

TYPE 1

Forward

12 (b)

$\Phi\left(\frac{\mu + \sigma - \mu}{\sigma}\right) = 0.8413$ $P(z < 1) = 0.8413 \times 2 = 0.6826$ $0.6826 \times 800 = 546 \text{ (accept 547)}$ <p>OR</p> $SR \ 800 \times 2/3 = 533 \text{ or } 534$	B1	0.8413 (p) seen or implied (can use their own numbers)
	M1	finding the correct area i.e. $2p - 1$
	A1 3	correct answer, must be a positive integer
	SR B1	for 2/3
	B1	for 533 or 534 or B2 if 533 or 534 and no working

19 (i)	$P(X > 0) = 1 - \Phi\left(\frac{0 - -15.1}{\sqrt{62}}\right)$	M1	Standardising, sq rt, no cc
	$= 1 - \Phi(1.918)$	M1	Prob < 0.5 after use of normal tables
	$= 1 - 0.9724$		
	$= 0.0276 \text{ or answer rounding to}$	A1 [3]	Correct answer

22	$20p = 1.6 \quad p = 0.08$	M1	Equation relating $20p$ to the mean
	$P(X > 2) = 1 - \{(0.92)^{20} + {}^{20}C_1(0.08)(0.92)^{19} + {}^{20}C_2(0.08)^2(0.92)^{18}\}$	A1	Correct p can be implied
	$= 1 - (0.1887 + 0.3281 + 0.2711)$	M1	Bin expression involving $p^x(1-p)^{20-x} {}^{20}C_x$ any p
	$= 0.212$	M1	Subtracting 2 or 3 binomial probs from 1, one of which is $P(0)$
		A1 [5]	Correct answer

28 (i)	$P(x > 10.9) = P(z > \frac{10.9 - 11}{0.095})$	M1	Standardising, no cc, no sq rt
	$= P(z > -1.0526)$		
	$= 0.8538 \text{ (0.854)}$	A1 [2]	Rounding to correct answer
(ii)	$P(\text{at least } 2 < 10.9) = 1 - P(0, 1)$	M1	Bin expression with \sum powers = 6, 6C_x , $p + q = 1$.
	$= 1 - (0.8538)^6 - {}^6C_1(0.1462)(0.8538)^5$	A1ft	Reasonably correct unsimplified expression ft their (i)
	$= 0.215$	A1 [3]	Rounding to correct answer

33 (i) $P(X > 20) = P(z > -6.4/3.7)$ $= P(z > -1.730)$ $= 0.9582$ Number of students = 335 or 336	M1	Standardising no cc no sq rt
	A1	Prob rounding to 0.958
	A1ft	Correct answer ft their prob, must be integer
	[3]	
(ii) $P(\text{very slow}) = 0.05$ $P(0, 1, 2) =$ $(0.95)^8 + {}^8C_1(0.05)^1(0.95)^7 + {}^8C_2(0.05)^2(0.95)^6$ $= 0.6634 + 0.2793 + 0.0515$ $= 0.994$	B1	0.05 or 0.95 seen
	M1	Binomial term with ${}^8C_r p^r (1-p)^{8-r}$ seen any p
	M1	Correct expression for $P(0, 1, 2)$, p close to 0.05
	A1	Answer rounding to 0.994
	[4]	

39

(b) $P(X < a + 33) = 0.75$ $z = 0.674$ $\frac{a + 33 - 33}{\sqrt{21}} = 0.674$ $a = 3.09$	M1	Using 0.75 oe
	A1	± 0.674 seen
	M1	Standardising, no cc, must have sq rt
	A1	Correct answer
	[4]	

52 $z_1 = \frac{30 - 28.3}{\sqrt{4.5}} = 0.8014$ $z_2 = \frac{25 - 28.3}{\sqrt{4.5}} = -1.5556$ $\Phi_1 - (1 - \Phi_2) = 0.7884 + 0.9401 - 1$ $= 0.729$	M1	Standardising at least one value, sq rt.ess; no cc
	M1	$\Phi_1 + \Phi_2 - 1$ oe
	A1	Correct answer
	[3]	

62 (i) $\Phi\left(\frac{84.5 - 82}{\sqrt{126}}\right) - \Phi\left[\frac{83.5 - 82}{\sqrt{126}}\right]$ $= \Phi(0.2227) - \Phi(0.1336)$ $= 0.5883 - 0.5533$ $= 0.0350$	M1	Standardising using 83.5 or 84.5, must have square root
	M1	Subtracting two probabilities, both > 0.5 or both < 0.5
	A1	Correct answer
(ii) $P(x > 87) = 1 - \Phi\left(\frac{87 - 82}{\sqrt{126}}\right) = 1 - \Phi(0.445)$ $= 1 - 0.6718 = 0.3282$ $P(0, 1) = (0.6718)^5 + {}_5C_1(0.3282)(0.6718)^4$ $= 0.471$	M1	Standardising, no cc, must have square root
	A1	Correct probability
	M1	Any binomial term of form ${}_5C_x p^x (1-p)^{5-x}$, $x \neq 0$
(iii) $P(x < 87) = 0.6718$ $P(x < k) = 0.9718$ $z = 1.908$ or 1.909 $1.909 = \pm \frac{k - 82}{\sqrt{126}}$ $k = 103$	A1	Correct answer
	M1	Finding $P(x < 87)$, value > 0.5
	M1	Adding 0.3 to their 0.6718 or equivalent
	A1	Correct z
	M1	Equation with k , 82 or 81.5 or 82.5, $\sqrt{126}$, and a z -value
	A1	Correct answer rounding to 103
	[5]	

Type 2

Reverse

12 (a)	$\frac{5.2 - 2s}{s} = -1.282$	M1	Equation with \pm correct LHS seen here or later, can be μ or s , no cc
		B1	± 1.282 seen accept ± 1.28 or anything in between
		M1	solving their equation with recognisable z -value and only 1 unknown occurring twice
	$s = 7.24$ or 7.23	A1 4	correct final answer

19

(ii) $z = -1.22$	B1	$z = \pm 1.22$
$-1.22 = \frac{0 - \mu}{\sqrt{40}}$	M1	an equation in μ , recognisable z , $\sqrt{40}$, no cc
$\mu = 7.72$ c.a.o	A1 [3]	correct answer c.w.o from same sign on both sides

20 (i) $z = 0.674$ $\frac{1002 - \mu}{8} = 0.674$ $\mu = 997$	B1 M1 A1 [3]	± 0.674 or rounding to, seen, e.g. 0.6743 Standardising and attempting to solve for μ , must use recognisable z -value, no cc, no sq rt, no sq Correct answer rounding to 997
(ii) $P(2) = 3 \times \frac{225}{900} \times \frac{224}{899} \times \frac{675}{898}$ $= 0.140$ OR $\frac{{}^{225}C_2 \times {}^{675}C_1}{{}^{900}C_3}$	M1 A1 [2]	$900 \times 899 \times 898$ or ${}^{900}C_3$ seen in denom Correct answer not 0.141 or 0.14

24 (i) mean = 51	B1 [1]	
(ii) $z = \pm 0.674$ $\pm(63 - 51) / \sigma = 0.674$ $\sigma = 17.8$	B1 M1 A1 [3]	Correct z Standardising, no cc, no $\sqrt{\sigma}$, no σ^2 Correct answer

<p>26 (+/-) 1.045, (+/-) 0.313</p> <p>$20.9 - \mu = -0.313 \sigma$ $30 - \mu = 1.045 \sigma$</p> <p>$\sigma = 6.70$ $\mu = 23.0$</p>	<p>B1, B1</p> <p>M1</p> <p>A1 A1</p> <p>[5]</p>	<p>1 correct z-value, the other correct z-value.</p> <p>Valid attempt to solve 2 equations relating to μ, σ, 30, 20.9. No $\sqrt{\sigma}$, σ^2</p> <p>correct answer correct answer</p>
<p>29 (i) $P(X < 2\mu) = P\left(z < \frac{2\mu - \mu}{\sigma}\right)$</p> <p>$= P(z < \mu/\sigma) = P(z < 5/3)$</p> <p>$= 0.952$</p>	<p>M1</p> <p>A1</p> <p>A1 [3]</p>	<p>Standardising, and attempt to get 1 variable, no cc, no $\sqrt{\quad}$, no sq</p> <p>$\pm 5/3$ seen oe</p> <p>Rounding to correct answer</p>
<p>(ii) $P\left(X < \frac{\mu}{3}\right) = P\left(z < \frac{-2\mu}{3\sigma}\right)$</p> <p>$\frac{-2\mu}{3\sigma} = 1.047$ $\mu = -1.57\sigma$</p>	<p>M1</p> <p>B1</p> <p>A1 [3]</p>	<p>standardising attempt resulting in $z \leq -$ some μ/σ</p> <p>allow $\pm \left(\frac{\mu/3 - \mu}{\sigma}\right)$</p> <p>$\pm 1.047$ seen</p> <p>correct single number, answer must have a minus sign and $\mu = \dots\sigma$</p>
<p>39 (a) $z > \frac{2\mu - \mu}{\sigma} = \frac{\mu}{\sigma} = \frac{7\sigma^2}{3\sigma}$</p> <p>$\frac{7\sigma}{3} = 1.272$</p> <p>$\sigma = 0.545$ $\mu = 0.693$</p>	<p>M1</p> <p>M1 B1 A1</p> <p>[4]</p>	<p>Standardising attempt resulting in $z > \text{some } \mu/\sigma$</p> <p>Substituting to eliminate μ or σ</p> <p>1.272 seen</p> <p>Both answers correct</p>
<p>61 $z = -1.036 = \frac{5.6 - 93}{\sigma}$</p> <p>$\sigma = 3.57$</p>	<p>B1 M1</p> <p>A1 3</p>	<p>$\pm (1.036 \text{ to } 1.037)$ seen</p> <p>Equation with 5.6 or 13.0, 9.3, σ and a z value, no cc</p> <p>Correct final answer</p>

TYPE 3

Binomial----> Normal

<p>2 (i) $c \int_0^5 t(25 - t^2) dt = 1$</p> $c \left[\frac{25t^2}{2} - \frac{t^4}{4} \right]_0^5 = 1$ $c \left[\frac{625}{2} - \frac{625}{4} \right] = 1 \Rightarrow c = \frac{4}{625}$	<p>M1</p> <p>A1</p> <p>A1 3</p>	<p>For equating to 1 and a sensible attempt to integrate</p> <p>For correct integration and correct limits</p> <p>For given answer correctly obtained</p>
<p>(ii) $\int_2^4 ct(25 - t^2) dt = \left[\frac{25ct^2}{2} - \frac{ct^4}{4} \right]_2^4 = c[136] - c[46]$</p> $= \frac{72}{125} \quad (0.576)$	<p>M1*</p> <p>M1*dep</p> <p>A1 3</p>	<p>For attempting to integrate $f(t)$ between 2 and 4 (or attempt 2 and 4)</p> <p>For subtracting their value when $t = 2$ from their value when $t = 4$</p> <p>For correct answer</p>
<p>(iii) $\int_0^5 ct^2(25 - t^2) dt = \left[\frac{4}{625} \times \frac{25t^3}{3} - \frac{4}{625} \times \frac{t^5}{5} \right]_0^5$</p> $= \frac{8}{3}$	<p>M1*</p> <p>A1</p> <p>M1*dep</p> <p>A1 4</p>	<p>For attempting to integrate $tf(t)$, no limits needed</p> <p>For correct integrand can have c (or their c)</p> <p>For subtracting their value when $t=0$ from their value when $t=5$</p> <p>For correct answer</p>

<p>4 (i) constant p, independent trials, fixed number of trials, only two outcomes</p>	<p>B1</p> <p>B1 2</p>	<p>For an option</p> <p>For a second option</p>
<p>(ii) $P(X < 4) =$</p> $0.72^{14} + {}_{14}C_1 \times 0.28 \times 0.72^{13}$ $+ {}_{14}C_2 \times 0.28^2 \times 0.72^{12}$ $+ {}_{14}C_3 \times 0.28^3 \times 0.72^{11}$ $(= 0.0101 + 0.0548 + 0.1385 + 0.2154)$ $= 0.419$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 4</p>	<p>For adding with some C in $P(0 + 1 + 2 + 3)$ or $P(1 + 2 + 3)$ or $P(0 + 1 + 2 + 3 + 4)$ or $P(1 + 2 + 3 + 4)$</p> <p>For 0.28 and 0.72 to powers which sum to 14</p> <p>Need 2 or more terms</p> <p>For completely correct unsimplified form</p> <p>For correct final answer</p> <p>NB 0.418 is A0 if PA # 1 or A1 if PA # 2</p>

<p>(iii) $\mu = 50 \times 0.28 (= 14)$</p> <p>$\sigma^2 = 50 \times 0.28 \times 0.72 (= 10.08)$</p> <p>$P(\text{more than } 18) = 1 - \Phi\left(\frac{18.5 - 14}{\sqrt{10.08}}\right)$</p> <p>$= 1 - \Phi(1.417)$</p> <p>$= 1 - 0.9218 \text{ or } 0.9217$</p> <p>$= 0.0782 \text{ or } 0.0783$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>For 14 and 10.08 seen, can be implied</p> <p>For standardising with or without cc, must have sq root</p> <p>For continuity correction 17.5 or 18.5 AND a final answer < 0.5</p> <p>4 For correct answer</p> <p>NB 0.078 is A0 if RE # 1 or A1 if RE # 2</p>
---	---	--

<p>5 $\mu = 160, \sigma^2 = 96$</p> <p>$P(\leq 165) = \Phi\left(\frac{164.5 - 160}{\sqrt{96}}\right) = \Phi(0.4593)$</p> <p>$= 0.677$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>For 160 and 96 seen or implied by 9.798</p> <p>For standardising, must have square root</p> <p>For continuity correction, either 165.5 or 164.5</p> <p>For using tables and finding correct area (i.e. > 0.5)</p> <p>[5] For correct answer</p>
--	--	--

<p>9</p> <p>(i) $1 - P(0, 1, 2)$</p> <p>$= 1 - ((0.9)^4 + (0.09)(0.9)^3 \times {}_{14}C_1 + (0.09)^2(0.9)^2 \times {}_{14}C_2)$</p> <p>$= 1 - (0.2670 + 0.3698 + 0.2377)$</p> <p>$= 0.126$</p>	<p>M1</p> <p>B1</p> <p>B1</p> <p>A1</p>	<p>For $1 - P(0, 1, 2)$</p> <p>Correct numerical expression for P(0) or P(1)</p> <p>Correct numerical expression for P(2)</p> <p>4 Correct answer</p>
<p>(ii) $\mu = 200 \times 0.76 = 152$</p> <p>$\sigma^2 = 200 \times 0.76 \times 0.24 = 36.48$</p> <p>$P(X > 155)$</p> <p>$= 1 - \Phi\left(\frac{155.5 - 152}{\sqrt{36.48}}\right) = 1 - \Phi(1.5795)$</p> <p>$= 1 - 0.9188 = 0.281$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>For both mean and variance correct</p> <p>For standardising, with or without cc, must have $\sqrt{\quad}$ on denom</p> <p>For use of continuity correction 154.5 or 155.5</p> <p>For finding an area < 0.5 from their z</p> <p>5 For answer rounding to 0.281</p>

11 (i) $(0.6)^{10} \times (0.4)^{10} \times {}_{20}C_{10}$ $= 0.117$	M1 A1 2	3 term binomial expression involving ${}_{20}C_{\text{something}}$ and powers summing to 20 Correct final answer
<hr/> (ii) $P(18, 19, 20)$ $= (0.6)^{18} (0.4)^2 {}_{20}C_2 + (0.6)^{19} (0.4)^1 {}_{21}C_1$ $+ (0.6)^{20}$ $= 0.003087 + 0.000487 + 0.00003635$ $= 0.00361$ OR using normal approx $N(12, 4.8)$ $z = \frac{17.5 - 12}{\sqrt{4.8}}$ $= 2.51$ Prob = $1 - 0.9940 = 0.0060$	M1 A1 A1 M1 A1 A1 3	Summing three or 4 binomial expressions One correct unsimplified expression allow 0.4 0.6 muddle Correct answer Standardising, cc 16.5 or 17.5, their mean, $\sqrt{\quad}$ (their var) 2.51 seen 0.0060 seen must be 0.0060
<hr/> (iii) $\mu = 150 \times 0.60 = 90$ $\sigma^2 = 150 \times 0.60 \times 0.40 = 36$ $P(88 < X < 97)$ $= \Phi\left(\frac{97.5 - 90}{6}\right) - \Phi\left(\frac{87.5 - 90}{6}\right)$ $= \Phi(1.25) - \Phi(-0.4166)$ $= 0.8944 - (1 - 0.6616)$ $= 0.556$	B1 M1 M1 A1 M1 A1 6	For seeing 90 and 36 For standardising, with or without cc, must have sq rt on denom one continuity correction 97.5 or 96.5 or 87.5 or 88.5 0.8944 or 0.6616 or 0.3384 or 0.3944 or 0.1616 seen subtracting a probability from their standardised 97 prob correct answer

13 (i)	$P(\geq 3) = 1 - P(0, 1, 2)$	M1	For attempt at $1 - P(0, 1, 2)$ or $1 - P(0, 1, 2, 3)$ or $P(3...15)$ or $P(4...15)$
	$= 1 - (6/7)^{15} - {}_{15}C_1 (1/7) (6/7)^{14} - {}_{15}C_2 (1/7)^2 (6/7)^{13}$	M1	For 1 or more terms with 1/7 and 6/7 to powers which sum to 15 and ${}_{15}C_{\text{something}}$
	$(= 1 - 0.0990 - 0.2476 - 0.2889)$	A1	Completely correct unsimplified form
	$= 0.365$ (accept 0.364)	A1 4	Correct final answer
	(ii) $\mu = 56 \times 1/7 (= 8)$	B1	8 and 6.857 or 6.86 or 2.618 seen or implied
	$\sigma^2 = 56 \times 1/7 \times 6/7 (= 6.857)$		
	$P(\text{more than } 7) = 1 - \Phi\left(\frac{7.5 - 8}{\sqrt{6.857}}\right)$	M1	Standardising attempt with or without cc, must have square root
	$= \Phi\left(\frac{8 - 7.5}{\sqrt{6.857}}\right) = \Phi(0.1909)$	M1	Continuity correction either 7.5 or 6.5
	$= 0.576$	M1	Final answer > 0.5 (award this if the long way is used and the final answer is > 0.5)
		A1 5	Correct final answer

15 (i) $P(X=5) = (0.65)^5 \times (0.35)^2 \times {}_7C_5$	M1	Expression with 3 terms, powers summing to 7 and a ${}_7C$ term
	A1 2	Correct answer
	B1	32.5 and 11.375 seen or implied
	M1	standardising, with or without cc, must have sq rt
	M1	for continuity correction 28.5 or 29.5
	M1	correct area ie < 0.5 must be from a normal approx
	A1 5	correct answer
(ii) $\mu = 50 \times 0.65 (= 32.5),$		
$\sigma^2 = 50 \times 0.65 \times 0.35 (= 11.375)$		
$P(\text{fewer than } 29) = \Phi\left(\frac{28.5 - 32.5}{\sqrt{11.375}}\right)$		
$= 1 - \Phi(1.186)$		
$= 1 - 0.8822$		
$= 0.118$		
(iii) $0.65 n \geq 8$	M1	equality or inequality with np and 8
smallest $n = 13$	A1 2	correct answer

17 (i)	$(0.05)(0.75)(0.15)$ $= 0.00563$ (9 / 1600)	M1 B1 A1 3	Multiplying 3 probs only, no Cs 0.05 or 0.15 or $1/5 \times 1/4$ seen Correct answer
(ii)	$P(\text{at least } 8) = P(8, 9, 10)$ $= {}_{10}C_8(0.75)^8(0.25)^2 + {}_{10}C_9(0.75)^9(0.25) + (0.75)^{10}$ $= 0.526$	B1 M1 A1 3	Binomial expression involving $(0.75)^r(0.25)^{10-r}$ and a C, $r \neq 0$ or 10 Correct unsimplified expression can be implied Correct answer
(iii)	$\mu = 90 \times 0.75 = 67.5$ $\sigma^2 = 90 \times 0.75 \times 0.25 = 16.875$ $P(X > 60)$ $= 1 - \Phi\left(\frac{60.5 - 67.5}{\sqrt{16.875}}\right) = \Phi(1.704)$ $= 0.956$	B1 M1 M1 M1 A1 5	90×0.75 (67.5) and $90 \times 0.75 \times 0.25$ (16.875 or 16.9) seen For standardising, with or without cc, must have $\sqrt{\quad}$ on denom For use of continuity correction 60.5 or 59.5 For finding an area > 0.5 from their z For answer rounding to 0.956

18	mean $= 200 \times 0.08 = 16$ var $= 14.72$ $P(X \geq 15) = 1 - \Phi\left(\frac{14.5 - 16}{\sqrt{14.72}}\right)$ $= \Phi(0.391)$ $= 0.652$	B1 M1 M1 M1 A1 [5]	For both 16 and 14.7 seen For standardising, with or without cc, must have $\sqrt{\quad}$ in denom For use of continuity correction 14.5 or 15.5 For finding a prob > 0.5 from their z , legit For answer rounding to 0.652 c.w.o
19 (i)	$P(X > 0) = 1 - \Phi\left(\frac{0 - -15.1}{\sqrt{62}}\right)$ $= 1 - \Phi(1.918)$ $= 1 - 0.9724$ $= 0.0276$ or answer rounding to	M1 M1 A1 [3]	Standardising, sq rt, no cc Prob < 0.5 after use of normal tables Correct answer
(ii)	$z = -1.22$ $-1.22 = \frac{0 - \mu}{\sqrt{40}}$ $\mu = 7.72$ c.a.o	B1 M1 A1 [3]	$z = \pm 1.22$ an equation in μ , recognisable z , $\sqrt{40}$, no cc correct answer c.w.o from same sign on both sides

21 (i) $P(X < 3) = P(0) + P(1) + P(2)$ $= (0.84)^{11} + (0.16)(0.84)^{10} \times {}^{11}C_1 +$ $(0.16)^2(0.84)^9 \times {}^{11}C_2$ $= 0.1469 + 0.30782 + 0.2931$ $= 0.748$	M1 M1 A1 [3]	Binomial term with ${}^{11}C_r p^r (1-p)^{11-r}$ seen Correct expression for $P(0, 1, 2)$ or $P(0, 1, 2, 3)$ Can have wrong p Correct final answer. Normal approx M0 M0 A0
(ii) $\mu = 125 \times 0.64 = 80$ $\sigma^2 = 125 \times 0.64 \times 0.36 = 28.8$ $P(X > 73) = 1 - \Phi\left(\frac{73.5 - 80}{\sqrt{28.8}}\right)$ $= \Phi(1.211)$ $= 0.887$	B1 M1 M1 M1 A1 [5]	80 and 28.8 or 5.37 seen standardising, with or without cc, must have sq rt in denom continuity correction 73.5 or 72.5 only correct region (> 0.5 if mean > 73.5 , vv if mean < 73.5) correct answer

<p>23 (i) $\frac{{}^4C_2 \times {}^7C_1}{{}^{11}C_3} = 0.255$</p> <p>OR $\frac{4}{11} \times \frac{3}{10} \times \frac{7}{9} \times 3$</p> <p>$= 0.255$ (14/55) (42/165)</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Using 2 combs mult for numerator and 1 comb for denom</p> <p>Correct denom or num unsimplified</p> <p>Correct answer</p> <p>Multiplying 3 correct probs</p> <p>Mult by 3 or Σ their 3 options</p> <p>Correct answer</p>
<p>(ii) $P(3^{\text{rd}} \text{ is orange}) = P(P, P, O)$ $+ P(P, O, O) + P(O, P, O)$ $+ P(O, O, O)$</p> <p>$= \frac{4}{11} \times \frac{3}{10} \times \frac{7}{9} + \frac{4}{11} \times \frac{7}{10} \times \frac{6}{9}$ $+ \frac{7}{11} \times \frac{4}{10} \times \frac{6}{9} + \frac{7}{11} \times \frac{6}{10} \times \frac{5}{9}$</p> <p>$= \left[\frac{14}{165} + \frac{28}{165} + \frac{28}{165} + \frac{7}{33} \right]$</p> <p>$= 7/11$ (0.636)</p> <p>OR using a tree diagram</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>Summing four 3-factor options with or without replacement</p> <p>At least 3 correct unsimplified options</p> <p>Correct answer. Award B3 if the correct answer is stated with no working.</p>
<p>(iii) $P(P O) = \frac{P(P \cap O)}{P(O)}$</p> <p>$= \frac{P(P, P, O) + P(P, O, O)}{P(O)}$</p> <p>$= \frac{28/110}{7/11} = \frac{28}{70} = \frac{4}{10} = 0.4$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Substituting in cond prob formula with at least one 3-factor product in num, and denom their (ii) or 7/11</p> <p>Summing exactly 2 three-factor products in num</p> <p>Correct answer</p>
<p>(iv) $\mu = 121 \times \frac{4}{11} = 44$</p> <p>$\sigma^2 = 121 \times \frac{4}{11} \times \frac{7}{11} = 28$</p> <p>$P(X < 39) = \Phi\left(\frac{38.5 - 44}{\sqrt{28}}\right)$</p> <p>$= \Phi(-1.039)$</p> <p>$= 1 - 0.8506$</p> <p>$= 0.149$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>44 and 28 or 5.29 seen</p> <p>Standardising, with or without cc, must have sq rt on denom</p> <p>cc either 39.5 or 38.5</p> <p>Correct area “1 – Φ” seen</p> <p>Correct answer</p>

<p>25 $P(\text{total } 7) = P(3,4 \text{ or } 4,3) = 2/16$</p> <p>$P(\text{total } 8) = P(4,4) = 1/16$</p> <p>$P(7 \text{ or more}) = 3/16$</p> <p>Expected $200 \times \frac{3}{16} = 37.5$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>[4]</p>	<p>Attempt to find $P(7) + P(8)$</p> <p>3/16 seen</p> <p>Multiplying their prob by 200</p> <p>Correct final answer ft their prob</p>
---	---	---

32 mean = $200 \times 2/15 (= 26.67)$ (80/3) variance = $200 \times 2/15 \times 13/15 (= 23.11)$ (208/9) $P(21 < X < 35) =$ $P\left(\frac{21.5 - 26.67}{\sqrt{23.11}} < z < \frac{34.5 - 26.67}{\sqrt{23.11}}\right)$ $= P(-1.075 < z < 1.629)$ $= 0.8589 + 0.9483 - 1$ $= 0.807$	B1 M1 M1 M1 A1 [5]	mean and variance correct standardising, \pm , with or without cc, must have sq rts continuity corrections 20.5 or 21.5, 34.5 or 35.5 $\Phi_1 + \Phi_2 - 1$ answer rounding to 0.807
--	---	--

35 (i) constant/given prob, independent trials, fixed/given no. of trials, only two outcomes	B1 B1 [2]	One option correct Three options correct
(ii) $P(8, 9, 0, 1) =$ ${}^9C_8(0.3)^8(0.7) + (0.3)^9 + (0.7)^9 + {}^9C_1(0.3)(0.7)^8$ $= 0.196$	M1 A1 A1 [3]	One term seen involving $(0.3)^x(0.7)^{9-x}({}^9C_x)$ Correct unsimplified expression Correct answer
(iii) mean = $90 \times 0.3 = 27$ var = 18.9 $P(X > 35) = 1 - \Phi\left(\frac{35.5 - 27}{\sqrt{18.9}}\right)$ $= 1 - \Phi(1.955) = 0.0253$ $P(X < 27) = \Phi\left(\frac{26.5 - 27}{\sqrt{18.9}}\right) = 1 - \Phi(0.115)$ $= 0.4542$ Total prob = 0.480 accept 0.48	B1 M1 M1 M1 A1 [5]	Expressions for 27 and 18.9 (4.347) seen Standardising one expression, must have sq rt in denom, cc not necessary Continuity correction applied at least once $(1 - \Phi_1) + (1 - \Phi_2)$ accept $(0.0329 + 0.5)$ if no cc Rounding to correct answer

38 $18p = 2.7$ $p = 0.15$ $P(2, 3, 4) =$ ${}^{18}C_2 \times (0.15)^2(0.85)^{16} + {}^{18}C_3(0.15)^3(0.85)^{15}$ $+ {}^{18}C_4(0.15)^4(0.85)^{14}$ $= 0.655$	B1 M1 A1 A1 [4]	Correct value for p Summing 3 binomial probs o.e Correct unsimplified answer Correct answer
---	------------------------------	--

42 (i) $np = 24, npq = 4.8$ $z = \pm \left(\frac{24.5 - 24}{\sqrt{4.8}} \right) = 0.228$ Prob = 0.590	B1 M1 M1 A1 [4]	24 and 4.8 or $\sqrt{4.8}$ seen can be unsimplified Standardising, need sq rt, cc not necessary Continuity correction 24.5 or 25.5 used Correct answer must be from 24.5
(ii) np and nq both > 5 .	B1 [1]	Need both

<p>46 $\mu = 250 \times 0.86 = 215$</p> <p>$\sigma^2 = 250 \times 0.86 \times 0.14 = 30.1$</p> <p>$P(X > 210) = 1 - \Phi\left(\frac{210.5 - 215}{\sqrt{30.1}}\right)$</p> <p>$= \Phi(0.820)$</p> <p>$= 0.794$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1 [5]</p>	<p>250×0.86 and $250 \times 0.86 \times 0.14$ seen o.e</p> <p>Standardising, with or without cc, must have sq rt in denom</p> <p>Continuity correction 210.5 or 209.5 only</p> <p>Correct region (> 0.5) ft their mean</p> <p>Correct answer</p>
--	---	---

<p>54 (i) $P(2 < X < 12) = 1 - P(0, 1, 2, 12)$</p> <p>$= 1 - (0.35)^{12} - (0.65)(0.35)^{11} {}_{12}C_1 - (0.65)^2(0.35)^{10} {}_{12}C_2 - (0.65)^{12}$</p> <p>$= 1 - 0.0065359$</p> <p>$= 0.993$</p> <p>(ii) $1 - (0.87)^n > 0.95$</p> <p>$0.05 > (0.87)^n$</p> <p>$n = 22$</p>	<p>M1</p> <p>A1</p> <p>A1 [3]</p> <p>M1</p> <p>M1</p> <p>A1 [3]</p>	<p>Using binomial with ${}_{12}C_{\text{something}}$ and powers summing to 12, $\Sigma p = 1$</p> <p>Correct unsimplified answer</p> <p>Accept 0.994 from correct working only</p> <p>Equality or inequality in (0.87 or 0.78 or 0.35), power n or $n - 1$, 0.95 or 0.05</p> <p>Attempt to solve an equation with a power in (can be implied)</p> <p>Correct answer</p>
---	---	---

TYPE 4

Advanced Mix

1 (i) $\bar{x} = 375.3$ $\sigma^2_{n-1} = 8.29$	B1 M1 A1 3	For correct mean (3.s.f) For legit method involving $n-1$, can be implied For correct answer
(ii) $p = 0.19$ or equiv. $0.19 \pm 2.055 \times \sqrt{\frac{0.19 \times 0.81}{200}}$ $0.133 < p < 0.247$	B1 M1 B1 A1 4	For correct p For correct form $p \pm z \times \sqrt{\frac{pq}{n}}$ either/both sides For $z = 2.054$ or 2.055 For correct answer

3 (i) $z = 0.674$ or 0.675 allow 0.67 to 0.675 $\frac{52 - \mu}{5} = 0.674$ $\mu = 48.6$	B1 M1 A1	For correct z , can be + or - For an equation relating 52, 5, μ and any $z \neq 0.5987$ or 0.7734 ish For correct answer
(ii) $z_1 = \frac{40 - 48.63}{5} = -1.726$ $z_2 = \frac{46 - 48.63}{5} = 0.526$ prob = $0.9578 - 0.7005 = 0.2573$ $(0.2573)^4$ = 0.00438 or 4.38×10^{-3} accept 0.00449×10^{-3} NB 0.0045 gets A0 and RE #1	M1 M1 M1 A1 ft 4	For standardising 40 or 46, 5 or $\sqrt{5}$ in denom or 5^2 with their mean, no cc For subtracting two probs consistent with their mean ie usually $\Phi_1 - \Phi_2$ or $(1 - \Phi_1) - (1 - \Phi_2)$ but could be of type $\Phi_1 - (1 - \Phi_2)$ if their mean is in between 40 and 46 For raising their answer above to a power 4 For correct answer

<p>6 (i) $z_1 = 0.02/0.15 = 0.1333$</p> <p>$z_2 = -0.08/0.15 = -0.5333$</p> <p>area = $\Phi(0.1333) - \Phi(-0.533)$ $= \Phi(0.1333) - [1 - \Phi(0.5333)]$ $= 0.5529 + 0.7029 - 1$ $= 0.256$</p> <p>Prob all 4 = $(0.256)^4$ (0.00428 to 0.00430)</p> <p>(ii) $z = \pm 1.282$ or 1.28 or 1.281</p> <p>$\pm 1.282 = \frac{b}{0.15}$</p> <p>limits between 1.71 and 2.09</p>	M1	For standardising one value, no cc
	M1	For standardising the other value, no cc. SR ft on no sq rt
	M1	For finding correct area (i.e. two Φ s - 1)
	A1	For correct answer
	A1ft [5]	For correct answer, ft from their (i) , if $p < 1$, allow 0.0043
	B1	For correct z, + or - or both
	M1	For seeing an equation involving + or - of their z, b, 0.15 (their z can only be 0.842 or 0.84 or 0.841)
	A1ft [3]	both limits needed, ft 1.77 to 2.03 on 0.842 only

<p>7 (i) $1.282 = (5130 - \mu) / 40.6$</p> <p>$\mu = 5080$ (5078) rounding to 5080</p> <p>(ii) $P(< 5000) = \Phi[(5000 - 5078) / 40.6]$ $= \Phi(-1.921)$ $= 1 - 0.9727$ $= 0.0273$ or 2.73%</p> <p>(iii) $\mu = 60$, var = 54 $P(\text{fewer than } 65) = \Phi(64.5 - 60) / \sqrt{54}$ $= \Phi(0.6123)$ $= 0.730$ accept 0.73</p>	B1	For ± 1.282 seen, or 1.28, 1.281, not 1.29 or 1.30
	M1	For standardising, with or without sq rt, squared, no cc
	A1	For correct answer
	M1	For standardising, criteria as above, can include cc
	M1	For correct area found using tables ie < 0.5 ft on wrong (i)
	A1	For correct answer, accept 0.0274
	B1	For 60 and 54 seen (could be sd or variance)
	M1	For using 64.5 or 65.5 in a standardising process
	M1	For standardising, must have $\sqrt{54}$ (their 54) in denom
	A1	For correct answer

8 (i) $1.645 = \frac{50 - 38}{\sigma}$ $\sigma = 7.29$	B1 M1 A1	3	Using $z = \pm 1.645$ or 1.65 Equation with 38, 50, σ and a recognisable z -value Correct answer
(ii) $z = \frac{30 - 38}{\text{their } \sigma} (-1.097)$ $P(z < 30) = 1 - \Phi(1.097)$ $= 1 - 0.8637$ $= 0.136$	M1 M1 A1	3	Standardising, no cc Finding correct area ie < 0.5 Correct answer
(iii) $1 - (0.95)^9$ $= 0.370$	B1 B1	2	$(0.95)^9$ seen correct answer

10 (i) heights, weights, times etc of something	B1	1	Any sensible set of data, must be qualified
(ii) $z = 0.64 = \frac{\mu - 10}{\sqrt{21}}$ $\mu = 12.9$	B1 M1 A1	3	$z = \pm 0.64$ seen equation relating 10, $\sqrt{21}$, 21, μ and their z or $1 -$ their z , (must be a recognisable z value ie not 0.77) correct answer
(iii) $z = \frac{22 - 12.9}{\sqrt{21}}$ $= 1.986$ $P(X > 22) = 1 - \Phi(1.986)$ $= 1 - 0.9765$ $= 0.0235$ $300 \times 0.0235 = 7.05$ answer = 7	M1 M1ft M1 A1	4	standardising, with or without sq rt, no cc, must be their mean correct area ie < 0.5 , ft on their mean > 22 mult by 300 correct answer, accept 7 or 8 must be integer

14 (i) $z = \pm 1.68$ $z = \frac{5.5 - 4.5}{\sigma}$ $\sigma = 0.595$ accept 25/42	B1 M1 A1	3	Number rounding to 1.68 seen Standardising and attempting to solve with their z , ; must be z value, no cc, no σ^2 , no $\sqrt{\sigma}$ Correct answer
(ii) $z_1 = \frac{3.8 - 4.5}{0.5952} = -1.176$ $z_2 = \frac{4.8 - 4.5}{0.5952} = 0.504$ prob = $\Phi(0.504) - (1 - \Phi(1.176))$ $= 0.6929 - (1 - 0.8802)$ $= 0.573$	M1 A1ft M1 A1	4	For standardising 3.8 or 4.8, mean 4.5 not 5.5, their σ or $\sqrt{\sigma}$ or σ^2 in denom One correct z -value, ft on their σ Correct area ie $\Phi_1 + \Phi_2 - 1$ or $\Phi_1 - \Phi_2$ if μ taken to be 5.5 Correct answer only

16	(i)	$-0.674 = \frac{7 - \mu}{2.6}$ $\mu = 8.75$	B1 M1 M1 A1 4	± 0.674 seen only Standardising must have a recognisable z-value, no cc and 2.6 For solving their equation with recognisable z-value, μ and 2.6 not $1 - 0.674$ or 0.326, allow cc Correct answer
	(ii)	$P(X > 6.2) = P\left(z > \frac{6.2 - 6.5}{2.6}\right)$ $= P(z > -0.1154)$ $= 0.546$	M1 M1 A1 3	Standardising, no cc on the 6.2 prob > 0.5 Correct answer

27	(i)	$P(X = 2) = (0.25)^2 \times (0.75)^6 \times {}^8C_2$ $= 0.311$	M1 A1 [2]	3 term binomial expression involving 8C something, powers summing to 8 correct answer
	(ii)	$12 \times 0.25 = 3, < 5$ so not possible	B1 [1]	
	(iii)	mean = $40 \times 0.25 (= 10)$ variance = $40 \times 0.25 \times 0.75 (= 7.5)$ $P(X \text{ at least } 13) = P\left(z > \frac{12.5 - 10}{\sqrt{7.5}}\right)$ $= P(z > 0.913)$ $= 1 - \Phi(0.913)$ $= 1 - 0.8194$ $= 0.181$	B1 M1 M1 M1 A1 [5]	40×0.25 and $40 \times 0.25 \times 0.75$ seen, o.e. standardising, \pm , with or without cc, must have sq rt continuity correction 12.5 or 13.5 correct area, i.e. < 0.5 legit correct answer

31	(i)	$0.431 = \frac{135 - \mu}{\sigma}$ $-0.842 = \frac{127 - \mu}{\sigma}$ $\sigma = 6.29$ $\mu = 132$	B1 B1 M1 A1 A1 [5]	One $\pm z$ -value correct, accept 0.430 A second $\pm z$ -value correct Solving two equations relating μ , σ , 135, 127 and their z-values (must be z-values) Correct answer accept 6.28 Correct answer
	(ii)	$P(X < 145) = P\left(z < \frac{145 - 132.3}{6.284}\right)$ $= P(z < 2.023)$ $= 0.978$	M1 M1 A1 [3]	Standardising no sq rt no cc Correct use of normal tables Answer rounding to 0.978 or 0.979
	(iii)	$p = 1/3$ $P(\text{at least } 2) = 1 - P(0, 1)$ $= 1 - [(2/3)^8 + {}^8C_1 \times (1/3)^1 (2/3)^7]$ $= 0.805$	M1 A1 A1 [3]	Binomial expression with powers summing to 8 and ${}^8C_{\text{something}}$. (any p) Correct unsimplified expression Answer rounding to 0.805

34 (i) Zotoc: $z = \frac{367 - 320}{21.6} = 2.176$ Ganmor: $z = \frac{367 - 350}{7.5} = 2.267$ $P(\text{Zotoc}) = 0.985$ $P(\text{Ganmor}) = 0.988$	M1 A1 A1 [3]	Standardising either car's fuel, no cc, no sq, no $\sqrt{}$ Correct answer Correct answer
(ii) $z = 0.23$ $0.23 = \frac{x - 320}{21.6}$ $x = 324.968$ $d = 4.97$	B1 M1 M1ind A1 [4]	± 0.23 seen Standardising either car, no cc, no sq rt, no sq $320 + d - 320$ i.e. just d on num Correct answer, -4.97 gets A0

37 (i) $z = 0.807$ $0.807 = \frac{10 - 8.2}{\sigma}$ $s = 2.23$	B1 M1 A1 [3]	0.807 seen standardising, must have σ , no sq rt, no cc and a z-value correct answer
(ii) $P(> 1 \text{ min from mean}) = P(\text{mod } z > \frac{1}{2.23})$ $= P(z > 0.4484)$ $= (1 - 0.6729) \times 2$ $= 0.654$	M1 M1 A1 [3]	standardising, their sd, no cc and adding two areas using $1 - \Phi(z)$ correct answer

<p>(iii) $P(> 2 \text{ longer}) = 1 - P(0, 1, 2 \text{ longer})$</p> $= 1 - \{(0.79)^6 + {}^6C_1(0.21)(0.79)^5 + {}^6C_2(0.21)^2(0.79)^4\}$ $= 0.112$	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>binomial term ${}^6C_x p^x (1-p)^{6-x}$</p> <p>correct unsimplified answer</p> <p>correct answer</p>
<p>(iv) $\mu = 35 \times 0.5 = 17.5$ $\sigma^2 = 35 \times 0.5 \times 0.5 = 8.75$</p> $P(X < 16) = \Phi\left(\frac{15.5 - 17.5}{\sqrt{8.75}}\right)$ $= 1 - \Phi(0.676)$ $= 1 - 0.7505$ $= 0.2495 \text{ (0.249 or 0.250)}$ <p>OR ${}^{35}C_0 0.5^0 0.5^{35} + {}^{35}C_1 0.5^1 0.5^{34} + {}^{35}C_2 0.5^2 0.5^{33} + \dots$ $= 8582372584/2^{35} = 0.250$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>17.5 and 8.75 or $\sqrt{8.75}$ seen</p> <p>standardising, with or without cc, must have sd in denom</p> <p>continuity correction 15.5 or 16.5 only, seen</p> <p>using $1 - \Phi(z)$</p> <p>correct answer</p> <p>binomial term ${}^{35}C_x 0.5^x 0.5^{35-x}$</p> <p>at least 2 correct terms ($x \geq 0$) seen</p> <p>summing 16 or 17 terms</p> <p>correct expression</p> <p>correct answer</p>

<p>43 (i) $z = -1.282$</p> $P(x < 20) = P\left(z < \frac{20 - \mu}{0.8}\right)$ $-1.282 = \frac{20 - \mu}{0.8}$ $\mu = 21.0 \text{ cm (21.0256)}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>± 1.282 or ± 1.281 seen</p> <p>Standardising, no cc, must have 0.8, must be a z-value</p> <p>Correct answer</p>
<p>(ii) $P(21.5 < x < 22.5)$</p> $= P\left(\frac{21.5 - 21.03}{0.8} < z < \frac{22.5 - 21.03}{0.8}\right)$ $= \Phi(1.8375) - \Phi(0.5875)$ $= 0.9670 - 0.7217$ $= 0.2453$ <p>$P(< 2) = P(0) + P(1)$ $= (0.7547)^4 + (0.2453)^1 (0.7547)^3 {}^4C_1$</p> $= 0.746$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>2 attempts at standardising with their mean, must have 0.8 oe</p> <p>Subtracting 2 Φs ft their mean</p> <p>Needn't be entirely accurate, rounding to 0.24 or 0.25</p> <p>Binomial term with ${}^4C_r p^r (1-p)^{4-r}$ seen $r \neq 0$, any $p < 1$</p> <p>Bin expression for $P(0) + P(1)$, any $p < 1$</p> <p>Accept 3sf rounding to 0.75</p>

44 (i) $z = \pm 1.751$ $\pm \frac{20 - \mu}{\mu/4} = 1.751$ $\mu = 13.9$	B1 M1 A1 [3]	Correct z Standardising no cc, no sqrt, must be a z -value Correct answer
(ii) $P(X < 10) = P(z < \pm \frac{10 - 13.91}{13.91/4})$ $= P(z < -1.124)$ $= 1 - 0.8694$ $= 0.131$ $P(10 < X < 20) = 0.96 - 0.131$ $= 0.829$ or 0.830	M1 M1 A1 [3]	Standardising attempt with 10, their μ and their $\mu/4$, no cc, no sqrt “ $\Phi_1 + \Phi_2 - 1$ ”, ft their mean Correct answer
(iii) $\mu = 250 \times 0.96 = 240$ $\sigma^2 = 250 \times 0.96 \times 0.04 = 9.6$ $P(\geq 235) = 1 - \Phi\left(\pm \frac{234.5 - 240}{\sqrt{9.6}}\right)$ $= \Phi(1.775)$ $= 0.962$	B1 M1 M1 M1 A1 [5]	240 and 9.6 or sq rt 9.6 seen unsimplified Standardising, with or without cc, must have sq rt in denom Continuity correction 234.5 or 235.5 only Correct region > 0.5 , ft their mean Correct answer
45 (i) $(0.75)^n < 0.06$ $n > 9.78$ $n = 10$	M1* M1dep* A1 [3]	Equation or inequality with 0.75^n and 0.06 or 0.94 seen Attempt at solving by trial and error (can be implied) or using logarithms correctly Correct answer
(ii) $E(X) = 14 \times 0.75$ or 10.5 Try $P(10) = {}^{14}C_{10}(0.75)^{10}(0.25)^4 = 0.220$ $P(11) = {}^{14}C_{11}(0.75)^{11}(0.25)^3 = 0.240$ (mode is) 11 OR	M1 M1 A1 [3] M1 M1 A1	Evaluating binomial probability for an integer value directly above or below their mean Evaluating the other binomial probability Correct answer Evaluating binomial $P(n)$ and $P(n + 1)$ Evaluating binomial $P(10)$, $P(11)$ and $P(12)$ Correct answer
(iii) $P(> 11)$ $= {}^{14}C_{12}(0.75)^{12}(0.25)^2 + {}^{14}C_{13}(0.75)^{13}(0.25)^1 + (0.75)^{14}$ $= 0.281$ $P(3) = {}^5C_3 (0.2811)^3 (0.7189)^2$ $= 0.115$	M1 M1 A1 M1 A1 [5]	A binomial term of the form ${}^{14}C_n p^n (1 - p)^{14 - n}$ seen, $n \neq 0$ or 14 Summing binomial $P(12, 13, 14)$ or $P(11, 12, 13, 14)$ Correct answer 0.280 – 0.282 A binomial term of the form ${}^5C_3 p^3 (1 - p)^2$ seen, any p Correct answer

