TYPE 1 : EASY WITHOUT DIAGRAM.

1 **(i)** Find the roots of the equation $z^2 - z + 1 = 0$, giving your answers in the form $x + iy$, where x and y are real. [2]

(ii) Obtain the modulus and argument of each root. [3]

(iii) Show that each root also satisfies the equation $z^3 = -1$. [2]

9709/03/M/J/04/Q8

(i) $z^2 - z + 1 = 0$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$z = \frac{1 \pm \sqrt{-3}}{2}$$

$$z = \frac{1 \pm \sqrt{-1 \times 3}}{2} = \frac{1 \pm \sqrt{-1}\sqrt{3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i \quad , \quad z = \frac{1}{2} - \frac{\sqrt{3}}{2} i$$

(ii) $z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$$R = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

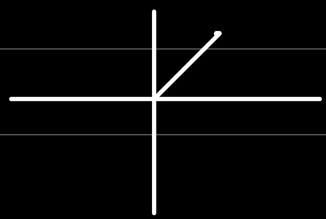
$R = 1$

$z = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

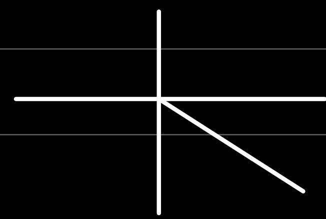
$$R = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$$

$R = 1$

ARGUMENT



$$\alpha = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = 60$$



$$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\arg(\theta) = \alpha = 60$$

$$z = \frac{1}{2} - \frac{\sqrt{3}}{2} i$$

$$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\arg(\theta) = -\alpha = -60$$

(iii) $z^3 = -1$

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$\boxed{(\sqrt{3})^3 = 3^{3/2} = 3^{1+\frac{1}{2}} = 3^1 \cdot 3^{\frac{1}{2}} = 3\sqrt{3}}$$

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^3 = \left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^2\left(\frac{\sqrt{3}i}{2}\right)^1 + {}^3C_2\left(\frac{1}{2}\right)^1\left(\frac{\sqrt{3}i}{2}\right)^2 + {}^3C_3\left(\frac{1}{2}\right)^0\left(\frac{\sqrt{3}i}{2}\right)^3$$

$$= \frac{1}{8} + (3)\left(\frac{1}{4}\right)\left(\frac{\sqrt{3}i}{2}\right) + (3)\left(\frac{1}{2}\right)\left(\frac{3i^2}{4}\right) + (1)(1)\left(\frac{3\sqrt{3}i^3}{8}\right)$$

$$= \frac{1}{8} + \frac{3\sqrt{3}}{8} i + \frac{9}{8} i^2 + \frac{3\sqrt{3}}{8} i^2 \cdot i$$

$$= \frac{1}{8} + \frac{3\sqrt{3}}{8} i + \frac{9}{8}(-1) + \frac{3\sqrt{3}}{8}(-1) i$$

$$= \frac{1}{8} - \frac{9}{8} + \cancel{\frac{3\sqrt{3}}{8}} i - \cancel{\frac{3\sqrt{3}}{8}} i$$

$$z^3 = -1$$

Let's do the second one using exp form.

$z = \frac{1}{2} - \frac{\sqrt{3}}{2} i \qquad R = 1 \ , \quad \alpha = -\frac{\pi}{3}$

$z = 1 e^{i\left(-\frac{\pi}{3}\right)}$

$z^3 = \left(1 e^{i\left(-\frac{\pi}{3}\right)}\right)^3 = 1 e^{i(-\pi)}$

Now use polar form to go back.

$z^3 = 1\left(\cos(-\pi) + i \sin(-\pi)\right)$

$z^3 = 1\left(-1 + i(0)\right)$

$z^3 = -1$

2 **(a)** The complex number z is given by $z = \frac{4 - 3i}{1 - 2i}$.

(i) Express z in the form $x + iy$, where x and y are real. [2]

(ii) Find the modulus and argument of z. [2]

(b) Find the two square roots of the complex number $5 - 12i$, giving your answers in the form $x + iy$, where x and y are real. [6]

9709/03/O/N/07/Q8

(i) $z = \frac{4-3i}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{4 + 8i - 3i - 6i^2}{(1+2i)(1-2i)}$

$= \frac{4 + 5i - 6i^2}{(1)^2 - (2i)^2}$

$$= \frac{4 + 5i - 6(-1)}{1 - 4i^2}$$

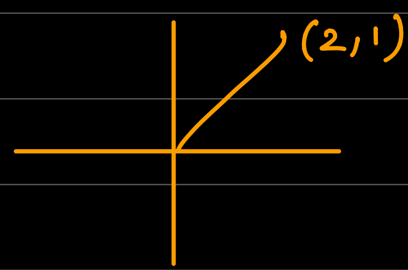
$$= \frac{10 + 5i}{1 - 4(-1)}$$

$$= \frac{10 + 5i}{5}$$

$$\boxed{z = 2 + i} \quad (2, 1)$$

Modulus $= R = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

Argument: $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$



$\alpha = 0.4636$ rad.

$Arg(\theta) = \alpha = 0.4636$.

SQUARE ROOTS OF $5 - 12i$ in form $x + iy$.

$$\sqrt{5 - 12i} = x + iy$$

$$5 - 12i = (x + iy)^2$$

$$5 - 12i = x^2 + 2xyi + i^2y^2$$

$$5 - 12i = x^2 - y^2 + 2xyi$$

$x^2 - y^2 = 5$

$2xyi = -12i$

$y = -6$

$$x^2-\left(\frac{-6}{x}\right)^2=5$$

$$x^2-\frac{36}{x^2}=5$$

$$\boxed{x^2=a}$$

$$a-\frac{36}{a}=5$$

$$a^2-36=5a$$

$$a^2-5a-36=0$$

$$a^2-9a+4a-36=0$$

$$a(a-9)+4(a-9)=0$$

$$a=9,-4.$$

$x^2=9$, $x^2=-4$

$x=3$, $x=-3$ No solutions

$y=\frac{-6}{3}$, $y=\frac{-6}{-3}$ x & y must be real.

$y=-2$ $y=2$

$$\text{Root}=x+iy=3+i(-2)=3-2i$$

$$-3+i(2)=-3+2i$$

1 The variable complex number z is given by

$$z = 1 + \cos 2\theta + \mathrm{i}\sin 2\theta,$$

where θ takes all values in the interval $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. (first or fourth Quadrant)

(i) Show that the modulus of z is $2\cos\theta$ and the argument of z is θ. [6]

(ii) Prove that the real part of $\frac{1}{z}$ is constant. [3]

9709/32/M/J/10/Q8

$$z = \underbrace{1 + \cos 2\theta}_{\text{Real}} + \underbrace{i \sin 2\theta}_{\text{Imag.}}$$

Modulus: $\sqrt{(1+\cos 2\theta)^2 + (\sin 2\theta)^2}$

$\sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$

$\sqrt{1 + 2\cos 2\theta + 1}$

$\sqrt{2 + 2\cos 2\theta}$

$\sqrt{2(1 + \cos 2\theta)}$

$\sqrt{2(1 + 2\cos^2\theta - 1)}$

$\sqrt{2(2\cos^2\theta)}$

$\sqrt{4\cos^2\theta}$

$2\cos\theta$ (shown).

Argument : $\alpha = \tan^{-1}\left(\dfrac{\sin 2\theta}{\underbrace{1+\cos 2\theta}_{w1}}\right)$

(w1) $\dfrac{\sin 2\theta}{1+\cos 2\theta}$

$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$

$\dfrac{\cancel{2}\sin\theta\,\cancel{\cos\theta}}{\cancel{2}\cos^{\cancel{2}}\theta}$

$\tan\theta$

$\alpha = \tan^{-1}(\tan\theta)$

$\alpha = \theta$

$\arg(\theta) = \alpha = \theta.$

(ii) $\dfrac{1}{z} = \dfrac{1}{1+\cos 2\theta + i\sin 2\theta} \times \dfrac{1+\cos 2\theta - i\sin 2\theta}{1+\cos 2\theta - i\sin 2\theta}$

$= \dfrac{1+\cos 2\theta - i\sin 2\theta}{(\underline{1+\cos 2\theta} + \underline{i\sin 2\theta})(\underline{1+\cos 2\theta} - \underline{i\sin 2\theta})}$

$= \dfrac{1+\cos 2\theta - i\sin 2\theta}{(1+\cos 2\theta)^2 - (i\sin 2\theta)^2}$

Real

$\dfrac{1+\cos 2\theta}{(1+\cos 2\theta)^2 - (i\sin 2\theta)^2} - \dfrac{i\sin 2\theta}{(1+\cos 2\theta)^2 - (i\sin 2\theta)^2}$

Now work with real part only.

$$\frac{1+\cos 2\theta}{1+2\cos 2\theta+\cos^2 2\theta - i^2\sin^2 2\theta}$$

$$\frac{1+\cos 2\theta}{1+2\cos 2\theta+(\cos^2 2\theta+\sin^2 2\theta)}$$

$$\frac{1+\cos 2\theta}{2+2\cos 2\theta}$$

$$\frac{\cancel{1+\cos 2\theta}}{2(\cancel{1+\cos 2\theta})}$$

$\frac{1}{2}$ Shown that it is Constant.

3 (i) Find the roots of the equation

$$z^2+(2\sqrt{3})z+4=0,$$

giving your answers in the form $x+iy$, where x and y are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation

$$z^6=-64.$$ [3]

9709/33/M/J/11/Q7

(i) $$z = \frac{-(2\sqrt{3}) \pm \sqrt{(2\sqrt{3})^2-4(1)(4)}}{2(1)}$$

$$= \frac{-2\sqrt{3}\pm\sqrt{-4}}{2}$$

$\sqrt{-4} = \sqrt{-1}\times\sqrt{4}$

$= i(2)$

$= 2i$

$$= \frac{-2\sqrt{3} \pm 2i}{2}$$

$$= -\sqrt{3} \pm i$$

Roots : $-\sqrt{3} + i$ and $-\sqrt{3} - i$

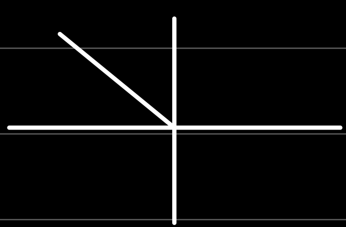
Modulus $= \sqrt{(-\sqrt{3})^2 + (1)^2}$

$= 2$

Argument: $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha = \frac{\pi}{6}$

$z = -\sqrt{3} + i \qquad (-\sqrt{3}, 1)$



$\arg(\theta) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

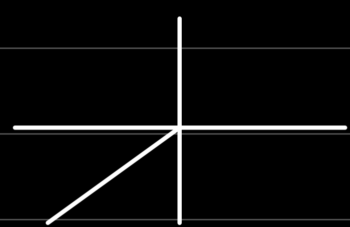
Modulus $= \sqrt{(-\sqrt{3})^2 + (-1)^2}$

$= 2$

Argument: $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha = \frac{\pi}{6}$

$z = -\sqrt{3} - i \qquad (-\sqrt{3}, -1)$



$\arg(\theta) = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}$

(ii) For z^6, take to EXP FORM.

$z = -\sqrt{3} + i$

$R = 2$

$\theta = \frac{5\pi}{6}$

$z = 2e^{\frac{5\pi}{6}i}$

$z = -\sqrt{3} - i$

$R = 2$

$\theta = -\frac{5\pi}{6}$

$z = 2e^{-\frac{5\pi}{6}i}$

$$z^6 = \left(2e^{\frac{5\pi}{6}i}\right)^6$$
$$= 64e^{5\pi i}$$

$$z^6 = \left(2e^{-\frac{5\pi}{6}i}\right)^6$$
$$z^6 = 64e^{-5\pi i}$$

$$z^6 = 64(\cos 5\pi + i\sin 5\pi)$$
$$= 64(-1 + i(0))$$
$$z^6 = -64$$

$$z^6 = 64(\cos(-5\pi) + i\sin(-5\pi))$$
$$z^6 = 64(-1 + i(0))$$
$$z^6 = -64.$$

4 **Throughout this question the use of a calculator is not permitted.**

The complex number u is defined by

$$u = \frac{1+2i}{1-3i}.$$

(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Show on a sketch of an Argand diagram the points A, B and C representing the complex numbers u, $1 + 2i$ and $1 - 3i$ respectively. [2]

(iii) By considering the arguments of $1 + 2i$ and $1 - 3i$, show that

$$\tan^{-1}2 + \tan^{-1}3 = \tfrac{3}{4}\pi.$$ [3]

9709/32/M/J/12/Q7

(i) $$u = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$\frac{1+3i+2i+6i^2}{(1-3i)(1+3i)}$$

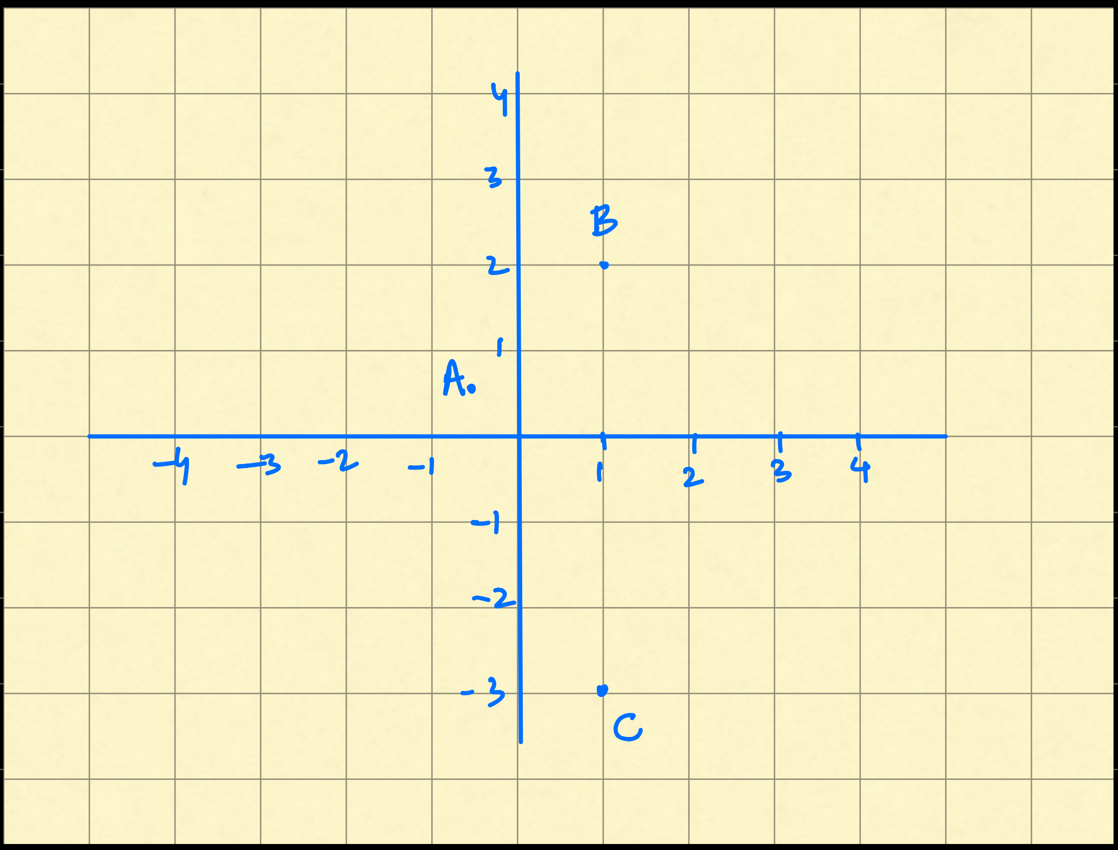
$$\frac{1+5i+6(-1)}{(1)^2-(3i)^2}$$

$$\frac{-5+5i}{1-9i^2}$$

$$\frac{-5+5i}{1+9}$$

$$\frac{-5+5i}{10}$$

$$u = -\frac{1}{2} + \frac{1}{2}i$$



$$u = \frac{1+2i}{1-3i}$$

$$\arg(u) = \arg(1+2i) - \arg(1-3i)$$

$u = \frac{-1}{2} + \frac{1}{2}i$	$1+2i$	$1-3i$
$\alpha = \tan^{-1}\left(\frac{1/2}{1/2}\right)$	$\alpha = \tan^{-1}\left(\frac{2}{1}\right)$	$\alpha = \tan^{-1}\left(\frac{3}{1}\right)$
$\alpha = \frac{\pi}{4}$	$\text{Arg} = \alpha = \tan^{-1}(2)$	$\text{Arg} = -\alpha = -\tan^{-1}(3)$
$\text{Arg} = \pi - \frac{\pi}{4}$		

$$\text{Arg} = \frac{3\pi}{4}$$

$$\arg(u) = \arg(1+2i) - \arg(1-3i)$$

$$\frac{3\pi}{4} = \tan^{-1}(2) - (-\tan^{-1}3)$$

$$\boxed{\frac{3\pi}{4} = \tan^{-1}2 + \tan^{-1}3}$$

5 The complex number z is defined by $z = a + ib$, where a and b are real. The complex conjugate of z is denoted by z^*.

(i) Show that $|z|^2 = zz^*$ and that $(z - ki)^* = z^* + ki$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10i| = 2|z - 4i|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2iz^* + 2iz - 12 = 0.$$

Hence show that $|z - 2i| = 4$. [5]

(iii) Describe the set of points geometrically. = circle radius = 4 centre (0, 2) [1]

9709/33/M/J/13/Q7

$z = a + bi$ $\qquad z^* = a - bi$

$|z|^2$ $\qquad zz^* = (a+bi)(a-bi)$

Modulus $= |z| = \sqrt{a^2+b^2}$ $\qquad = a^2 - (bi)^2$

$\qquad = a^2 - b^2 i^2$

$\boxed{|z|^2 = a^2+b^2}$ $\qquad \boxed{zz^* = a^2+b^2}$

Hence $|z|^2 = zz^*$

$z = a + bi$ $\qquad z^* = a - bi$

$(z - ki)^* = z^* + ki$

$(a + \underline{bi} - \underline{ki})^* = a - bi + ki$

$a - bi + ki = a - bi + ki$ (shown).

$|z - 10i| = 2|z - 4i|$ $\qquad |z|^2 = zz^*$

SQUARE BOTH SIDES $\qquad |z-10i|^2 = (z-10i)(z-10i)^*$

$|z - 10i|^2 = 4|z-4i|^2$ $\qquad |z-4i|^2 = (z-4i)(z-4i)^*$

$(z-10i)(z-10i)^* = 4(z-4i)(z-4i)^*$

$(z-10i)(z^*+10i) = 4(z-4i)(z^*+4i)$ $\qquad (z-ki)^* = z^* + ki$

$zz^* + 10iz - 10iz^* - 100i^2 = 4[zz^* + 4iz - 4iz^* - 16i^2]$ $\qquad (z-10i)^* = z^* + 10i$

$zz^* + 10iz - 10iz^* + 100 = 4zz^* + 16iz - 16iz^* + 64$ $\qquad (z-4i)^* = z^* + 4i$

$0 = 3zz^* + 6iz - 6iz^* - 36$

$3(zz^* + 2iz - 2iz^* - 12) = 0$

$zz^* + 2iz - 2iz^* - 12 = 0$

$zz^* + 2iz - 2iz^* - 12 = 0 \longrightarrow |z-2i| = 4$

square both.

$|z-2i|^2 = 16$ $\qquad |z|^2 = zz^*$

$(z-2i)(z-2i)^* = 16$ $\qquad |z-2i|^2 = (z-2i)(z-2i)^*$

$(z-2i)(z^*+2i) = 16$

$zz^* - 2iz^* + 2iz - 4i^2 = 16$ $\qquad (z-ki)^* = z^* + ki$

$zz^* - 2iz^* + 2iz + 4 = 16$ $\qquad (z-2i)^* = z^* + 2i$

$zz^* - 2iz^* + 2iz - 12 = 0$

Otherwise: $zz^* + 2iz - 2iz^* - 12 = 0$

$zz^* + 2iz - 2iz^* + 4 = 12 + 4$

$zz^* + 2iz - 2iz^* - 4i^2 = 16$

$z(z^* + 2i) - 2i(z^* + 2i) = 16$

$(z-2i)(z^*+2i) = 16$

$$|z-2i|^2 = 16$$

$$|z-2i| = 4.$$

7 (a) It is given that $-1 + (\sqrt{5})i$ is a root of the equation $z^3 + 2z + a = 0$, where a is real. Showing your working, find the value of a, and write down the other complex root of this equation. [4]

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\frac{w-1}{w+1} = i\tan\theta$. [4]

9709/32/M/J/14/Q7

ROOTS ALWAYS LIVE AS CONJUGATE PAIRS.

One root $= -1+\sqrt{5}i$

other root $= -1-\sqrt{5}i$

$$z^3 + 2z + a = 0$$

$$(-1+\sqrt{5}i)^3 + 2(-1+\sqrt{5}i) + a = 0$$

$$(-1+\sqrt{5}i)((-1+\sqrt{5}i)^2 + 2) + a = 0$$

$$(-1+\sqrt{5}i)(1 + 5i^2 - 2(1)(\sqrt{5}i) + 2) + a = 0$$

$$(-1+\sqrt{5}i)(-2-2\sqrt{5}i) + a = 0$$

$$2 + \cancel{2\sqrt{5}i} - \cancel{2\sqrt{5}i} - 2(5)i^2 + a = 0$$

$$2 - 10(-1) + a = 0$$

$$12 + a = 0$$

$$a = -12$$

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\frac{w-1}{w+1} = i\tan\theta$. [4]

9709/32/M/J/14/Q7

$$z = R(\cos\theta + i\sin\theta)$$

$$w = 1(\cos 2\theta + i\sin 2\theta)$$

$$w = \cos 2\theta + i\sin 2\theta$$

$$\frac{\omega-1}{\omega+1} = \frac{\cos 2\theta + i\sin 2\theta - 1}{\cos 2\theta + i\sin 2\theta + 1}$$

$$= \frac{\cos 2\theta - 1 + i\sin 2\theta}{\cos 2\theta + 1 + i\sin 2\theta} \times \frac{\cos 2\theta + 1 - i\sin 2\theta}{\cos 2\theta + 1 - i\sin 2\theta}$$

$$= \frac{(\cos 2\theta - 1 + i\sin 2\theta)(\cos 2\theta + 1 - i\sin 2\theta)}{(\cos 2\theta + 1 + i\sin 2\theta)(\cos 2\theta + 1 - i\sin 2\theta)}$$

$$= \frac{\cos^2 2\theta + \cancel{\cos 2\theta} - \cancel{i\sin 2\theta\cos 2\theta} - \cancel{\cos 2\theta} - 1 + i\sin 2\theta + \cancel{i\sin 2\theta\cos 2\theta} + i\sin 2\theta - i^2\sin^2 2\theta}{(\cos 2\theta + 1)^2 - (i\sin 2\theta)^2}$$

$$\frac{\cancel{\cos^2 2\theta} \cancel{-1} + 2i\sin 2\theta + \cancel{\sin^2 2\theta}}{\cos^2 2\theta + 2\cos 2\theta + 1 + \sin^2 2\theta}$$

$$\frac{2i\sin 2\theta}{2 + 2\cos 2\theta}$$

$$\frac{i\sin 2\theta}{1 + \cos 2\theta}$$

$$\frac{i(2\sin\theta\cos\theta)}{1 + 2\cos^2\theta - 1}$$

$$\frac{i(\cancel{2}\sin\theta\,\cancel{\cos\theta})}{\cancel{2}\cos^{\cancel{2}}\theta}$$

$$i\tan\theta.$$

2 **(a)** The complex number u is defined by $u = \dfrac{5}{a+2\mathrm{i}}$, where the constant a is real.

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) Find the value of a for which $\arg(u^*) = \frac{3}{4}\pi$, where u^* denotes the complex conjugate of u. [3]

$$u = \frac{5}{a+2i} \times \frac{a-2i}{a-2i}$$

$$= \frac{5(a-2i)}{(a+2i)(a-2i)}$$

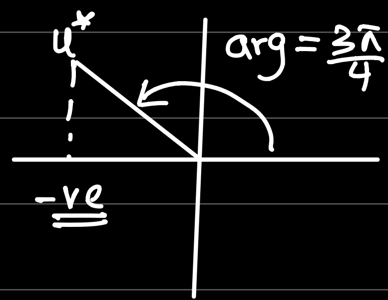
$$= \frac{5a - 10i}{a^2 - 4i^2}$$

$$= \frac{5a - 10i}{a^2+4}$$

$$u = \frac{5a}{a^2+4} - \frac{10}{a^2+4}i$$

$$u^* = \frac{\overset{5(2)}{5a}}{a^2+4} + \frac{10}{a^2+4}i \longrightarrow \arg(u^*) = \frac{3\pi}{4}$$

$$\alpha = \tan^{-1}\left(\frac{\frac{10}{a^2+4}}{\frac{5a}{a^2+4}}\right) = \tan^{-1}\left(\frac{2}{a}\right)$$



$$\arg(u^*) = \pi - \alpha$$

$$\frac{3\pi}{4} = \pi - \tan^{-1}\left(\frac{2}{a}\right)$$

$$\tan^{-1}\left(\frac{2}{a}\right) = \pi - \frac{3\pi}{}$$

$$\tan^{-1}\left(\frac{2}{a}\right) = \frac{\pi}{4}$$

$$\frac{2}{a} = \tan\left(\frac{\pi}{4}\right)$$

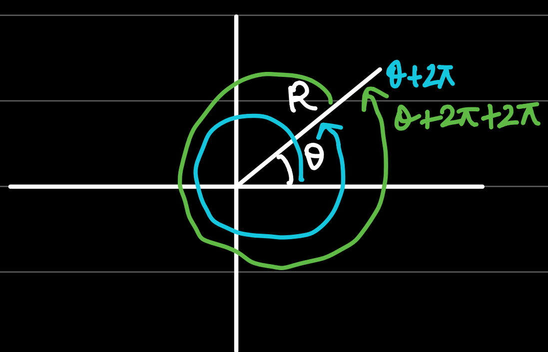
$$\frac{2}{a} = 1$$

$$a = 2$$ BUT WAIT

Since u^* is in Second quadrant

$$\frac{5a}{a^2+4} \text{ should be } -ve$$

Hence $\boxed{a = -2}$



Exp $\Rightarrow$ $z = Re^{i\theta}$

$Re^{i(\theta+2\pi)}$

$Re^{i(\theta+2\pi(2))}$

$$z = Re^{i(\theta+2\pi(k))}$$

$k = 0, 1, 2, 3 \ldots\ldots$

Now if you need n^{th} Root

$$z^{\frac{1}{n}} = \left(Re^{i(\theta+2\pi(k))}\right)^{\frac{1}{n}} = R^{\frac{1}{n}} e^{i\left(\frac{\theta+2\pi(k)}{n}\right)}$$

$K = 0, 1, \dots n$

6 The complex number z is defined by $z = \dfrac{9\sqrt{3} + 9\mathrm{i}}{\sqrt{3} - \mathrm{i}}$. Find, showing all your working,

(i) an expression for z in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, [5]

(ii) the two square roots of z, giving your answers in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

9709/31/M/J/14/Q5

$$z = \frac{9\sqrt{3} + 9i}{\sqrt{3} - i} \times \frac{\sqrt{3} + i}{\sqrt{3} + i}$$

$$= \frac{(9\sqrt{3})(\sqrt{3}) + 9\sqrt{3}i + 9\sqrt{3}i + 9i^2}{(\sqrt{3})^2 - i^2}$$

$$= \frac{27 + 18\sqrt{3}i - 9}{4}$$

$$= \frac{18 + 18\sqrt{3}i}{4}$$

$$= \frac{9}{2} + \frac{9}{2}\sqrt{3}i$$

$$\text{Modulus} = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{9}{2}\sqrt{3}\right)^2} = 9$$

$$\text{Argument: } \alpha = \tan^{-1}\left(\frac{\cancel{\frac{9}{2}}\sqrt{3}}{\cancel{\frac{9}{2}}}\right) = \frac{\pi}{3}$$

$$\text{First Quad: } \arg(\theta) = \alpha = \frac{\pi}{3}$$

$$z = 9e^{i\left(\frac{\pi}{3}\right)}$$

(ii) In Real $\quad z = 9e^{i\left(\frac{\pi}{3} + 2\pi \cdot k\right)}$

Square Root: $\quad z^{\frac{1}{2}} = 9^{\frac{1}{2}} e^{i\left(\frac{\frac{\pi}{3}+2\pi k}{2}\right)}$

$$z^{\frac{1}{2}} = 3e^{i\left(\frac{\frac{\pi}{3}+2\pi k}{2}\right)}$$

$k=0$

$$= 3e^{i\left(\frac{\frac{\pi}{3}+2\pi(0)}{2}\right)}$$

$$\boxed{z^{\frac{1}{2}} = 3e^{i\left(\frac{\pi}{6}\right)}}$$

$$z^{\frac{1}{2}} = 3e^{i\left(\frac{\frac{\pi}{3}+2\pi k}{2}\right)}$$

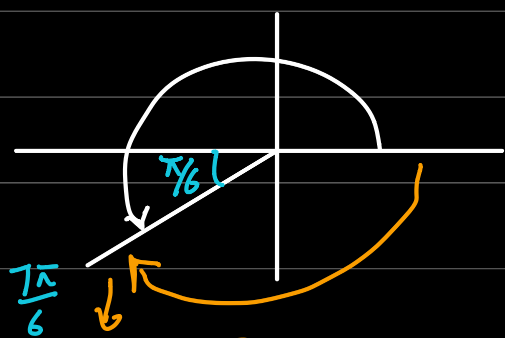
$k=1$

$$= 3e^{i\left(\frac{\frac{\pi}{3}+2\pi(1)}{2}\right)}$$

$$= 3e^{i\left(\frac{7\pi}{6}\right)}$$

Now an argument cannot be $\frac{7\pi}{6}$ since

$$-\pi < \arg < \pi$$



$\arg = -\frac{5\pi}{6}$

$$z^{\frac{1}{2}} = 3e^{-\frac{5\pi}{6}i}$$

LOCUS

FROM 1 POINT = CIRCLE

FROM 2 POINTS = PERPENDICULAR BISECTOR.

WE JUST NEED TO LEARN TO READ SYMBOLS.

$|z-(a+bi)|$ ⇒ correct form

Distance of z from (a, b)

$|z-3+2i| = |z-(3-2i)|$

Distance of z from (3, −2)

$|z+3-4i| = |z-(-3+4i)|$

Distance of z from (−3, 4)

$|z+2+5i| = |z-(-2-5i)|$

Distance of z from (−2, −5)

$|z-3i| = |z-(0+3i)|$

Distance of z from (0, 3)

$|z+2| = |z-(-2+0i)|$

Distance of z from (−2, 0)

$|z| = |z-(0+0i)|$

Distance of z from $(0,0)$

$\arg(z-(a+bi))$

angle of z from (a,b) $\longrightarrow$ (a,b) is the temporary origin.

$|z-(a+bi)|$ $\Rightarrow$ correct form

Distance of z from (a,b)

$|z-(3+2i)| = 4$

Distance of z from $(3,2)$ is 4

Locus from 1 point = Circle.

Circle : radius $= 4$

centre $= (3,2)$

$|z-(1+2i)| = |z-(3+4i)|$

Distance of z from $(1,2)$ is equal to Distance of z from $(3,4)$

REPHRASE : z is equidistant from $(1,2)$ and $(3,4)$

LOCUS FROM TWO POINTS = PERPENDICULAR BISECTOR.

FOR LOCUS:

TIP 1: USE EXACT AND SAME SCALE ON AXES.

TIP 2: ONCE THE ARGAND DIAGRAM IS MADE, USE OLEVELS TRIG/CIRCLES TO FIGURE OUT REST.

1 The complex numbers $1 + 3i$ and $4 + 2i$ are denoted by u and v respectively.

(i) Find, in the form $x + iy$, where x and y are real, the complex numbers $u - v$ and $\frac{u}{v}$. [3]

(ii) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram, with origin O, the points A, B and C represent the numbers u, v and $u - v$ respectively.

(iii) State fully the geometrical relationship between OC and BA. [2]

(iv) Prove that angle $AOB = \frac{1}{4}\pi$ radians. [2]

9709/03/O/N/04/Q6

(i) $u - v = (1+3i) - (4+2i) = -3 + i$

$$\frac{u}{v} = \frac{1+3i}{4+2i} \times \frac{4-2i}{4-2i}$$

$$\frac{4 - 2i + 12i - 6i^2}{4^2 - (2i)^2}$$

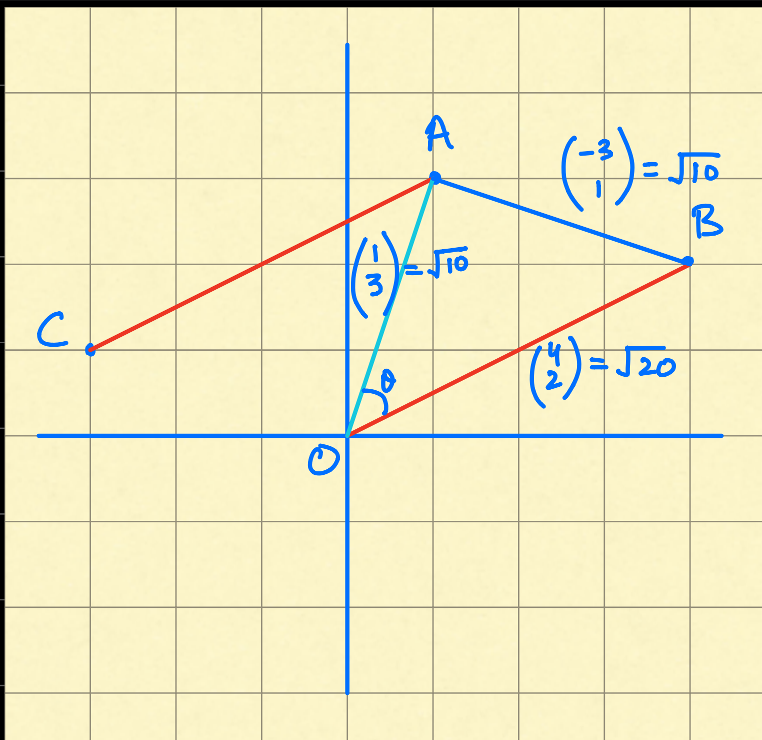
$$\frac{4 + 10i + 6}{16 - 4i^2}$$

$$\frac{10 + 10i}{20}$$

$$\frac{u}{v} = \frac{1}{2} + \frac{1}{2}i$$

(ii) argument: $\alpha = \tan^{-1}\left(\frac{1/2}{1/2}\right) = \frac{\pi}{4}$

FIRST QUADRANT $\Rightarrow \arg(\theta) = \alpha = \frac{\pi}{4}$



$u = 1+3i$ (A)

$v = 4+2i$ (B)

$u-v = -3+i$ (C)

OC & BA are equal and parallel. (PARLLELOGRAM)

COS RULE:

$$\cos\theta = \frac{(\sqrt{20})^2 + (\sqrt{10})^2 - (\sqrt{10})^2}{2\sqrt{20}\sqrt{10}}$$

$$\cos\theta = \frac{20}{2\sqrt{200}}$$

$$\cos\theta = \frac{20}{2\sqrt{2}\sqrt{100}}$$

$$\cos\theta = \frac{\cancel{20}}{\cancel{2}\sqrt{2}(\cancel{10})}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

3 The equation $2x^3 + x^2 + 25 = 0$ has one real root and two complex roots.

(i) Verify that $1 + 2i$ is one of the complex roots. [3]

(ii) Write down the other complex root of the equation. [1]

(iii) Sketch an Argand diagram showing the point representing the complex number $1 + 2i$. Show on the same diagram the set of points representing the complex numbers z which satisfy

$$|z| = |z - 1 - 2i|.$$ [4]

9709/03/O/N/05/Q7

(i) $2(1+2i)^3 + (1+2i)^2 + 25 = 0$

$(1+2i)^2[2(1+2i)+1] + 25 = 0$

$(1+4i+4i^2)(2+4i+1) + 25 = 0$

$(4i-3)(4i+3) + 25 = 0$

$(4i)^2 - (3)^2 + 25 = 0$

$16i^2 - 9 + 25 = 0$

$-16 - 9 + 25 = 0$

$0 = 0$ proven.

one complex root $= 1+2i$

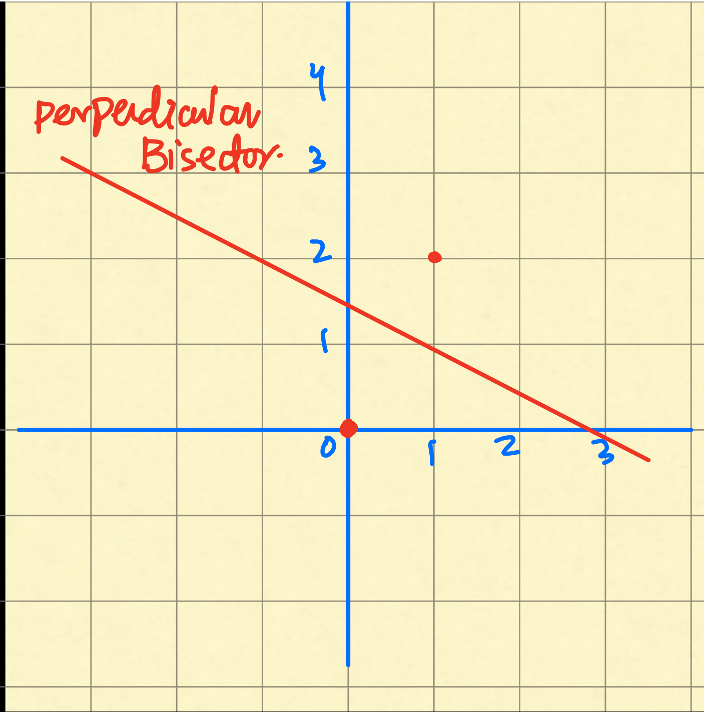
other complex root $= 1-2i$

(iii) $|z| = |z-1-2i|$

$|z-(0+0i)| = |z-(1+2i)|$

Distance of z from $(0,0)$ is equal to distance of z from $(1,2)$

Locus from two points = perpendicular Bisector.



5 The complex number $\frac{2}{-1+\mathrm{i}}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z - u^2| < |z - u|$. [4]

9709/03/M/J/07/Q8

(i) $u = \dfrac{2}{-1+i} \times \dfrac{-1-i}{-1-i}$

$= \dfrac{-2-2i}{(-1)^2 - i^2}$

$= \dfrac{-2-2i}{1+1}$

$u = -1-i$

Modulus $= \sqrt{(-1)^2+(-1)^2} = \sqrt{2}$

arg $\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

3rd Quad: arg $= \alpha - \pi = \frac{\pi}{4} - \pi = -3\pi$

$u^2 = (-1-i)^2$

$= [-(1+i)]^2$

$= +(1+2i+i^2)$

$= 1+2i-1$

$u^2 = 2i = 0+2i$

Modulus $= \sqrt{0^2+2^2}$

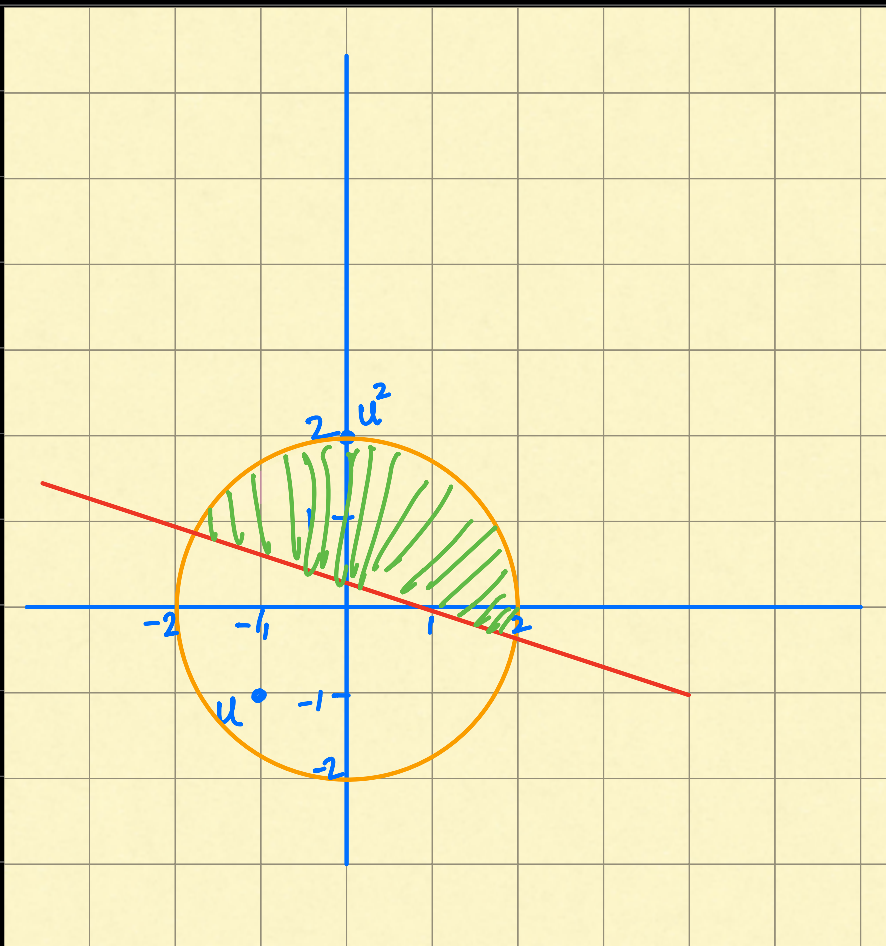
$= 2$

arg: $\alpha = \tan^{-1}\left(\frac{2}{0}\right) = \frac{\pi}{2}$

(0,2)

4 4

$\arg = \frac{\pi}{2}$



$|z| < 2$

$|z-(0+0i)| < 2$

Distance of z from $(0,0)$ is less than 2.

Locus from 1 point = circle

shade inside circle.

$|z-u^2| < |z-u|$

$|z-(0+2i)| < |z-(-1-i)|$

Distance of z from $(0,2)$ is less than distance of z from $(-1,-1)$.

10 **Throughout this question the use of a calculator is not permitted.**

(a) Solve the equation $(1+2i)w^2 + 4w - (1-2i) = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]

$c = -(1-2i)$

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z-1-i| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

9709/31/O/N/16/Q9

$$w = \frac{-4 \pm \sqrt{(4)^2 - 4(1+2i)(1-2i)}}{2(1+2i)}$$

$$w = \frac{-4 \pm \sqrt{16 - 4[(1)^2 - (2i)^2]}}{2(1+2i)}$$

$$w = \frac{-4 \pm \sqrt{\cancel{16} - 4 - \cancel{16}}}{2(1+2i)}$$

$$w = -4 \pm \sqrt{-1 \times 4}$$

Blunder!

$$\frac{\quad}{2(1+2i)}$$

$$\frac{-4 \pm 2i}{2(1+2i)}$$

$$w = \frac{-2 \pm i}{1+2i}$$

$$w = \frac{-2+i}{1+2i} \quad \text{or} \quad w = \frac{-2-i}{1+2i}$$

Now Rationalize.

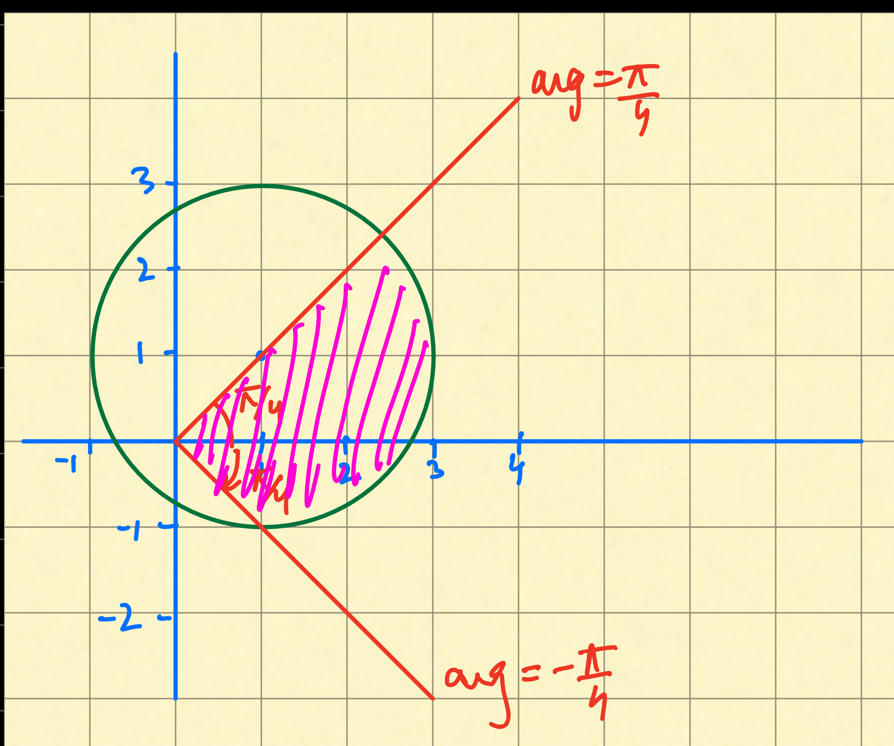
(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - i| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

9709/31/O/N/16/Q9

$$|z - 1 - i| \leqslant 2 \qquad -\frac{\pi}{4} \leq \arg z < \frac{\pi}{4}$$

$$|z - (1+i)| \leqslant 2$$

$$-\frac{\pi}{4} \leq \arg(z - (0+0i)) \leq \frac{\pi}{4}$$



11 **(a)** The complex number u is given by $u = 8 - 15\mathrm{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \mathrm{i}b$, where the numbers a and b are real and exact. [5]

(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

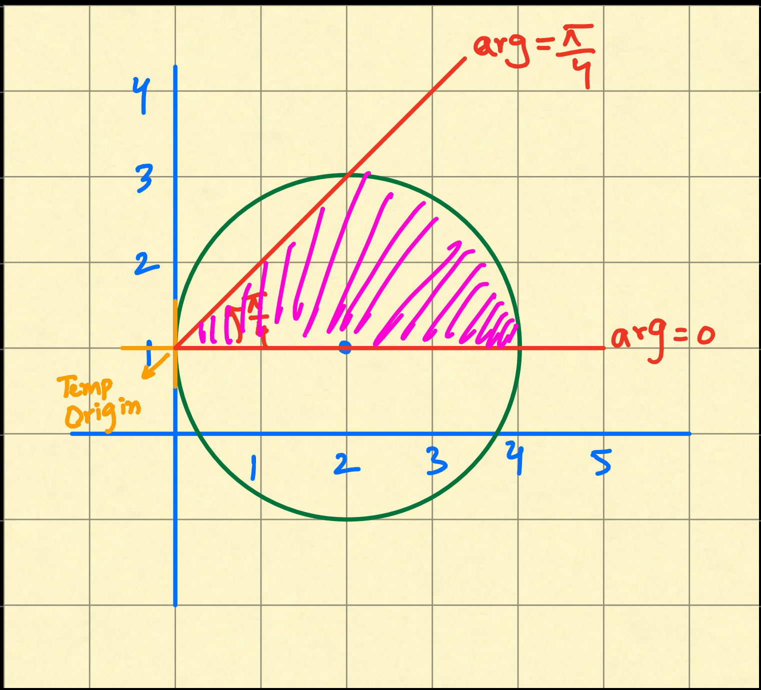
9709/31/O/N/17/Q7

(b) $|z-2-i| \leq 2$ $\qquad$ $0 < \arg(z-i) \leq \frac{\pi}{4}$

$|z-(2+i)| \leq 2$

$0 < \arg[z-(0+i)] \leq \frac{\pi}{4}$

$\cdot(0,1)$ is temp. origin.



12 **Throughout this question the use of a calculator is not permitted.**

The complex number $1 - (\sqrt{3})\mathrm{i}$ is denoted by u.

$z = a + bi$

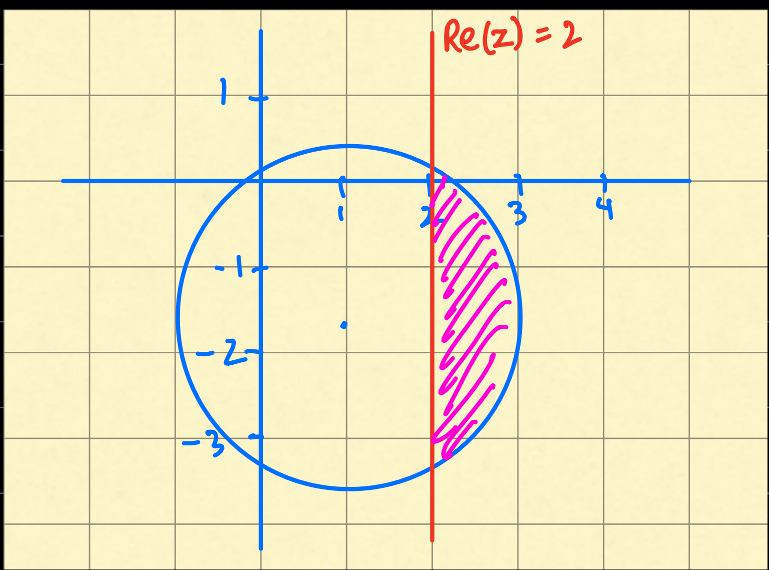
$Re(z) = a, \; Im(z) = b$

(i) Find the modulus and argument of u. [2]

(ii) Show that $u^3 + 8 = 0$. [2]

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leqslant 2$ and $\mathrm{Re}\, z \geqslant 2$, where $\mathrm{Re}\, z$ denotes the real part of z. [4]

9709/32/O/N/17/Q7



$|z-u| \leq 2$

$|z-(1-\sqrt{3}i)| \leq 2$

TYPE 4 : WITH DIAGRAM HARD .

3 The complex number w is given by $w = -\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$.

(i) Find the modulus and argument of w. [2]

(ii) The complex number z has modulus R and argument θ, where $-\frac{1}{3}\pi < \theta < \frac{1}{3}\pi$. State the modulus and argument of wz and the modulus and argument of $\frac{z}{w}$. [4]

(iii) Hence explain why, in an Argand diagram, the points representing z, wz and $\frac{z}{w}$ are the vertices of an equilateral triangle. [2]

(iv) In an Argand diagram, the vertices of an equilateral triangle lie on a circle with centre at the origin. One of the vertices represents the complex number $4 + 2\mathrm{i}$. Find the complex numbers represented by the other two vertices. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [4]

9709/03/O/N/08/Q10

(i) Modulus : $\sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

Argument $\Rightarrow$ $\alpha = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \frac{\pi}{3}$

Second Quadrant $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\boxed{z}$ Modulus $= R$

Argument $= \theta$

$\boxed{w}$ Modulus $= 1$

Argument $= \frac{2\pi}{3} = 120°$

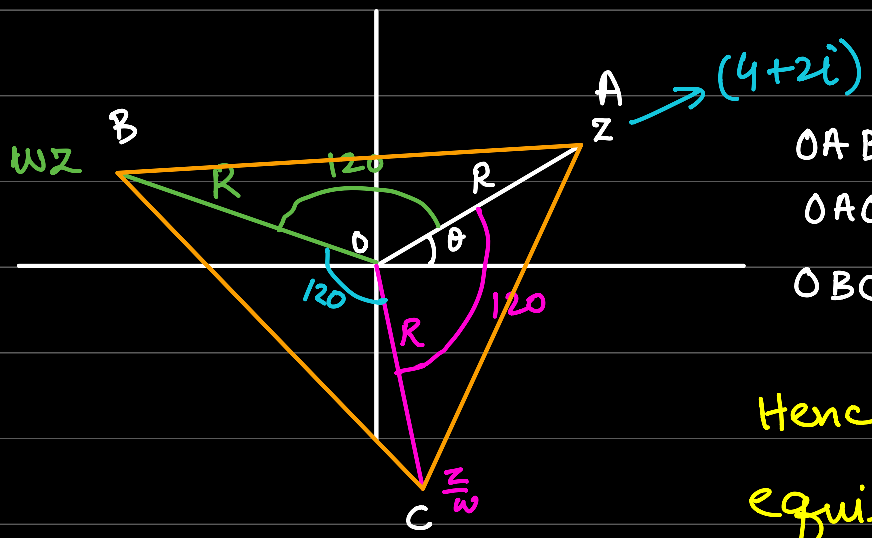
$\boxed{wz}$ Modulus $= R \times 1 = R$

Argument $= \theta + \frac{2\pi}{3} = \theta + 120°$

$\boxed{z}$ Modulus $= \frac{R}{} = R$

ω

$$\text{Argument} = \theta - \frac{2\pi}{3} = \theta - 120^\circ$$



OAB, OAC, OBC $\Big\}$ congruent triangles

SAS property.

Hence, $AB = AC = CB$

equilateral triangle.

(iv) $z = 4+2i$

Other two vertices are

$$\omega z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)(4+2i), \quad \frac{z}{\omega} = \frac{4+2i}{\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)}$$

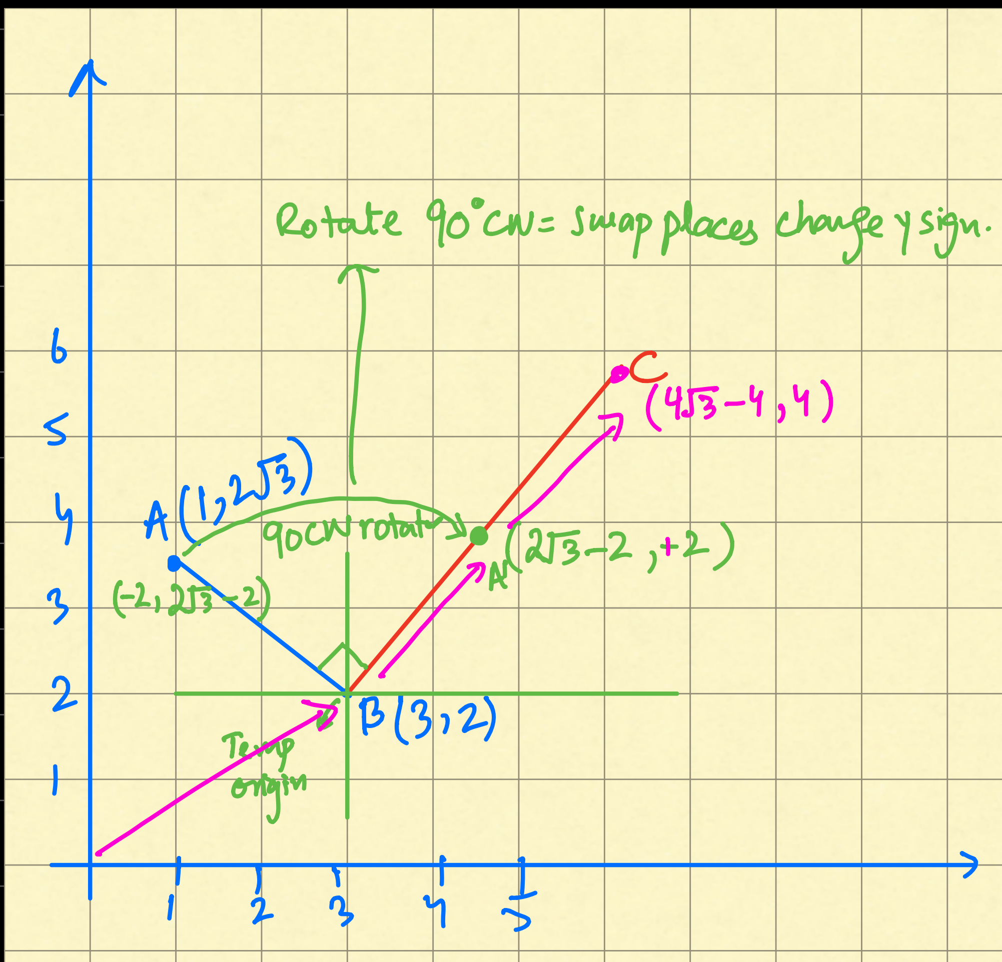
solve further for final Ans.

(b) The complex numbers u and v are given by $u = 1 + (2\sqrt{3})\mathrm{i}$ and $v = 3 + 2\mathrm{i}$. In an Argand diagram, u and v are represented by the points A and B. A third point C lies in the first quadrant and is such that $BC = 2AB$ and angle $ABC = 90°$. Find the complex number z represented by C, giving your answer in the form $x + \mathrm{i}y$, where x and y are real and exact. [4]

9709/33/M/J/17/Q11

$BC = 2AB$

$BC = 2AB$



Correct coordinates of C

$\vec{OC} = \vec{OB} + \vec{BC}$

$$\vec{OC} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 4\sqrt{3} - 4 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4\sqrt{3} - 1 \\ 6 \end{pmatrix}$$

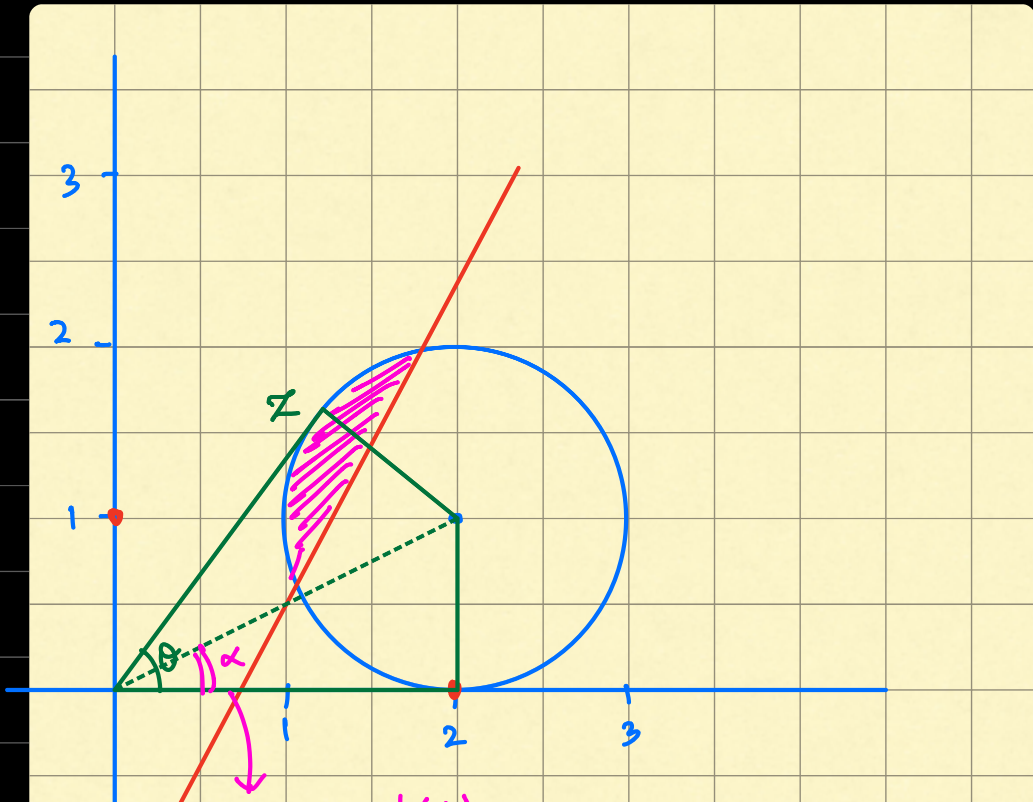
$C = (4\sqrt{3} - 1, 6)$

15 **(a)** The complex number $\dfrac{3-5\mathrm{i}}{1+4\mathrm{i}}$ is denoted by u. Showing your working, express u in the form $x+\mathrm{i}y$, where x and y are real. [3]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z-2-\mathrm{i}| \leqslant 1$ and $|z-\mathrm{i}| \leqslant |z-2|$. [4]

(ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

9709/33/M/J/14/Q7



$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.565$

$\text{Max arg} = \theta = 2\alpha = 2(26.565) = 53.13$

$|z-2-i| \leqslant 1$

$|z-(2+i)| \leqslant 1$

$|z-i| \leqslant |z-2|$

$|z-(0+i)| < |z-(2+0i)|$

$\arg(z)$

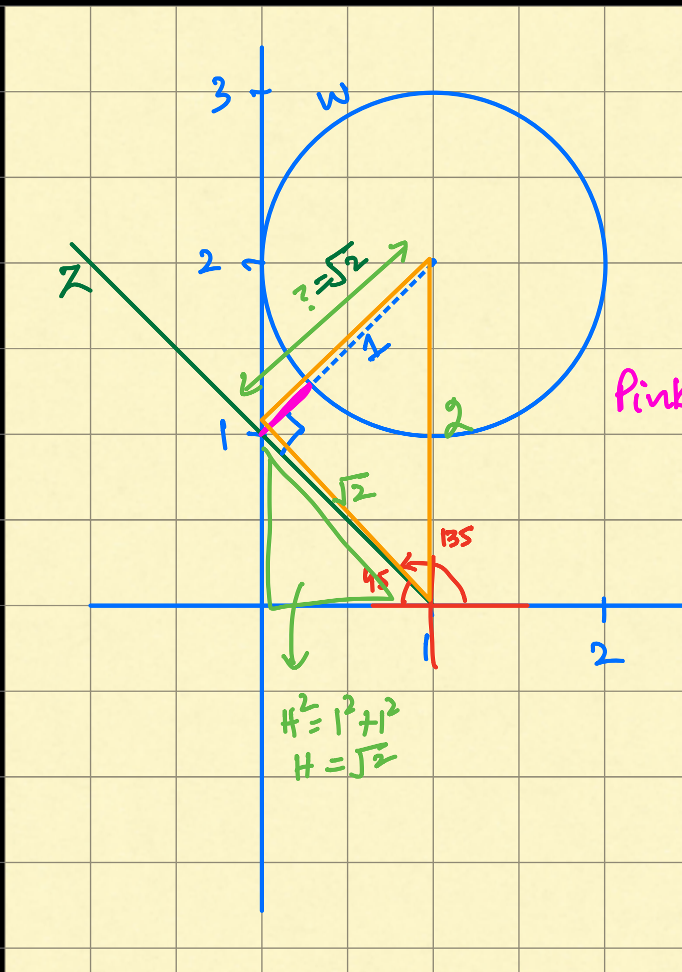
$\arg(z-(0+0i))$

21 **(a)** Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

(b) **(i)** On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2\mathrm{i}| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]

(ii) Calculate the least value of $|w - z|$ for points on these loci. [2]

9709/31/M/J/16/Q10



$x^2 + (\sqrt{2})^2 = 2^2$

$x^2 + 2 = 4$

$x^2 = 2$

$x = \sqrt{2}$

Pink length $= \sqrt{2} - 1$

$= 1.4142 - 1$

$= 0.4142.$

$|w - 1 - 2i| = 1$

$|w - (1 + 2i)| = 1$

$\arg(z - (1 + 0i)) = \frac{3\pi}{4}$

$|w - z|$

Distance of w from z

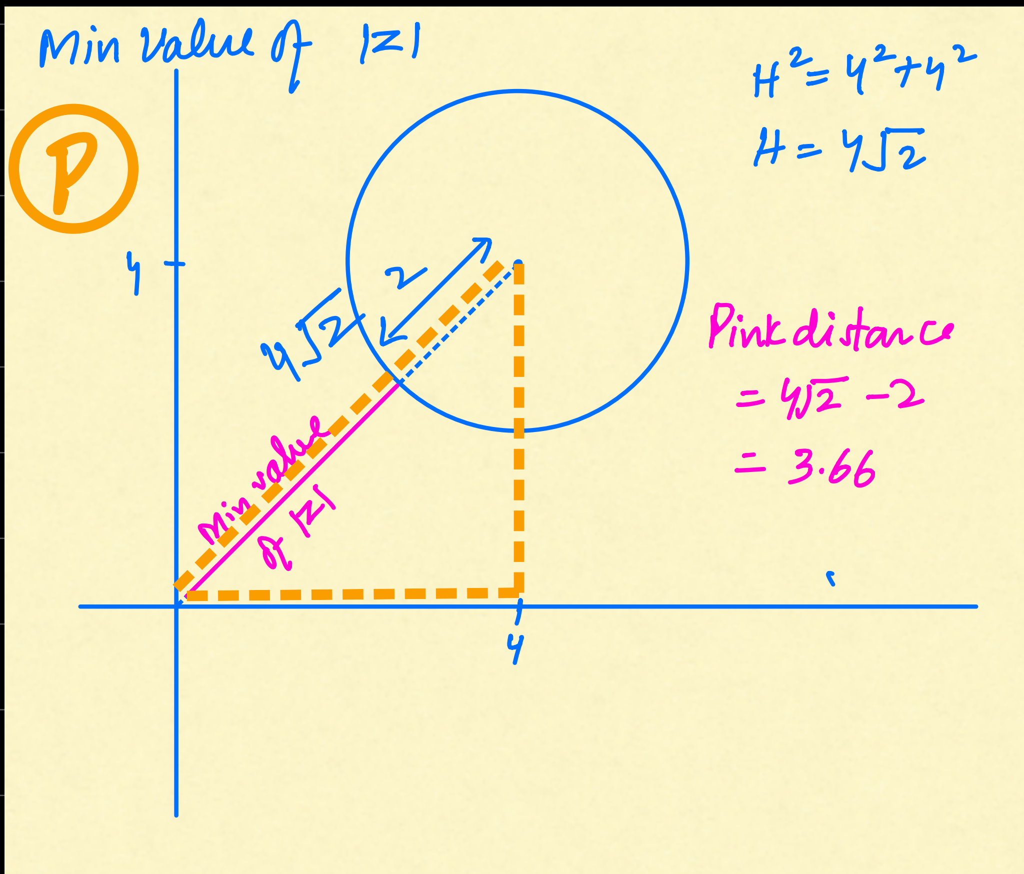
(b) **(i)** Sketch an Argand diagram showing the region R consisting of points representing the complex numbers z where

$$|z - 4 - 4\mathrm{i}| \leqslant 2. \qquad [2]$$

(ii) For the complex numbers represented by points in the region R, it is given that

$$\underbrace{p}_{\text{Min value of }|z|} \leqslant |z| \leqslant \underbrace{q}_{\text{Max value of }|z|} \quad \text{and} \quad \alpha \leqslant \arg z \leqslant \beta.$$

Find the values of p, q, α and β, giving your answers correct to 3 significant figures. [6]

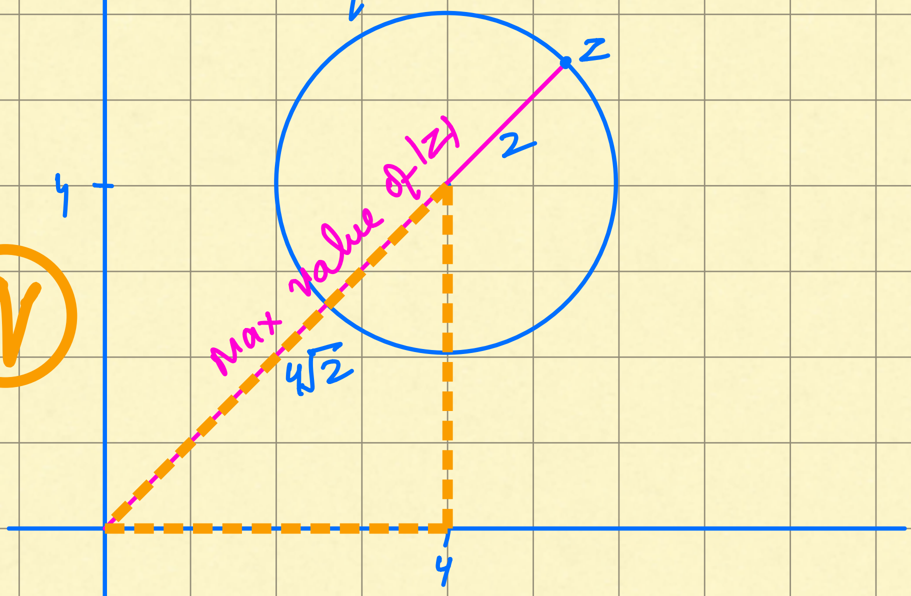


$|z - (4+4i)| = 2$

$|z| = |z - (0+0i)|$

(a)

Max value of $|z| = |z-(0+0i)|$

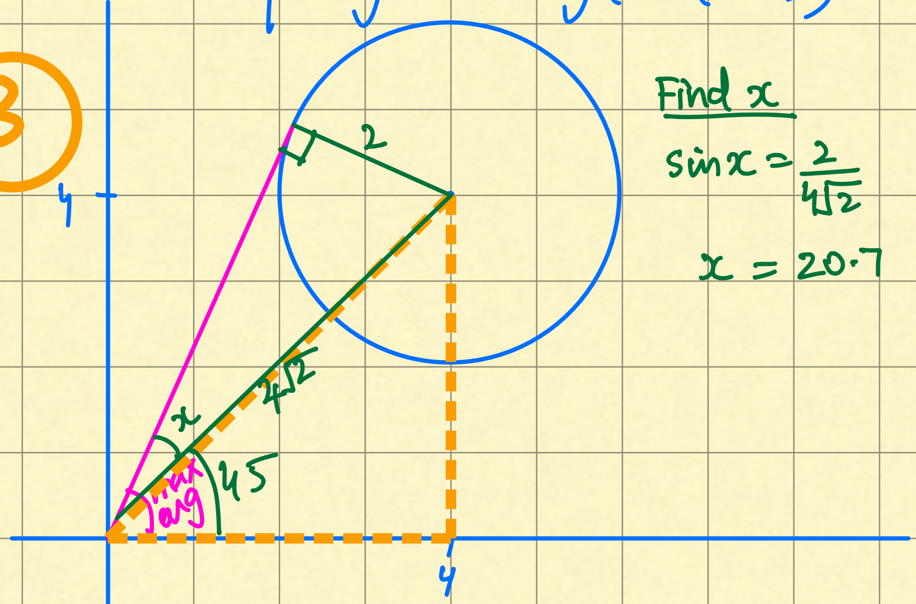


Max value of $|z| = 4\sqrt{2}+2$

$= 7.66$

(B)

max value of $\arg(z) = \arg(z-(0+0i))$



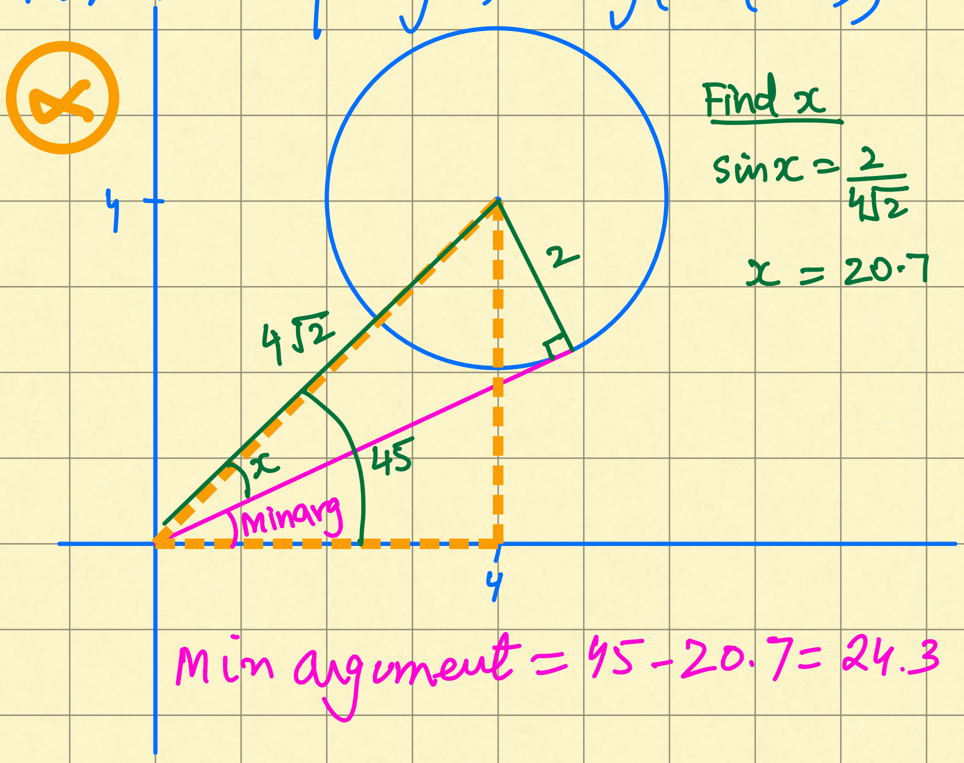
Find x

$\sin x = \frac{2}{4\sqrt{2}}$

$x = 20.7$

Max argument $= 20.7 + 45 = 65.7$

max value of $\arg(z) = \arg(z-(0+0i))$



Find x

$\sin x = \frac{2}{4\sqrt{2}}$

$x = 20.7$

Min argument $= 45 - 20.7 = 24.3$

Tomm:

→ Complex + Polynomials

→ Differential EQ

→ INTEGRATION

→ INEQUALITIES.

TYPES : COMPLEX NO. WITH POLYNOMIALS.

ROOTS OF AN EQUATION :
IF COMPLEX , THEY LIVE IN CONJUGATE PAIRS.

Quadratic : (Two Roots) $\rightarrow$ RR or CC
Cubic : (Three Roots) $\rightarrow$ RRR , RCC
x^4 : (4 Roots) $\rightarrow$ RRRR, RR CC, CC CC

1 The polynomial $\mathrm{p}(z)$ is defined by

$$\mathrm{p}(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z + 2)$ is a factor of $\mathrm{p}(z)$. $z+2=0$, $z=-2$

(i) Find the value of m. $=6$ [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $\mathrm{p}(z) = 0$, [5]

(b) the six roots of the equation $\mathrm{p}(z^2) = 0$. [6]

9709/33/O/N/10/Q10

$$p(-2) = 0$$

$$0 = (-2)^3 + m(-2)^2 + 24(-2) + 32$$

$$m = 6$$

$$P(z) = z^3 + 6z^2 + 24z + 32$$

$$z^3 + 6z^2 + 24z + 32 \equiv (z+2)(az^2 + bz + c)$$

Max power | Constant | Middle Power

$z^3 = az^3$ | $32 = 2c$ | $6z^2 = 2az^2 + bz^2$

$a = 1$ | $c = 16$ | $6 = 2a + b$

$$6 = 2(1) + b$$

$$b = 4$$

$$p(z) = (z+2)(z^2 + 4z + 16)$$

$$p(z) = 0$$

$$(z+2)(z^2 + 4z + 16) = 0$$

$$z + 2 = 0$$

$$\boxed{z = -2}$$

$$z^2 + 4z + 16 = 0$$

$$z = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{-48}}{2}$$

$$= \frac{-4 \pm \sqrt{16 \times 3 \times -1}}{2}$$

$$= \frac{-4 \pm 4\sqrt{3}\, i}{2}$$

$$z = -2 \pm 2\sqrt{3}\, i$$

$$\boxed{z = -2}, \quad \boxed{z = -2 + 2\sqrt{3}\, i}, \quad \boxed{z = -2 - 2\sqrt{3}\, i}$$

When we solved

$$p(z) = 0 \implies \boxed{z = -2} \; \boxed{z = -2 + 2\sqrt{3}\, i} \; \boxed{z = -2 - 2\sqrt{3}\, i}$$

$$p(t) = 0 \implies \boxed{t = -2} \; \boxed{t = -2 + 2\sqrt{3}\, i} \; \boxed{t = -2 - 2\sqrt{3}\, i}$$

$$p(x) = 0 \implies \boxed{x = -2} \; \boxed{x = -2 + 2\sqrt{3}\, i} \; \boxed{x = -2 - 2\sqrt{3}\, i}$$

$$p(z^2) = 0 \implies \boxed{z^2 = -2} \; \boxed{z^2 = -2 + 2\sqrt{3}\, i} \; \boxed{z^2 = -2 - 2\sqrt{3}\, i}$$

$z^2 = -2$

$z = \pm\sqrt{-2}$

$z = \pm\sqrt{2 \times -1}$

$z = \pm\sqrt{2}\, i$

$\boxed{z = \sqrt{2}\, i}$ $\boxed{z = -\sqrt{2}\, i}$

$z^2 = -2 + 2\sqrt{3}\, i$

$z = \sqrt{-2 + 2\sqrt{3}\, i}$

$R = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$

arg: $\alpha = \tan^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right) = \frac{\pi}{3}$

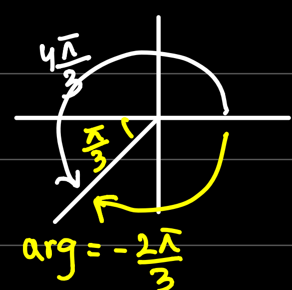
2^{nd} Quad: $\arg = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$$z^2 = 4e^{i\left(\frac{2\pi}{3} + 2\pi(k)\right)}$$

$$z = 2e^{i\left(\frac{\frac{2\pi}{3} + 2\pi(k)}{2}\right)}$$

$k = 0$ $\qquad$ $k = 1$

$$z = 2e^{i\left(\frac{\pi}{3}\right)}, \quad z = 2e^{i\left(\frac{4}{3}\pi\right)}$$



$$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right), \quad z = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$

$$z = 2\left(\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right)\right), \quad z = 2\left(\frac{-1}{2} + i\left(\frac{-\sqrt{3}}{2}\right)\right)$$

$\boxed{z = 1 + \sqrt{3}\, i}$, $\boxed{z = -1 - \sqrt{3}\, i}$

Repeat same process with $z^2 = -2 - 2\sqrt{3}\, i$

$z = \sqrt{-2 - 2\sqrt{3}\, i}$

$\sqrt{-2 - 2\sqrt{3}\, i} = x + yi$

$-2 - 2\sqrt{3}\, i = (x + iy)^2$

9709/31/O/N/12

4 The polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $f(x)$.

(i) Find the value of a. [2]

(ii) Show that, when a has this value, the equation $f(x) = 0$ has only one real root. [3]

9709/33/O/N/13/Q3

(i) $x+2=0$ is a factor

$x=-2$

$f(-2)=0$

$0 = (-2)^3 + a(-2)^2 - a(-2) + 14$

$a = -1$

$f(x) = x^3 - x^2 + x + 14$

$x^3 - x^2 + x + 14 \equiv (x+2)(ax^2 + bx + c)$

Max | Constant | middle.

$x^3 = ax^3$ | $14 = 2c$ | $-x^2 = 2ax^2 + bx^2$

$a = 1$ | $c = 7$ | $-1 = 2a + b$

$-1 = 2(1) + b$

$b = -3$

$x^3 - x^2 + x + 14 \equiv (x+2)(x^2 - 3x + 7)$

$f(x) = 0$

$(x+2)(x^2 - 3x + 7) = 0$

$x+2=0$, $x^2 - 3x + 7 = 0$

$x=-2$ should have no real roots.

one Real Root

$b^2 - 4ac$

$(-3)^2 - 4(1)(7)$

$9 - 28$

-19 (No Real Roots)

$x = 2$ is a solution of $f(x)$

Factor $\Rightarrow$? $(x-2)$ is a factor.

$x = \sqrt{3} \rightarrow$ factor $= x - \sqrt{3}$

$x = 2 + \sqrt{3} \longrightarrow$ factor $\Rightarrow (x - 2 - \sqrt{3})$

$x = -1 + \sqrt{5} \longrightarrow$ factor $\Rightarrow (x + 1 - \sqrt{5})$

6 **Throughout this question the use of a calculator is not permitted.**

It is given that the complex number $-1 + (\sqrt{3})i$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. $\boxed{-1-\sqrt{3}i}$ [1]

(ii) Find the value of k and the third root of the equation. [6]

9709/32/M/J/19/Q5

$x = -1 + \sqrt{3}i \longrightarrow$ Factor $= (x+1-\sqrt{3}i)$

$x = -1 - \sqrt{3}i \longrightarrow$ Factor $= (x+1+\sqrt{3}i)$

$$kx^3+5x^2+10x+4 \equiv (x+1-\sqrt{3}i)(x+1+\sqrt{3}i)(ax+b)$$

$$\left[(x+1)^2 - (\sqrt{3}i)^2\right](ax+b)$$

$$(x^2+2x+1-3i^2)(ax+b)$$

$$kx^3+5x^2+10x+4 \equiv (x^2+2x+4)(ax+b)$$

Max

$kx^3 = ax^3$

$k = a$

$k = 2$

Constant

$4 = 4b$

$b = 1$

Middle.

$5x^2 = bx^2 + 2ax^2$

$5 = b + 2a$

$5 = 1 + 2a$

$a = 2$

$\boxed{k = 2}$

Third factor $= (2x+1)$

Third root : $2x+1 = 0$

$x = -\frac{1}{2}$.