ALEVELS P3

INTEGRATION WITH PARTIAL
I2

1 (a) Show that $\displaystyle\int_3^4 \frac{3x}{(x+1)(x-2)}\,dx = \ln 5.$ [6]

9709/03/O/N/04

2 Let $f(x) = \dfrac{7x+4}{(2x+1)(x+1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^2 f(x)\,dx = 2 + \ln\tfrac{5}{3}.$ [5]

9709/03/O/N/06

3 Let $f(x) \equiv \dfrac{x^2+3x+3}{(x+1)(x+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^3 f(x)\,dx = 3 - \tfrac{1}{2}\ln 2.$ [4]

9709/03/M/J/08

4 **(i)** Express $\dfrac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part **(i)**, show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}.$$ [2]

(iii) Hence show that $\displaystyle\int_0^1 \frac{4}{(x+1)^2(x+3)^2}\,dx = \tfrac{7}{12} - \ln\tfrac{3}{2}.$ [5]

9709/31/M/J/10

5 **(i)** Find the values of the constants A, B, C and D such that

$$\frac{2x^3-1}{x^2(2x-1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x-1}.$$ [5]

(ii) Hence show that

$$\int_1^2 \frac{2x^3-1}{x^2(2x-1)}\,dx = \tfrac{3}{2} + \tfrac{1}{2}\ln\left(\tfrac{16}{27}\right).$$ [5]

6 Show that $\displaystyle\int_0^7 \frac{2x+7}{(2x+1)(x+2)}\,dx = \ln 50.$ [7]

9709/33/O/N/10

7 Let $f(x) = \dfrac{12+8x-x^2}{(2-x)(4+x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{2-x} + \dfrac{Bx+C}{4+x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 f(x)\,dx = \ln\left(\tfrac{25}{2}\right)$. [5]

9709/31/O/N/11

8 By first expressing $\dfrac{4x^2+5x+3}{2x^2+5x+2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2+5x+3}{2x^2+5x+2}\,dx = 8 - \ln 9.$$ [10]

9709/31/M/J/12

9 Let $f(x) = \dfrac{4x^2-7x-1}{(x+1)(2x-3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_2^6 f(x)\,dx = 8 - \ln\left(\tfrac{49}{3}\right)$. [5]

9709/33/M/J/12

10 Let $f(\boldsymbol{x}) = \dfrac{6+6\boldsymbol{x}}{(2-\boldsymbol{x})(2+\boldsymbol{x}^2)}$.

(i) Express $f(\boldsymbol{x})$ in the form $\dfrac{\boldsymbol{A}}{2-\boldsymbol{x}} + \dfrac{\boldsymbol{Bx}+\boldsymbol{C}}{2+\boldsymbol{x}^2}$. [4]

(ii) Show that $\displaystyle\int_{-1}^1 f(\boldsymbol{x})\,d\boldsymbol{x} = 3\ln 3$. [5]

9709/33/M/J/14

11 Let $f(\boldsymbol{x}) = \dfrac{11\boldsymbol{x}+7}{(2\boldsymbol{x}-1)(\boldsymbol{x}+2)^2}$.

(i) Express $f(\boldsymbol{x})$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_1^2 f(\boldsymbol{x})\,d\boldsymbol{x} = \tfrac{1}{4} + \ln\left(\tfrac{9}{4}\right)$. [5]

9709/33/M/J/15

12 Let $f(x) = \dfrac{4x^2 + 12}{(x+1)(x-3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/31/M/J/16

13 Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x+1)(x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x)\,dx = 8 - \ln 3$. [5]

9709/32/M/J/16

14 Let $f(x) = \dfrac{10x - 2x^2}{(x+3)(x-1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/33/M/J/16

15 Let $f(x) = \dfrac{3x^2 + x + 6}{(x+2)(x^2+4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9709/33/O/N/16

16 Let $f(x) = \dfrac{3x^2 - 4}{x^2(3x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_1^2 f(x)\,dx = \ln\left(\frac{25}{8}\right) - 1$. [5]

9709/33/M/J/17

17 Let $\mathrm{f}(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $\mathrm{f}(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\displaystyle\int_1^4 \mathrm{f}(x)\,\mathrm{d}x = 6 + \tfrac{1}{2}\ln\left(\tfrac{16}{7}\right)$. [5]

9709/31/O/N/17

18 Let $\mathrm{f}(x) = \dfrac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\displaystyle\int_{-1}^0 \mathrm{f}(x)\,\mathrm{d}x = 1 + \tfrac{1}{2}\ln\left(\tfrac{3}{4}\right)$. [5]

9709/31/O/N/18

19 Let $\mathrm{f}(x) = \dfrac{10x + 9}{(2x+1)(2x+3)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \tfrac{1}{2}\ln\tfrac{9}{5} + \tfrac{1}{5}$. [5]

9709/32/M/J/19

20 Let $\mathrm{f}(x) = \dfrac{x^2 + x + 6}{x^2(x+2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence, showing full working, show that the exact value of $\displaystyle\int_1^4 \mathrm{f}(x)\,\mathrm{d}x$ is $\tfrac{9}{4}$. [5]

9709/31/O/N/19

21 Let $\mathrm{f}(x) = \dfrac{2x^2 + x + 8}{(2x-1)(x^2+2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\displaystyle\int_1^5 \mathrm{f}(x)\,\mathrm{d}x$, giving the answer in the form $\ln c$, where c is an integer. [5]

9709/32/O/N/19