NORMAL DISTRIBUTION

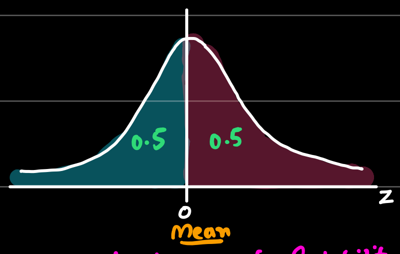
(15 MARKS) (2 QUESTIONS)

Discrete ??? (Fixed Outcomes)
1- Binomial Distribution
2- Geometric Distribution.

CONDITIONS: 1- CONTINOUS DATA (OUTCOMES)

eg: length of leaves
heights of students

150 cm 151 cm



Area under the graph = Probability

2- Symmetrical data

Data is divided evenly on both sides of mean.

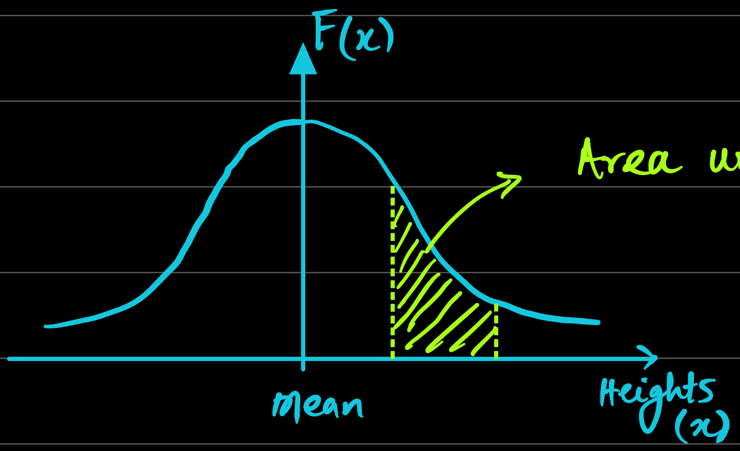
3- n is large

we will discuss this later on in chapter.

IT IS ALWAYS GIVEN WHEN YOU HAVE TO USE THE NORMAL DISTRIBUTION.

Experiment: Heights of students in a class

X = heights = Data

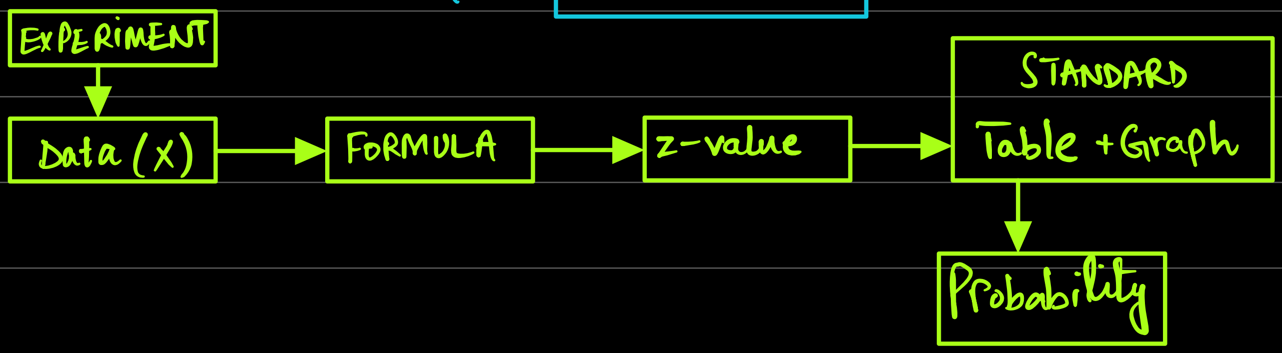


Area under graph = PROBABILITY

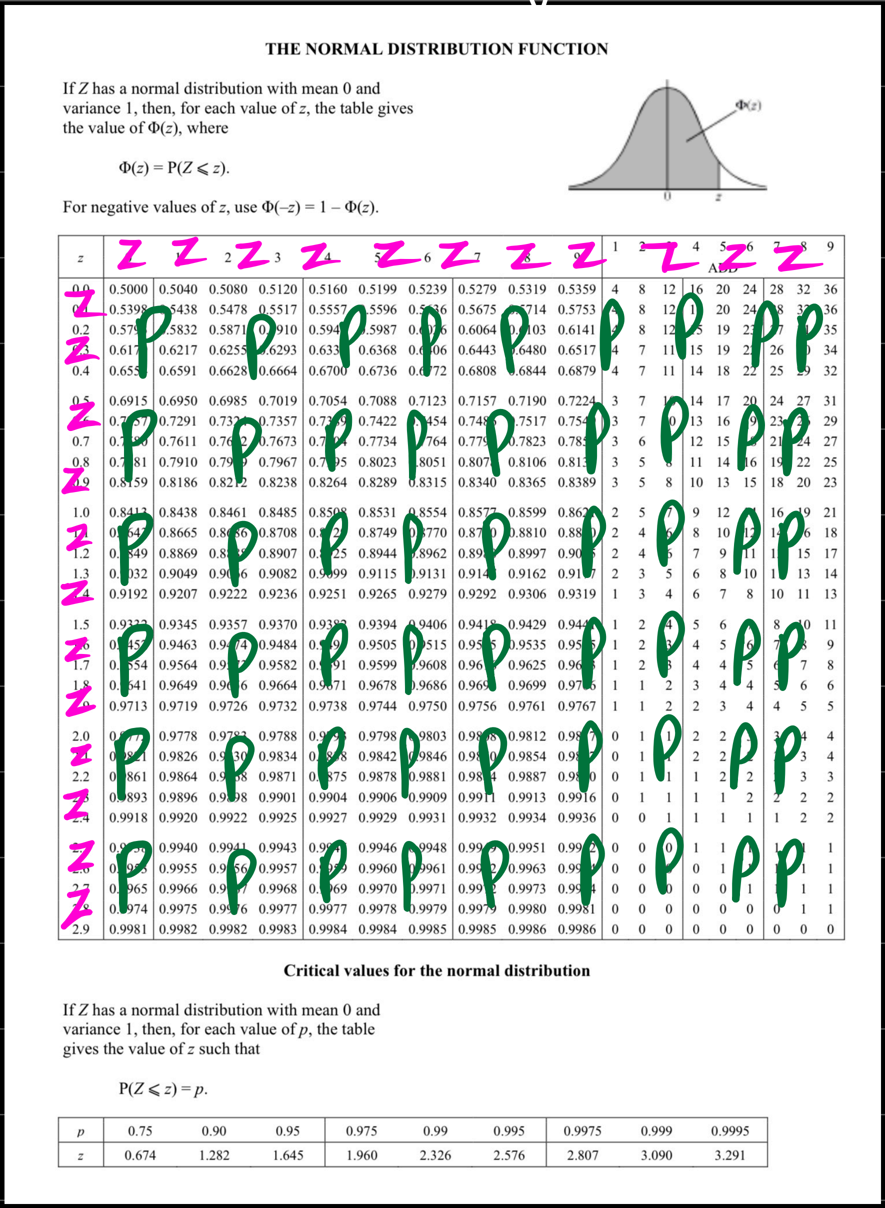
Each time you do a new experiment, a new graph is to made. Finding area under graphs is complicated.

MATHS PEOPLE DID NOT LIKE THIS AT ALL!

So we designed a standard graph which can work on any experiment. THAT WAS CALLED A Z-TABLE



This table is given in EXAM.



THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z, the table gives the value of $\Phi(z)$, where

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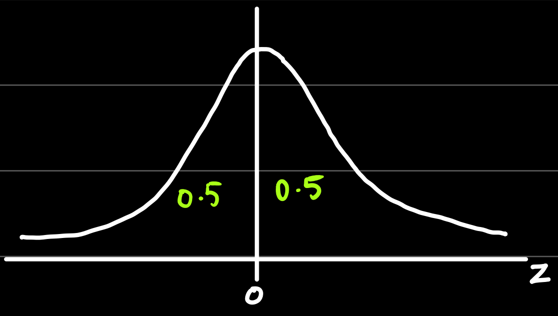
z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	[illegible]	0.5675	0.5714	0.5753	4	8	12	[illegible]	20	24	[illegible]	[illegible]	36
0.2	[illegible]	0.5832	0.5871	[illegible]	[illegible]	0.5987	[illegible]	0.6064	[illegible]	0.6141	4	8	12	[illegible]	19	[illegible]	[illegible]	[illegible]	35
0.3	[illegible]	0.6217	0.6255	0.6293	[illegible]	0.6368	[illegible]	0.6443	0.6480	0.6517	4	7	11	15	19	[illegible]	26	[illegible]	34
0.4	[illegible]	0.6591	0.6628	0.6664	0.6700	0.6736	[illegible]	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	[illegible]	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	[illegible]	14	17	20	24	27	31
0.6	[illegible]	0.7291	0.7324	0.7357	[illegible]	0.7422	[illegible]	[illegible]	0.7517	[illegible]	3	7	[illegible]	13	16	[illegible]	23	[illegible]	29
0.7	[illegible]	0.7611	[illegible]	0.7673	[illegible]	0.7734	[illegible]	[illegible]	0.7823	[illegible]	3	6	[illegible]	12	15	[illegible]	21	24	27
0.8	[illegible]	0.7910	[illegible]	0.7967	[illegible]	0.8023	0.8051	[illegible]	0.8106	[illegible]	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	[illegible]	2	5	7	9	12	[illegible]	16	19	21
1.1	[illegible]	0.8665	[illegible]	0.8708	[illegible]	0.8749	0.8770	[illegible]	0.8810	[illegible]	2	4	6	8	10	12	[illegible]	[illegible]	18
1.2	[illegible]	0.8869	[illegible]	0.8907	[illegible]	0.8944	0.8962	[illegible]	0.8997	[illegible]	2	4	6	7	9	11	[illegible]	15	17
1.3	[illegible]	0.9049	[illegible]	0.9082	[illegible]	0.9115	0.9131	[illegible]	0.9162	[illegible]	2	3	5	6	8	10	[illegible]	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	[illegible]	0.9345	0.9357	0.9370	[illegible]	0.9394	0.9406	[illegible]	0.9429	[illegible]	1	2	4	5	6	[illegible]	8	10	11
1.6	[illegible]	0.9463	[illegible]	0.9484	[illegible]	0.9505	0.9515	[illegible]	0.9535	[illegible]	1	2	3	4	5	6	7	[illegible]	9
1.7	[illegible]	0.9564	[illegible]	0.9582	[illegible]	0.9599	0.9608	[illegible]	0.9625	[illegible]	1	2	3	4	4	5	6	7	8
1.8	[illegible]	0.9649	[illegible]	0.9664	[illegible]	0.9678	0.9686	[illegible]	0.9699	[illegible]	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	[illegible]	0.9778	[illegible]	0.9788	[illegible]	0.9798	0.9803	[illegible]	0.9812	[illegible]	0	1	1	2	2	[illegible]	[illegible]	4	4
2.1	[illegible]	0.9826	[illegible]	0.9834	[illegible]	0.9842	0.9846	[illegible]	0.9854	[illegible]	0	1	1	2	2	2	[illegible]	3	4
2.2	[illegible]	0.9864	[illegible]	0.9871	[illegible]	0.9878	0.9881	[illegible]	0.9887	[illegible]	0	1	1	1	2	2	[illegible]	3	3
2.3	[illegible]	0.9896	[illegible]	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	[illegible]	0.9940	0.9941	0.9943	[illegible]	0.9946	0.9948	[illegible]	0.9951	[illegible]	0	0	0	1	1	[illegible]	1	1	1
2.6	[illegible]	0.9955	[illegible]	0.9957	[illegible]	0.9960	0.9961	[illegible]	0.9963	[illegible]	0	0	[illegible]	0	1	[illegible]	1	1	1
2.7	[illegible]	0.9966	[illegible]	0.9968	[illegible]	0.9970	0.9971	[illegible]	0.9973	[illegible]	0	0	0	0	0	1	[illegible]	1	1
2.8	[illegible]	0.9975	[illegible]	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

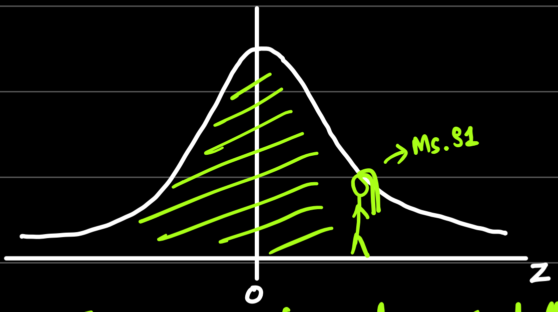
Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

$$P(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291



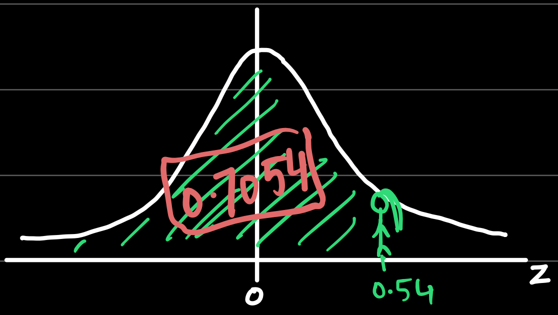


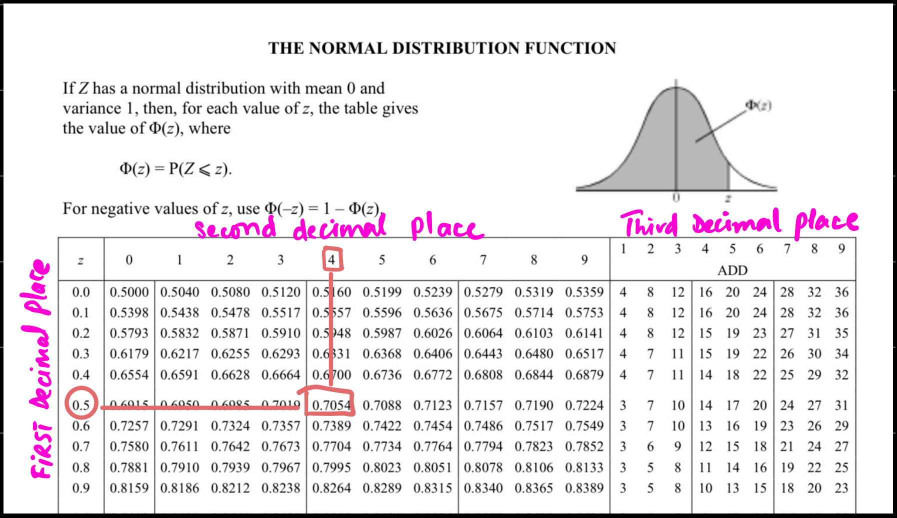
This z-table always tells area to left side of z-value

USING TABLE TO FIND PROBABILITIES

(i) $P(Z < 0.54)$

left




THE NORMAL DISTRIBUTION FUNCTION

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second decimal place

Third Decimal place

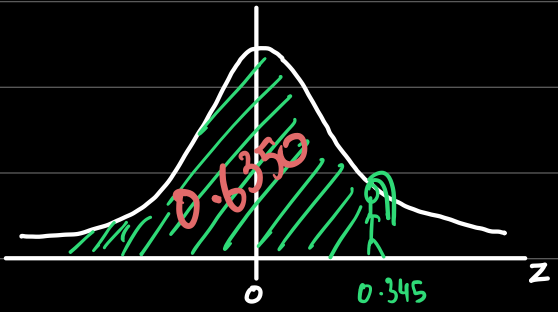
First Decimal place

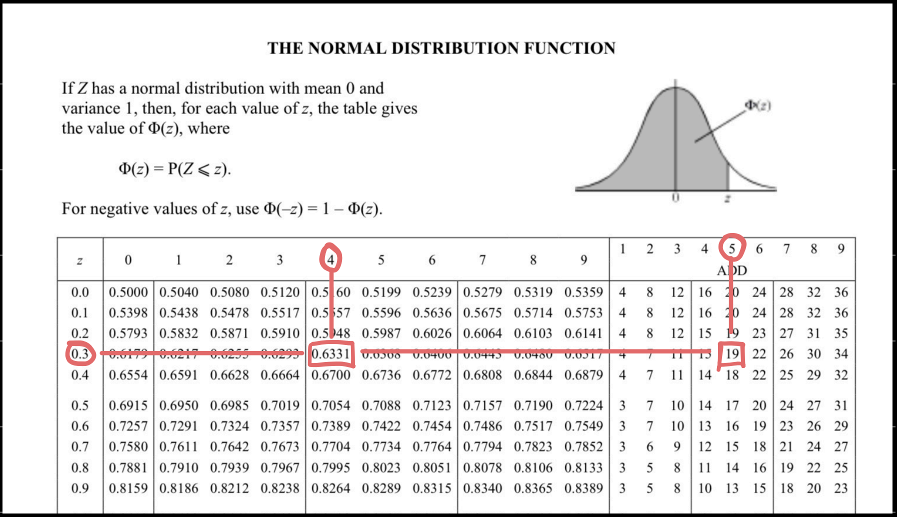
z	0	1	2	3	4	5	6	7	8	9	ADD								
											1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

$P(Z < 0.54) = 0.7054$

(ii) $P(Z < 0.345)$

left

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											1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

Table value at 0.34 = 0.6331

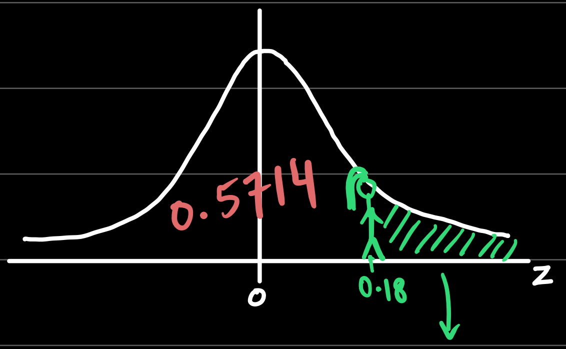
Add for third dp +19

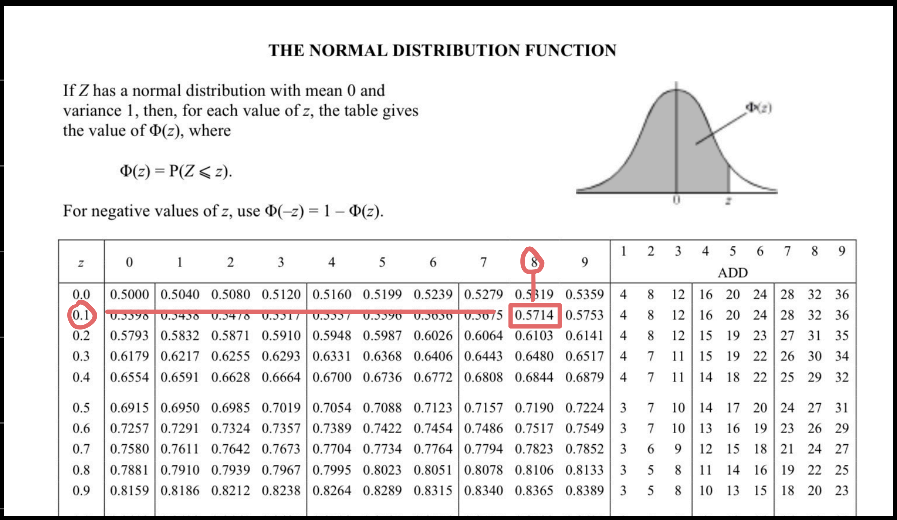
Table value for 0.345 = 0.6350

$P(Z < 0.345) = 0.6350$

iii) $P(Z > 0.18)$

right





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z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
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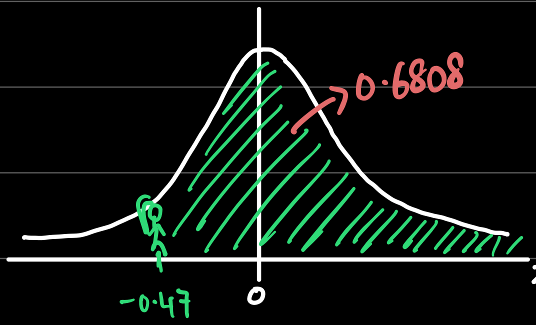
$= 1 - 0.5714$

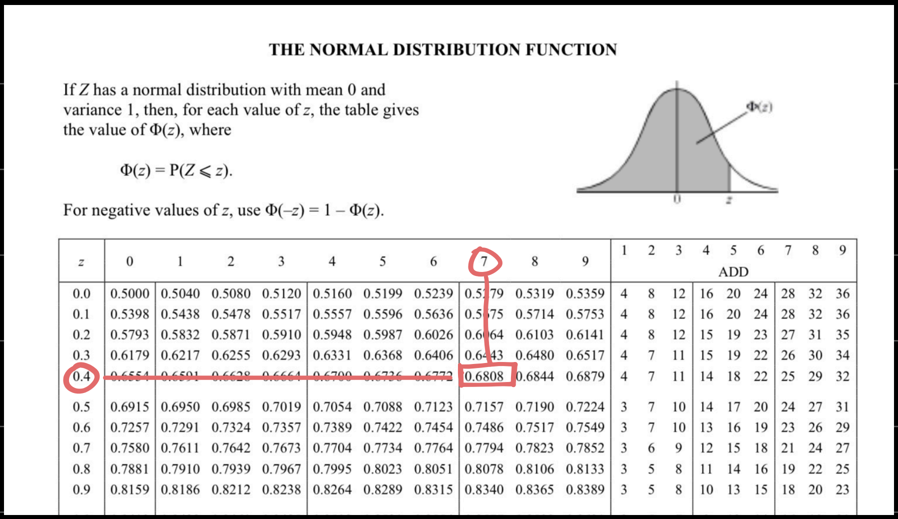
$=$

$P(Z > 0.18) = 0.4286$

(iv) $P(Z > -0.47)$

right





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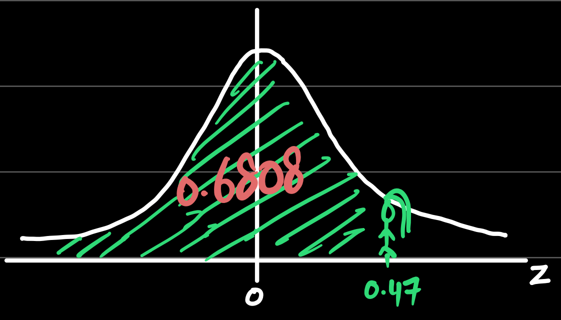
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

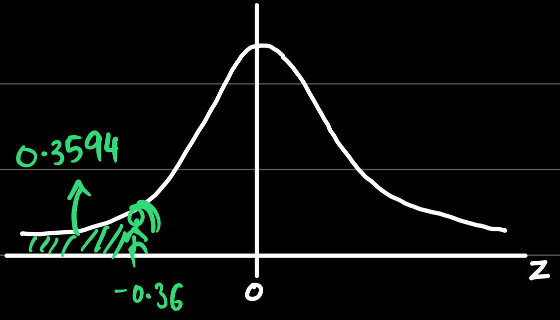
There is no negative z-value on table. Flip Diagram.

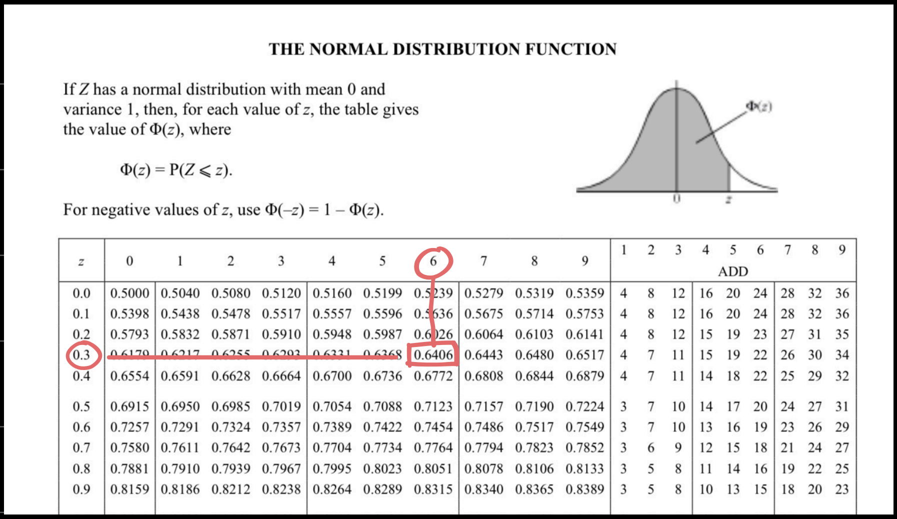


$P(Z > -0.47) = 0.6808$

(v) $P(z < -0.36)$

Left





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											ADD								
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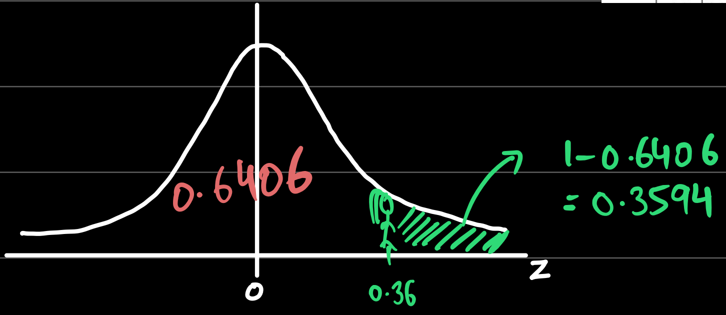
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23



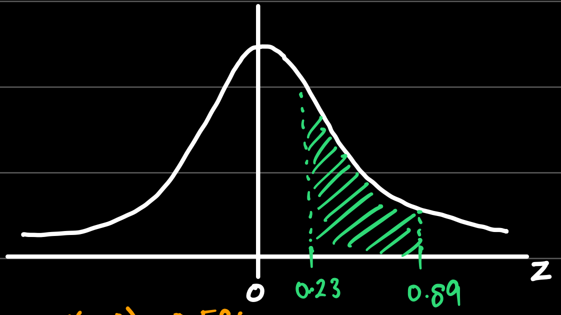
In Normal Distribution

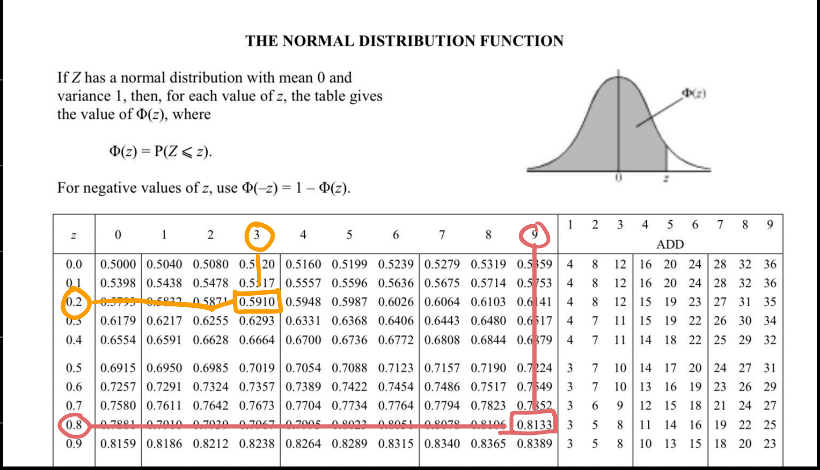
$X < 45$ and $X \leqslant 45$
are exactly same

In Binomial and geometric
$X < 45$ and $X \leqslant 45$
are two separate ideas

$P(z < -0.36) = 0.3594$

(vi) $P(0.23 < z < 0.89)$





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$\phi(0.23) = 0.5910$

Table value of $z \Rightarrow$ Symbol $\Rightarrow \phi(z)$

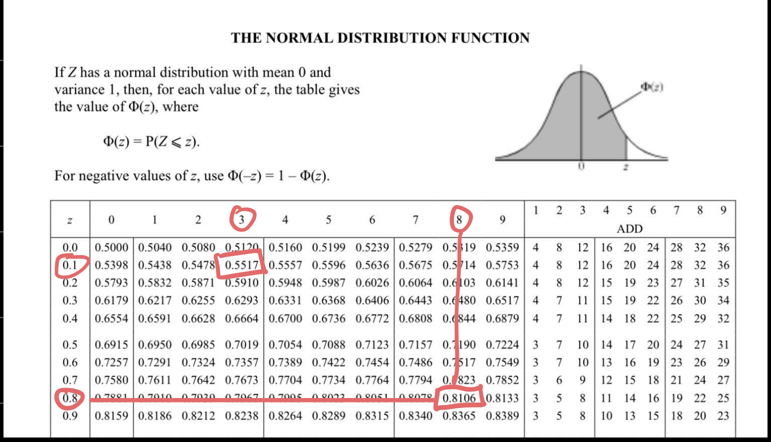
$\phi(0.89) = 0.8133$

$\phi(0.89) = 0.8133$

$$P(0.23 < z < 0.89) = 0.8133 - 0.5910$$

$$= 0.2223$$

(vii) $P(-0.88 < Z < -0.13)$



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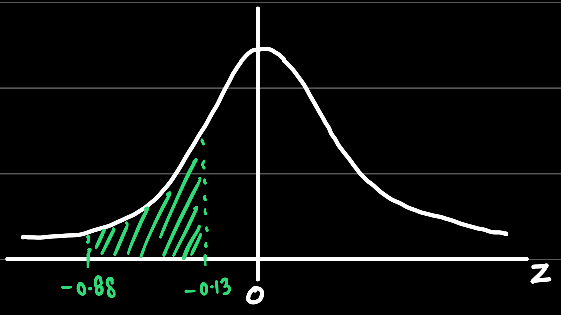
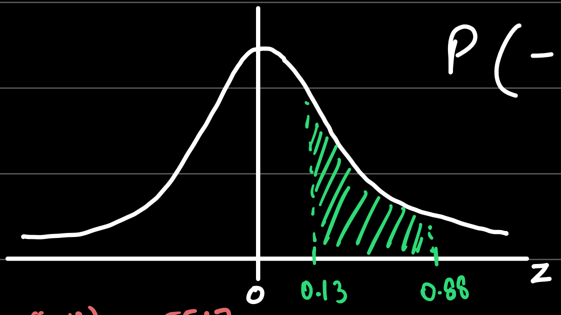




$P(-0.88 < Z < -0.13) = 0.8106 - 0.5517$

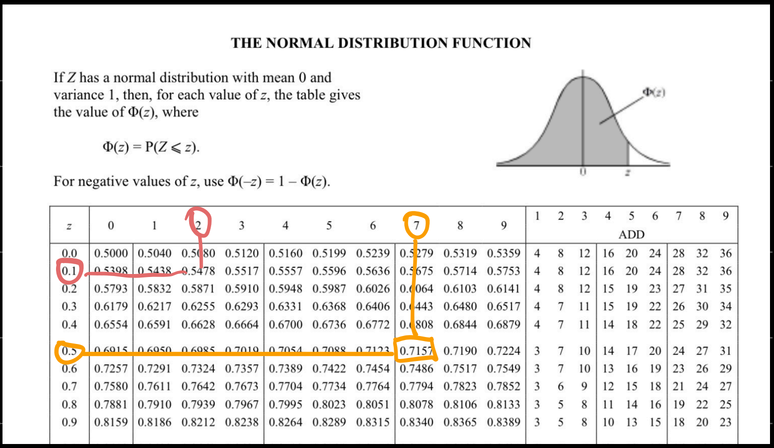
$= 0.2589.$

$\phi(0.13) = 0.5517$

$\phi(0.88) = 0.8106$

ADVANCED

(viii) $P(-0.12 < Z < 0.57)$



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0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
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0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
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0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
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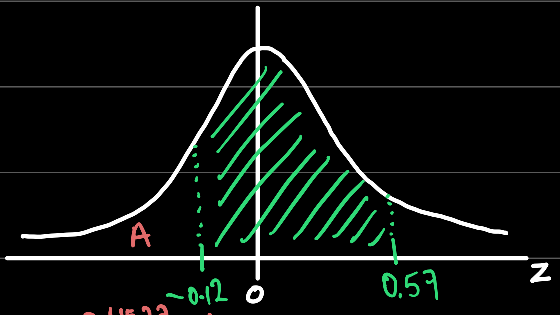
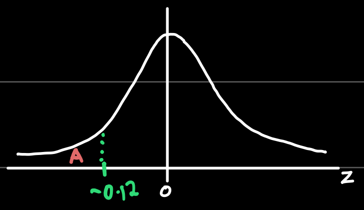
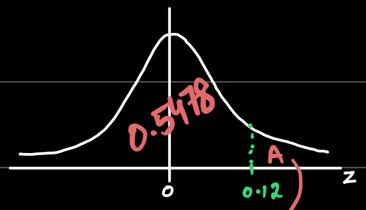


0.4522

$\phi(0.57) = 0.7157$





$A = 1 - 0.5478$

$A = 0.4522$

$P(-0.12 < Z < 0.57) = 0.7157 - 0.4522$

$= 0.2635$

TYPE 1: FORWARD WORKING

Data(x) → FORMULA → Z-VALUES → DIAGRAM + TABLE → PROBABILITY

x, μ, σ

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

x = Data value (height etc)

μ = Mean

σ = Standard deviation.

Q:) X is normally distributed such that X has mean 36 and SD 12.

$\mu = 36$ $\sigma = 12$

In S1 write all probabilities to 4d.p.

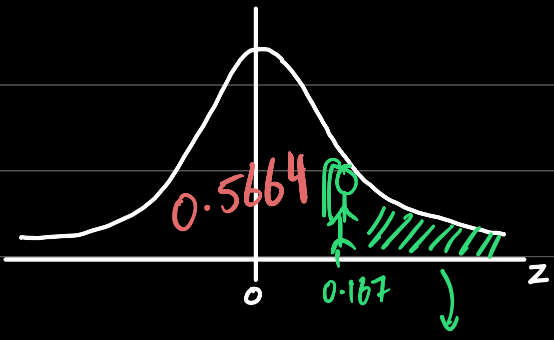
Find all z-values up to 3 dp

(i) $P(X > 38)$

$$z = \frac{x - \mu}{\sigma} = \frac{38 - 36}{12} = 0.167$$

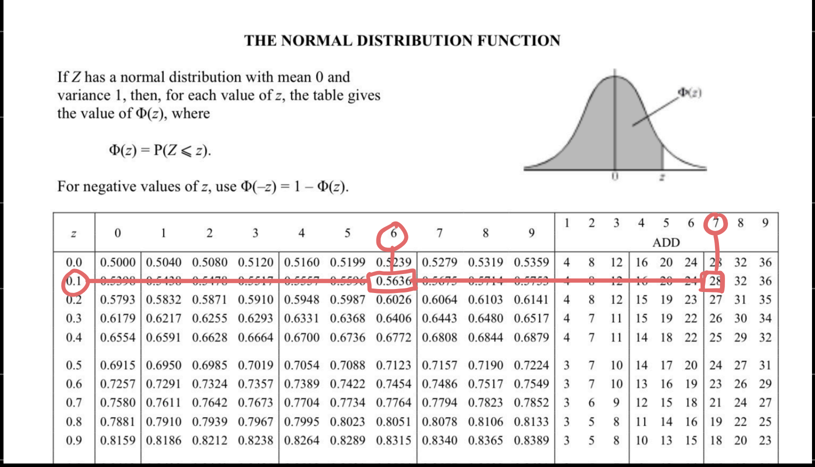
$P(Z > 0.167)$

right



$1 - 0.5664$

$P(X > 38) = 0.4336$



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0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

$\Phi(0.16) = 0.5636$

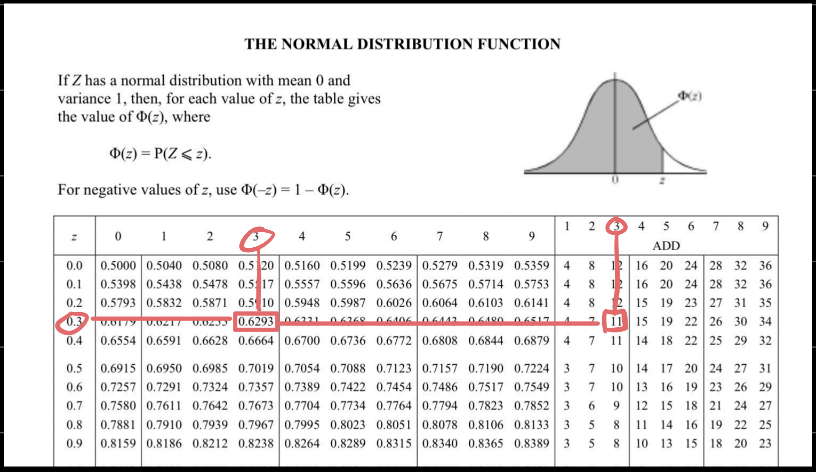
Add 3rd dp: $+28$

$\Phi(0.167) = 0.5664$

(ii) $P(X > 32)$

$$Z = \frac{X-\mu}{\sigma} = \frac{32-36}{12} = -0.333$$

$P(Z > -0.333)$



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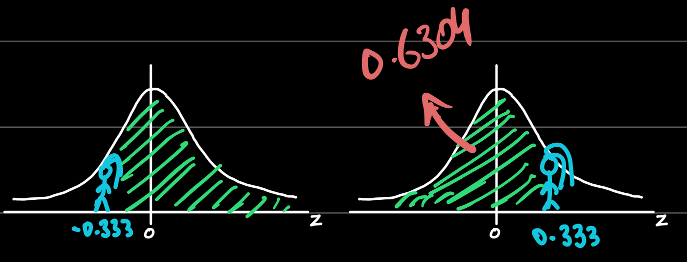


$\Phi(0.33) = 0.6293$

Add third dp : $+ 11$

$\Phi(0.333) = 06304$

$P(X > 32) = 0.6304$

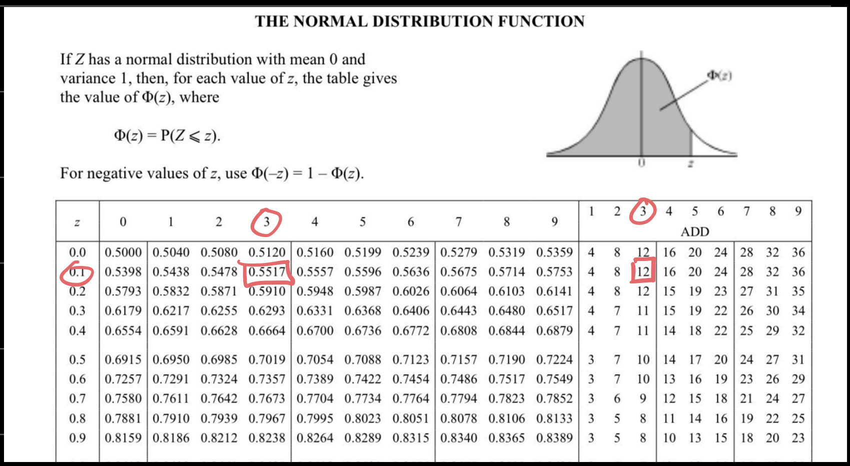
Q: X is normally distributed

Mean = 30 , SD = 15

(i) $P(X > 32)$

$$Z = \frac{X-\mu}{\sigma} = \frac{32-30}{15} = 0.133$$

$P(Z > 0.133)$



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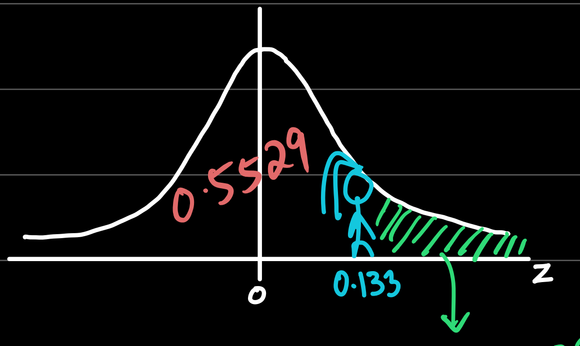
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23



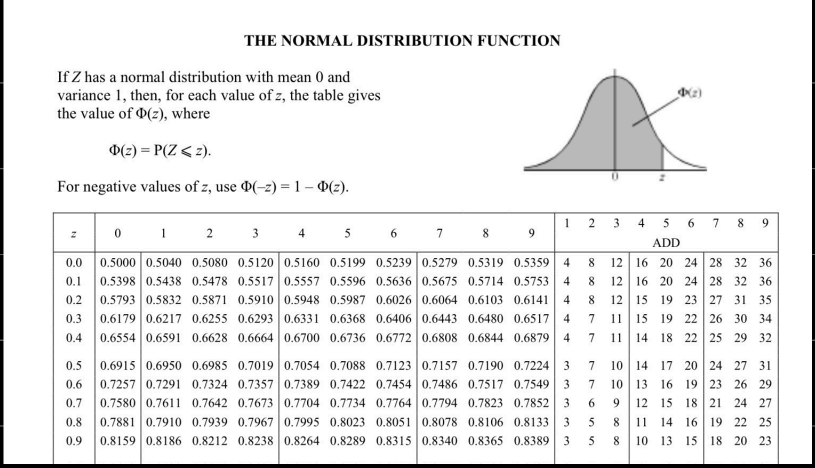
$\Phi(0.13) = 0.5517$

Add third dp : $+ 12$

0.5529

$= 1 - 0.5529$

$= 0.4471$



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0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
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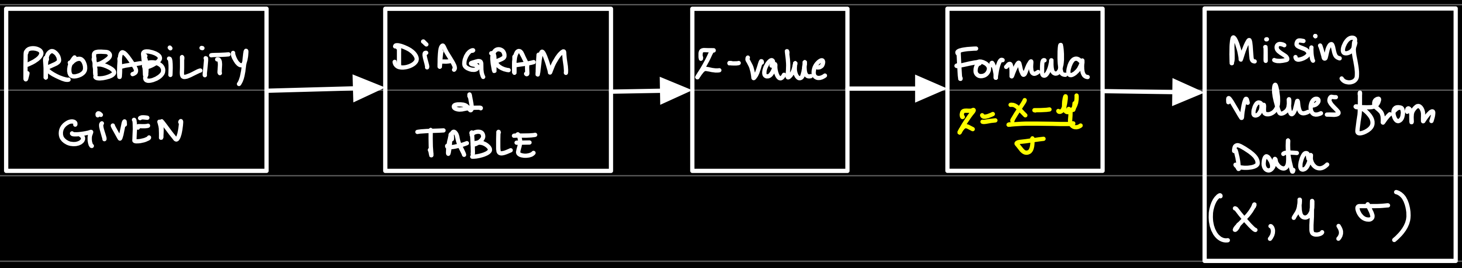
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

TYPE 2 : REVERSE WORKING

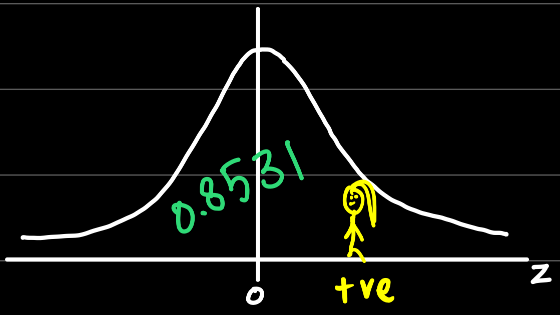


FIRST STEP IN REVERSE WORKING IS TO DECIDE +/− SIGN OF z. (DRAW MS. S1 FOR HELP.)

1. $P(z < \boxed{}) = 0.8531$

Left

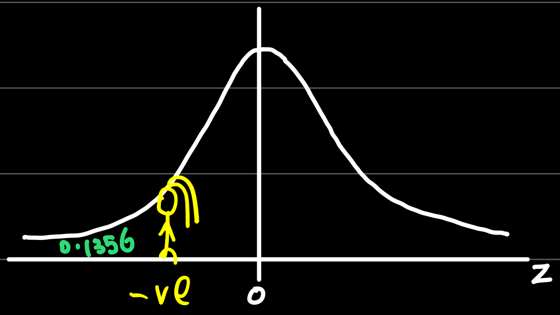
area is more than half



2. $P(z < \boxed{}) = 0.1356$

Left

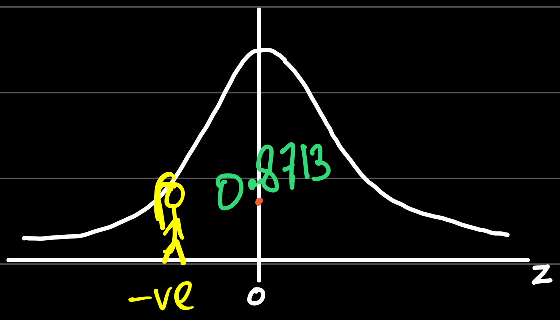
area is less than half.



$\boxed{3}$ $P(z > \boxed{}) = 0.8713$

right

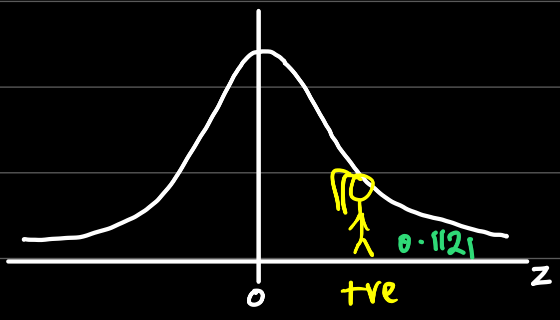
area is more than half (0.5)



$\boxed{4}$ $P(z > \boxed{}) = 0.1121$

right

area is less than half (0.5)

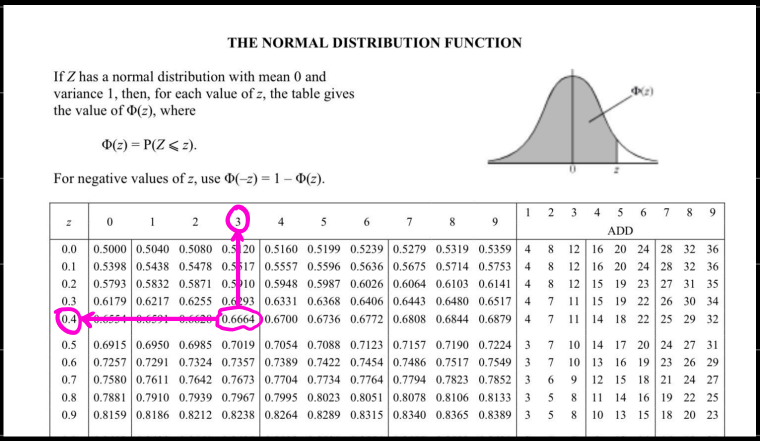


REVERSE WORKING

Q: Find a in all of following:

Mean = 12   SD = 8

(i) $P(X > a) = 0.6664$



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0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
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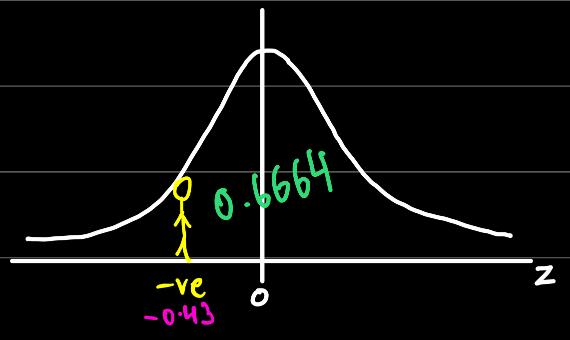
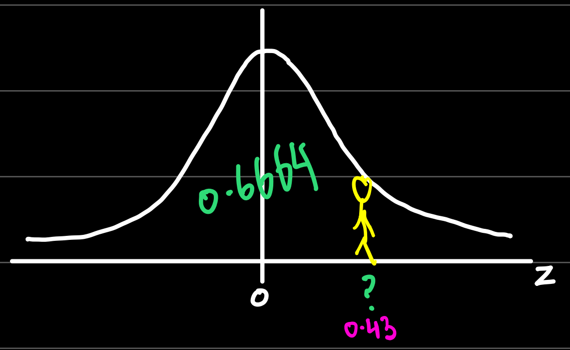
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

$P(Z > \boxed{-ve = -0.43}) = 0.6664$

right

area is more than half.



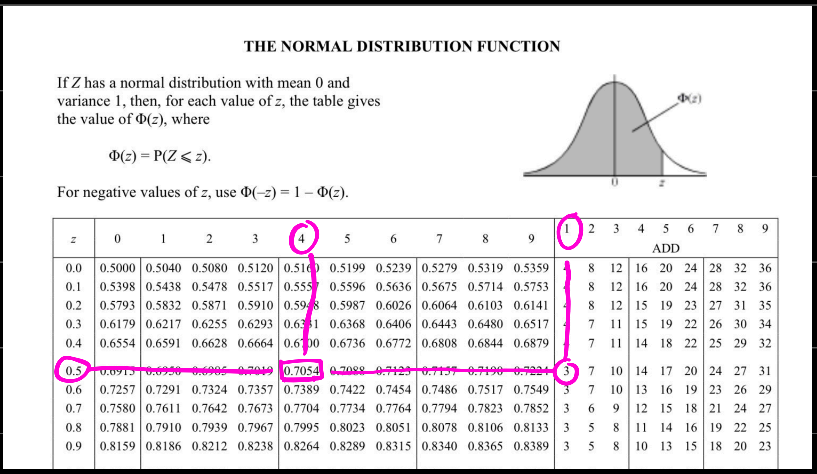


$$z = \frac{x - \mu}{\sigma}$$

$$-0.43 = \frac{a - 12}{8}$$

$$\boxed{a = 8.56}$$

(ii) $P(X < a) = 0.7057$



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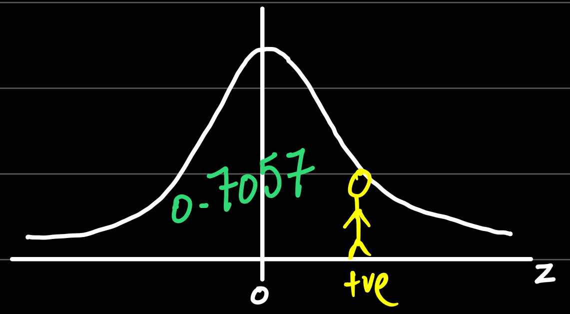
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

$P(Z < \boxed{+0.541}) = 0.7057$

left

area is more than half



$\phi(0.54) = 0.7054$

Third dp = 1 ⟶ $+ \; ?3$

$$\underline{0.7057}$$

$$z = \frac{x - \mu}{\sigma}$$

$$+0.541 = \frac{a - 12}{8}$$

$$8(0.541) = a - 12$$

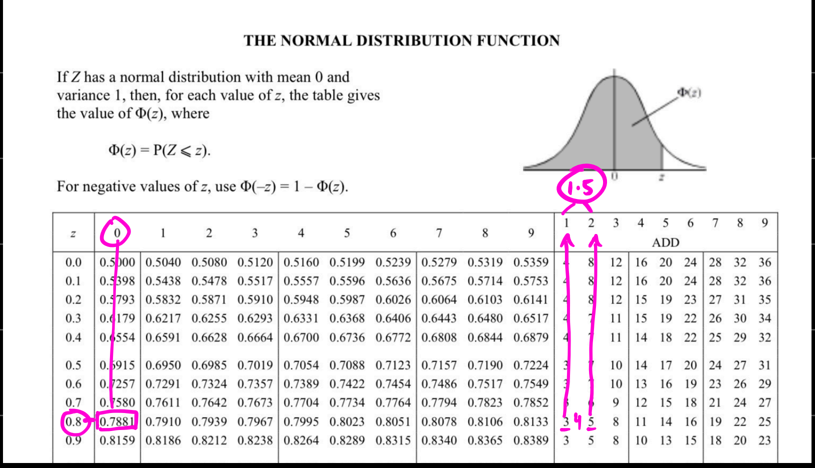
$$a = 16.328$$

(iii) $P(X < a) = 0.7885$

$P(Z < \boxed{+0.8015}) = 0.7885$

left

area is more than half.



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1.5

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
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0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
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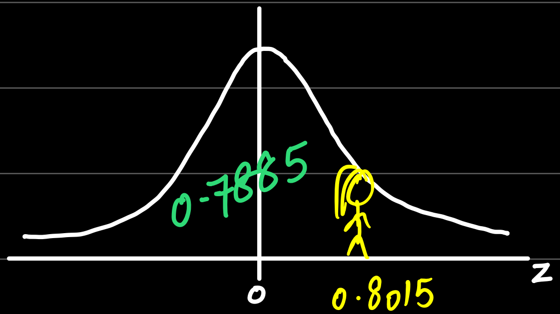
3 4 5

$$\varnothing(0.80) = 0.7881$$

$$\begin{array}{r} +4 \\ \hline 0.7885 \end{array}$$

$z = 0.8015$



$$z = \frac{X - \mu}{\sigma}$$

$$0.8015 = \frac{a - 12}{8}$$

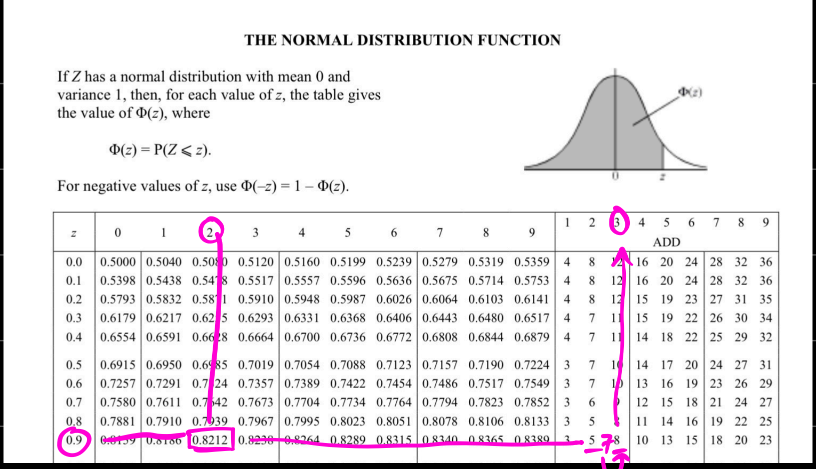
$a = \boxed{}$

(iv) $P(X > a) = 0.1781$

$P(Z > \boxed{+0.923}) = 0.1781$

right

area is less than half.



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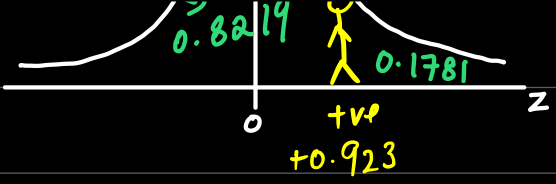
7

$1 - 0.1781$

$\varnothing(0.92) = 0.8212$ Jump to closer



$$+7$$
$$0.8219$$

one if not in middle.

$$z = \frac{X - \mu}{\sigma}$$

$$0.923 = \frac{a - 12}{8}$$

$$a = \boxed{}$$

PARAMETERS

SYMBOLS

BINOMIAL	NORMAL.
$X \underset{\text{is distributed}}{\sim} \overset{\text{Binomially}}{B}(\underset{\text{repeats}}{n}, \underset{\text{Prob. of Success}}{p})$	$X \underset{\text{distributed}}{\sim} \overset{\text{Normally}}{N}(\underset{\text{mean}}{\mu}, \underset{\text{variance}}{\sigma^2})$
$X \sim B(12, 0.2)$ Binomial Distribution, $n=12$, $p=0.2$	$X \sim N(12, \overset{\text{Be careful this is variance}}{36})$ Normal Distribution, $\mu = 12$, $\sigma^2 = 36$ $\sigma = 6$ $SD = 6$

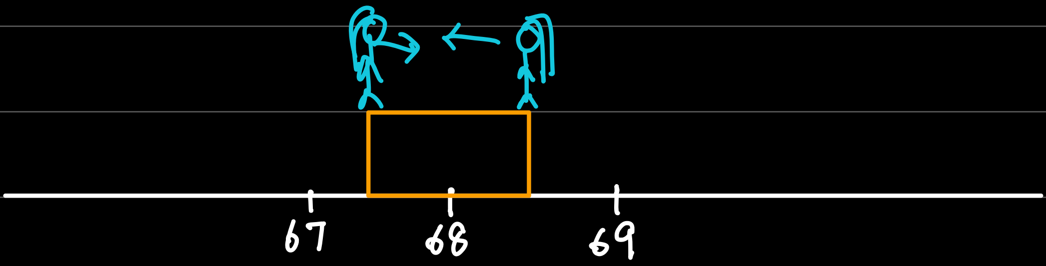
TYPE 3 BINOMIAL TO NORMAL APPROXIMATION

(n becomes too large)

BINOMIAL ⟶ **NORMAL DISTRIBUTION**

n, p, q

1. $\mu = np$
2. $SD = \sigma = \sqrt{npq}$
3. Correction of continuation.



BINOMIAL ⟶ NORMAL

$P(X > 68) \longrightarrow P(X > 68.5)$

$P(X \overset{\text{right}}{\geq} 68) \longrightarrow P(X > 67.5)$

$P(X < 68) \longrightarrow P(X < 67.5)$

$P(X \overset{\text{left}}{\leq} 68) \longrightarrow P(X < 68.5)$

$P(X = 68) \longrightarrow P(67.5 < X < 68.5)$

IN BINOMIAL & GEOMETRIC $>$ or $\geq$ are two different scenarios.	In NORMAL, IT DOES NOT MATTER IF YOU USE $>$ or $\geq$.

CONDITIONS FOR WHICH THIS APPROXIMATION is JUSTIFIED : $\boxed{np > 5}$ and $\boxed{nq > 5}$

Q. A dice is thrown 140 times. X denotes the random variable for number of times dice lands on a multiple of 3.

$n = 140$, $p = \frac{2}{6} = \frac{1}{3}$, $q = \frac{2}{3}$

X

(i) Find probability that dice lands on multiple of ③ exactly twice.

$$P(X=2) = {}^{140}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{138} = \boxed{\quad\quad}$$

(ii) Find the probability that dice lands on multiple of 3 more than 68 times.

$$P(X > 68) = P(X=69) + P(X=70) \ldots\ldots\ P(X=140)$$

This is where you need Binomial to normal approximation.

$n = 140$

$p = 1/3$

$q = 2/3$

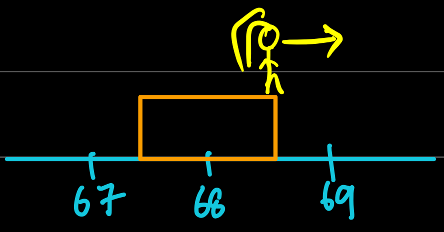
$P(X > 68)$

right

1) $\mu = 140\left(\frac{1}{3}\right) = \frac{140}{3}$

2) $\sigma = \sqrt{140\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} = \sqrt{\frac{280}{9}}$

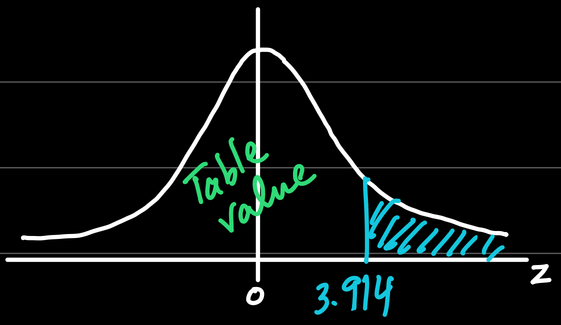
3)



$P(X > 68.5)$

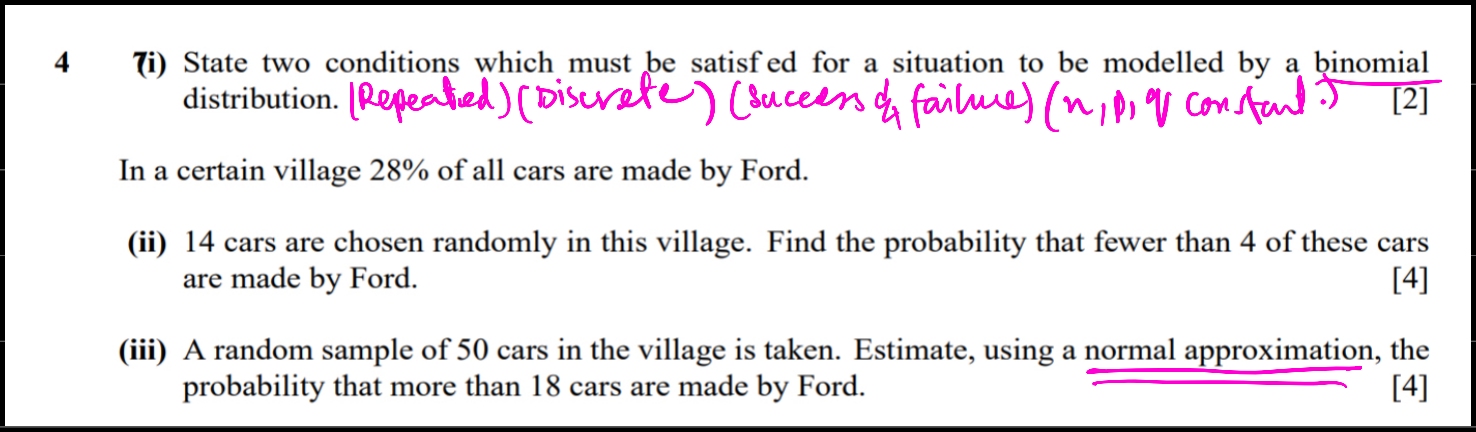
$$z = \frac{68.5 - \frac{140}{3}}{\sqrt{\frac{280}{9}}} =$$

$$P(z > \boxed{3.9143})$$



$$\underline{1 - \phi(3.914)}$$

$$= \boxed{\quad\quad}$$



4 **7i)** State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. (Repeated) (Discrete) (success & failure) (n, p, q constant.) [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

(iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford. [4]

(ii) Binomial : $n = 14$ $\quad p = 28\% = 0.28$, $q = 0.72$

$$P(X < 4) = P(X = 3, 2, 1, 0)$$

$$= {}^{14}C_3(0.28)^3(0.72)^{11} + {}^{14}C_2(0.28)^2(0.72)^{12} + {}^{14}C_1(0.28)^1(0.72)^{13} + {}^{14}C_0(0.28)^0(0.72)^{14}$$

$$= \boxed{\quad\quad\quad}$$

(iii) Binomial : $n = 50$ $\quad p = 28\% = 0.28$, $q = 0.72$

$$P(X > 18) = P(X = 19, 20, 21 \ldots\ldots 50)$$

$n = 50$

$p = 0.28$

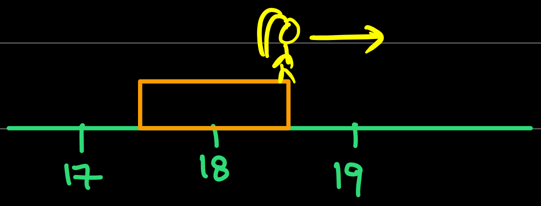
$q = 0.72$

$P(X > 18)$

right

$\mu = np = 50(0.28) = 14$

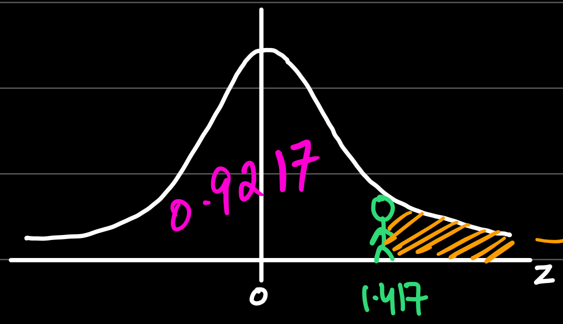
$\sigma = \sqrt{50(0.28)(0.72)} = \sqrt{10.08}$



$P(X > 18.5)$

$$z = \frac{18.5 - 14}{\sqrt{10.08}} = 1.417$$

$P(z > 1.417)$



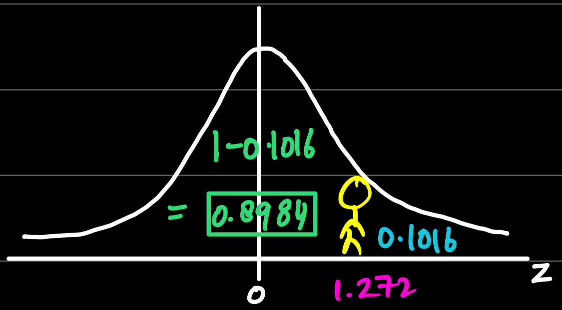
$1 - 0.9217 = 0.0783$

$P(X > 18) = 0.0783$

39 **(a)** The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

$3\mu = 7\sigma^2$

$P(X > 2\mu) = 0.1016$



$P(z > \boxed{+1.272}) = 0.1016$

right

area is less than half

$$z = \frac{X - \mu}{\sigma}$$

$$1.272 = \frac{2\mu - \mu}{\sigma}$$

$\boxed{\mu = 1.272\sigma}$ $\boxed{3\mu = 7\sigma^2}$

$3(1.272\sigma) = 7\sigma^2$

$\sigma = 0.545$

$\mu = 1.272(0.545)$

$\mu = 0.6934$