ALEVELS P3

MARKING SCHEME

Integration

A11

1   **(i)**   State or imply $\dfrac{du}{dx} = \sec^2 x$ B1

Express integrand in terms of u and du M1

Integrate to obtain $\dfrac{u^{n+1}}{n+1}$ or equivalent   A1

Substitute correct limits correctly to confirm given result $\dfrac{1}{n+1}$ A1   [4]

  (ii)   **(a)**   Use $\sec^2 x = 1 + \tan^2 x$ twice   M1

Obtain integrand $\tan^4 x + \tan^2 x$ A1

Apply result from part **(i)** to obtain $\frac{1}{3}$ A1   [3]

Or
Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from **(i)**   M1

Obtain $\int u^2 \mathrm{d}u$ A1

Apply limits correctly and obtain $\frac{1}{3}$ A1

  (b)   Arrange, perhaps implied, integrand to
$t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$ B1

Attempt application of result from part **(i)** at least twice   M1

Obtain $\dfrac{1}{8} + \dfrac{4}{6} + \dfrac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent   A1   [3]

2   **(i)**   Use product rule   M1
Obtain derivative in any correct form   A1
Differentiate first derivative using the product rule   M1
Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$ A1
Verify the given statement   A1   **5**

  (ii)   Integrate and reach $kx\sin\frac{1}{2}x + l\int \sin\frac{1}{2}x\,\mathrm{d}x$ M1*

Obtain $2x\sin\frac{1}{2}x - 2\int \sin\frac{1}{2}x\,\mathrm{d}x$, or equivalent   A1

Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$ A1

Use correct limits $x = 0$, $x = \pi$ correctly   M1(dep*)
Obtain answer $2\pi - 4$, or exact equivalent   A1   **5**

3	(i)	Either	Substitute $x = -1$ and evaluate	**M1**	
			Obtain 0 and conclude $x + 1$ is a factor	**A1**	
		Or	Divide by $x + 1$ and obtain a constant remainder	**M1**	
			Obtain remainder = 0 and conclude $x + 1$ is a factor	**A1**	[2]

	(ii)	Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$	**M1**	
		Obtain complete quotient $4x^2 - 5x - 6$	**A1**	
		State form $\dfrac{A}{x+1} + \dfrac{B}{x-2} + \dfrac{C}{4x+3}$	**A1**	
		Use relevant method for finding at least one constant	**M1**	
		Obtain one of $A = -2, B = 1, C = 8$	**A1**	
		Obtain all three values	**A1**	
		Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
		Obtain $-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$, condoning no use of modulus signs and absence of $\dots + c$	**A1**	[8]

4(i)	Remove logarithms correctly and obtain $e^x = \dfrac{1-y}{y}$	**B1**
	Obtain the **given answer** $y = \dfrac{e^{-x}}{1 + e^{-x}}$ following full working	**B1**
	Total:	2

(ii)	State integral $k\ln(1 + e^{-x})$ where $k = \pm 1$	***M1**
	State correct integral $-\ln(1 + e^{-x})$	**A1**
	Use limits correctly	**DM1**
	Obtain the **given answer** $\ln\left(\dfrac{2e}{e+1}\right)$ following full working	**A1**
	Total:	4