C1 Without Diagram Easy Answers P3

1 (**i**) *EITHER*: Solve the quadratic and use $\sqrt{-1} = \mathrm{i}$ M1

Obtain roots $\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$ and $\frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}$ or equivalent A1

OR: Substitute $x + \mathrm{i}y$ and solve for x or y M1

Obtain correct roots A1 **2**

(**ii**) State that the modulus of each root is equal to 1 B1√

State that the arguments are $\frac{1}{3}\pi$ and $-\frac{1}{3}\pi$ respectively B1√ + B1√ **3**

[Accept degrees and $\frac{5}{3}\pi$ instead of $-\frac{1}{3}\pi$. Accept a modulus in the form $\sqrt{\frac{p}{q}}$ or $\sqrt{n}$, where p, q, n are integers. An answer which only gives roots in modulus-argument form earns B1 for both the implied moduli and B1 for both the implied arguments.]

(**iii**) *EITHER*: Verify $z^3 = -1$ for each root B1 + B1

OR: State $z^3 + 1 = (z+1)(z^2 - z + 1)$ B1

Justify the given statement B1

OR: Obtain $z^3 = z^2 - z$ B1

Justify the given statement B1 **2**

2 **(a) (i)** *EITHER*: Carry out multiplication of numerator and denominator by 1 + 2i, or equivalent — M1

Obtain answer 2 + i, or any equivalent of the form $(a + ib)/c$ — A1

*OR*1: Obtain two equations in x and y, and solve for x or for y — M1

Obtain answer 2 + i, or equivalent — A1

*OR*2: Using the correct processes express z in polar form — M1

Obtain answer 2 + i, or equivalent — A1 **[2]**

(ii) State that the modulus of z is $\sqrt{5}$ or 2.24 — B1

State that the argument of z is 0.464 or 26.6° — B1 **[2]**

(b) *EITHER*: Square $x + iy$ and equate real and imaginary parts to 5 and −12 respectively — M1

Obtain $x^2 - y^2 = 5$ and $2xy = -12$ — A1

Eliminate one variable and obtain an equation in the other — M1

Obtain $x^4 - 5x^2 - 36 = 0$ or $y^4 + 5y^2 - 36 = 0$, or 3-term equivalent — A1

Obtain answer 3 –2i — A1

Obtain second answer –3 + 2i and no others — A1

[SR: Allow a solution with $2xy = 12$ to earn the second A1 and thus a maximum of 3/6.]

OR: Convert 5 –12i to polar form (R, θ) — M1

Use the fact that a square root has the polar form $(\sqrt{R}, \frac{1}{2}\theta)$ — M1

Obtain one root in polar form, e.g. $(\sqrt{13}, -0.588)$ or $(\sqrt{13}, -33.7°)$ — A1 + A1

Obtain answer 3 –2i — A1

Obtain answer –3 + 2i and no others — A1 **[6]**