DE3 With Proof Chain Rule Answers
P3

1 (i)	State or imply $\frac{dV}{dt} = 1000\frac{dh}{dt}$	B1	
	State or imply $\frac{dV}{dt} = 30 - k\sqrt{h}$ or $\frac{dh}{dt} = 0.03 - m\sqrt{h}$	B1	
	Show that $k = 10$ or $m = 0.01$ and justify the given equation [Allow the first B1 for the statement that 0.03 = 30/1000.]	B1	**3**
(ii)	Separate variables and attempt integration of $\frac{x-3}{x}$ with respect to x	M1*	
	Obtain $x - 3\ln x$, or equivalent	A1	
	Obtain $0.005t$, or equivalent	A1	
	Use $x = 3$, $t = 0$ in the evaluation of a constant or as limits in an answer involving $\ln x$ and kt	M1(dep*)	
	Obtain answer in any correct form e.g. $t = 200(x - 3 - 3\ln x + 3\ln 3)$ [To qualify for the first M mark, an attempt to solve the earlier differential equation in h and t must involve correct separation of variables, the use of a substitution such as $\sqrt{h} = u$, and an attempt to integrate the resulting function of u.]	A1	**5**
(iii)	Substitute $x = 1$ and calculate t	M1	
	Obtain answer $t = 259$ correctly	A1	**2**

2	(**i**) State or obtain $\frac{dV}{dt} = 4h^2\frac{dh}{dt}$, or $\frac{dV}{dh} = 4h^2$, or equivalent	B1	
	State or imply $\frac{dV}{dt} = 20 - kh^2$	B1	
	Use the given values to evaluate k	M1	
	Show that $k = 0.2$, or equivalent, and obtain the given equation [The M1 is dependent on at least one B mark having been earned.]	A1	[4]
	(**ii**) Fully justify the given identity	B1	[1]
	(**iii**) Separate variables correctly and attempt integration of both sides	M1	
	Obtain terms $-20h$ and t, or equivalent	A1	
	Obtain terms $a\ln(10 + h) + b\ln(10 - h)$, where $ab \neq 0$, or $k\ln\left(\frac{10+h}{10-h}\right)$	M1	
	Obtain correct terms, i.e. with $a = 100$ and $b = -100$, or $k = 2000/20$, or equivalent	A1	
	Evaluate a constant and obtain a correct expression for t in terms of h	A1	[5]

3 **(i)** State or imply $V = \pi h^3$ B1

State or imply $\frac{dV}{dt} = -k\sqrt{h}$ B1

Use $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$, or equivalent M1

Obtain the given equation A1 **[4]**

[The M1 is only available if $\frac{dV}{dh}$ is in terms of h and has been obtained by a correct method.]

[Allow B1 for $\frac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\frac{k}{3\pi}$ has been justified.]

(ii) Separate variables and integrate at least one side M1

Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent A1

Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$ M1

Use $t = 60, h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$ M1

Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$ A1

(ii) Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent A1 **[6]**

(iii) Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$ B1 **[1]**