ALEVELS P3

COMPLEX NUMBERS WITH POLYNOMIAL
C5

1 The polynomial $p(z)$ is defined by

$$p(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z + 2)$ is a factor of $p(z)$.

(i) Find the value of m. [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $p(z) = 0$, [5]

(b) the six roots of the equation $p(z^2) = 0$. [6]

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2 The polynomial $p(x)$ is defined by

$$p(x) = x^3 - 3ax + 4a,$$

where a is a constant.

(i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of a. [2]

(ii) When a has this value,

(a) factorise $p(x)$ completely, [3]

(b) find all the roots of the equation $p(x^2) = 0$. [2]

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3 The complex number $1 + (\sqrt{2})i$ is denoted by $\boldsymbol{u}$. The polynomial $\boldsymbol{x}^4 + \boldsymbol{x}^2 + 2\boldsymbol{x} + 6$ is denoted by $p(\boldsymbol{x})$.

(i) Showing your working, verify that $\boldsymbol{u}$ is a root of the equation $p(\boldsymbol{x}) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $p(\boldsymbol{x}) = 0$. [6]

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4 The polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $f(x)$.

(i) Find the value of a. [2]

(ii) Show that, when a has this value, the equation $f(x) = 0$ has only one real root. [3]

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5 The polynomial $4x^4 + ax^2 + 11x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 2$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the real roots of the equation $p(x) = 0$. [2]

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6 **Throughout this question the use of a calculator is not permitted.**

It is given that the complex number $-1 + (\sqrt{3})\mathrm{i}$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

(ii) Find the value of k and the third root of the equation. [6]

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