I6 WITH SUBSTITUTION MARKING SCHEME

1 **(i)** State or imply $dx = \sec^2\theta \, d\theta$ **or** $\frac{dx}{d\theta} = \sec^2\theta$ **B1**

Substitute for x and dx throughout the integral **M1**

Obtain integral in terms of θ in any correct form **A1**

Reduce to the given form correctly **A1** **4**

(ii) State integral $\frac{1}{2}\sin 2\theta$ **B1**

Use limits $\theta = 0$ and $\theta = \frac{1}{4}\pi$ correctly in integral of the form $k \sin 2\theta$ **M1**

Obtain answer ½ or 0.5 **A1** **3**

2 (i) State $2(3y^2)\frac{dy}{dx}$ as derivative of $2y^3$, or equivalent B1

State $3x\frac{dy}{dx} + 3y$ as derivative of $3xy$, or equivalent B1

Solve for $\frac{dy}{dx}$ M1

Obtain given answer correctly A1 4

[The M1 is dependent on at least one of the B marks being obtained.]

(ii) State or imply that the coordinates satisfy $y - x^2 = 0$ B1

Obtain an equation in x (or in y) M1

Solve and obtain $x = 1$ only (or $y = 1$ only) A1

Substitute x- (or y-)value in $y - x^2 = 0$ or in the equation of the curve M1

Obtain $y = 1$ only (or $x = 1$ only) A1 5

[SR: If B1 is earned and (1, 1) stated to be the only solution with no other evidence, award B2. If the point is also shown to lie on the curve award a further B2.]

3 (i) State or imply $du = \frac{1}{2\sqrt{x}}dx$, or $2u\,du = dx$, or $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, or equivalent B1

Substitute for x and dx throughout the integral M1

Obtain the given form of indefinite integral correctly with no errors seen A1 3

(ii) Attempting to express the integrand as $\frac{A}{u} + \frac{B}{4-u}$, use a correct method to find either A or B M1*

Obtain $A = \frac{1}{2}$ and $B = \frac{1}{2}$ A1

Integrate and obtain $\frac{1}{2}\ln u - \frac{1}{2}\ln(4-u)$, or equivalent A1√ + A1√

Use limits $u = 1$ and $u = 2$ correctly, or equivalent, in an integral of the form $c \ln u + d \ln(4-u)$ M1(dep*)

Obtain given answer correctly following full and exact working A1 6

4 **(i)** State or imply $\frac{dx}{d\theta} = 2\sec^2\theta$ or $dx = 2\sec^2\theta \, d\theta$ B1

Substitute for x and dx throughout M1

Obtain any correct form in terms of θ A1

Obtain the given form correctly (including the limits) A1 [4]

(ii) Use cos 2A formula, replacing integrand by $a + b\cos 2\theta$, where $ab \neq 0$ M1*

Integrate and obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$ A1

Use limits $\theta = 0$ and $\theta = \frac{1}{4}\pi$ M1(dep*)

Obtain answer $\frac{1}{8}(\pi + 2)$, or exact quivalent A1 [4]

5 **(i)** State or imply $\mathrm{d}x = 2\cos\theta\,\mathrm{d}\theta$, or $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$, or equivalent — B1
Substitute for x and $\mathrm{d}x$ throughout the integral — M1
Obtain the given answer correctly, having changed limits and shown sufficient working — A1 [3]

(ii) Replace integrand by $2 - 2\cos 2\theta$, or equivalent — B1
Obtain integral $2\theta - \sin 2\theta$, or equivalent — B1√
Substitute limits correctly in an integral of the form $a\theta \pm b\sin 2\theta$, where ab Þ 0 — M1
Obtain answer $\dfrac{1}{3}\pi - \dfrac{\sqrt{3}}{2}$ or exact equivalent — A1 [4]
[The f.t. is on integrands of the form $a + c\cos 2\theta$, where ac Þ 0.]

6 **(i)** State or imply $\mathrm{d}x = 2t\,\mathrm{d}t$ or equivalent — B1
Express the integral in terms of x and $\mathrm{d}x$ — M1
Obtain given answer $\displaystyle\int_1^5 (2x-2)\ln x\,\mathrm{d}x$, including change of limits **AG** — A1 [3]

(ii) Attempt integration by parts obtaining $(ax^2 + bx)\ln x \pm \displaystyle\int (ax^2+bx)\frac{1}{x}\,\mathrm{d}x$ or equivalent — M1
Obtain $(x^2 - 2x)\ln x - \displaystyle\int (x^2-2x)\frac{1}{x}\,\mathrm{d}x$ or equivalent — A1
Obtain $(x^2 - 2x)\ln x - \frac{1}{2}x^2 + 2x$ — A1
Use limits correctly having integrated twice — M1
Obtain $15\ln 5 - 4$ or exact equivalent — A1 [5]
[Equivalent for M1 is $(2x-2)(ax\ln x + bx) - \displaystyle\int (ax\ln x + bx)\,2\mathrm{d}x$]

7 **(i)** State or imply $2u\,\mathrm{d}u = -\mathrm{d}x$, or equivalent — B1
Substitute for x and $\mathrm{d}x$ throughout
Obtain integrand $\dfrac{-10u}{6-u^2+u}$, or equivalent — A1
Show correct working to justify the change in limits and obtain the given answer correctly — A1 [4]

(ii) State or imply the form of fractions $\dfrac{A}{3-u} + \dfrac{B}{2+u}$ and use a relevant method to find A or B — M1
Obtain $A = 6$ and $B = -4$ — A1
Integrate and obtain $-6\ln(3-u) - 4\ln(2+u)$, or equivalent — A1√ + A1√
Substitute limits correctly in an integral of the form $a\ln(3-u) + b\ln(2+u)$ — M1
Obtain the given answer correctly having shown sufficient working — A1 [6]
[The f.t. is on A and B.]

8	(i)	State or imply du = 2cos2x dx or equivalent	B1	
		Express integrand in terms of u and du	M1	
		Obtain $\int \frac{1}{2}u^3\left(1-u^2\right)$ du or equivalent	A1	
		Integration to obtain an integral of the form $k_1u^4 + k_2u^6, k_1, k_2 \neq 0$	M1	
		Use limits 0 and 1 or (if reverting to x) 0 and $\frac{1}{4}\pi$ correctly	DM1	
		Obtain $\frac{1}{24}$, or equivalent	A1	[6]
	(ii)	Use 40 and upper limit from part **(i)** in appropriate calculation	M1	
		Obtain k = 10 with no errors seen	A1	[2]
9	(a)	Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{2}x^2 \mathrm{d}x$	M1*	
		Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2 \ \mathrm{d}x$	A1	
		Obtain $2x^2 \ln x - x^2$	A1	
		Use limits, having integrated twice	M1 (dep*)	
		Confirm given result $56 \ln 2 - 12$	A1	[5]
	(b)	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x} = 4\cos 4x$	B1	
		Carry out complete substitution except limits	M1	
		Obtain $\int (\frac{1}{4} - \frac{1}{4}u^2)\ \mathrm{d}u$ or equivalent	A1	
		Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2	M1	
		Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]
10	(i)	Use correct quotient or chain rule to differentiate sec x	M1	
		Obtain given derivative, sec x tan x, correctly	A1	
		Use chain rule to differentiate y	M1	
		Obtain the given answer	A1	[4]
	(ii)	Using $\mathrm{d}x\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$, or equivalent, express integral in terms of θ and dθ	M1	
		Obtain $\int \sec\theta\ \mathrm{d}\theta$	A1	
		Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
		Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]

11	Carry out complete substitution including the use of $\frac{du}{dx} = 3$	M1	
	Obtain $\int \left(\frac{1}{3} - \frac{1}{3u} \right) du$	A1	
	Integrate to obtain form $k_1 u + k_2 \ln u$ or $k_1 u + k_2 \ln 3u$ where $k_1 k_2 \neq 0$	M1	
	Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and + c	A1	[4]

12	State $\frac{du}{dx} = 3\sec^2 x$ or equivalent	B1	
	Express integral in terms of u and du (accept unsimplified and without limits)	M1	
	Obtain $\int \frac{1}{3} u^{\frac{1}{2}}\, du$	A1	
	Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3} u^{\frac{3}{2}}$	M1	
	Obtain $\frac{14}{9}$	A1	[5]

13	**(i)**	Substitute for x and dx throughout using $u = \sin x$ and $du = \cos x\, dx$, or equivalent	M1	
		Obtain integrand e^{2u}	A1	
		Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
		Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
		Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**
	(ii)	Use chain rule or product rule	M1	
		Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x} \cos x - e^{2\sin x} \sin x$	A1 + A1	
		Equate derivative to zero and obtain a quadratic equation in $\sin x$	M1	
		Solve a 3-term quadratic and obtain a value of x	M1	
		Obtain answer 0.896	A1	**6**

14	State or imply $\dfrac{du}{dx} = e^x$	B1	
	Substitute throughout for x and dx	M1	
	Obtain $\displaystyle\int \frac{u}{u^2 + 3u + 2}\,du$ or equivalent (ignoring limits so far)	A1	
	State or imply partial fractions of form $\dfrac{A}{u+2} + \dfrac{B}{u+1}$, following their integrand	B1	
	Carry out a correct process to find at least one constant for their integrand	M1	
	Obtain correct $\dfrac{2}{u+2} - \dfrac{1}{u+1}$	A1	
	Integrate to obtain $a\ln(u+2) + b\ln(u+1)$	M1	
	Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their A and B	A1√	
	Apply appropriate limits and use at least one logarithm property correctly	M1	
	Obtain given answer $\ln\dfrac{8}{5}$ legitimately	A1	[10]
	SR for integrand $\dfrac{u^2}{u(u+1)(u+2)}$		
	State or imply partial fractions of form $\dfrac{A}{u} + \dfrac{B}{u+1} + \dfrac{C}{u+2}$	(B1)	
	Carry out a correct process to find at least one constant	(M1)	
	Obtain correct $\dfrac{2}{u+2} - \dfrac{1}{u+1}$	(A1)	
	…complete as above.		
15 **(i)**	State or imply $du = -\dfrac{1}{2\sqrt{x}}dx$, or equivalent	B1	
	Substitute for x and dx throughout	M1	
	Obtain integrand $\dfrac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
	Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
(ii)	Integrate and obtain at least two terms of the form $a\ln u, bu,$ and cu^2	M1*	
	Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
	Substitute limits correctly	M1(dep*)	
	Obtain the given answer correctly having shown sufficient working	A1	**[4]**

16 State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent **B1**
Use identity $\sin 2x = 2\sin x\cos x$ **B1**
Carry out complete substitution, for x and $\mathrm{d}x$ **M1**
Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent **A1**

Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$ **M1***
Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$ **A1**

Apply correct limits correctly **dep M1***
Obtain $\frac{20}{3}$ or exact equivalent **A1** [8]

17 **(i)** State or imply $\mathrm{d}u = 2x\,\mathrm{d}x$, or equivalent **B1**
Substitute for x and $\mathrm{d}x$ throughout **M1**
Reduce to the given form and justify the change in limits **A1**
[3]

(ii) Convert integrand to a sum of integrable terms and attempt integration **M1**
Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent **A1 + A1**
(deduct A1 for each error or omission)
Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2} **M1**
Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent **A1**
[5]

18 **(i)**	State or imply $\mathrm{d}u = \frac{1}{2\sqrt{x}}\mathrm{d}x$ Substitute for x and $\mathrm{d}x$ throughout Justify the change in limits and obtain the given answer	**B1** **M1** **A1**	[3]
(ii)	Convert integrand into the form $A + \frac{B}{u\quad 1}$ Obtain integrand A =1, B = –2 Integrate and obtain $u - 2\ln(u+1)$ Substitute limits correctly in an integral containing terms au and $b\ln(u + 1)$, where $ab \neq 0$ Obtain the given answer following full and correct working [The f.t. is on A and B.]	**M1*** **A1** **A1**√ + **A1**√ **DM1** **A1**	[6]

19(i)	State or imply $\mathrm{d}u = -\sin x\ \mathrm{d}x$	**B1**
	Using correct double angle formula, express the integral in terms of u and $\mathrm{d}u$	**M1**
	Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2 \,\mathrm{d}u$ with no errors seen	**A1**
	Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
	Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
	Total:	**6**

19(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in cos and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2$ 1, or equivalent	**A1**
	Obtain answer 0.32	**A1**

20(i)	Use the chain rule	**M1**
	Obtain correct derivative in any form	**A1**
	Use correct trigonometry to express derivative in terms of $\tan x$	**M1**
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4\tan x}{4 + \tan^2 x}$, or equivalent	**A1**
	Total:	**4**
20(ii)	Equate derivative to –1 and solve a 3–term quadratic for $\tan x$	**M1**
	Obtain answer x=1.11 and no other in the given interval	**A1**
	Total:	**2**

21(i)	Use product rule	**M1***
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**
	Obtain answer $x = 0.685$	**A1**
		5
21(ii)	Use the given substitution and reach $a\int u^2 - u^4)\mathrm{d}u$	**M1**
	Obtain correct integral with $a = 5$ and limits 0 and 1	**A1**
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	**M1**
	Obtain answer $\frac{2}{3}$	**A1**

22(i)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**
	Obtain answer $x = 1.26$	**A1**
		5

22(ii)	Using $\mathrm{d}u = \pm \sin x\,\mathrm{d}x$ express integrand in terms of u and $\mathrm{d}u$	**M1**	
	Obtain integrand $\mathrm{e}^{u}\left(u^{2}-1\right)$	**A1**	OE
	Commence integration by parts and reach $a\mathrm{e}^{u}\left(u^{2}-1\right)+b\int u\mathrm{e}^{u}\,\mathrm{d}u$	***M1**	
	Obtain $\mathrm{e}^{u}\left(u^{2}-1\right)-2\int u\mathrm{e}^{u}\,\mathrm{d}u$	**A1**	OE
	Complete integration, obtaining $\mathrm{e}^{u}\left(u^{2}-2u+1\right)$	**A1**	OE
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**	
	Obtain answer $\frac{4}{\mathrm{e}}$, or exact equivalent	**A1**	
		7	