I1 WITH DIFFERENTIATION MARKING SCHEME

1 (**i**)	State x-coordinate of A is 1	B1	**1**
(**ii**)	Use product or quotient rule	M1	
	Obtain derivative in any correct form e.g. $-\frac{2\ln x}{x^3}+\frac{1}{x}.\frac{1}{x^2}$	A1	
	Equate derivative to zero and solve for ln x	M1	
	Obtain $x = e^{\frac{1}{2}}$ or equivalent (accept 1.65)	A1	
	Obtain $y = \frac{1}{2e}$ or exact equivalent not involving ln	A1	**5**
	[SR: if the quotient rule is misused, with a 'reversed' numerator or x^2 instead of x^4 in the denominator, award M0A0 but allow the following M1A1A1.]		
(**iii**)	Attempt integration by parts, going the correct way	M1	
	Obtain $-\frac{\ln x}{x}+\int\frac{1}{x}.\frac{1}{x}\,dx$ or equivalent	A1	
	Obtain indefinite integral $-\frac{\ln x}{x}-\frac{1}{x}$	A1	
	Use x-coordinate of A and e as limits, having integrated twice	M1	
	Obtain exact answer $1-\frac{2}{e}$, or equivalent	A1	**5**
	[If u = ln x is used, apply an analogous scheme to the result of the substitution.]		
2 **(i)**	Use product or quotient rule	M1*	
	Obtain first derivative $2xe^{-\frac{1}{2}x}-\frac{1}{2}x^2e^{-\frac{1}{2}x}$ or equivalent	A1	
	Equate derivative to zero and solve for non-zero x	M1(dep*)	
	Obtain answer $x = 4$	A1	**4**
(ii)	Integrate by parts once, obtaining $kx^2e^{-\frac{1}{2}x}+l\int xe^{-\frac{1}{2}x}\,dx$, where $kl \neq 0$	M1	
	Obtain integral $-2x^2e^{-\frac{1}{2}x}+4\int xe^{-\frac{1}{2}x}\,dx$, or any unsimplified equivalent	A1	
	Complete the integration, obtaining $-2(x^2+4x+8)e^{-\frac{1}{2}x}$ or equivalent	A1	
	Having integrated by parts twice, use limits x = 0 and x = 1 in the complete integral	M1	
	Obtain simplified answer $16-26e^{-\frac{1}{2}}$ or equivalent	A1	**5**

3	(i)	Use quotient or product rule	**M1**	
		Obtain derivative in any correct form	**A1**	
		Equate derivative to zero and solve for x or x^2	**M1**	
		Obtain $x = 1$ correctly	**A1**	**4**
		[Differentiating $(x^2 + 1)y = x$ using the product rule can also earn the first **M1A1**.]		
		[SR: if the quotient rule is misused, with a 'reversed' numerator or v instead of v^2 in the denominator, award **M0A0** but allow the following **M1A1**.]		
	(ii)	Obtain indefinite integral of the form $k\ln(x^2 + 1)$, where $k = ½$, 1 **or** 2	**M1***	
		Use limits $x = 0$ and $x = p$ correctly, or equivalent	**M1(dep*)**	
		Obtain answer ½ $\ln(p^2 + 1)$	**A1**	**3**
		[Also accept $-\ln\cos\theta$ or $\ln\cos\theta$, where $x = \tan\theta$, for the first **M1***.]		
	(iii)	Equate to 1 and convert equation to the form $p^2 + 1 = \exp(1/k)$	**M1**	
		Obtain answer $p = 2.53$	**A1**	**2**

4	(i)	Use product rule	M1	
		Obtain derivative in any correct form e.g. $\frac{x^{\frac{1}{2}}}{x} + \frac{x^{-\frac{1}{2}}}{2}.\ln x$	A1	
		Equate derivative to zero and solve for $\ln x$	M1	
		Obtain $x = e^{-2}$ (or $\frac{1}{e^2}$) or equivalent	A1	4
	(ii)	*EITHER*: Attempt integration by parts with $u = \ln x$	M1	
		Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \int \frac{2}{3}x^{\frac{3}{2}}.\frac{1}{x}\mathrm{d}x$, or equivalent	A1	
		OR: Attempt integration by parts with $u = x^{\frac{1}{2}}$	M1	
		Obtain $x^{\frac{1}{2}}(x\ln x - x) - \int (x\ln x - x).\frac{x^{-\frac{1}{2}}}{2}\mathrm{d}x$	A1	
		Obtain indefinite integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1	
		Use $x = 1$ and $x = 4$ as limits	M1	
		Obtain answer 4.28	A1	5

5	**(i)**	Either use correct product or quotient rule, or square both sides, use correct product rule and make a reasonable attempt at applying the chain rule	M1	
		Obtain correct result of differentiation in any form	A1	
		Set derivative equal to zero and solve for x	M1	
		Obtain $x = \frac{1}{2}$ only, correctly	A1	[4]
	(ii)	State or imply the indefinite integral for the volume is $\pi\int e^{-x}(1+2x)\mathrm{d}x$	B1	
		Integrate by parts and reach $\pm e^{-x}(1+2x) \pm \int 2e^{-x}\mathrm{d}x$	M1	
		Obtain $-e^{-x}(1+2x) + \int 2e^{-x}\mathrm{d}x$, or equivalent	A1	
		Complete integration correctly, obtaining $-e^{-x}(1+2x) - 2e^{-x}$, or equivalent	A1	
		Use limits $x = -\frac{1}{2}$ and $x = 0$ correctly, having integrated twice	M1	
		Obtain exact answer $\pi\ 2\sqrt{e} - 3$, or equivalent	A1	[6]
		[If π omitted initially or 2π or $\pi/2$ used, give B0 and then follow through.]		

6	**(i)** **EITHER** Use product and chain rule	M1	
	Obtain correct derivative in any form	A1	
	OR Square and differentiate LHS by chain rule and RHS by product rule or as powers	M1	
	Obtain correct result in any form	A1	
	Set $\frac{dy}{dx}$ equal to zero and make reasonable attempt to solve for $x \neq 0$	M1	
	Obtain answer $x = \sqrt{\frac{2}{3}}$, or exact equivalent, correctly	A1	**4**
	(ii) State or imply $dx = \cos\theta \, d\theta$ or $\frac{dx}{d\theta} = \cos\theta$	B1	
	Substitute for x and dx throughout the integral $\int y dx$	M1	
	Obtain the given form correctly with no errors seen	A1	**3**
	(iii) Attempt integration and reach indefinite integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$	M1*	
	Obtain indefinite integral $\frac{1}{8}\theta - \frac{1}{32}\sin 4\theta$, or equivalent	A1	
	Substitute limits correctly	M1(dep*)	
	Obtain exact answer $\frac{1}{16}\pi$	A1	**4**
	[Working to carry out the change of limits is needed for the A mark in **(ii)** but, if omitted, can be earned retrospectively if it is seen in part **(iii)**]		
7	**(i)** Use correct product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and find non-zero x	M1	
	Obtain $x = \exp -\frac{1}{3}$, or equivalent	A1	
	Obtain $y = -1/(3e)$, or any ln-free equivalent	A1	[5]
	(ii) Integrate and reach $kx^4 \ln x + l\int x^4 . \frac{1}{x} dx$	M1	
	Obtain $\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 \, dx$	A1	
	Obtain integral $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$, or equivalent	A1	
	Use limits $x = 1$ and $x = 2$ correctly, having integrated twice	M1	
	Obtain answer $4\ln 2 - \frac{15}{16}$, or exact equivalent	A1	[5]
8	**(i)** Obtain derivative of form $k\cos 3x \sin 3x$, any constant k	M1	
	Obtain $-24\cos 3x \sin 3x$ or unsimplified equivalent	A1	
	Obtain $-6\sqrt{3}$ or exact equivalent	A1	[3]
	(ii) Express integrand in the form $a + b\cos 6x$, where $ab \neq 0$	M1	
	Obtain $2 + 2\cos 6x$ o.e.	A1	
	Obtain $2x + \frac{1}{3}\sin 6x$ or equivalent, condoning absence of $+ c$, ft on a, b	A1√	[3]

9	**(i)**	Attempt integration by parts and reach $\pm x^2e^{-x} \pm \int 2xe^{-x}dx$	M1*	
		Obtain $-x^2e^{-x} + \int 2xe^{-x}dx$, or equivalent	A1	
		Integrate and obtain $-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$, or equivalent	A1	
		Use limits $x = 0$ and $x = 3$, having integrated by parts twice	M1(dep*)	
		Obtain the given answer correctly	A1	[5]
	(ii)	Use correct product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for non-zero x	M1	
		Obtain $x = 2$ with no errors send	A1	[4]
	(iii)	Carry out a complete method for finding the x-coordinate of P	M1	
		Obtain answer $x = 1$	A1	[2]
10	**(i)**	Use product and chain rule	M1	
		Obtain correct derivative in any form, e.g. $15\sin^2 x\cos^3 x - 10\sin^4 x\cos x$	A1	
		Equate derivative to zero and obtain a relevant equation in one trigonometric function	M1	
		Obtain $2\tan^2 x = 3$, $5\cos^2 x = 2$, or $5\sin^2 x = 3$	A1	
		Obtain answer $x = 0.886$ radians	A1	[5]
	(ii)	State or imply $du = -\sin x\, dx$, or $\frac{du}{dx} = -\sin x$, or equivalent	B1	
		Express integral in terms of u and du	M1	
		Obtain $\pm \int 5(u^2 - u^4)du$, or equivalent	A1	
		Integrate and use limits $u = 1$ and $u = 0$ (or $x = 0$ and $x = \frac{1}{2}\pi$)	M1	
		Obtain answer $\frac{2}{3}$, or equivalent, with no errors seen	A1	[5]
11	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = e^{-\frac{1}{2}}$, or equivalent	A1	
		Obtain answer $y = -\frac{1}{2}e^{-1}$, or equivalent	A1	[5]
	(ii)	Attempt integration by parts reaching $kx^3 \ln x \pm k \int x^3 . \frac{1}{x} dx$	M1*	
		Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\, dx$, or equivalent	A1	
		Integrate again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent	A1	
		Use limits $x = 1$ and $x = e$, having integrated twice	M1(dep*)	
		Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent	A1	[5]
		[SR: An attempt reaching $ax^2(x\ln x - x) + b\int 2x(x\ln x - x)\,dx$ scores M1. Then give the first A1 for $I = x^2(x\ln x - x) - 2I + \int 2x^2\,dx$, or equivalent.]		

12 (i)	Differentiate to obtain $4\cos\frac{1}{2}x - \frac{1}{2}\sec^2\frac{1}{2}x$	B1	
	Equate to zero and find value of $\cos\frac{1}{2}x$	M1	
	Obtain $\cos\frac{1}{2}x = \frac{1}{2}$ and confirm $\alpha = \frac{2}{3}\pi$	A1	[3]
(ii)	Integrate to obtain $-16\cos\frac{1}{2}x\ \ldots$	B1	
	$\ldots\ +2\ln\cos\frac{1}{2}x$ or equivalent	B1	
	Using limits 0 and $\frac{2}{3}\pi$ in $a\cos\frac{1}{2}x + b\ln\cos\frac{1}{2}x$	M1	
	Obtain $8 + 2\ln\frac{1}{2}$ or exact equivalent	A1	[4]
13 (i)	Use correct product rule	M1	
	Obtain derivative in any correct form, e.g. $\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}$	A1	
	Carry out a complete method to form an equation of the tangent at $x = 1$	M1	
	Obtain answer $y = x - 1$	A1	[4]
(ii)	State or imply that the indefinite integral for the volume is $\pi\int x(\ln x)^2\,\mathrm{d}x$	B1	
	Integrate by parts and reach $ax^2(\ln x)^2 + b\int x^2.\frac{\ln x}{x}\,\mathrm{d}x$	M1*	
	Obtain $\frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,\mathrm{d}x$, or unsimplified equivalent	A1	
	Attempt second integration by parts reaching $cx^2\ln x + d\int x^2.\frac{1}{x}\,\mathrm{d}x$	M1(dep*)	
	Complete the integration correctly, obtaining $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2$	A1	
	Substitute limits $x = 1$ and $x = \mathrm{e}$, having integrated twice	M1(dep*)	
	Obtain answer $\frac{1}{4}\pi(\mathrm{e}^2 - 1)$, or exact equivalent	A1	[7]
	[If π omitted, or 2π or $\pi/2$ used, give B0 and then follow through.] [Integration using parts $x\ln x$ and $\ln x$ is also viable.]		

14		**Either**		
	(i)	Use integration by parts and reach an expression $kx^2 \ln x \pm n \int x^2 . \frac{1}{x} dx$	M1	
		Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$ or equivalent	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Or		
		Use Integration by parts and reach an expression $kx(x\ln x - x) \pm m \int x\ln x - x dx$	M1	
		Obtain $I = (x^2 \ln x - x^2) - I + \int x dx$	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Substitute limits correctly and equate to 22, having integrated twice	DM1*	
		Rearrange and confirm given equation $a = \sqrt{\dfrac{87}{2\ln a - 1}}$	A1	[5]
	(ii)	Use iterative process correctly at least once	M1	
		Obtain final answer 5.86	A1	
		Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)	A1	
		($6 \rightarrow 5.8030 \rightarrow 5.8795 \rightarrow 5.8491 \rightarrow 5.8611 \rightarrow 5.8564$)		[3]

15	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$	A1	
		Equate derivative to zero and use a double angle formula	M1*	
		Reduce equation to one in a single trig function	M1(dep*)	
		Obtain a correct equation in any form, e.g. $10 \cos^3 x = 6 \cos x$, $4 = 6 \tan^2 x$ or $4 = 10 \sin^2 x$	A1	
		Solve and obtain $x = 0.685$	A1	[6]
	(ii)	Using $du = \pm \cos x \, dx$, or equivalent, express integral in terms of u and du	M1	
		Obtain $\int 4u^2(1-u^2) du$, or equivalent	A1	
		Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$	M1	
		Obtain answer $\dfrac{8}{15}$ (or 0.533)	A1	[4]

16 (i)	Use product rule to find first derivative	M1	
	Obtain $2xe^{2-x} - x^2e^{2-x}$	A1	
	Confirm $x = 2$ at M	A1	[3]
(ii)	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x}\,dx$	*M1	
	Obtain $-x^2e^{2-x} + \int 2xe^{2-x}\,dx$	A1	
	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$	*M1	
	Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$	A1	
	Use limits 0 and 2 having integrated twice	M1 dep *M	
	Obtain $2e^2 - 10$	A1	[6]

17 **(i)**	Use the quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	**A1**	[4]
(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
	State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
	Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
	State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent	**A1**	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$	**M1**	

18 **(i)**	Use the correct product rule	**M1**	
	Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \sqrt{5} - 1$ only	**A1**	[4]
(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\,dx$	**M1***	
	Obtain $2e^{\frac{1}{2}x}(2 \quad - x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent	**A1**	
	Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent	**A1**	
	Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice	**DM1**	
	Obtain answer 24 – 8e, or <u>exact</u> simplified equivalent	**A1**	[5]

19(i)	Use correct quotient rule or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = 2$	**A1**
	Total:	**4**
19(ii)	State or imply ordinates 1.6487…, 1.3591…, 1.4938…	**B1**
	Use correct formula, or equivalent, with $h = 1$ and three ordinates	**M1**
	Obtain answer 2.93 only	**A1**
	Total:	**3**
19(iii)	Explain why the estimate would be less than E	**B1**
	Total:	**1**

20(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
20(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\ dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\ dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

21(i)	Integrate by parts and reach $lxe^{-\frac{1}{2}x} + m\int e^{-\frac{1}{2}x}\,dx$	M1*
	Obtain $-2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x}\,dx$	A1
	Complete the integration and obtain $-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}$, or equivalent	A1
	Having integrated twice, use limits and equate result to 2	M1(dep*)
	Obtain the given equation correctly	A1
		5
21(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
21(iii)	Use the iterative formula $a_{n+1} = 2\ln(a_n + 2)$ correctly at least once	M1
	Obtain final answer 3.36	A1
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	A1
		3

22(i)	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{-3\sin x(2+\sin x) - 3\cos x\cos x}{(2+\sin x)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	M1	
	Use $\cos^2 x + \sin^2 x = 1$ and solve for $\sin x$	M1	$6\sin x - 3 = 0 \Rightarrow \sin x = \ldots$
	Obtain coordinates $x = -\pi/6$ and $y = \sqrt{3}$ ISW	A1 + A1	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	
22(ii)	State indefinite integral of the form $k\ln(2 + \sin x)$	M1*	
	Substitute limits correctly, equate result to 1 and obtain $3\ln(2 + \sin a) - 3\ln 2 = 1$	A1	or equivalent
	Use correct method to solve for a	M1(dep*)	Allow for a correct method to solve an incorrect equation, so long as that equation has a solution. $1 + \frac{1}{2}\sin a = e^{1/3} \Rightarrow a = \sin^{-1}\left[2\ e^{1/3} - 1\right]$ Can be implied by 52.3°
	Obtain answer $a = 0.913$ or better	A1	Ignore additional solutions. Must be in radians.
		4	

23(i)	Use correct quotient rule	**M1**	
	Obtain $\frac{dy}{dx} = -\mathrm{cosec}^2 x$ correctly	**A1**	AG
		2	
23(ii)	Integrate by parts and reach $ax \cot x + b\int \cot x \, dx$	***M1**	
	Obtain $-x\cot x + \int \cot x \, dx$	**A1**	OE
	State $\pm \ln \sin x$ as integral of $\cot x$	**M1**	
	Obtain complete integral $-x\cot x + \ln \sin x$	**A1**	OE
	Use correct limits correctly	**DM1**	$0 + 0 + \frac{\pi}{4} - \ln\frac{1}{\sqrt{2}}$
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	**A1**	AG
		6	