I5 SINGLE PART

P3

1 Find the exact value of the constant k for which $\int_1^k \frac{1}{2x-1}\,\mathrm{d}x = 1$. [4]

2 Use integration by parts to show that

$$\int_2^4 \ln x\,\mathrm{d}x = 6\ln 2 - 2.$$ [4]

3 Show that $\int_0^{\pi} x^2 \sin x\,\mathrm{d}x = \pi^2 - 4$. [5]

4 Show that $\int_0^1 (1-x)\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = 4\mathrm{e}^{-\frac{1}{2}} - 2$. [5]

5 Find the exact value of $\int_1^4 \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x$. [5]

6 **(a)** Find $\int (4 + \tan^2 2x)\,\mathrm{d}x$. [3]

7 Find the exact value of $\int_0^{\frac{1}{2}} x\mathrm{e}^{-2x}\,\mathrm{d}x$. [5]

8 Find the exact value of $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x\,\mathrm{d}x$. [5]

9 Find the exact value of $\int_0^{\frac{1}{2}\pi} \theta \sin \tfrac{1}{2}\theta\,\mathrm{d}\theta$. [4]

10 Showing all necessary working, find the value of $\int_0^{\frac{1}{6}\pi} x\cos 3x\,\mathrm{d}x$, giving your answer in terms of π. [5]

11 **(i)** Find $\int \frac{\ln x}{x^3}\,\mathrm{d}x$. [3]

12 Show that $\int_0^{\frac{1}{4}\pi} x^2\cos 2x\,\mathrm{d}x = \frac{1}{32}(\pi^2 - 8)$. [5]