D3 Implicit
P3

1 Find the gradient of the curve with equation

$$2x^2 - 4xy + 3y^2 = 3,$$

at the point (2, 1). [4]

2 The equation of a curve is $x^3 + 2y^3 = 3xy$.

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{2y^2 - x}$. [4]

(ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

3 The equation of a curve is $xy(x + y) = 2a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [8]

4 The equation of a curve is $x^3 - x^2y - y^3 = 3$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [4]

(ii) Find the equation of the tangent to the curve at the point (2, 1), giving your answer in the form $ax + by + c = 0$. [2]

5 The equation of a curve is

$$x \ln y = 2x + 1.$$

(i) Show that $\frac{dy}{dx} = -\frac{y}{x^2}$ [4]

(ii) Find the equation of the tangent to the curve at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$. [4]

6 The equation of a curve is $3x^2 - 4xy + y^2 = 45$.

(i) Find the gradient of the curve at the point $(2, -3)$. [4]

(ii) Show that there are no points on the curve at which the gradient is 1. [3]

7 The equation of a curve is $\ln(\boldsymbol{xy}) - \boldsymbol{y}^3 = 1$.

(i) Show that $\dfrac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}\boldsymbol{x}} = \dfrac{\boldsymbol{y}}{\boldsymbol{x}(3\boldsymbol{y}^3 - 1)}$. [4]

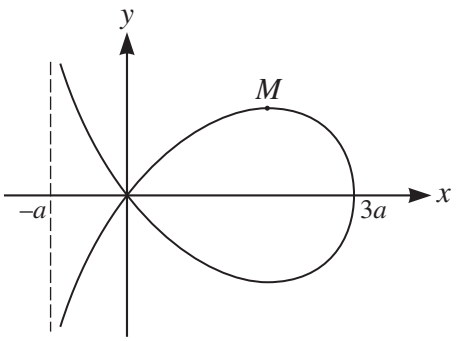
(ii) Find the coordinates of the point where the tangent to the curve is parallel to the $\boldsymbol{y}$-axis, giving each coordinate correct to 3 significant figures. [4]

8 For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \dfrac{1 + x^2}{1 + \mathrm{e}^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

9



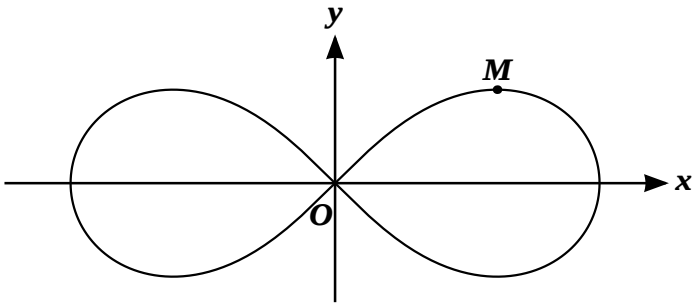
The diagram shows the curve with equation

$$\boldsymbol{x}^3 + \boldsymbol{xy}^2 + \boldsymbol{ay}^2 - 3\boldsymbol{ax}^2 = 0,$$

where $\boldsymbol{a}$ is a positive constant. The maximum point on the curve is $\boldsymbol{M}$. Find the $\boldsymbol{x}$-coordinate of $\boldsymbol{M}$ in terms of $\boldsymbol{a}$. [6]

10 A curve has equation $3e^{2x}y + e^{x}y^3 = 14$. Find the gradient of the curve at the point (0, 2). [5]

11



The diagram shows the curve $(\boldsymbol{x}^2 + \boldsymbol{y}^2)^2 = 2(\boldsymbol{x}^2 - \boldsymbol{y}^2)$ and one of its maximum points $\boldsymbol{M}$. Find the coordinates of $\boldsymbol{M}$. [7]

12 The equation of a curve is $x^3 - 3x^2y + y^3 = 3$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x^2 - 2xy}{x^2 - y^2}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

13 The equation of a curve is $xy(x - 6y) = 9a_3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [7]

14 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

(ii) Hence show that there are two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

15 The equation of a curve is $x^3y - 3xy^3 = 2a^4$, where a is a non-zero constant.

(i) Show that $\frac{dy}{dx} = \frac{3x^2y - 3y^3}{9xy^2 - x^3}$. [4]

(ii) Hence show that there are only two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

16 The equation of a curve is $x^2(x + 3y) - y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(ii) Hence find the exact coordinates of the two points on the curve at which the gradient of the normal is 1. [4]

17 The equation of a curve is $2x^3 - y^3 - 3xy^2 = 2a^3$, where a is a non-zero constant.

(i) Show that $\frac{dy}{dx} = \frac{2x^2 - y^2}{y^2 + 2xy}$. [4]

(ii) Find the coordinates of the two points on the curve at which the tangent is parallel to the y-axis. [5]

18 Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates $(1, 3)$. [4]

19 The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

