DE1 WITHOUT PROOF QUESTIONS

1 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^3 + 1}{y^2},$$

obtaining an expression for y in terms of x. [6]

9709/03/M/J/04

2 **(i)** Using partial fractions, find

$$\int \frac{1}{y(4-y)}\,dy.$$ [4]

(ii) Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = y(4-y),$$

obtaining an expression for y in terms of x. [4]

(iii) State what happens to the value of y if x becomes very large and positive. [1]

9709/3/M/J/05

3 Given that $y = 2$ when $x = 0$, solve the differential equation

$$y\frac{dy}{dx} = 1 + y^2,$$

obtaining an expression for y^2 in terms of x. [6]

9709/03/O/N/06

4 The number of insects in a population t days after the start of observations is denoted by N. The variation in the number of insects is modelled by a differential equation of the form

$$\frac{dN}{dt} = kN\cos(0.02t),$$

where k is a constant and N is taken to be a continuous variable. It is given that $N = 125$ when $t = 0$.

(i) Solve the differential equation, obtaining a relation between N, k and t. [5]

(ii) Given also that $N = 166$ when $t = 30$, find the value of k. [2]

(iii) Obtain an expression for N in terms of t, and find the least value of N predicted by this model. [3]

9709/03/O/N/07

5 **(i)** Express $\dfrac{100}{x^2(10-x)}$ in partial fractions. [4]

(ii) Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \tfrac{1}{100}x^2(10-x),$$

obtaining an expression for t in terms of x. [6]

9709/03/M/J/09

6 The temperature of a quantity of liquid at time t is θ. The liquid is cooling in an atmosphere whose temperature is constant and equal to A. The rate of decrease of θ is proportional to the temperature difference $(\theta - A)$. Thus θ and t satisfy the differential equation

$$\frac{d\theta}{dt} = -k(\theta - A),$$

where k is a positive constant.

(i) Find, in any form, the solution of this differential equation, given that $\theta = 4A$ when $t = 0$. [5]

(ii) Given also that $\theta = 3A$ when $t = 1$, show that $k = \ln\frac{3}{2}$. [1]

(iii) Find θ in terms of A when $t = 2$, expressing your answer in its simplest form. [3]

9709/32/O/N/09

7 Given that $y = 0$ when $x = 1$, solve the differential equation

$$xy\frac{dy}{dx} = y^2 + 4,$$

obtaining an expression for y^2 in terms of x. [6]

9709/31/M/J/10

8 The variables x and t are related by the differential equation

$$e^{2t}\frac{dx}{dt} = \cos^2 x,$$

where $t \geqslant 0$. When $t = 0$, $x = 0$.

(i) Solve the differential equation, obtaining an expression for x in terms of t. [6]

(ii) State what happens to the value of x when t becomes very large. [1]

(iii) Explain why x increases as t increases. [1]

9709/32/M/J/10

9 The number of birds of a certain species in a forested region is recorded over several years. At time t years, the number of birds is N, where N is treated as a continuous variable. The variation in the number of birds is modelled by

$$\frac{dN}{dt} = \frac{N(1800 - N)}{3600}.$$

It is given that $N = 300$ when $t = 0$.

(i) Find an expression for N in terms of t. [9]

(ii) According to the model, how many birds will there be after a long time? [1]

9709/31/M/J/11

10 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

9709/31/O/N/10

11 During an experiment, the number of organisms present at time t days is denoted by N, where N is treated as a continuous variable. It is given that

$$\frac{dN}{dt} = 1.2e^{-0.02t}N^{0.5}.$$

When $t = 0$, the number of organisms present is 100.

(i) Find an expression for N in terms of t. [6]

(ii) State what happens to the number of organisms present after a long time. [1]

9709/33/O/N/11

12 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation and hence find the value of y when $x = 0.5$, giving your answer correct to 2 decimal places. [8]

9709/31/M/J/12

13 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{2x+y},$$

and $y = 0$ when $x = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

9709/32/M/J/12

14 The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

9709/31/O/N/12

15 The variables x and y are related by the differential equation

$$(x^2 + 4)\frac{dy}{dx} = 6xy.$$

It is given that $y = 32$ when $x = 0$. Find an expression for y in terms of x. [6]

9709/33/O/N/12

16 **(i)** Express $\dfrac{1}{x^2(2x+1)}$ in the form $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{2x+1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x+1)\frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

9709/32/M/J/13

17 The variables x and t satisfy the differential equation

$$t\frac{dx}{dt} = \frac{k - x^3}{2x^2},$$

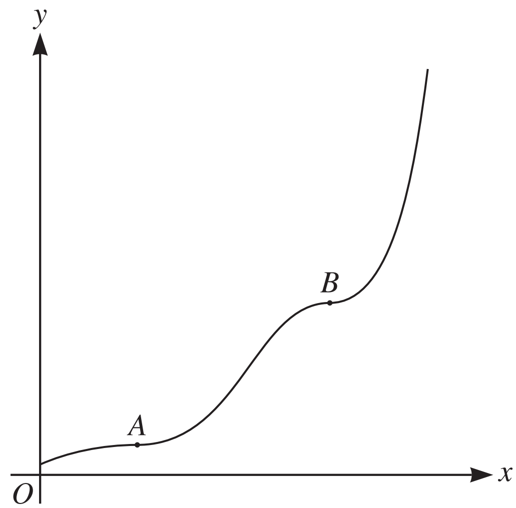
for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x as t becomes large. [1]

9709/33/M/J/13

18



A particular solution of the differential equation

$$3y^2\frac{dy}{dx} = 4(y^3 + 1)\cos^2 x$$

is such that $y = 2$ when $x = 0$. The diagram shows a sketch of the graph of this solution for $0 \leqslant x \leqslant 2\pi$; the graph has stationary points at A and B. Find the y-coordinates of A and B, giving each coordinate correct to 1 decimal place. [10]

9709/33/O/N/13

19 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6ye^{3x}}{2 + e^{3x}}.$$

Given that $y = 36$ when $x = 0$, find an expression for y in terms of x. [6]

9709/31/M/J/14

20 The variables $\boldsymbol{x}$ and θ satisfy the differential equation

$$2\cos^2\theta\frac{d\boldsymbol{x}}{d\theta} = \sqrt{(2\boldsymbol{x} + 1)},$$

and $\boldsymbol{x} = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for $\boldsymbol{x}$ in terms of θ. [7]

9709/33/M/J/14

21 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{dR}{dx} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

9709/31/O/N/14

22 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

9709/33/O/N/14

23 Given that $\boldsymbol{y} = 1$ when $\boldsymbol{x} = 0$, solve the differential equation

$$\frac{d\boldsymbol{y}}{d\boldsymbol{x}} = 4\boldsymbol{x}(3\boldsymbol{y}^2 + 10\boldsymbol{y} + 3),$$

obtaining an expression for $\boldsymbol{y}$ in terms of $\boldsymbol{x}$. [9]

9709/31/M/J/15

24 The number of organisms in a population at time t is denoted by x. Treating x as a continuous variable, the differential equation satisfied by x and t is

$$\frac{dx}{dt} = \frac{xe^{-t}}{k + e^{-t}},$$

where k is a positive constant.

(i) Given that $x = 10$ when $t = 0$, solve the differential equation, obtaining a relation between x, k and t. [6]

(ii) Given also that $x = 20$ when $t = 1$, show that $k = 1 - \dfrac{2}{e}$. [2]

(iii) Show that the number of organisms never reaches 48, however large t becomes. [2]

9709/32/M/J/15

25 The number of micro-organisms in a population at time $\boldsymbol{t}$ is denoted by $\boldsymbol{M}$. At any time the variation in $\boldsymbol{M}$ is assumed to satisfy the differential equation

$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}\boldsymbol{t}} = \boldsymbol{k}(\sqrt{\boldsymbol{M}})\cos(0.02\boldsymbol{t}),$$

where $\boldsymbol{k}$ is a constant and $\boldsymbol{M}$ is taken to be a continuous variable. It is given that when $\boldsymbol{t} = 0$, $\boldsymbol{M} = 100$.

(i) Solve the differential equation, obtaining a relation between $\boldsymbol{M}$, $\boldsymbol{k}$ and $\boldsymbol{t}$. [5]

(ii) Given also that $\boldsymbol{M} = 196$ when $\boldsymbol{t} = 50$, find the value of $\boldsymbol{k}$. [2]

(iii) Obtain an expression for $\boldsymbol{M}$ in terms of $\boldsymbol{t}$ and find the least possible number of micro-organisms. [2]

9709/33/M/J/15

26 The variables x and θ satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = (x+2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

9709/31/O/N/15

27 The variables x and y satisfy the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = y(1-2x^2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x in a form not involving logarithms. [6]

9709/31/M/J/16

28 The variables x and θ satisfy the differential equation

$$(3+\cos 2\theta)\frac{\mathrm{d}x}{\mathrm{d}\theta} = x\sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

(ii) State the least value taken by x. [1]

9709/32/M/J/16

29 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{-2y} \tan^2 x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and it is given that $y = 0$ when $x = 0$. Solve the differential equation and calculate the value of y when $x = \frac{1}{4}\pi$. [8]

9709/33/M/J/16

30 A large field of area $4\ \text{km}^2$ is becoming infected with a soil disease. At time t years the area infected is $x\ \text{km}^2$ and the rate of growth of the infected area is given by the differential equation $\frac{dx}{dt} = kx(4-x)$, where k is a positive constant. It is given that when $t = 0$, $x = 0.4$ and that when $t = 2$, $x = 2$.

(i) Solve the differential equation and show that $k = \frac{1}{4}\ln 3$. [9]

(ii) Find the value of t when 90% of the area of the field is infected. [2]

9709/31/O/N/16

31 **(i)** Express $\frac{1}{x(2x+3)}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

9709/31/M/J/17

32 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = 4\cos^2 y \tan x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

9709/31/O/N/17

33 The variables x and y satisfy the differential equation

$$(x+1)\frac{dy}{dx} = y(x+2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x. [7]

9709/32/O/N/17

34 In a certain chemical reaction the amount, x grams, of a substance is decreasing. The differential equation relating x and t, the time in seconds since the reaction started, is

$$\frac{dx}{dt} = -kx\sqrt{t},$$

where k is a positive constant. It is given that $x = 100$ at the start of the reaction.

(i) Solve the differential equation, obtaining a relation between x, t and k. [5]

(ii) Given that $t = 25$ when $x = 80$, find the value of t when $x = 40$. [3]

9709/31/M/J/18

35 **(i)** Express $\dfrac{1}{4-y^2}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x\frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

9709/33/M/J/18

36 The coordinates (x, y) of a general point on a curve satisfy the differential equation

$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

9709/31/O/N/18

37 **(i)** Differentiate $\dfrac{1}{\sin^2\theta}$ with respect to θ. [2]

(ii) The variables x and θ satisfy the differential equation

$$x\tan\theta\frac{dx}{d\theta} + \operatorname{cosec}^2\theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ. [6]

9709/31/M/J/19

38 The variables x and y satisfy the differential equation $\dfrac{dy}{dx} = xe^{x+y}$. It is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation, obtaining y in terms of x. [7]

(ii) Explain why x can only take values that are less than 1. [1]

9709/32/M/J/19

39 The variables x and y satisfy the differential equation

$$(x+1)y\frac{dy}{dx} = y^2 + 5.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation obtaining an expression for y^2 in terms of x. [7]

9709/33/M/J/19

40 The variables x and θ satisfy the differential equation

$$\sin\tfrac{1}{2}\theta\frac{dx}{d\theta} = (x+2)\cos\tfrac{1}{2}\theta$$

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos\theta$. [8]

9709/32/O/N/19

41 The variables x and t satisfy the differential equation $5\frac{dx}{dt} = (20 - x)(40 - x)$. It is given that $x = 10$ when $t = 0$.

(i) Using partial fractions, solve the differential equation, obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x when t becomes large. [1]

9709/33/O/N/19