



ZAIÑEMATICS

FOR THE LOVE OF MATHS

A - Level

P1 Notes

By

Zain Afaq

Compiled By Rafay Mushtaq



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ZAIÑEMATICS

FOR THE LOVE OF MATHS

3| Coordinate Geometry

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ZAIÑMATICS

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Coordinate Geometry

P1

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COORDINATE GEOMETRY

Monday, 6 July 2020 5:15 PM

Chapter 1. Coordinate Geometry (7+4 marks).

merged with other topics.

15% of

P1.

$$A(x_1, y_1) \quad B(x_2, y_2)$$

$$\text{MIDPOINT} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{DISTANCE} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{GRADIENT} = \frac{y_2 - y_1}{x_2 - x_1} \quad (\text{SLOPE}) \quad m$$

GRADIENTS (STEEPNESS)

① Horizontal line

$$\text{gradient} = 0$$

② Vertical line.

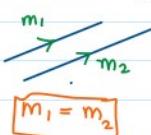
$$\text{gradient} = \infty$$

③

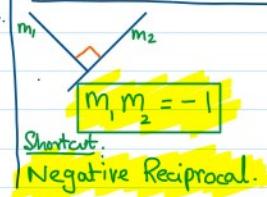
will move left to right on line



PARALLEL LINES, have same gradient.



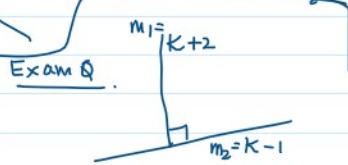
(90°) PERPENDICULAR LINES



$$m_1 = 3 \rightarrow m_2 = -\frac{1}{3}$$

$$m_1 = -\frac{1}{5} \rightarrow m_2 = +5$$

$$m_1 = \frac{3}{4} \rightarrow m_2 = -\frac{4}{3}$$



Exam Q.

$$m_1 \times m_2 = -1$$

$$(k+2)(k-1) = -1$$

Quadratic Solve.

$$k = \square$$

EQUATION OF A LINE

SLOPE-INTERCEPT FORM

$$y = mx + c$$

POINT-SLOPE FORM (MOST IMPORTANT)

$$y - y_1 = m(x - x_1)$$

OPTIONAL

TWO POINT FORM.

$$y - y_1 = \frac{x - x_1}{x_2 - x_1}$$

SLOPE-INTERCEPT FORM

$$y = mx + c$$

Given: ① Slope (m)
② y -intercept (c)

POINT-SLOPE FORM (MOST IMPORTANT)

$$y - y_1 = m(x - x_1)$$

Given:

- ① Slope (m)
- ② Point on line ((x_1, y_1))

TWO POINT FORM

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Given Two points on line.
 (x_1, y_1) (x_2, y_2) .

Q Find equation of line that has gradient 3 and it cuts y -axis at 1

$$y = mx + c$$

$$y = 3x + 1$$

Never used in A Levels

Q Find equation of line with slope 2 and it passes through $(1, 5)$.

$$m = 2, (1, 5)$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 2(x - 1)$$

$$y - 5 = 2x - 2$$

$$y = 2x + 3$$

Q Find equation of line that passes through $(1, 3)$ and $(7, 12)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{12 - 3} = \frac{x - 1}{7 - 1}$$

$$\frac{y - 3}{9} = \frac{x - 1}{6}$$

$$2(y - 3) = 3(x - 1)$$

$$2y - 6 = 3x - 3$$

$$2y = 3x + 3$$

$$y = \frac{3}{2}x + \frac{3}{2}$$

Q Find equation of line with gradient 4 and it passes through $(2, 7)$

$$m = 4$$

$$(2, 7)$$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 4(x - 2)$$

$$y - 7 = 4x - 8$$

$$y = 4x - 8 + 7$$

$$y = 4x - 1$$

Q Find equation of line that passes through $(-2, 5)$ and has slope 5.

$$m = 5, (-2, 5)$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 5(x - (-2))$$

$$y - 5 = 5(x + 2)$$

$$y = 5x + 10 + 5$$

$$y = 5x + 15$$

A LEVELS IS ABOUT DETAILED CLEAR WORKINGS

SIMULTANEOUS EQUATION:-

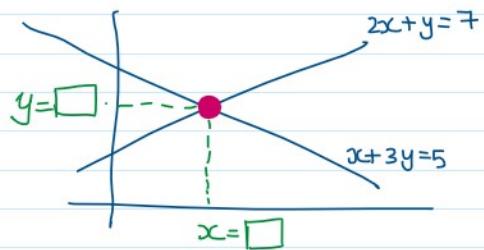
$$1/ 2x + y = 7$$



CONCEPT:-

Q Levels

$$\begin{aligned} 2x + y &= 7 \\ x + 3y &= 5 \end{aligned}$$



$$x = \square, y = \square$$

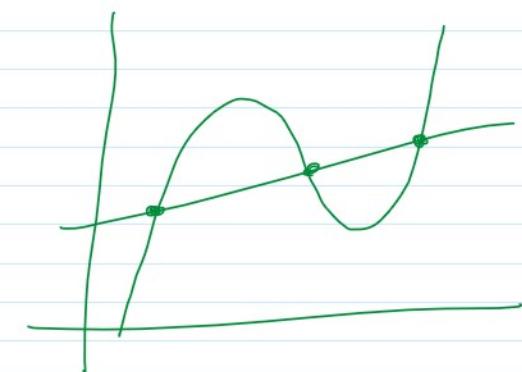
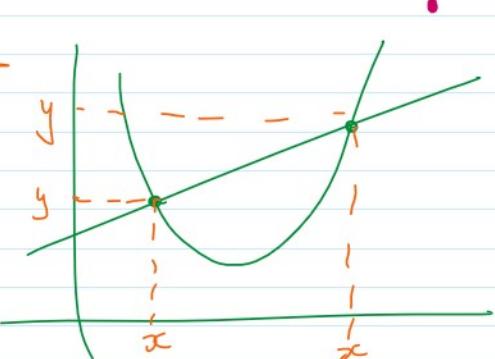
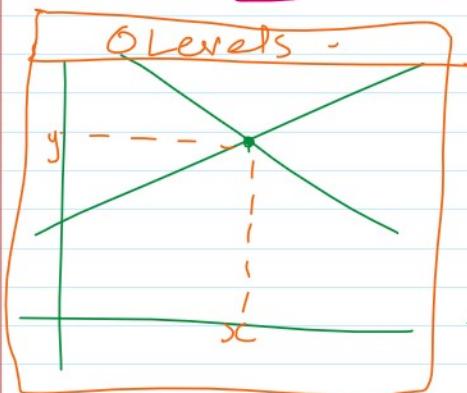
SIMULTANEOUS SOLVING WILL ALWAYS GIVE YOU POINTS OF INTERSECTION OF TWO GRAPHS.

BEST WAY TO SOLVE FOR POINT OF INTERSECTION (SIMULTANEOUS SOLVING).

- 1) Make y subject for both equations and put them equal.
- 2) SUBSTITUTION WILL ALWAYS WORK.
- 3) ELIMINATION WILL NOT ALWAYS BE AVAILABLE.
(Specially when x or y has a power).

HOW MANY POINTS OF INTERSECTION DO YOU EXPECT IF TWO GRAPHS MEET

IT DEPENDS!



If power of x and y is 1

LINEAR GRAPH

If either x or y or both have any other power than 1,

NON LINEAR GRAPH

CIRCLE

LINEAR GRAPH

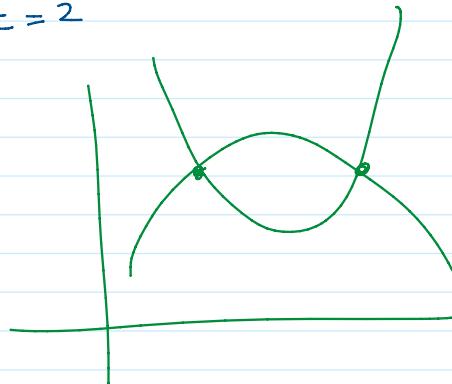
STRAIGHT LINE

$$y = 2x + 5$$

$$2x - 3y + 7 = 0$$

$$y = 7$$

$$x = 2$$



NON LINEAR GRAPH

CURVE

$$y = x^2 + 2x$$

$$y = 3x^3$$

$$y = \frac{1}{x}$$

$$y = x^{-1}$$

Curve.

$$\sqrt{y} = 2x + 4$$

1) Class Notes .

2) Past papers (Topical).

I will upload these
sheet.

Class

~~Book~~

Paper

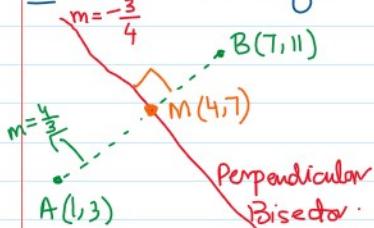
How TO FIND EQUATION OF PERPENDICULAR 90°
on Two Points (Lines)

BISECTOR
Equal two parts.

HOW TO FIND EQUATION OF PERPENDICULAR BISECTOR OF TWO POINTS (4 Marks)

BISECTOR
Equal two parts.

Q Find the equation of perpendicular bisector of A(1, 3) and B(7, 11).



STEP 1
Find coordinates of midpoint M.

$$M = \left(\frac{1+7}{2}, \frac{3+11}{2} \right)$$

$$M = (4, 7)$$

STEP 2
gradient of AB.

$$m_{AB} = \frac{11-3}{7-1} = \frac{8}{6} = \frac{4}{3}$$

Negative reciprocal.

$$m' = -\frac{3}{4}$$

STEP 3: x_1, y_1
 $m = -\frac{3}{4}$, $M(4, 7)$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -\frac{3}{4}(x - 4)$$

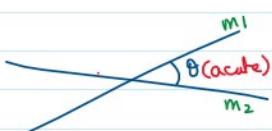
$$4y - 28 = -3x + 12$$

$$4y = -3x + 40$$

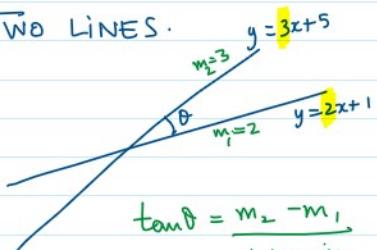
Note: No need to make y subject in A Levels.

Symbol = means perpendicular

ANGLE BETWEEN TWO LINES.
(ACUTE).



$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$



$$\tan \theta = \frac{1}{7}$$

$$\theta = \tan^{-1}(\frac{1}{7})$$

While taking inverse ignore any -ve sign.

$$m_1 = 3, m_2 = 2$$

$$\tan \theta = \frac{2-3}{1+2(3)}$$

$$\tan \theta = -\frac{1}{7}$$

$$\theta = \tan^{-1}(\frac{1}{7})$$

$$\theta = 8.13^\circ$$

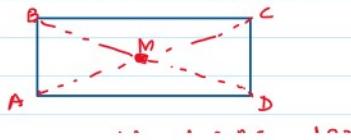
Reason: Discuss later in trigonometry.

$$\theta = 8.13^\circ$$

SPECIAL SHAPES

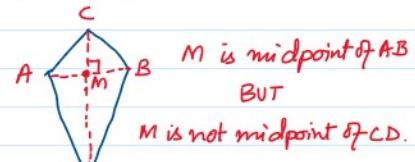
DIAGONALS MEET AT MIDPOINT

- 1) SQUARE
- 2) RHOMBUS
- 3) RECTANGLE
- 4) PARALLELOGRAM.



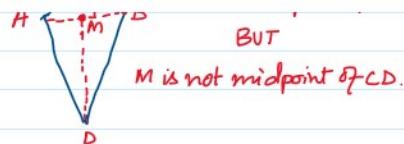
DIAGONALS MEET AT 90°

- 1) SQUARE
- 2) RHOMBUS
- 3) KITE



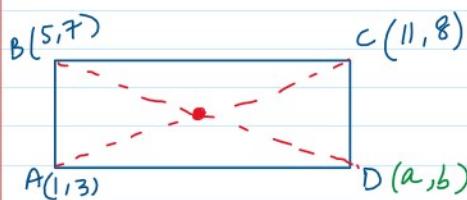


M is midpoint of AC and BD.



BUT
M is not midpoint of CD.

Q Find coordinates of D (1 marks).



Don't take (x, y)
since there is a
chance of mix up.

Midpoint of AC = Midpoint of BD.

$$\left(\frac{1+11}{2}, \frac{3+8}{2} \right) = \left(\frac{a+5}{2}, \frac{b+7}{2} \right)$$

$$\begin{aligned} \frac{11+1}{2} &= \frac{a+5}{2} \\ 12 &= a+5 \\ a &= 7 \end{aligned} \quad \begin{aligned} \frac{3+8}{2} &= \frac{b+7}{2} \\ 11 &= b+7 \\ b &= 4 \end{aligned}$$

D · (7, 4)

HOW TO FIND GRADIENT OF A LINE FROM ITS EQUATION.

STEP 1 = Make y subject.

STEP 2 = Now compare with $y = mx + c$

$$2y + 3x = 7$$

$$2y = -3x + 7$$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$y = mx + c$$

~~x~~ , ~~-3~~ , ~~$\frac{7}{2}$~~ , ~~$-\frac{3}{2}$~~
 ~~1.5~~ , ~~-1.5~~

IF A POINT LIES ON A LINE, IT MUST SATISFY ITS EQUATION

(o-1 levels)
Q Check if (2, 3) lies
on line $y = 2x + 1$
or not?

A levels.
Q Given that (3, k) lies on
 $2x + 3y = 12$, find k. (1 or 2 marks)

$$x \sim \dots \sim 1 \quad (2, 3)$$

$$(3, k)$$

$$2x + 3y = 12$$

or not?

$$y = 2x + 1 \quad (2, 3)$$

$$3 = 2(2) + 1$$

$$3 \neq 5$$

No, $(2, 3)$ does not lie on

$$y = 2x + 1$$

$$(3, k)$$

x y

$$2x + 3y = 12$$

$$2(3) + 3(k) = 12$$

$$6 + 3k = 12$$

$$3k = 6$$

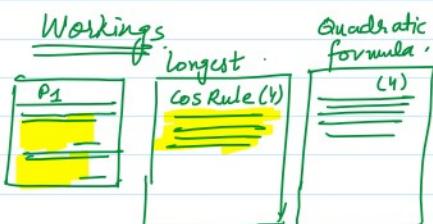
$$\boxed{k = 2}$$

IF A LINE CUTS X-Axis , put $y=0$

IF A LINE CUTS Y-Axis , put $x=0$.

O-levels:

Length of chapter ↓



r/s

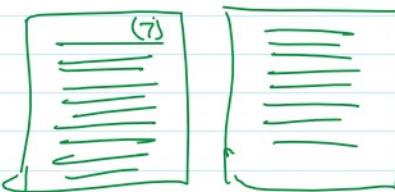
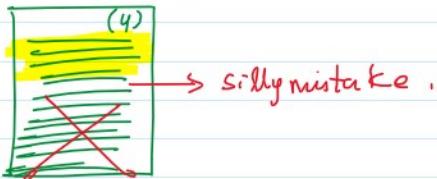
A-levels

Length of Chapter ↑
STAMINA ↑

Workings.

P1 + M1.

- Normally go upto 1 page for 4-5 marks.
- Go upto 2 pages for 6-8 marks





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Binomial P1

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BINOMIAL

Tuesday, 18 August 2020 5:06 PM

Coordinate Geometry Q1 — Q10 (Wed) (19 Aug 2020)

BINOMIAL (4-6 Marks)Total terms in a binomial = $n+1$

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2 + {}^n C_3 a^{n-3} b^3 + \dots$$

$\begin{matrix} {}^n C_r \\ \text{Calculator} \end{matrix}$ → Button beneath Alpha **Incr** } Meaning will come in **SL**

$\begin{matrix} \text{SHIFT} \\ \div \end{matrix}$ } ${}^6 C_3 = 6 \text{ Incr}^3$
 $6 C 3 = 20$

Expand first three terms in following.

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2$$

$$\textcircled{1} (2+3x)^5 = 2^5 + {}^5 C_1 (2)^{5-1} (3x)^1 + {}^5 C_2 (2)^{5-2} (3x)^2$$

$$\text{Calculator} = [5C1 \times 2^4 \times 3] [5C2 \times 2^3 \times 3^2]$$

$$[32 + 240x + 720x^2]$$

$$\textcircled{2} (5-4x)^4 = (5)^4 + {}^4 C_1 (5)^{4-1} (-4x)^1 + {}^4 C_2 (5)^{4-2} (-4x)^2$$

$$[4C1 \times 5^3 \times (-4)] [4C2 \times 5^2 \times (-4)^2]$$

$$625 - 2000x + 2400x^2$$

$$\textcircled{3} (3+5x)^3$$

$$(3)^3 + {}^3 C_1 (3)^{3-1} (5x)^1 + {}^3 C_2 (3)^{3-2} (5x)^2$$

$$[3C2 \times 3^1 \times 5^2]$$

$$27 + 135x + 225x^2$$

$$\textcircled{4} (x + \frac{1}{x})^7$$

$$(x)^7 + {}^7 C_1 (x)^{7-1} \left(\frac{1}{x}\right)^1 + {}^7 C_2 (x)^{7-2} \left(\frac{1}{x}\right)^2$$

$$x^7 + (7) x^5 \left(\frac{1}{x}\right) + (21) (x^3) \left(\frac{1}{x}\right)$$

$$x^7 + 7x^5 + 21x^3$$

(v) Expand first three terms in expansion

$$\left(3x - \frac{2}{x}\right)^5$$

$$(3x)^5 + {}^5 C_1 (3x)^{5-1} \left(\frac{-2}{x}\right)^1 + {}^5 C_2 (3x)^{5-2} \left(\frac{-2}{x}\right)^2$$

$$243x^5 + (5)(81x^4) \left(\frac{-2}{x}\right) + (10)(27x^3) \left(\frac{4}{x^2}\right)$$

$$243x^5 - 810x^3 + 1080x$$

TYPE 1 QUESTION

(a) Expand $(2-x)^6$ upto first three terms.

$$(a+b)^n = \text{Total term} = n+1$$

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2 + {}^n C_3 a^{n-3} b^3 + \dots$$

$$(2-x)^6 = (2)^6 + {}^6 C_1 (2)^{6-1} (-x)^1 + {}^6 C_2 (2)^{6-2} (-x)^2$$

$$= [64 - 192x + 240x^2]$$

Hence

(i) Find coefficient of x^2 in expansion

$$(3x+5)(2-x)^6 \rightarrow x^2$$

$$3x \times -192x = -576x^2$$

$$+5 \times 240x^2 = 1200x^2$$

$$\boxed{624x^2}$$

$$(2-x)^6 = \boxed{64 - 192x + 240x^2}$$

(ii) Find coefficient of (x^2) in expansion of $\frac{7}{7 \text{ terms}} (3-7x+2x^2)(2-x)^6$

$$\begin{array}{rcl} 3 & \times & 240x^2 = 720x^2 \\ -7x & \times & -192x = 1344x^2 \\ +2x^2 & \times & 64 = 128x^2 \\ \hline & & 2192x^2 \end{array}$$

(iii) Hence

Find coefficient of x^3 in expansion

of

$$(2x+x^2)(2-x)^6 \rightarrow x^3$$

$$\begin{array}{rcl} 2x & \times & 240x^2 = 480x^3 \\ +x^2 & \times & -192x = -192x^3 \\ \hline & & 288x^3 \end{array}$$

(iv) Find coefficient of x in expansion of

$$(7x+3)(2-x)^6 \rightarrow x$$

$$\begin{array}{rcl} 7x & \times & 64 = 448x \\ +3 & \times & -192x = -576x \\ \hline & & -128x \end{array}$$

25 (i) Find the first 3 terms in the expansion of $(2x-x^2)^6$ in ascending powers of x . [3]

(ii) Hence find the coefficient of x^8 in the expansion of $(2+x)(2x-x^2)^6$. [2]

$$\begin{aligned} (i) \quad (2x-x^2)^6 &= (2x)^6 + {}^6 C_1 (2x)^5 (-x^2)^1 + {}^6 C_2 (2x)^4 (-x^2)^2 \\ &= 64x^6 + (6)(32x^5)(-x^2) + (15)(16x^4)(x^4) \\ &= 64x^6 - 192x^7 + 240x^8 \end{aligned}$$

$$(2+x)(2x-x^2)^6 \rightarrow x^8$$

(ii)

→ student think

$$(2+x)(2x-x^2)^6 \longrightarrow x^8$$

(ii)

$$\begin{array}{rcl} 2 & \times & 240x^8 \\ +x & \times & -192x^7 \\ \hline & = & 480x^8 \\ & = & -192x^8 \\ & \hline & 288x^8 \end{array}$$

student think
1 step
In Real
M1 = Complete Method.

VARIATION : Reverse Working.

- 9 (i) Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x . [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1+2x+ax^2)(2-x)^6$ is 48, find the value of the constant a . [3]

$$\begin{aligned} (i) (2-x)^6 &= (2)^6 + {}^6C_1(2)^5(-x)^1 + {}^6C_2(2)^4(-x)^2 \\ &= 64 - 192x + 240x^2 \end{aligned}$$

$$(ii) (1+2x+ax^2)(2-x)^6 \longrightarrow 48x^2$$

$$\begin{array}{rcl} 1 & \times & 240x^2 = 240x^2 \\ +2x & \times & -192x = -384x^2 \\ +ax^2 & \times & 64 = \frac{64a x^2}{48x^2} \end{array}$$

$$\begin{aligned} 240 - 384 + 64a &= 48 \\ 64a &= 48 + 384 - 240 \\ 64a &= 192 \\ a &= 3 \end{aligned}$$

- 31 (i) Find the first three terms when $(2+3x)^6$ is expanded in ascending powers of x . [3]

- (ii) In the expansion of $(1+ax)(2+3x)^6$, the coefficient of x^2 is zero. Find the value of a . [2]

$$\begin{aligned} (i) (2+3x)^6 &= (2)^6 + {}^6C_1(2)^5(3x)^1 + {}^6C_2(2)^4(3x)^2 \\ &= 64 + 576x + 2160x^2 \end{aligned}$$

$$(ii) (1+ax)(2+3x)^6 \longrightarrow 0x^2$$

$$\begin{array}{rcl} 1 & \times & 2160x^2 = 2160x^2 \\ +ax & \times & 576x = 576ax^2 \\ \hline & & 0x^2 \end{array}$$

$$\begin{aligned} 2160 + 576a &= 0 \\ 576a &= -2160 \\ a &= -3.75 \end{aligned}$$

NEW VOCAB

Coefficient of x^2 is zero = No term of x^2 = $0x^2$

Coefficient of x is zero = No term of x = $0x$

- 3 (i) Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x . [3]

- (ii) Find the value of k for which there is no term in x^2 in the expansion of $(1+kx)(2-x)^6$. [2]

$$\begin{aligned} (i) (2-x)^6 &= (2)^6 + {}^6C_1(2)^5(x)^1 + {}^6C_2(2)^4(-x)^2 \\ &= 64 - 192x + 240x^2 \end{aligned}$$

$$(ii) (1+kx)(2-x)^6 \longrightarrow 0x^2$$

$$\begin{array}{r} \times M1 \\ \begin{array}{r} 1 \\ + kx \\ \hline \end{array} \times \begin{array}{r} 240x^2 \\ -192x \\ \hline 0 \end{array} = \begin{array}{r} 240x^2 \\ -192kx^2 \\ \hline 0 \end{array} \end{array} \quad \begin{array}{l} 240 - 192k = 0 \\ 240 = 192k \\ k = \frac{240}{192} \\ \boxed{k = 1.25} \end{array}$$

12 (i) Find the first three terms, in descending powers of x , in the expansion of $\left(x - \frac{2}{x}\right)^6$. [3]

(ii) Find the coefficient of x^4 in the expansion of $(1+x^2)\left(x - \frac{2}{x}\right)^6$. [2]

$$\begin{aligned} (i) \quad \left(x - \frac{2}{x}\right)^6 &= (x)^6 + {}^6C_1 (x)^5 \left(-\frac{2}{x}\right)^1 + {}^6C_2 (x)^4 \left(-\frac{2}{x}\right)^2 \\ &= x^6 + (6) (x^5) \left(\frac{-2}{x}\right) + (15) (x^4) \left(\frac{4}{x^2}\right) \\ &= x^6 - 12x^4 + 60x^2 \end{aligned}$$

$$\begin{aligned} (ii) \quad (1+x^2) \left(x - \frac{2}{x}\right)^6 &\longrightarrow x^4 \\ \begin{array}{r} 1 \\ + x^2 \end{array} \times \begin{array}{r} -12x^4 \\ 60x^2 \end{array} &= \begin{array}{r} -12x^4 \\ 60x^4 \\ \hline 48x^4 \end{array} \end{aligned}$$

10 (i) Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in ~~descending~~^{2 terms} powers of x . [3]

(ii) Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x^2}\right) \left(2x - \frac{3}{x}\right)^5$. [2]

$$\begin{aligned} (i) \quad (2x)^5 + {}^5C_1 (2x)^4 \left(-\frac{3}{x}\right)^1 + {}^5C_2 (2x)^3 \left(-\frac{3}{x}\right)^2 \\ (5)(16x^4) \left(\frac{-3}{x}\right) &\quad (10)(8x^3) \left(\frac{9}{x^2}\right) \\ \boxed{32x^5 - 240x^3 + 720x} \end{aligned}$$

$$(ii) \quad 1 \times 720x = 720x$$

$$\begin{array}{r} + \frac{2}{x^2} \\ \hline \end{array} \times -240x^3 = \frac{-480x}{240x}$$

14 (i) Find the first 3 terms in the expansion, in ~~ascending~~^{descending} powers of x , of $(1-2x^2)^8$. [2]

(ii) Find the coefficient of x^4 in the expansion of $(2-x^2)(1-2x^2)^8$. [2]

$$(i) \quad (1-2x^2)^8 = (1)^8 + {}^8C_1 (1)^7 (-2x^2)^1 + {}^8C_2 (1)^6 (-2x^2)^2$$

$$= 1 - 16x^2 + 112x^4$$

$$(ii) (2-x^2)(1-2x^2)^8 \rightarrow x^4$$

$$2 \times 112x^4 = 224x^4$$

$$-x^2 \times -16x^2 = \frac{16x^4}{240x^4}$$

11 (i) Find the first 3 terms in the expansion of $(1+ax)^5$ in ascending powers of x . [2]

(ii) Given that there is no term in x in the expansion of $(1-2x)(1+ax)^5$, find the value of the constant a . [2]

(iii) For this value of a , find the coefficient of x^2 in the expansion of $(1-2x)(1+ax)^5$. [3]

$$(i) (1)^5 + \frac{5}{1}(1)^4(ax) + \frac{5}{2}(1)^3(ax)^2$$

$$1 + 5\left(\frac{2}{5}x\right) + 10\left(\frac{2}{5}\right)^2x^2$$

$$1 + 2x + \frac{8}{5}x^2$$

$$1 \times 5ax \\ -2x \times 1$$

$$= \frac{5ax}{0x}$$

$$5a - 2 = 0 \\ a = \frac{2}{5}$$

$$(ii) (1-2x)(1+ax)^5 \rightarrow 0x$$

$$1 \times 10a^2x^2 = 10a^2x^2 \\ -2x \times 5ax = -10ax^2 \\ \underline{(10a^2 - 10a)x^2}$$

$$\begin{aligned} & 10a^2 - 10a \\ & 10\left(\frac{2}{5}\right)^2 - 10\left(\frac{2}{5}\right) \\ & = -2.4x^2 \end{aligned}$$

HOW TO EXTRACT

ONE

PARTICULAR TERM.

Term No

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

POWER OF X

Q) find 5th term in expansion of $(2-x)^6$

$$T_r = {}^6 C_r (2)^{6-r} (-x)^r$$

$$T_5 = {}^6 C_4 (2)^{6-4} (-x)^4$$

$$= (5)(4)(x^4)$$

$$T_5 = 60x^4$$

Q) find the term of x^2 in expansion of $(3x-2)^6$

$$T_{r+1} = {}^6 C_r (3x)^{6-r} (-2)^r$$

$$= {}^6 C_r \cdot 3^{6-r} \cdot x^{6-r} \cdot (-2)^r$$

$$x^{6-r} = x$$

$$6-r = 2$$

$$6 = r+2$$

$$r = 4$$

$$T_{4+1} = {}^6 C_4 (3x)^{6-4} (-2)^4$$

$$= (15)(3x)^2(16)$$

$$= (15)(9x^2)(16)$$

$$= 2160x^2$$

STEPS

1) Expand all powers and isolate x terms

2) Equate isolated x -terms to your required term. Find r .

3) Put value of r in first step.

r cannot be decimal or fraction or negative.

Q Find the term of x^2 in expansion of $\left(3x + \frac{2}{x}\right)^8$

$$\rightarrow T_{r+1} = {}^8C_r (3x)^{8-r} \left(\frac{2}{x}\right)^r$$

$${}^8C_r \cdot 3^{8-r} \cdot x^{8-r} \cdot \frac{2^r}{x^r}$$

$$\frac{2^r}{x^r} = x^2$$

$$x^{8-2r} = x^2$$

$$8-2r = 2$$

$$6 = 2r$$

$$\boxed{r=3}$$

$$T_{3+1} = {}^8C_3 (3x)^{8-3} \left(\frac{2}{x}\right)^3$$

$$T_4 = (56)(243x^5) \left(\frac{8}{x^3}\right)$$

$$T_4 = 108864x^2$$

5 Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$. [3]

$$\rightarrow T_{r+1} = {}^6C_r (x)^{6-r} \left(\frac{2}{x}\right)^r$$

$$= {}^6C_r \cdot x^{6-r} \cdot \frac{2^r}{x^r}$$

$$\frac{x^{6-r}}{x^r} = x^2$$

$$x^{6-r-r} = x^2$$

$$6-2r = 2$$

$$4 = 2r$$

$$\boxed{r=2}$$

$$T_{2+1} = {}^6C_2 (x)^{6-2} \left(\frac{2}{x}\right)^2$$

$$= 15 \cdot x^4 \cdot \frac{4}{x^2}$$

$$= 60x^2$$

Coefficient = 60.

7 Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2} + \frac{2}{x}\right)^6$. [3]

$$\rightarrow T_{r+1} = {}^6C_r \left(\frac{x}{2}\right)^{6-r} \left(\frac{2}{x}\right)^r$$

$${}^6C_r \cdot \frac{x^{6-r}}{2^{6-r}} \cdot \frac{2^r}{x^r}$$

$$\frac{x^{6-r}}{x^r} = \left(\frac{x^2}{x}\right)$$

$$x^{6-2r} = x^2$$

$$6-2r = 2$$

$$T_{2+1} = {}^6C_2 \left(\frac{x}{2}\right)^{6-2} \left(\frac{2}{x}\right)^2$$

$$= (15) \left(\frac{x^4}{16}\right) \left(\frac{4}{x^2}\right)$$

$$= \frac{15}{4} x^2$$

$$\text{Coefficient} = \frac{15}{4} = 3.75$$

$$\begin{aligned}x^{6-2r} &= x^2 \\6-2r &= 2 \\4 &= 2r \\r &= 2\end{aligned}$$

$$\text{Coefficient} = \frac{15}{4} = 3.75.$$

$$\frac{\sqrt{2}}{3} = 0.6666666666666666$$

✓ ✓

- 16 Find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^7$.

[3]

$$\begin{aligned}T_{n+1} &= {}^7C_n (x)^{7-n} \left(\frac{2}{x^2}\right)^n \\&= {}^7C_n \cdot x^{7-n} \cdot \frac{2^n}{x^{2n}} \\&\quad \begin{aligned}\frac{x^{7-n}}{x^{2n}} &= x \\7-n-2n &= 1 \\7-3n &= 1 \\6 &= 3n \\n &= 2\end{aligned}\end{aligned}$$

$$\begin{aligned}T_{2+1} &= {}^7C_2 (x)^{7-2} \left(\frac{2}{x^2}\right)^2 \\&= (21) x^5 \cdot \frac{4}{x^4} \\&= 84x\end{aligned}$$

$$\text{Coefficient} = 84$$

- 22 Find the coefficient of x^6 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^7$.

[4]

$$\begin{aligned}T_{n+1} &= {}^7C_n (2x^3)^{7-n} \left(-\frac{1}{x^2}\right)^n \\&= {}^7C_n \cdot 2^{7-n} x^{3(7-n)} \cdot \frac{(-1)^n}{(x^2)^n} \\&\quad \begin{aligned}\frac{x^{21-3n}}{x^{2n}} &= x^6 \\21-5n &= 6 \\5n &= 15 \\n &= 3\end{aligned}\end{aligned}$$

$$\begin{aligned}T_{3+1} &= {}^7C_3 (2x^3)^{7-3} \left(-\frac{1}{x^2}\right)^3 \rightarrow \frac{(-1)^3}{(x^2)^3} \\&= (35)(16x^12) \frac{-1}{x^6} \\&= -560x^6\end{aligned}$$

- 27 Find the coefficient of x^3 in the expansion of $(2 - \frac{1}{2}x)^7$.

[3]

$$\begin{aligned}T_{n+1} &= {}^7C_n (2)^{7-n} \left(-\frac{1}{2}x\right)^n \\&= {}^7C_n \cdot 2^{7-n} \cdot \frac{(-1)^n}{2^n} x^n\end{aligned}$$

$$\begin{aligned}T_{3+1} &= {}^7C_3 (2)^4 \left(-\frac{1}{2}x\right)^3 \\&= (35)(16) \left(-\frac{1}{8}x^3\right) \\&= -70x^3\end{aligned}$$

$$\begin{array}{c} x^r = x^3 \\ \boxed{r=3} \end{array} \quad \left| \begin{array}{l} 2^r \\ = -70x^3 \end{array} \right.$$

New Vocabulary :

Term Independent of $x \rightarrow x^0$

- 15 Find the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^9$. [3]

Required $\stackrel{\downarrow}{=} x^0$

$$\begin{aligned} \rightarrow T_{n+1} &= {}^9C_n (x)^{9-n} \left(-\frac{1}{x^2}\right)^n \\ &= {}^9C_n \cdot x^{9-n} \cdot \frac{(-1)^n}{x^{2n}} \\ &\frac{x^{9-n}}{x^{2n}} = x^0 \rightarrow \text{Term independent of } x. \rightarrow = \boxed{-84} \\ &x^{9-3n} = x^0 \\ &9-3n = 0 \\ &\boxed{n=3} \end{aligned} \quad \begin{aligned} T_{3+1} &= {}^9C_3 (x)^{9-3} \left(-\frac{1}{x^2}\right)^3 \\ &= 84 \cancel{x} \frac{(-1)}{\cancel{x}} \\ &= \cancel{84} \end{aligned}$$

- 19 Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^6$. [3]

$$\begin{aligned} \rightarrow T_{n+1} &= {}^6C_n (2x)^{6-n} \left(\frac{1}{x^2}\right)^n \\ &= {}^6C_n \cdot 2^{6-n} \cdot x^{6-n} \cdot \frac{1}{(x^2)^n} \\ &\frac{x^{6-n}}{x^{2n}} = x^0 \\ &x^{6-3n} = x^0 \\ &6-3n = 0 \\ &\boxed{n=2} \end{aligned} \quad \begin{aligned} T_{2+1} &= {}^6C_2 (2x)^4 \left(\frac{1}{x^2}\right)^2 \\ &= (15)(16x^4) \left(\frac{1}{x^4}\right) \\ &= \boxed{240} \end{aligned}$$



ZAIÑEMATICS

FOR THE LOVE OF MATHS

Sequences

P1

Compiled by Rafay Mushtaq



zainematics



zainematics



zainematics

20

Very Easy . (7 MARKS) .

(Nothing Like O Levels)

ARITHMETIC :- (AP)

$$1, 4, 7, 10, \dots \quad d = \frac{10-7}{7-4} = 3$$

$$60, 55, 50, \dots \quad d = \frac{55-50}{50-60} = -5$$

Common difference = $d = \frac{\text{Next term}}{\text{Term}} - \frac{\text{Previous Term}}$

FORMULAS

$$\square \text{ nth term} = a + (n-1)d$$

First term. Term No. common difference

[2] Sum to first n terms

$$S_n = \frac{n}{2} (a + l)$$

Last term (if given).

[3] If last term is not given.

then formula for sum is:

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Q: $4, 7, 10, 13, \dots \quad (61) -$ AP $a = 4, d = 13-10 = +3$
 $n=20$

(i) Find the 20^{th} term in this sequence.

$$\text{nth term} = a + (n-1)d$$

$$20^{\text{th}} \text{ term} = 4 + (20-1)(+3)$$

$$20^{\text{th}} \text{ term} = 61$$

(ii) Find sum of first 20 terms.

$$S_{20} = \frac{n}{2} (a + l)$$

last term.

$$= \frac{20}{2} (4 + 61)$$

$$S_{20} = 650$$

(iii) Find sum of first 30 terms in sequence.

$$S_{30} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{30}{2} [2(4) + (30-1)(3)]$$

$$= 15(8 + 87)$$

$$S_{30} = 1425$$

$$S_n = \frac{n}{2} (a + l)$$

\downarrow

$$\text{nth term} = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

GEOMETRIC PROGRESSION (GP)

$$2, 4, 8, 16, 32, 64, \dots$$

$$\frac{2}{2} = \frac{4}{2} = \frac{8}{4} = \dots = r$$

$$r = \frac{64}{32} = \frac{16}{8} = 2$$

$$9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$$

$$\frac{9}{3} = \frac{3}{3} = \frac{1}{3} = \dots = r$$

$$r = \frac{3}{9} = \frac{1}{3} = \boxed{\frac{1}{3}}$$

$$\square \text{ nth term} = a \cdot r^{n-1}$$

Dot stands for multiply

[2] SUM OF FIRST n-TERMS

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$$

you have to pick formula for which

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \quad r = \frac{1}{3}, \frac{1}{3} = \boxed{\frac{1}{3}}$$

COMMON RATIO = $r = \frac{\text{NextTerm}}{\text{PreviousTerm}}$

$r-1$ $1-r$
you have to pick formula for which denominator is positive.

$$r=5 \quad (\text{First})$$

$$r=\frac{1}{2} \quad (\text{second})$$

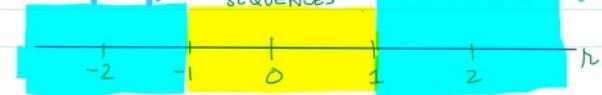
3. SUM TO INFINITY.

EXISTS ONLY FOR CONVERGING SEQUENCES.

DIVERGING.

CONVERGING SEQUENCES

DIVERGING.



IMP $\rightarrow -1 < r < 1$

$$S_{\infty} = \frac{a}{1-r}$$

If you apply this formula on a diverging sequence, It will give an answer but it will be absurd.

EXAM TRICK

	AP	GP
FIRST	a	a
SECOND	$a+d$	ar^1
THIRD	$a+2d$	ar^2
FOURTH	$a+3d$	ar^3
TENTH	$a+9d$	ar^9
100 th term.	$a+99d$	ar^{99}
25 th term.	$a+24d$	ar^{24}

8th term of GP is 45
 $ar^7 = 45$

15th term of AP is 70
 $a+14d = 70$

TYPE 1 = FORMULAS ONLY -

- I A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, (r)

(ii) the sum of the first ten terms.

[2]

[2]

GP

$$a = 64$$

$$S_{\infty} = 256$$

$$(i) S_{\infty} = \frac{a}{1-r}$$

$$256 = \frac{64}{1-r}$$

$$(ii) S_{10} = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}$$

$$r = \frac{3}{4} \quad \begin{matrix} \nearrow \\ \text{+ve denominator} \end{matrix}$$

$$256 = \frac{64}{1-r}$$

$$1-r = \frac{64}{256}$$

$$1-r = \frac{1}{4}$$

$$r = \boxed{\frac{3}{4}}$$

$$r = \boxed{\frac{3}{4}}$$

↑
+ve denominator

$$S_{10} = \frac{64(1-0.75^10)}{1-0.75}$$

$$= 241.5837$$

- (b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find
- the first term of the progression,
 - the sum to infinity.

GP

$$r = -\frac{2}{3}$$

$$S_3 = 35$$

$$S_n = \frac{a(r^n-1)}{r-1}, \quad \boxed{a(1-r^n)}$$

$$S_3 = \frac{a(1-r^3)}{1-r}$$

$$35 = a \frac{(1 - (-\frac{2}{3})^3)}{1 - (-\frac{2}{3})}$$

$$\frac{175}{3} = a \left(\frac{35}{27}\right)$$

$$\boxed{a = 45}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{45}{1 - (-\frac{2}{3})}$$

$$\boxed{S_\infty = 27}$$

- 14 The first term of a geometric progression is 12 and the second term is -6. Find

- the tenth term of the progression,
- the sum to infinity.

[3]
[2]

GP $a = 12$, $ar = -6$

$$12 r = -6$$

$$\boxed{r = -\frac{1}{2}}$$

$$\textcircled{i} \text{ Tenth term} = ar^9 = (12) \left(-\frac{1}{2}\right)^9 = 12 \left(\frac{-1}{512}\right)$$

$$= -\frac{3}{128}$$

$$\textcircled{ii} \text{ } S_\infty = \frac{a}{1-r} = \frac{12}{1 - \left(-\frac{1}{2}\right)} = \boxed{8}$$

2 Find

- the sum of the first ten terms of the geometric progression 81, 54, 36, ..., 32nd term [3]
- the sum of all the terms in the arithmetic progression 180, 175, 170, ..., 25. [3]

AP 180, 175, 170, ..., 25

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ..., 25. [3]

i) GP

$$81, 54, 36, \dots$$

$$a = 81, r = \frac{54}{81} = \frac{2}{3}$$

$$S_{10} = \frac{a(1-r^n)}{1-r} = \frac{81(1-(\frac{2}{3})^{10})}{1-\frac{2}{3}}$$

$$S_{10} = 238.786$$

AP

$$180, 175, 170, \dots, 25$$

$$a = 180, d = -5$$

$$\begin{aligned} S_n &= \frac{n}{2} (a+l) \\ &= \frac{32}{2} (180+25) \\ &= 3280 \end{aligned}$$

for n , apply n th term formula on last term.

$$n$$
th term = $a + (n-1)d$

$$25 = 180 + (n-1)(-5)$$

$$-155 = (-5)(n-1)$$

$$31 = n-1$$

$$n = 32$$

- 6 (a) Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]
 (b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive.
 Find
 (i) the value of x ,
 (ii) the sum to infinity of the progression.

DO

~~DO~~

$$105, 112, 119, \dots, 399$$

$$S_n = \frac{n}{2} (a+l)$$

$$= \frac{43}{2} (105+399)$$

$$n$$
th term = $a + (n-1)d$

$$399 = 105 + (n-1)(7)$$

$$294 = 7(n-1)$$

$$42 = n-1$$

$$n = 43$$

- b) GP
 common ratio stays same -
 Find (ii) twice and put it equal.

$$\begin{aligned} \frac{x}{144} &= \frac{64}{x} \\ x^2 &= 64 \times 144 \\ x &= \sqrt{64 \times 144} \\ x &= 96 \end{aligned}$$

$$144, 96, 64$$

$$r = \frac{96}{144}$$

$$\begin{aligned} S_\infty &= \frac{a}{1-r} = \frac{144}{1-\frac{96}{144}} \\ &= 432 \end{aligned}$$

- 7 The second term of a geometric progression is 3 and the sum to infinity is 12.

- (i) Find the first term of the progression.

An arithmetic progression has the same first and second terms as the geometric progression.

- (ii) Find the sum of the first 20 terms of the arithmetic progression.

i) GP

$$\text{second term} = 3$$

$$ar = 3$$

$$r = \frac{3}{a}$$

$$\text{Sum to infinity} = 12$$

$$\frac{a}{1-r} = 12$$

$$a = 12 \left(1 - \frac{3}{a}\right)$$

[4] Note: First write in words,
 Then put all available values
 The equate.

The
GP

$$\text{First} = 3$$

$$\text{Second} = 3$$

An
AP

$$\text{First} = a$$

$$\text{Second} = a+d$$

$$n = \frac{3}{\alpha}$$

$$\alpha = 12 \left(1 - \frac{3}{\alpha}\right)$$

$$a = 12 - \frac{36}{\alpha}$$

$$\alpha^2 = 12\alpha - 36$$

$$\alpha^2 - 12\alpha + 36 = 0$$

$$\alpha^2 - 6\alpha - 6\alpha + 36 = 0$$

$$\alpha(\alpha-6) - 6(\alpha-6) = 0$$

$$(\alpha-6)(\alpha-6) = 0$$

$$\alpha = 6$$

Second = 3 Second = $a+d$

$$a=6, \quad a+d=3$$

$$6+d=3$$

$$d=-3$$

$\boxed{\text{AP}}$ $a=6, \quad d=-3$

$$S_{20} = \frac{20}{2} \left[2(6) + (20-1)(-3) \right] = -450$$

- 9 The first term of a geometric progression is 81 and the fourth term is 24. Find

- (i) the common ratio of the progression,
(ii) the sum to infinity of the progression.

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

- (iii) Find the sum of the first ten terms of the arithmetic progression.

$\boxed{\text{GP}}$ First term = 81 Fourth term = 24

(i) $a = 81$

$$\alpha r^3 = 24$$

$$81 r^3 = 24$$

$$r^3 = \frac{8}{27}$$

$$r = \frac{2}{3}$$

(ii) $S_\infty = \frac{a}{1-r} = \frac{81}{1-\frac{2}{3}} = 243$

$\boxed{\text{This GP}}$ $\boxed{\text{An AP}}$

Second = $ar = (81)\left(\frac{2}{3}\right)$ First = a

Third = $ar^2 = (81)\left(\frac{2}{3}\right)^2$ Fourth = $a+3d$

Do not put equal before putting all known values.

$81\left(\frac{2}{3}\right) = a$ $81\left(\frac{2}{3}\right)^2 = a+3d$

$a = 54$ $72\left(\frac{4}{9}\right) = 54+3d$

$$72\left(\frac{4}{9}\right) = 54+3d$$

$$-18 = 3d$$

$$d = -6$$

$S_{10} = \frac{n}{2} [2a + (n-1)d]$

$$= \frac{10}{2} [2(54) + (10-1)(-6)]$$

$$= 270$$

- 26 The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

- (i) Find the common difference of the progression.

The first term, the ninth term and the n th term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

- (ii) Find the common ratio of the geometric progression and the value of n .

$\boxed{\text{AP}}$ First = 12 $S_9 = 135$

$$a = 12$$

$$\frac{9}{2} [2(12) + (9-1)d] = 135$$

$$24 + 8d = 30$$

$$8d = 6$$

$$d = \frac{3}{4}$$

$\boxed{\text{THIS AP}}$ $\boxed{\text{ANOTHER GP}}$

First = $a = 12$ First = a

Ninth = $a+8d = 12+8\left(\frac{3}{4}\right)$ Second = ar

n th term = $a+(n-1)d$ Third = ar^2

$12+8\left(\frac{3}{4}\right)$ $12+(n-1)\left(\frac{3}{4}\right) = ar^2$

$a = 12$ $12+(n-1)\left(\frac{3}{4}\right) = 12\left(\frac{3}{2}\right)^2$

$12+8\left(\frac{3}{4}\right) = ar$

$18 = 12r$

$$\boxed{u = \frac{3}{4}} \quad \left\{ \begin{array}{l} \boxed{\alpha = 12} \\ \boxed{n = \frac{3}{2}} \end{array} \right. \quad \begin{array}{l} 12 + 8 \left(\frac{3}{4} \right) = 12 \cdot 2 \\ 18 = 12r \end{array} \quad \begin{array}{l} 12 + (n-1) \left(\frac{3}{4} \right) = 12 \left(\frac{3}{2} \right)^2 \\ n = 21 \end{array}$$

TYPE 2 : Sum.

- 25 (a) In an arithmetic progression, the sum of the first n terms, denoted by S_n , is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference.

[3]

$$S_n = n^2 + 8n$$

Method. First put $n=1$

$$n=1, S_1 = (1)^2 + 8(1) = 9 \quad \boxed{a=9}$$

S_1 means sum of one term

That means it is first term.

$$n=2, S_2 = (2)^2 + 8(2) = 20 \quad \boxed{S_2 \text{ means sum of first two terms.}}$$

S_2 = First Term + Second term.

$$20 = 9 + \text{Second term.}$$

$$\text{Second term} = 11.$$

First term = 9
Common Difference = $d = 2$

$$\boxed{\text{AP}} \quad 9, 11, \dots \quad d = \text{Next-prev} = 11 - 9 = \boxed{2}$$

- 32 (a) In an arithmetic progression, the sum, S_n , of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression.

[3]

$$S_n = 2n^2 + 8n$$

$$n=1, S_1 = 2(1)^2 + 8(1) = 10$$

S_1 , first term = 10

$$n=2, S_2 = 2(2)^2 + 8(2) = 24$$

S_2 = First + Second term.

$$24 = 10 + \text{Second term.}$$

Second term = 14

$$\boxed{\text{AP} = 10, 14, \dots}$$

$$\boxed{\text{Common difference} = 14 - 10 = 4}$$

TYPE 3 : SCENARIO BASED :-

LAYMAN APPROACH

Read as if you have no idea of AP/GP chapter.

Predict first three terms by common sense.

Day/year	n	Terms.	Check AP $d = \text{Next-Prev}$	Check GP $r = \frac{\text{Next}}{\text{Prev}}$
2012	1	<input type="text"/>		
2013	2	<input type="text"/> <input type="text"/>		
2014	3	<input type="text"/> <input type="text"/> <input type="text"/>		

2013	2	
2014	3	
⋮	⋮	⋮
2020	9	



- (b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]

(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

Year,	n	Terms	Check AP $d = \text{Next} - \text{Prev.}$	Check GP $r = \frac{\text{Next}}{\text{Prev.}}$
2012	1	4000	200	$\frac{4200}{4000} = 1.05$
2013	2	4200	210	$\frac{4410}{4200} = 1.05$
2014	3	4410	X	
2022	11			

① Grant in 2022 = $(1)^{th}$ term of GP = $a \cdot r^{n-1}$
 $= (4000)(1.05)^{10}$
 $= 6515.5785.$

$$\begin{aligned} 4000 \times 5\% &= 200 \\ (i) S_{11} &= \frac{a(r^n - 1)}{r - 1} \\ &= \frac{4000(1.05^{11} - 1)}{1.05 - 1} \\ &= 56827.14865 \end{aligned}$$

- 5 Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was \$5000. Find

(i) the grant given in 2011, [3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive. [2]

Years	n	Terms	Check AP $d = N - P$	Check GP $r = \frac{N}{P}$
2001	1	5000	250	$r = \frac{5250}{5000} = 1.05$
2002	2	5250	262.5	$= \frac{5512.5}{5250} = 1.05$
2003	3	5512.5	X	
2011	11			

① Grant in 2011 = 11^{th} term = $a \cdot r^{n-1}$
 $= (5000)(1.05)^{10}$
 $= 8184.473$

- 18 A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

- (i) if Model 1 is used, [4]
(ii) if Model 2 is used. [3]



ZAIÑMATICS

FOR THE LOVE OF MATHS

Quadratics

P1

Compiled by Rafay Mushtaq



[zainematics](#)



[zainematics](#)



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QUADRATICS (P1)

Saturday, October 17, 2020 2:12 AM

Links with: 1) COORDINATE (9+)
2) FUNCTIONS (11+)

QUADRATIC EQUATION:

Max power of $x = 2$

$$y = 9x^2 + 3x + 5$$

$$y = 6x - 2x^2$$

$$y = 8x^2$$

1 STANDARD FORM

$$y = ax^2 + bx + c$$

SHAPE

y-intercept.

2 COMPLETED SQUARE

VERTEX FORM
CONICAL FORM,

$$y = a(x-b)^2 + c$$

SHAPE

Min value if $a > 0$

Max value if $a < 0$.

TURNING POINT =

$$\begin{cases} x-b=0 \\ x=b \end{cases} \quad \begin{cases} y=c \end{cases}$$

3 ROOT FORM

$$y = a(x-b)(x-c)$$

SHAPE

x -intercepts.
(ROOTS)

$$\begin{cases} x-b=0 \\ x=b \end{cases}$$

$$\begin{cases} x-c=0 \\ x=c \end{cases}$$

STANDARD TO **COMPLETED SQUARE FORM.**

Q Express $y = 4x^2 - 24x + 40$ in form $a(x-b)^2 + c$

$$4[x^2 - 6x + (3)^2 - (3)^2 + 10]$$

Half

$$4[(x-3)^2 - 9 + 10]$$

$$4[(x-3)^2 + 1] = 4(x-3)^2 + 4$$

$$a(x-b)^2 + c$$

$$\begin{cases} a=4 \\ b=3 \\ c=4 \end{cases}$$

Q Express $2x^2 + 7x - 12$ in form $a(x-b)^2 + c$.

Q Express $2x^2 + 7x - 12$ in form $a(x-b)^2 + c$.

$$\begin{aligned}
 & 2 \left[x^2 + \frac{7}{2}x + \underbrace{\left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2}_{\text{Half}} - 6 \right] \\
 & 2 \left[\left(x + \frac{7}{4}\right)^2 - \frac{49}{16} - 6 \right] \\
 & 2 \left[\left(x + \frac{7}{4}\right)^2 - \frac{145}{16} \right] \\
 & 2 \left(x + \frac{7}{4} \right)^2 - \frac{145}{8} \\
 & a(x-b)^2 + c
 \end{aligned}$$

$a=2$
 $b = -\frac{7}{4}$
 $c = -\frac{145}{8}$

Q Express $y = 3x^2 - 24x + 5$ in form $a(x+b)^2 - c$

$$\begin{aligned}
 & 3 \left[x^2 - 8x + \underbrace{\left(4\right)^2 - \left(4\right)^2}_{\text{Half}} + \frac{5}{3} \right] \\
 & 3 \left[(x-4)^2 - 16 + \frac{5}{3} \right] \\
 & 3 \left[(x-4)^2 - \frac{43}{3} \right] \\
 & 3(x-4)^2 - \frac{43}{3} \\
 & a(x+b)^2 - c
 \end{aligned}$$

$$a=3, \quad +b=-4 \quad -c=-43$$

$$c = 43$$

Q Express $4x^2 - 24x + 44$ in form $(2x-a)^2 + b$.

STEP1: Completed square form.

$$\begin{aligned}
 & 4 \left[x^2 - 6x + \underbrace{\left(3\right)^2 - \left(3\right)^2}_{\text{Half}} + 11 \right] \\
 & 4 \left[(x-3)^2 - 9 + 11 \right] \\
 & 4 \left[(x-3)^2 + 2 \right]
 \end{aligned}
 \quad \left| \begin{array}{l}
 4(x-3)^2 + 8 \\
 2^2(x-3)^2 + 8 \\
 [2(x-3)]^2 + 8 \\
 (2x-6)^2 + 8 \\
 (2x-a)^2 + b \\
 -b = -a \\
 a = 6 \\
 +b = 8 \\
 b = 8
 \end{array} \right.$$

Q Express $9x^2 - 36x + 20$ in form $(3x+a)^2 + b$

Q Express $9x^2 - 36x + 20$ in form $(3x+a)^2 + b$

$$\left. \begin{array}{l} 9\left[x^2 - 4x + (2)^2 - (2)^2 + \frac{20}{9}\right] \\ 9\left[(x-2)^2 - 4 + \frac{20}{9}\right] \\ 9\left[(x-2)^2 - \frac{16}{9}\right] \end{array} \right| \begin{array}{l} 9(x-2)^2 - 16 \\ 3^2(x-2)^2 - 16 \\ [3(x-2)]^2 - 16 \\ (3x-6)^2 - 16 \\ (3x+a)^2 + b \end{array}$$

$a = -6, b = -16.$

STANDARD \longrightarrow ROOT FORM.
 $y = ax^2 + bx + c$ $y = a(x-b)(x-c)$
 FACTORIZATION.

$$\begin{aligned} y &= x^2 - 7x + 12 \\ &= x^2 - 3x - 4x + 12 \\ &\quad x(x-3) - 4(x-3) \\ y &= 1(x-3)(x-4) \\ y &= a(x-b)(x-c) \\ a &= 1 \quad \boxed{-3 = -b} \quad \boxed{-4 = -c} \\ &\quad \boxed{b=3} \quad \boxed{c=4} \end{aligned}$$

SKETCH

1 SHAPE
(ALL FORMS)



2 TURNING POINT (VERTEX) (COMPLETED SQUARE FORM).

$$y = 2(x-3)^2 - 4$$

\downarrow
 $x-3=0$

$$\text{TURNING POINT} = \boxed{x=3} \quad \boxed{y=-4} = (3, -4)$$

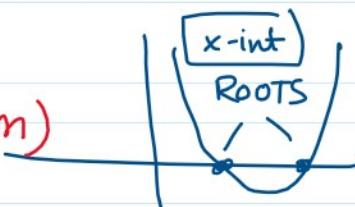
3 y-intercept ($x=0$) (STANDARD FORM)

$$y = 2x^2 + 7x + 1 \rightarrow \text{y intercept.}$$

4 x-intercepts ($y=0$) (ROOT FORM)

ONLY USE X-INTERCEPTS IF QUESTION REQUIRES.

$$y = 3(x-2)(x+6)$$



$$y = 3(x-2)(x+6)$$

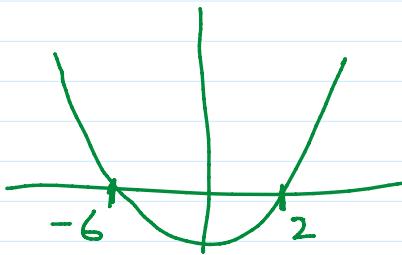
$$x-2=0$$

$$x+6=0$$

x -INTERCEPTS

$$x=2$$

$$x=-6$$



Q

SKETCH:

$$y = 4x^2 - 24x + 40$$

Shape

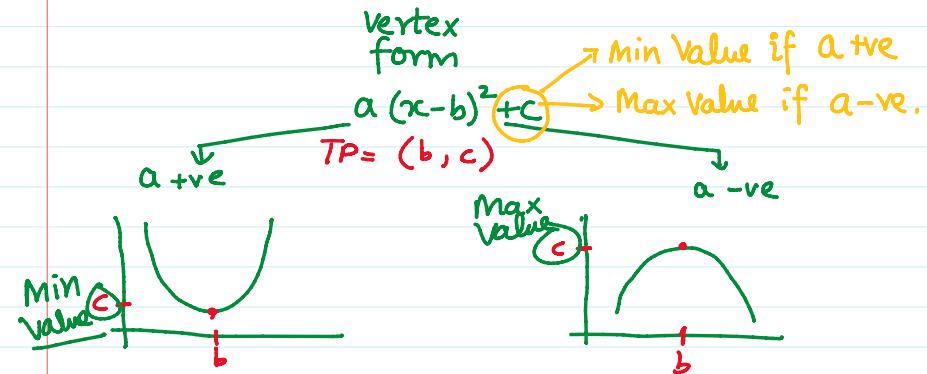
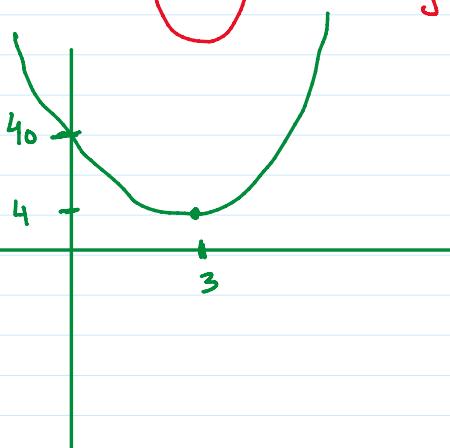
$$y = 4(x-3)^2 + 4$$

y-intercept

$$x-3=0$$

Turning point.

$$x=3 \quad y=4$$



ROOTS (x -intercepts)
 $y = 0$

$$y = 2x^2 - 3x + 8$$

$$\sim -2 \quad 2x+8$$

... (Turn and)

$$y = 2x^2 - 3x + 8$$

$$0 = 2x^2 - 3x + 8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

+ve (Two Ans)

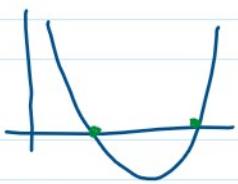
Zero (1 Ans)

-ve (No Ans)

DISCRIMINANT

$$b^2 - 4ac$$

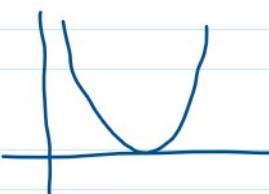
positive.
(2 Ans)



$$b^2 - 4ac > 0$$

TWO DISTINCT
REAL ROOTS

Zero
(1 Ans)



$$b^2 - 4ac = 0$$

TWO EQUAL/REPEAT
REAL ROOTS

Negative
(No Ans)



$$b^2 - 4ac < 0$$

NO REAL
ROOT
(TWO NOT REAL
ROOTS)

ONLY
Real

$$b^2 - 4ac > 0$$

$$b^2 - 4ac = 0$$

$$b^2 - 4ac \geq 0$$

INEQUALITY

Q Find set of values of k for which

$y = 3x^2 - 2x - 2k$ has no real roots

$$a=3, b=-2, c=-2k$$

$$b^2 - 4ac < 0$$

$$(-2)^2 - 4(3)(-2k) < 0$$

$$4 + 24k < 0$$

$$24k < -4$$

$$k < -\frac{4}{24}$$

$$\boxed{k < -\frac{1}{6}}$$

Q Find value of k for which $y = kx^2 - 9x + 2$ has two real repeated roots.

$$b^2 - 4ac = 0$$

$$(-9)^2 - 4(k)(2) = 0$$

$$81 - 8k = 0$$

$$81 = 8k$$

$$\boxed{k = \frac{81}{8}}$$

INTERSECTION OF A LINE AND A QUADRATIC CURVE

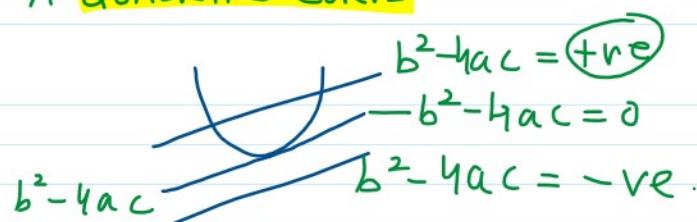
$$y = 2x^2 + 6x - 5 \rightarrow y = 2x + 1$$

STEP 1: EQUATE BOTH

$$2x^2 + 6x - 5 = 2x + 1$$

$$2x^2 + 4x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$(4)^2 - 4(2)(-6)$
+ 54 (Two points of intersection).

+ve (Two Ans)

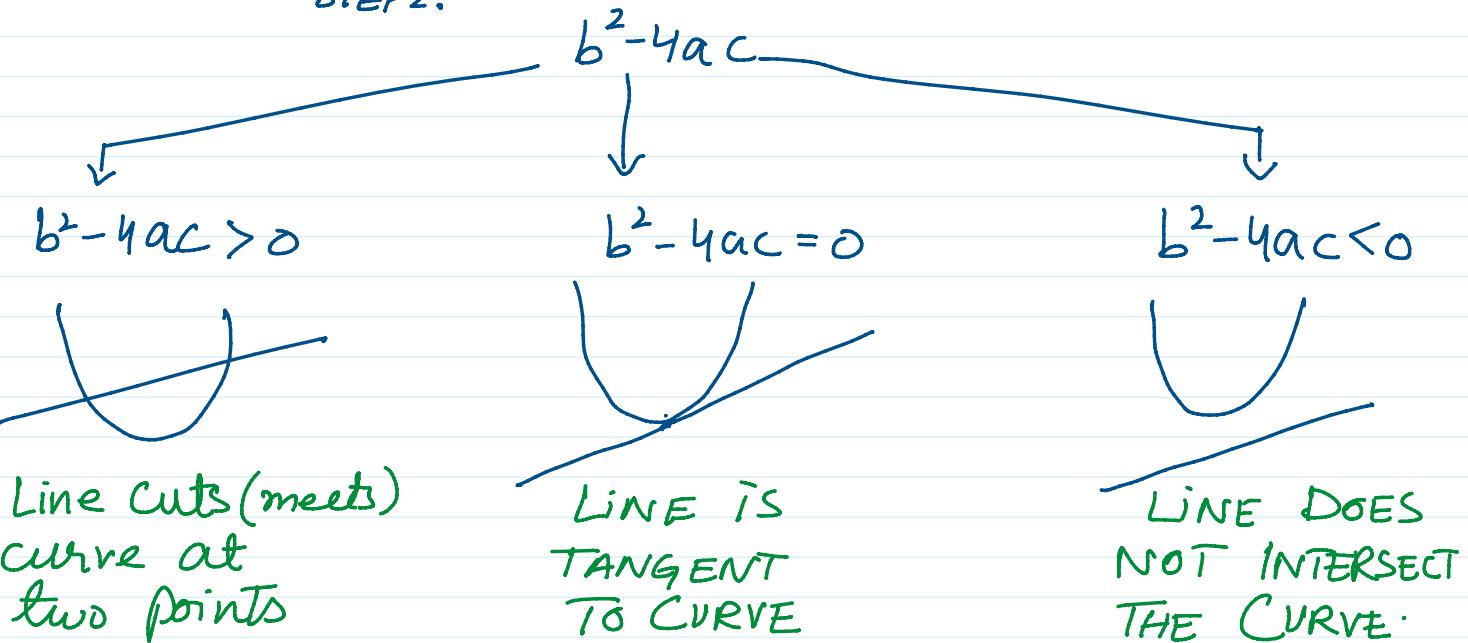
= 0 (1 Ans)

-ve (No Ans)

LINE + QUADRATIC CURVE

STEP1: EQUATE BOTH.

STEP2:



INEQUALITIES

LINEAR

$$\begin{aligned} \boxed{1} \quad 2x - 3 &> 7 \\ 2x &> 10 \\ x &> 5 \end{aligned}$$

$$\boxed{2} \quad -2x < 6$$

$$x > \frac{6}{-2}$$

$$x > -3$$

$$\begin{aligned} \boxed{3} \quad \frac{x}{-3} &> 6 \\ x &< (6)(-3) \\ x &< -18 \end{aligned}$$

QUADRATIC

$$x^2 > 16$$

YOU ARE NOT ALLOWED TO TAKE SQUARE ROOT ON AN INEQUALITY.

$$(x-2)(x-5) > 0$$

$$\cancel{x-2} > 0 \quad \cancel{x-5} > 0$$

YOU CANNOT DO EITHER/OR STEP ON AN INEQUALITY.

STEP1: FACTORIZE.

STEP2: SKETCH

SHAPE

x-intercepts

STEP3: Colour the relevant part of curve.

STEP4: Write inequality for x-values of coloured parts of curve only.

$$\begin{aligned} \textcircled{Q} \quad x^2 - 6x - 16 &< 0 \\ x^2 - 8x + 2x - 16 &< 0 \\ x(x-8) + 2(x-8) &< 0 \end{aligned}$$

STEP1: FACTORIZE.

STEP2: SKETCH

SHAPE

x-intercepts

$$x^2 - 8x + 2x - 16 < 0$$

$$x(x-8) + 2(x-8) < 0$$

$$(x-8)(x+2) < 0$$

$\downarrow y < 0$ (below x-axis)



$$-2 < x < 8$$

STEP2: SKETCH x-intercepts

STEP3: Colour the relevant part of curve.

STEP4: Write inequality for x-values of coloured parts of curve only.

Q

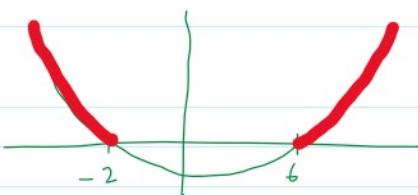
$$x^2 - 4x - 12 > 0$$

$$x^2 - 6x + 2x - 12 > 0$$

$$x(x-6) + 2(x-6) > 0$$

$$(x-6)(x+2) > 0$$

$y > 0$ (above x-axis)

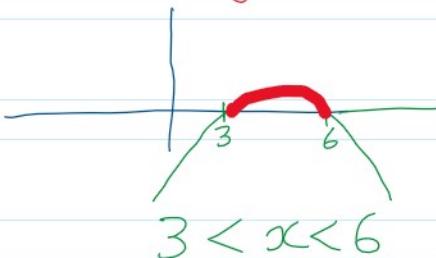


$$x < -2 \quad \text{or} \quad x > 6$$

Q

$$(x-3)(6-x) > 0$$

$y > 0$



$$x^2 < 16$$

$$x^2 - 16 < 0$$

$$x^2 - 4^2 < 0$$

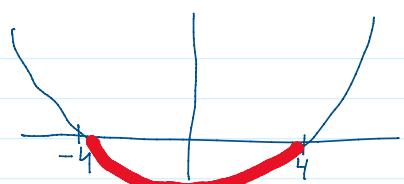
$$(x+4)(x-4) < 0$$

STEP1: FACTORIZE.

STEP2: SKETCH SHAPE x-intercepts

STEP3: Colour the relevant part of curve.

STEP4: Write inequality for x-values of coloured parts of curve only.



$$-4 < x < 4$$



ZAINEMATICS

FOR THE LOVE OF MATHS

Functions

P1

Compiled by Rafay Mushtaq



zainematics



zainematics



zainematics

FUNCTIONS (P1)

(10 MARKS)

FUNCTIONS
ALONE

WITH
QUADRATICS

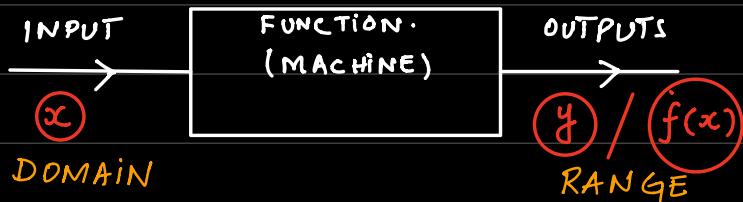
WITH
TRIG

WITH
DIFFERENTIATION

VERY LONG BUT EASY CHAPTER.

MEMORISE EVERY DETAIL!

FUNCTIONS ARE NUMBER MACHINES.



$$f(x) = 2x + 3 \quad g(x) = x^2 - 5$$

Name of function *value of input (x)*

$$f(1) = 2(1) + 3 = 5$$

$$f(4) = 2(4) + 3 = 11$$

$$g(3) = (3)^2 - 5 = 4$$

$$f(k) = 2k + 3$$

$$g(t-1) = (t-1)^2 - 5$$

INVERSE OF A FUNCTION:

(2 MARKS IN O LEVELS) (3 MARKS IN A LEVELS)

THERE ARE ADDITIONAL STEPS. BE CAREFUL!

$$f(x) = 2x - 5$$

STEP 1:

$$f(x) = y$$

$$y = 2x - 5$$

$$y + 5 = 2x$$

$$f(x) = y$$

STEP 2:

$$x = \frac{y+5}{2}$$

$$f^{-1}(y) = x$$

STEP 3:

$$f^{-1}(y) = x$$

STEP 4:

$$f^{-1}(y) = \frac{y+5}{2}$$

$$f^{-1}(x) = \frac{x+5}{2}$$

$$\text{Q: } f(x) = 2x + 8$$

$$\begin{aligned}f(x) &= y \\y &= 2x + 8 \\y - 8 &= 2x \\x &= \frac{y - 8}{2}\end{aligned}$$

$$f^{-1}(y) = x$$

$$f^{-1}(y) = \frac{y - 8}{2}$$

$$f^{-1}(x) = \frac{x - 8}{2}$$

$$\text{Q: } g(x) = x^3 - 8$$

$$\begin{aligned}g(x) &= y \\y &= x^3 - 8 \\y + 8 &= x^3 \\x^3 &= y + 8 \\x &= \sqrt[3]{y + 8} \\g^{-1}(y) &= x\end{aligned}$$

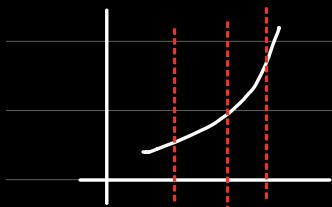
$$g^{-1}(y) = \sqrt[3]{y + 8}$$

$$g^{-1}(x) = \sqrt[3]{x + 8}$$

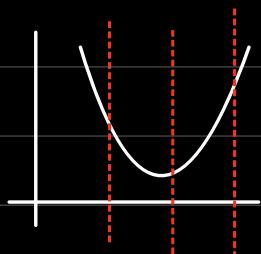
TEST

VERTICAL LINE TEST

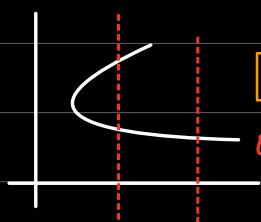
CHECKS WHETHER A GRAPH IS
A FUNCTION OR NOT?



PASS IT IS FUNCTION
ONE-ONE FUNCTION



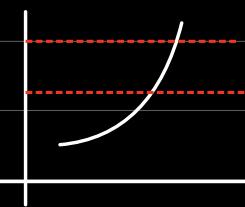
PASS IT IS FUNCTION
MANY-ONE FUNCTION



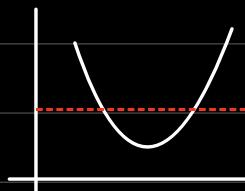
FAIL NOT A FUNCTION
ONE-MANY.

HORIZONTAL LINE TEST

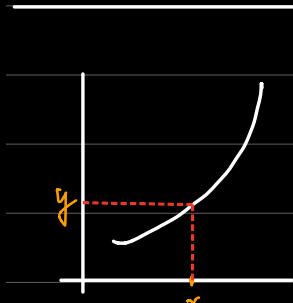
CHECKS IF THE INVERSE OF
A FUNCTION EXISTS OR NOT?



PASS INVERSE EXISTS
ONE-ONE FUNCTION

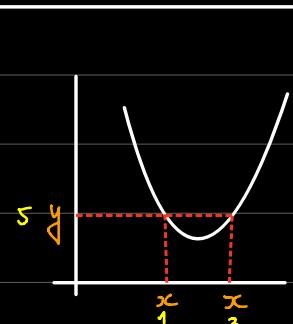


FAIL INVERSE
DOES NOT EXIST.
MANY-ONE FUNCTION



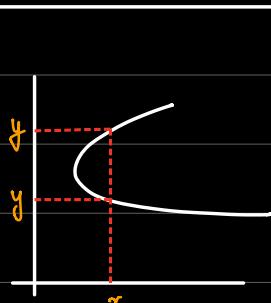
ONE - ONE FUNCTION

For one input you
will get one output.



MANY TO ONE FUNCTION

For Many inputs you
can get one output.

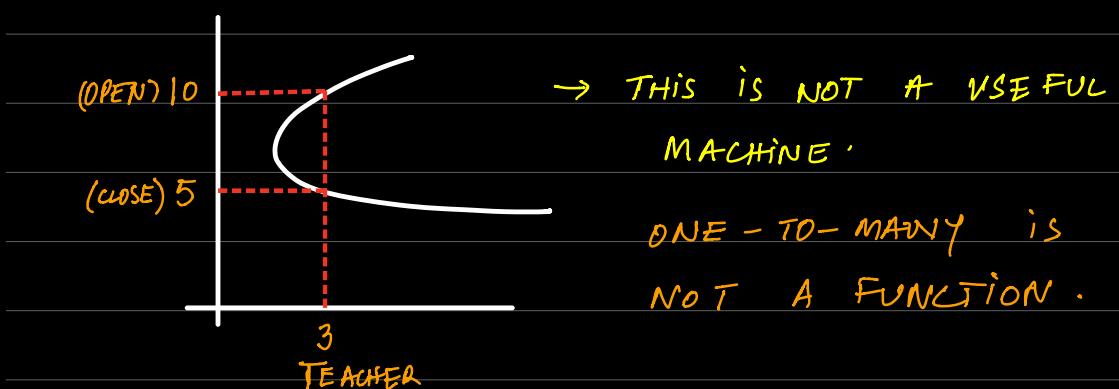
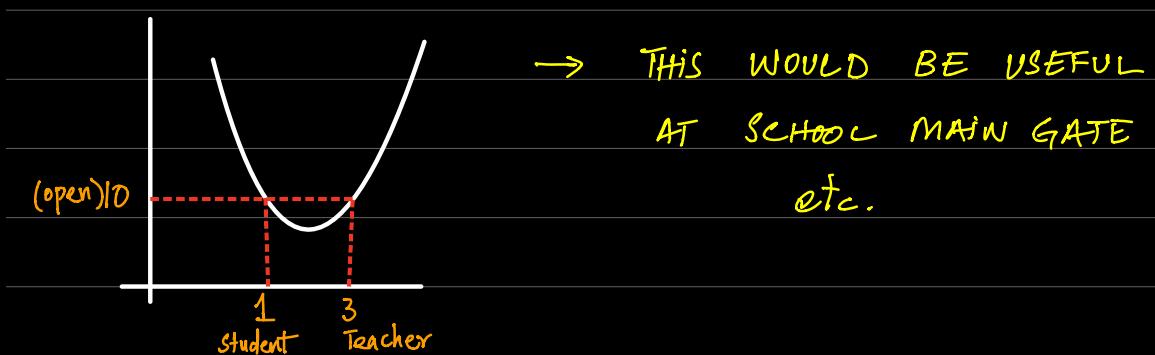
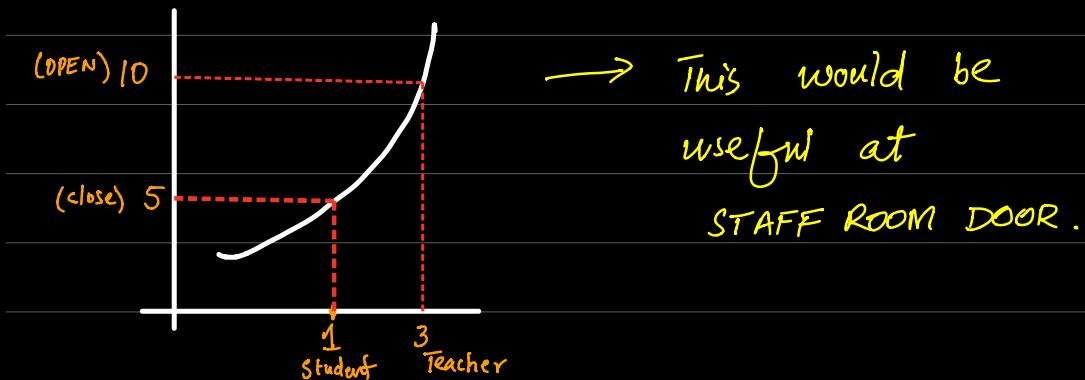


ONE TO MANY

one input can
get you many outputs.

ACCESS POINTS

$$\begin{array}{ll} \text{INPUT} = 1 \text{ (STUDENT)} & \text{OUTPUT} = 5 \text{ (CLOSE)} \\ = 3 \text{ (TEACHER)} & = 10 \text{ (OPEN)} \end{array}$$



COMPOSITE FUNCTIONS

(FUNCTION WITHIN A FUNCTION)

$$f(x) = 2x + 5$$

$$g(x) = x^2 + 3$$

$$\begin{aligned} fg(x) &= f(g(x)) = 2g(x) + 5 \\ &= 2(x^2 + 3) + 5 \\ &= 2x^2 + 6 + 5 \\ &= 2x^2 + 11 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ESSENTIAL WORKING.}$$

$$gf(x) = (2x+5)^2 + 3$$

$$ff(x) = 2(2x+5) + 5$$

$$gg(x) = (x^2 + 3)^2 + 3$$

STEP 1: Write the first function with () instead of x .

STEP 2: Write value of second function inside the bracket.

Q:

$$f(x) = 2x + 3$$

Given that $ff(t) = 21$, find t .

$$ff(x) = 2(2x+3) + 3 = 4x + 6 + 3 = 4x + 9$$

$$ff(x) = 4x + 9$$

$$ff(t) = 4t + 9$$

$$21 = 4t + 9$$

$$4t = 12$$

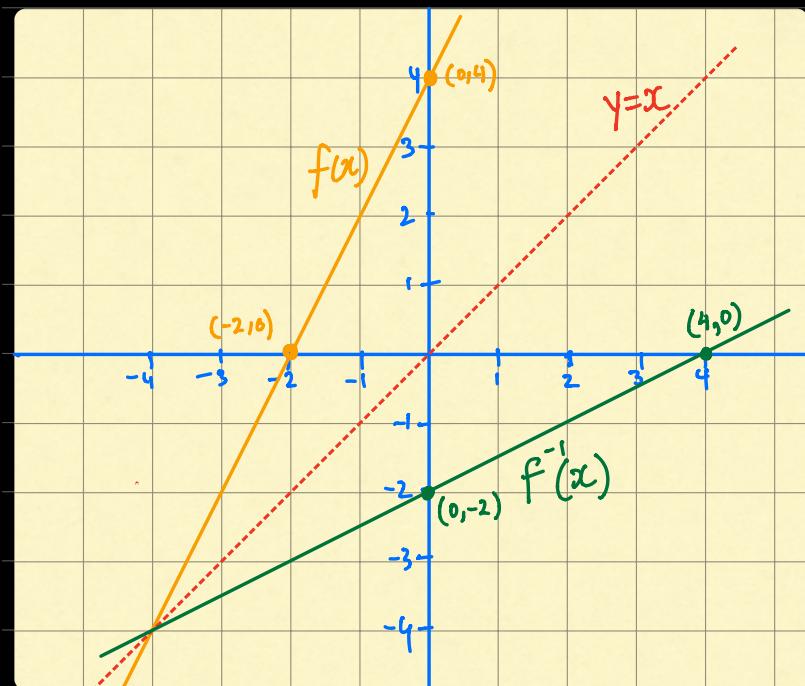
$$t = 3$$

SKETCH OF $f(x)$ and $f^{-1}(x)$, ON SAME DIAGRAM.

Q: $f(x) = 2x + 4$

Sketch $f(x)$ and $f^{-1}(x)$ on same diagram
making clear the relationship between them.
(3marks).

IMP: THIS HAS TO BE DRAWN TO-SCALE.



$$f(x) = 2x + 4$$

$$y = 2x + 4$$

STRAIGHT LINE.

x-intercept $y=0$

$$0 = 2x + 4$$

$$x = -2$$

y-intercept $x=0$

$$y = 2(0) + 4$$

$$y = 4$$

$f^{-1}(x)$ is reflection of $f(x)$ in line $y=x$.

Q: $f(x) = 2x^2 - 8x + 14$

(a) Express $f(x)$ in form $a(x-b)^2 + c$

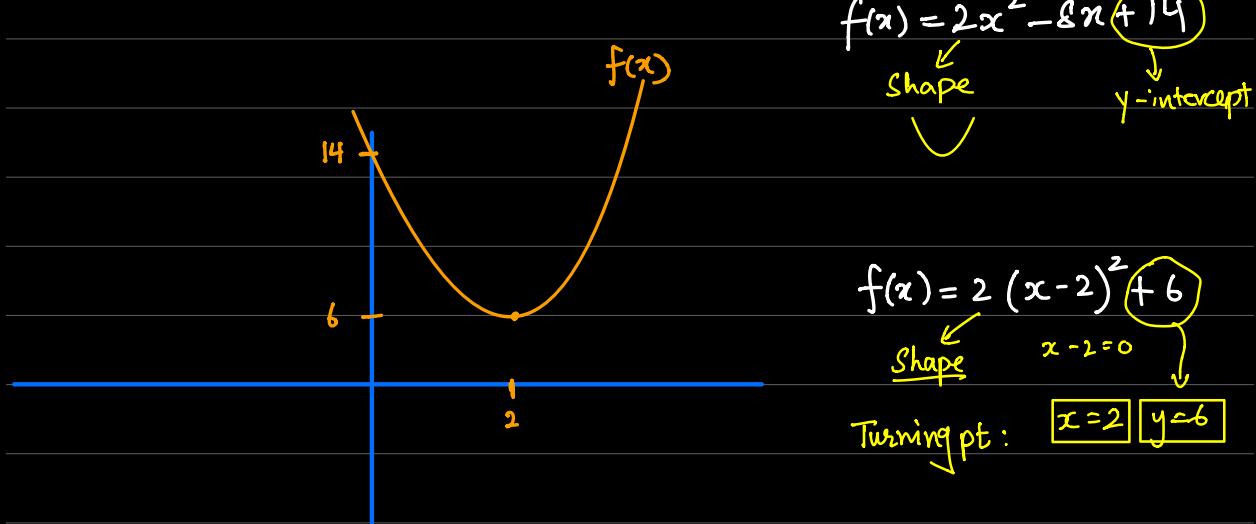
$$2 \left[x^2 - 4x + 2^2 - (2)^2 + 7 \right]$$

$$2 \left[(x-2)^2 - 4 + 7 \right]$$

$$2 \left[(x-2)^2 + 3 \right]$$

$$2(x-2)^2 + 6$$

(b) Sketch $f(x)$.



(c) State with a reason whether $f^{-1}(x)$ exists or not?

No, inverse does not exist because it is not a one-one function.

DOMAIN VALUES OF X FOR WHICH A FUNCTION DOES NOT GO CRAZY.

TYPE1

$$f(x) = \sqrt{x}$$

Domain:

$$\begin{aligned}x &\geq 0 \\x &\in \mathbb{R}\end{aligned}$$

$$f(x) = \sqrt{x-2}$$

Domain:

$$\begin{aligned}x &\geq 2 \\x &\in \mathbb{R}\end{aligned}$$

GENERAL RULE

$$f(x) = \sqrt{\boxed{t}}$$

$x \in \mathbb{R}$

\downarrow belongs to Real Numbers
 \downarrow Any no.

Domain:

$$\boxed{\quad} \geq 0 \text{ and solve.}$$

$$x \in \mathbb{R}$$

$$f(x) = \sqrt{2x+3}$$

$$\text{Domain: } 2x+3 \geq 0$$

$$2x \geq -3$$

$$x \geq \frac{-3}{2} \text{ and } x \in \mathbb{R}$$

TYPE2

$$f(x) = \frac{3}{x}$$

Domain: $x \neq 0$
 $x \in \mathbb{R}$

$$f(x) = \frac{3}{x-2}$$

Domain: $x \neq 2$
 $x \in \mathbb{R}$

GENERAL RULE

$$f(x) = \frac{\text{Anything}}{\boxed{\quad}}$$

Domain: $\boxed{\quad} \neq 0$ solve
 $x \in \mathbb{R}$

$$f(x) = \frac{2x+5}{3x-1}$$

Domain:
 $3x - 1 \neq 0$

$$3x \neq 1$$

$$x \neq \frac{1}{3} \quad \text{and } x \in \mathbb{R}$$

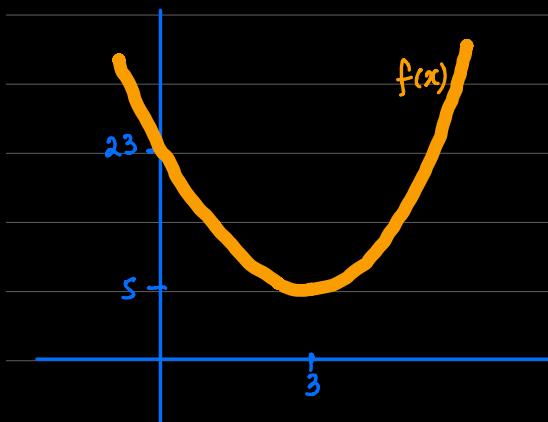
RANGE: VALUES OF y THAT GRAPH OF $f(x)$ COVERS.

GOLDEN RULE: NEVER TELL RANGE OF A FUNCTION
 WITHOUT LOOKING AT ITS GRAPH

FIND RANGE OF FOLLOWING FUNCTIONS:

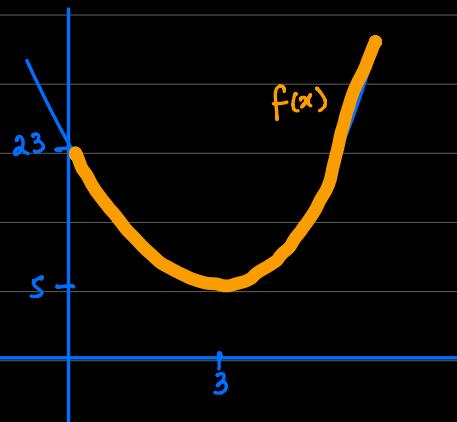
(a) $f(x) = 2(x-3)^2 + 5 \quad \underline{x \in \mathbb{R}}$

\downarrow
 You are
 allowed
 all
 values
 of x .



Range: $y \geq 5$
 $f(x) \geq 5$

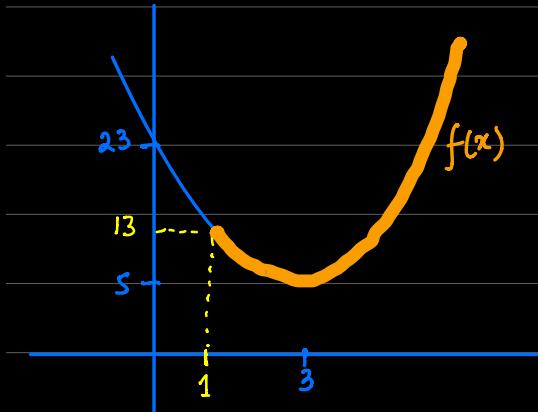
(b) $f(x) = 2(x-3)^2 + 5 \quad x > 0$



Range: $y \geq 5$
 $f(x) \geq 5$

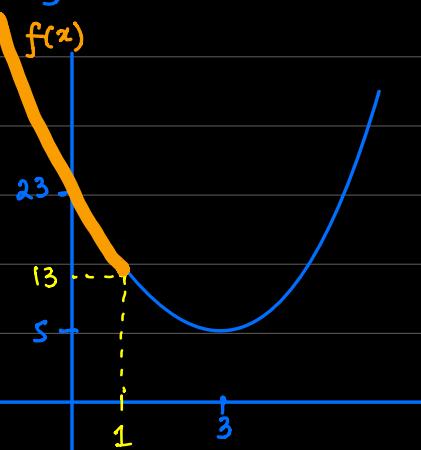
Range is usually written
 by name of function.

(c) $f(x) = 2(x-3)^2 + 5 \quad x \geq 1$
 put $x=1$, $y=2(1-3)^2+5$
 $y=13$



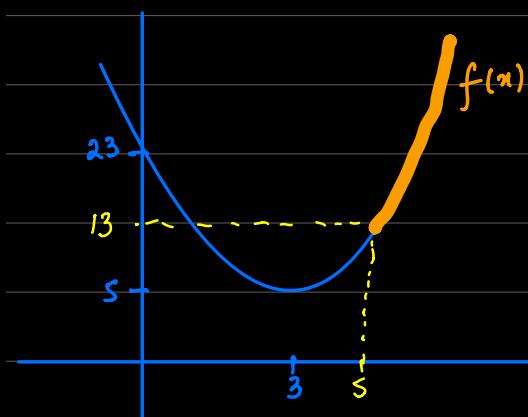
Range: $y \geq 5$
 $f(x) \geq 5$

(d) $f(x) = 2(x-3)^2 + 5 \quad x < 1$



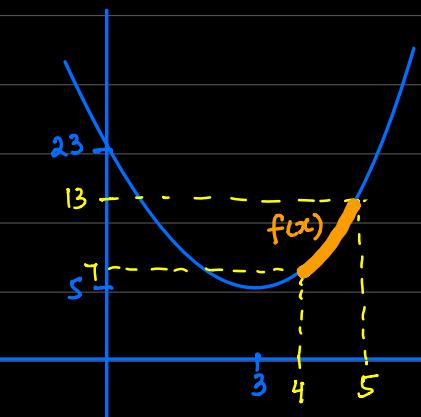
Range: $y \geq 13$
 $f(x) \geq 13$

(e) $f(x) = 2(x-3)^2 + 5 \quad x \geq 5$
 put $x=5$, $y=2(5-3)^2+5$
 $y=13$



Range: $y \geq 13$
 $f(x) \geq 13$

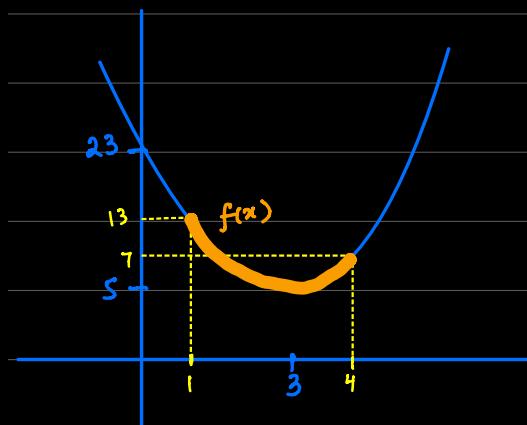
(f) $f(x) = 2(x-3)^2 + 5 \quad 4 < x \leq 5$
 put $x=4$, $y=2(4-3)^2+5=7$



Range: $7 < y \leq 13$
 $7 < f(x) \leq 13$

(g)

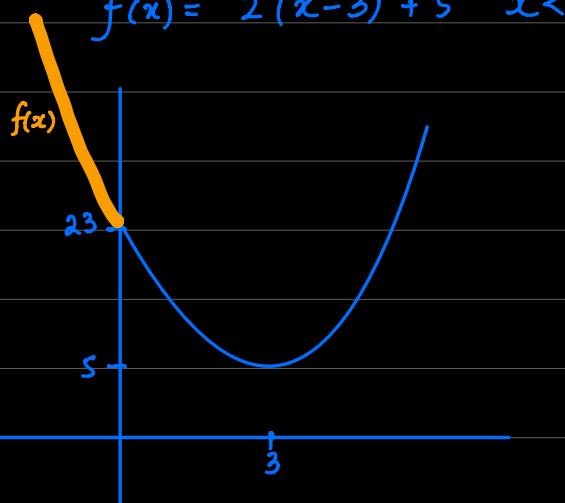
$$f(x) = 2(x-3)^2 + 5 \quad 1 < x < 4$$



Range:	$5 \leq y < 13$
	$5 \leq f(x) < 13$

(h)

$$f(x) = 2(x-3)^2 + 5 \quad x < 0$$



Range :	$y > 23$
	$f(x) > 23$



ZAIÑMATICS

FOR THE LOVE OF MATHS

Circular Measure

P1

Compiled by Rafay Mushtaq



zainematics



zainematics



zainematics

TRIGONOMETRY.

CIRCULAR
MEASURE
(8 MARKS)

PURE
TRIGONOMETRY.
(8 MARKS).

BASICS :-

UNITS

ANGLE.

DEGREES

$$180^\circ$$

=

RADIANS
 π rad.

Q Convert 110° to radians.

$$180^\circ \rightarrow \pi \text{ rad.}$$

$$110^\circ \rightarrow x$$

$$180x = 110\pi$$

$$x = \frac{110\pi}{180}$$

$$x = 1.919 \text{ rad.}$$

Q Convert $\frac{3\pi}{2}$ rad to degrees

$$180^\circ \rightarrow \pi \text{ rad}$$

$$x \rightarrow \frac{3\pi}{2}$$

$$\pi x = 180 \times \frac{3\pi}{2}$$

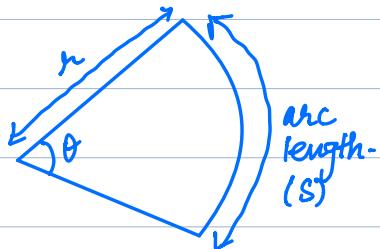
$$x = 270^\circ$$

FAMOUS ANGLES

$$180^\circ \rightarrow \pi \text{ rad.}$$

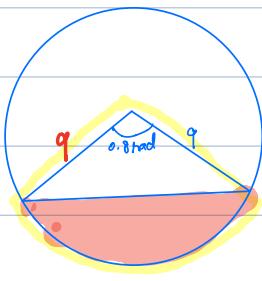
$$\begin{aligned}
 90^\circ &\longrightarrow \frac{\pi}{2} \\
 45^\circ &\longrightarrow \frac{\pi}{4} \\
 30^\circ &\longrightarrow \frac{\pi}{6} \\
 60^\circ &\longrightarrow \frac{\pi}{3} \\
 360^\circ &\longrightarrow 2\pi
 \end{aligned}$$

DEGREES	0	30	45	60	90
RADIANS	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ infinite



DEGREE (θ)	RADIANS (θ)
$\text{Arc length} = \frac{\theta^\circ}{360^\circ} \times 2\pi r$	Arc length $s = r\theta$
$\text{Area of Sector} = \frac{\theta^\circ}{360^\circ} \times \pi r^2$	Area of Sector $A = \frac{1}{2} r s$ $A = \frac{1}{2} r^2 \theta$

2



Find Shaded area?

Shaded Area = Sector - Triangle

$$\frac{1}{2}r^2\theta - \frac{1}{2} \times \text{neighboring sides} \times \sin(\text{Angle})$$

CASIO fx 82MS

CASIO fx 350MS

$$\begin{aligned} & \frac{1}{2}(9)^2(0.8) - \frac{1}{2} [9][9] \sin(0.8) \\ &= 32.4 - 29.05 \\ &= 3.35 \end{aligned}$$

calculator mode change.

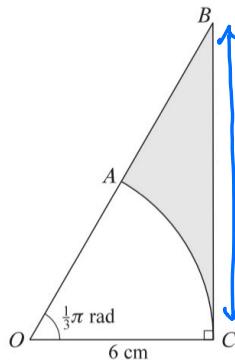
WHEN TO CHANGE CALCULATOR TO RADIANS MODE

ONLY WHEN

 \sin [] \cos [] \tan [] \sin' [] \cos' [] \tan' []

AND ITS A RADIANS QUESTION.

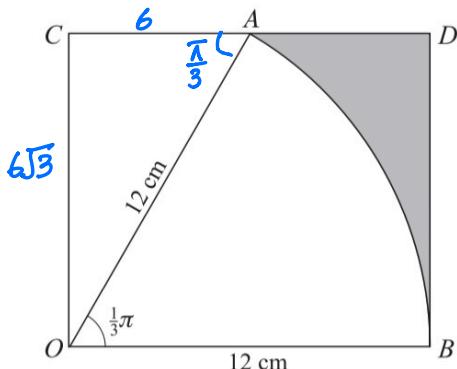
$$\begin{aligned} \text{Shaded Area} &= \text{Triangle} - \text{Sector} \\ &= \frac{1}{2}(OC)(BC) - \frac{1}{2}r^2\theta \\ &= \frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) \\ &= 18\sqrt{3} - 6\pi \end{aligned}$$



$$\begin{aligned} \tan \frac{\pi}{3} &= \frac{BC}{6} \\ \sqrt{3} &= \frac{BC}{6} \\ BC &= 6\sqrt{3} \end{aligned}$$

6

$$\begin{aligned} \sin \frac{\pi}{3} &= \frac{OC}{12} \\ \frac{\sqrt{3}}{2} &= \frac{OC}{12} \\ OC &= 6\sqrt{3} \\ \cos \frac{\pi}{3} &= \frac{AC}{12} \\ \frac{1}{2} &= \frac{AC}{12} \\ AC &= 6 \end{aligned}$$



In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b . [6]

exact form.

SHADED = RECTANGLE - TRIANGLE - SECTOR.
AREA

$$(12)(6\sqrt{3}) - \frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right)$$

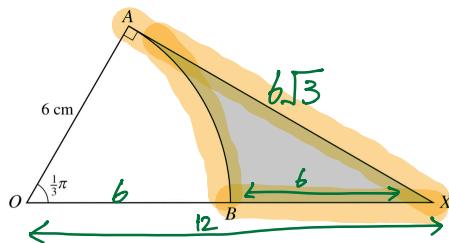
$$72\sqrt{3} - 18\sqrt{3} - 24\pi$$

$$54\sqrt{3} - 24\pi$$

$$a\sqrt{3} - b\pi$$

$$\boxed{a=54}, \quad -b = -24$$

$$\boxed{b=24}$$



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A , and OBX is a straight line.

$$\tan\left(\frac{1}{3}\pi\right) = \frac{AX}{6}$$

$$\sqrt{3} = \frac{AX}{6}$$

$$\boxed{AX = 6\sqrt{3}}$$

- (i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

- (ii) the area of the shaded region, [3]
 (iii) the perimeter of the shaded region. [4]

Area of shaded region = Triangle - Sector

$$\frac{1}{2}(OA)(AX) - \frac{1}{2}r^2\theta$$

$$\frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$$

$$\boxed{18\sqrt{3} - 6\pi}$$

$$\text{Perimeter} = AX + BX + \text{Arc } AB$$

$$6\sqrt{3} + (6) + (2\pi)$$

$$= \boxed{6 + 6\sqrt{3} + 2\pi}$$

$$\boxed{BX}$$

$$OX^2 = OA^2 + AX^2$$

$$OX^2 = 6^2 + (6\sqrt{3})^2$$

$$= 36 + 108$$

$$\boxed{\text{Arc } AB}$$

$$s = r\theta$$

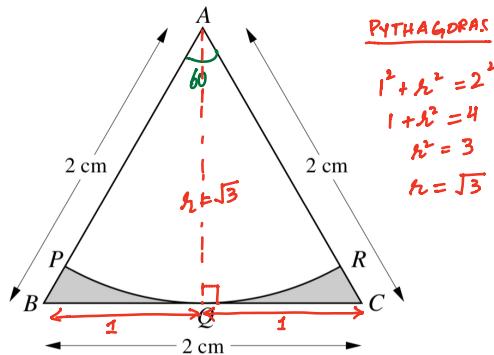
$$= 6\left(\frac{\pi}{3}\right)$$

$$= 2\pi$$

$$OX^2 = 144$$

$$OX = 12$$

22



In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q . An arc of a circle with centre A touches BC at Q , and meets AB at P and AC at R . Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

$$\text{Shaded Area} = \frac{1}{2} \pi \left[\frac{1}{2} \sin(60^\circ) - \frac{\frac{60}{360} \times \pi r^2}{\frac{1}{2}} \right]$$

$$2 \left(\frac{\sqrt{3}}{2} \right) - \frac{60}{360} \times \pi (\sqrt{3})^2$$

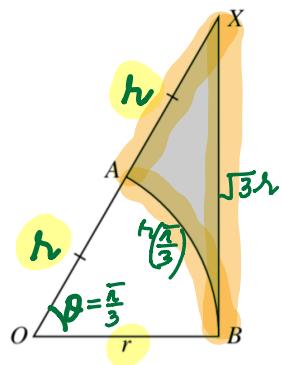
$$\sqrt{3} - \frac{1}{6} \pi (3)$$

$$= \boxed{\sqrt{3} - \frac{\pi}{2}}$$

$$\cos \theta = \frac{r}{2r}$$

$$\cos \theta = \frac{1}{2}$$

$$\boxed{\theta = \frac{\pi}{3}}$$



PYTHAGORAS

$$(2r)^2 = h^2 + BX^2$$

$$4r^2 = h^2 + BX^2$$

$$BX^2 = 3r^2$$

$$BX = \sqrt{3r^2} = \sqrt{3}r$$

$$BX = \sqrt{3}r$$

In the diagram, AB is an arc of a circle with centre O and radius r . The line XB is a tangent to the circle at B and A is the mid-point of OX .

$$\text{Perimeter} = hr + \sqrt{3}r + \frac{\pi r}{3}$$

$$\begin{aligned}\text{Arclength } AB &= s = r\theta \\ &= r\left(\frac{\pi}{3}\right)\end{aligned}$$

- (i) Show that angle $AOB = \frac{1}{3}\pi$ radians.

[2]

Express each of the following in terms of r , π and $\sqrt{3}$:

- (ii) the perimeter of the shaded region,

[3]

- (iii) the area of the shaded region. = Triangle - Sector.

[2]

$$\frac{1}{2}(r)(r\sqrt{3}) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$$

$$\boxed{\frac{r^2\sqrt{3}}{2} - \frac{r^2\pi}{6}}$$



ZAINEMATICS

FOR THE LOVE OF MATHS

Trigonometry

P1

Compiled by Rafay Mushtaq



zainematics



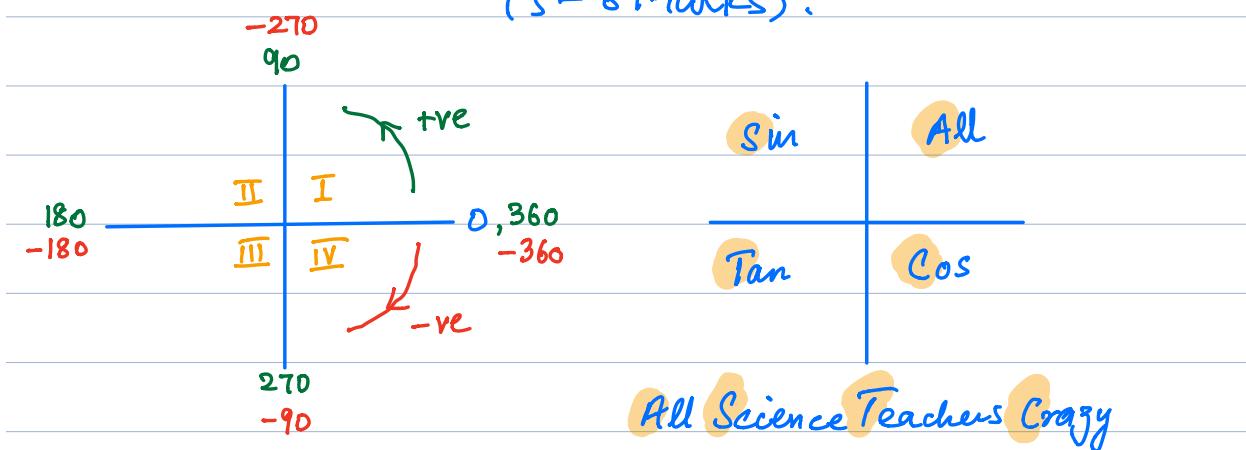
zainematics



zainematics

TRIGONOMETRY:

(5-8 marks).

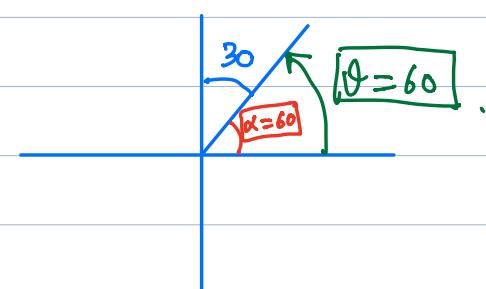
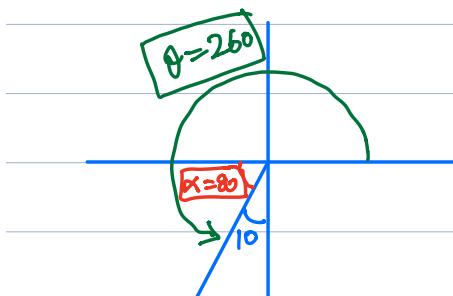
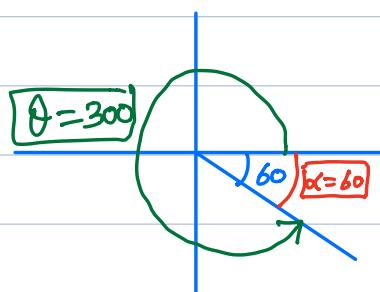
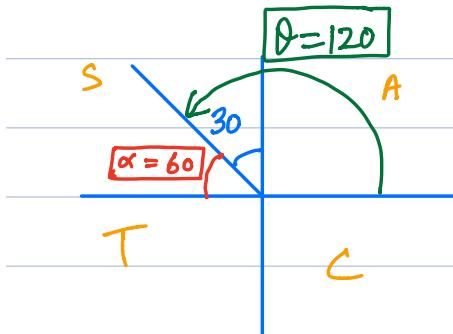
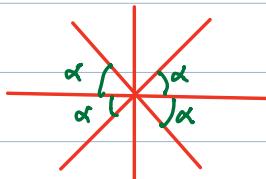


EVERY ANGLE HAS TWO MAIN VALUES.

θ = ORIGINAL ANGLE
 (STARTS FROM +ve x-axis)
 ACW (+ve) CW (-ve)

BASIC ANGLE: (α)

ACUTE ANGLE MADE WITH
 X-AXIS .



sin/cos/tan

$$\left[\begin{array}{l} \text{TRIG RATIO OF} \\ \text{ANY ANGLE } (\theta) \end{array} \right] = \left[\begin{array}{l} \text{TRIG RATIO OF ITS BASIC ANGLE } (\alpha) \\ \text{AFTER ADJUSTING QUADRANT +/- SIGN.} \end{array} \right]$$

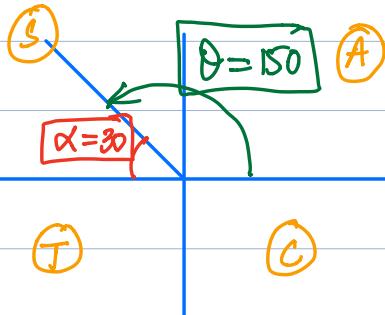
Q. WITHOUT USING CALCULATOR EVALUATE.

(i) $\cos 150^\circ$



$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\boxed{\cos 150^\circ = -\frac{\sqrt{3}}{2}}$$



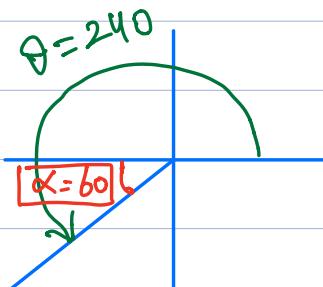
[2] $\tan 240^\circ$



$$\tan 60^\circ = \sqrt{3}$$



$$\tan 240^\circ = +\sqrt{3}$$



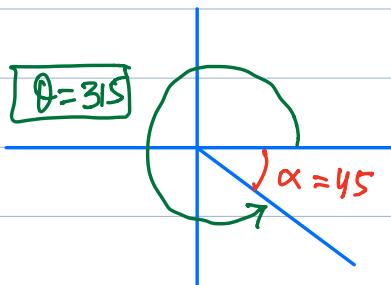
[3] $\sin 315^\circ$



$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$



$$\boxed{\sin 315^\circ = -\frac{1}{\sqrt{2}}}$$

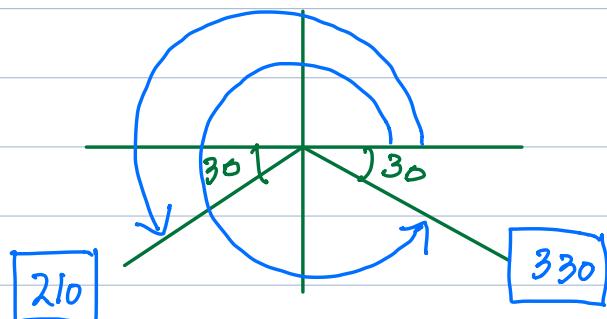


EQUATION SOLVING.

① Permission to take inverse
 III. $\sin x = -\frac{1}{2}$ $0 < x < 360$

$x = \sin^{-1}\left(-\frac{1}{2}\right)$
 ③ Basic Angle. ↳ ② ignore +/- sign while taking inverse

$$x = 30^\circ$$



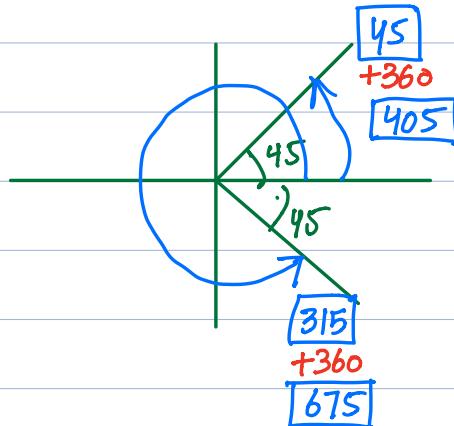
$$x = 210^\circ, 330^\circ$$

2 $\cos x = \frac{1}{\sqrt{2}}$ $0 < x < 720$

$$x = \cos^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$x = 45^\circ$$

$$x = 45, 315, 405, 675$$

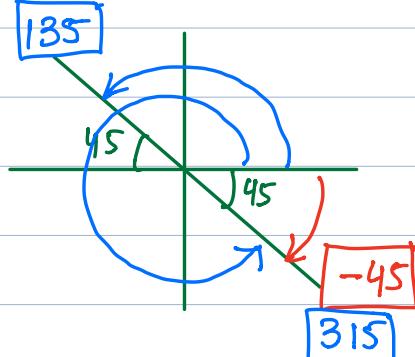


3 $\tan x = -1$ $-180 < x < 360$

$$x = \tan^{-1}(-1)$$

$$x = 45^\circ$$

$x = -45, 135, 315$



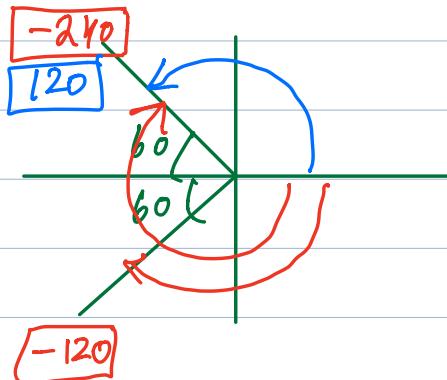
NOTE: IF YOUR RANGE IS IN NEGATIVE
ALWAYS FIND NEGATIVE ANGLES
FIRST.

4

$$\cos x = -\frac{1}{2}$$

$$-360 < x < 180$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$



$$x = -240^\circ, -120^\circ, 120^\circ$$

IF WE DO NOT HAVE PERMISSION TO
TAKE INVERSE :
RANGE CHANGE.

$$\sin 2x = \frac{1}{2}$$

$$0 < x < 360^\circ$$

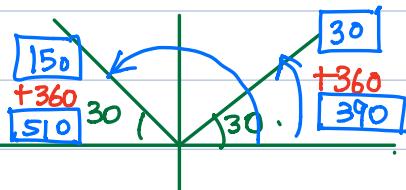
$$2x = A$$

$$0 < 2x < 720^\circ$$

$$\sin A = \frac{1}{2}$$

$$0 < A < 720^\circ$$

$$x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$



$$A = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$

$$2x = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$

$$x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$$

2

$$\cos(x+70) = \frac{1}{2} \quad 0 < x < 360$$

+70

$$70 < x+70 < 430$$

$$A = x+70$$

$$\cos A = \frac{1}{2}$$

$$70 < A < 430$$

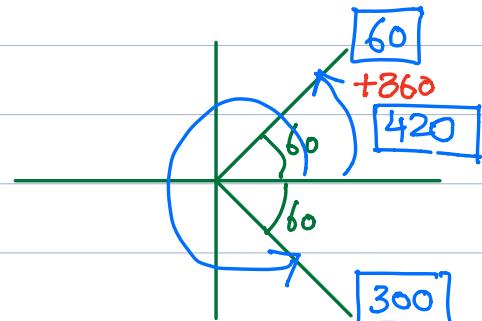
$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

out of range

$$A = \cancel{60}, 300, 420$$

$$x+70 = 300, 420$$

$$x = 230, 350$$



3

$$\cos(x-80) = \frac{1}{\sqrt{2}}$$

$$0 < x < 360$$

-80

$$A = x-80$$

$$-80 < x-80 < 280$$

$$\cos A = \frac{1}{\sqrt{2}}$$

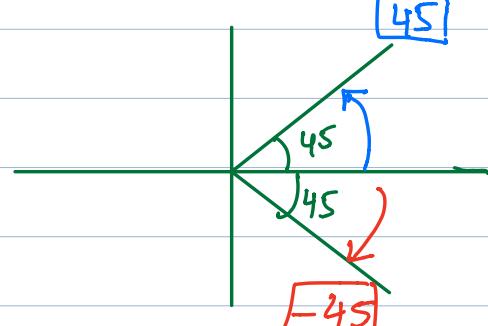
$$-80 < A < 280$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45$$

$$A = -45, 45$$

$$x-80 = -45, 45$$

$$x = 35, 125$$



$$\boxed{4} \quad \sin(2x+40) = \frac{1}{2}$$

$0 < x < 180$

$\times 2$

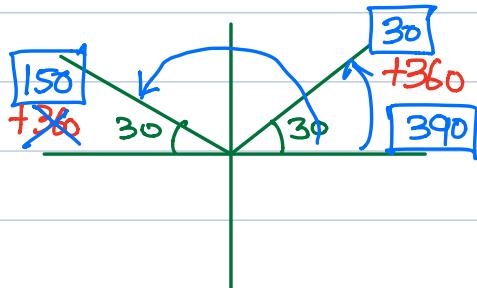
$0 < 2x < 360$

$+40$

$40 < 2x+40 < 400$

$$\sin A = \frac{1}{2} \quad 40 < A < 400$$

$$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$



$$A = \cancel{30}, 150, 390$$

$$2x+40 = 150, 390$$

$$2x = 110, 350$$

$$\boxed{x = 55, 175}$$

IDENTITIES: (17)

RECIPROCAL

$$\frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\frac{1}{\tan \theta} = \cot \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

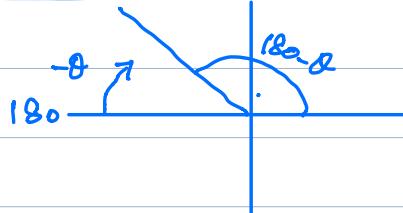
$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

θ = Acute angle

$$\sin(180 - \theta) = \sin \theta$$

$$\cos(180 - \theta) = -\cos \theta$$

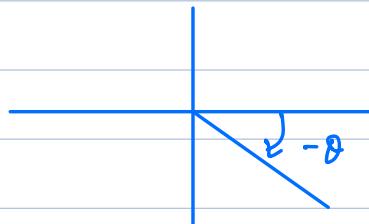
$$\tan(180 - \theta) = -\tan \theta$$



$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

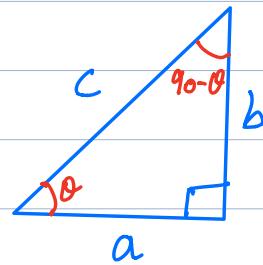


$$\sin(90 - \theta) = \cos \theta$$

$$\cos(90 - \theta) = \sin \theta$$

$$\tan(90 - \theta) = \frac{1}{\tan \theta}$$

(M1, P1)



SYMBOLS.

$$\sin^2 \theta = (\sin \theta)^2$$

$$\sin x^2 = \sin(x^2)$$

$$\sin^2 30^\circ \rightarrow (\sin 30) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

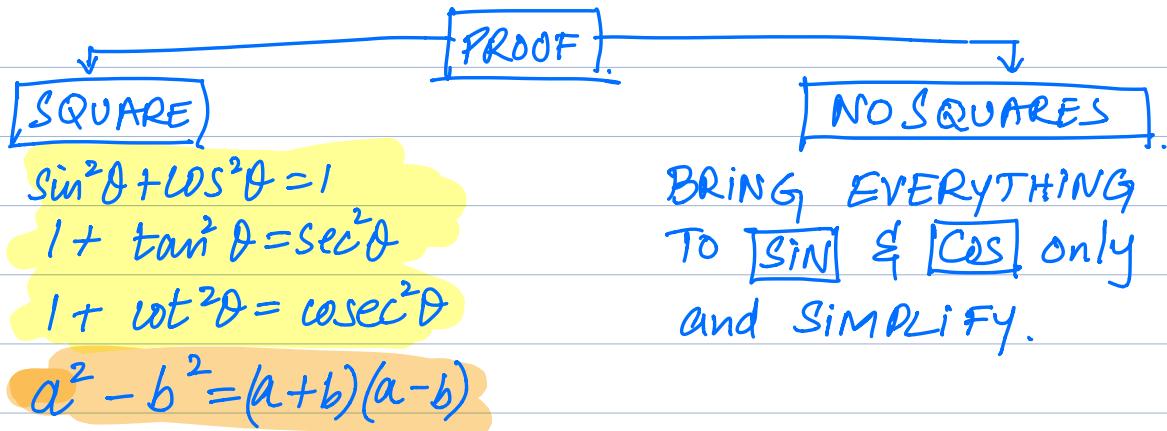
$$\sin 30^\circ \rightarrow \sin(30^\circ) = \sin(90^\circ) = \boxed{1}$$

Ex

$$\begin{array}{l} 2 \cos(180 - \theta) = 1 \\ 2(-\cos \theta) = 1 \\ -2 \cos \theta = 1 \\ \cos \theta = -\frac{1}{2} \end{array} \quad \left| \quad \begin{array}{l} \underline{\underline{Q}} \quad 2 \cos(-\theta) = 1 \\ \cos(-\theta) = \frac{1}{2} \\ \cos \theta = \frac{1}{2} \end{array} \right.$$

PROVING IDENTITIES

- 1) YOU ARE ALLOWED TO WORK WITH ONLY ONE SIDE AND PROVE IT EQUAL TO OTHER SIDE.
- 2) YOU ARE NOT ALLOWED TO SIMPLIFY BOTH SIDES AT SAME TIME.



32 (i) Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [2]

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} \quad (\text{shown}) \quad (\text{Q.E.D.})$$

13 Prove the identity

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} \equiv \frac{2}{\cos x}. \quad [4]$$

$$\frac{(1+\sin x)(1+\sin x) + (\cos x)(\cos x)}{\cos x (1+\sin x)}$$

$$\frac{1+2\sin x + \sin^2 x + \cos^2 x}{\cos x (1+\sin x)}$$

$$\frac{1+2\sin x + 1}{\cos x (1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x (1+\sin x)}$$

$$\frac{2(1+\sin x)}{\cos x (1+\sin x)} = \frac{2}{\cos x} \quad (\text{Proven})$$

- 38 (i) Show that $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \equiv \frac{1}{\sin^2 \theta - \cos^2 \theta}$. [3]

$$\frac{\sin \theta (\sin \theta - \cos \theta) + \cos \theta (\sin \theta + \cos \theta)}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}$$

$$\frac{\sin^2 \theta - \sin \theta \cos \theta + \sin \theta \cos \theta + \cos^2 \theta}{(\sin \theta)^2 - (\cos \theta)^2}$$

$$\frac{1}{\sin^2 \theta - \cos^2 \theta}$$

- 27 (i) Prove the identity $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta} \right)^2 \equiv \frac{1 - \cos \theta}{1 + \cos \theta}$. [3]

$$\left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2$$

$$\left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

$$\frac{(1 - \cos \theta)^2}{\sin^2 \theta} \quad \begin{array}{l} \text{Sin}^2 \theta + \cos^2 \theta = 1 \\ \sin^2 \theta = 1 - \cos^2 \theta \end{array}$$

$$\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$\frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

21 (i) Prove the identity $\frac{\sin x \tan x}{1 - \cos x} \equiv 1 + \frac{1}{\cos x}$.

$$\frac{\sin x \cdot \frac{\sin x}{\cos x}}{1 - \cos x}$$

$$\frac{\sin x \cdot \frac{\sin x}{\cos x}}{1 - \cos x} \div (1 - \cos x)$$

$$\frac{\sin^2 x}{\cos x} \times \frac{1}{(1 - \cos x)}$$

$$\frac{\sin^2 x}{\cos x (1 - \cos x)} \\ \frac{1 - \cos^2 x}{\cos x (1 - \cos x)}$$

$$\frac{(1 + \cos x)(1 - \cos x)}{\cos x (1 - \cos x)}$$

$$\frac{1 + \cos x}{\cos x}$$

$$\frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$\boxed{\frac{1}{\cos x} + 1}$$

shown.

CORRECT

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

WRONG.

$$\frac{c}{a+b} = \frac{c}{a} + \frac{c}{b}$$

DONE!



ZAIÑEMATICS

FOR THE LOVE OF MATHS

Differentiation

P1

Compiled by Rafay Mushtaq



zainematics



zainematics



zainematics

TRIG
↓
5 types.

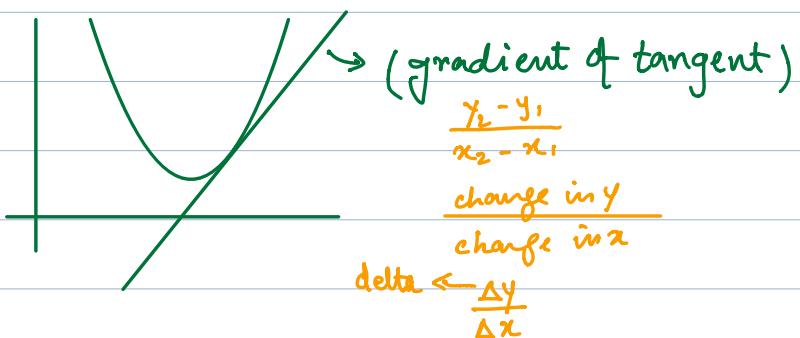
FUNCTIONS
5 TYPES

DIFF
3 TYPES

INTEGRATION
3 TYPES.

DIFFERENTIATION GRADIENT OF TANGENT (CURVE)

DEFINITION:



SYMBOLS

$$y \xrightarrow{\text{diff}} \frac{dy}{dx}$$

$$y \xrightarrow{\text{diff}} y'$$

HOW TO DIFF :-

BASIC RULES :-

1] $x \longrightarrow 1$

$2x \longrightarrow 2$

$10x \longrightarrow 10$

$12x \longrightarrow 12$

2] Alone constant $\longrightarrow 0$

$6 \longrightarrow 0$

$4 \longrightarrow 0$

$100 \longrightarrow 0$

e.g. $y = 5 - 3x$

$$\frac{dy}{dx} = 0 - 3 = -3$$

POWER RULE :- (THREE STEP PROCESS)

$$(\square)^n \longrightarrow \textcircled{1} n (\square)^{n-1} \times \square'$$

① $y = x^7$
 $\frac{dy}{dx} = 7x^6 \times 1$
 $\frac{dy}{dx} = 7x^6$

② $y = 12x^3$
 $\frac{dy}{dx} = 36x^2 \times 1$
 $\frac{dy}{dx} = 36x^2$

③ $y = (12x)^3$
 $\frac{dy}{dx} = 3(12x)^2 \times 12$
 $\frac{dy}{dx} = 36(12x)^2$

④ $y = 9x^3 - 12x + 7$
 $\frac{dy}{dx} = 27x^2(1) - 12 + 0$
 $\frac{dy}{dx} = 27x^2 - 12$

⑤ $y = (3x-1)^5$
 $\frac{dy}{dx} = 5(3x-1)^4 \times (3-0)$
 $\frac{dy}{dx} = 15(3x-1)^4$

⑥ $y = 6x^2 - 9x + 8$
 $\frac{dy}{dx} = 12x^1(1) - 9 + 0$
 $\frac{dy}{dx} = 12x - 9$

⑦ $y = (2x+5)^3 - 9x^2$
 $\frac{dy}{dx} = 3(2x+5)^2(2+0) - 18x^1(1)$
 $\frac{dy}{dx} = 6(2x+5)^2 - 18x$

YOU CANNOT DIFFERENTIATE IN DENOMINATOR!

$$[8] \quad y = 3x^2 - \frac{2}{x^3}$$

$$y = 3\cancel{x}^2 - 2\cancel{x}^{-3}$$

$$\frac{dy}{dx} = 6x(1) + 6x^{-4}(1)$$

$$\frac{dy}{dx} = 6x + \frac{6}{x^4}$$

$$[9] \quad y = \frac{12}{(2x-3)^2}$$

$$y = 12(2x-3)^{-2}$$

$$\frac{dy}{dx} = -24(2x-3)^{-3}(2-0)$$

$$\frac{dy}{dx} = \frac{-48}{(2x-3)^3}$$

$$[9] \quad y = \frac{9x^2 + 2}{x}$$

$$y = \frac{9x^2}{x} + \frac{2}{x}$$

$$y = 9x + \frac{2}{x}$$

$$y = 9x + 2\cancel{x}^{-1}$$

$$\frac{dy}{dx} = 9 + (-2)x^{-2}(1)$$

$$\boxed{\frac{dy}{dx} = 9 - \frac{2}{x^2}}$$

OUTCOMES :-

$\frac{dy}{dx}$ = gradient of TANGENT (curve).

Q $y = x^2 - 2x + 4$

i) Find expression for $\frac{dy}{dx}$

$$y = \boxed{x^2} - 2x + 4$$

$$\frac{dy}{dx} = 2x^1(1) - 2 + 0 = \boxed{2x - 2}$$

ii) Find gradient of tangent at $x = 2$

$$\frac{dy}{dx} = 2x - 2 \quad x = 2 \\ = 2(2) - 2$$

gradient of tangent = $\boxed{2 = m_T}$

iii) Find equation of tangent at $x = 2$

For y coordinate put $x = 2$ in curve equation.

$$y = 2^2 - 2(2) + 4 = 4$$

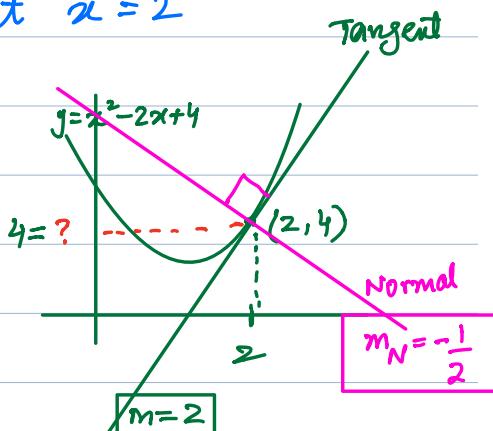
Point $(2, 4)$, $m_T = 2$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 2)$$

$$y - 4 = 2x - 4$$

$\boxed{y = 2x}$ TANGENT.



iv) Find equation of normal to curve at $x=2$.

$$m_T = 2 \longrightarrow m_N = -\frac{1}{2}$$

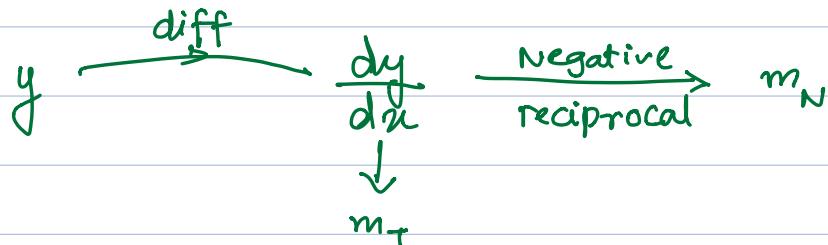
$$y - 4 = -\frac{1}{2}(x - 2)$$

Point $(2, 4)$

$$2y - 8 = -x + 2$$

$$2y = -x + 10$$

You cannot tell gradient of normal directly.



- 2 Find the gradient of the curve $y = \frac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

$$\boxed{\frac{dy}{dx}}$$

$$y = 12(x^2 - 4x)^{-1}$$

$$\begin{aligned} &\sqrt{x^2} \\ &2x^1(1) \end{aligned}$$

$$\frac{dy}{dx} = -12(x^2 - 4x)^{-2} \times (2x - 4)$$

$$\frac{dy}{dx} = \frac{-12(2x-4)}{(x^2-4x)^2} \quad x = 3$$

$$= \frac{-12(2(3)-4)}{(3^2-4(3))^2} = -\frac{8}{3}$$

22 A curve has equation $y = \frac{4}{3x-4}$ and $P(2, 2)$ is a point on the curve. [4]

(i) Find the equation of the tangent to the curve at P . [4]

(ii) Find the angle that this tangent makes with the x -axis. [2]

$$\text{(i)} \quad y = 4(3x-4)^{-1}$$

$$\frac{dy}{dx} = -4(3x-4)^{-2}(3)$$

$$\boxed{\frac{dy}{dx} = \frac{-12}{(3x-4)^2}}$$

$$x = 2$$

$$m_T = \frac{-12}{(3(2)-4)^2} = \frac{-12}{4} = -3$$

$$m_T = -3, P(2, 2)$$

$$y - 2 = -3(x - 2)$$

$$y = -3x + 6 + 2$$

$$\boxed{y = -3x + 8}$$

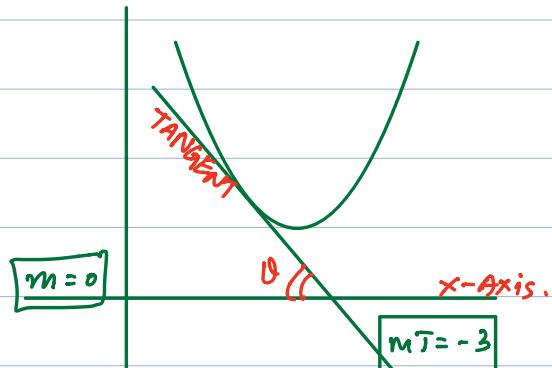
$$\text{(ii)} \quad \tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan \theta = \frac{0 - (-3)}{1 + (0)(-3)}$$

$$\tan \theta = 3$$

$$\theta = \tan^{-1}(3)$$

$$\theta = 71.565$$



Please wait. I'm about to
reach academy. In traffic hn.

11 The equation of a curve is $y = \frac{12}{x^2 + 3}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point $P(1, 3)$. [3]

$$y = 12(x^2 + 3)^{-1}$$

$$\frac{dy}{dx} = -12(x^2 + 3)^{-2}(2x)$$

$$\frac{dy}{dx} = \frac{-24x}{(x^2 + 3)^2}$$

$$m_T = \frac{-24(1)}{(1^2 + 3)^2} = \frac{-24}{16} = -\frac{3}{2}$$

$$m_N = \frac{2}{3}, P(1, 3)$$

$$y - 3 = \frac{2}{3}(x - 1)$$

$$3y - 9 = 2x - 2$$

$$3y = 2x + 7$$

10 The equation of a curve is $y = 5 - \frac{8}{x}$

(i) Show that the equation of the normal to the curve at the point $P(2, 1)$ is $2y + x = 4$. [4]

point of intersection (simultaneously).
This normal meets the curve again at the point Q .

Diff

(ii) Find the coordinates of Q .

(iii) Find the length of PQ .

$$y = 5 - 8x^{-1}$$

$$\frac{dy}{dx} = 0 - (-8)x^{-2}(1) = \frac{8}{x^2}$$

$$m_T = \frac{8}{2^2} = \frac{8}{4} = 2$$

$$m_N = -\frac{1}{2}, P(2, 1)$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$2y - 2 = -x + 2$$

$$2y = -x + 4$$

$$2y + x = 4$$

coordinates
geo. { [3]
[2]

$$\text{回 } \boxed{Q} \quad y = 5 - \frac{8}{x} \quad 2y + x = 4$$

$$5 - \frac{8}{x} = \frac{4-x}{2}$$

$$\frac{5x-8}{x} = \frac{4-x}{2}$$

$$10x - 16 = 4x - x^2$$

$$x^2 + 6x - 16 = 0$$

$$x^2 + 8x - 2x - 16 = 0$$

$$x(x+8) - 2(x+8) = 0$$

$$(x-2)(x+8) = 0$$

$$x = 2 \quad , \quad x = -8$$

Point P

Point Q

$$y = \frac{4 - (-8)}{2}$$

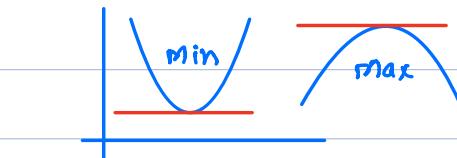
$$y = 6$$

$$Q(-8, 6)$$

$$P(2, 1) \quad Q(-8, 6)$$

$$\begin{aligned} \text{Length } PQ &= \sqrt{(-8-2)^2 + (6-1)^2} \\ &= \sqrt{125} \\ &= \underline{\quad} \end{aligned}$$

STATIONARY POINT
 TURNING POINT
 VERTEX
 MAXIMUM POINT
 MINIMUM POINT
 CRITICAL POINT (SPT)



TANGENT = HORIZONTAL = Gradient is zero.

GRADIENT OF TANGENT IS ZERO

$$\frac{dy}{dx} = 0$$

Q. $y = x^2 + 12x - 3$

Find x-coordinates of its stationary point.

$$\frac{dy}{dx} = 2x + 12$$

$$0 = 2x + 12$$

$$2x = -12$$

$$x = -6$$

NATURE OF A STATIONARY POINT



$$\begin{array}{c}
 y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2} \\
 \text{+ve MIN} \quad \text{-ve MAX.}
 \end{array}$$

$$\begin{array}{c}
 y \xrightarrow{\text{diff}} y' \xrightarrow{\text{diff}} y'' \\
 \text{+ve MIN} \quad \text{-ve MAX.}
 \end{array}$$

7 The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x . [3]

(ii) Find the x -coordinates of the two stationary points and determine the nature of each stationary point. [5]

$$(i) \quad y = (2x - 3)^3 - 6x$$

$$\frac{dy}{dx} = 3(2x-3)^2(2) - 6$$

$$\frac{dy}{dx} = 6(2x-3)^2 - 6$$

$$\frac{d^2y}{dx^2} = 12(2x-3)^1(2) - 0$$

$$\frac{d^2y}{dx^2} = 24(2x-3)$$

(ii) STATIONARY POINTS

$$\frac{dy}{dx} = 0$$

$$6(2x-3)^2 - 6 = 0$$

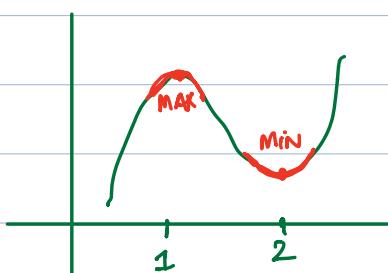
$$6(2x-3)^2 = 6$$

$$\sqrt{(2x-3)^2} = \pm 1$$

$$2x-3 = \pm 1$$

$$x = \frac{3 \pm 1}{2}$$

$$x = 2, \quad x = 1$$



$$x = 2$$

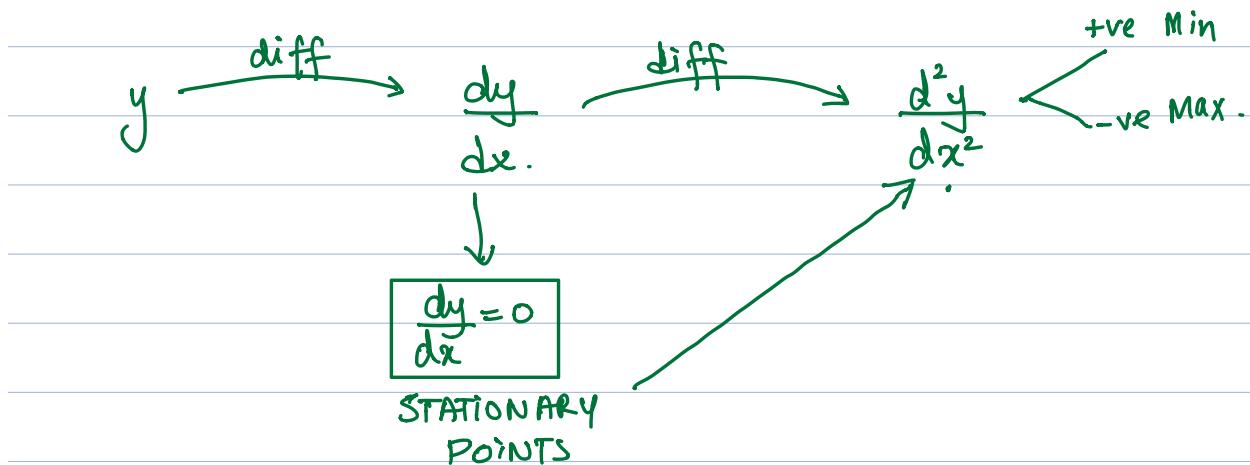
NATURE :-

$$\frac{d^2y}{dx^2} = 24(2x-3)$$

$$\frac{d^2y}{dx^2} = 24(2(2)-3) = 24(+)(\text{Min})$$

$$x = 1$$

$$\frac{d^2y}{dx^2} = 24(2(1)-3) = -24(-)(\text{Max})$$



52 A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

(i) $y = 8x^{-1} + 2x$

$$\frac{dy}{dx} = -8x^{-2}(1) + 2$$

$$\frac{dy}{dx} = -8x^{-2} + 2$$

$$\frac{d^2y}{dx^2} = 16x^{-3}(1)$$

$$\frac{d^2y}{dx^2} = \frac{16}{x^3}$$

(ii) STATIONARY POINT: - NATURE

$$\frac{dy}{dx} = 0$$

$$-8x^{-2} + 2 = 0$$

$$-\frac{8}{x^2} + 2 = 0$$

$$2 = \frac{8}{x^2}$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = 2, -2$$

$$x=2 \quad \frac{d^2y}{dx^2} = \frac{16}{2^3} = 2$$

\oplus
Min

$$x=-2 \quad \frac{d^2y}{dx^2} = \frac{16}{(-2)^3}$$

= -2

Max

20 A curve has equation $y = \frac{1}{x-3} + x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [2]

(ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]

$$y = (x-3)^{-1} + x$$

$$\frac{dy}{dx} = -1(x-3)^{-2}(1) + 1$$

$$\boxed{\frac{dy}{dx} = -1(x-3)^{-2} + 1}$$

$$\frac{d^2y}{dx^2} = 2(x-3)^{-3}(1) + 0$$

$$\boxed{\frac{d^2y}{dx^2} = 2(x-3)^{-3}}$$

MAX / MIN POINT (TURNING POINT).

$$0 = -1(x-3)^{-2} + 1$$

$$0 = \frac{-1}{(x-3)^2} + 1$$

$$\frac{1}{(x-3)^2} = 1$$

$$(x-3)^2 = 1$$

$$x-3 = \pm 1$$

$$x = 3 \pm 1$$

$$x = 4, \quad x = 2$$

$$y = \frac{1}{4-3} + 4 \quad y = \frac{1}{2-3} + 2$$

$$y = 5 \quad y = 1$$

$$B(4, 5) \quad A(2, 1)$$

$$x = 4, \quad \frac{d^2y}{dx^2} = \frac{2}{(4-3)^3} = 2 \text{ (+ve) min } B \quad \cup$$

$$x = 2, \quad \frac{d^2y}{dx^2} = \frac{2}{(2-3)^3} = -2 \text{ (-ve) max } A \quad \cap$$

- 30 The curve $y = \frac{10}{2x+1} - 2$ intersects the x -axis at A . The tangent to the curve at A intersects the y -axis at C .

(i) Show that the equation of AC is $5y + 4x = 8$. [5]

(ii) Find the distance AC . [2]

$$\boxed{A} \quad x\text{-axis}, y=0, \quad 0 = \frac{10}{2x+1} - 2$$

$$2 = \frac{10}{2x+1}$$

$$4x+2 = 10$$

$$A = (2, 0)$$

$$\boxed{x=2}$$

TANGENT AT A:

$$y = 10(2x+1)^{-1} - 2$$

$$\frac{dy}{dx} = 10(-1)(2x+1)^{-2}(2) - 0$$

$$\frac{dy}{dx} = \frac{-20}{(2x+1)^2} \quad x=2$$

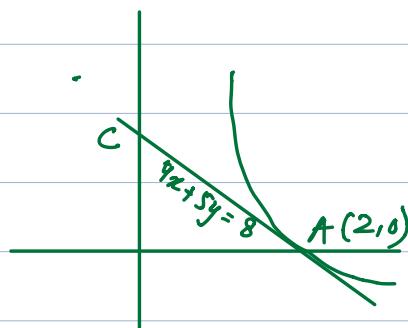
$$m_T = \frac{-20}{(2(2)+1)^2} = -\frac{4}{5}$$

$$m = -\frac{4}{5} \quad A(2, 0)$$

$$y-0 = -\frac{4}{5}(x-2)$$

$$\boxed{5y = -4x + 8} \quad \text{Tangent}$$

$$4x + 5y = 8$$



C y-intercept: $x = 0$

$$4(0) + 5y = 8$$

$$y = 1.6$$

$$AC = \sqrt{(0 - 2)^2 + (1.6 - 0)^2} = \boxed{\quad}$$

12

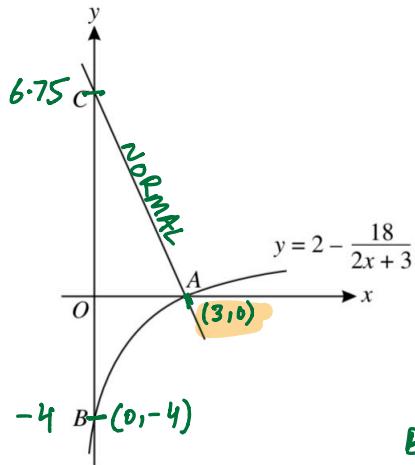
A x-axis. $y = 0$

$$0 = 2 - \frac{18}{2x+3}$$

$$\frac{18}{2x+3} = 2$$

$$18 = 4x + 6$$

$$x = 3$$



B $x = 0$, y-axis

$$y = 2 - \frac{18}{2(0)+3}$$

$$y = -4$$

C $x = 0$, y-axis

$$9(0) + 4y = 27$$

$$4y = 27$$

$$y = 6.75$$

$$BC = 6.75 + 4 = \boxed{10.75}$$

The diagram shows part of the curve $y = 2 - \frac{18}{2x+3}$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

(ii) Find the length of BC. Normal [2]

$$y = 2 - 18(2x+3)^{-1}$$

$$\frac{dy}{dx} = 0 - 18(-1)(2x+3)^{-2}(2)$$

$$\frac{dy}{dx} = \frac{36}{(2x+3)^2} \quad x = 3$$

$$m_T = \frac{36}{(2(3)+3)^2} = \frac{4}{9}$$

Normal

$$m = -\frac{9}{4}, A(3, 0)$$

$$y - 0 = -\frac{9}{4}(x - 3)$$

$$4y = -9x + 27$$

$$9x + 4y = 27 \quad \boxed{AC}$$

- 5 A curve has equation $y = \frac{k}{x}$. Given that the gradient of the curve is -3 when $x = 2$, find the value of the constant k .

$$\frac{dy}{dx} = -3, \quad x = 2$$

[3]

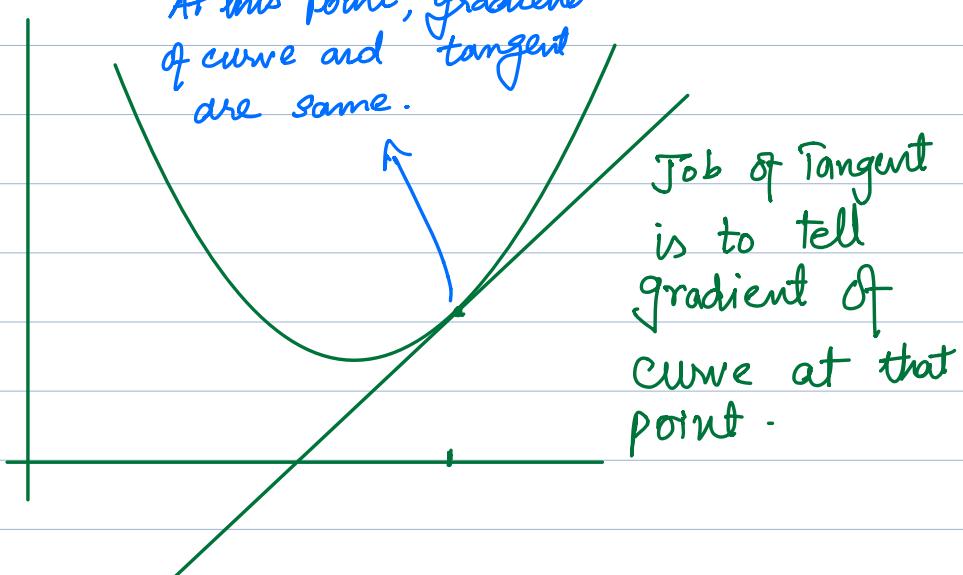
$$y = kx^{-1}$$

$$\frac{dy}{dx} = k(-1)x^{-2}$$

$$\frac{dy}{dx} = -\frac{k}{x^2}$$

$$-3 = -\frac{k}{(2)^2}$$

$$k = 12$$



$$(eg) \quad y = 3\sqrt{x} + 2x \rightarrow y = 3\sqrt{x} + 2x$$

$$(eg) \quad y = 3\sqrt{(x+5)} \rightarrow y = 3\sqrt{x+5}$$

$$y = \sqrt{2x+3}$$

$$y = (2x+3)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} (2x+3)^{\frac{-1}{2}} \times (2)$$

$$\frac{dy}{dx} = \frac{1}{(2x+3)^{\frac{1}{2}}} = \frac{1}{\sqrt{2x+3}}$$

RATE OF CHANGE

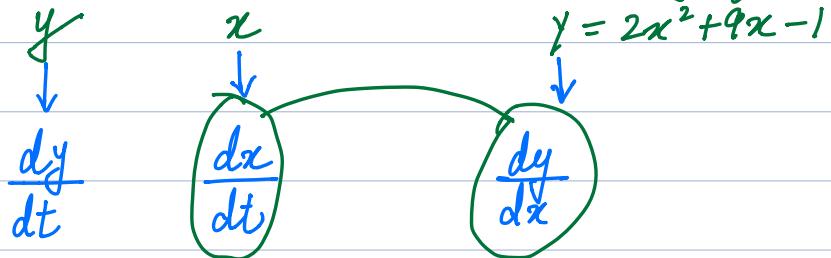
Rate of change of $x = \frac{dx}{dt}$

Rate of change of volume = $\frac{dv}{dt}$

Rate of change of Area = $\frac{dA}{dt}$

LAYOUT

Two main variables + Connecting equation

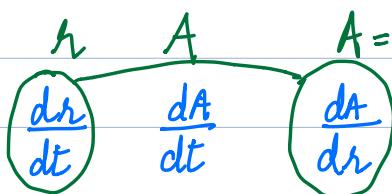


CHAIN RULE:

$$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$$

Q. A circular water pond is expanding such that its radius is increasing at rate of 4 m/s. Find rate of increase of its area when $r = 15$.

connecting equation.



$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r^1 (1)$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$\boxed{\frac{dA}{dr} = 2\pi r}$$

data in question

$$\frac{dr}{dt} = 4$$

$$\frac{dA}{dt} = ?$$

$$r = 15$$

$$\frac{dA}{dt} = 2\pi r \times 4$$

$$\frac{dA}{dt} = 2\pi(15) \times 4$$

$$\frac{dA}{dt} = 120\pi = \boxed{\quad}$$

- 21 The volume of a spherical balloon is increasing at a constant rate of 50 cm^3 per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

MAIN VARIABLES

$$r \quad V$$

CONN. EQ.

$$V = \frac{4}{3}\pi r^3$$

data in question

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 50$$

$$\left(\frac{dr}{dt} \right) \frac{dV}{dt} \left(\frac{dV}{dr} \right)$$

$$\frac{dV}{dr} = \frac{4}{3}\pi(3)r^2(1)$$

$$\frac{dr}{dt} = ?$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dr}{dt} \times \frac{dV}{dr}$$

$$50 = \frac{dr}{dt} \times 4\pi r^2$$

$$50 = \frac{dr}{dt} \times 4\pi(10)^2$$

$$\frac{dr}{dt} = \frac{50}{400\pi} = \boxed{\quad} .$$

- 31 An oil pipeline under the sea is leaking oil and a circular patch of oil has formed on the surface of the sea. At midday the radius of the patch of oil is 50 m and is increasing at a rate of 3 metres per hour. Find the rate at which the area of the oil is increasing at midday. [4]

$A = \pi r^2$	$A = \pi r^2$	<u>Data</u>
$\frac{dA}{dt} = \frac{dr}{dt} \times \frac{dA}{dr}$	$\frac{dA}{dr} = 2\pi r$	$r = 50$
$\frac{dA}{dt} = 3 \times 2\pi r$	$\frac{dA}{dt} = ?$	$\frac{dr}{dt} = 3$
$= 3 \times 2\pi (50)$		
$= 300\pi$		

- 11 The equation of a curve is $y = \frac{12}{x^2 + 3}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

point $P(1, 3)$.

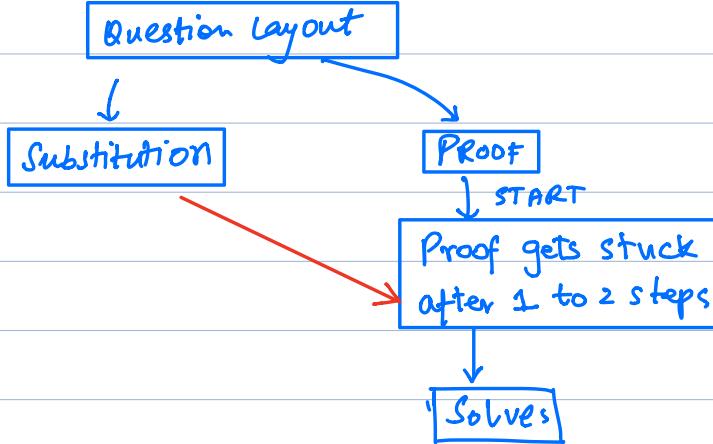
- (iii) A point is moving along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y -coordinate as the point passes through $P(x=1, y=3)$. [2]

$y = 12(x^2 + 3)^{-1}$ $\frac{dy}{dx} = -12(x^2 + 3)^{-2}(2x)$ $\frac{dy}{dx} = \frac{-24x}{(x^2 + 3)^2}$	x y $y = \frac{12}{x^2 + 3}$ $\frac{dx}{dt}$ $\frac{dy}{dt}$ $\frac{dy}{dx}$	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $\frac{dy}{dt} = \frac{-24x}{(x^2 + 3)^2} \times 0.012$ $\frac{dy}{dt} = \frac{-24(1)}{(1^2 + 3)^2} \times 0.012 = -0.018$
---	---	---

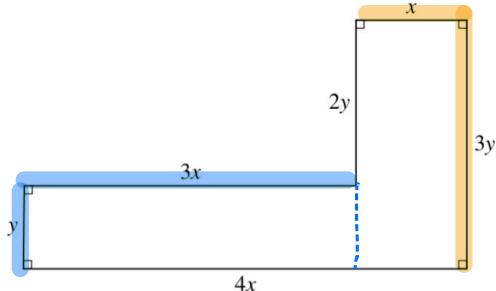
at
Point P
 $x = 1$

SCENARIO BASED

ALWAYS ATTEMPT THE DIFFERENTIATION PART FIRST.



26



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

- (i) Find an expression for y in terms of x . [1]
- (ii) Given that the area of the garden is $A \text{ m}^2$, show that $A = 48x - 8x^2$. [2]
- (iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

$$(i) P = y + 2y + 3y + 4x + 3x + x$$

$$P = 6y + 8x$$

$$48 = 6y + 8x$$

$$y = \frac{48 - 8x}{6}$$

$$(ii) A = (y)(3x) + (x)(3y)$$

$$A = 3xy + 3xy$$

$$A = 6xy$$

$$A = 6x \left(\frac{48 - 8x}{6} \right)$$

$$A = 48x - 8x^2$$

(iii)

$$A = 48x - 8x^2$$
$$\boxed{\frac{dA}{dx} = 48 - 16x}$$

Max
↓
Stationary point.
↓
diff = 0

$$0 = 48 - 16x$$

$$16x = 48$$

$$x = 3$$

$$A = 48(3) - 8(3)^2 = 72 \text{ (Max-value of area).}$$

$$\frac{dA}{dx} = 48 - 16x$$

$$\frac{d^2A}{dx^2} = -16 \text{ (Max).}$$

- 13 A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is 96 cm 2 .

- (i) Express h in terms of x and show that the volume, V cm 3 , of the block is given by

$$V = 24x - \frac{1}{2}x^3.$$

[3]

Given that x can vary,

- (ii) find the stationary value of V , [3]

- (iii) determine whether this stationary value is a maximum or a minimum. [2]

Total SA = 96



$$2(x^2) + 4(x)(h) = 96$$

$$2x^2 + 4xh = 96$$

$$h = \frac{96 - 2x^2}{4x}$$

$$V = (x)(x)(h)$$

$$V = x^2h$$

$$V = x^2 \left(\frac{96 - 2x^2}{4x} \right)$$

$$V = \frac{96x - 2x^3}{4}$$

$$V = \frac{96x}{4} - \frac{2x^3}{4}$$

$$V = 24x - \frac{1}{2}x^3$$

(ii) $V = 24x - \frac{1}{2}x^3$

$$\frac{dV}{dx} = 24 - \frac{1}{2}(3)(x)^2(1)$$

$$\boxed{\frac{dV}{dx} = 24 - \frac{3}{2}x^2}$$

$$0 = 24 - \frac{3}{2}x^2$$

$$\frac{3}{2}x^2 = 24$$

$$x^2 = 16$$

$$x = 4$$

$$V = 24(4) - \frac{1}{2}(4)^3$$

$$V = 64$$

(iii) $\frac{dV}{dx} = 24 - \frac{3}{2}x^2$

$$\frac{d^2V}{dx^2} = 0 - \frac{3}{2}(2x)$$

$$\frac{d^2V}{dx^2} = -3x \quad \boxed{x=4}$$

$$\frac{d^2V}{dx^2} = -3(4) = -12 \quad (\text{Max})$$

DIF FERENTIATION

QUESTIONS:

[TYPE 1] EQUATION OF TANGENT AND NORMAL .

[TYPE 2] STATIONARY POINTS AND ITS NATURE .

[TYPE 3] SCENARIO BASED PROOF QUESTION

[TYPE 4] RATE OF CHANGE .

[TYPE 5] INCREASING / DECREASING FUNCTIONS .



ZAINEMATICS

FOR THE LOVE OF MATHS

Integration P1

Compiled by Rafay Mushtaq



zainematics



zainematics



zainematics

99

INTEGRATION

- OUTCOMES :
- 1) AREA UNDER GRAPH
 - 2) VOLUME OF ROTATION
 - 3) Reverse working.

$$y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2}$$

integration *integration*

SYMBOL

$$\int \square dx$$

BASE RULE :

$$1 \longrightarrow x$$

$$\text{alone constant}(k) \longrightarrow Kx$$

$$2 \longrightarrow 2x$$

$$5 \longrightarrow 5x$$

POWER RULE :

$$(\boxed{\quad})^n \longrightarrow \frac{(\boxed{\quad})^{n+1}}{n+1}$$

RULES:

1) YOU ARE NOT ALLOWED TO INTEGRATE POWER UNLESS "DIFFERENTIATION OF BOX"
 \square' IS PRESENT OUTSIDE OPERATOR.

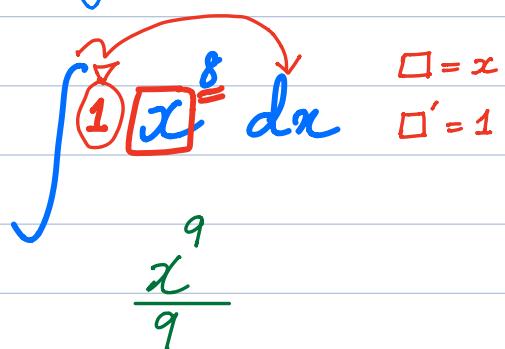
2) ONCE THIS CONDITION IS FULFILLED,
THREE THINGS WILL DISAPPEAR,

$$\int \square' dx$$

AND POWER IS ALLOWED TO BE
INTEGRATED.

EXAMPLE:

1) $\int x^8 dx$


$$\int 1 \boxed{x^8} dx$$
$$\square = x$$
$$\square' = 1$$
$$\frac{x^9}{9}$$

$$2) \int (x+5)^7 dx$$

$$\int \textcircled{1} (x+5)^7 dx \quad \square = x+5 \\ \square' = 1$$

$$\frac{(x+5)^8}{8}$$

$$3) \int (2x+4)^6 dx$$

$$\frac{1}{2} \int \textcircled{2} (2x+4)^6 dx \quad \square = 2x+4 \\ \square' = 2$$

$$\frac{1}{2} \times \frac{(2x+4)^7}{7}$$

$$\frac{(2x+4)^7}{14}$$

$$3) \int (5x+8)^7 dx$$

$$\frac{1}{5} \int 5(5x+8)^7 dx$$

$$\square = 5x+8$$

$$\square' = 5$$

$$\frac{1}{5} \times \frac{(5x+8)^8}{8}$$

$$\frac{(5x+8)^8}{40}$$

RULES:

i) YOU ARE NOT ALLOWED TO INTEGRATE POWER UNLESS "DIFFERENTIATION OF BOX" \square' IS PRESENT OUTSIDE OPERATOR.

ii) ONCE THIS CONDITION IS FULFILLED, THREE THINGS WILL DISAPPEAR,

$$\int \square' dx$$

AND POWER IS ALLOWED TO BE INTEGRATED.

$$4) \int x(x^2+5)^3 dx$$

$$\frac{1}{2} \int 2x(x^2+5)^3 dx$$

$$\square = x^2+5$$

$$\square' = 2x$$

$$\frac{1}{2} \times \frac{(x^2+5)^4}{4}$$

$$\frac{(x^2+5)^4}{8}$$

$$5) \int x^2 (9x^3 + 3)^4 dx$$

$$\frac{1}{27} \int (27x^2) (9x^3 + 3)^4 dx$$

$$\square = 9x^3 + 3$$

$$\square' = 27x^2$$

$$\frac{1}{27} \times \frac{(9x^3 + 3)^5}{5}$$

$$6) \int (2x+3)^7 dx$$

$$\frac{1}{2} \int (2)(2x+3)^7 dx$$

$$\square = 2x+3$$

$$\square' = 2$$

$$\frac{1}{2} \times \frac{(2x+3)^8}{8}$$

THERE ARE ONLY 2 TYPES OF INTEGRATION
THAT YOU WILL FACE IN P1.

$$(\square^n)$$

In this case, you have to be careful about conditions of integration.

There is no whole power. Power is on each x term individually

you can integrate without checking conditions.

$$\text{eg 1) } \int 4x^7 dx$$

No whole power.

Power is directly on x-term.

You can integrate directly.

$$\int 4x^{\boxed{7}} dx$$

$$\frac{4x^8}{8}$$

$$2) \int (x^3 + 2x^2 + 5) dx$$

No whole power
Treat as separate terms.

$$\frac{x^4}{4} + \frac{2x^3}{3} + 5x$$

$$\underline{\underline{Q}} \quad \int x(3x^2+9)^5 dx$$

$$\frac{1}{6} \int \cancel{6x} (\boxed{3x^2+9})^5 dx$$

$\Delta = 3x^2+9$
 $\Delta' = 6x$

$$\frac{1}{6} \times \frac{(3x^2+9)^6}{6}$$

$$\underline{\text{Q}} \quad \int \sqrt{2x+1} dx$$

$\frac{1}{2} \left\{ 2(2x+1)^{\frac{1}{2}} dx \right.$

$\square = 2x+1$
 $\square' = 2$

$$\frac{1}{2} \times \frac{(2x+1)^{1.5}}{1.5}$$

$$\frac{(2x+1)^{1.5}}{3}$$

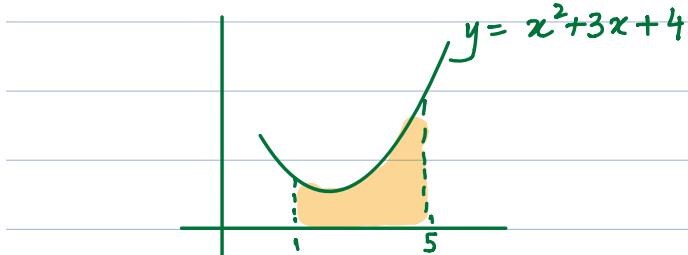
$$\underline{\text{Q}} \quad \int (2x^1 + 5) dx$$

$$\frac{2x^2}{2} + 5x$$

$$x^2 + 5x$$

AREA UNDER GRAPH.

(Area between curve and x-axis)



$$\text{Area} = \int_{1}^{5} (x^2 + 3x + 4) dx$$

Modulus

$$|3| = 3$$

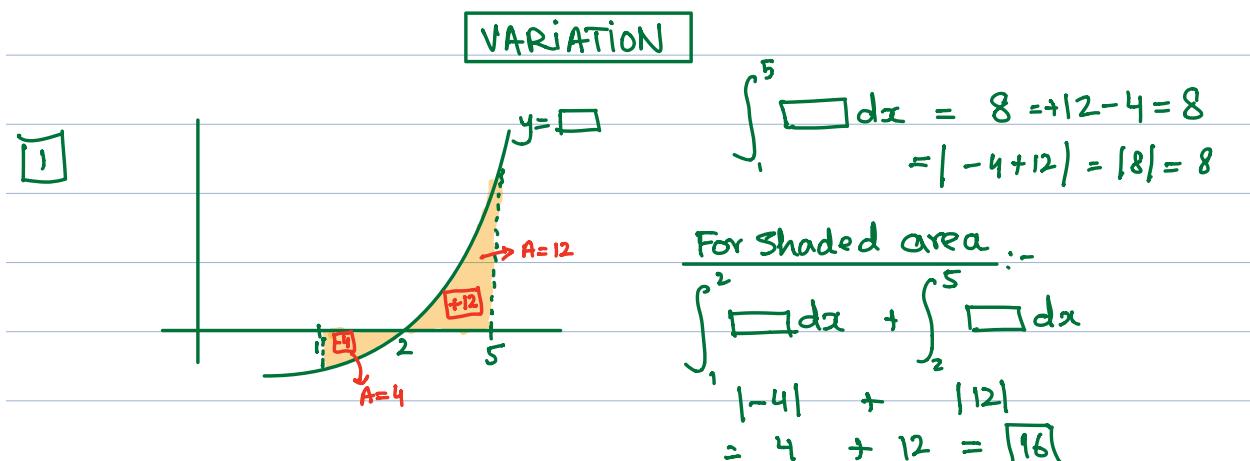
$$|-3| = 3$$

Modulus $\left| \frac{x^3}{3} + \frac{3x^2}{2} + 4x \right|_1^5$

$$\left| \left(\frac{5^3}{3} + \frac{3(5)^2}{2} + 4(5) \right) - \left(\frac{1^3}{3} + \frac{3(1)^2}{2} + 4(1) \right) \right|$$

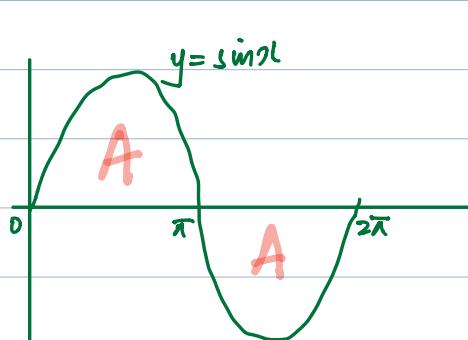
$$= |99.2 - 5.83|$$

Area = 93.37



Integration considers areas below x -axis as negative. And it gives us net answer.

Hence if you have some portion of shaded area below x -axis, you will need to split the process.

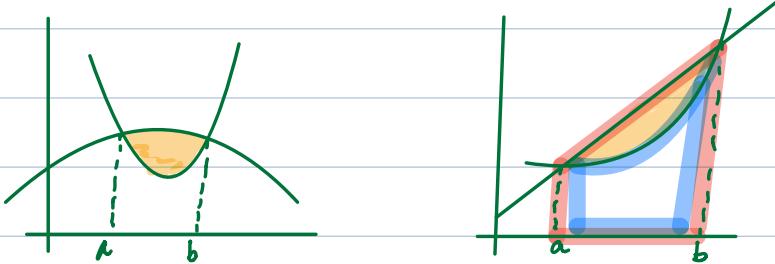


$$\int_0^{2\pi} \sin x dx = 0$$

$$\int_0^{\pi} \sin x dx = A$$

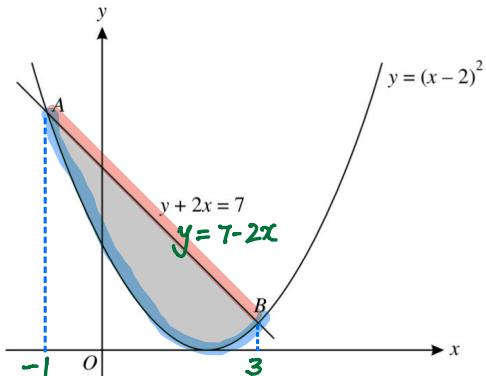
$$\int_{\pi}^{2\pi} \sin x dx = (-A) = A$$

2 Area between two graphs



Area between two graphs = $\int_a^b (\text{upper graph}) dx - \int_a^b (\text{lower graph}) dx$.

21



The diagram shows the curve $y = (x - 2)^2$ and the line $y + 2x = 7$, which intersect at points A and B . Find the area of the shaded region. [8]

Points of intersection:

$$y = (x - 2)^2 \quad y = 7 - 2x$$

$$(x - 2)^2 = 7 - 2x$$

$$x^2 - 4x + 4 = 7 - 2x$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x - 3) + 1(x - 3) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1$$

$$x = 3$$

, *upper graph* *lower graph.*

Shaded Area = $\int_{-1}^3 (7-2x) dx - \int_{-1}^3 (x-2)^{\frac{2}{3}} dx$ $\square = x-2$
 $\square' = 1$

$$= \left| 7x - \frac{2x^2}{2} \right|_{-1}^3 - \left| \frac{(x-2)^3}{3} \right|_{-1}^3$$

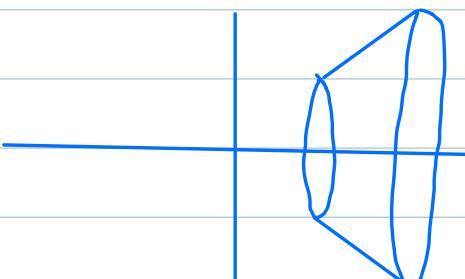
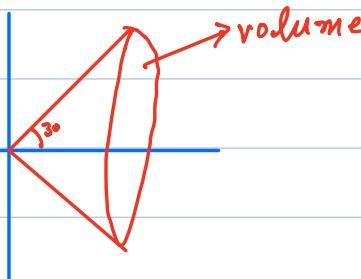
$$\left| (7(3) - (3)^2) - (7(-1) - (-1)^2) \right| - \left| \frac{(3-2)^3}{3} - \frac{(-1-2)^3}{3} \right|$$

$$(12 - (-6)) - \left(\frac{1}{3} + 9 \right)$$

$$= 18 - \frac{28}{3}$$

Shaded area. = $\frac{26}{3}$

VOLUME OF ROTATION 360°

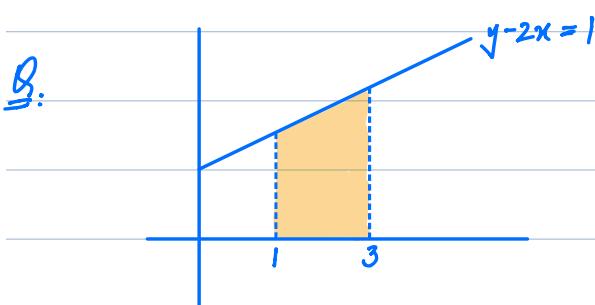


ABOUT x-Axis :-

- 1) Make y - subject .
- 2) Square both sides .
- 3) Integrate RHS
- 4) Apply limits
- 5) Multiply by π

ABOUT y-Axis :-

- 1) Make x - subject .
- 2) Square both sides .
- 3) Integrate RHS
- 4) Apply limits
- 5) Multiply by π



Find the volume when the shaded region is rotated 360° about x-axis .

- Step1: Make y Subject
 Step2: Square both sides
 Step3: Integrate RHS.

$$\begin{aligned}
 y &= 2x + 1 \\
 y^2 &= (2x+1)^2 \\
 \frac{1}{2} \int_1^3 (2x+1)^2 dx & \quad \square = 2x+1 \\
 & \quad \square' = 2
 \end{aligned}$$

$$\frac{1}{2} \times \frac{(2x+1)^3}{3}$$

Step4: Apply limits

$$\left| \frac{(2x+1)^3}{6} \right|_1^3$$

$$\left| \frac{(2(3)+1)^3}{6} - \frac{(2(1)+1)^3}{6} \right|$$

$$\frac{316}{6}$$

$$52 \frac{2}{3}$$

Step5: Multiply by π

$$\text{Volume} = 52 \frac{2}{3} \times \pi = 165.45$$

IN THIS WORKING SOMETIMES THE CONDITION FOR INTEGRATION WILL NOT BE GETTING SATISFIED. IN THAT CASE OPEN THE BRACKETS USING $(a+b)^2 = a^2 + 2ab + b^2$.

eg

$$\int (2x - x^2)^2 dx$$

$$\square = 2x - x^2$$

$$\square' = 2 - 2x$$

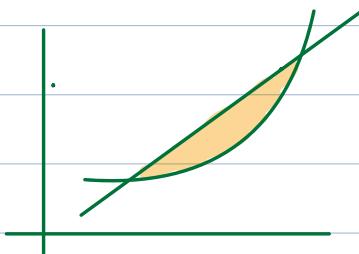
$$\int ((2x)^2 - 2(2x)(x^2) + (x^2)^2) dx$$

we cannot introduce a variable term.

$$\int (4x^2 - 4x^3 + x^4) dx \quad \text{now integrate individually.}$$

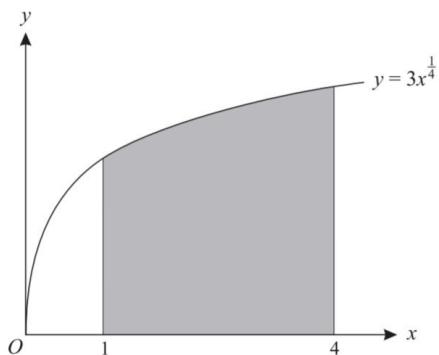
$$\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}$$

VOLUME OF SHADeD REGION BETWEEN TWO GRAPHS.



$$\begin{aligned} \text{Volume of shaded region} &= \text{Volume of upper graph} - \text{Volume of lower graph.} \end{aligned}$$

12



The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π . [4]

$$y = 3x^{\frac{1}{4}}$$

$$y^2 = (3x^{\frac{1}{4}})^2$$

$$y^2 = 9x^{\frac{1}{2}}$$

$$\int 9x^{\frac{1}{2}} dx$$

$$\frac{9x^{1.5}}{1.5}$$

$$[6x^{1.5}]_1^4$$

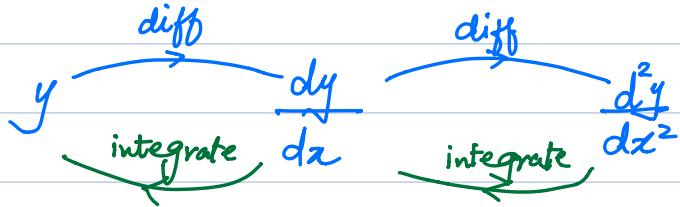
$$| 6(4)^{1.5} - 6(1)^{1.5} |$$

$$| 48 - 6 | \\ = 42$$

$\times \pi$

$$\text{Volume} = 42\pi$$

REVERSE STEP:



Constant of Integration (+c)

$$y = x^2 - 5x + 8$$

(diff) integrate!

$$\frac{dy}{dx} = 2x - 5$$

$$y = \int (2x - 5) dx$$

$$y = \frac{2x^2}{2} - 5x$$

$$y = x^2 - 5x + C$$

- NOTE:-
- 1) IF YOU ARE INTEGRATING "WITH LIMITS" (AREA OR VOLUME), NO NEED TO PUT $+C$
 - 2) IF YOU ARE INTEGRATING "WITHOUT LIMITS" (REVERSE WORKING), ALWAYS USE $+C$

$$| (\quad \cancel{+C}) - (\quad +C) |$$

Q $\frac{dy}{dx} = 3x + 5$. Given that curve passes through $(1, 5)$, Find equation of curve.

$$y = \int (3x + 5) dx$$

$$y = \frac{3x^2}{2} + 5x + C$$

$$\begin{aligned} x &= 1 \\ y &= 5 \end{aligned}$$

$$5 = \frac{3(1)^2}{2} + 5(1) + C$$

$$C = -\frac{3}{2}$$

Never use
 $y - y_1 = m(x - x_1)$
 for these points
 Since that
 is equation
 of straight line.
 NOT of curve.

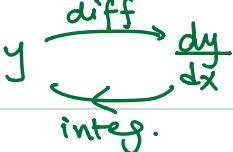
EQUATION OF CURVE:

$$y = \frac{3x^2}{2} + 5x - \frac{3}{2}$$

- 7 A curve is such that $\frac{dy}{dx} = \frac{16}{x^3}$, and $(1, 4)$ is a point on the curve.

(i) Find the equation of the curve.

$$\underline{y}$$



[4]

$$y = \int \frac{16}{x^3} dx$$

$$y = \int 16x^{-3} dx$$

$$y = \frac{16x^{-2}}{-2} + C$$

$$\boxed{y = -\frac{8}{x^2} + C} \quad \begin{matrix} x=1 \\ y=4 \end{matrix}$$

$$y = -\frac{8}{(1)^2} + C$$

$$C = 12$$

CURVE

$$\boxed{y = -\frac{8}{x^2} + 12}$$

- 15 A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q . Find

(i) the equation of the curve,

[3]

$$y = \int (4-x) dx$$

$$y = 4x - \frac{x^2}{2} + C$$

$$\boxed{y = 4x - \frac{x^2}{2} + C} \quad \begin{matrix} x=2 \\ y=9 \end{matrix}$$

$$9 = 4(2) - \frac{2^2}{2} + C$$

$$9 = 8 - 2 + C$$

$$C = 3$$

$$\boxed{y = 4x - \frac{x^2}{2} + 3}$$



- 44 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at (2, 12).

(i) Find the equation of the curve.

$$\frac{dy}{dx} = 0, x=2, y=12$$

[6]

$$\frac{dy}{dx} = \int (-4x) dx$$

$$\frac{dy}{dx} = -4 \frac{x^2}{2} + C$$

$$\frac{dy}{dx} = -2x^2 + C$$

$$\frac{dy}{dx} = 0, x=2$$

$$0 = -2(2)^2 + C$$

$$C = 8$$

$$\frac{dy}{dx} = -2x^2 + 8$$

$$y = \int (-2x^2 + 8) dx$$

$$y = -\frac{2x^3}{3} + 8x + C$$

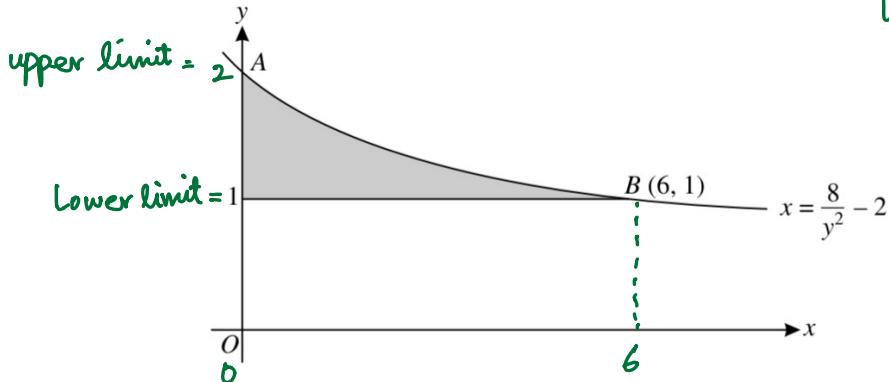
$$x=2 \\ y=12$$

$$12 = -\frac{2(2)^3}{3} + 8(2) + C$$

$$C = \frac{16}{3} - 4 = \frac{4}{3}$$

$$y = -\frac{2x^3}{3} + 8x + \frac{4}{3}$$

43



A

$$\begin{aligned} x &= 0 \\ 0 &= \frac{8}{y^2} - 2 \\ 2 &= \frac{8}{y^2} \\ 2y^2 &= 8 \\ y^2 &= 4 \\ y &= 2 \end{aligned}$$

The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y -axis at the point A . The point $B(6, 1)$ lies on the curve. The shaded region is bounded by the curve, the y -axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the y-axis. [6]

$$x = \frac{8}{y^2} - 2$$

$$x^2 = \left(\frac{8}{y^2} - 2 \right)^2$$

$$\int \left(\frac{8}{y^2} - 2 \right)^2 dy$$

$$\left(\frac{8}{y^2} \right)^2 - 2 \left(\frac{8}{y^2} \right)(2) + (2)^2$$

$$\frac{64}{y^4} - \frac{32}{y^2} + 4$$

$$\int (64y^{-4} - 32y^{-2} + 4) dy$$

$$\left[\frac{64y^{-3}}{-3} - \frac{32y^{-1}}{-1} + 4y \right]_1^2$$

$$\left| \left(\frac{64(2)^{-3}}{-3} - \frac{32(2)^{-1}}{-1} + 4(2) \right) - \left(\frac{64(1)^{-3}}{-3} - \frac{32(1)^{-1}}{-1} + 4(1) \right) \right|$$

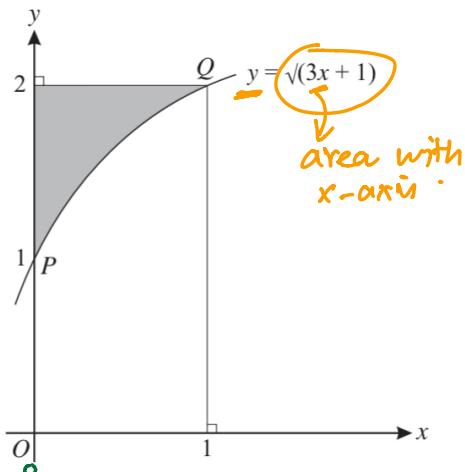
$$\left| \frac{64}{3} - \left(\frac{44}{3} \right) \right|$$

$$\frac{20}{3}$$

$$\times \pi$$

$$\text{Volume} = \frac{20\pi}{3}$$

17



The diagram shows the curve $y = \sqrt{3x + 1}$ and the points $P(0, 1)$ and $Q(1, 2)$ on the curve. The shaded region is bounded by the curve, the y -axis and the line $y = 2$.

- (i) Find the area of the shaded region.

[4]

$$\begin{aligned} \text{Shaded area} &= \text{Rectangle} - \text{Area under curve} \\ &= (2)(1) - \frac{14}{9} \end{aligned}$$

$$= 2 - \frac{14}{9} = \boxed{\frac{4}{9}}$$

Area under curve = $\frac{1}{3} \int_{0}^{1} (3x+1)^{\frac{1}{2}} dx$

$\square = 3x+1$
 $\square' = 3$

$$\frac{1}{3} \times \frac{(3x+1)^{1.5}}{1.5}$$

$$\left| \frac{(3x+1)^{1.5}}{4.5} \right|_0^1$$

$$\left| \frac{(3(1)+1)^{1.5}}{4.5} - \frac{(3(0)+1)^{1.5}}{4.5} \right|$$

$$\frac{16}{9} - \frac{2}{9}$$

$$= \frac{14}{9}$$

Method 2: To Find area between curve and y-axis,

make x-subject

$$y = \sqrt{3x+1}$$

$$y^2 = 3x+1$$

$$x = \frac{y^2 - 1}{3}$$

area with y-axis.

Integrate and put limits of y-axis now.

$$\int \frac{y^2 - 1}{3} dy$$

$$\frac{1}{3} \int (y^2 - 1) dy$$

$$\frac{1}{3} \left| \frac{y^3}{3} - y \right|^2$$

whenever there is
a constant common
in [DIFF] OR [INTEG]
always take it
outside .

$$\frac{1}{3} \left| \left(\frac{2^3}{3} - 2 \right) - \left(\frac{1^3}{3} - 1 \right) \right|$$

$$\frac{1}{3} \left| \frac{2}{3} - \left(-\frac{2}{3} \right) \right|$$

$$\frac{1}{3} \left| \frac{4}{3} \right| = \boxed{\frac{4}{9}}$$

Friday } Sequences
Saturday }

Sunday } Quadratics Marathon. (3 hours).

