ALEVELS P3

COMPLEX NUMBERS WITH DIAGRAM (EASY)

C3

1 The complex numbers $1 + 3\mathrm{i}$ and $4 + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers $u - v$ and $\frac{u}{v}$. [3]

(ii) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram, with origin O, the points A, B and C represent the numbers u, v and $u - v$ respectively.

(iii) State fully the geometrical relationship between OC and BA. [2]

(iv) Prove that angle $AOB = \frac{1}{4}\pi$ radians. [2]

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2 **(i)** Solve the equation $z^2 - 2\mathrm{i}z - 5 = 0$, giving your answers in the form $x + \mathrm{i}y$ where x and y are real. [3]

(ii) Find the modulus and argument of each root. [3]

(iii) Sketch an Argand diagram showing the points representing the roots. [1]

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3 The equation $2x^3 + x^2 + 25 = 0$ has one real root and two complex roots.

(i) Verify that $1 + 2\mathrm{i}$ is one of the complex roots. [3]

(ii) Write down the other complex root of the equation. [1]

(iii) Sketch an Argand diagram showing the point representing the complex number $1 + 2\mathrm{i}$. Show on the same diagram the set of points representing the complex numbers z which satisfy

$$|z| = |z - 1 - 2\mathrm{i}|.$$ [4]

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4 The complex number $2 + \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) Show, on a sketch of an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u + u^*$ respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

(ii) Express $\frac{u}{u^*}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u}{u^*}$, or otherwise, prove that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right).$$ [2]

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5 The complex number $\dfrac{2}{-1+\mathrm{i}}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z - u^2| < |z - u|$. [4]

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6 **(i)** Solve the equation $z^2 + (2\sqrt{3})\mathrm{i}z - 4 = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [3]

(ii) Sketch an Argand diagram showing the points representing the roots. [1]

(iii) Find the modulus and argument of each root. [3]

(iv) Show that the origin and the points representing the roots are the vertices of an equilateral triangle. [1]

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7 The complex number z is given by

$$z = (\sqrt{3}) + \mathrm{i}.$$

(i) Find the modulus and argument of z. [2]

(ii) The complex conjugate of z is denoted by z^*. Showing your working, express in the form $x + \mathrm{i}y$, where x and y are real,

(a) $2z + z^*$,

(b) $\dfrac{\mathrm{i}z^*}{z}$.

[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z and $\mathrm{i}z^*$ respectively. Prove that angle $AOB = \frac{1}{6}\pi$. [3]

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8 The complex number w is defined by $w = 2 + \mathrm{i}$.

(i) Showing your working, express w^2 in the form $x + \mathrm{i}y$, where x and y are real. Find the modulus of w^2. [3]

(ii) Shade on an Argand diagram the region whose points represent the complex numbers z which satisfy

$$|z - w^2| \leqslant |w^2|.$$ [3]

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9 The complex number u is defined by $u = \dfrac{(1+2\mathrm{i})^2}{2+\mathrm{i}}$.

(i) Without using a calculator and showing your working, express u in the form $x + \mathrm{i}y$, where x and y are real. [4]

(ii) Sketch an Argand diagram showing the locus of the complex number z such that $|z - u| = |u|$. [3]

9709/31/M/J/12/Q4

10 **Throughout this question the use of a calculator is not permitted.**

(a) Solve the equation $(1 + 2\mathrm{i})w^2 + 4w - (1 - 2\mathrm{i}) = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - \mathrm{i}| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

9709/31/O/N/16/Q9

11 **(a)** The complex number u is given by $u = 8 - 15\mathrm{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \mathrm{i}b$, where the numbers a and b are real and exact. [5]

(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

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12 **Throughout this question the use of a calculator is not permitted.**

The complex number $1 - (\sqrt{3})\mathrm{i}$ is denoted by u.

(i) Find the modulus and argument of u. [2]

(ii) Show that $u^3 + 8 = 0$. [2]

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leqslant 2$ and $\operatorname{Re} z \geqslant 2$, where $\operatorname{Re} z$ denotes the real part of z. [4]

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13 **(i)** Showing all working and without using a calculator, solve the equation $z^2 + (2\sqrt{6})z + 8 = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

(ii) Sketch an Argand diagram showing the points representing the roots. [1]

(iii) The points representing the roots are A and B, and O is the origin. Find angle AOB. [3]

(iv) Prove that triangle AOB is equilateral. [1]

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14 **Throughout this question the use of a calculator is not permitted.**

The complex numbers $-3\sqrt{3} + \mathrm{i}$ and $\sqrt{3} + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real and exact, the complex numbers uv and $\dfrac{u}{v}$. [5]

(ii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]

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15 **Throughout this question the use of a calculator is not permitted.**

The complex number u is defined by

$$u = \frac{4\mathrm{i}}{1 - (\sqrt{3})\mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

(ii) Find the exact modulus and argument of u. [2]

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

9709/33/M/J/19/Q8