C4 With Diagram Hard Answers P3

1

(i)	*EITHER*: Multiply numerator and denominator by 2 + i , or equivalent	M1	
	Simplify numerator to 5 + 5i or denominator to 5	A1	
	Obtain answer 1 + i	A1	
	OR: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain $x = 1$	A1	
	Obtain $y = 1$	A1	
	OR: Using correct processes express u in polar form	M1	
	Obtain $u = \sqrt{2}\ (\cos 45° + \mathrm{i} \sin 45°)$, or equivalent	A1	
	Obtain answer 1 + i	A1	3
(ii)	State that the modulus is $\sqrt{2}$ or 1.41	B1√	
	State that the argument is 45° or $\frac{1}{4}\pi$ (or 0.785)	B1√	2
(iii)	Show the point representing u in a relatively correct position	B1√	
	Show a circle with centre at the point representing u	B1√	
	Indicate or imply the radius is 1	B1	3
	[NB: If the Argand diagram has unequal scales the locus is not circular in appearance, but an ellipse with centre u and equal axes parallel to the axes of the diagram earns B1√, and B1 if both semi-axes are indicated or implied to be equal to 1. In such a situation only award B1√ for a circle with centre u and a horizontal or vertical radius indicated or implied to be 1.]		
(iv)	Carry out complete strategy for calculating min| z | for the locus	M1	
	Obtain answer $\sqrt{2} - 1$ (or 0.414)	A1√	2
	[The f.t. is on the value of u.]		

2 **(i)** Find modulus of $2\cos\theta - 2\mathrm{i}\sin\theta$ and show it is equal to 2 — B1
Show a circle with centre at the point representing **i** — B1
Show a circle with radius 2 — B1 [3]

(ii) Substitute for z and multiply numerator and denominator by the conjugate of $z + 2 - \mathrm{i}$, or equivalent — M1
Obtain correct real denominator in any form — A1
Identify and obtain correct unsimplified real part in terms of $\cos\theta$, e.g. $(2\cos\theta + 2)/(8\cos\theta + 8)$ — A1
State that real part equals $\frac{1}{4}$ — A1 [4]

3 **(i)** State that the modulus of w is 1 — B1
State that the argument of w is $\frac{2}{3}\pi$ or 120° (accept 2.09, or 2.1) — B1 [2]

(ii) State that the modulus of wz is R — B1√
State that the argument of wz is $\theta + \frac{2}{3}\pi$ — B1√
State that the modulus of z/w is R — B1√
State that the argument of z/w is $\theta - \frac{2}{3}\pi$ — B1√ [4]

(iii) State or imply the points are equidistant from the origin — B1
State or imply that two pairs of points subtend $\frac{2}{3}\pi$ at the origin, or that all three pairs subtend equal angles at the origin — B1 [2]

iv) (Multiply 4 + 2i by w and use $\mathrm{i}^2 = -1$ — M1
Obtain $-(2 + \sqrt{3}) + (2\sqrt{3} - 1)\mathrm{i}$, or exact equivalent — A1
Divide 4 + 2i by w, multiplying numerator and denominator by the conjugate of w, or equivalent — M1
Obtain $-(2 - \sqrt{3}) - (2\sqrt{3} + 1)\mathrm{i}$, or exact equivalent — A1 [4]
[Use of polar form of 4 + 2i can earn M marks and then A marks for obtaining exact x + iy answers.]
[SR: If answers only seen in polar form, allow B1+B1 in **(i)**, B1√ + B1√ in **(ii)**, but A0 + A0 in **(iv)**.]

4 **(i)** Substitute $x = -2 + i$ in the equation and attempt expansion of $(-2 + i)^3$ M1
Use $i^2 = -1$ correctly at least once and solve for k M1
Obtain $k = 20$ A1 [3]

(ii) State that the other complex root is $-2 - i$ B1 [1]

(iii) Obtain modulus $\sqrt{5}$ B1
Obtain argument 153.4° or 2.68 radians B1 [2]

(iv) Show point representing u in relatively correct position in an Argand diagram B1
Show vertical line through $z = 1$ B1
Show the correct half-lines from u of gradient zero and 1 B1
Shade the relevant region B1 [4]
[SR: For parts **(i)** and **(ii)** allow the following alternative method:
State that the other complex root is $-2 - i$ B1
State quadratic factor $x^2 + 4x + 5$ B1
Divide cubic by 3-term quadratic, equate remainder to zero and solve for k, or, using 3-term quadratic, factorise cubic and obtain k M1
Obtain $k = 20$ A1]

5 **(i)** Obtain modulus $\sqrt{8}$ B1
Obtain argument $\frac{1}{4}\pi$ or 45° B1 [2]

(ii) Show 1, i and u in relatively correct positions on an Argand diagram B1
Show the perpendicular bisector of the line joining 1 and i B1
Show a circle with centre u and radius 1 B1
Shade the correct region B1 [4]

(iii) State or imply relevance of the appropriate tangent from O to the circle B1 √
Carry out complete strategy for finding $|z|$ for the critical point M1
Obtain answer $\sqrt{7}$ A1 [3]

6 **(i)** Either: Multiply numerator and denominator by (1 − 2i), or equivalent M1
Obtain –3i A1
State modulus is 3 A1
Refer to u being on negative imaginary axis or equivalent and confirm argument as $-\frac{1}{2}\pi$ A1

Or: Using correct processes, divide moduli of numerator and denominator M1
Obtain 3 A1
Subtract argument of denominator from argument of numerator M1
Obtain $-\tan^{-1}\frac{1}{2} - \tan^{-1}2$ or –0.464 – 1.107 and hence $-\frac{1}{2}\pi$ or –1.57 A1 [4]

(ii) Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction B1
Use correct trigonometry to find required value M1
Obtain $\frac{3}{2}\sqrt{2}$ or equivalent A1 [3]

(iii) Show, or imply, locus is a circle with centre $(1 + i)u$ and radius 1 M1
Use correct method to find distance from origin to furthest point of circle M1
Obtain $3\sqrt{2}+1$ or equivalent A1 [3]

7 **(a)** *EITHER*: Square $x + iy$ and equate real and imaginary parts to 1 and $-2\sqrt{6}$ respectively M1*
Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$ A1
Eliminate one variable and find an equation in the other M1(dep*)
Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent A1
Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$ A1 [5]

OR: Denoting $1 - 2\sqrt{6}i$ by $R\text{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\text{cis}(\frac{1}{2}\theta)$ and find values of R and either $\cos\theta$ or $\sin\theta$ or $\tan\theta$ M1*
Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta)$, and $\cos\theta = \frac{1}{5}$ or $\sin\theta = -\frac{2\sqrt{6}}{5}$ or $\tan\theta = -2\sqrt{6}$ A1
Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$ M1(dep*)
Obtain $\cos\frac{1}{2}\theta = \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta = \pm\sqrt{\frac{2}{5}}$, or equivalent A1
Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$, or equivalent A1
[Condone omission of ± except in the final answers.]

(b) Show point representing 3i on a sketch of an Argand diagram B1
Show a circle with centre at the point representing 3i and radius 2 B1√
Shade the interior of the circle B1√
Carry out a complete method for finding the greatest value of arg z M1
Obtain answer 131.8° or 2.30 (or 2.3) radians A1 [5]
[The f.t. is on solutions where the centre is at the point representing –3i.]

8	**(i)**	Use correct method for finding modulus of their w^2 or w^3 or both	M1	
		Obtain $\lvert w^2 \rvert = 2$ and $\lvert w^3 \rvert = 2\sqrt{2}$ or equivalent	A1	
		Use correct method for finding argument of their w^2 or w^3 or both	M1	
		Obtain $\arg(w^2) = -\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ and $\arg(w^3) = \frac{1}{4}\pi$	A1ft	[4]
	(ii)	Obtain centre $-\frac{1}{2} - \frac{1}{2}i$ (their w^2)	B1ft	
		Calculate the diameter or radius using $\lvert w - w^2 \rvert$ w21 or right-angled triangle or cosine rule or equivalent	M1	
		Obtain radius $\frac{1}{2}\sqrt{10}$ or equivalent	A1	
		Obtain $\lvert z + \frac{1}{2} + \frac{1}{2}i \rvert = \frac{1}{2}\sqrt{10}$ or equivalent	A1ft	[4]

9 **(a)** EITHER: Eliminate u or w and obtain an equation in w or in u M1

Obtain a quadratic in u or w, e.g. $u^2 - 4iu - 5 = 0$ or $w^2 + 4iw - 5 = 0$ A1

Solve a 3-term quadratic for u or for w M1

OR1: Having squared the first equation, eliminate u or w and obtain an equation in w or u M1

Obtain a 2-term quadratic in u or w, e.g. $u^2 = -3 + 4i$ A1

Solve a 2-term quadratic for u or for w M1

OR2: Using $u = a + ib$, $w = c + id$, equate real and imaginary parts and obtain 4 equations in a, b, c and d M1

Obtain 4 correct equations A1

Solve for a and b, or for c and d M1

Obtain answer $u = 1 + 2i$, $w = 1 - 2i$ A1

Obtain answer $u = -1 + 2i$, $w = -1 - 2i$ and no other A1 [5]

(b) **(i)** Show point representing $2 - 2i$ in relatively correct position B1

Show a circle with centre $2 - 2i$ and radius 2 B1√

Show line for $\arg z = -\frac{1}{4}\pi$ B1

Show line for $\mathrm{Re}\, z = 1$ B1

Shade the relevant region B1 [5]

(ii) State answer $2 + \sqrt{2}$, or equivalent (accept 3.41) B1 [1]

10 **(a)** Expand and simplify as far as $\mathrm{i}w^2 = -8\mathrm{i}$ or equivalent — B1

Obtain first answer $\mathrm{i}\sqrt{8}$, or equivalent — B1

Obtain second answer $-\mathrm{i}\sqrt{8}$, or equivalent and no others — B1 [3]

(b) **(i)** Draw circle with centre in first quadrant — M1

Draw correct circle with interior shaded or indicated — A1 [2]

(ii) Identify ends of diameter corresponding to line through origin and centre — M1

Obtain $p = 3.66$ and $q = 7.66$ — A1

Show tangents from origin to circle — M1

Evaluate $\sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ — M1

Obtain $\alpha = \frac{1}{4}\pi - \sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 0.424 — A1

Obtain $\beta = \frac{1}{4}\pi + \sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 1.15 — A1 [6]

11 **(a)** State or imply $3a + 3b\mathrm{i} + 2\mathrm{i}(a - b\mathrm{i}) = 17 + 8\mathrm{i}$ — B1

Consider real and imaginary parts to obtain two linear equations in a and b — M1*

Solve two simultaneous linear equations for a or b — M1 (dep*)

Obtain $7 - 2\mathrm{i}$ — A1 [4]

(b) Either: Show or imply a triangle with side 2 — B1

State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$ — B1

State or imply argument is $\frac{1}{4}\pi$ — B1

Use sine rule or equivalent to find r — M1

Obtain $6.69\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$ — A1

Or: State $y = x$. — B1

State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$ — B1

State or imply argument is $\frac{\pi}{4}$ — B1

Solve for x or y. — M1

Obtain $6.69e^{\frac{1}{4}\pi i}$ — A1 [5]

12 **(a)** Substitute $w = x + iy$ and state a correct equation in x and y — B1

Use $i^2 = -1$ and equate real parts — M1

Obtain $y = -2$ — A1

Equate imaginary parts and solve for x — M1

Obtain $x = 2\sqrt{2}$, or equivalent, only — A1 [5]

(b) Show a circle with centre 2i — B1

Show a circle with radius 2 — B1

Show half line from –2 at $\frac{1}{4}\pi$ to real axis — B1

Shade the correct region — B1

Carry out a complete method for calculating the greatest value of $|z|$ — M1

Obtain answer 3.70 — A1 [6]

13 **(a)** *EITHER:* Solve for u or for v — M1

Obtain $u = \frac{2i-6}{1-2i}$ or $v = \frac{5}{1-2i}$, or equivalent — A1

Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent

Or: Set u or v equal to $x + iy$, obtain two equations by equating real and imaginary parts and solve for x or for y — M1

OR: Using $a + ib$ and $c + id$ for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d — M1

Obtain $b + 2d = 2$, $a + 2c = 0$, $a + d = 0$ and $-b + c = 3$, or equivalent — A1

Solve for one unknown — M1

Obtain final answer $u = -2 - 2i$, or equivalent — A1

Obtain final answer $v = 1 + 2i$, or equivalent — A1 **[5]**

(b) Show a circle with centre –i — B1

Show a circle with radius 1 — B1

Show correct half line from 2 at an angle of $\frac{3}{4}\pi$ to the real axis — B1

Use a correct method for finding the least value of the modulus — M1

Obtain final answer $\frac{3}{\sqrt{2}} - 1$, or equivalent, e.g. 1.12 (allow 1.1) — A1 **[5]**

14 **(a)** Solve using formula, including simplification under square root sign — M1*

Obtain $\frac{-2 \pm 4\mathrm{i}}{2(2-\mathrm{i})}$ or similarly simplified equivalents — A1

Multiply by $\frac{2+\mathrm{i}}{2+\mathrm{i}}$ or equivalent in at least one case — M1(d*M)

Obtain final answer $-\frac{4}{5}+\frac{3}{5}\mathrm{i}$ — A1

Obtain final answer –i — A1 [5]

(b) Show w in first quadrant with modulus and argument relatively correct — B1

Show w^3 in second quadrant with modulus and argument relatively correct — B1

Show w* in fourth quadrant with modulus and argument relatively correct — B1

Use correct method for area of triangle — M1

Obtain 10 by calculation — A1 [5]

15 **(a)** *EITHER*: Multiply numerator and denominator by 1 – 4i, or equivalent, and use $\mathrm{i}^2 = -1$ — M1

Simplify numerator to –17 –17i , or denominator to 17 — A1

Obtain final answer –1 –i — A1

OR: Using $\mathrm{i}^2 = -1$, obtain two equations in x and y, and solve for x or for y — M1

Obtain $x = -1$ or $y = -1$, or equivalent — A1

Obtain final answer –1 – i — A1 **3**

(b) (i) Show a point representing 2 + i in relatively correct position — B1

Show a circle with centre 2 + i and radius 1 — B1

Show the perpendicular bisector of the line segment joining i and 2 — B1

Shade the correct region — B1 **4**

(ii) State or imply that the angle between the tangents from the origin to the circle is required — M1

Obtain answer 0.927 radians (or 53.1°) — A1 **2**

16 **(i)** Either Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent — B1

Multiply by $\frac{3+4\mathrm{i}}{3+4\mathrm{i}}$ and simplify to $x+\mathrm{i}y$ form or equivalent — M1

Confirm given answer $2+4\mathrm{i}$ — A1

Or Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent — B1

Obtain two equations in x and y and solve for x or y — M1

Confirm given answer $2+4\mathrm{i}$ — A1 [3]

(ii) Identify $4+4$ or $-4+4\mathrm{i}$ as point at either end or state $p=2$ or state $p=-6$ — B1

Use appropriate method to find both critical values of p — M1

State $-6 \leqslant p \leqslant 2$ — A1 [3]

(iii) Identify equation as of form $|z-a|=a$ or equivalent — M1

Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$ — A1

State $|z-5|=5$ — A1 [3]

17 **(i)** Square $x+\mathrm{i}y$ and equate real and imaginary parts to -1 and $4\sqrt{3}$ — M1

Obtain $x^2-y^2=-1$ and $2xy=4\sqrt{3}$ — A1

Eliminate one unknown and find an equation in the other — M1

Obtain $x^4+x^2-12=0$ or $y^4-y^2-12=0$, or three term equivalent — A1

Obtain answers $\pm(\sqrt{3}+2\mathrm{i})$ — A1 **[5]**

[If the equations are solved by inspection, give B2 for the answers and B1 for justifying them]

(ii) Show a circle with centre $-1+4\sqrt{3}$ in a relatively correct position — B1

Show a circle with radius 1 and centre not at the origin — B1

Carry out a complete method for calculating the greatest value of arg z — M1

Obtain answer 1.86 or 106.4° — A1 **[4]**

18 **(i)** *EITHER*: Substitute for u in $\frac{\mathrm{i}}{u}$ and multiply numerator and denominator by $1+\mathrm{i}$ — M1

Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent — A1

OR: Substitute for u, obtain two equations in x and y and solve for x or for y — M1

Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent — A1 **2**

(ii) Show a point representing u in a relatively correct position — B1

Show the bisector of the line segment joining u to the origin — B1

Show a circle with centre at the point representing i — B1

Show a circle with radius 2 — B1 **4**

(iii) State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270° — B1

State or imply the intersection in the first quadrant represents 2 + i — B1

State argument 0.464, (0.4636)or equivalent, e.g. 26.6° (26.5625) — B1 **3**

19 **(i)** Show u in a relatively correct position **B1**
Show u^* in a relatively correct position **B1**
Show $u^* - u$ in a relatively correct position **B1**
State or imply that $OABC$ is a parallelogram **B1** [4]

(ii) *EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent **M1**
Simplify the numerator to 8 + 6i or the denominator to 10 **A1**
Obtain final answer $\frac{4}{5} + \frac{3}{5}\text{i}$, or equivalent **A1**
OR: Substitute for u, obtain two equations in x and y and solve for x or for y **M1**
Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent **A1**
Obtain final answer $\frac{4}{5} + \frac{3}{5}\text{i}$, or equivalent **A1** [3]

(iii) State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$ **B1**
Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$ **M1**
Fully justify the given statement using exact values **A1** [3]

20 **(a)** Either Find w using conjugate of $1+3\mathrm{i}$ **M1**

Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent **A1**

Square $x+\mathrm{i}y$ form to find w^2 **M1**

Obtain $w^2=\frac{48-14\mathrm{i}}{25}$ and confirm modulus is 2 **A1**

Use correct process for finding argument of w^2 **M1**

Obtain –0.284 radians or $-16.3°$ **A1**

Or 1 Find w using conjugate of $1+3\mathrm{i}$ **M1**

Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent **A1**

Find modulus of w and hence of w^2 **M1**

Confirm modulus is 2 **A1**

Find argument of w and hence of w^2 **M1**

Obtain –0.284 radians or $-16.3°$ **A1**

Or 2 Square both sides to obtain $(-8+6\mathrm{i})w^2=-12+16\mathrm{i}$ **B1**

Find w^2 using relevant conjugate **M1**

Use correct process for finding modulus of w^2 **M1**

Confirm modulus is 2 **A1**

Use correct process for finding argument of w^2 **M1**

Obtain –0.284 radians or $-16.3°$ **A1**

Or 3 Find modulus of LHS and RHS **M1**

Find argument of LHS and RHS **M1**

Obtain $\sqrt{10}\,\mathrm{e}^{1.249\mathrm{i}}$ $w=\sqrt{20}\,\mathrm{e}^{1.107\mathrm{i}}$ or equivalent **A1**

Obtain $w=\sqrt{2}\,\mathrm{e}^{-0.1419\mathrm{i}}$ or equivalent **A1**

Use correct process for finding w^2 **M1**

Obtain 2 and –0.284 radians or $-16.3°$ **A1**

Or 4 Find moduli of $2+4\mathrm{i}$ and $1+3\mathrm{i}$ **M1**

Obtain $\sqrt{20}$ and $\sqrt{10}$ **A1**

Obtain $\left|w^2\right|=2$ correctly **A1**

Find $\arg(2+4\mathrm{i})$ and $\arg(1+3\mathrm{i})$ **M1**

Use correct process for $\arg(w^2)$ **A1**

Obtain –0.284 radians or $-16.3°$ **A1**

Or 5 Let $w=a+\mathrm{i}b$, form and solve simultaneous equations in a and b **M1**

$a=\frac{7}{5}$ and $b=-\frac{1}{5}$ **A1**

Find modulus of w and hence of w^2 **M1**

Confirm modulus is 2 **A1**

Find argument of w and hence of w^2 **M1**

Obtain –0.284 radians or $-16.3°$ **A1**

Or 6	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
	Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
	Use $\left\|w^2\right\| = w\overline{w}$	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of w and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	[6]

(b)	Draw circle with centre the origin and radius 5	**B1**	
	Draw straight line parallel to imaginary axis in correct position	**B1**	
	Use relevant trigonometry on a correct diagram to find argument(s)	**M1**	
	Obtain $5\mathrm{e}^{\pm\frac{1}{3}\pi\mathrm{i}}$ or equivalents in required form	**A1**	[4]

21	(**a**) Square $x+\mathrm{i}y$ and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively	**M1**	
	Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$	**A1**	
	Eliminate one variable and find an equation in the other	**M1**	
	Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent	**A1**	
	Obtain answers $\pm(3 - \mathrm{i}\sqrt{2})$	**A1**	
			[5]

(**b**)	(**i**) Show point representing 1 + 2i	**B1**	
	Show circle with radius 1 and centre 1 + 2i	**B1**	
	Show a half line from the point representing 1	**B1**	
	Show line making the correct angle with the real axis	**B1**	
			[4]

	(**ii**) State or imply the relevance of the perpendicular from 1 + 2i to the line	**M1**	
	Obtain answer $\sqrt{2} - 1$ (or 0.414)	**A1**	
			[2]

22 **(i)** *EITHER*: Divide by denominator and obtain quadratic remainder M1
Obtain $A = 1$ A1
Use any relevant method to obtain B, C or D M1
Obtain one correct answer A1
Obtain $B = 2$, $C = 1$ and $D = -3$ A1
OR: Reduce RHS to a single fraction and equate numerators, or equivalent M1
Obtain $A = 1$ A1
Use any relevant method to obtain B, C or D M1
Obtain one correct answer A1
Obtain $B = 2$, $C = 1$ and $D = -3$ A1 [5]
[SR: If $A = 1$ stated without working give B1.]

(ii) Integrate and obtain $x + 2\ln x - \frac{1}{x} - \frac{3}{2}\ln(2x-1)$, or equivalent B3√
(The f.t. is on A, B, C, D. Give B2√ if only one error in integration; B1√ if two.)
Substitute limits correctly in the complete integral M1
Obtain given answer correctly following full and exact working A1 [5]

23 **(i)** *EITHER*: Multiply numerator and denominator of $\frac{u}{v}$ by 2 + i, or equivalent **M1**
Simplify the numerator to −5 +5i or denominator to 5 **A1**
Obtain final answer −1 + I **A1**

OR: Obtain two equations in x and y and solve for x or for y **(M1**
Obtain $x = -1$ or $y = 1$ **A1**
Obtain final answer −1 + I **A1)**
[3]

(ii) Obtain $u + v = 1 + 2i$ **B1**
In an Argand diagram show points A, B, C representing u, v and $u + v$ respectively **B1√**
State that OB and AC are parallel **B1**
State that $OB = AC$ **B1**
[4]

(iii) Carry out an appropriate method for finding angle AOB, e.g. find $\arg(u / v)$ **M1**
Show sufficient working to justify the given answer $\frac{3}{4}\pi$ **A1**
[2]

7	**(i)**	State modulus $2\sqrt{2}$, or equivalent	**B1**	
		State argument $-\frac{1}{3}\pi$ (or $-60°$)	**B1**	[2]
	(ii) **(a)**	State answer $3\sqrt{2}+\sqrt{6}\,\mathrm{i}$	**B1**	
	(b)	*EITHER:* Substitute for z and multiply numerator and denominator by conjugate of $\mathrm{i}z$	**M1**	
		Simplify the numerator to $4\sqrt{3}+4\,\mathrm{i}$ or the denominator to 8	**A1**	
		Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	**A1**	
		OR: Substitute for z, obtain two equations in x and y and solve for x or for y	**M1**	
		Obtain $x=\frac{1}{2}\sqrt{3}$ or $y=\frac{1}{2}$	**A1**	
		Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	**A1**	[4]
	(iii)	Show points A and B in relatively correct positions	**B1**	
		Carry out a complete method for finding angle AOB, e.g. calculate the argument of $\frac{z^*}{\mathrm{i}z}$	**M1**	
		Obtain the given answer	**A1**	[3]

25(i)	State that $u - 2w = -7 - \mathrm{i}$	**B1**
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by $3 - 4\mathrm{i}$, or equivalent	**(M1**
	Simplify the numerator to $25 + 25\mathrm{i}$ or denominator to 25	**A1**
	Obtain final answer $1 + \mathrm{i}$	**A1)**
	OR: Obtain two equations in x and y and solve for x or for y	**(M1**
	Obtain $x = 1$ or $y = 1$	**A1**
	Obtain final answer $1 + \mathrm{i}$	**A1)**
	Total:	**4**
25(ii)	Find the argument of $\frac{u}{w}$	**M1**
	Obtain the given answer	**A1**
	Total:	**2**
25(iii)	State that OB and CA are parallel	**B1**
	State that $CA = 2OB$, or equivalent	**B1**
	Total:	**2**

26(i)	*EITHER:* Substitute $x = 2 - i$ (or $x = 2 + i$) in the equation and attempt expansions of x^2 and x^3	**(M1**
	Equate real and/or imaginary parts to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*1: Substitute $x = 2 - i$ in the equation and attempt expansions of x^2 and x^3	**(M1**
	Substitute $x = 2 + i$ in the equation and add/subtract the two equations	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*2: Factorise to obtain $(x-2+i)(x-2-i)(x-p)$ $\left(=\left(x^2-4x+5\right)(x-p)\right)$	**(M1**
	Compare coefficients	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*3: Obtain the quadratic factor ($x^2 - 4x + 5$)	**(M1**
	Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*4: Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$	**(M1**
	Solve for γ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*5: Factorise as $(x - (2-i))(x^2 + ex + g)$ and compare coefficients to form an equation in a and b	**(M1**
	Equate real and/or imaginary parts to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	Total:	**4**

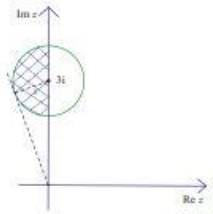
26(ii)	Show a circle with centre $2-\mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 1 and centre not at the origin	**B1**
	Show the perpendicular bisector of the line segment joining 0 to $-\mathrm{i}$	**B1**
	Shade the correct region	**B1**
	Total:	**4**

27(a)	Solve for z or for w	**M1**
	Use $\mathrm{i}^2=-1$	**M1**
	Obtain $w=\dfrac{\mathrm{i}}{2-\mathrm{i}}$ or $z=\dfrac{2+\mathrm{i}}{2-\mathrm{i}}$	**A1**
	Multiply numerator and denominator by the conjugate of the denominator	**M1**
	Obtain $w=-\frac{1}{5}+\frac{2}{5}\mathrm{i}$	**A1**
	Obtain $z=\frac{3}{5}+\frac{4}{5}\mathrm{i}$	**A1**
	Total:	**6**
27(b)	*EITHER:* Find $\pm\left[2+\left(2-2\sqrt{3}\right)\mathrm{i}\right]$	**(B1**
	Multiply by 2i (or –2i)	**M1***
	Add result to v	**DM1**
	Obtain answer $4\sqrt{3}-1+6\mathrm{i}$	**A1)**
	OR: State $\dfrac{z-v}{v-u}=k\mathrm{i}$, or equivalent	**(M1**
	State $k=2$	**A1**
	Substitute and solve for z even if i omitted	**M1**
	Obtain answer $4\sqrt{3}-1+6\mathrm{i}$	**A1)**
	Total:	**4**

28(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $3x - y = 1$ and $3y - x = 5$	**A1**
	Solve and obtain answer $z = 1 + \mathbf{2(i)}$	**A1**
	Total:	**4**
28(b)	Show a circle with radius 3	**B1**
	Show the line $y = 2$ extending in both quadrants	**B1**
	Shade the correct region	**B1**
	Carry out a complete method for finding the greatest value of arg z	**M1**
	Obtain answer 2.41	**A1**
	Total:	**5**

29(i)	*EITHER*: Multiply numerator and denominator by 1 + 2i, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y		**M1**	
		Obtain quotient $-\frac{4}{5} + \frac{7}{5}$i, or equivalent	**A1**	
		Use correct method to find either r or θ	**M1**	
		Obtain $r = 1.61$	**A1**	
		Obtain $\theta = 2.09$	**A1**	
	OR:	Find modulus or argument of 2 + 3i or of 1 – 2i	**B1**	
		Use correct method to find r	**M1**	
		Obtain $r = 1.61$	**A1**	
		Use correct method to find θ	**M1**	
		Obtain $\theta = 2.09$	**A1**	
			5	
29(ii)	Show a circle with centre 3 – 2i		**B1**	
	Show a circle with radius 1		**B1ft**	Centre not at the
	Carry out a correct method for finding the least value of $\|z\|$		**M1**	
	Obtain answer $\sqrt{13} - 1$		**A1**	
			4	

)(a)(i)	Multiply numerator and denominator by 1 + 2i, or equivalent	**M1**	Requires at least one of $2+10i+12i^2$ and $1-4i^2$ together with use of $i^2=-1$. Can be implied by $\frac{-10+10i}{5}$
	Obtain quotient – 2 + 2i	**A1**	
	Alternative		
	Equate to $x + iy$, obtain two equations in x and y and solve for x or for y	**M1**	$x+2y=2, \quad y-2x=6$
	Obtain quotient – 2 + 2i	**A1**	
		2	
(a)(ii)	Use correct method to find either r or θ	**M1**	If only finding θ, need to be looking for θ in the correct quadrant
	Obtain $r=2\sqrt{2}$, or exact equivalent	**A1ft**	ft their $x + iy$
	Obtain $\theta=\frac{3}{4}\pi$ from exact work	**A1ft**	ft on $k(-1 + i)$ for $k > 0$ Do not ISW
		3	

(b)	Show a circle with centre 3i	**B1**	
	Show a circle with radius 1	**B1ft**	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and **shade** the correct region	**B1**	
	Carry out a correct method for calculating greatest value of arg z	**M1**	e.g. $\arg z=\frac{\pi}{2}+\sin^{-1}\frac{1}{3}$
	Obtain answer 1.91	**A1**	
		5	

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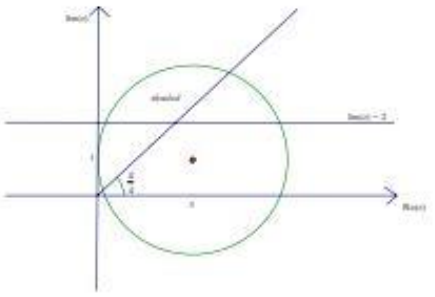
(a)	Square $a + ib$ and equate real and imaginary parts to -3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2 - b^2 = -3$ and $2ab = -2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4 + 3a^2 - 10 = 0$, or $b^4 - 3b^2 - 10 = 0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm\left(\sqrt{2} - \sqrt{5}\mathrm{i}\right)$, or exact equivalent	A1	
		5	
(b)	Show point representing $3 + \mathrm{i}$ in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y = 2$	B1	
	Shade the correct region	B1	
		5	

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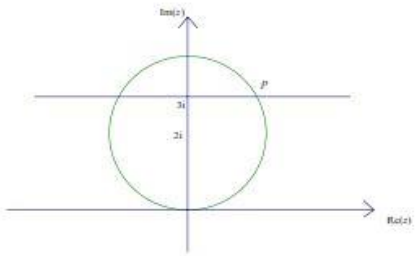
a)	Substitute and obtain a correct horizontal equation in x and y in any form	B1	$zz^* + \mathrm{i}z - 2z^* = 0 \Rightarrow$ $x^2 + y^2 + \mathrm{i}x - y - 2x + 2\mathrm{i}y = 0$ Allow if still includes brackets and/or i^2
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero OE	*M1	For their horizontal equation
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	A1	Allow $\mathrm{i}x + 2\mathrm{i}y = 0$
	Solve for x or for y	DM1	
	Obtain answer $\frac{6}{5} - \frac{3}{5}\mathrm{i}$ and no other	A1	OE, condone $\frac{1}{5}(6 - 3\mathrm{i})$
		5	

(b)(i)	Show a circle with centre 2i and radius 2	**B1**	
	Show horizontal line $y = 3$ – in first and second quadrant	**B1**	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully 'correct' diagram.
		2	
b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	**M1**	$\left(z = \sqrt{3} + 3\text{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or 60°)	**A1**	**SC:** Allow B2 for 60° with no working
		2	

33(i)	Obtain answer $w = \frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}$	**B1**
		1
33(ii)	Show point representing u	**B1**
	Show point representing v in relatively correct position	**B1**
		2
33(iii)	Explain why the moduli are equal	**B1**
	Explain why the arguments are equal	**B1**
	Use $\text{i}^2 \quad -1$ and obtain $2uw$ in the given form	**M1**
	Obtain answer $1 - 2\sqrt{3} + \left(2 + \sqrt{3}\right)\text{i}$	**A1**
		4