Partial Derivatives

(14.3)

- All rules of normal derivatives apply here
- Numbers and other variables are treated as constants.

14.3: 1-54, 81-85

14.4: 1-32, 34-36

14.5: 1-34

$\rightarrow \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ | $\frac{\partial f}{\partial x} = f_x$ | $\frac{\partial}{\partial x} \approx \frac{d}{dx}$

Q- $f(x,y) = x^2 + 3xy + y - 1$; $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ at $(4,-5)$

$$\frac{\partial f}{\partial x} = 2x + 3(1 \cdot y + x \cdot 0) + 0 + 0$$

$$= 2x + 3y$$

$$\rightarrow 2(4) + 3(-5) = 8 - 15 = \underline{\underline{-7}}$$

$$\frac{\partial f}{\partial y} = 3x + 1$$

$$\rightarrow 3(4) + 1 = \underline{\underline{13}}$$

Q- $f(x,y) = y\sin xy$; $\frac{\partial f}{\partial y}$?

$$\frac{\partial f}{\partial y} = y(\cos xy \times x) + \sin xy$$

$$= xy\cos xy + \sin xy$$

Q- $f(x,y) = \frac{2y}{y+\cos x}$, f_x, f_y ?

$$f_x = \frac{(y+\cos x)(0) - 2y(-\sin x)}{(y+\cos x)^2} \quad , \quad f_y = \frac{y+\cos x(2) - 2y(1)}{(y+\cos x)^2}$$

$$= \frac{2y\sin x}{(y+\cos x)^2} \qquad = \frac{2y+2\cos x-2y}{(y+\cos x)^2} = \frac{2\cos x}{(y+\cos x)^2}$$

Q- $yz - \ln z = x + y$; $\frac{\partial z}{\partial x}$

$$\frac{\partial}{\partial x}(yz - \ln z) = \frac{\partial}{\partial x}(x+y)$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\cdot\frac{\partial z}{\partial x} = \frac{\partial x}{\partial x} + 0$$; y is treated as constant

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{z}\right) = 1$$

$$\frac{\partial z}{\partial x} = \frac{z}{yz-1}$$

- Second order Partial Derivatives: Differentiating twice $\rightarrow \frac{\partial}{\partial x} \rightarrow \frac{\partial^2}{\partial x^2}$ / $\frac{\partial f}{\partial x} \rightarrow \frac{\partial f^2}{\partial x^2} = f_{xx}$
- $\frac{\partial f}{\partial x} \rightarrow \frac{\partial^2 f}{\partial x \partial y} = f_{xy}$ or f_{yx}

OR $\longrightarrow \frac{\partial^2 f}{\partial y \partial x}$

Q- $f(x,y) = x\cos y + ye^{x}$; $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y \partial x}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$

$\frac{\partial f}{\partial x} = \cos y + ye^{x}$, $\frac{\partial f}{\partial y} = -x\sin y + e^{x}$

$\frac{\partial^2 f}{\partial x^2} = ye^{x}$ $\frac{\partial^2 f}{\partial y^2} = -x\cos y$

$\frac{\partial^2 f}{\partial x \partial y} = -\sin y + e^{x} = \frac{\partial^2 f}{\partial x \partial y}$