

**ALEVEL P3**

**A4 LOGRITHMS**

**MARK SCHEME**

1	(i) Coeff of $x^3 = 6C3 \times 2^3 = 160$	B1 B1 B1	B1 for 6C3 B1 for 2 <sup>3</sup> B1 for 160
	(ii) Term in $x^2 = 6C2 \times 2^2 = 60$ reqd coeff = $1 \times (i) - 3 \times 60$ $\rightarrow -20$	M1 A1 [3]	B1 for 60 (could be given in (i)) Needs to consider 2 terms co

- 2 Use law for subtraction or addition of logarithms, or the equivalent in exponentials M1  
 Use  $\ln e = 1$  or  $e = \exp(1)$  M1  
 Obtain a correct equation free of logarithms e.g.  $\frac{1+x}{x} = e$  or  $1 + x = ex$  A1  
 Obtain answer  $x = 0.58$  (allow 0.582 or answer rounding to it) A1 4
- 3 Use law for the logarithm of a product or quotient, or the logarithm of a power M1  
 Obtain  $\ln x = \ln 4 - \ln 3$ , or equivalent A1  
 Obtain answer  $y = \frac{\ln 4 - \ln x}{\ln 3}$ , or equivalent A1 3
- 4 State or imply at any stage that  $3^{-x} = \frac{1}{3^x}$ , or that  $3^{-x} = \frac{1}{u}$  where  $u = 3^x$  B1  
 Convert given equation into the 3-term quadratic in  $u$  (or  $3^x$ ):  $u^2 - 2u - 1 = 0$  B1  
 Solve a 3-term quadratic, obtaining one or two roots M1  
 Obtain root  $\frac{2 + \sqrt{8}}{2}$ , or a simpler equivalent, or decimal value in [2.40, 2.42] A1  
 Use a correct method for finding the value of  $x$  from a positive root M1  
 Obtain  $x = 0.802$  only A1 6
- 5 EITHER State or imply  $e^x + 1 = e^{2x}$ , or  $1 + e^{-x} = e^x$ , or equivalent B1  
 Solve this equation as a quadratic in  $u = e^x$ , or in  $e^x$ , obtaining one or two roots M1  
 Obtain root  $\frac{1}{2}(1 + \sqrt{5})$ , or decimal in [1.61, 1.62] A1  
 Use correct method for finding  $x$  from a positive root M1  
 Obtain  $x = 0.481$  and no other answer A1  
 [For the solution 0.481 with no working, award B3 (for 0.48 give B2). However a suitable statement can earn the first B1 in addition, giving a maximum of 4/5 (or 3/5) in such cases.]
- OR State an appropriate iterative formula, e.g.  $x_{n+1} = \frac{1}{2} \ln(1 + e^{x_n})$  or B1  
 $x_{n+1} = \frac{1}{3} \ln(e^{x_n} + e^{2x_n})$  B1  
 Use the iterative formula correctly at least once M1  
 Obtain final answer 0.481 A1  
 Show sufficient iterations to justify its accuracy to 3 d.p., or show there is a sign change in the value of a relevant function in the interval (0.4805, 0.4815) A1  
 Show that the equation has no other root A1 5

[For the solution 0.585 with no relevant working, award B1 and a further B1 if 0.585 is shown to be the only root.]

- 10 Use law for the logarithm of a power, a quotient, or a product correctly at least once M1  
 Use  $\ln e = 1$  or  $e = \exp(1)$  M1  
 Obtain a correct equation free of logarithms, e.g.  $1 + x^2 = ex^2$  A1  
 Solve and obtain answer  $x = 0.763$  only A1 [4]  
 [For the solution  $x = 0.763$  with no relevant working give B1, and a further B1 if 0.763 is shown to be the only root.]  
 [Treat the use of logarithms to base 10 with answer 0.333 only, as a misread.]  
 [SR: Allow iteration, giving B1 for an appropriate formula, e.g.  $x_{n+1} = \exp((\ln(1 + x_n^2) - 1)/2)$ , M1 for using it correctly once, A1 for 0.763, and A1 for showing the equation has no other root but 0.763.]
- 11 (i) Use law for the logarithm of a product or quotient M1  
 Use  $\log_2 32 = 5$  or  $2^5 = 32$  M1  
 Obtain  $x^2 + 5x - 32 = 0$ , or horizontal equivalent A1 [3]  
 (ii) Solve a 3-term quadratic equation M1  
 Obtain answer  $x = 3.68$  only, or exact equivalent, e.g.  $\frac{\sqrt{153} - 5}{2}$  A1 [2]
- 12 Use law for the logarithm of a product, power or quotient M1\*  
 Obtain a correct linear equation, e.g.  $(2x - 1)\ln 5 = \ln 2 + x \ln 3$  A1  
 Solve a linear equation for  $x$  M1(dep\*)  
 Obtain answer  $x = 1.09$  A1 [4]  
 [SR: Reduce equation to the form  $a^x = b$  M1\*, obtain  $\left(\frac{25}{3}\right)^x = 10$  A1, use correct method to calculate value of  $x$  M1(dep\*), obtain answer 1.09 A1.]
- 13 Rearrange as  $e^{2x} - e^x - 6 = 0$ , or  $u^2 - u - 6 = 0$ , or equivalent B1  
 Solve a 3-term quadratic for  $e^x$  or for  $u$  M1  
 Obtain simplified solution  $e^x = 3$  or  $u = 3$  A1  
 Obtain final answer  $x = 1.10$  and no other A1 [4]
- 14 State or imply  $4 - 2^x = -10$  and 10 B1  
 Use correct method for solving equation of form  $2^x = a$  M1  
 Obtain 3.81 A1 [3]

<b>15</b>	<i>EITHER:</i> Use law of the logarithm of a power or quotient and remove logarithms Obtain a 3-term quadratic equation $x^2 - x - 3 = 0$ , or equivalent Solve 3-term quadratic obtaining 1 or 2 roots Obtain answer 2.30 only	M1 A1 M1 A1
	<i>OR1:</i> Use an appropriate iterative formula, e.g. $x_{n+1} = \exp\left(\frac{1}{2} \ln(3x_n + 4)\right) - 1$ correctly at least once Obtain answer 2.30 Show sufficient iterations to at least 3 d.p. to justify 2.30 to 2 d.p., or show there is a sign change in the interval (2.295, 2.305) Show there is no other root	M1 A1 A1 A1
	<i>OR2:</i> Use calculated values to obtain at least one interval containing the root Obtain answer 2.30 Show sufficient calculations to justify 2.30 to 3 s.f., e.g. show it lies in (2.295, 2.305) Show there is no other root	M1 A1 A1 A1
		[4]
<b>16</b>	Use law of the logarithm of a power and a product or quotient and remove logarithms Obtain a correct equation in any form, e.g. $\frac{2x+3}{x^2} = 3$ Solve 3-term quadratic obtaining at least one root Obtain final answer 1.39 only	M1 A1 M1 A1
<b>17</b>	<i>EITHER</i> Use laws of indices correctly and solve for $5^x$ or for $5^{-x}$ or for $5^{x-1}$	M1
	$\frac{5}{5-1}$	
	Obtain $5^x$ or for $5^{-x}$ or for $5^{x-1}$ in any correct form, e.g. $5^x =$ Use correct method for solving $5^x = a$ , or $5^{-x} = a$ , or $5^{x-1} = a$ , where $a > 0$ Obtain answer $x = 1.14$	A1 M1 A1
<b>18</b>	State or imply $\ln e = 1$ Apply at least one logarithm law for product or quotient correctly (or exponential equivalent) Obtain $x + 5 = ex$ or equivalent and hence $\frac{5}{e-1}$	B1 M1 A1 [3]

19	(i) <u>Either</u>	State or imply non-modular equation $(4x-1)^2 = (x-3)^2$ or pair of linear equations $4x-1 = \pm(x-3)$ Solve a three-term quadratic equation or two linear equations Obtain $-\frac{2}{3}$ and $\frac{4}{5}$	B1 M1 A1
	<u>Or</u>	Obtain value $-\frac{2}{3}$ from inspection or solving linear equation Obtain value $\frac{4}{5}$ similarly	B1 B2 [3]
	(ii)	State or imply at least $4^y = \frac{4}{5}$ , following a positive answer from part (i) Apply logarithms and use $\log a^b = b \log a$ property Obtain $-0.161$ and no other answer	B1✓ M1 A1 [3]
20		Use law for the logarithm of a product, quotient or power Use $\ln e = 1$ or $\exp(l) = 3$ Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$ Rearrange as $y = (ex^3 - 1)^{-1}$ , or equivalent	M1 M1 A1 A1 [4]
21	EITHER:	State or imply non-modular equation $2^2(3^x - 1)^2 = (3^x)^2$ , or pair of equations $2(3^x - 1) = \pm 3^x$ Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$ ) OR: Obtain $3^x = 2$ by solving an equation or by inspection Obtain $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$ ) by solving an equation or by inspection Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$ ), where $a > 0$ Obtain final answers $0.631$ and $-0.369$	M1 A1 B1 B1 M1 A1 [4]
22		Apply at least one logarithm property correctly Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent <b>without logarithm</b> involved Rearrange to express $x$ in terms of $a$ Obtain $\frac{16}{a-8}$ or equivalent	*M1 A1 M1 d*M A1 [4]

23	Remove logarithms and obtain $5 - e^{-2x} = e^{\frac{1}{2}}$ , or equivalent Obtain a correct value for $e^{-2x}$ , $e^{2x}$ , $e^{-x}$ or $e^x$ , e.g. $e^{2x} = 1/(5 - e^{\frac{1}{2}})$ Use correct method to solve an equation of the form $e^{2x} = a$ , $e^{-2x} = a$ , $e^x = a$ or $e^{-x} = a$ where $a > 0$ . [The M1 is dependent on the correct removal of logarithms.] Obtain answer $x = -0.605$ only.	B1 B1 M1 A1      4
24	Use law of the logarithm of a quotient or product or $2 = \log_{10} 100$ Remove logarithms and obtain $x + 9 = 100x$ , or equivalent Obtain answer $x = \frac{1}{11}$	M1 A1 A1      3
25	Use law of the logarithm of a power Obtain a correct linear equation in any form, e.g. $x = (x - 2) \ln 3$ Obtain answer $x = 22.281$	M1 A1 A1      [3]
26	Use law for the logarithm of a power at least once Obtain correct linear equation, e.g. $5x\ln 2 = (2x + 1)\ln 3$ Solve a linear equation for $x$ Obtain $x = 0.866$	*M1 A1 M1 dep *M A1      [4]
27	Use laws of indices correctly and solve for $u$ Obtain $u$ in any correct form, e.g. $u = \frac{16}{16 - 1}$ Use correct method for solving an equation of the form $4^x = a$ , where $a > 0$ Obtain answer $x = 0.0466$	M1 A1 M1 A1      [4]
28	Use law for the logarithm of a product, quotient or power Obtain a correct equation free of logarithms, e.g. $\frac{x+4}{x^2} = 4$ Solve a 3-term quadratic obtaining at least one root Obtain final answer $x = 1.13$ only	M1 A1 M1 A1      4
29	State or imply $1+u=u^2$ Solve for $u$ Obtain root $\frac{1}{2}(1 + \sqrt{5})$ , or decimal in [1.61, 1.62] Use correct method for finding $x$ from a positive root Obtain $x = 0.438$ and no other answer	B1 M1 A1 M1 A1      [5]

- 30 (i) EITHER: State or imply non-modular equation  $(2(x - 1))^2 = (3x)^2$ , or pair of linear equations  
 $2(x - 1) = \pm 3x$  B1  
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1  
 Obtain answers  $x = -2$  and  $x = \frac{2}{5}$  A1
- OR: Obtain answer  $x = -2$  by inspection or by solving a linear equation (B1)  
 Obtain answer  $x = \frac{2}{5}$  similarly B2)  
 [3]
- (ii) Use correct method for solving an equation of the form  $5^x = a$  or  $5^{x+1} = a$ , where  $a > 0$  M1  
 Obtain answer  $x = -0.569$  only A1  
 [2]
- 31 Use law of the logarithm of a product, power or quotient M1\*  
 Obtain a correct linear equation, e.g.  $(3x - 1)\ln 4 = \ln 3 + x\ln 5$  A1  
 Solve a linear equation for  $x$  DM1\*  
 Obtain answer  $x = 0.975$  A1 [4]

32	Solve for $3^x$ and obtain $3^x = \frac{18}{7}$  Use correct method for solving an equation of the form $3^x = a$ , where $a > 0$ Obtain answer $x = 0.860$ 3 d.p. only	B1 M1 A1	[3]
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33	Use law of the logarithm of a quotient  Remove logarithms and obtain a correct equation, e.g. $e^z = \frac{y+2}{y+1}$  Obtain answer $y = \frac{2 - e^z}{e^z - 1}$ , or equivalent	M1 A1 A1	[3]
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34	Use law of the logarithm of a power or a quotient	M1	
	Remove logarithms and obtain a correct equation in $x$ . e.g. $x^2 + 1 = ex^2$	A1	
	Obtain answer 0.763 and no other	A1	
<b>Total:</b>			<b>3</b>

35	Rearrange as $3u^2 + 4u - 4 = 0$ , or $3e^{2x} + 4e^x - 4 = 0$ , or equivalent	B1	
	Solve a 3-term quadratic for $e^x$ or for $u$	M1	
	Obtain $e^x = \frac{2}{3}$ or $u = \frac{2}{3}$	A1	
	Obtain answer $x = -0.405$ and no other	A1	
<b>Total:</b>			<b>4</b>

36	Use law for the logarithm of a power or a quotient on the given equation	<b>M1</b>
	Use $\log_2 8 = 3$ or $2^3 = 8$	<b>M1</b>
	Obtain $x^2 - 8x - 8 = 0$ , or horizontal equivalent	<b>A1</b>
	Solve a 3-term quadratic equation	
	Obtain final answer $x = 8.90$ only	

37	Use law for the logarithm of a product, quotient or power	<b>M1</b>
	Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$	<b>A1</b>
	Solve for $x$	<b>M1</b>
	Obtain answer $x = 1.52$ only	<b>A1</b>
		<b>4</b>

38	<i>EITHER:</i> State or imply non-modular equation $3^2(2^x - 1)^2 = (2^x)^2$ , or pair of equations $3(2^x - 1) = \pm 2^x$	<b>M1</b>
	Obtain $2^x = \frac{3}{2}$ and $2^x = \frac{3}{4}$ or equivalent	<b>A1</b>
	<i>OR:</i> Obtain $2^x = \frac{3}{2}$ by solving an equation	<b>B1</b>
	Obtain $2^x = \frac{3}{4}$ by solving an equation	<b>B1</b>
	Use correct method for solving an equation of the form $2^x = a$ , where $a > 0$	<b>M1</b>
	Obtain <b>final</b> answers $x = 0.585$ and $x = -0.415$ only	<b>A1</b>
		<b>4</b>

39	State or imply $u^2 = u + 5$ , or equivalent in $5^x$	B1
	Solve for $u$ , or $5^x$	M1
	Obtain root $\frac{1}{2}(1 + \sqrt{21})$ , or decimal in [2.79, 2.80]	A1
	Use correct method for finding $x$ from a positive root	M1
	Obtain answer $x = 0.638$ and no other answer	A1
	<b>Total:</b>	<b>5</b>

40	Rearrange the equation in the form $a e^{2x} = b$ or $a e^x = b e^{-x}$	M1
	Obtain correct equation in either form with $a = 2$ and $b = 5$	A1
	Use correct method to solve for $x$	M1
	Obtain answer $x = 0.46$	A1
		4

41	Substitute and obtain 3-term quadratic $3u^2 - 4u - 1 = 0$ , or equivalent	B1
	Solve a 3 term quadratic for $u$	M1
	Obtain root $(\sqrt{7} - 2) / 3$ , or decimal in [0.21, 0.22]	A1
	Use correct method for finding $x$ from a positive value of $e^x$	M1
	Obtain answer $x = -1.536$ only	A1
		5

42	Use law for the logarithm of a product, quotient or power	M1
	Obtain a correct equation free of logarithms	A1
	Solve a 3-term quadratic obtaining at least one root	M1
	Obtain answer $x = 4.30$ only	A1
		4

43	State or imply $u^2 - u - 12 (= 0)$ , or equivalent in $3^x$	B1
	Solve for $u$ , or for $3^x$ , and obtain root 4	B1
	Use a correct method to solve an equation of the form $3^x = a$ where $a > 0$	M1
	Obtain final answer $x = 1.26$ <b>only</b>	A1
		4

44	Use law of the logarithm of a product or quotient	M1
	Use law of the logarithm of power <b>twice</b>	M1
	Obtain a correct linear equation in $x$ , e.g. $(3-2x)\ln 5 = \ln 4 + x\ln 7$	A1
	Obtain answer $x = 0.666$	A1
		4

45	State 1 $e^{2y} = e^x$	B1
	Make $y$ the subject	M1
	Obtain answer $y = \frac{1}{2} \ln(e^x - 1)$	A1
		3

46	Remove logarithms and state $4 - 3^x = e^{1.2}$ , or equivalent	B1
	Use correct method to solve an equation of the form $3^x = a$ , where $a > 0$ .	M1
	Obtain answer $x = -0.351$ only	A1
		3

47	Reduce the equation to a horizontal equation in $3^{3x}$ , $3^{3x+1}$ or $27^x$	M1
	Simplify and reach $3(3^{3x}) - 5 = 3 \cdot 27^x - 5$ , or equivalent	A1
	Use correct method for finding $x$ from a positive value of $3^{3x}$ , $3^{3x-1}$ or $27^x$	M1
	Obtain answer $x = 0.155$	A1
		4