Class Notes

Chapter 6: The Link Layer and Local Area Networks (LANs)

Multiple Access Links and Protocols:

(1) Point-to-point links
   a. PPP protocols
   b. High-level data link protocol (HDLC)
(2) Broadcast Links
   a. Problem: How to fairly grant access to the shared medium?
(3) Classroom analogy:

“Give everyone a chance to speak.”

“Don’t speak until you are spoken to.”

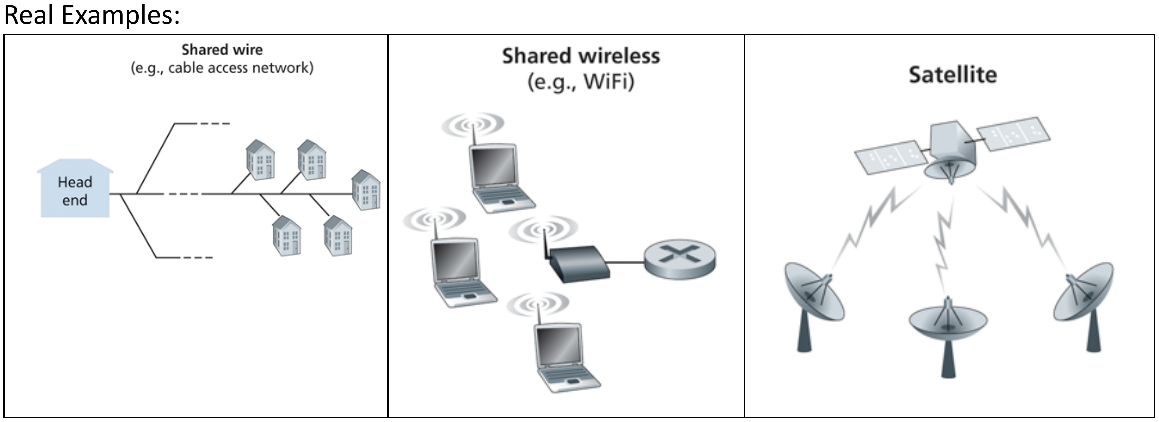
“Don’t monopolize the conversation.”

“Raise your hand if you have a question.”

“Don’t interrupt when someone is speaking.”

“Don’t fall asleep when someone is talking.”

(4) Real Examples:

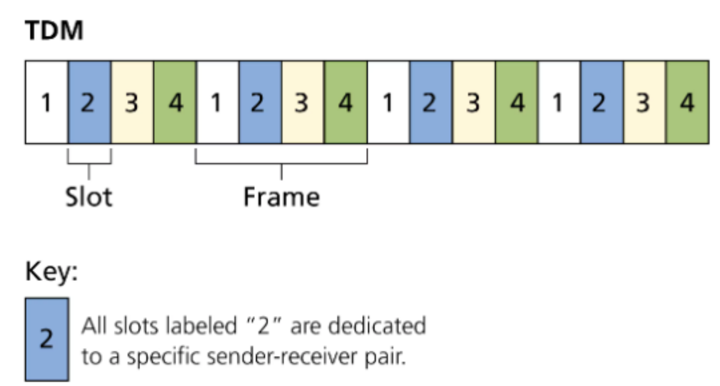


(5) Why we need a **<u>multiple access protocol to coordinate access to the channel</u>**?
   a. Collision: Multiple transmitters sending at the same time
   b. If you don’t do anything, communication channel might be virtually useless.
(6) <u>Three broad classes</u> of MAC protocols:
   a. Channel partitioning protocols
   b. Random Access protocols
   c. Taking-turns protocols

(7) **Goals:** If a shared link has capacity **R** bps, then we desire the following characteristics from a MAC protocol:
   a. **[Single User gets full bandwidth]** When only one node has data to send, that node has a throughput of R bps.
   b. **[Equitable Sharing]** When M nodes have data to send, each of these nodes has a throughput of R/M bps. This need not necessarily imply that each of the M nodes always has an instantaneous rate of R/M, but rather that each node should have an average transmission rate of R/M over some suitably defined interval of time.
   c. **[Fault-Tolerant]** The protocol is decentralized; that is, there is no master node that represents a single point of failure for the network.
   d. **[Low Hardware Cost]** The protocol is simple, so that it is inexpensive to implement.

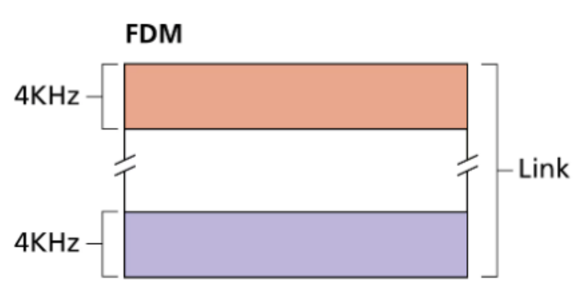
Channel partitioning protocols:

(1) Time Division Multiplexing (TDM)



(2) Evaluate based on our 4-step criterion
   a. No collisions, decentralized, If N nodes, R/N bandwidth for everyone
   b. A single user still gets R/N bandwidth because its turns come at a specific time

(3) Frequency Division Multiplexing (FDM)



(4) Same as TDM

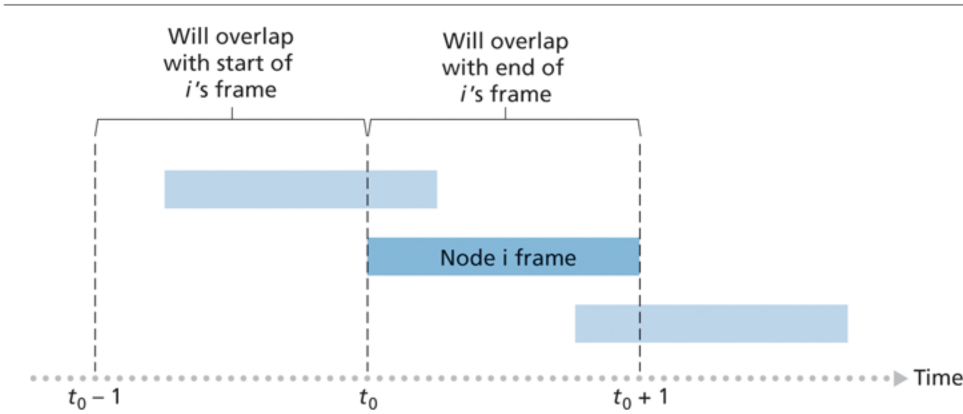
(5) Code Division Multiple Access (CDMA)
   a. Assigns different codes to each transmitting node
   b. Each node uses its unique code to encode its data bits
   c. If codes are carefully chosen, multiple nodes can transmit simultaneously without interfering with each other
   d. CDMA was heavily used in 3G but then left for more advanced technologies in 4G and 5G due to multiple challenges with CDMA when number of participants increase in the system. CDMA was also complex to implement.

Random access protocols:

Pure – Aloha has maximum efficiency of only 18%:

(1) Invented by a Ph.D. Engineer Norman Abramson in 1970s.
(2) Any node can transmit at any time. There is still a notion of frame (time a node can transmit. For example, time: L bits / R bps)
(3) Collision window is now worth two slots. Frame lengths were not required to be of the same length in the protocol, but for our analysis we will <u>assume</u> frame sizes are the same (primarily for simplifying our analyses).
(4) Understand the concept of vulnerable time.

FIGURE 6.11 INTERFERING TRANSMISSIONS IN PURE ALOHA



Derivation of Pure-Aloha Max Efficiency:

1. Define the Variables

First, let's explicitly define what we are measuring.

- N: The total number of nodes in the network.
- T: The time it takes to send one frame (Frame Time). We normalize this to $T = 1$.
- p: The probability that a specific node starts transmitting during a unit time interval.
- S **(Throughput)**: The average number of **successful** frames per frame time.
- G **(Offered Load)**: The total average number of frames transmitted (attempted) by the entire network per frame time.
  - Relationship: $G = N \cdot p$ (and conversely, $p = \frac{G}{N}$).

2. The Vulnerable Time (The "Danger Zone")

In Pure ALOHA, a node can transmit at any time t.

- If Node A transmits at time t, it finishes at $t + 1$.
- Collision occurs if any other node starts transmitting between $t - 1$ and $t + 1$.
- Therefore, the **Vulnerable Time is 2 unit intervals**.

3. Probability of Success for ONE Node

Let's look at a single node, Node i, that wants to send a frame.

For Node i to succeed, two things must happen:

1. Node i transmits. (Probability $= p$).
2. **No other node** transmits during the vulnerable time (duration of 2).

There are $N - 1$ other nodes. The probability that **one specific other node** does *not* transmit in a single time unit is $(1 - p)$. Therefore, the probability that this specific other node does *not* transmit for **2** time units is $(1 - p)^2$.

Since there are $N - 1$ other nodes, and they act independently, the probability that **ALL** of them remain silent for 2 time units is:

$$P(\text{others silent}) = [(1-p)^2]^{N-1} = (1-p)^{2(N-1)}$$

So, the probability that Node i has a successful transmission (P_{succ_i}) is:

$$P_{succ\_i} = p \cdot (1-p)^{2(N-1)}$$

4. Calculating Total System Throughput (S)

P_{succ_i} is just the chance for *one* guy. The Total Throughput S is the sum of successes from *all* N nodes in the system.

$$S = \sum_{1}^{N} P_{succ\_i} = N \cdot [p \cdot (1-p)^{2(N-1)}]$$

Now, we introduce the Load variable G. We know that $G = N \cdot p$. Let's substitute this into the equation:

$$S = G \cdot (1-p)^{2(N-1)}$$

Next, we eliminate p by using $p = G/N$:

$$S = G \cdot \left(1 - \frac{G}{N}\right)^{2(N-1)}$$

Some discussion that why we were allowed to add success_i to calculate S:

1. Successful Transmissions are "Mutually Exclusive"

This is the key. In the ALOHA protocol, is it possible for Node A to have a **successful** transmission and Node B to have a **successful** transmission at the exact same time?

No. If Node A and Node B overlap even by a microsecond, **both fail**. They become a collision.

Therefore, the event "Node A succeeds" and the event "Node B succeeds" are **disjoint (mutually exclusive)** events. They cannot happen simultaneously.

- **Rule of Sum:** For mutually exclusive events A and B, the probability of (A or B) is exactly $P(A) + P(B)$.
- Because we have N mutually exclusive possibilities (Node 1 wins OR Node 2 wins OR ... Node N wins), we can simply sum them up.

For example, if I flip a coin (p = 0.5) and you flip a coin (p = 0.5), the probability that *at least one* of us gets heads is not 0.5 +0.5 = 1 (it's actually 0.75).

2. We are calculating "Expectation", not just Probability

Strictly speaking, S is defined as the **Expected Value (E)** of the number of successful transmissions per frame time.

Let X_i be an "indicator variable" for Node i:

- $X_i = 1$ if Node i succeeds.
- $X_i = 0$ if Node i fails.

We want to find the expected total number of successes (X) for the whole network:

$$X = X_1 + X_2 + \cdots + X_N$$

There is a fundamental rule in statistics called the **Linearity of Expectation**:

> "The expectation of a sum is the sum of the expectations." (This works even if the variables are dependent!)

$$E[X] = E[X_1] + E[X_2] + \cdots + E[X_N]$$

Since $E[X_i]$ for a binary variable is just the probability $P(\text{success})$, we get:

$$S = \sum P_{success\_i}$$

5. Applying the Limit (The Infinite Node Assumption)

Real networks have many users sending infrequent data. We model this by letting the number of nodes N approach infinity ($N \to \infty$).

We use the standard calculus limit definition of the natural base e:

$$\lim_{x\to\infty}\left(1-\frac{a}{x}\right)^{bx} = e^{-ab}$$

Let's look at our equation:

$$S = G \cdot \lim_{N\to\infty}\left(1-\frac{G}{N}\right)^{2N-2}$$

We can ignore the "-2" in the exponent because as N becomes infinite, subtracting 2 changes nothing. The exponent effectively behaves like $2N$.

Using the limit rule where $x = N$, $a = G$, and $b = 2$:

$$S = G \cdot e^{-2G}$$

6. Maximizing Efficiency

Now we have the clean equation for Pure ALOHA efficiency:

$$S(G) = Ge^{-2G}$$

To find the maximum, we take the derivative with respect to G and set it to zero.

Product Rule: $(uv)' = u'v + uv'$

- $u = G \rightarrow u' = 1$
- $v = e^{-2G} \rightarrow v' = -2e^{-2G}$

$$\frac{dS}{dG} = (1)(e^{-2G}) + (G)(-2e^{-2G})$$

$$\frac{dS}{dG} = e^{-2G} - 2Ge^{-2G}$$

Factor out e^{-2G}:

$$e^{-2G}(1 - 2G) = 0$$

Since e^{-2G} is never zero, we solve:

$$1 - 2G = 0$$

$$2G = 1$$

$$G = 0.5$$

7. Final Calculation

The system is most efficient when the traffic load $G = 0.5$. Substitute $G = 0.5$ back into the main equation $S = Ge^{-2G}$:

$$S_{max} = 0.5 \cdot e^{-2(0.5)}$$

$$S_{max} = 0.5 \cdot e^{-1}$$

$$S_{max} = \frac{1}{2e}$$

$$S_{max} \approx 0.184$$

Result: The maximum efficiency is **18.4%**.

Don't be put off why G is a fraction: Remember:

- **Definition**: G represents the **average number of transmission attempts per packet time (T)**. It is not a discrete count of attempts in a single time unit but a statistical measure over the system as a whole.
  - It includes **all transmission attempts** in the system: both new transmissions and retransmissions due to collisions.
  - For example, if there are 10 users in the system, and each user tries to transmit on average once every 20 packet times, then $G = 10 \times \frac{1}{20} = 0.5$.

Efficiency in Slotted Aloha (Invented by Robert Metcalfe in 1972):

(1) Slotted Aloha has efficiency of 37%.

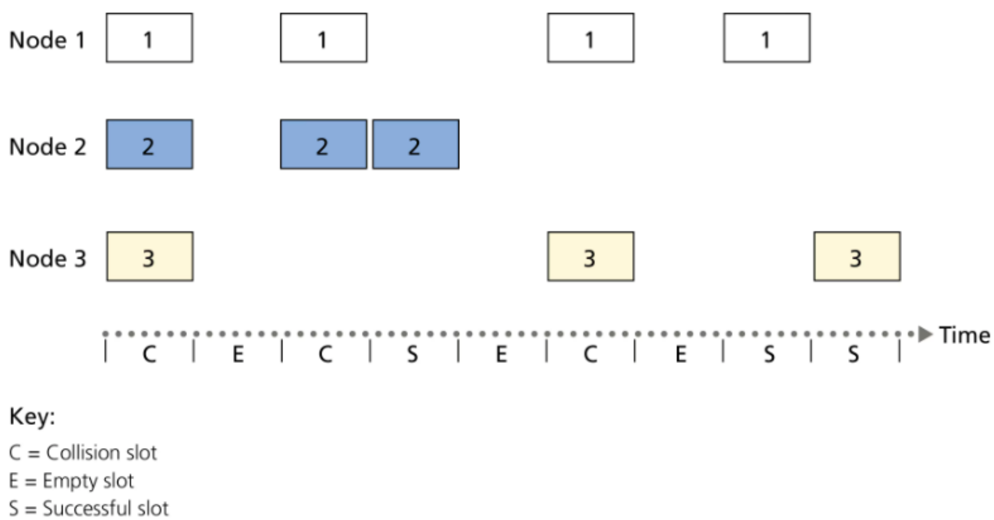


Figure 6.10: Nodes 1, 2, and 3 collide in the first slot. Node 2 finally succeeds in the fourth slot, node 1 in the eighth slot, and node 3 in the ninth slot

(2) Details:

1. The Setup & Assumptions

- N **Nodes:** An infinite population of users.
- **Time Slots:** Time is divided into discrete intervals of length $T = 1$ (one frame transmission time).
- **Synchronization:** Nodes can **only** start transmitting at the beginning of a time slot.
- p**:** Probability a node attempts to transmit in a given slot.

2. The Vulnerable Time

This is the critical difference from Pure ALOHA.

- Because transmissions are synchronized to the start of a slot, "partial overlaps" are impossible.
- A collision only occurs if two nodes pick the **same specific slot**.
- Therefore, the **Vulnerable Time = 1 Frame Time** (not 2).

3. Probability of Success for One Node

Let's focus on **Node** i. For Node i to be successful in a specific slot:

1. Node i must transmit (Probability $= p$).
2. The other $N - 1$ nodes must **not** transmit in that same slot.

- Probability one specific other node is silent: $(1 - p)$
- Probability all $N - 1$ other nodes are silent: $(1 - p)^{N-1}$

$$P_{success\_i} = p \cdot (1 - p)^{N-1}$$

(Note: The exponent is simply $N - 1$ here, whereas in Pure ALOHA it was $2(N - 1)$).

4. Calculating Throughput (S)

The total network throughput S is the sum of success probabilities for all N nodes.

$$S = N \cdot P_{success\_i}$$

$$S = N \cdot p \cdot (1 - p)^{N-1}$$

Substitute Total Load (G): Recalling that $G = N \cdot p$ (and $p = G/N$), we substitute:

$$S = G \cdot \left(1 - \frac{G}{N}\right)^{N-1}$$

5. Applying the Limit ($N \to \infty$)

We assume infinite nodes to model random traffic. We apply the limit to the term in the parenthesis.

$$S = G \cdot \lim_{N \to \infty} \left(1 - \frac{G}{N}\right)^{N-1}$$

Simplifying the Limit:

1. **The Exponent:** We can treat the exponent $N - 1$ effectively as N, because subtracting 1 is negligible at infinity (multiplicative factor of 1).
2. **The Formula:** We use the identity we proved earlier: $\lim_{x \to \infty}(1 - \frac{a}{x})^x = e^{-a}$.
   - Here, $a = G$.

$$S = G \cdot e^{-G}$$

6. Maximizing Efficiency

We now have the throughput equation for Slotted ALOHA: $S(G) = Ge^{-G}$. To find the max, derive with respect to G and set to 0.

Product Rule:

- $u = G \to u' = 1$
- $v = e^{-G} \to v' = -e^{-G}$

$$\frac{dS}{dG} = (1)(e^{-G}) + (G)(-e^{-G})$$

$$\frac{dS}{dG} = e^{-G} - Ge^{-G}$$

$$e^{-G}(1 - G) = 0$$

Since $e^{-G} \neq 0$:

$$1 - G = 0$$

$$G = 1$$

Result: The optimal load for Slotted ALOHA is $G = 1$ frame per slot.

7. Final Calculation

Substitute $G = 1$ back into the throughput equation:

$$S_{max} = 1 \cdot e^{-(1)}$$

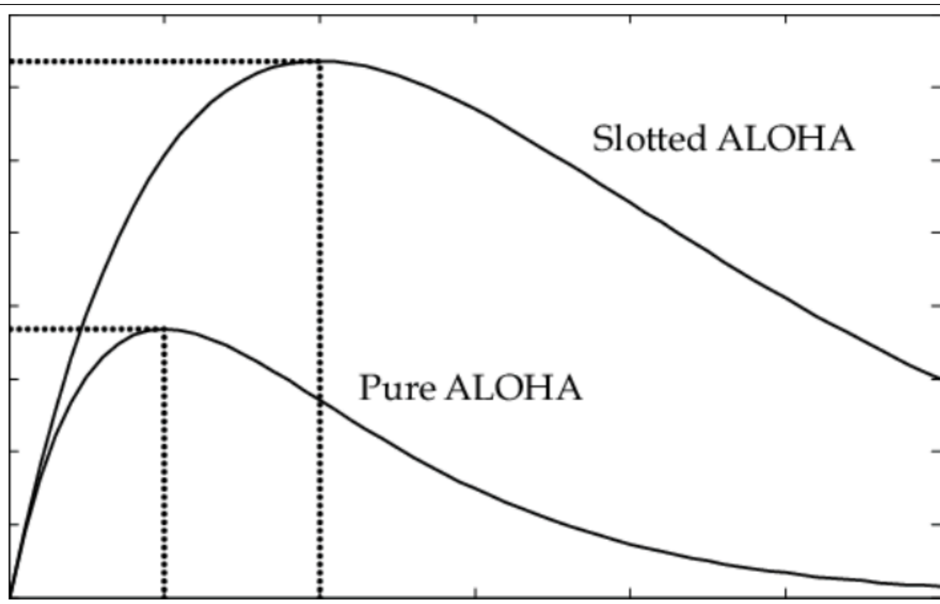
$$S_{max} = \frac{1}{e}$$

$$S_{max} \approx 0.368$$

Comparison of Pure-Aloha and Slotted-Aloha:

Summary of Differences

Step	Pure ALOHA	Slotted ALOHA	Why?
Vulnerable Time	2	1	Slots prevent partial overlap.
Exponent	$2(N-1)$	$1(N-1)$	Reflects vulnerable time.
Limit Result	e^{-2G}	e^{-G}	Math consequence of exponent.
Throughput Eq	$S = Ge^{-2G}$	$S = Ge^{-G}$	
Optimal Load	$G = 0.5$	$G = 1.0$	
Max Efficiency	$1/2e \approx 18\%$	$1/e \approx 37\%$	



Throughput versus offered traffic for Pure and Slotted ALOHA

How to Read This Graph

- **Vertical Axis (S):** This represents **Throughput** (Efficiency). The higher the curve, the more successful packets are getting through.
- **Horizontal Axis (G):** This represents **Offered Load** (Traffic). Moving right means more users are trying to send data.
- **The Difference:** Notice how the **Slotted ALOHA** curve (usually the higher one) reaches a peak of **0.368**, whereas the **Pure ALOHA** curve peaks much lower at **0.184**. This visualizes exactly why synchronization (slots) doubles the efficiency.

In Pure-Aloha and slotted-Aloha:

- Nodes neither sense the medium before sending
- Nor they stop sending when collision happens
- Sender relies on ACK from the receiver to know if packet was successfully through