D3 Implicit Answers
P3

1

1 (i) $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ divides by $\cos^2\theta$ $\rightarrow \tan^2\theta + 3\tan\theta = 4$	M1 A1 [2]	Knowing to divide by $\cos^2\theta$ Correct quadratic (not nec = 0)
(ii) Solution $\tan\theta = 1$ or $\tan\theta = -4$	M1	Correct solution of quadratic = 0
$\rightarrow \theta = 45°$ or $104.0°$	A1 A1 [3]	Correct only for each one.

2

6 (i) State $2(3y^2)\frac{dy}{dx}$ as derivative of $2y^3$, or equivalent	B1	
State $3x\frac{dy}{dx} + 3y$ as derivative of $3xy$, or equivalent	B1	
Solve for $\frac{dy}{dx}$	M1	
Obtain given answer correctly [The M1 is dependent on at least one of the B marks being obtained.]	A1	4
(ii) State or imply that the coordinates satisfy $y - x^2 = 0$	B1	
Obtain an equation in x (or in y)	M1	
Solve and obtain $x = 1$ only (or $y = 1$ only)	A1	
Substitute x- (or y-)value in $y - x^2 = 0$ or in the equation of the curve	M1	
Obtain $y = 1$ only (or $x = 1$ only) [SR: If B1 is earned and (1, 1) stated to be the only solution with no other evidence, award B2. If the point is also shown to lie on the curve award a further B2.]	A1	5

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3 **EITHER** State $x^2 \frac{dy}{dx} + 2xy$, or equivalent, as derivative of x^2y B1

State $y^2 + 2xy\frac{dy}{dx}$, or equivalent, as derivative of xy^2 B1

OR State $xy(1 + \frac{dy}{dx})$, or equivalent, as a term in an attempt to apply the product rule B1

State $(y + x\frac{dy}{dx})(x + y)$, or equivalent, in an attempt to apply the product rule B1

Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero M1

Obtain a horizontal equation, e.g. $y^2 = -2xy$, or $y = -2x$, or equivalent A1√

Explicitly reject $y = 0$ as a possibility A1

Obtain an equation in x (or in y) M1

Obtain $x = a$ A1

Obtain $y = -2a$ only A1 [8]

[The first M1 is dependent on at least one B mark having been earned.]

[SR: for an attempt using $(x + y) = 2a^3 / xy$, the B marks are given for the correct derivatives of the two sides of the equation, and the M1 for setting $\frac{dy}{dx}$ equal to zero.]

[SR: for an attempt which begins by expressing y in terms of x, give M1A1 for a reasonable attempt at differentiation, M1A1√ for setting $\frac{dy}{dx}$ equal to zero and obtaining an equation free of surds, A1 for solving and obtaining $x = a$; then M1 for obtaining an equation for y, A1 for $y = -2a$ and A1 for finding and rejecting $y = a$ as a possibility.]

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4 **(i)** State $2xy + x^2 \frac{dy}{dx}$ as derivative of x^2y B1

State $3y^2 \frac{dy}{dx}$ as derivative of y^3 B1

Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ M1

Obtain answer $\frac{3x^2 - 2xy}{x^2 + 3y^2}$, or equivalent A1 [4]

(ii) Find gradient of tangent at (2, 1) and form equation of tangent M1

Obtain answer $8x - 7y - 9 = 0$, or equivalent A1√ [2]

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5 **(i)** *EITHER*: State or imply $\frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln y$ B1

State correct derivative of LHS, e.g. $\ln y + \frac{x}{y}\frac{dy}{dx}$ B1

Differentiate RHS and obtain an expression for $\frac{dy}{dx}$ M1

Obtain given answer A1

OR 1: State $\ln y = \frac{2x+1}{x}$, or equivalent, and differentiate both sides M1

State correct derivative of LHS, e.g. $\frac{1}{y}\frac{dy}{dx}$ B1

State correct derivative of RHS, e.g. $-1/x^2$ B1

Rearrange and obtain given answer A1

OR 2: State $y = \exp(2+1/x)$, or equivalent, and attempt differentiation by chain rule M1

State correct derivative of RHS, e.g. $-\exp(2+1/x)/x^2$ B1 + B1

Obtain given answer A1 [4]

[The B marks are for the exponential term and its multiplier.]

(ii) State or imply $x = -\frac{1}{2}$ when $y = 1$ B1

Substitute and obtain gradient of –4 B1√

Correctly form equation of tangent M1

Obtain final answer $y + 4x + 1 = 0$, or equivalent A1 [4]

6 **(i)** Obtain $2y\frac{dy}{dx}$ as derivative of y^2 B1

Obtain $-4y - 4x\frac{dy}{dx}$ as derivative of $-4xy$ B1

Substitute $x = 2$ and $y = -3$ and find value of $\frac{dy}{dx}$

(dependent on at least one B1 being earned and $\frac{d(45)}{dx} = 0$) M1

Obtain $\frac{12}{7}$ or equivalent A1 [4]

(ii) Substitute $\frac{dy}{dx} = 1$ in an expression involving $\frac{dy}{dx}$, x and y and obtain $ay = bx$ M1

Obtain $y = x$ or equivalent A1

Uses $y = x$ in original equation and demonstrate contradiction A1 [3]

7 **(i)** *EITHER*: State or imply $\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln xy$, or equivalent — B1

State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3, or equivalent — B1

Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ — M1

Obtain the given answer — A1

OR Obtain $xy = \exp(1 + y^3)$ and state or imply $y + x\frac{dy}{dx}$ as derivative of xy — B1

State or imply $3y^2\frac{dy}{dx}\exp(1 + y^3)$ as derivative of $(1 + y^3)$ — B1

Equate derivatives and solve for $\frac{dy}{dx}$ — M1

Obtain the given answer — A1 [4]

[The M1 is dependent on at least one of the B marks being earned]

(ii) Equate denominator to zero and solve for y — M1*

Obtain $y = 0.693$ only — A1

Substitute found value in the equation and solve for x — M1(dep*)

Obtain $x = 5.47$ only — A1 [4]

8 **(i)** Use correct quotient rule or equivalent — M1

Obtain $\frac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent — A1

Substitute $x = 0$ and obtain $-\frac{1}{2}$ or equivalent — A1 [3]

(ii) Differentiate y^3 and obtain $3y^2\frac{dy}{dx}$ — B1

Differentiate $5xy$ and obtain $5y + 5x\frac{dy}{dx}$ — B1

Obtain $6x^2 + 5y + 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ — B1

9 *EITHER*: State $2ay\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of ay^2 B1

State $y^2 + 2xy\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of xy^2 B1

Equate derivative of LHS to zero and set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero M1

Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent A1

Eliminate y and obtain an equation in x M1

Solve for x and obtain answer $x = \sqrt{3}a$ A1

OR1: Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x + a}$ and attempt differentiation of one side B1

Use correct quotient or product rule to differentiate RHS M1

Obtain correct derivative of RHS in any form A1

Set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero and obtain an equation in x M1

Obtain a correct horizontal equation free of surds A1

Solve for x and obtain answer $x = \sqrt{3a}$ A1

OR2: Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x + a}\right)^{\frac{1}{2}}$ and differentiation of RHS B1

Use correct quotient or product rule and chain rule M1

Obtain correct derivative in any form A1

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10 Differentiate y^3 to obtain $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ B1

Use correct product rule at least once *M1

Obtain $6\mathrm{e}^{2x}y + 3\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^{x}y^3 + 3\mathrm{e}^{x}y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of LHS A1

Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ M1(d*M)

Obtain $-\frac{4}{3}$ or equivalent as **final answer** A1 [5]

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11 Obtain correct derivative of RHS in any form B1

Obtain correct derivative of LHS in any form B1

Set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero and obtain a horizontal equation M1

Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work A1

By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2 M1

Obtain $x = \frac{1}{2}\sqrt{3}$ A1

Obtain $y = \frac{1}{2}$ A1 **7**

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[3]

12 (**i**) State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2 y$ **B1**

State $3y^2 \frac{dy}{dx}$ as derivative of y^3 **B1**

Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ **M1**

Obtain the given answer **A1**

[4]

(**ii**) Equate numerator to zero **M1***
Obtain $x = 2y$, or equivalent **A1**
Obtain an equation in x or y **DM1***
Obtain the point $(-2, -1)$ **A1**
State the point (0, 1.44) **B1**

[5]

13	*EITHER*:	*EITHER*:	State $2xy + x^2 \frac{dy}{dx}$, or equivalent, as derivative of $x^2 y$	**B1**	
			State $6y^2 + 12xy \frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	**B1**	
		OR:	Differentiating LHS using correct product rule, state term $xy(1 - 6\frac{dy}{dx})$, or equivalent	**B1**	
			State term $(y + x\frac{dy}{dx})(x - 6y)$, or equivalent	**B1**	
			Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	**M1***	
			Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**	
			Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	**A1**	
			Obtain an equation in x or y	**DM1**	
			Obtain answer $(-3a, -a)$	**A1**	
	OR:		Rearrange to $y = \frac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x-6y)^2} \times .....$	**B1**	
			State term $(x - 6y) + x(1 - 6y')$, or equivalent	**B1**	
			Justify division by $x(x - 6y)$	**B1**	
			Set $\frac{dy}{dx}$ equal to zero	**M1***	
			Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**	
			Obtain an equation in x or y	**DM1**	
			Obtain answer $(-3a, -a)$	**A1**	[7]

Question		
14(i)	State or imply $y^3 + 3xy^2 \frac{dy}{dx}$ as derivative of xy^3	**B1**
	State or imply $4y^3 \frac{dy}{dx}$ as derivative of y^4	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
		4
14(ii)	Equate numerator to zero	***M1**
	Obtain $y = -2x$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	**A1**
		4

Question		
15(i)	State or imply $3x^2y + x^3 \frac{dy}{dx}$ as derivative of x^3y	**B1**
	State or imply $9xy^2 \frac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer AG	**A1**
		4
15(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	**M1**
	Obtain answer $x = a$ and $y = -a$	**A1**
	Obtain answer $x = -a$ and $y = a$	**A1**
	Consider and reject $y = 0$ and $x = y$ as possibilities	**B1**
		4

Question			
16(i)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**	
	State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	**B1**	$3x^2+6xy+3x^2\frac{dy}{dx}-3y^2\frac{dy}{dx}=0$
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**	Given answer so check working carefully
	Obtain the given answer	**A1**	
		4	
16(ii)	Equate derivative to – 1 and solve for y	**M1***	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	**M1(dep*)**	
	Obtain answer $(1, -2)$	**A1**	
	Obtain answer $(\sqrt[3]{3}, 0)$	**B1**	Must be exact e.g.
		4	

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Question		
17(i)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	**B1**
	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	**B1**
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
	Total:	**4**
17(ii)	Equate denominator to zero and solve for y	**M1***
	Obtain $y = 0$ and $x = a$	**A1**
	Obtain $y = \alpha x$ and substitute in curve equation to find x or to find y	**M1(dep*)**
	Obtain $x = -a$	**A1**
	Obtain $y = 2a$	**A1**
	Total:	**5**

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uestion			
18	State or imply $3y^2 + 6xy\frac{dy}{dx}$ as derivative of $3xy^2$	**B1**	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	**B1**	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	**M1**	$\left(\frac{dy}{dx} = \frac{x^2 + y^2}{y^2 - 2xy}\right)$ For incorrect derivative nee substitution
	Obtain final answer $\frac{10}{3}$ or equivalent	**A1**	3.33 or better. Allow $\frac{30}{9}$ ISW after correct an
		4	

Question			
19	State $4\ y + 2x^2\ \frac{\mathrm{d}y}{\mathrm{d}}$, or equivalent, as derivative of $2\ ^2y$	**B1**	
	State $y^2 + 2xy\ \frac{\mathrm{d}y}{\mathrm{d}}$, or equivalent, as derivative of y^2	**B1**	
	Equate attempted derivative of LHS to zero and set $\frac{\mathrm{d}y}{\mathrm{d}}$ equal to zero (or set numerator equal to zero)	***M1**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - 4xy}{2\ ^2 - 2xy}$
	Reject $y = 0$	**B1**	Allow from $y^2 \quad kxy = 0$
	Obtain $y = 4x$	**A1**	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	**DM1**	$8\ ^3 \quad 16x^3 = a^3$ or $\frac{y^3}{8} - \frac{y^3}{4} = a^3$
	Obtain $y = -2a$	**A1**	With no errors seen
		7	