# A LEVEL (P6) RANDOM VARIABLE

# QUESTION'S

**1** Two fair dice are thrown. Let the random variable $X$ be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

**(i)** Copy and complete the following table to show the probability distribution of $X$. [3]

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | | | | | | |

**(ii)** Find $\mathrm{E}(X)$. [2]

**2** A box contains five balls numbered 1, 2, 3, 4, 5. Three balls are drawn randomly at the same time from the box.

**(i)** By listing all possible outcomes (123, 124, etc.), find the probability that the sum of the three numbers drawn is an odd number. [2]

The random variable $L$ denotes the largest of the three numbers drawn.

**(ii)** Find the probability that $L$ is 4. [1]

**(iii)** Draw up a table to show the probability distribution of $L$. [3]

**(iv)** Calculate the expectation and variance of $L$. [3]

**3** A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable $X$ is the number of dice that fall on a green face.

**(i)** Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

**(ii)** Draw up a table for the probability distribution of $X$, giving your answers correct to 4 decimal places. [5]

**4** In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

**(i)** Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made. [1]

**(ii)** Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures. [2]

**(iii)** Draw up a probability distribution table for Mario's profit. [3]

**(iv)** Calculate his expected profit. [2]

**5** 32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

**(i)** How many teams play in only 1 match? [1]

**(ii)** How many teams play in exactly 2 matches? [1]

**(iii)** Draw up a frequency table for the numbers of matches which the teams play. [3]

**(iv)** Calculate the mean and variance of the numbers of matches which the teams play. [4]

**6** The discrete random variable $X$ has the following probability distribution.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.26 | $q$ | $3q$ | 0.05 | 0.09 |

**(i)** Find the value of $q$. [2]

**(ii)** Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

**7** A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

**(i)** Find the probability that the three peppers are all different colours. [3]

**(ii)** Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]

**(iii)** The number of **green** peppers taken is denoted by the discrete random variable $X$. Draw up a probability distribution table for $X$. [5]

**8** The random variable $X$ takes the values $-2$, 0 and 4 only. It is given that $\mathrm{P}(X = -2) = 2p$, $\mathrm{P}(X = 0) = p$ and $\mathrm{P}(X = 4) = 3p$.

**(i)** Find $p$. [2]

**(ii)** Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [4]

**9** Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

**(i)** Draw a tree diagram to illustrate this situation. [3]

**(ii)** Let $X$ be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of $X$. [4]

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | | $\frac{1}{4}$ | | | |

**(iii)** Calculate the expected number of unanswered phone calls on a day. [2]

**10** A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.

**(i)** Find the probability of obtaining at least 7 odd numbers in 8 throws of the die. [4]

The die is thrown twice. Let $X$ be the sum of the two scores. The following table shows the possible values of $X$.

Second throw

| | | 1 | 3 | 5 | | 6 | 6 |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 4 | 6 | | 7 | 7 |
| | 3 | 4 | 6 | 8 | | 9 | 9 |
| First | 5 | 6 | 8 | 10 | 10 | 11 | 11 |
| throw | 5 | 6 | 8 | 10 | 10 | 11 | 11 |
| | 6 | 7 | 9 | 11 | 11 | 12 | 12 |
| | 6 | 7 | 9 | 11 | 11 | 12 | 12 |

**(ii)** Draw up a table showing the probability distribution of $X$. [3]

**(iii)** Calculate $E(X)$. [2]

**(iv)** Find the probability that $X$ is greater than $E(X)$. [2]

**11** Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable $X$ denote Gohan's score.

**(i)** Show that $P(X = 2) = \frac{5}{16}$. [2]

**(ii)** The table below shows the probability distribution of $X$.

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{5}{16}$ | $\frac{1}{16}$ | $\frac{3}{8}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{16}$ |

Calculate $E(X)$ and $\mathrm{Var}(X)$. [4]

**12** The probability distribution of the random variable $X$ is shown in the following table.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.08 | $p$ | 0.12 | 0.16 | $q$ | 0.22 |

The mean of $X$ is 1.05.

**(i)** Write down two equations involving $p$ and $q$ and hence find the values of $p$ and $q$. [4]

**(ii)** Find the variance of $X$. [2]

**13** In a particular discrete probability distribution the random variable $X$ takes the value $\frac{120}{r}$ with probability $\frac{r}{45}$, where $r$ takes all integer values from 1 to 9 inclusive.

**(i)** Show that $\mathrm{P}(X = 40) = \frac{1}{15}$. [2]

**(ii)** Construct the probability distribution table for $X$. [3]

**(iii)** Which is the modal value of $X$? [1]

**(iv)** Find the probability that $X$ lies between 18 and 100. [2]

**14** The probability distribution of the discrete random variable $X$ is shown in the table below.

| $x$ | $-3$ | $-1$ | $0$ | $4$ |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $a$ | $b$ | 0.15 | 0.4 |

Given that $\mathrm{E}(X) = 0.75$, find the values of $a$ and $b$. [4]

**15** A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable $X$ represents the number of geese chosen.

**(i)** Draw up the probability distribution of $X$. [3]

**(ii)** Show that $\mathrm{E}(X) = \frac{8}{7}$ and calculate $\mathrm{Var}(X)$. [3]

**(iii)** When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

**16** Set $A$ consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4.

Set $B$ consists of the seven digits 0, 0, 0, 0, 2, 2, 2.

One digit is chosen at random from each set. The random variable $X$ is defined as the sum of these two digits.

**(i)** Show that $P(X = 2) = \frac{3}{7}$. [2]

**(ii)** Tabulate the probability distribution of $X$. [2]

**(iii)** Find $E(X)$ and $Var(X)$. [3]

**(iv)** Given that $X = 2$, find the probability that the digit chosen from set $A$ was 2. [2]

**17** Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

**(i)** Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable $X$ is Sanket's score.

**(ii)** Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of $X$.

| $x$ | 4 | 6 | 7 | 8 | 10 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{3}{9}$ | $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{1}{9}$ |

**(iii)** Given that $E(X) = \frac{58}{9}$, find $Var(X)$. [2]

Sanket throws the die twice.

**(iv)** Find the probability that the total of the scores on the two throws is 16. [2]

**(v)** Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

**18** The discrete random variable $X$ takes the values 1, 4, 5, 7 and 9 only. The probability distribution of $X$ is shown in the table.

| $x$ | 1 | 4 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $P(X = x)$ | $4p$ | $5p^2$ | $1.5p$ | $2.5p$ | $1.5p$ |

Find $p$. [3]

**19** In a probability distribution the random variable $X$ takes the value $x$ with probability $kx$, where $x$ takes values 1, 2, 3, 4, 5 only.

**(i)** Draw up a probability distribution table for $X$, in terms of $k$, and find the value of $k$. [3]

**(ii)** Find $\mathrm{E}(X)$. [2]

**20** The possible values of the random variable $X$ are the 8 integers in the set $\{-2, -1, 0, 1, 2, 3, 4, 5\}$. The probability of $X$ being 0 is $\frac{1}{10}$. The probabilities for all the other values of $X$ are equal. Calculate

**(i)** $\mathrm{P}(X < 2)$, [2]

**(ii)** the variance of $X$, [3]

**(iii)** the value of $a$ for which $\mathrm{P}(-a \leqslant X \leqslant 2a) = \frac{17}{35}$. [1]

**21** Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

**(i)** Show that the probability that the score is 6 is 0.08. [1]

**(ii)** Draw up a probability distribution table for the score. [2]

**(iii)** Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

**(iv)** Find the probability that Judy wins with the second choice of cards. [3]

**(v)** Find an expression for the probability that Judy wins with the $n$th choice of cards. [2]

**22** The probability that Sue completes a Sudoku puzzle correctly is 0.75.

**(i)** Sue attempts $n$ Sudoku puzzles. Find the least value of $n$ for which the probability that she completes all $n$ puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by $X$.

**(ii)** Find the value of $X$ that has the highest probability. You may assume that this value is one of the two values closest to the mean of $X$. [3]

**(iii)** Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]

**23** A team of 4 is to be randomly chosen from 3 boys and 5 girls. The random variable $X$ is the number of girls in the team.

**(i)** Draw up a probability distribution table for $X$. [4]

**(ii)** Given that $\mathrm{E}(X) = \frac{5}{2}$, calculate $\mathrm{Var}(X)$. [2]

**24** Bag $A$ contains 4 balls numbered 2, 4, 5, 8. Bag $B$ contains 5 balls numbered 1, 3, 6, 8, 8. Bag $C$ contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

**(i)** Find the probability that exactly two of the selected balls have the same number. [5]

**(ii)** Given that exactly two of the selected balls have the same number, find the probability that they are both numbered 2. [2]

**(iii)** Event $X$ is 'exactly two of the selected balls have the same number'. Event $Y$ is 'the ball selected from bag $A$ has number 2'. Showing your working, determine whether events $X$ and $Y$ are independent or not. [2]

**25** A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

**(i)** One rope is chosen at random. Find the expectation and variance of its length. [4]

**(ii)** Two ropes are chosen at random. Find the probability that they have different lengths. [2]

**(iii)** Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

**26** A spinner has 5 sides, numbered 1, 2, 3, 4 and 5. When the spinner is spun, the score is the number of the side on which it lands. The score is denoted by the random variable $X$, which has the probability distribution shown in the table.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.3 | 0.15 | $3p$ | $2p$ | 0.05 |

**(i)** Find the value of $p$. [1]

A second spinner has 3 sides, numbered 1, 2 and 3. The score when this spinner is spun is denoted by the random variable $Y$. It is given that $\mathrm{P}(Y = 1) = 0.3$, $\mathrm{P}(Y = 2) = 0.5$ and $\mathrm{P}(Y = 3) = 0.2$.

**(ii)** Find the probability that, when both spinners are spun together,

**(a)** the sum of the scores is 4, [3]

**(b)** the product of the scores is less than 8. [3]

**27** The random variable $X$ has the probability distribution shown in the table.

| $x$ | 2 | 4 | 6 |
|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.5 | 0.4 | 0.1 |

Two independent values of $X$ are chosen at random. The random variable $Y$ takes the value 0 if the two values of $X$ are the same. Otherwise the value of $Y$ is the larger value of $X$ minus the smaller value of $X$.

**(i)** Draw up the probability distribution table for $Y$. [4]

**(ii)** Find the expected value of $Y$. [1]

**28** The six faces of a fair die are numbered 1, 1, 1, 2, 3, 3. The score for a throw of the die, denoted by the random variable $W$, is the number on the top face after the die has landed.

**(i)** Find the mean and standard deviation of $W$. [3]

**(ii)** The die is thrown twice and the random variable $X$ is the sum of the two scores. Draw up a probability distribution table for $X$. [4]

**(iii)** The die is thrown $n$ times. The random variable $Y$ is the number of times that the score is 3. Given that $\mathrm{E}(Y) = 8$, find $\mathrm{Var}(Y)$. [3]

**29** Ashok has 3 green pens and 7 red pens. His friend Rod takes 3 of these pens at random, without replacement. Draw up a probability distribution table for the number of green pens Rod takes. [4]

**30** A fair tetrahedral die has four triangular faces, numbered 1, 2, 3 and 4. The score when this die is thrown is the number on the face that the die lands on. This die is thrown three times. The random variable $X$ is the sum of the three scores.

**(i)** Show that $\mathrm{P}(X = 9) = \frac{10}{64}$. [3]

**(ii)** Copy and complete the probability distribution table for $X$. [3]

| $x$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $\frac{1}{64}$ | $\frac{3}{64}$ | | | $\frac{12}{64}$ | | | | | |

**(iii)** Event $R$ is 'the sum of the three scores is 9'. Event $S$ is 'the product of the three scores is 16'. Determine whether events $R$ and $S$ are independent, showing your working. [5]

**31** The discrete random variable $X$ has the following probability distribution.

| $x$ | $-3$ | 0 | 2 | 4 |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $p$ | $q$ | $r$ | 0.4 |

Given that $\mathrm{E}(X) = 2.3$ and $\mathrm{Var}(X) = 3.01$, find the values of $p$, $q$ and $r$. [6]

**32** Fiona uses her calculator to produce 12 random integers between 7 and 21 inclusive. The random variable $X$ is the number of these 12 integers which are multiples of 5.

**(i)** State the distribution of $X$ and give its parameters. [3]

**(ii)** Calculate the probability that $X$ is between 3 and 5 inclusive. [3]

Fiona now produces $n$ random integers between 7 and 21 inclusive.

**(iii)** Find the least possible value of $n$ if the probability that none of these integers is a multiple of 5 is less than 0.01. [3]

**33** Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and 2 boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

**(i)** Find the probability that the two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag. [2]

**(ii)** Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that the sweet taken from Susan's bag was also a chocolate. [4]

**(iii)** The random variable $X$ is the number of times a chocolate is taken. State the possible values of $X$ and draw up a table to show the probability distribution of $X$. [5]

**34** The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers greater than 14. [4]

**35** James has a fair coin and a fair tetrahedral die with four faces numbered 1, 2, 3, 4. He tosses the coin once and the die twice. The random variable $X$ is defined as follows.

- If the coin shows a **head** then $X$ is the **sum** of the scores on the two throws of the die.
- If the coin shows a **tail** then $X$ is the score on the **first throw** of the die only.

**(i)** Explain why $X = 1$ can only be obtained by throwing a tail, and show that $P(X = 1) = \frac{1}{8}$. [2]

**(ii)** Show that $P(X = 3) = \frac{3}{16}$. [4]

**(iii)** Copy and complete the probability distribution table for $X$. [3]

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{8}$ | | $\frac{3}{16}$ | | $\frac{1}{8}$ | | $\frac{1}{16}$ | $\frac{1}{32}$ |

Event $Q$ is 'James throws a tail'. Event $R$ is 'the value of $X$ is 7'.

**(iv)** Determine whether events $Q$ and $R$ are exclusive. Justify your answer. [2]

**36** Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

**(i)** Show that P(the sum of the numbers on the three cards is 11) = $\frac{1}{2}$. [3]

**(ii)** Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Event $R$ is 'the sum of the numbers on the three cards is 11'. Event $S$ is 'the number on the first card taken is a 3'.

**(iii)** Determine whether events $R$ and $S$ are independent. Justify your answer. [3]

**(iv)** Determine whether events $R$ and $S$ are exclusive. Justify your answer. [1]

**37** Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

**(i)** Find the probability that the two digits chosen are equal. [3]

**(ii)** Find the probability that one digit is a 5 and one digit is not a 5. [3]

**(iii)** Find the probability that the first digit Dayo chose was a 5, given that the second digit he chose is not a 5. [4]

**(iv)** The random variable $X$ is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of $X$. [3]

**38** A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random variable $X$ represents the number of paperback books she chooses.

**(i)** Show that the probability that she chooses exactly 2 paperback books is $\frac{3}{14}$. [2]

**(ii)** Draw up the probability distribution table for $X$. [3]

**(iii)** You are given that $\mathrm{E}(X) = 3$. Find $\mathrm{Var}(X)$. [2]

**39** Coin $A$ is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin $B$ is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin $A$ is thrown twice and coin $B$ is thrown once.

**(i)** Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]

**(ii)** Draw up the probability distribution table for the number of heads obtained. [4]

**(iii)** Find the expectation of the number of heads obtained. [2]

**40** A pet shop has 6 rabbits and 3 hamsters. 5 of these pets are chosen at random. The random variable $X$ represents the number of hamsters chosen.

**(i)** Show that the probability that exactly 2 hamsters are chosen is $\frac{10}{21}$. [2]

**(ii)** Draw up the probability distribution table for $X$. [4]

**41** The number of phone calls, $X$, received per day by Sarah has the following probability distribution.

| $x$ | 0 | 1 | 2 | 3 | 4 | $\geqslant 5$ |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 0.24 | 0.35 | $2k$ | $k$ | 0.05 | 0 |

**(i)** Find the value of $k$. [2]

**(ii)** Find the mode of $X$. [1]

**(iii)** Find the probability that the number of phone calls received by Sarah on any particular day is more than the mean number of phone calls received per day. [3]

**42** Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

**(i)** Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

**(ii)** The random variable $X$ is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for $X$ and find $E(X)$. [4]

**43** The number of books read by members of a book club each year has the binomial distribution $B(12, 0.7)$.

**(i)** State the greatest number of books that could be read by a member of the book club in a particular year and find the probability that a member reads this number of books. [2]

**(ii)** Find the probability that a member reads fewer than 10 books in a particular year. [3]

**44** **(a)** Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if

**(i)** there are no restrictions, [2]

**(ii)** the number made is an even number. [4]

**(b)** Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

**45** A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

**(i)** Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable $S$.

**(ii)** By listing all possible selections (126, 246 and so on) draw up the probability distribution table for $S$. [5]

**46** A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

**(i)** Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

**(ii)** Construct the probability distribution table for the number of white rabbits in the sample. [3]

**(iii)** Find the expected value of the number of white rabbits in the sample. [1]

**47** Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

**(i)** Draw a fully labelled tree diagram to illustrate this situation. [3]

**(ii)** Let $X$ be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of $X$. [4]

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | | 0.24 | | |

**(iii)** Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

**48** A fair spinner $A$ has edges numbered 1, 2, 3, 3. A fair spinner $B$ has edges numbered −3, −2, −1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let $X$ be the sum of the numbers for the two spinners.

**(i)** Copy and complete the table showing the possible values of $X$. [1]

| | | Spinner $A$ | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 3 |
| Spinner $B$ | −3 | −2 | | | |
| | −2 | | | 1 | |
| | −1 | | | | |
| | 1 | | | | |

**(ii)** Draw up a table showing the probability distribution of $X$. [3]

**(iii)** Find $\text{Var}(X)$. [3]

**(iv)** Find the probability that $X$ is even, given that $X$ is positive. [2]