National University of Computer and Emerging Sciences, Lahore Campus

Course:	Multivariable Calculus	Course Code:	MT1008
Program:	CS, DS, SE	Semester:	Fall 2024
		Total Marks:	10
Sub. Date:	15-Feb-24		
Section:	All	Name:	
Exam:	Assignment-I	Roll No:	

Book: Thomas Calculus by G. B. Thomas, 13th Edition

Q# 01: $f(x,y) = \sqrt{25 - x^2 - y^2}$, c = 0,1,2,3,4

(a) find the function's domain
(b) find the function's range
(c) describe the function's level curves
(d) find the boundary of the function's domain
(e) determine if the domain is an open region, a closed region, or neither, and decide if the domain is bounded or unbounded.

Q# 02: Find and sketch the domain of the following function.

$$h(x,y) = \frac{1}{ln(4-x^2-y^2)}$$

Q# 03: At what points (x,y) in the plane the following function is continuous?

$$g(x,y) = \frac{x+y}{2+cosx}$$

Q# 04: By considering different paths of approach, show that the following function have no limit as $(x,y) \to (0,0)$?

$$f(x,y) = \frac{xy}{|xy|}$$

Q# 05: Show that the following limit does not exist.

$$lim_{(x,y)\to(1,-1)} \frac{xy+1}{x^2-y^2}$$

Q# 06: Define $f(0,0)$ in a way that extends f to be continuous at the origin.

$$f(x,y) = ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2 + y^2}\right)$$

Q# 07: Justify with valid reason that why the function $f(x,y,z) = x + y - z$ is continuous at every point (x_0, y_0, z_0)?

Q# 08: Under what condition the paritial derivative exists for the following function? Also find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$$f(x,y) = \sum_{n=0}^{\infty}(xy)^n$$

Q# 09: Give a counter example which shows that partial derivative doesnot imply continuity.
Also explain that under what condition(s) for a multivariable function $w_{xy} = w_{yx}$?

Q# 10: evaluate $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, and $\frac{\partial u}{\partial z}$.

$$u = \frac{p-q}{q-r}, p = x + y + z, q = x - y + z, r = x + y - z.$$

Q# 11: Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the indicated point.

$$sin\,(x + y) + sin\,(y + z) + sin\,(x + z) = 0,\ (\pi, \pi, \pi)$$

Q# 12: In what direction are the derivatives of the following at the indicated point equal to zero?

(a) $f(x, y) = xy + y^2 \; at \; P(3, 2)$

(b) $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2} at \; P(1, 1)$

Q# 13: How is the derivative of a differentiable function $f(x, y, z)$ at a point P_0 in the direction of a unit vector u related to the scalar component of $(\nabla f)P_0$ in the direction of u? Give reasons for your answer.