Promise	Reading time (80%)	Solving Time (20%)

PROBABILITY

Study of Chance

Chance of rain = 80% $\longrightarrow$ Probability $= \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$

Chance of no rain = 20% $\longrightarrow$ Probability $= \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$

Impossible — Certain

0 — $\frac{1}{2}$ — 1

less probable | more probable

$\downarrow$

Equally likely

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}.$$

SAMPLE SPACES

List of all the possible outcomes in an experiment.

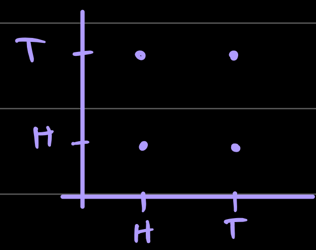
Event	Sample Space	Total.
1- Toss a coin	H, T	2

2 - Roll a dice — 1, 2, 3, 4, 5, 6 — 6

3. Toss Two coins — 4

HH TH

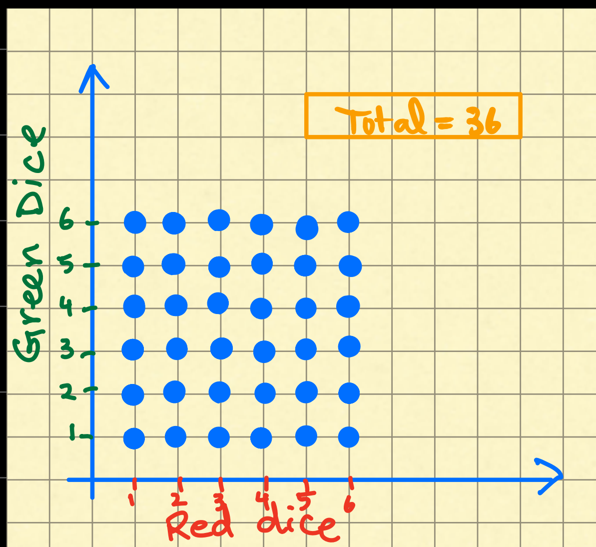
HT TT



P(both tails) = $\frac{1}{4}$

P(a head and a tail) = $\frac{2}{4} = \frac{1}{2}$

4. Roll two fair dices



(4, 6) v/s (6, 4)
R G R G

(5, 5)
R G

P(both same score) = $\frac{6}{36} = \frac{1}{6}$

P(sum of scores is 10) = $\frac{3}{36} = \frac{1}{12}$

(6, 4) (5, 5) (4, 6)

5- Toss a coin and roll a fair dice.

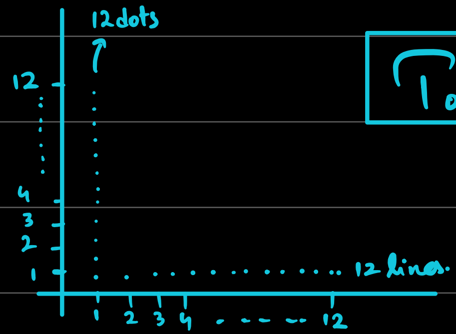
H4

$\boxed{12}$

H1	T1
H2	T2
H3	T3
H4	T4
H5	T5
H6	T6

6- Two 12 sided dices are rolled.

What is the probability that product of both scores is 24.



$\boxed{\text{Total} = 144}$

(4, 6) (6, 4) (2, 12) (12, 2) (3, 8) (8, 3)

$P(\text{product is } 24) = \frac{6}{144} = \frac{1}{24}$

Since we cannot make sample spaces for bigger experiments;

LISTING IS A PROPER TECHNIQUE

TREE DIAGRAMS

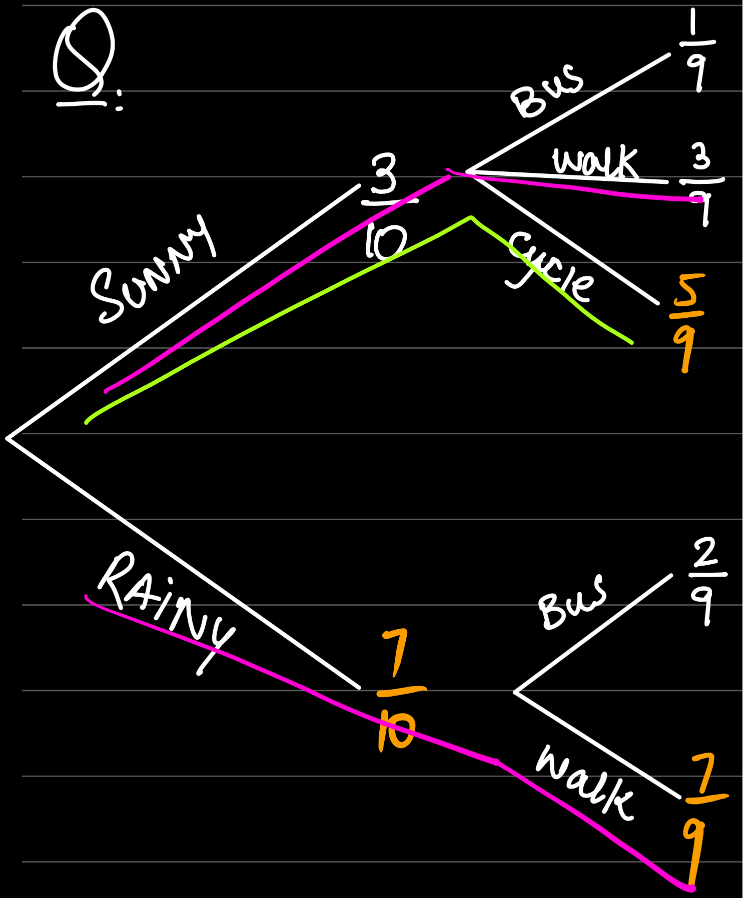
Rules:

1- Sum of all branches starting from one point is ALWAYS 1.

2- Branch total = Multiply probability.

3- More than one branch = Add individual branch total.

Q:



(i) $P(\text{sunny and cycle}) = \underset{\text{S}}{\frac{3}{10}} \times \underset{\text{C}}{\frac{5}{9}} = \frac{1}{6}$

S and C

(ii) $P(\text{walk})$

S and W or R and W

$$= \left(\frac{3}{10} \times \frac{3}{9}\right) + \left(\frac{7}{10} \times \frac{7}{9}\right)$$

$$= \frac{9}{90} + \frac{49}{90} = \frac{58}{90} = \frac{29}{45}$$

(iii) $P(\text{Not walk}) = 1 - \frac{29}{45}$

$$= \frac{16}{45}$$

Total = 45
Walk = 29
Not walk = 16

$P(W) = \frac{29}{45}$
$P(NW) = \frac{16}{45}$

AND = MULTIPLY
OR = ADD

CONDITIONAL PROBABILITY

A = First event B = Second event

Given that second event (B) has already happened, find the probability of the first event (A).

$$P(A \mid B) = \frac{P(A \text{ and } B \text{ together})}{P(B)}$$

A: To be found
|: given that
B: Already happened

(iv) Given that Ali goes to School by bus, find the probability that it was raining.

Weather (A)
Rain ?
Sunny

Transport (B)
Bus ✓
Walk
Cycle.

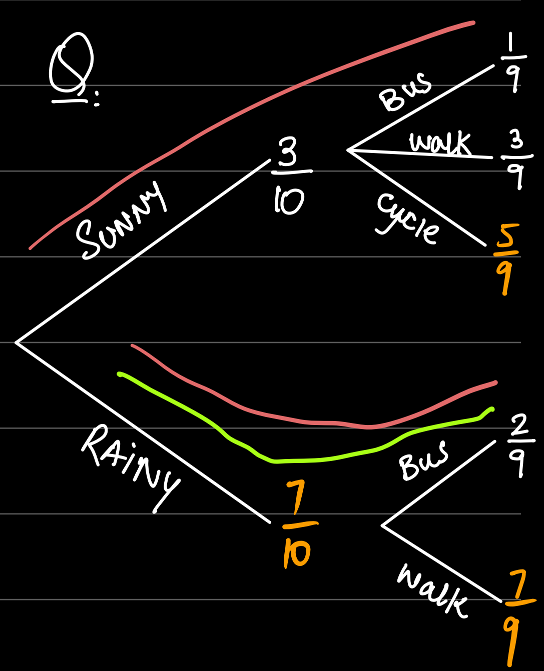
$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(\text{Rain} \mid \text{Bus}) = \frac{P(\text{Rain and Bus})}{P(\text{Bus})}$$

$$= \frac{\frac{7}{10} \times \frac{2}{9}}{\left(\frac{3}{10} \times \frac{1}{9}\right) + \left(\frac{7}{10} \times \frac{2}{9}\right)}$$

$$= \frac{14}{17}$$

Q:



R = Red

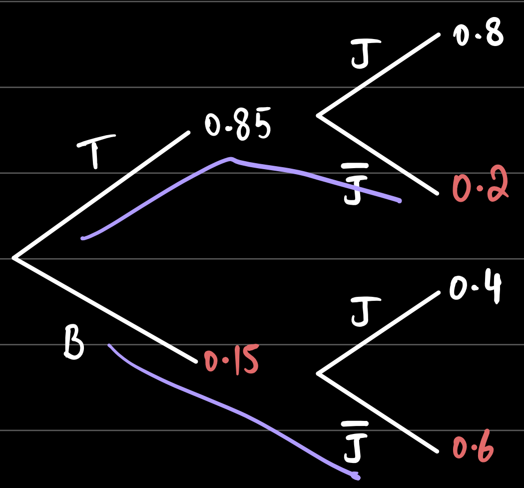
$\bar{R}$ = Not Red.

10 Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

(i) Draw a fully labelled tree diagram to show this information. [2]

second event has happened First event.

(ii) Given that Maria did **not** have jam for breakfast, find the probability that she had toast. [4]



$$P(T \mid \bar{J}) = \frac{P(T \text{ and } \bar{J})}{P(\bar{J})}$$

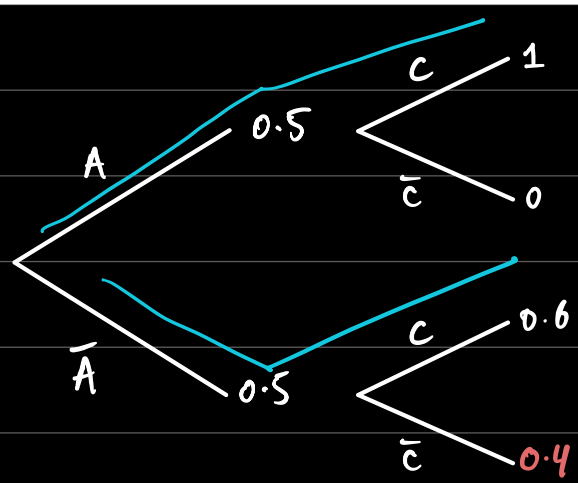
$$= \frac{(0.85)(0.2)}{(0.85)(0.2) + (0.15)(0.6)}$$

$$= \frac{17}{26}$$

6 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]



(i) $P(C) = (0.5)(1) + (0.5)(0.6)$

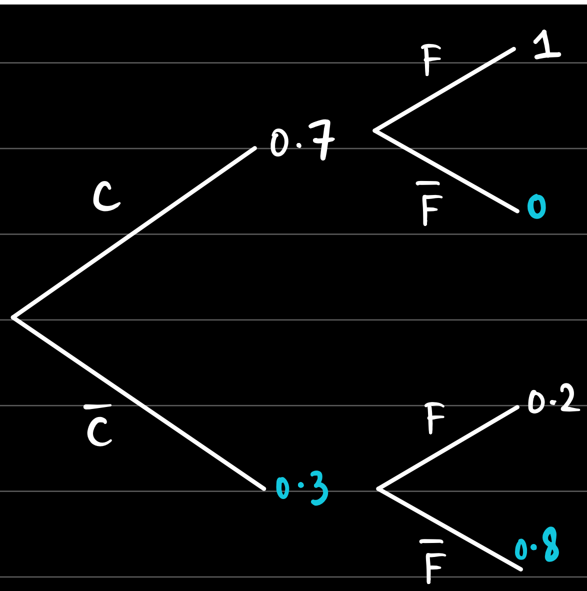
$$= 0.8$$

(ii) $P(A \mid C) = \dfrac{P(A \text{ and } C)}{P(C)}$

$$= \frac{(0.5)(1)}{0.8}$$

$$= \frac{5}{8}$$

17 When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]



$$P(C|F) = \frac{P(C \text{ and } F)}{P(F)}$$

$$= \frac{(0.7)(1)}{(0.7)(1) + (0.3)(0.2)}$$

$$= \frac{35}{38}$$

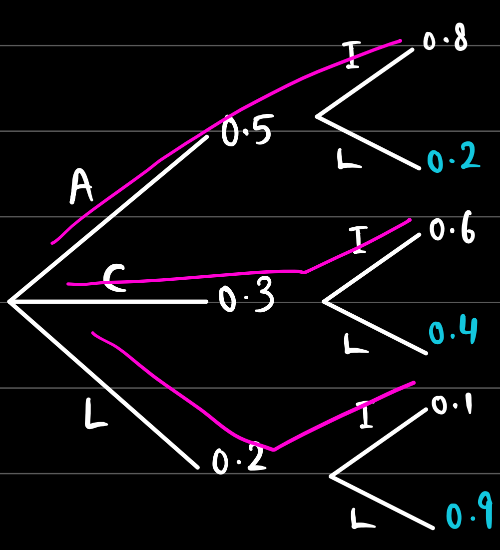
$$= 0.92105$$

21 Fabio drinks coffee each morning. He chooses Americano, Cappucino or Latte with probabilities 0.5, 0.3 and 0.2 respectively. If he chooses Americano he either drinks it immediately with probability 0.8, or leaves it to drink later. If he chooses Cappucino he either drinks it immediately with probability 0.6, or leaves it to drink later. If he chooses Latte he either drinks it immediately with probability 0.1, or leaves it to drink later.

(i) Find the probability that Fabio chooses Americano and leaves it to drink later. [1]

Second event happened

(ii) Fabio drinks his coffee immediately. Find the probability that he chose Latte. first event [4]



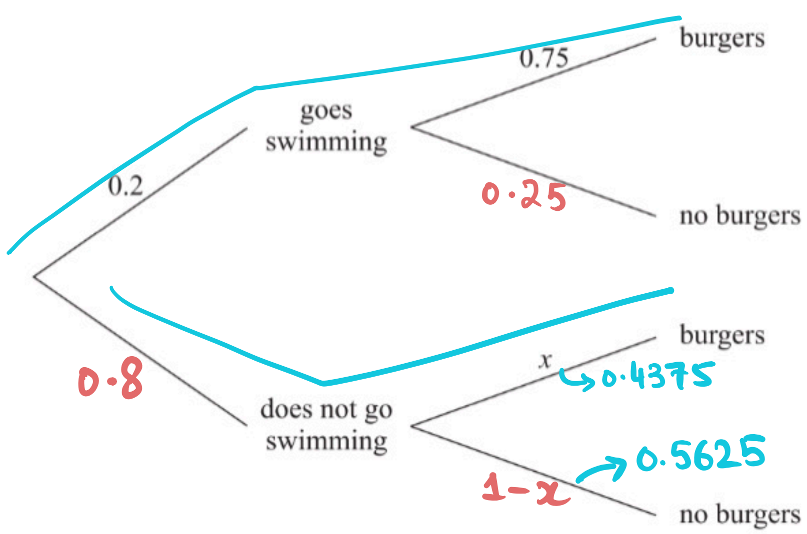
(i) $P(A \text{ and } L) = 0.5 \times 0.2 = 0.1$

(ii) $P(L|I) = \frac{P(L \text{ and } I)}{P(I)}$

$$= \frac{(0.2)(0.1)}{(0.5)(0.8) + (0.3)(0.6) + (0.2)(0.1)}$$

$$= \frac{1}{30}$$

4 The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.



The probability that Henk has burgers for supper on any day is 0.5.

(i) Find x. second event First event. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

The probability that Henk has burgers for supper on any day is 0.5.

$$P(B) = 0.5$$

$$(0.2)(0.75) + (0.8)(x) = 0.5$$

$$x = 0.4375$$

(ii) $$P(S \mid B) = \frac{P(S \text{ and } B)}{P(B)}$$

$$= \frac{(0.2)(0.75)}{0.5}$$

$$= 0.3$$

9 At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

Elephant — HH | HT → Jungle Tractor.
Camel — TT | TH

(i) Find the probabilities that he goes on each of the three rides. [2]

The probabilities that Ravi is frightened on each of the rides are as follows:

elephant ride $\frac{6}{10}$, camel ride $\frac{7}{10}$, jungle tractor ride $\frac{8}{10}$.

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened. [2]

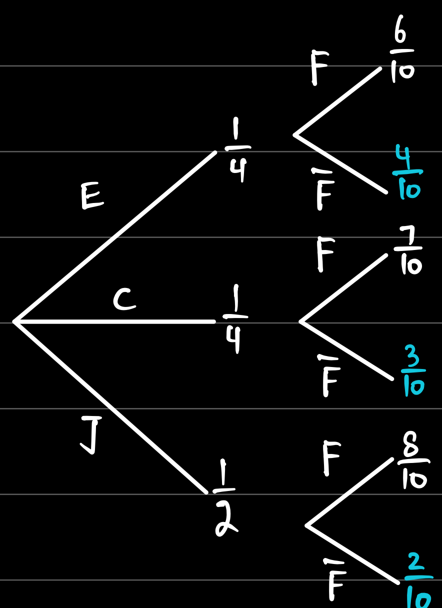
Ravi goes on a ride.

(iii) Find the probability that he is frightened. [2]

(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride. [3]

Second event — First event.

(i) $P(E) = \frac{1}{4}$ $P(C) = \frac{1}{4}$ $P(JT) = \frac{2}{4} = \frac{1}{2}$



$$P(F) = \left(\frac{1}{4} \times \frac{6}{10}\right) + \left(\frac{1}{4} \times \frac{7}{10}\right) + \left(\frac{1}{2} \times \frac{8}{10}\right)$$

$$= \frac{29}{40} = 0.725$$

$P(F) = 0.725$

$P(\bar{F}) = 1 - 0.725 = 0.275$

(iv) $$P(C \mid \bar{F}) = \frac{P(C \text{ and } \bar{F})}{P(\bar{F})}$$

$$= \frac{\left(\frac{1}{4} \times \frac{3}{10}\right)}{0.275}$$

$$= \frac{3}{11}$$

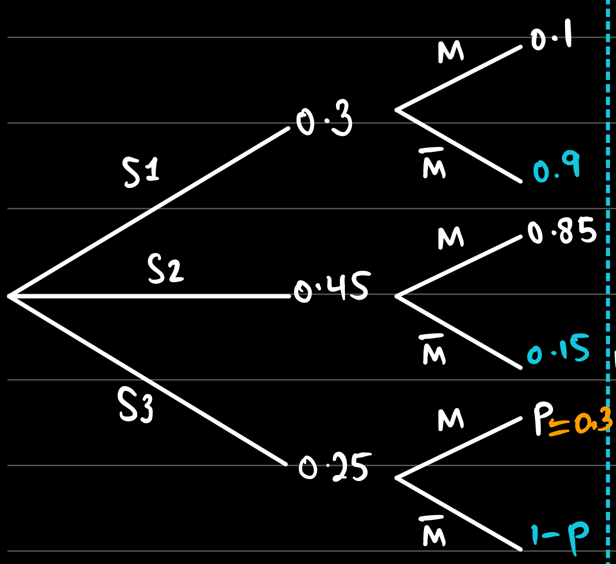
19 Maria has 3 pre-set stations on her radio. When she switches her radio on, there is a probability of 0.3 that it will be set to station 1, a probability of 0.45 that it will be set to station 2 and a probability of 0.25 that it will be set to station 3. On station 1 the probability that the presenter is male is 0.1, on station 2 the probability that the presenter is male is 0.85 and on station 3 the probability that the presenter is male is p. When Maria switches on the radio, the probability that it is set to station 3 and the presenter is male is 0.075.

(i) Show that the value of p is 0.3. [1]

second event

(ii) Given that Maria switches on and hears a male presenter, find the probability that the radio was set to station 2. [4]

first event



(i)

$P(S3 \text{ and } M) = 0.075$

$(0.25)(p) = 0.075$

$p = 0.3$

(ii) $P(S2 \mid M) = \dfrac{P(S2 \text{ and } M)}{P(M)}$

$= \dfrac{(0.45)(0.85)}{(0.3)(0.1) + (0.45)(0.85) + (0.25)(0.3)}$

$= \dfrac{51}{65}$

1 signal = Ishara

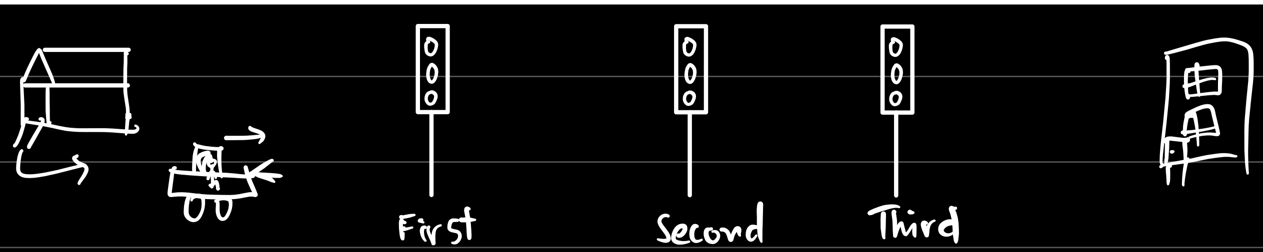
8 There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information. [2]

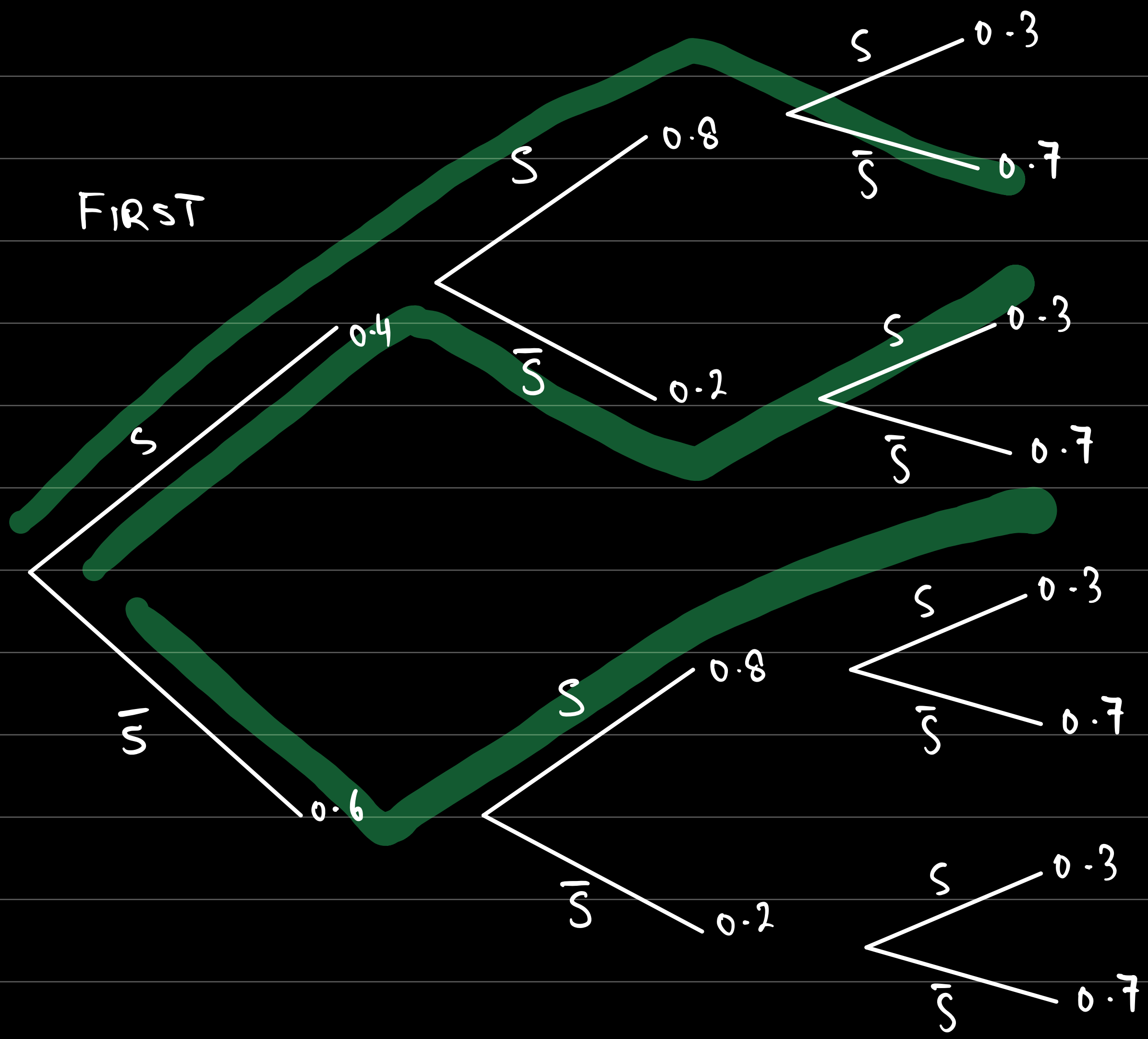
(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set. [2]

(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights. [3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights. [3]



FIRST



(ii) Find the probability that Karinne has to stop at each of the first two ~~sets of lights~~ signals but does not have to stop at the third ~~set.~~ signal [2]

$P(SS\bar{S}) = (0.4)(0.8)(0.7) = 0.224$

(iii) Find the probability that Karinne has to stop at exactly two of the three ~~sets of lights~~. Signals [3]

$$P(\text{stops at exactly Two}) = \underset{(0.4)(0.8)(0.7)}{SS\bar{S}} \text{ or } \underset{+\,(0.4)(0.2)(0.3)}{S\bar{S}S} \text{ or } \underset{+\,(0.6)(0.8)(0.3)}{\bar{S}SS}$$

$$= 0.392$$

(iv) Find the probability that Karinne has to stop at the first ~~set of lights~~ signal, given that she has to stop at exactly two ~~sets of lights~~. Signals [3]

$SS\bar{S}$ or $S\bar{S}S$

$$P\left(\text{stops at first} \,\middle|\, \text{stops at exactly two}\right) = \frac{P(\text{stops at first AND Stops at exctly two})}{P(\text{stops at exactly two})}$$

$SS\bar{S}$ or $S\bar{S}S$

$$= \frac{(0.4)(0.8)(0.7) + (0.4)(0.2)(0.3)}{0.392}$$

$$= \frac{31}{49}$$

General Advice

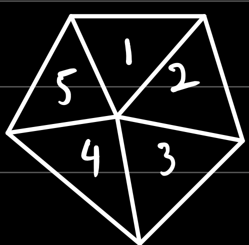
Always write probabilities up to 4 dp.

15 A fair five-sided spinner has sides numbered 1, 2, 3, 4, 5. Raj spins the spinner and throws two fair dice. He calculates his score as follows.

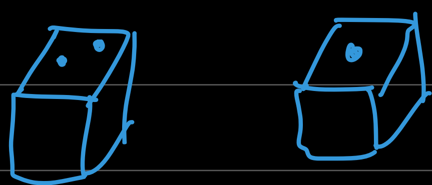
- If the spinner lands on an **even-numbered** side, Raj **multiplies** the two numbers showing on the dice to get his score.
- If the spinner lands on an **odd-numbered** side, Raj **adds** the numbers showing on the dice to get his score.

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side.

[6]



Spinner



$\boxed{\text{Spinner is even}}$

2 Dice scores get multiplied.

Total outcomes = 36

$\boxed{\text{Score 12}}$ (2,6) (6,2) (3,4) (4,3)

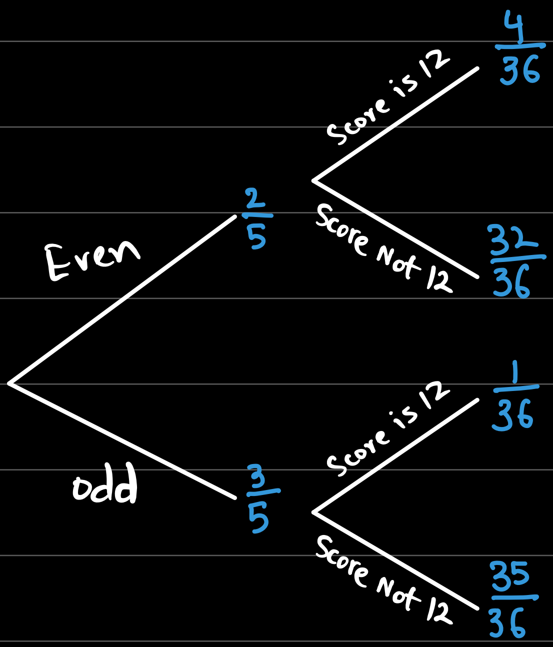
$\boxed{\text{Spinner is odd}}$

2 Dice Scores are added.

Total outcomes = 36

$\boxed{\text{Score 12}}$ (6,6)

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side.



$$P(\text{even} \mid \text{score is } 12) = \frac{P(\text{even AND score is } 12)}{P(\text{score is } 12)}$$

$$= \frac{\frac{2}{5} \times \frac{4}{36}}{\left(\frac{2}{5} \times \frac{4}{36}\right) + \left(\frac{3}{5} \times \frac{1}{36}\right)}$$

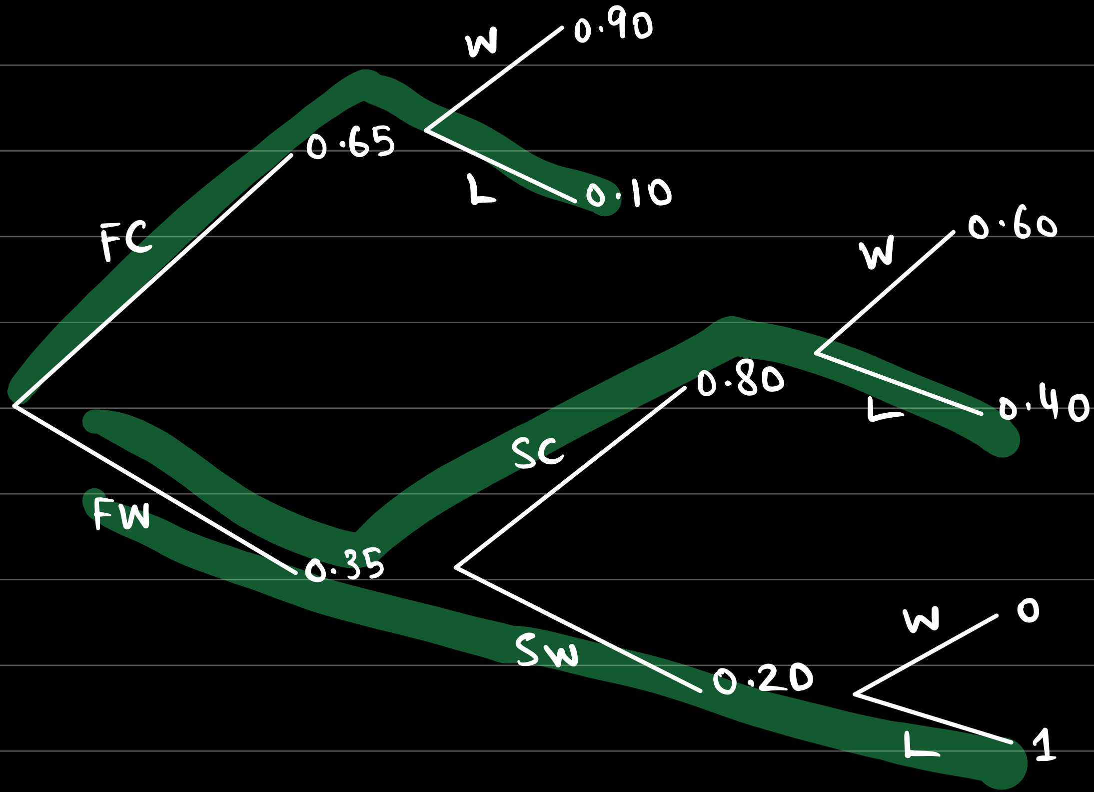
$$= \frac{8}{11}$$

1 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]



(ii) Using your tree diagram, find the probability that Don loses the point. [3]

FC L FW SC L FW SW L

$$P(L) = (0.65)(0.1) + (0.35)(0.8)(0.4) + (0.35)(0.2)(1)$$

$$P(L) = 0.247$$

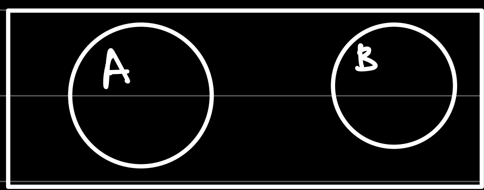
(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]

$$P(FC|L) = \frac{P(FC \text{ and } L)}{P(L)} = \frac{(0.65)(0.1)}{0.247} = \frac{5}{19}$$

MUTUALLY EXCLUSIVE	INDEPENDENT EVENTS

MUTUALLY EXCLUSIVE

Events that can never happen together in an experiment.



(1–2 marks)

you will have to show from experiment if two events can occur together or not.

(NO FORMULA)

INDEPENDENT EVENTS

$$P(\underset{\text{together}}{A \text{ and } B}) = P(A) \times P(B)$$

(4–5 marks)

Find these probabilities seperately and if they satisfy this equation, only then these events are independent.

(You cannot think and tell).

12 Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24.

R : both of the numbers are greater than 8.

(i) Find P(Q). [2]

(ii) Find P(R). [2]

(iii) Are events Q and R exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

Total outcomes = 144

$\boxed{Q}$ Product is 24

(2,12) (12,2)

(3,8) (8,3)

(4,6) (6,4)

$P(Q) = \frac{6}{144} = \frac{1}{24}$

$\boxed{R}$ Both numbers greater than 8

(9,9)	(10,9)	(11,9)	(12,9)
(9,10)	(10,10)	(11,10)	(12,10)
(9,11)	(10,11)	(11,11)	(12,11)
(9,12)	(10,12)	(11,12)	(12,12)

$P(Q) = \frac{16}{144} = \frac{1}{9}$

(iii) Since there are no two numbers greater than 8 with product of 24, Q and R are exclusive.

(iv) $P(Q \text{ and } R) = P(Q) \times P(R)$
together

$\frac{0}{144} \neq \frac{1}{24} \times \frac{1}{9}$

Hence Q and R are not independent.

3
13
20
22
24
28
29

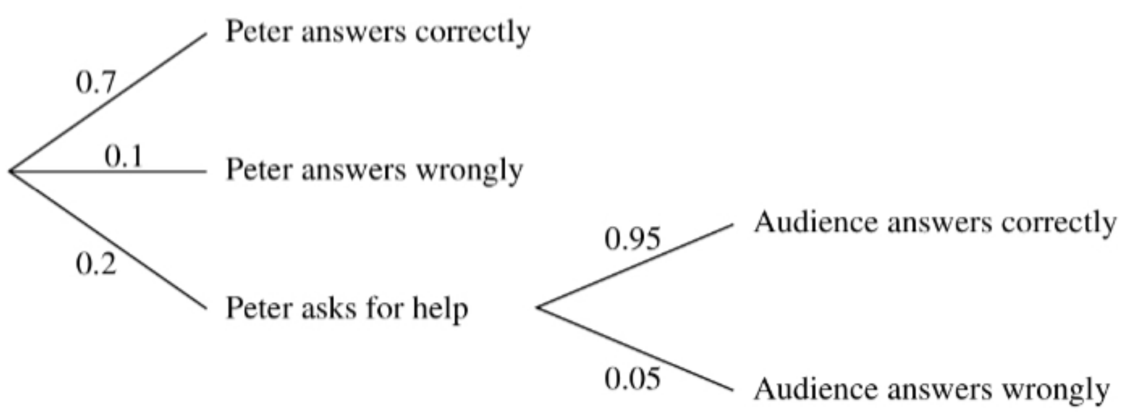
Do full probability sheet except these

11 In a television quiz show Peter answers questions one after another, stopping as soon as a question is answered wrongly.

- The probability that Peter gives the correct answer himself to any question is 0.7.
- The probability that Peter gives a wrong answer himself to any question is 0.1.
- The probability that Peter decides to ask for help for any question is 0.2.

On the first occasion that Peter decides to ask for help he asks the audience. The probability that the audience gives the correct answer to any question is 0.95. This information is shown in the tree diagram below.

(i) $P(FC) = 0.7 + (0.2)(0.95) = 0.89$

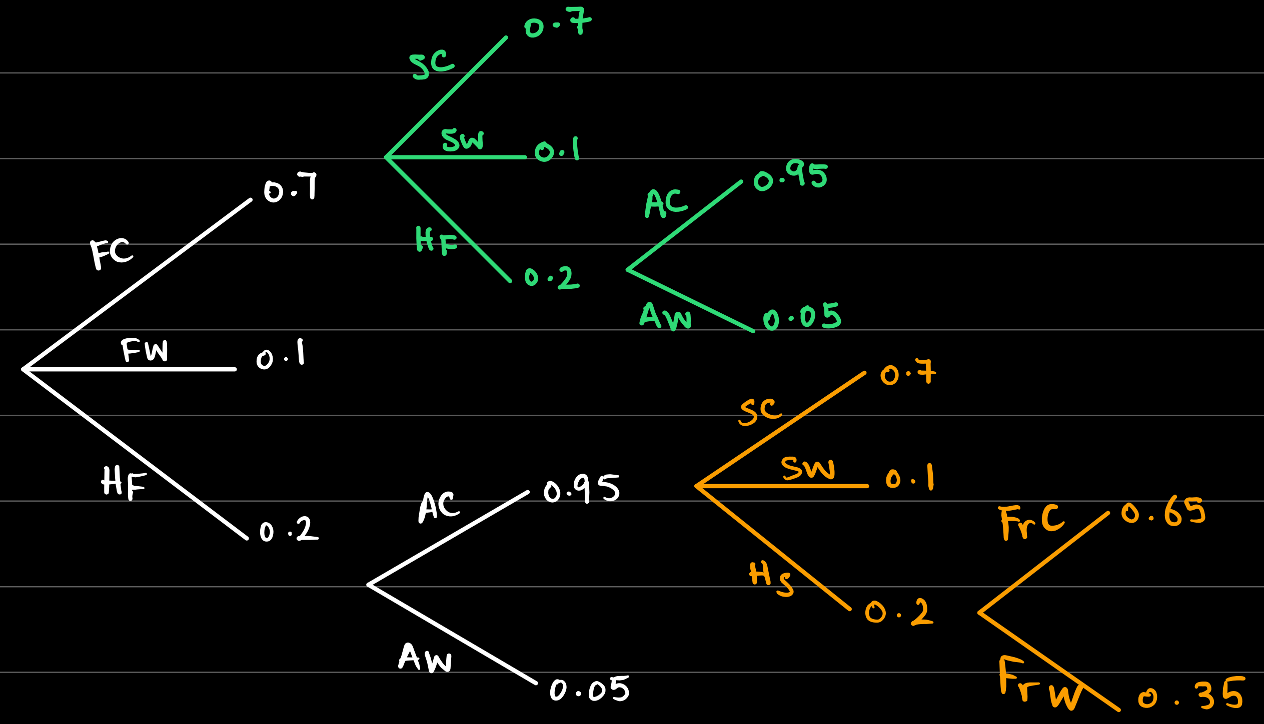


(i) Show that the probability that the first question is answered correctly is 0.89. [1]

On the second occasion that Peter decides to ask for help he phones a friend. The probability that his friend gives the correct answer to any question is 0.65.

(ii) Find the probability that the first two questions are both answered correctly. [6]

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience. [3]



(ii) Find the probability that the first two questions are both answered correctly. [6]

$(FC)(SC) = (0.7)(0.7) = 0.49$

$(FC)(HF)(AC) = (0.7)(0.2)(0.95) = 0.133$

$(HF)(AC)(SC) = (0.2)(0.95)(0.7) = 0.133$

$(HF)(AC)(HS)(FrC) = (0.2)(0.95)(0.2)(0.65) = 0.0247$

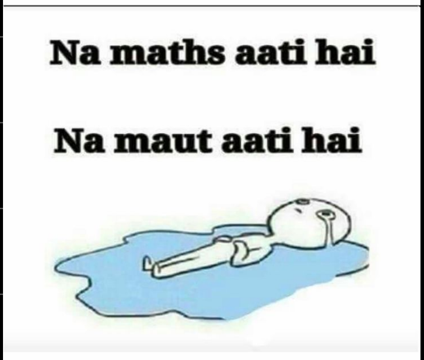
P(both questions answered correctly $= 0.7807$

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience. [3]

$$P\left(\text{Asking Audience} \,\middle|\, \text{both Q's correct}\right) = \frac{P\left(\text{Asking Audience AND both Q's correct}\right)}{P(\text{both correct})}$$

$$= \frac{0.133 + 0.133 + 0.0247}{0.7807}$$

$$= 0.372$$



3 Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	$\boxed{358}$	305

Total Male = 618

Total Female = 663

Total Unemployed = 564

Total Employed = 717

$\boxed{\text{Total} = 1281}$

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find P(M). [2]

(ii) Find P(M and E). [1]

(iii) Are M and E independent events? Justify your answer. [3]

(iv) Given that the person chosen is unemployed, find the probability that the person is female. [2]

$\frac{358}{564}$

(i) $P(M) = \frac{\text{Total male}}{\text{Total}} = \frac{618}{1281} = 0.482$

(ii) $P(M \text{ and } E) = \frac{\text{Male who are employed}}{\text{Total}} = \frac{412}{1281} = 0.322$

(iii) IF M and E are independent

$P(M \text{ and } E) = P(M) \times P(E)$

$\frac{412}{1281} \neq \frac{618}{1281} \times \frac{717}{1281}$

No they are not independent.

$P(M) = \frac{618}{1281}$

$P(M \text{ and } E) = \frac{412}{1281}$

$P(E) = \frac{\text{Employed}}{\text{Total}} = \frac{717}{1281}$

Concept

AND means Multiply
only if we are sure these are independent events.

Tree diagrams are for sure independent event.

$$P(M \text{ and } E) = P(M) \times P(E)$$

22 Ronnie obtained data about the gross domestic product (GDP) and the birth rate for 170 countries. He classified each GDP and each birth rate as either 'low', 'medium' or 'high'. The table shows the number of countries in each category.

		Birth rate			
		Low	Medium	High	
GDP	Low	3	5	45	GLT = 53
	Medium	20	42	12	GMT = 74
	High	35	8	0	GHT = 43
		BRLT = 58	BRMT = 55	BRHT = 57	Total count = 170

One of these countries is chosen at random.

(i) Find the probability that the country chosen has a medium GDP. [1]

(ii) Find the probability that the country chosen has a low birth rate, given that it does not have a medium GDP. [2]

$= \frac{\text{Fav}}{\text{Total}} = \frac{35+3}{53+43} = \frac{38}{96}$

(iii) State with a reason whether or not the events 'the country chosen has a high GDP' and 'the country chosen has a high birth rate' are exclusive. [2]

Because there are no countries with high GDP and high birth rate together. P(A and B) = 0

One country is chosen at random from those countries which have a medium GDP and then a different country is chosen at random from those which have a medium birth rate.

(iv) Find the probability that both countries chosen have a medium GDP and a medium birth rate. [3]

(i) $P(MG) = \frac{MGT}{\ } = \frac{74}{\ }$

Total 170

		Birth rate		
		Low	Medium	High
GDP	Low	3	5	45
	Medium	20	42	12
	High	35	8	0