A9 Differentiation Answers
P3

1 **10** **(i)** Use product rule — M1

Obtain correct derivative $\cos 2x - 2x\sin 2x$ — A1

Equate derivative to zero and obtain given answer correctly — A1 — **3**

(ii) Use the iterative formula correctly at least once — M1

Obtain final answer 0.43 — A1

Show sufficient iterations to at least 3d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.425, 0.435) — A1 — **3**

(iii) Attempt integration by parts and obtain $\pm kx\sin 2x \pm \int l\sin 2x\,dx$, where $k, l = \frac{1}{2}, 1$, or 2 — M1*

Obtain $\frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$ — A1

Obtain indefinite integral $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ — A1

Use limits $x = 0$ and $x = \frac{1}{4}\pi$ having integrated twice — M1(dep)*

Obtain answer $\frac{1}{8}\pi - \frac{1}{4}$, or exact equivalent — A1 — **5**

2 Use product rule — M1

Obtain derivative in any correct form — A1

Form equation of tangent at $x = \frac{1}{4}\pi$ correctly — M1

Simplify answer to $y = x$, or $y - x = 0$ — A1 — **4**

[SR: The misread $y = x\sin x$ can only earn M1M1.]

3 **(i)** Use quotient or product rule to differentiate $(1 - x)/(1 + x)$ — M1

Obtain correct derivative in any form — A1

Use chain rule to find $\frac{dy}{dx}$ — M1

Obtain a correct expression in any form — A1

Obtain the gradient of the normal in the given form correctly — A1 — [5]

(ii) Use product rule — M1

Obtain correct derivative in any form — A1

Equate derivative to zero and solve for x — M1

Obtain $x = \frac{1}{2}$ — A1 — [4]

4 **(i)** Obtain $\dfrac{k\cos 2x}{1+\sin 2x}$ for any non-zero constant k — M1

Obtain $\dfrac{2\cos 2x}{1+\sin 2x}$ — A1 — [2]

(ii) Use correct quotient or product rule — M1

Obtain $\dfrac{x\sec^2 x - \tan x}{x^2}$ or equivalent — A1 — [2]

5	**(i)** *EITHER*: State $\dfrac{dx}{dt} = \sec^2 t / \tan t$, or equivalent	B1	
	State $\dfrac{dy}{dt} = 2\sin t \cos t$, or equivalent	B1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	M1	
	Obtain correct answer in any form, e.g. $2\sin^2 t \cos^2 t$	A1	
	OR: Obtain $y = e^{2x} / (1 + e^{2x})$, or equivalent	B1	
	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form, e.g. $2e^{2x} / (1 + e^{2x})^2$	A1	
	Obtain correct derivative in terms of t in any form, e.g. $(2\tan^2 t) / (1 + \tan^2 t)^2$	A1	[4]
	(ii) State or imply $t = \dfrac{1}{4}\pi$ when $x = 0$	B1	
	Form the equation of the tangent at $x = 0$	M1	
	Obtain correct answer in any horizontal form, e.g. $y = \dfrac{1}{2}x + \dfrac{1}{2}$	A1	[3]
	[SR: If the *OR* method is used in part **(i)**, give B1 for stating or implying $y = \dfrac{1}{2}$ or $\dfrac{dy}{dx} = \dfrac{1}{2}$ when $x = 0$.]		

6	Use correct quotient or product rule or equivalent	M1	
	Obtain $\dfrac{(1+e^{2x}).2e^{2x} - e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x = \ln 3$ into attempt at first derivative and show use of relevant logarithm property at least once in a correct context	M1	
	Confirm given answer $\frac{9}{50}$ legitimately	A1	[4]

7	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Justify the given statement	A1	**[3]**

8	**(i)**	Use of product or quotient rule	M1	
		Obtain $-5\mathrm{e}^{-\frac{1}{2}x}\sin 4x + 40\mathrm{e}^{-\frac{1}{2}x}\cos 4x$	A1	
		Equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and obtain tan4z = k or R cos($4x \pm \alpha$)	M1	
		Obtain $\tan 4x = 8$ or $\sqrt{65}\cos\left(4x \pm \tan^{-1}\frac{1}{8}\right)$	A1	
		Obtain 0.362 or 20.7°	A1	
		Obtain 1.147 or 65.7°	A1	[6]
	(ii)	State or imply that x-coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45°	B1	
		Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi$. 25	M1	
		Obtain $n > \frac{4}{\pi}(25 - 0.362) + 1$, following through on their value of x_1	A1√	
		n = 33	A1	[4]

9	**(i)**	State or imply that the derivative of e^{-2x} is $-2\mathrm{e}^{-2x}$	**B1**	
		Use product or quotient rule	**M1**	
		Obtain correct derivative in any form	**A1**	
		Use Pythagoras	**M1**	
		Justify the given form	**A1**	[5]
	(ii)	Fully justify the given statement	**B1**	[1]
	(iii)	State answer $x = \frac{1}{4}\pi$	**B1**	[1]

10	Use correct quotient rule or equivalent to find first derivative	**M1***	
	Obtain $\frac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
	Substitute $x = \frac{1}{4}\pi$ to find gradient	**dep M1***	
	Obtain $-\frac{3}{2}$	**A1**	
	Form equation of tangent at $x = \frac{1}{4}\pi$	**M1**	
	Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent	**A1**	[6]

11	Use correct quotient or product rule Obtain correct derivative in any form Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	M1 A1 A1 A1	[4]

12(i)	Use the chain rule	M1
	Obtain correct derivative in any form	A1
	Use correct trigonometry to express derivative in terms of $\tan x$	M1
	Obtain $\frac{dy}{dx} = -\frac{4\tan x}{4+\tan^2 x}$, or equivalent	A1
	Total:	**4**
12(ii)	Equate derivative to –1 and solve a 3–term quadratic for $\tan x$	M1
	Obtain answer x=1.11 and no other in the given interval	A1
	Total:	**2**

Question	Answer	Marks	Guidance
13	Use correct quotient rule	M1	Allow use of correct product rule on $x\times(1+\ln x)^{-1}$
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{(1+\ln x) - x\times\frac{1}{x}}{(1+\ln x)^2} = \left(\frac{1}{1+\ln x} - \frac{1}{(1+\ln x)^2}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in $\ln x$ or $(1+\ln x)$	M1	Horizontal form. Accept $\ln x = \frac{1}{4}(1+\ln x)^2$
	Reduce to $(\ln x)^2 - 2\ln x + 1 = 0$	A1	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in $\ln x$ for x	M1	Must see working if solving incorrect quadratic
	Obtain answer $x = \mathrm{e}$	A1	Accept e^1
	Obtain answer $y = \frac{1}{2}\mathrm{e}$	A1	Exact only with no decimals seen before the exact value. Accept $\frac{\mathrm{e}^1}{2}$ but not $\frac{\mathrm{e}}{1+\ln \mathrm{e}}$
		7	

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14(i)	Use the quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Reduce to $-\dfrac{2e^{-x}}{\left(1-e^{-x}\right)^2}$, or equivalent, and explain why this is always negative	**A1**	
		3	

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14(ii)	Equate derivative to – 1 and obtain the given equation	**B1**	
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	**B1**	
	Solve for *a*	**M1**	
	Obtain answer $a = \ln\left(2+\sqrt{3}\right)$ and no other	**A1**	
		4	