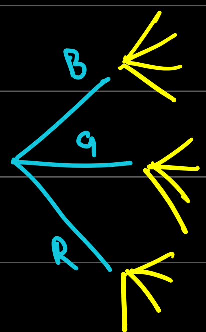
ARRANGEMENTS, PERMUTATIONS & COMBINATIONS

(10 Marks) (Time Saving) (workings are small).

ARRANGEMENTS

1 3 shirts 4 pants.

How many different dress settings are possible?



$3 \times 4 = 12$ ways

SHIRT		PANT
3	$\times$	4

$= 12$ ways.

EVENTS FOLLOWED BY EACH OTHER
NUMBER OF WAYS OF BOTH ARE OBTAINED BY MULTIPLYING THIER INDIVDUAL WAYS.

2 MENU

Starters: 1. ______
2. ______
3. ______

MAIN: 1. ______
2. ______
3. ______
4. ______
5. ______

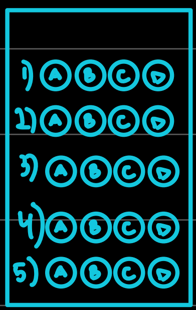
Drinks: 1. ______

Starter		MAIN		DRINK		DESSERT
3	$\times$	5	$\times$	4	$\times$	2

$= 120$ ways.

2. ______
3. ______
4. ______
Desserts: 1. ______
2. ______

$\boxed{3}$ MCQ EXAM CONTAINS 5 QUESTIONS.
EACH QUESTION HAS 4 OPTIONS (A, B, C, D)



Q1		Q2		Q3		Q4		Q5
4	×	4	×	4	×	4	×	4

= 1024 ways.

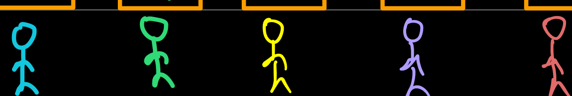
Thought Process

1- How many events do we have.
2- Fill the no. of ways of each event.
3- If they are followed by each other, MULTIPLY.

Q: 5 friends A, B, C, D and E Stand in a straight line.

(i) How many different arrangements are possible.

$\boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 5! = 120$ ways



FACTORIAL

$3! = 3 \times 2 \times 1 = 6$

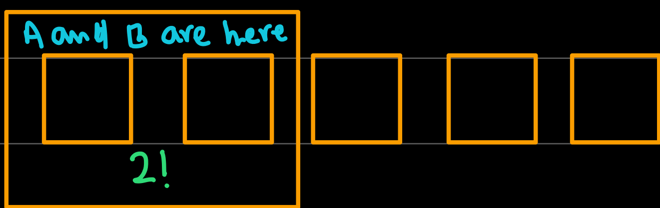
$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! =$

Factorial is used when you have to ARRANGE objects in a line and

OBJECTS = SPACES. $\boxed{n!}$

(ii) A and B want to stand together.



$4!$

$= 2! \times 4!$

$= 48$ ways.

(iii) A and B do not want to stand together.

A and B do not want to sit together $=$ TOTAL $-$ A and B sit together

$= 120 - 48$

$= 72$ ways.

This method to separate things is not universal. This method can only separate

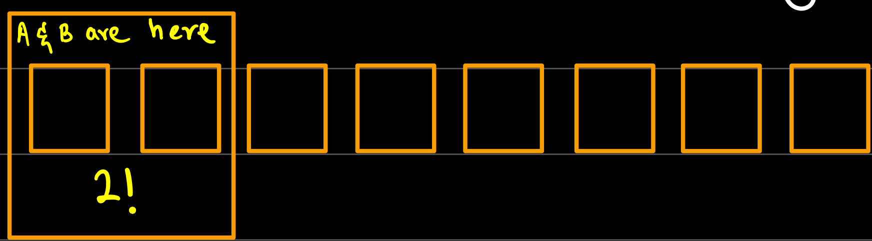
TWO THINGS (eg A & B). WE WILL STUDY THE UNIVERSAL TECHNIQUE TO SEPARATE LATER

Q. 8 friends go to watch a movie. They sit in same row with 8 seats. Find how many arrangements are possible if:

(i) No restrictions $= 8!$

$= 40320$ ways.

(ii) A and B want to sit together.

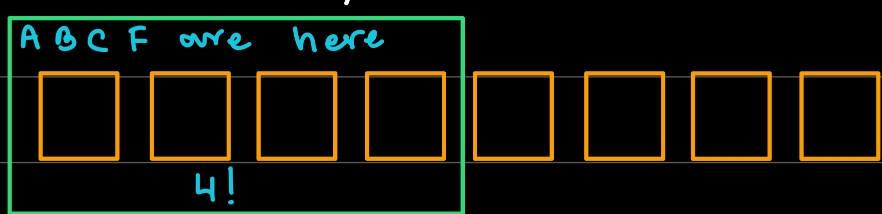


7!

$= 2! \times 7!$

$= 10080$ ways.

(iii) A, B, C, and F want to sit together.



5!

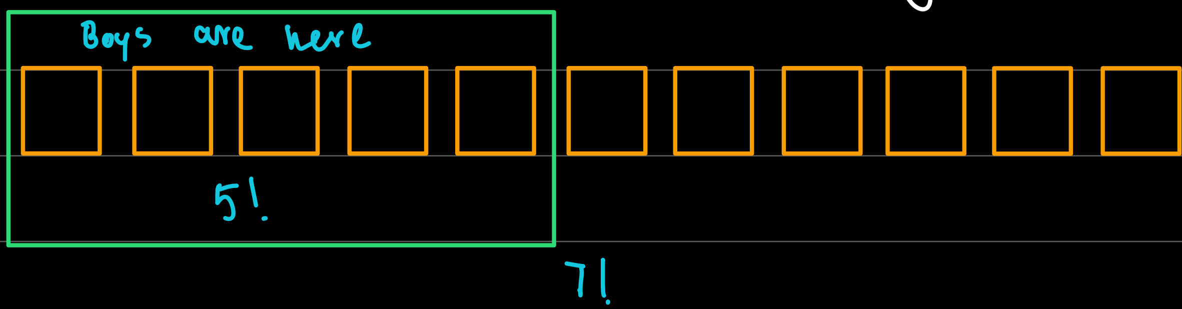
$= 4! \times 5!$

$= 2880$

Q. 5 boys and 6 girls stand in a straight line. Find the number of arrangements if

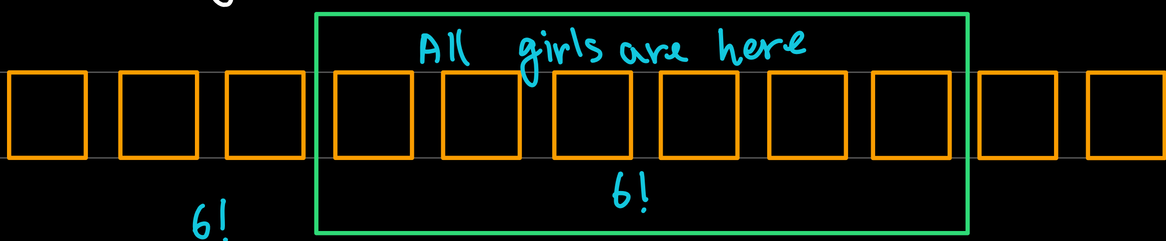
(i) No restrictions: $11! = 39916800$ ways

(ii) All boys want to stand together.



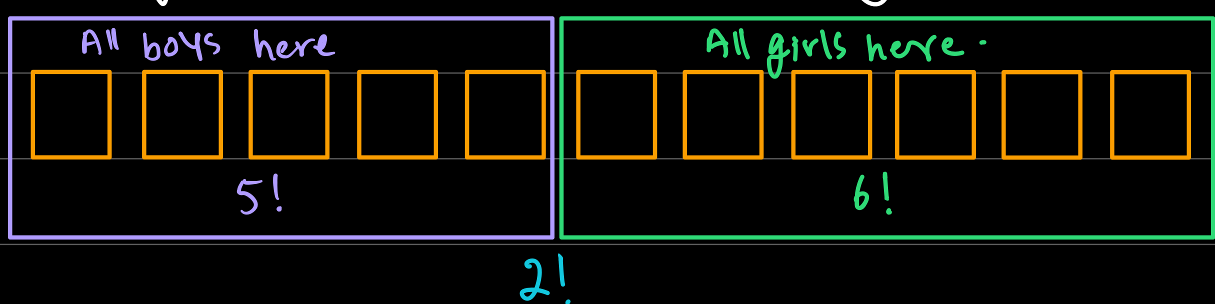
$= 5! \times 7!$

(iii) All girls want to stand together



$= 6! \times 6! = \boxed{\quad}$

(iv) All boys want to stand together and all girls want to stand together.



$= 5! \times 6! \times 2!$

(v) No two girls want to stand next to each other.

G	B	G	B	G	B	G	B	G	B	G
6	5	5	4	4	3	3	2	2	1	1

$= 5! \times 6!$

$=$

MEMORIZE THIS CASE (n/n-1) ALTERNATING.

$A = n$, $B = n-1$

To seperate A, we use alternating pattern.

Q:- On a family picnic, there are 5 men 8 women and 4 children. They stand in a straight line for a photograph. Find the number of arrangements if:

(i) No restrictions: $17!$

(ii) All women want to stand together:

_ _ _ _ _ _ _ _ | _ _ _ _ _ _ _ _ _ _

All women are here $= 8!$ $\quad$ $10!$

$= 8! \times 10!$

(iii) all men want to stand together and all women want to stand together.

All women are here $= 8!$	All men are here $= 5!$	$6!$

$$8! \times 5! \times 6!$$

(iv) All adults want to stand together and all children want to stand together.

All adults are here $= 13!$	All child $= 4!$

$2!$

$$= 13! \times 4! \times 2!$$

$=$

(v) All men want to stand together, all women want to stand together and all children want to stand together.

All women here	All men here	Children here
$8!$	$5!$	$4!$

$3!$

$$\boxed{8! \times 5! \times 4! \times 3!}$$

HOW TO TREAT IDENTICAL REPEATING ITEMS.

Method: 1) FIRST TREAT EVERYTHING NORMALLY AS IF THERE IS NO REPEAT.

2) NOW DIVIDE WITH ALL THE REPEATS $r!$

Q) THE Alphabets of word CROSS are arranged in a straight line. Find number of possible arrangements if :

identical repeat

CROSS

Total = 5

(i) No restrictions =

(ii) if all consonants should be together.

Q) THE Alphabets of word ARRANGEMENTS are arranged in a straight line. Find number of possible arrangements if :

(i) No restrictions =

ARRANGEMENTS
2R, 2A, 2E, 2N

(ii) All vowels should be together

$\boxed{\text{OBJECTS} = \text{SPACES}}$

Factorial $n!$

OBJECTS ≠ SPACES

PERMUTATIONS

ORDER MATTERS

$$^{n}P_{r}$$

Class = 30,
we need 3 student to give First, second and third position

		$\xrightarrow{\text{swap}}$	
First	=	A	B
Second	=	B	C
Third	=	C	A

This shows order matters

$$= {}^{30}P_{3}$$

KEY WORDS: $^{n}P_{r}$

ARRANGEMENTS

COMBINATION

ORDER DOES NOT MATTER

$$^{n}C_{r}$$

Class = 30,
we need 3 student to make a football team.

A, B, C $\xrightarrow{\text{swap}}$ A, C, B

order does not matter

$$= {}^{30}C_{3}$$

KEY WORDS: $^{n}C_{r}$

COMBINATIONS
CHOOSE
DIVIDE
SHARE
SELECT
TEAM
DELEGATION

ARRANGEMENTS

Object = spaces → FACTORIAL $\boxed{n!}$

Object ≠ spaces → PERMUTATION $\boxed{{}^{n}P_{r}}$

Q. Out of 10 students 3 students contest for positions of president, vice president & secretary of a society.
Find the number of arrangements if:

(i) No restrictions

(ii). Two students are brothers and either both of them win a position or none of them will win.

$$\boxed{{}^{n}P_{r} = {}^{n}C_{r} \times r!}$$

${}^{n}P_{r}$ → chooses things along with Arrangements paying attention to order

ABC, ACB, BAC are all different arrangements

${}^{n}C_{r}$ → choosing without paying attention to order

ABC, ACB, CAB are all same combination.

$r!$ → Arrange them

example: A, B, C, D, E

We need 3 alphabets.

Permutation v/s COMBINATION

1) ABC ACB BAC BCA CAB CBA

2) ABD — — — — —

3) ABE _ _ _ _ _

4) BCD _ _ _ _ _

5) BCE _ _ _ _ _

6) CDE _ _ _ _ _

7) ACD _ _ _ _ _

8) ACE _ _ _ _ _

9) ADE _ _ _ _ _

10) BDE _ _ _ _ _

$^5P_3 = 60$ $^5C_3 = 10$

$\boxed{\text{we need 3 alphabets from 5 such that their order matters.}}$

choosing with order

$^5P_3$

choose without order AND THEN ORDER

$^5C_3 \times 3!$

$^nP_r = {^nC_r} \times r!$

$60 = 10 \times 6$

$60 = 60$

UNIVERSAL TECHNIQUE TO SEPERATE COMPLETELY

Q. 4 Boys and 7 girls stand in a straight line. Find the number of arrangements if.

(i) All girls stand together.

(ii) No two boys stand next to each other.

we want all boys separate.

Step 1: First make girls stand.

STEP 2: Place boys on positions of stars.

Stars = 8, Boys = 4 $\Big\}\ {}^{n}P_{r}$

$$* \ G \ * \ G \ * \ G \ * \ G \ * \ G \ * \ G \ * \ G \ *$$

$$7! \times {}^{8}P_{4} = 8467200 \text{ ways.}$$

Q ADDITIONAL

Find number of arrangements if

ADDITIONAL

2D, 2A, 2I

(i) No restrictions $= \dfrac{10!}{\underset{D}{2!}\ \underset{A}{2!}\ \underset{I}{2!}}$

(ii) All vowels are next to each other.

$\boxed{\text{Vowels} = \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ 5!}$ $\underset{6!}{\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }} = \dfrac{5! \times 6!}{2!\,2!\,2!}$

(iii) No two vowels are next to each other.

we need to separate vowels completely.

Make consonants stand first.

ADDITIONAL

2D, 2A, 2I

* C * C * C * C * C *

$$\frac{5! \times {}^{6}P_{5}}{\underset{D}{2!}\ \underset{A}{2!}\ \underset{I}{2!}} =$$

Q: 3 History books and 7 philosophy books are arranged in a straight line. Find the number of arrangements if:

(i) No restrictions 10!

(ii) All history books are next to each other.

$\boxed{3!}$ _ _ _ _ _ _ _ _ $= 3! \times 8!$

$8!$ $= 241920$ ways.

(iii) Not all history books are next to each other.

NOT ALL TOGETHER = TOTAL − ALL TOGETHER

$= 10! - (3! \times 8!)$

$= 3386880$ ways

This means that all 3 of them should not be together but two of them can be next to each other.

This part does not mean that all history books are separated.

(iv) No two history books are next to each other.

This means that all history books should be separated. (Star method)

$* \underline{P} * \underline{P} * \underline{P} * \underline{P} * \underline{P} * \underline{P} * \underline{P} *$

$7! \times {}^{8}P_{3}$

IMPORTANT:

NOT All next to each other = TOTAL − ALL NEXT

TO EACH OTHER

NO TWO NEX TO EACH OTHER = (STAR METHOD TO SEPARATE)

SELECTION, CHOOSING, SHARING, combinations CANNOT HANDLE IDENTICAL REPEATS LIKE WE DO IN ARRANGEMENTS (ie we divide by r! there)

YOU CANNOT APPLY ^nC_r IF THERE ARE IDENTICAL REPEATS. YOU HAVE TO MANAGE THE IDENTICAL REPEATS MANUALLY.

21 **(i)** Find the number of different ways that the 9 letters of the word HAPPINESS can be arranged in a line. [1]

(ii) The 9 letters of the word HAPPINESS are arranged in random order in a line. Find the probability that the 3 vowels (A, E, I) are not all next to each other. [4]

(iii) Find the number of different selections of 4 letters from the 9 letters of the word HAPPINESS which contain no Ps and either one or two Ss. [3]

combination

(i) HAPPINESS $= \dfrac{9!}{2!\,2!} = 90720$ ways.

2P, 2S

P S

(ii) ALL VOWELS NEXT TO EACH OTHER $= \dfrac{3! \times 7!}{2!\,2!} = 7560$

$\boxed{\text{vowels} = 3!}$

7!

NOT ALL NEXT TO EACH OTHER = TOTAL − ALL NEXT TO EACH OTHER

$= 90720 - 7560$

NOT ALL NEXT TO EACH OTHER = 83160

$$P(\text{NOT ALL NEXT TO EACH OTHER}) = \frac{\text{NOT ALL NEXT TO EACH OTHER}}{\text{TOTAL}}$$

$$= \frac{83160}{90720}$$

$$= \frac{11}{12}$$

(iii) Find the number of different selections of 4 letters from the 9 letters of the word HAPPINESS which contain no Ps and either one or two Ss. [3]

nCr

nCr cannot handle identical repeats so you cannot do $\cancel{\frac{9C4}{2!\,2!}}$ This is not possible.

NO P and one S or NO P and 2 S

HAP~~P~~INES~~S~~ HA~~PP~~INES~~S~~

$\boxed{S}\,\boxed{}\,\boxed{}\,\boxed{}$ $\boxed{S}\,\boxed{S}\,\boxed{}\,\boxed{}$

1 $\;^5C_3$ 1 1 $\;^5C_2$

$(1 \times {}^5C_3) + (1 \times 1 \times {}^5C_2)$

$= 20$ ways.

$\boxed{\text{TYPE 1}}$ ALPHABETS

8 (i) Find the number of ways in which all twelve letters of the word REFRIGERATOR can be arranged

(a) if there are no restrictions, [2]

(b) if the Rs must all be together. [2]

(ii) How many different selections (^nC_r) of four letters from the twelve letters of the word REFRIGERATOR contain no Rs and two Es? [3]

(i) REFRIGERATOR $= \dfrac{12!}{\underset{R}{4!}\ \underset{E}{2!}} = 9979200.$

4R , 2E

(ii) $\boxed{\_\ \_\ \overset{4!}{\_}\ \_}$ $\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{9!}$ $= \dfrac{4! \times 9!}{\underset{R}{4!}\ \underset{E}{2!}} = 181440$

(iii). NO R and two E

$\cancel{R}\cancel{E}F\cancel{R}IG\cancel{E}\cancel{R}ATO\cancel{R}$

$\underset{1}{\boxed{E}}\ \underset{1}{\boxed{E}}\ \underset{^6C_2}{\boxed{}\ \boxed{}}$ $= 1 \times 1 \times {^6C_2} = {^6C_2} = 15$

29 **(i)** In how many ways can all 9 letters of the word TELEPHONE be arranged in a line if the letters P and L must be at the ends? [2]

How many different selections of 4 letters can be made from the 9 letters of the word TELEPHONE if

(ii) there are no Es, [1]

(iii) there is exactly 1 E, [2]

(iv) there are no restrictions? [4]

$\boxed{P}$ _ _ _ _ _ _ _ $\boxed{L}$ OR $\boxed{L}$ _ _ _ _ _ _ _ $\boxed{P}$

$1 \quad \frac{7!}{3!_{E}} \quad 1 \qquad\qquad 1 \quad \frac{7!}{3!_{E}} \quad 1$

$$\left(1 \times \frac{7!}{3!} \times 1\right) + \left(1 \times \frac{7!}{3!} \times 1\right)$$

$$= 1680$$

How many different selections of 4 letters can be made from the 9 letters of the word TELEPHONE if

$\to {}^nC_r$

(ii) there are no Es, [1]

(iii) there is exactly 1 E, [2]

(iv) there are no restrictions? [4]

(ii) NO E

T~~E~~L~~E~~PHON~~E~~

$\square\square\square\square$

${}^6C_4$

= 15 ways

(iii) Exactly 1 E

T~~E~~L~~E~~PHON~~E~~

? ?

$\boxed{E}\square\square\square$

$1 \quad {}^6C_3$

= 20 ways.

(iv) If there are no restriction and you select 4 alphabets from TELEPHONE

How many repeat cases are Possible.

Selecting 4 letters from TELEPHON without restriction =

NO E	Exactly 1 E	Exactly 2 E	EXACTLY 3 E
15	20	15	6

$= 15 + 20 + 15 + 6 = \boxed{56}$

Exactly two E

T~~E~~L~~E~~PHON~~E~~

?

$\boxed{E}\ \boxed{E}\ \boxed{\ }\ \boxed{\ }$

$1 \times 1 \times {}^6C_2$

$= 15$

Exactly 3 E

T~~E~~L~~E~~PHON~~E~~

$\boxed{E}\ \boxed{E}\ \boxed{E}\ \boxed{\ }$

$1 \times 1 \times 1 \times {}^6C_1$

$= 6$

39 Find the number of different ways in which all 8 letters of the word TANZANIA can be arranged so that

(i) all the letters A are together, [2]

(ii) the f rst letter is a consonant (T, N, Z), the second letter is a vowel (A, I), the third letter is a consonant, the fourth letter is a vowel, and so on alternately. [3]

4 of the 8 letters of the word TANZANIA are selected. How many possible selections contain

(iii) exactly 1 N and 1 A, [2]

(iv) exactly 1 N? [3]

(i) TANZANIA

3A , 2N

$\boxed{\_\ \overset{3!}{\_}\ \_}\ \_\ \_\ \_\ \_\ \_$

6!

$= \dfrac{3! \times 6!}{\underset{A}{3!}\ \underset{N}{2!}} = 360$ ways.

(ii) CONSONANTS

TNZN

VOWELS

AAIA

C V C V C V C V
4 4 3 3 2 2 1 1

$$\frac{4! \times 4!}{3!\ 2!} = 48 \text{ ways.}$$

A N

4 of the 8 letters of the word TANZANIA are selected. How many possible selections contain

ncr

(iii) exactly 1 N and 1 A, [2]

(iv) exactly 1 N? [3]

TANZANIA

3A, 2N

(iii) Exactly 1 N and 1 A

TANZANIA

? ? ?

$\boxed{N}\ \boxed{A}\ \boxed{\ }\ \boxed{\ }$

$1 \times 1 \times {}^3C_2 = 3$ ways

You could also do this manually.

LISTING

NATZ
NATI
NAIZ

$\boxed{\text{EXACTLY 1 N}}$

LISTING

TANZANIA

1N, NOA

$\boxed{N}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

$1 \quad {}^3C_3 = 1$

NTZI

1N, 1A (from last part) = 3

NATZ
NATI
NAIZ

1N, 2A

TANZANIA

$\boxed{N}\ \boxed{A}\ \boxed{A}\ \boxed{\ }$

NAAT

$1 \times 1 \times 1 \times {}^{3}C_{1} = 3$ NAAZ
NAAI

1N, 3A TANZANIA

$\boxed{N}\ \boxed{A}\ \boxed{A}\ \boxed{A} = 1$ NAAA

$1 \times 1 \times 1 \times 1$

8 ways.

TYPE 2: NUMBERS

9 The six digits 4, 5, 6, 7, 7, 7 can be arranged to give many different 6-digit numbers.

(i) How many different 6-digit numbers can be made? [2]

(ii) How many of these 6-digit numbers start with an odd digit and end with an odd digit? [4]

(i) Spaces = objects $\frac{6!}{3!} = 120$ ways.

7 repeat

(ii) $5\ \boxed{4,6,7,7}\ 7$ or $7\ \boxed{4,6,7,7}\ 5$ or $7\ \boxed{4,5,6,7}\ 7$

No of ways: $\left(1 \times \frac{4!}{2!} \times 1\right) + \left(1 \times \frac{4!}{2!} \times 1\right) + \left(1 \times 4! \times 1\right)$

two 7 repeat

$12 + 12 + 24$

$= 48$ ways.

12 **(a)** Find how many numbers between 5000 and 6000 can be formed from the digits 1, 2, 3, 4, 5 and 6

(i) if no digits are repeated, [2]

(ii) if repeated digits are allowed. [2]

1, 2, 3, 4, $\cancel{5}$, 6

(i) $\boxed{5}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

No. of way: $(1 \times 5 \times 4 \times 3) = 60$

(ii) $\boxed{5}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

No. of ways $(1 \times 6 \times 6 \times 6) = 216$

Repeat is allowed means all 6 numbers are available for each place.

If question asks for different numbers that can be made : ITS ARRANGEMENT

13 **(a)** **(i)** Find how many different four-digit numbers can be made using only the digits 1, 3, 5 and 6 with no digit being repeated. [1]

(ii) Find how many different odd numbers greater than 500 can be made using some or all of the digits 1, 3, 5 and 6 with no digit being repeated. [4]

(i) $4! = 24$ ways.

(ii) Three digit number OR Four Digit Number.

1, 3, 5, 6

odd
more than 500

$\boxed{5/6}\ \boxed{\ }\ \boxed{1/3/5}$

5 can come on both first & last digit

1, 3, 5, 6

odd

$\boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{1/3/5}$

$3! \quad \times 3$

First Digit is 5

$\boxed{5}\ \boxed{}\ \boxed{1/3}$ $\qquad$ $\boxed{6}\ \boxed{}\ \boxed{1/3/5}$

$(1 \times 2 \times 2) + (1 \times 2 \times 3) + (3! \times 3)$

$4 + 6 + 18$

$= 28$ ways.

32) (b) (i) The digits of the number 1 244 687 can be rearranged to give many different 7-digit numbers. How many of these 7-digit numbers are even? [4]

(ii) How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, 2, 4, 6, 7, 8? [2]

(i) ends with 2 $\qquad$ ends with 4 $\qquad$ ends with 6 $\qquad$ ends with 8

$\boxed{1,4,4,6,8,7}\ 2$ OR $\boxed{1,2,4,6,8,7}\ 4$ OR $\boxed{1,2,4,4,8,7}\ 6$ OR $\boxed{1,2,4,4,6,7}\ 8$

$\left(\frac{6!}{2!} \times 1\right) + \left(6! \times 1\right) + \left(\frac{6!}{2!} \times 1\right) + \left(\frac{6!}{2!} \times 1\right)$

$= 360 + 720 + 360 + 360$

$= 1800$ ways.

(ii) 1, $\cancel{2}$, 4, 6, 7, 8 $\qquad$ $\boxed{2}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}$

$1 \times {}^5P_4$

$= 120$ ways.

41 Nine cards are numbered 1, 2, 2, 3, 3, 4, 6, 6, 6.

(i) All nine cards are placed in a line, making a 9-digit number. Find how many different 9-digit numbers can be made in this way

(a) if the even digits are all together, [4]

(b) if the f rst and last digits are both odd. [3]

combination

(ii) Three of the nine cards are chosen and placed in a line, making a 3-digit number. Find how many different numbers can be made in this way

(a) if there are no repeated digits, [2]

(b) if the number is between 200 and 300. [2]

(i) $\boxed{\text{Even} = 6!}$

$4!$ _ _ _ $= \dfrac{6! \times 4!}{2!\,2!\,3!} = 720.$

(ii) $1\ \boxed{22\,34\,666}\ 3$ OR $3\ \boxed{22\,34\,666}\ 1$ OR $3\ \boxed{1224666}\ 3$

$\left(1 \times \dfrac{7!}{2!\,3!} \times 1\right) + \left(1 \times \dfrac{7!}{2!\,3!} \times 1\right) + \left(1 \times \dfrac{7!}{2!\,3!} \times 1\right)$

$420 + 420 + 420$

$= 1260$

$^5C_3$ arranged $= 3!$

(ii) Three of the nine cards are chosen and placed in a line, making a 3-digit number. Find how many different numbers can be made in this way

arrangement.

(a) if there are no repeated digits, [2]

(b) if the number is between 200 and 300. [2]

(ii) (a) 1 2 3 4 6

$^5P_3 = \underbrace{^5C_3}_{\text{chosen}} \times \underbrace{3!}_{\text{placed in a line}}$

arranged.

(b) 1 2 2 3 3 4 6 6 6

Let's do listing. between 200 and 300

212	221	231	241	261
213	223	232	242	262
214	224	234	243	263
216	226	233	246	264
		236		266

$4 + 4 + 5 + 4 + 5 = 22$ ways.

40 Find how many different numbers can be made from some or all of the digits of the number 1 345 789 if

(i) all seven digits are used, the odd digits are all together and no digits are repeated, [2]

(ii) the numbers made are even numbers between 3000 and 5000, and no digits are repeated, [3]

(iii) the numbers made are multiples of 5 which are less than 1000, and digits can be repeated. [3]

(i) $\underbrace{\boxed{\text{odds: } 5!}}_{3!}\ \_\ \_ = 5! \times 3! = 720$

(ii) $\boxed{3/4}\ \boxed{}\ \boxed{}\ \boxed{4/8}$

Starts with 4

$\boxed{4}\ \boxed{}\ \boxed{}\ \boxed{8}$

Starts with 3

$\boxed{3}\ \boxed{}\ \boxed{}\ \boxed{4/8}$

$(1 \times {}^5P_2 \times 1) + (1 \times {}^5P_2 \times 2)$

$20 + 40$

$= 60$ ways.

(iii) one digit | two digit | Three digit

$$\boxed{5} \qquad \boxed{}\,\boxed{5} \qquad \boxed{}\,\boxed{}\,\boxed{5}$$

$$(1) + (7 \times 1) + (7 \times 7 \times 1)$$

$$1 + 7 + 49$$

$$= 57.$$

$$\underset{\text{study}}{5}\quad \underset{\text{sleep}}{5}\quad \underset{\text{Ent}}{2}\quad \underset{\text{study}}{5}\quad \underset{\text{sleep}}{5}\quad \underset{\text{Ent}}{2} = 24 \text{ hours.}$$