I5 SINGLE PART ANSWERS PAPER 3

1 Obtain indefinite integral of the form $a\ln(2x-1)$, where $a = \frac{1}{2}$, 1, or 2 — M1

Use limits and obtain equation $\frac{1}{2}\ln(2k-1) = 1$ — A1

Use correct method for solving an equation of the form $a\ln(2k-1) = 1$, where $a = \frac{1}{2}$, 1, or 2, for k — M1

Obtain answer $k = \frac{1}{2}(e^2+1)$, or exact equivalent — A1 **[4]**

2 Using 1 and ln x as parts reach $x\ln x \pm \int x.\frac{1}{x}\,dx$ — M1*

Obtain indefinite integral $x\ln x - x$ — A1

Substitute correct limits correctly — M1(dep*)

Obtain given answer — A1 **[4]**

3 Integrate by parts and reach $\pm x^2\cos x \pm \int 2x\cos x\,dx$ — M1

Obtain $-x^2\cos x + \int 2x\cos x\,dx$, or equivalent — A1

Complete the integration, obtaining $-x^2\cos x + 2x\sin x + 2\cos x$, or equivalent — A1

Substitute limits correctly, having integrated twice — M1

Obtain the given answer correctly — A1 [5]

4 Attempt integration by parts and reach $k(1-x)e^{-\frac{1}{2}x} \pm k\int e^{-\frac{1}{2}x}\,dx$, or equivalent — M1

Obtain $-2(1-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$, or equivalent — A1

Integrate and obtain $-2(1-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$, or equivalent — A1

Use limits $x = 0$ and $x = 1$, having integrated twice — M1

Obtain the given answer correctly — A1 [5]

5 *EITHER:* Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,\mathrm{d}x$ M1*

Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,\mathrm{d}x$, or equivalent A1

Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent A1
Substitute limits $x = 1$ and $x = 4$, having integrated twice M1(dep*)
Obtain answer $4(\ln 4 - 1)$, or exact equivalent A1

OR1: Using $u = \ln x$, or equivalent, integrate by parts and reach $ku\mathrm{e}^{\frac{1}{2}u} - m\int \mathrm{e}^{\frac{1}{2}u}\,\mathrm{d}u$ M1*

Obtain $2u\mathrm{e}^{\frac{1}{2}u} - 2\int \mathrm{e}^{\frac{1}{2}u}\,\mathrm{d}u$, or equivalent A1

Integrate again and obtain $2u\mathrm{e}^{\frac{1}{2}u} - 4\mathrm{e}^{\frac{1}{2}u}$, or equivalent A1
Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice M1(dep*)
Obtain answer $4\ln 4 - 4$, or exact equivalent A1

OR2: Using $u = \sqrt{x}$, or equivalent, integrate and obtain $ku\ln u - m\int u.\frac{1}{u}\,\mathrm{d}u$ M1*

Obtain $4u\ln u - 4\int 1\,\mathrm{d}u$, or equivalent A1

Integrate again and obtain $4u\ln u - 4u$, or equivalent A1

Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int \ln u\,\mathrm{d}u$ as $u\ln u \pm u$ M1(dep*)
Obtain answer $8\ln 2 - 4$, or exact equivalent A1

OR3: Integrate by parts and reach $I = \frac{x\ln x \pm x}{\sqrt{x}} + k\int \frac{x\ln x \pm x}{x\sqrt{x}}\,\mathrm{d}x$ M1*

Obtain $I = \frac{x\ln x - x}{\sqrt{x}} + \frac{1}{2}I - \frac{1}{2}\int \frac{1}{\sqrt{x}}\,\mathrm{d}x$ A1

Integrate and obtain $I = 2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent A1
Substitute limits $x = 1$ and $x = 4$, having integrated twice M1(dep*)
Obtain answer $4\ln 4 - 4$, or exact equivalent A1 **[5]**

6 **(a)** Use identity $\tan^2 2 \quad \sec^2 2 \quad 1$
Obtain integral of form $ax + b\tan 2x$ M1
Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of $+c$ $x = x -$ B1 A1 [3]

(b) State $\sin x\cos\frac{1}{2}\pi + \cos x\ \sin\frac{1}{6}\pi$ B1

Simplify integrand to $\cos\frac{1}{6}\pi + \frac{\cos x\sin\frac{1}{6}\pi}{\sin x}$ or equivalent B1
Integrate to obtain at least term of form $a\ln(\sin x)$ *M1
Apply limits and simplify to obtain two terms M1 dep *M
Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent A1 [5]

7 Integrate by parts and reach $ax\mathrm{e}^{-2x} + b\int \mathrm{e}^{-2x}\ \mathrm{d}x$ **M1**

Obtain $-\frac{1}{2}x\mathrm{e}^{-2x} + \frac{1}{2}\int \mathrm{e}^{-2x}\ \mathrm{d}x$, or equivalent **A1**

Complete the integration correctly, obtaining $-\frac{1}{2}x\mathrm{e}^{-2x} - \frac{1}{4}\mathrm{e}^{-2x}$, or equivalent **A1**

Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice **M1**

Obtain answer $\frac{1}{4} - \frac{1}{2}\mathrm{e}^{-1}$, or exact equivalent **A1**

[5]

8 Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x\ \mathrm{d}x$ **M1***

Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent **A1**

Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent **A1**

Use limits correctly having integrated twice **DM1***

Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen **A1** [5]

Question		
9	Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ \mathrm{d}\theta$	***M1**
	Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	**A1**
	Substitute limits correctly, having integrated twice	**DM1**
	Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	**A1**
	Total:	**4**

10	Integrate by parts and reach $ax\sin 3x + b\int \sin 3x \mathrm{d}x$	**M1***
	Obtain $\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x \mathrm{d}x$, or equivalent	**A1**
	Complete the integration and obtain $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$, or equivalent	**A1**
	Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	**M1(dep*)**
	Obtain answer $\frac{1}{18}(\pi - 2)$ OE	**A1**
	Total:	**5**

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11	Integrate by parts and reach $a\ \frac{\ln x}{x^2} + b\int \frac{1}{x}.\frac{1}{x^2}\ \mathrm{d}x$	**M1***	
	Obtain $\pm\ \frac{1}{2}\frac{\ln x}{x^2}\ \pm \int \frac{1}{x}.\frac{1}{2x^2}\ \mathrm{d}x$, or equivalent	**A1**	
	Complete integration correctly and obtain $-\frac{\ln x}{2x^2} - \frac{1}{4x^2}$, or equivalent	**A1**	Condone without '+ *C* ' ISW
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12	Commence integration and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	**M1***	
	Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent	**A1**	
	Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	**A1**	
	Use limits correctly, having integrated twice	**DM1**	
	Obtain given answer correctly	**A1**	
		5	