

A14 Vectors

P3

- 1 The lines l and m have vector equations

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

respectively.

- (i) Show that l and m do not intersect. [4]

The point P lies on l and the point Q has position vector $2\mathbf{i} - \mathbf{k}$.

- (ii) Given that the line PQ is perpendicular to l , find the position vector of P . [4]

- (iii) Verify that Q lies on m and that PQ is perpendicular to m . [2]

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- 2 With respect to the origin O , the points A and B have position vectors given by

$$\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + s(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

- (i) Prove that the line l does not intersect the line through A and B . [5]

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- 3 The points A and B have position vectors, relative to the origin O , given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.$$

The line l passes through A and is parallel to OB . The point N is the foot of the perpendicular from B to l .

- (i) State a vector equation for the line l . [1]

- (ii) Find the position vector of N and show that $BN = 3$. [6]

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- 4 The points A and B have position vectors, relative to the origin O , given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

- (i) Show that l does not intersect the line passing through A and B . [4]

- (ii) The point P lies on l and is such that angle PAB is equal to 60° . Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P . [6]

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- 5 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

respectively.

- (i) Show that l and m intersect. [4]

- (ii) Calculate the acute angle between the lines. [3]

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- 6 With respect to the origin O , the points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line AB and OP is perpendicular to AB .

- (i) Find a vector equation for the line AB . [1]

- (ii) Find the position vector of P . [4]

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- 7 With respect to the origin O , the lines l and m have vector equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

- (i) Prove that l and m do not intersect. [4]

- (ii) Calculate the acute angle between the directions of l and m . [3]

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- 8 With respect to the origin O , the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B , and $\overrightarrow{AP} = \lambda \overrightarrow{AB}$.

- (i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

- (ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ , find the value of λ for which OP bisects the angle AOB . [5]

- (iii) When λ has this value, verify that $AP : PB = OA : OB$. [1]

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- 9 The point P has coordinates $(-1, 4, 11)$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(i) Find the perpendicular distance from P to l .

[4]

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- 10 The lines l and m have equations $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$ respectively, where a and b are constants.

(i) Given that l and m intersect, show that

$$2a - b = 4. \quad [4]$$

(ii) Given also that l and m are perpendicular, find the values of a and b . [4]

(iii) When a and b have these values, find the position vector of the point of intersection of l and m . [2]

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- 11 With respect to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}.$$

The plane m is parallel to \overrightarrow{OC} and contains A and B .

(ii) Find the length of the perpendicular from C to the line through A and B . [5]

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- 12 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix},$$

where p is a constant. It is given that the lines intersect.

(i) Find the value of p and determine the coordinates of the point of intersection. [5]

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13 Referred to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

- (i) Find the exact value of the cosine of angle BAC . [4]

- (ii) Hence find the exact value of the area of triangle ABC . [3]

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14 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

- (i) Show that the length of the perpendicular from A to l is 15. [5]

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15 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

- (i) Show that the lines intersect for all values of a . [4]

- (ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a . [4]

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16 The straight line l_1 passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

- (i) Show that the lines l_1 and l_2 are skew. [6]

- (ii) Find the acute angle between the direction of the line l_2 and the direction of the x -axis. [3]

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17 The points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

- (i) Show that l does not intersect the line passing through A and B . [5]

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- 18 The points A , B and C have position vectors, relative to the origin O , given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C .

- (i) Find a vector equation for the line passing through A and B . [2]

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- 19 The points A , B and C have position vectors, relative to the origin O , given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral $ABCD$ is a parallelogram.

- (i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

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- 20 The points A and B have position vectors, relative to the origin O , given by $\overrightarrow{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{k}$. The line l has vector equation $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

- (i) Show that the line passing through A and B does not intersect l . [4]

- (ii) Show that the length of the perpendicular from A to l is $\frac{1}{\sqrt{2}}$. [5]

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- 21 The line l has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$.

- (i) Find the position vectors of the two points on the line whose distance from the origin is $\sqrt{10}$. [5]

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- 22 Relative to the origin O , the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. The line l has equation $\mathbf{r} = 9\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

- (i) Find the position vector of the foot of the perpendicular from A to l . Hence find the position vector of the reflection of A in l . [5]

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- 23 The points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + m\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$, where m is a constant.

- (i) Given that the line l intersects the line passing through A and B , find the value of m . [5]

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- 24 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

- (i) Show that the lines do not intersect. [3]

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- 25 The point P has position vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

- (i) Find the length of the perpendicular from P to l , giving your answer correct to 3 significant figures. [5]

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- 26 Two lines l and m have equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

- (i) Show that the lines are skew. [4]

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- 27 The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

- (i) Show that l does not intersect the line passing through A and B . [5]

The point P , with parameter t , lies on l and is such that angle PAB is equal to 120° .

- (ii) Show that $3t^2 + 8t + 4 = 0$. Hence find the position vector of P . [6]

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- 28 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

- (i) Show that l does not intersect the line passing through A and B . [5]

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29 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

- (i) The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l .
[5]

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30 Two lines l and m have equations $\mathbf{r} = a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively, where a is a constant. It is given that the lines intersect.

- (i) Find the value of a .
[4]

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