I4 WITH TRIG IDENTITY QUESTIONS

1 **(i)** Prove the identity

$$\sin^2\theta\cos^2\theta \equiv \tfrac{1}{8}(1-\cos 4\theta). \qquad [3]$$

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^2\theta\cos^2\theta\,d\theta. \qquad [3]$$

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2 **(i)** Using the expansions of $\cos(3x-x)$ and $\cos(3x+x)$, prove that

$$\tfrac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x. \qquad [3]$$

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x\,dx = \tfrac{1}{8}\sqrt{3}. \qquad [3]$$

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3 **(i)** Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, [3]

(b) find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \cos^4\theta\,d\theta$. [3]

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4 **(i)** By differentiating $\dfrac{1}{\cos \boldsymbol{x}}$, show that if $\boldsymbol{y} = \sec \boldsymbol{x}$ then $\dfrac{d\boldsymbol{y}}{d\boldsymbol{x}} = \sec \boldsymbol{x} \tan \boldsymbol{x}$. [2]

(ii) Show that $\dfrac{1}{\sec \boldsymbol{x} - \tan \boldsymbol{x}} \equiv \sec \boldsymbol{x} + \tan \boldsymbol{x}$. [1]

(iii) Deduce that $\dfrac{1}{(\sec \boldsymbol{x} - \tan \boldsymbol{x})^2} \equiv 2\sec^2 \boldsymbol{x} - 1 + 2\sec \boldsymbol{x} \tan \boldsymbol{x}$. [2]

(iv) Hence show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec \boldsymbol{x} - \tan \boldsymbol{x})^2}\,d\boldsymbol{x} = \tfrac{1}{4}(8\sqrt{2} - \pi)$. [3]

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5 **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta \, d\theta = \frac{1}{2}\ln 3$. [4]

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6 **(i)** Prove the identity $\tan 2\theta - \tan\theta \equiv \tan\theta \sec 2\theta$. [4]

(ii) Hence show that $\int_0^{\frac{1}{6}\pi} \tan\theta \sec 2\theta \, d\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

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7 **(i)** Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

(iii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1+\sin\theta}{1-\sin\theta} \, d\theta$. [4]

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8 **(i)** Show that $\dfrac{2\sin x - \sin 2x}{1-\cos 2x} \equiv \dfrac{\sin x}{1+\cos x}$. [4]

(ii) Hence, showing all necessary working, find $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2\sin x - \sin 2x}{1-\cos 2x} \, dx$, giving your answer in the form $\ln k$. [4]

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9 **(i)** By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3\sin x - 4\sin^3 x$. [4]

(ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx$. [4]

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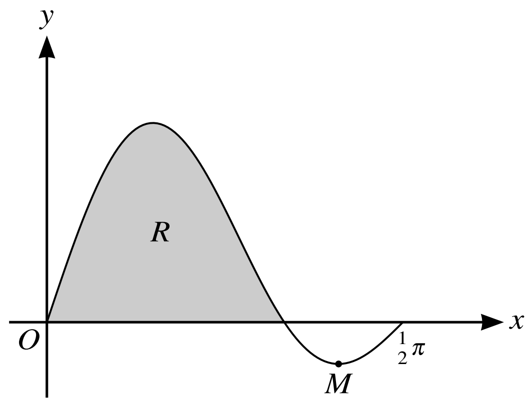

The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

(iii) Using the result of part **(i)**, express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

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11 Let $\mathrm{f}(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $\mathrm{f}(\theta) = \tan \theta$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \mathrm{f}(\theta)\,\mathrm{d}\theta = \tfrac{1}{2}\ln \tfrac{3}{2}$. [4]

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12 **(i)** By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

(ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leqslant x \leqslant \pi$. [2]

(iii) Find the exact value of $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x\,\mathrm{d}x$. [4]

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