec x, or equivalent — B1
Using perimeters, obtain a correct equation in x, e.g. $2a$ cosec $x + ax$ cosec $x = 4a$, or $2r + rx = 4a$ — B1
Deduce the given form of equation correctly — B1 [3]

(ii) Use the iterative formula correctly at least once — M1
Obtain final answer 0.76 — A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the value of $\sin x - \frac{1}{4}(2 + x)$ in the interval (0.755, 0.765) — A1 [3]

10 (**i**) Integrate by parts and reach $kx\mathrm{e}^{\frac{1}{2}x} - k\int \mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x$ — M1
Obtain $2x\mathrm{e}^{\frac{1}{2}x} - 2\int \mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x$ — A1
Complete the integration, obtaining $2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$, or equivalent — A1
Substitute limits correctly and equate result to 6, having integrated twice — M1
Rearrange and obtain $a = \mathrm{e}^{-\frac{1}{2}a} + 2$ — A1 [5]

(**ii**) Make recognizable sketch of a relevant exponential graph, e.g. $y = \mathrm{e}^{-\frac{1}{2}x} + 2$ — B1
Sketch a second relevant straight line graph, e.g. $y = x$, or curve, and indicate the root — B1 [2]

(**iii**) Consider sign of $x - \mathrm{e}^{-\frac{1}{2}x} - 2$ at $x = 2$ and $x = 2.5$, or equivalent — M1
Justify the given statement with correct calculations and argument — A1 [2]

(**iv**) Use the iterative formula $x_{n+1} = 2 + \mathrm{e}^{-\frac{1}{2}x_n}$ correctly at least once, with $2 \leq x_n \leq 2.5$ — M1
Obtain final answer 2.31 — A1
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (2.305, 2.315) — A1 [3]

11 **(i)** Compare signs of $x^3 - 2x - 2$ when $x = 1$ and $x = 2$, or equivalent — M1
Complete the argument with correct calculations — A1 **2**

(ii) State or imply the equation $x = (2x^3 + 2) / (3x^2 - 2)$ — B1
Rearrange this in the form $x^3 - 2x - 2 = 0$, or work *vice versa* — B1 **2**

(iii) Use the iterative formula correctly at least once with $x_n > 0$ — M1
Obtain final answer 1.77 — A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change
In the interval (1.765, 1.775) — A1 **3**

12 **(i)** Evaluate, or consider the sign of, $x^3 - 8x - 13$ for two integer values of x, or equivalent — M1
Conclude $x = 3$ and $x = 4$ with no errors seen — A1 [2]

(ii) Use the iterative formula correctly at least once — M1
Obtain final answer 3.43 — A1
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.425, 3.435) — A1 [3]

13 **(i)** Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}r^2 \sin\theta$, or equivalent, form an equation — M1
Obtain a correct equation in r and x and/or $x/2$ in any form — A1
Obtain the given equation correctly — A1 [3]

(ii) Consider the sign of $x - (\frac{3}{4}\pi - \sin x)$ at $x = 1.3$ and $x = 1.5$, or equivalent — M1
Complete the argument with correct calculations — A1 [2]

(iii) Use the iterative formula correctly at least once — M1
Obtain final answer 1.38 — A1
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.375, 1.385) — A1 [3]

14 **(i)** Use correct quotient or product rule — M1
Obtain correct derivative in any form — A1
Equate derivative to zero and solve for x — M1
Obtain the given answer correctly — A1 [4]

(ii) Use the iterative formula correctly at least once — M1
Obtain final answer 4.49 — A1
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the interval (4.485, 4.495) — A1 [3]

15 **(i)** Make recognisable sketch of a relevant graph over the given range B1
Sketch the other relevant graph on the same diagram and justify the given statement B1 [2]

(ii) Consider sign of $4x^2 - 1 - \cot x$ at $x = 0.6$ and $x = 1$, or equivalent M1
Complete the argument correctly with correct calculated values A1 [2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.73 A1
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.725, 0.735) A1 [3]

16 **(i)** Attempt integration by parts M1
Obtain $-x^{-1}\ln x + \int \frac{1}{x^2}\,\mathrm{d}x$, $\frac{x\ln x - x}{x^2} + 2\int \frac{\ln x}{x^2}\,\mathrm{d}x - 2\int \frac{1}{x^2}\,\mathrm{d}x$ or equivalent A1
Obtain $-x^{-1}\ln x - x^{-1}$ or equivalent A1
Use limits correctly, equate to $\frac{2}{5}$ and attempt rearrangement to obtain a in terms of $\ln a$ M1
Obtain given answer $a = \frac{5}{3}(1 + \ln a)$ correctly A1 [5]

(ii) Use valid iterative formula correctly at least once M1
Obtain final answer 3.96 A1
Show sufficient iterations to › 4 dp to justify accuracy to 2 dp or show sign change in interval (3.955, 3.965) A1 [3]
[4 → 3.9772 → 3.9676 → 3.9636 → 3.9619]

SR: Use of $a_{n+1} = \mathrm{e}^{\left(\frac{3}{5}a_n - 1\right)}$ to obtain 0.50 also earns 3/3.

17 **(i)** State or imply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$ B1
Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent M1
Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$ A1 [3]

(ii) Use iterative formula correctly at least once M1
Obtain value for θ of 2.11 A1
Show sufficient iterations to justify value of θ or show sign change in interval (2.105, 2.115) A1
Use correct trigonometry to find an expression for the length of AB M1
e.g. $20\sin 1.055$ or $\sqrt{200 - 200\cos 2.11}$
Hence 17.4 A1 [5]
[2.1 → 2.1198 → 2.1097 → 2.1149 → 2.1122]

18 **(i)** State or imply $CT = r \tan x$ or $OT = r \sec x$, or equivalent — B1
Using correct area formulae, form an equation in r and x — M1
Obtain the given answer correctly — A1 [3]

(ii) Use the iterative formula correctly at least once — M1
Obtain the final answer 1.35 — A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p. , or show there is a sign change in the interval (1.345, 1.355) — A1 [3]

19 **(i)** Make recognisable sketch of a relevant graph over the given range — B1
Sketch the other relevant graph and justify the given statement — B1 [2]

(ii) Consider the sign of $\cot x - \left(1 + x^2\right)$ at $x = 0.5$ and $x = 0.8$, or equivalent — M1
Complete the argument with correct calculated values — A1 [2]

(iii) Use the iterative formula correctly at least once with $0.5 \le x_n \le 0.8$ — M1
Obtain final answer 0.62 — A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625) — A1 [3]

20 **(i)** Make recognisable sketch of a relevant graph over the given interval — B1
Sketch the other relevant graph and justify the given statement — B1 [2]

(ii) Consider the sign of $\sec x - (3 - \frac{1}{2} x^2)$ at $x = 1$ and $x = 1.4$, or equivalent — M1
Complete the argument with correct calculated values — A1 [2]

(iii) Convert the given equation to $\sec x = 3 - \frac{1}{2}x^2$ or work *vice versa* — B1

(iv) Use a correct iterative formula correctly at least once — M1
Obtain final answer 1.13 — A1
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135) — A1 [3]
[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]

21	**(i)** **Either**		
	Use integration by parts and reach an expression $kx^2 \ln x \pm n\int x^2 . \frac{1}{x} dx$	M1	
	Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$ or equivalent	A1	
	Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
	Or		
	Use Integration by parts and reach an expression $kx(x\ln x - x) \pm m\int x\ln x - x dx$	M1	
	Obtain $I = (x^2 \ln x - x^2) - I + \int x dx$	A1	
	Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
	Substitute limits correctly and equate to 22, having integrated twice	DM1*	
	Rearrange and confirm given equation $a = \sqrt{\frac{87}{2\ln a - 1}}$	A1	[5]
	(ii) Use iterative process correctly at least once	M1	
	Obtain final answer 5.86	A1	
	Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)	A1	
	(6 → 5.8030 → 5.8795 → 5.8491 → 5.8611 → 5.8564)		[3]

22 **(i)** Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero — M1
Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$ — A1
Obtain $kt(t^3 + et + f) = 0$ or equivalent — M1
Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$ — A1 [4]

(ii) Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent — M1
Justify the given statement with correct calculations (–0.06 and 0.02) — A1 [2]

(iii) Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$ — M1
Obtain final answer 1.276 — A1
Show sufficient iterations to justify answer or show there is a change of sign in interval (1.2755, 1.2765) — A1 [3]

(iv) Evaluate $\tan^{-1}$ (answer from part **(iii)**) to obtain at least one value — M1
Obtain –2.24 and 0.906 — A1
State $-\pi$, 0 and π — B1 [3]
[SR If A0, B0, allow B1 for any 3 roots]

23 **(i)** Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}bh$, form an equation an a and θ M1

Obtain given answer A1 [2]

(ii) Use the iterative formula correctly at least once M1

Obtain answer $\theta = 1.32$ A1

Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change in the interval (1.315, 1.325) A1 [3]

24 **(i)** Substitute for x and dx throughout the integral M1

Obtain $\int 2u\cos u\,\mathrm{d}u$ A1

Integrate by parts and obtain answer of the form $au\sin u + b\cos u$, where $ab \neq 0$ M1

Obtain $2u\sin u + 2\cos u$ A1

Use limits $u = 0$, $u = p$ correctly and equate result to 1 M1

Obtain the given answer A1 [6]

(ii) Use the iterative formula correctly at least once M1

Obtain final answer $p = 1.25$ A1

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.245, 1.255) A1 [3]

25 **(i)** Use correct product or quotient rule and use chain rule at least once M1

Obtain derivative in any correct form A1

Equate derivative to zero and solve an equation with at least two non-zero terms for real x M1

Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent A1 [4]

(ii) State a suitable equation, e.g. $\alpha = \sqrt{(\ln(4 + 8\alpha^2))}$ B1

Rearrange to reach $e^{\alpha^2} = 4 + 8\alpha^2$ B1

Obtain $\frac{1}{2} = e^{-\frac{1}{2}\alpha^2}\sqrt{(1 + 2\alpha^2)}$, or work *vice versa* B1 [3]

(iii) Use the iterative formula correctly at least once M1

Obtain final answer 1.86 A1

Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865) A1 [3]

26 **(i)** Find y for $x = -2$ M1
Obtain 0 and conclude that $\alpha = -2$ A1 [2]

(ii) Either Find cubic factor by division or inspection or equivalent M1
Obtain $x^3 + 2x - 8$ A1
Rearrange to confirm given equation $x = \sqrt[3]{8 - 2x}$ A1
Or Derive cubic factor from given equation and form product with $(x - \alpha)$ M1
$(x+2)(x^3 + 2x - 8)$ A1
Obtain quartic $x^4 + 2x^3 + 2x^2 - 4x - 16\ (= 0)$ A1
Or Derive cubic factor from given equation and divide the quartic by the cubic M1
$(x^4 + 2x^3 + 2x^2 - 4x - 16) \div (x^3 + 2x - 8)$ A1
Obtain correct quotient and zero remainder A1 [3]

(iii) Use the given iterative formula correctly at least once M1
Obtain final answer 1.67 A1
Show sufficient iterations to at least 4 d.p. to justify answer 1.67 to 2 d.p. or show there is a change of sign in interval (1.665, 1.675) A1 [3]

27 **(i)** Use the iterative formula correctly at least once M1
Obtain final answer 3.6840 A1
Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405) A1 [3]

(ii) State a suitable equation, e.g. $x = \dfrac{x(x^3 + 100)}{2(x^3 + 25)}$ B1
State that the value of α is $3\sqrt{50}$, or exact equivalent B1 [2]

28 **(i)** State the correct derivatives $2e^{2x-3}$ and $2/x$ B1
Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$ M1
Obtain the given result correctly (or work *vice versa*) A1 [3]

(ii) Consider the sign of $a - \dfrac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent M1
Complete the argument with correct calculated values A1 [2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.35 A1
Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) A1 [3]

29 **(i)** State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$ B1

Use correct formula to express the area of sector ABC in terms of r and θ M1

Use correct area formulae to express the area of a segment in terms of r and θ M1

State a correct equation in r and θ in any form A1

Obtain the given answer A1 **[5]**

[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]

(ii) Use the iterative formula correctly at least once M1

Obtain final answer 0.95 A1

Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955) A1 **[3]**

30 **(i)** Use integration by parts to obtain $ax\mathrm{e}^{-\frac{1}{2}x} + \int b\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$ M1*

Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} + \int 8\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$ or unsimplified equivalent A1

Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} - 16\mathrm{e}^{-\frac{1}{2}x}$ A1

Use limits correctly and equate to 9 M1(d*M)

Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly A1 [5]

(ii) Use correct iteration formula correctly at least once M1

Obtain final answer 3.77 A1

Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775) A1 [3]

$[3.5 \to 3.6766 \to 3.7398 \to 3.7619 \to 3.7696 \to 3.7723]$

31 **(i)** Sketch $y = \mathrm{cosec}\,x$ for at least $0, x, \pi$ B1

Sketch $y = x(\pi - x)$ for at least $0, x, \pi$ B1

Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales B1 [3]

(ii) Use $\mathrm{cosec}x = \frac{1}{\sin x}$ and commence rearrangement M1

Obtain given equation correctly, showing sufficient detail A1 [2]

(iii) **(a)** Use the iterative formula correctly at least once M1

Obtain final answer 0.66 A1

Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (0.655, 0.665) A1 [3]

(b) Obtain 2.48 B1 [1]

32 **(i)** Use correct arc formula and form an equation in r and x — M1
Obtain a correct equation in any form — A1
Rearrange in the given form — A1 **3**

(ii) Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$ — M1
Complete the argument correctly with correct calculated values — A1 **2**

(iii) Use the iterative formula correctly at least once — M1
Obtain final answer 1.21 — A1
Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205,1.215) — A1 **3**

33 **(i)** Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$ — M1
Complete the argument correctly with correct calculated values — A1 **2**

(ii) State or imply $\alpha = \frac{1}{2}\ln(1 + 10/\alpha)$ — B1
Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work *vice versa* — B1 **2**

(iii) Use the iterative formula correctly at least once — M1
Obtain final answer 1.14 — A1
Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145) — A1 **3**

34 **(i)** Integrate and reach $bx\ln 2x - c\int x.\frac{1}{x}\,dx$, or equivalent — M1*

Obtain $x\ln 2x - \int x.\frac{1}{x}\,dx$, or equivalent — A1
Obtain integral $\ln 2x - x$, or equivalent — A1
Substitute limits correctly and equate to 1, having integrated twice — M1(dep*)
Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$ — A1
Obtain the given answer — A1 **[6]**

(ii) Use the iterative formula correctly at least once — M1
Obtain final answer 1.94 — A1
Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945). — A1 **[3]**

35 **(i)** Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$ B1
Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$ B1
Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$ B1 [3]

(ii) Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$ M1
Complete argument correctly with correct calculations (–11.6 and 25.6) A1 [2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 3.377 A1
Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775) A1 [3]

(iv) Attempt value of $\ln(x+1)$ M1
Obtain 1.48 A1 [2]

36 **(i)** Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$ B1
Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
Obtain $\frac{d}{dx} = \frac{1}{2}(3t^2 + 2)(t + 2)$ A1
Identify value of t at the origin as –1 B1
Substitute to obtain $\frac{5}{2}$ as gradient at the origin A1 [5]

(ii) **(a)** Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2 + 2} - 2$ B1 [1]

(b) Use the iterative formula correctly at least once M1
Obtain value $p = -1.924$ or better (–1.92367…) A1
Show sufficient iterations to justify accuracy or show a sign change in appropriate interval A1
Obtain coordinates (–5.15, –7.97) A1 [4]

39 **(i)** State or imply $AT = r \tan x$ or $BT = r \tan x$ B1
Use correct arc formula and form an equation in r and x M1
Rearrange in the given form A1 **[3]**

(ii) Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.3$ M1
Complete the argument correctly with correct calculated values A1 **[2]**

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.11 A1
Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115) A1 **[3]**

38 **(i)** Integrate and reach $\pm x\sin x \mp \int \sin x \, dx$ M1*

Obtain integral $x\sin x + \cos x$ A1

Substitute limits correctly, must be seen since AG, and equate result to 0.5 M1(dep*)

Obtain the given form of the equation A1 **4**

(ii) *EITHER*: Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leqslant \frac{\pi}{2}$ M1

OR: Using limits correctly, consider the sign of $\left[x\sin x + \cos x\right]_0^a - 0.5$, or compare the value of $\left[x\sin x + \cos x\right]_0^a$ with 0.5, for $a = 1$ AND for another relevant value, e.g $a = 1.5 \leqslant \frac{\pi}{2}$. M1

Complete the argument, so change of sign, or above and below stated, both with correct calculated values A1 **2**

(iii) Use the iterative formula correctly at least once M1

Obtain final answer 1.2461 A1

Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615) A1 **3**

39 **(i)** Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent **M1**

Obtain the pair $x = 2$ and $x = 3$, with no errors seen **A1** [2]

(ii) State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$ **B1**

Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa* **B1** [2]

(iii) Use the iterative formula correctly at least once **M1**

Obtain final answer 2.219 **A1**

Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195) **A1** [3]

40 **(i)** Use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ and equate $\frac{dy}{dx}$ to 4 **M1**

Obtain $\frac{4p^3}{2p+3} = 4$ or equivalent **A1**

Confirm given result $p = \sqrt[3]{2p+3}$ correctly **A1** [3]

(ii) Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0 **M1**

Justify result with correct calculations and argument (–0.076 and 0.087 or –0.77 and 1 respectively) **A1** [2]

(iii) Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$ **M1**

Obtain final answer 1.89 **A1**

Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895) **A1** [3]

41 **(i)** Make recognizable sketch of a relevant graph **B1**

Sketch the other relevant graph and justify the given statement **B1**

[2]

(ii) State $x = \frac{1}{2}\ln(25/x)$ **B1**

Rearrange this in the form $5e^{-x} = \sqrt{x}$ **B1**

[2]

(iii) Use the iterative formula correctly at least once **M1**

Obtain final answer 1.43 **A1**

Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435) **A1**

[3]

42 **(i)** Use correct quotient or chain rule **M1**

Obtain correct derivative in any form **A1**

Obtain the given answer correctly **A1** [3]

(ii) State a correct equation, e.g. $-e^{-a} = -\operatorname{cosec} a \cot a$ **B1**

Rearrange it correctly in the given form **B1** [2]

(iii) Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ **M1**

Complete the argument correctly with correct calculated values **A1** [2]

(iv) Use the iterative formula correctly at least once **M1**

Obtain final answer 1.317 **A1**

Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1,3175) **A1** [3]

43 **(i)** Use the product rule **M1**

Obtain correct derivative in any form **A1**

Equate 2-term derivative to zero and obtain the given answer correctly **A1**

[3]

(ii) Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or compare values of relevant expressions at $p = 2$ and $p = 2.5$ **M1**

Complete the argument correctly with correct calculated values **A1**

[2]

(iii) Use the iterative formula correctly at least once **M1**

Obtain final answer 2.15 **A1**

Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145,2.155) **A1**

44 **(i)**	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	[2]
(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	**M1** **A1**	[2]
(iii)	State $x = 2\sin^{-1}\left(\dfrac{3}{x+3}\right)$ Rearrange this in the form $\text{cosec}\,\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\dfrac{x}{2} = \left(\dfrac{3}{x+3}\right)$ for first B1	**B1** **B1**	[2]
(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	[3]

45 **(i)**	Differentiate both equations and equate derivatives Obtain equation $\cos a - a\sin a = -\dfrac{k}{a^2}$ State $a\cos a = \dfrac{k}{a}$ and eliminate k Obtain the given answer showing sufficient working	**M1*** **A1 + A1** **DM1** **A1**	[5]
(ii)	Show clearly correct use of the iterative formula at least once Obtain answer 1.077 Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	**M1** **A1** **A1**	[3]
(iii)	Use a correct method to determine k Obtain answer $k = 0.55$	**M1** **A1**	[2]

46(i)	Use correct sector formula at least once and form an equation in r and x	**M1**
	Obtain a correct equation in any form	**A1**
	Rearrange in the given form	**A1**
	Total:	**3**
46(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
46(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.374	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	**A1**
	Total:	**3**

47(i)	Use correct product rule	**M1**
	Obtain correct derivative in any form $\left(y' = 2x\cos 2x - 2x^2 \sin 2x\right)$	**A1**
	Equate to zero and derive the given equation	**A1**
	Total:	**3**
47(ii)	Use the iterative formula correctly at least once e.g. $0.5 \to 0.55357 \to 0.53261 \to 0.54070 \to 0.53755$	**M1**
	Obtain final answer 0.54	**A1**
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	**A1**
	Total:	**3**

47(iii)	Integrate by parts and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	***M1**
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x.\frac{1}{2}\sin 2x \, dx$	**A1**
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x$, or equivalent	**A1**
	Substitute limits $x = 0$, $x = \frac{1}{4}\pi$, having integrated twice	**DM1**
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	**A1**
	Total:	**5**

48(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
48(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}\left(1/\left(1-x\right)\right)$ without suffices	**B1**
	Rearrange this as $\cot x = 1 - x$, or commence working *vice versa*	**B1**
	Total:	**2**
48(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 2.576 only	**A1**
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	**A1**
	Total:	**3**

49(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
49(ii)	Use an iterative formula correctly at least once	**M1**
	Show that (*B*) fails to converge	**A1**
	Using (*A*), obtain final answer 2.43	**A1**
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	**A1**
		4

50(i)	Integrate by parts and reach $ax^{\frac{3}{2}} \ln x + b\int x^{\frac{3}{2}}.\frac{1}{x}\ \mathrm{d}x$	***M1**
	Obtain $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{2}{3}\int x^{\frac{1}{2}}\ \mathrm{d}x$	**A1**
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	**A1**
	Substitute limits correctly and equate to 2	**DM1**
	Obtain the given answer correctly AG	**A1**
		5
50(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
50(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 3.031	**A1**
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	**A1**
		3

51(i)	Integrate by parts and reach $lx\mathrm{e}^{-\frac{1}{2}x} + m\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	**M1***
	Obtain $-2\ \mathrm{e}^{-\frac{1}{2}x} + 2\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	**A1**
	Complete the integration and obtain $-2x\mathrm{e}^{-\frac{1}{2}x} - 4\mathrm{e}^{-\frac{1}{2}x}$, or equivalent	**A1**
	Having integrated twice, use limits and equate result to 2	**M1(dep*)**
	Obtain the given equation correctly	**A1**
		5
51(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
51(iii)	Use the iterative formula $a_{n+1} = 21\ln(a_n + 2)$ correctly at least once	**M1**
	Obtain final answer 3.36	**A1**
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	**A1**
		3

52(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	**M1**	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2\sin\theta$
	State a correct equation in any form	**A1**	Given answer so check working carefully
	Obtain the given answer correctly	**A1**	
		3	
52(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	**M1**	e.g. $\mathrm{f}(\theta) = \frac{\pi}{2} + \sin\theta$ $\begin{cases}\mathrm{f}\ 2.2\ = 2.37... > 2.2\\ \mathrm{f}(2.4) = 2.24... < 2.4\end{cases}$ or $\mathrm{f}(\theta) = \theta - \frac{\pi}{2} - \sin\theta$ $\begin{cases}\mathrm{f}\ 2.2) = -0.17... < 0\\ \mathrm{f}(2.4)\ \ +0.15... > 0\end{cases}$
	Complete the argument correctly with correct calculated values	**A1**	
		2	

52(iii)	Use $\theta_{n+1} = \frac{1}{2}\pi + \sin\theta_n$ correctly at least once	**M1**	e.g.
	Obtain final answer 2.31	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)	**A1**	
		3	

2.2	2.3	2.4
2.3793	2.3165	2.2463
2.2614	2.3054	2.3512
2.3417	2.3129	2.2814
2.2881	2.3079	2.3288
2.3244		2.2970
2.3000		2.3185
2.3165		2.3041
2.3054		2.3138
2.3129		2.3072

53(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
53(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
53(iii)	Use iterative formula $x_{n+1} = \frac{3 + x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

54 (i)	Sketch a relevant graph, e.g. $y = x^3$	**B1**	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
54(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	**B1**	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	**B1**	
		2	

54(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.213	**A1**
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**
		3

55(i)	Use product rule on a correct expression	**M1**	Condone with $+\dfrac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	**A1**	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln\left(8 - x\right) - \dfrac{x}{8-x}$
	Equate derivative to 1 and obtain $x = 8 - \dfrac{8}{\ln\left(8 - x\right)}$	**A1**	**Given answer:** check carefully that it follows from correct working
			Condone the use of *a* for *x* throughout
		3	
55(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.9$ and $x = 3.1$	**M1**	$8 - \dfrac{8}{\ln 5.1} = 3.09 > 2.9, \quad 8 - \dfrac{8}{\ln 4.9} = 2.97 < 3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	**A1**	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

55(iii)	Use th $\ln(8-x_n)$ least two successive values. SR: Clear successive use of 0, 1, 2, 3 etc., or equivalent, scores M0.		2.9, 3.0897, 2.9728, 3.0460, 3.0006, 3.290, 3.0113, 3.0223, 3.0155 3.1, 2.9681, 3.0301, 2.9980, 3.0305, 3.0103, 3.0229, 3.0151 Allow M1 if values given to fewer than 4 dp
	Obtain final answer 3.02	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 3.02 to 2 d.p., or show there is a sign change in the interval (3.015, 3.025)	**A1**	Must have two consecutive values rounding correctly to 3.02
		3	

6(i)	Correct use of trigonometry to obtain AB	**B1**	$2r\cos x$ **AG**
		1	

56(i)	State at least one correct derivative	**B1**	$-2\sin\frac{1}{2}x$, $\frac{1}{(4-x)^2}$
	Equate product of derivatives to – 1	**M1**	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2} = (4-x)^2$	**A1**	
	Rearrange correctly to obtain $a = 4-\sqrt{2\sin\frac{a}{2}}$ **AG**	**A1**	
		4	
56(ii)	Calculate values of a relevant expression or pair of expressions at $a = 2$ and $a = 3$	**M1**	e.g. $a \quad 2 \quad 2 < 2.7027..$ $\begin{pmatrix}0.703\\-0.412\end{pmatrix}$ $\begin{pmatrix}2.317\\-0.995\end{pmatrix}$ $a = 3 \quad 3 > 2.587..$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	**A1**	
		2	
56(iii)	Use the iterative formula $a_{n+1} = 4-\sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	**M1**	
	Obtain final answer 2.611	**A1**	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	**A1**	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

57(i)	Correct use of trigonometry to obtain $AB \quad 2r\cos x$	**B1**	**AG**
		1	

57(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	**M1**	$\frac{1}{2}\times\frac{1}{2}\pi r^2 = \frac{1}{2}(2r\cos x)^2\, 2x$
	Obtain $x = \cos^{-1}\sqrt{\frac{\pi}{16}}$ correctly **AG**	**A1**	Via correct simplification e.g. from $\cos^2 x = \frac{\pi}{16x}$
		2	
57(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ Must be working in radians	**M1**	e.g. $x=1 \quad 1\to 1.11$ Accept $f(1)=1.11$ $x=1.5 \quad 1.5\to 1.20$ $f(1.5)=1.20$ $f(x) = x - \cos^{-1}\sqrt{\frac{\pi}{16x}}$: $f(1) = -0.111.., f(1.5) = 0.3..$ $f(x) = \cos x - \sqrt{\frac{\pi}{16x}}$: $f(1) \quad 0.097., f(1.5) = -0.291.$ For $16x\cos^2 x - \pi \quad f(1) = 1.529.., f(1.5) = -3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	**A1**	
		2	

58(i)	State $b = 3$	**B1**
		1
58(ii)	Commence division by $x - b$ and reach partial quotient $^3 + kx^2$	**M1**
	Obtain quotient $x^3 + x^2 + 3x + 2$	**A1**
	Equate quotient to zero and rearrange to make the subject a	**M1**
	Obtain the given equation	**A1**
		4
58(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}\left(2 + a_n^{\,2} + a_n^{\,3}\right)$ correctly at least once	**M1**
	Obtain final answer –0.715	**A1**
	Show sufficient iterations to 5 d.p. to justify –0.715 to 3 d.p., or show there is a sign change in the interval (–0.7145, –0.7155)	**A1**
		3

59(i)	Use correct product rule	**M1**	
	Obtain correct derivative in any form $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1}$	**A1**	
	Equate derivative to zero and derive $x = 1 + e^{\frac{1}{2(x-1)}}$ or $p = 1 + ^{\frac{1}{2(p-1)}}$	**A1**	AG
		3	
59(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.2$ and $x = 2.6$ $\mathrm{f}(x) = \ln(x-1) - \frac{1}{2(x-1)} \Rightarrow \mathrm{f}(2.2) = -0.234, \mathrm{f}(2.6) = 0.317$ $\mathrm{f}(x) = 2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1} \Rightarrow \mathrm{f}(2.2) = 0.005\ldots, \mathrm{f}(2.6) = -0.0017\ldots$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

59(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\frac{1}{2(p_n - 1)}\right)$ correctly at least once	**M1**
	Obtain final answer 2.42	**A1**
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	**A1**
		3

60(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int \sin\frac{1}{3}x\,dx$	***M1**	
	Obtain $3x\sin\frac{1}{3}x - 3\int \sin\frac{1}{3}x\,dx$	**A1**	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	**A1**	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	**DM1**	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}(-0) - 9$
	Obtain $a = \frac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	**A1**	With sufficient evidence to show how they reach the given equation
		5	
60(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	**M1**	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving $f(a) = 0$, $f(2.5) = 0.179$. and $f(3) = -0.173$ or if $f(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow f(2.5) = -0.13.., f(3) = 0.145...$
	Complete the argument correctly with correct calculated values	**A1**	Accept values to 1 sf. or better
		2	

60(iii)	Use the iterative process $a_{n+1} = a_{n+1}\dfrac{4-3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	**M1**
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	**A1**
	Obtain final answer 2.736	**A1**
		3

61(i)	Sketch a relevant graph, e.g. $y = \ln(x+2)$	**B1**	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
61(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

61(iii)	Use the iterative formula correctly at least **twice** using output from a previous iteration	**M1**
	Obtain final answer 1.23	**A1**
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	**A1**
		3