DE1 WITHOUT PROOF MARKING SCHEME

1 Separate variables and attempt to integrate M1

Obtain terms $\frac{1}{3}\ln(y^3+1)$ and x, or equivalent A1 + A1

Evaluate a constant or use limits $x = 0$, $y = 1$ with a solution containing terms $k\ln(y^3+1)$ and x, or equivalent M1

Obtain any correct form of solution e.g. $\frac{1}{3}\ln(y^3+1) = x + \frac{1}{3}\ln 2$ A1√

Rearrange and obtain $y = (2e^{3x} - 1)^{\frac{1}{3}}$, or equivalent A1 **6**

[f.t. is on $k \neq 0$.]

2 **(i)** Attempt to express integrand in partial fractions e.g. obtain A or B in $\dfrac{A}{y} + \dfrac{B}{4-y}$ **M1**

Obtain $\dfrac{1}{4}\left(\dfrac{1}{y} + \dfrac{1}{4-y}\right)$, or equivalent **A1**

Integrate and obtain $\frac{1}{4}\ln y - \frac{1}{4}\ln(4-y)$, or equivalent **A1√ + A1√** **4**

(ii) Separate variables correctly, integrate $\dfrac{A}{y} + \dfrac{B}{4-y}$ and obtain further term x, or equivalent **M1***

Use $y = 1$ and $x = 0$ to evaluate a constant, or as limits **M1(dep*)**

Obtain answer in any correct form **A1**

Obtain final answer $y = 4/(3e^{-4x} + 1)$, or equivalent **A1** **4**

(iii) State that y approaches 4 as x becomes very large **B1** **1**

3 Separate variables correctly and attempt to integrate one side M1

Obtain terms $\frac{1}{2}\ln(1+y^2)$ and x, or equivalent A1 + A1

Evaluate a constant or use limits $x = 0$, $y = 2$ with a solution containing terms $k\ln(1+y^2)$ and x, or equivalent M1

Obtain any correct form of solution, e.g. $\frac{1}{2}\ln(1+y^2) = x + \frac{1}{2}\ln 5$ A1

Rearrange and obtain $y^2 = 5e^{2x} - 1$, or equivalent A1 6

4 **(i)** Separate variables correctly and attempt integration of both sides M1*

Obtain term $\ln N$, or equivalent A1

Obtain term $\dfrac{k}{0.02}\sin(0.02t)$, or equivalent A1

Use $t = 0$, $N = 125$ to evaluate a constant, or as limits, in a solution containing terms of the form $a\ln N$ and $b\sin(0.02t)$, or equivalent M1

Obtain any correct form of solution, e.g. $\ln N = 50k\sin(0.02t) + \ln 125$ A1 **[5]**

(ii) Substituting $N = 166$ and $t = 30$, evaluate k M1(dep*)

Obtain $k = 0.0100479\ldots$(accept $k = 0.01$) A1 **[2]**

(iii) Rearrange and obtain $N = 125\exp(0.502\sin(0.02t))$, or equivalent B1

Set $\sin(0.02t) = -1$ in the expression for N, or equivalent M1

Obtain least value 75.6 (accept answers in the interval [75, 76]) A1 **[3]**

[For the B1, accept 0.5 following $k = 0.01$, and allow 4.8 or better for ln 125.]

5 **(i)** State or imply the form $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{10-x}$ B1

Use any relevant method to determine a constant M1

Obtain one of the values $A = 1$, $B = 10$, $C = 1$ A1

Obtain the remaining two values A1 **4**

[The form $\dfrac{Dx+E}{x^2}+\dfrac{C}{10-x}$ is acceptable and leads to $D = 1$, $E = 10$, $C = 1$]

(ii) Separate variables and attempt integration of both sides M1

Obtain terms $\ln x$, $-10/x$, $-\ln(10-x)$, or equivalent A1√ + A1√ + A1√

Evaluate a constant or use limits $x = 1$, $t = 0$ with a solution containing 3 of the terms $k\ln x$, l/x, $m\ln(10-x)$ and t, or equivalent M1

Obtain any correct expression for t, e.g. $t = \ln\left(\dfrac{9x}{10-x}\right)-\dfrac{10}{x}+10$ A1 **6**

[A separation of the form $\dfrac{a\mathrm{d}x}{x^2(10-x)} = b\mathrm{d}t$ is essential for the M1. The f.t. is on A, B, C]

[If A or B (D or E) omitted from the form of fractions, give B0M1A0A0 in **(i)**; M1A1√ A1√M1A0 in **(ii)**]

6 **(i)** Separate variables correctly B1

Integrate and obtain term $\ln(\theta - A)$, or equivalent B1

Integrate and obtain term $-kt$, or equivalent B1

Use $\theta = 4A$, $t = 0$ to determine a constant, or as limits M1

Obtain correct answer in any form, e.g. $\ln(\theta - A) = -kt + \ln 3A$, with no errors seen A1 [5]

(ii) Substitute $\theta = 3A$, $t = 1$ and justify the given statement B1 [1]

(iii) Substitute $t = 2$ and solve for θ in terms of A M1

Remove logarithms M1

Obtain answer $\theta = \frac{7}{3}A$, or equivalent, with no errors seen A1 [3]

[The M marks are only available if the solution to part **(i)** contains terms $a\ln(\theta - A)$ and bt.]

7 Separate variables correctly B1

Integrate and obtain term $\ln x$ B1

Integrate and obtain term $\frac{1}{2}\ln(y^2+4)$ B1

Evaluate a constant or use limits $y = 0$, $x = 1$ in a solution containing $a\ln x$ and $b\ln(y^2 + 4)$ M1

Obtain correct solution in any form, e.g. $\frac{1}{2}\ln(y^2+4) = \ln x + \frac{1}{2}\ln 4$ A1

Rearrange as $y^2 = 4(x^2-1)$, or equivalent A1 [6]

Q	Part	Answer	Mark	Total
8	**(i)**	Separate variables correctly and attempt integration of both sides	B1	
		Obtain term $\tan x$	B1	
		Obtain term $-\frac{1}{2}e_{-}^{2t}$	B1	
		Evaluate a constant or use limits $x = 0$, $t = 0$ in a solution containing terms $a \tan x$ and be^{-2t}	M1	
		Obtain correct solution in any form, e.g. $\tan x = \frac{1}{2} - \frac{1}{2}e^{-2t}$	A1	
		Rearrange as $x = \tan^{-1}(\frac{1}{2} - \frac{1}{2}e^{-2t})$, or equivalent	A1	[6]
	(ii)	State that x approaches $\tan^{-1}(\frac{1}{2})$	B1	[1]
	(iii)	State that $1 - e^{-2t}$ increases and so does the inverse tangent, or state that $e^{-2t}\cos^2 x$ is positive	B1	[1]
9	**(i)**	Separate variables correctly and integrate of at least one side	M1	
		Carry out an attempt to find A and B such that $\dfrac{1}{N(1800-N)} \equiv \dfrac{A}{N} + \dfrac{B}{1800-N}$, or equivalent	M1	
		Obtain $\dfrac{2}{N} + \dfrac{2}{1800-N}$ or equivalent	A1	
		Integrates to produce two terms involving natural logarithms	M1	
		Obtain $2 \ln N - 2 \ln (1800 - N) = t$ or equivalent	A1	
		Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a \ln N$, $b \ln(1800)$ and ct	M1	
		Obtain $2 \ln N - 2 \ln (1800 - N) = t - 2 \ln 5$ or equivalent	A1	
		Use laws of logarithms to remove logarithms	M1	
		Obtain $N = \dfrac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$ or equivalent	A1	[9]
	(ii)	State or imply that N approaches 1800	B1	[1]
10		Separate variables and attempt integration of at least one side	M1	
		Obtain term $\ln(x + 1)$	A1	
		Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm\frac{1}{2}$	M1	
		Obtain correct term $\frac{1}{2} \ln \sin 2\theta$	A1	
		Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$	M1	
		Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2}\ln \sin 2\theta - \frac{1}{2}\ln\frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm\frac{1}{2}$)	A1√	
		Rearrange and obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent	A1	[7]

11	**(i)**	Separate variables and attempt integration on both sides	M1*	
		Obtain $2N^{0.5}$ on left-hand side or equivalent	A1	
		Obtain $-60e^{-0.02t}$ on right-hand side or equivalent	A1	
		Use 0 and 100 to evaluate a constant or as limits in a solution containing terms $aN^{0.5}$ and $be^{-0.02t}$	DM1*	
		Obtain $2N^{0.5} = -60e^{-0.02t} + 80$ or equivalent	A1	
		Conclude with $N = (40 - 30e^{-0.02t})^2$ or equivalent	A1	[6]
	(ii)	State number approaches 1600 or equivalent, following expression of form $(c + de^{-0.02t})^n$	B1√	[1]

12 Separate variables correctly and attempt integration on at least one side M1

Obtain $\frac{1}{3}y^3$ or equivalent on left-hand side A1

Use integration by parts on right-hand side (as far as $axe^{3x} + \int be^{3x}\,dx$) M1

Obtain or imply $2xe^{3x} + \int 2e^{3x}\,dx$ or equivalent A1

Obtain $2xe^{3x} - \frac{2}{3}e^{3x}$ A1

Substitute $x = 0$, $y = 2$ in an expression containing terms Ay^3, Bxe^{3x}, Ce^{3x}, where $ABC \neq 0$, and find the value of c M1

Obtain $\frac{1}{3}y^3 = 2xe^{3x} - \frac{2}{3}e^{3x} + \frac{10}{3}$ or equivalent A1

Substitute $x = 0.5$ to obtain $y = 2.44$ A1 [8]

13 Separate variables correctly and attempt integration of both sides B1

Obtain term $-e^{-y}$, or equivalent B1

Obtain term $\frac{1}{2}e^{2x}$, or equivalent B1

Evaluate a constant, or use limits $x = 0$, $y = 0$ in a solution containing terms ae^{-y} and be^{2x} M1

Obtain correct solution in any form, e.g. $-e^{-y} = \frac{1}{2}e^{2x} - \frac{3}{2}$ A1

Rearrange and obtain $y = \ln(2/(3 - e^{2x}))$, or equivalent A1 [6]

14 Separate variables correctly and attempt integration of one side B1
Obtain term $\ln x$ B1
State or imply and use a relevant method to find A or B M1
Obtain $A = \frac{1}{2}$, $B = \frac{1}{2}$
Integrate and obtain $-\frac{1}{2}\ln(1-y) + \frac{1}{2}\ln(1+y)$, or equivalent A1$\sqrt{}$
[If the integral is directly stated as $k_1 \ln$ or $k_2 \ln$ give M1, and then A2 for $k_1 = \frac{1}{2}$ or $k_2 = -\frac{1}{2}$]
Evaluate a constant, or use limits $x = 2$, $y = 0$ in a solution containing terms $a \ln x$, $b \ln(1-y)$ and $c \ln(1 + y)$, where $abc \neq 0$ M1
[This M mark is not available if the integral of $1/(1-y^2)$ is initially taken to be of the form $k \ln(1-y^2)$]
Obtain solution in any correct form, e.g. $\frac{1}{2}\ln = \ln x - \ln 2$ A1
Rearrange and obtain $y =$, or equivalent, free of logarithms A1 [8]

15 Separate variables correctly and integrate one side M1
Obtain $\ln y = \ldots$ or equivalent A1
Obtain $= 3\ln(x^2+4)$ or equivalent A1
Evaluate a constant or use $x = 0$, $y = 32$ as limits in a solution containing terms $a \ln y$ and $b\ln\left(x^2+4\right)$ M1
Obtain $\ln y = 3\ln(x^2+4) + \ln 32 - 3\ln 4$ or equivalent A1
Obtain $y = \frac{1}{2}\left(x^2+4\right)$ or equivalent A1 [6]

16 **(i)** Use any relevant method to determine a constant M1
Obtain one of the values $A = 1$, $B = -2$, $C = 4$ A1
Obtain a second value A1
Obtain the third value A1 [4]

[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B. If only one is found by the rule, give B1M1A1A1.]

(ii) Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction M1

Obtain $\ln y = -\frac{1}{2} - 2\ln(2x+1) + c$, or equivalent A3$\sqrt{}$
Evaluate a constant, or use limits $x = 1$, $y = 1$, in a solution containing at least three terms of the form $k \ln y$, l/x, $m \ln x$ and $n \ln(2x + 1)$, or equivalent M1
Obtain solution $\ln y = -\frac{1}{2} - 2\ln x + 2\ln(2x+1) + c$, or equivalent A1
Substitute $x = 2$ and obtain $y = \frac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms A1 [7]
(The f.t. is on A, B, C. Give A2$\sqrt{}$ if there is only one error or omission in the integration; A1$\sqrt{}$ if two.)

17	**(i)**	Separate variables correctly and integrate at least one side	M1	
		Obtain term $\ln t$, or equivalent	B1	
		Obtain term of the form $a\ln(k-x^3)$	M1	
		Obtain term $-\frac{2}{3}\ln(k-x^3)$, or equivalent	A1	
		EITHER: Evaluate a constant or use limits $t=1$, $x=1$ in a solution containing $a\ln t$ and $b\ln(k-x^3)$	M1*	
		Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k-x^3)+\frac{2}{3}\ln(k-1)$	A1	
		Use limits $t=4$, $x=2$, and solve for k	M1(dep*)	
		Obtain $k=9$	A1	
		OR: Using limits $t=1$, $x=1$ and $t=4$, $x=2$ in a solution containing $a\ln t$ and $b\ln(k-x^3)$ obtain an equation in k	M1*	
		Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k-8)+\frac{2}{3}\ln(k-1)$	A1	
		Solve for k	M1(dep*)	
		Obtain $k=9$	A1	
		Substitute $k=9$ and obtain $x=(9-8t^{-\frac{3}{2}})^{\frac{1}{3}}$	A1	[9]
	(ii)	State that x approaches $9^{\frac{1}{3}}$, or equivalent	B1$\sqrt{}$	[1]
18		Use $2\cos^2 x = 1+\cos 2x$ or equivalent	B1	
		Separate variables and integrate at least one side	M1	
		Obtain $\ln\left(y^3+1\right)=\ldots$ or equivalent	A1	
		Obtain $\ldots = 2x+\sin 2x$ or equivalent	A1	
		Use $x=0$, $y=2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3+1\right)$, bx or $c\sin 2x$	M1*	
		Obtain $\ln\left(y^3+1\right)=2x+\sin 2x+\ln 9$ or equivalent e.g. implied by correct constant	A1	
		Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point	B1	
		Use correct process to find y-coordinate for at least one x-coordinate	M1(d*M)	
		Obtain 5.9	A1	
		Obtain 48.1	A1	[10]
19		Separate variables correctly and recognisable attempt at integration of at least one side	M1	
		Obtain $\ln y$, or equivalent	B1	
		Obtain $k\ln\left(2+e^{3x}\right)$	B1	
		Use $y(0)=36$ to find constant in $y=A\left(2+e^{3x}\right)^k$ or $\ln y = k\ln\left(2+e^{3x}\right)+c$ or equivalent	M1*	
		Obtain equation correctly without logarithms from $\ln y = \ln\left(A\left(2+e^{3x}\right)^k\right)$	*M1	
		Obtain $y=4\left(2+e^{3x}\right)^2$	A1	[6]

20		Separate variables correctly and attempt integration of at least one side	B1	
		Obtain term in the form $a\sqrt{(2x+1)}$	M1	
		Express $1/(\cos^2\theta)$ as $\sec^2\theta$	B1	
		Obtain term of the form $k\tan\theta$	M1	
		Evaluate a constant, or use limits $x = 0, \theta = \frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and k tan θ, $ak \neq 0$	M1	
		Obtain correct solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2}\tan\theta + \frac{1}{2}$	A1	
		Rearrange and obtain $x = \frac{1}{8}(\tan\theta + 1)^2 - \frac{1}{2}$, or equivalent	A1	**7**
21	**(i)**	Separate variables correctly and attempt to integrate at least one side	B1	
		Obtain term ln$R$	B1	
		Obtain $\ln x - 0.57x$	B1	
		Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a$ln$R$ and $b$ln$x$	M1	
		Obtain correct solution in any form	A1	
		Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57\ )}$	A1	**[6]**
	(ii)	Equate $\frac{\mathrm{d}R}{\mathrm{d}x}$ to zero and solve for x	M1	
		State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
		Obtain $R = 28.8$ (allow 28.9)	A1	**[3]**
22	**(i)**	Sensibly separate variables and attempt integration of at least one side	M1	
		Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\mathrm{d}x$	M1	
		Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\mathrm{d}x$ or equivalent	A1	
		Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent	A1	
		Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent	A1	[6]
	(ii)	Use $x = 0$ and $y = 100$ to find constant	M*1	
		Substitute 25 and calculate value of y	DM*1	
		Obtain 203	A1	[3]

23	Separate variables and factorise to obtain $\dfrac{\mathrm{d}y}{(3y+1)(y+3)} = 4x\,\mathrm{d}x$ or equivalent	B1	
	State or imply the form $\dfrac{A}{3y+1} + \dfrac{B}{y+3}$ and use a relevant method to find A or B	M1	
	Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$	A1	
	Integrate to obtain form $k_1\ln(3y+1) + k_2\ln(y+3)$	M1	
	Obtain correct $\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(\ \ =3) = 2x^2$ or equivalent	A1	
	Substitute $x = 0$ and $y = 1$ in equation of form $k_1\ln(3y+1) + k_2\ln(y+3) = k_3x^2 + c$ to find a value of c	M1	
	Obtain $c = 0$	A1	
	Use correct process to obtain equation without natural logarithm present	M1	
	Obtain $y = \dfrac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent	A1	[9]
24 (i)	Separate variables correctly and attempt integration of one side	B1	
	Obtain term $\ln x$	B1	
	Obtain term of the form $a\ln(k + \mathrm{e}^{-t})$	M1	
	Obtain term $-\ln(k + \mathrm{e}^{-t})$	A1	
	Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a\ln(k + \mathrm{e}^{-t})$ and $b\ln x$	M1*	
	Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k + \mathrm{e}^{-t}) + \ln(k+1)$	A1	**[6]**
(ii)	Substitute $x = 20$, $t = 1$ and solve for k	M1(dep*)	
	Obtain the given answer	A1	**[2]**
(iii)	Using $\mathrm{e}^{-t} \to 0$ and the given value of k, find the limiting value of x	M1	
	Justify the given answer	A1	**[2]**
25 (i)	Separate variables correctly and integrate one side	B1	
	Obtain term $2\sqrt{M}$, or equivalent	B1	
	Obtain term $50k\sin(0.02t)$, or equivalent	B1	
	Evaluate a constant of integration, or use limits $M = 100$, $t = 0$ in a solution with terms of the form $a\sqrt{M}$ and $b\sin(0.02t)$	M1*	
	Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k\sin(0.02t) + 20$	A1	5
(ii)	Use values $M = 196$, $t = 50$ and calculate k	M1(dep*)	
	Obtain answer $k = 0.190$	A1	2
(iii)	State an expression for M in terms of t, e.g. $M = (4.75\sin(0.02t) + 10)^2$	M1(dep*)	
	State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)	A1	2

26	Separate variables and integrate one side	**B1**	
	Obtain term $\ln(x+2)$	**B1**	
	Use cos $2A$ formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	**M1**	
	Obtain correct form $(1-\cos 4\theta)/2$, or equivalent	**A1**	
	Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	**A1**√	
	Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$	**M1**	
	Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	**A1**	
	Use correct method for solving an equation of the form $\ln(x+2) = f$	**M1**	
	Obtain answer $x = 0.962$	**A1**	[9]

27	Separate variables and attempt integration of at least one side	**M1***	
	Obtain term ln	**A1**	
	Obtain terms $\ln x - x^2$	**A1**	
	Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1***	
	Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$	**A1**	
	Obtain correct expression for y, free of logarithms, i.e. $y = 2x\exp(1-x^2)$	**A1**	
			[6]

28	**(i)**	Separate variables correctly and attempt integration of at least one side	**B1**	
		Obtain term $\ln x$	**B1**	
		Obtain term of the form $k\ln(3+\cos 2\theta)$, or equivalent	**M1**	
		Obtain term $-\frac{1}{2}\ln(3+\cos 2\theta)$, or equivalent	**A1**	
		Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution with terms $a\ln x$ and $b\ln(3+\cos 2\theta)$, where $ab \neq 0$	**M1**	
		State correct solution in any form, e.g. $\ln x = -\frac{1}{2}\ln(3+\cos 2\theta) + \frac{3}{2}\ln 3$	**A1**	
		Rearrange in a correct form, e.g. $x = \sqrt{\left(\dfrac{27}{3+\cos 2\theta}\right)}$	**A1**	[7]
	(ii)	State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60])	**B1**	[1]

29	Separate variables and make reasonable attempt at integration of either integral	**M1**	
	Obtain term $\frac{1}{2}e^2$	**B1**	
	Use Pythagoras	**M1**	
	Obtain terms $\tan x - x$	**A1**	
	Evaluate a constant or use $x = 0$, $y = 0$ as limits in a solution containing terms $ae^{\pm 2y}$ and $b\tan x$, ($ab \neq 0$)	**M1**	
	Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$	**A1**	
	Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where $a > 0$	**M1**	
	Obtain answer $y = 0.179$	**A1**	
			[8]

30	**(i)**	Separate variables correctly and integrate at least one side	**M1**	
		Integrate and obtain term kt, or equivalent	**A1**	
		Carry out a relevant method to obtain A and B such that $\dfrac{1}{(4-x)} \equiv \dfrac{A}{x} + \dfrac{B}{4-x}$, or equivalent	**M1***	
		Obtain $A = B = \dfrac{1}{4}$, or equivalent	**A1**	
		Integrate and obtain terms $\dfrac{1}{4}\ln x - \dfrac{1}{4}\ln(4-x)$, or equivalent	**A1**√	
		EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
		Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\dfrac{x}{4-x}\right) = 4kt - 8k$	**A1**	
		Use a second pair of limits and determine k	**DM1**	
		Obtain the given exact answer correctly	**A1**	
		OR: Use both pairs of limits in a definite integral	**M1***	
		Obtain the given exact answer correctly	**A1**	
		Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
		Obtain $\ln\dfrac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	**A1**	[9]
	(ii)	Substitute $x = 3.6$ and solve for t	**M1**	
		Obtain answer $t = 4$	**A1**	[2]

31(i)	Carry out a relevant method to obtain A and B such that $\dfrac{1}{x(2x \quad 3)} \equiv \dfrac{A}{x} + \dfrac{B}{2x+3}$, or equivalent	**M1**
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	**A1**
	Total:	**2**
31(ii)	Separate variables and integrate one side	**B1**
	Obtain term $\ln y$	**B1**
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	**B2 FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	**A1**
	Obtain answer $y = 1.29$ (3s.f. only)	**A1**
	Total:	**7**

32	Separate variables correctly and attempt integration of one side	**B1**
	Obtain term $\tan y$, or equivalent	**B1**
	Obtain term of the form $k \ln \cos x$, or equivalent	**M1**
	Obtain term $-4 \ln \cos x$, or equivalent	**A1**
	Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a \tan y$ and $b \ln \cos x$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\tan y = 4 \ln \sec x + 1$	**A1**
	Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a \tan y$ and $b \ln \cos x$, and use correct method to find x	**M1**
	Obtain answer $x = 0.587$	**A1**
		8

33	Separate variables and obtain $\int \frac{1}{y} \mathrm{d}y = \int \frac{x+2}{x+1} \mathrm{d}x$	**B1**
	Obtain term $\ln y$	**B1**
	Use an appropriate method to integrate $(x+2)/(x+1)$	***M1**
	Obtain integral $x + \ln(x+1)$, or equivalent, e.g. $\ln(x+1) + x + 1$	**A1**
	Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1**
	Obtain correct solution in x and y in any form e.g. $\ln y = x + \ln(x+1) - 1$	**A1**
	Obtain answer $y = (x+1)\mathrm{e}^{x-1}$	**A1**
		7

34(i)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln x$	**B1**
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms $a \ln x$ and $b\,t\sqrt{t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	**A1**
		5
34(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	**M1**
	Substitute $x = 40$ and eliminate k	**M1**
	Obtain answer $t = 64.1$	**A1**
		3

35(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	**M1**
	Obtain $A = B = \frac{1}{4}$	**A1**
	Total:	**2**
35(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln (2 + y)$ or $c \ln (2 - y)$	**M1**
	Obtain term $\ln x$	**B1**
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	**A1FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln (2 + y)$ and $c \ln (2 - y)$	**M1**
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	**A1**
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	**A1**
	Total:	**6**

36	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln y$	**B1**
	Obtain terms $2 \ln x - \frac{1}{2} x^2$	**B1+B1**
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**

37(i)	Use chain rule	**M1**	$k\cos\theta\sin^{-3}\theta\left(= -k\operatorname{cosec}^2\theta\cot\theta\right)$ Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	**A1**	e.g. $-2\operatorname{cosec}^2\theta\cot\theta$, $\frac{-2\cos\theta}{\sin^3\theta}$ Accept $\frac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
37(ii)	Separate variables correctly and integrate at least one side	**B1**	$\int x\,dx = \int -\operatorname{cosec}^2\theta\cot\theta\,d\theta$
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain term of the form $\frac{k}{\sin^2\theta}$	**M1***	or equivalent
	Obtain term $\frac{1}{2\sin^2\theta}$	**A1**	or equivalent
	Use $x = 4$, $\theta = \frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\frac{b}{\sin^2\theta}$, where $ab \neq 0$	**DM1**	Dependent on the preceding M1
	Obtain solution $x = \sqrt{\left(\operatorname{cosec}^2\theta + 12\right)}$	**A1**	or equivalent
		6	

38(i)	Separate variables correctly and attempt integration of at least one side	**B1**	$\int e^{-y}\,dy = \int xe^{x}\,dx$
	Obtain term $-e^{-y}$	**B1**	B0B1 is possible
	Commence integration by parts and reach $xe^{x} \pm \int e^{x}\,dx$	**M1**	B0B0M1A1 is possible
	Obtain $xe^{x} - e^{x}$	**A1**	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^{x} and ce^{x}, where $abc \neq 0$	**M1**	Must see this step
	Obtain correct solution in any form	**A1**	e.g. $e^{-y} = e^{x} - xe^{x}$
	Rearrange as $y = -\ln(1-x) - x$	**A1**	or equivalent e.g. $y = \ln\dfrac{1}{e^{x}(1-x)}$ ISW
		7	
38(ii)	Justify the given statement	**B1**	e.g. require $1-x>0$ for the ln term to exist, hence $x<1$ Must be considering the range of values of x, and must be relevant to *their* y involving $\ln(1-x)$
		1	

39	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln(x+1)$	**B1**
	Obtain term of the form $a\ln(y^2+5)$	**M1**
	Obtain term $\frac{1}{2}\ln(y^2+5)$	**A1**
	Use $y = 2$, $x = 0$ to determine a constant, or as limits, in a solution containing terms $a\ln(y^2+5)$ and $b\ln(x+1)$, where $ab \neq 0$	**M1**
	Obtain correct solution in any form	**A1**
	Obtain final answer $y^2 = 9(x+1)^2 - 5$	**A1**
		7

40	Separate variables correctly to obtain $\int \frac{1}{x+2}\,dx = \int \cot\frac{1}{2}\theta\, d\theta$	B1	Or equivalent integrands. Integral signs SOI
	Obtain term $\ln(x+2)$	B1	Modulus signs not needed.
	Obtain term of the form $k\ln\sin\frac{1}{2}\theta$	M1	
	Obtain term $2\ln\sin\frac{1}{2}\theta$	A1	
	Use $x=1$, $\theta=\frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p\ln(x+2)$ and $q\ln\left(\sin\frac{1}{2}\theta\right)$	M1	Reach C = an expression or a decimal value
	Obtain correct solution in any form e.g. $\ln(x+2)=2\ln\sin\frac{1}{2}\theta+\ln 12$	A1	$\ln 12 = 2.4849$….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln\frac{1}{2}$
	Remove logarithms and use correct double angle formula	M1	Need correct algebraic process. $\left(\frac{x+2}{12}=\frac{1-\cos\theta}{2}\right)$
	Obtain answer $x=4-6\cos\theta$	A1	
		8	

41(i)	Separate variables correctly and integrate one side	B1	
	Obtain term $0.2t$, or equivalent	B1	
	Carry out a relevant method to obtain A and B such that $\frac{1}{(20-x)(40-x)} = \frac{A}{20-x} + \frac{B}{40-x}$	*M1	OE
	Obtain $A=\frac{1}{20}$ and $B=-\frac{1}{20}$	A1	
	Integrate and obtain terms $-\frac{1}{20}\ln(20-x)+\frac{1}{20}\ln(40-x)$ OE	A1FT +A1FT	The FT is on A and B
	Use $x=10$, $t=0$ to evaluate a constant, or as limits	DM1	
	Obtain correct answer in any form	A1	
	Obtain final answer $x=\frac{60e^{4t}-40}{3e^{4t}-1}$	A1	OE
		9	
41(ii)	Stathat x approaches 20	B1	
		1	