A12 POLYNOMIALS
P3

1 The polynomial $2x^3 + ax^2 - 4$ is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

When a has this value,

(ii) factorise $p(x)$, [2]

(iii) solve the inequality $p(x) > 0$, justifying your answer. [2]

2 The polynomial $x^4 + 5x + a$ is denoted by $p(x)$. It is given that $x^2 - x + 3$ is a factor of $p(x)$.

(i) Find the value of a and factorise $p(x)$ completely. [6]

(ii) Hence state the number of real roots of the equation $p(x) = 0$, justifying your answer. [2]

3 The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, find the quadratic factor of $p(x)$. [2]

4 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $x^2 + x + 2$ is a factor of $p(x)$. Find the value of a and the other quadratic factor of $p(x)$. [4]

5 The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a. [3]

(ii) When a has this value, solve the inequality $p(x) < 0$, justifying your answer. [3]

6 The polynomial $2x^3 + ax^2 + bx - 4$, where a and b are constants, is denoted by $\mathrm{p}(x)$. The result of differentiating $\mathrm{p}(x)$ with respect to x is denoted by $\mathrm{p}'(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and of $\mathrm{p}'(x)$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

7 The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is 9.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

8 The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = 12x^3 + 25x^2 - 4x - 12.$$

(i) Show that $\mathrm{f}(-2) = 0$ and factorise $\mathrm{f}(x)$ completely. [4]

(ii) Given that

$$12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,$$

state the value of 3^y and hence find y correct to 3 significant figures. [3]

9 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x - 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $\mathrm{p}(x)$. [2]

10 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $\mathrm{p}(x)$. It is given that $\mathrm{p}(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation $\mathrm{p}(x) = 0$. [2]

11 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a and hence factorise $\mathrm{p}(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\dfrac{8x - 13}{\mathrm{p}(x)}$ in partial fractions. [5]

12 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

13 The polynomial $\boldsymbol{a}x^3 - 20x^2 + x + 3$, where $\boldsymbol{a}$ is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(3x + 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of $\boldsymbol{a}$. [3]

(ii) When $\boldsymbol{a}$ has this value, factorise $\mathrm{p}(x)$ completely. [3]

14 The polynomial $8x^3 + \boldsymbol{a}x^2 + \boldsymbol{b}x + 3$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$. [5]

(ii) When $\boldsymbol{a}$ and $\boldsymbol{b}$ have these values, find the remainder when $\mathrm{p}(x)$ is divided by $2x^2 - 1$. [3]

15 It is given that $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$. [1]

16 **(i)** The polynomial $f(x)$ is of the form $(x - 2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$. [2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of a and b. [5]

17 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b. [5]

18 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

19 The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

20 **(i)** Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

21 Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

22 The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b. [5]

23 The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b. [5]

24 The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is -24. Find the values of a and b. [5]

