Ex — 7.1

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} \cdot f(t) \cdot dt$$

* (i) $\mathcal{L}(1) = \frac{1}{s}$

(ii) $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$; $n = 1, 2, 3, \ldots$

(iii) $\mathcal{L}(e^{at}) = \frac{1}{s-a}$

(iv) $\mathcal{L}(\sin kt) = \frac{k}{s^2+k^2}$

(v) $\mathcal{L}(\cos kt) = \frac{s}{s^2+k^2}$

(vi) $\mathcal{L}(\sinh kt) = \frac{k}{s^2-k^2}$

(vii) $\mathcal{L}(\cosh kt) = \frac{s}{s^2-k^2}$

$\longleftrightarrow$

(i) $1 = \mathcal{L}^{-1}\left(\frac{1}{s}\right)$

(ii) $\mathcal{L}^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$; $n = 1, 2, 3, \ldots$

(iii) $\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

(iv) $\mathcal{L}^{-1}\left(\frac{k}{s^2+k^2}\right) = \sin kt$

(v) $\mathcal{L}^{-1}\left(\frac{s}{s^2+k^2}\right) = \cos kt$

(vi) $\mathcal{L}^{-1}\left(\frac{k}{s^2-k^2}\right) = \sinh kt$

(vii) $\mathcal{L}^{-1}\left(\frac{s}{s^2-k^2}\right) = \cosh kt$

Ex.1 $\mathcal{L}(1) = ?$

Sol:- $\mathcal{L}(1) = \int_0^\infty e^{-st} \cdot 1 \cdot dt$

$$= \lim_{b \to \infty} \int_0^b e^{-st} \cdot dt$$

$$= \lim_{b \to \infty} \left| \frac{e^{-st}}{-s} \right|_0^b$$

$$= \lim_{b \to \infty} \left(\frac{e^{-sb}}{-s} + \frac{e^{-s(0)}}{s} \right)$$

$$= \left(\frac{\frac{1}{e^\infty}}{-s} + \frac{1}{s} \right)$$

$$= 0 + \frac{1}{s}$$

$$= \frac{1}{s} \quad \text{Ans}$$

Ex. 2 $\quad \mathcal{L}(t) = ?$

Sol:- $\mathcal{L}(t) = \int_0^\infty e^{-st} \cdot t \cdot dt$

$$= \left| t \cdot \frac{e^{-st}}{-s} \right|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st} \cdot dt$$

$$= 0 + \frac{1}{s}\left| \frac{e^{-st}}{-s} \right|_0^\infty$$

$$= \frac{1}{s}\left[0 + \frac{1}{s} \right]$$

$$= \frac{1}{s^2}$$

Ex 3(a)

$\mathcal{L}(e^{-3t}) = ?$

$$\mathcal{L}(e^{-3t}) = \int_0^\infty e^{-st} \cdot e^{-3t}\, dt$$

$$= \int_0^\infty e^{-(s+3)t}\, dt$$

$$= \left| \frac{e^{-(s+3)t}}{-(s+3)} \right|_0^\infty$$

$$= 0 + \frac{1}{s+3}$$

$$= \frac{1}{s+3}$$

Ex. 3(b) $\mathcal{L}(e^{5t}) = ?$

$$\mathcal{L}(e^{5t}) = \int_0^\infty e^{-st} \cdot e^{5t}\, dt$$

$$= \int_0^\infty e^{(-s+5)t}\, dt$$

$$= \left. \frac{e^{(-s+5)t}}{-s+5} \right|_0^\infty$$

$$= \left. \frac{e^{-(s-5)t}}{(-s+5)} \right|_0^\infty$$

$$= 0 - \frac{1}{(-s+5)}$$

$$= \frac{1}{s-5}$$ Ans

Ex. 4

$$\mathcal{L}(\sin 2t) = ?$$

$$\mathcal{L}(\sin 2t) = \int_0^\infty \underset{II}{e^{-st}} \cdot \underset{I}{\sin 2t}\, dt$$

$$= \left| \sin 2t \cdot \frac{e^{-st}}{-s} \right|_0^\infty + \int_0^\infty \frac{e^{-st}}{s} \cdot \cos 2t \cdot (2)\, dt$$

$$= \cancel{\frac{\sin 2t}{s}}(0-0) + \frac{2}{s}\int_0^\infty \cos 2t \cdot e^{-st}\, dt$$

$$= \frac{2}{s}\left[\cos 2t \cdot \frac{e^{-st}}{-s}\right]_0^\infty + \frac{2}{s}\int \frac{e^{-st}}{s} \cdot (-\sin 2t) \cdot 2\, dt$$

$$\mathcal{L}(\sin 2t) = \frac{2}{s^2} - \frac{4}{s^2}\int e^{-st} \cdot \sin 2t \cdot dt$$

$$\mathcal{L}(\sin 2t) = \frac{2}{s^2} - \frac{4}{s^2}\mathcal{L}(\sin 2t)$$

$$\mathcal{L}(\sin 2t)\left(1 + \frac{4}{s^2}\right) = \frac{2}{s^2}$$

$$\mathcal{L}(\sin 2t)\left(\frac{s^2+4}{\cancel{s^2}}\right) = \frac{2}{\cancel{s^2}}$$

$$\mathcal{L}(\sin 2t) = \frac{2}{s^2+4}$$

Ex5 $\mathcal{L}(1+5t) = ?$

(a)

$$\mathcal{L}(1+5t) = \mathcal{L}(1) + \mathcal{L}(5t).$$

$$= \frac{1}{s} + \frac{5}{s^2}$$

(b) $\mathcal{L}(4e^{5t} - 10.\sin 2t) = \mathcal{L}(4e^{5t}) - \mathcal{L}(10.\sin 2t)$

$$= 4.\mathcal{L}(e^{5t}) - 10.\mathcal{L}(\sin 2t)$$

$$= 4.\frac{1}{s-5} - \frac{20}{s^2+4}$$

(c) $\mathcal{L}(20.e^{-3t} + 7t - 9) = 20\mathcal{L}(e^{-3t}) + 7\mathcal{L}(t) - 9\mathcal{L}(1)$

$$= \frac{20}{s+3} + \frac{7}{s^2} - \frac{9}{s}$$

$\mathcal{L}(f(t)) = ?$

$$f(t) = \begin{cases} 0 & 0 \le t < 3 \\ 2 & t \ge 3 \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^{\infty} e^{-st} \cdot f(t) \cdot dt$$

$$= \int_0^{3} e^{-st} \cdot 0 \, dt + \int_3^{\infty} e^{-st} \cdot 2 \, dt$$

$$= 0 + \left| 2 \frac{e^{-st}}{-s} \right|_3^{\infty}$$

$$= 0 + \frac{2e^{-3s}}{s}$$

$$= \frac{2e^{-3s}}{s} \quad ; \quad s > 0$$