T3 TRIG IDENTITY MARKING SCHEME

1 (i) Make relevant use of formula for $\sin 2\theta$ or $\cos 2\theta$ M1

Make relevant use of formula for $\cos 4\theta$ M1

Complete proof of the given result A1 **3**

2 **(i)**

EITHER: Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$ **B1**

Use double angle formulae to express LHS in terms of $\cos\theta$ (and maybe $\sin\theta$) **M1**

Obtain any correct expression in terms of $\cos\theta$ alone **A1**

Reduce correctly to the given form **A1**

OR: Use double angle formula to express RHS in terms of $\cos 2\theta$ **M1**

Express $\cos^2 2\theta$ in terms of $\cos 4\theta$. **B1**

Obtain any correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ **A1**

Reduce correctly to the given form **A1** **4**

(ii) Using the identity, carry out method for calculating one root **M1**

Obtain answer 27.2° (or 0.475 radians) or 27.3° (or 0.476 radians) **A1**

Obtain a second answer, e.g. 332.8° (or 5.81 radians) **A1**√

Obtain remaining answers, e.g. 152.8° and 207.2° (or 2.67 and 3.62 radians) and no others in range **A1**√ **4**

3 **(i)** Use $\cot A = 1/\tan A$ or $\cos A/\sin A$ and/or $\operatorname{cosec} A = 1/\sin A$ on at least two terms M1

Use a correct double angle formula or the $\sin(A - B)$ formula at least once M1

Obtain given result A1 **3**

(ii) Solve $\cot\theta = 2$ for θ and obtain answer 26.6° B1

Obtain answer 206.6° and no others in the given range B1√ **2**

[Ignore answers outside the given range. Treat answers given in radians as a misread]

4 **(i)** State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$ B1

Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent M1

Simplify and obtain the given identity correctly A1 [3]

5 **(i)** Express $\cos 4\theta$ as $2\cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2\sin^2 2\theta$ B1

Express $\cos 4\theta$ in terms of $\cos\theta$ M1

Obtain $8\cos^4\theta - 8\cos^2\theta + 1$ A1

Use $\cos 2\theta = 2\cos^2\theta - 1$ to obtain given answer $8\cos^4\theta - 3$ **AG** A1 [4]

(ii) **(a)** State or imply $\cos^4\theta = \frac{1}{2}$ B1

Obtain 0.572 B1

Obtain –0.572 B1 [3]

6 **(i)** Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ M1

Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$ M1

Obtain a correct expression in terms of $\sin\theta$ in any form A1

Obtain the given identity A1 **[4]**

[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]

(ii) Substitute for x and obtain the given answer B1 **[1]**

(iii) Carry out a correct method to find a value of x M1

Obtain answers 0.322, 0.799, –1.12 A1 + A1 + A1 **[4]**

[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]

7 **(i)** *EITHER*: Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$ **B1**

Use correct double angle formulae to express LHS in terms of $\sin\theta$ and/or $\cos\theta$ **M1**

Obtain a correct expression in terms of $\sin\theta$ alone **A1**

Reduce correctly to the given form **A1**

OR: Use correct double angle formula to express RHS in terms of $\cos 2\theta$ **M1**

Express $\cos^2 2\theta$ in terms of $\cos 4\theta$ **B1**

Obtain a correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ **A1**

Reduce correctly to the given form **A1** [4]

(ii) Use the identity and carry out a method for finding a root **M1**

Obtain answer 68.5° **A1**

Obtain a second answer, e.g. 291.5° **A1**√

Obtain the remaining answers, e.g. 111.5° and 248.5°, and no others in the given interval **A1**√ [4]

[Ignore answers outside the given interval. Treat answers in radians as a misread.]

8	**(i)**	*EITHER*: Use tan 2A formula to express LHS in terms of $\tan\theta$ Express as a single fraction in any correct form Use Pythagoras or cos 2A formula Obtain the given result correctly	**M1** **A1** **M1** **A1**	
		OR: Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ Express as a single fraction in any correct form Use Pythagoras or cos 2A formula or sin($A - B$) formula Obtain the given result correctly	**M1** **A1** **M1** **A1**	[4]

9	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and $\tan\theta$	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
	OR2: Express LHS in terms of secθ and tanθ	**(M1**
	Multiply numerator and denominator by secθ + tanθ and use Pythagoras	**M1**
	Complete the proof	**A1)**
	Total:	**3**

10	Express the LHS in terms of either cos x and sin x or in terms of tan x	**B1**
	Use Pythagoras	**M1**
	Obtain the given answer	**A1**
	Total:	**3**

11	Use correct tan($A \pm B$) formula and express the LHS in terms of tan x	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4

12	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	**M1**	$\dfrac{2\sin x - 2\sin x\cos x}{1-\left(2\cos^2 x - 1\right)}$
	Obtain a correct expression	**A1**	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	**M1**	
	Obtain the given RHS correctly *OR (working R to L)*:	**A1**	
	$\dfrac{\sin x}{1+\cos x}\times\dfrac{1-\cos x}{1-\cos x}=\dfrac{\sin x-\sin x\cos x}{1-\cos^2 x}$ M1A1 $=\dfrac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$		Given answer so check working carefully
	$=\dfrac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	

13(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
13(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

14(i)	State correct expansion of $\sin(2x+x)$	**B1**	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	**M1**	
	Obtain a correct expression in any form	**A1**	e.g. $2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	**A1**	Accept = for ≡
		4	

15	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	**M1**	
	Obtain a correct expression	**A1**	
	Obtain the given answer	**A1**	
		3	

16	Use $\cos(A+B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	**M1**	
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	**M1**	
	Obtain a correct expression in terms of $\cos x$ in any form	**A1**	
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	**A1**	AG
		4	