National University of Computer and Emerging Sciences, Lahore Campus

Course:	Multivariable Calculus	Course Code:	MT1008
Program:	CS, DS, SE	Semester:	Fall 2024
		Total Marks:	10
Sub. Date:	26-Mar-24		
Section:	All	Name:	
Exam:	Assignment-II	Roll No:	

Question#1:

Evaluate $\iiint\limits_{E} e^{-x^2-z^2}\, dV$ where E is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \le y \le 5$ and $z \le 0$.

Question#2:

D is bounded by the planes $y = 0, y = 2, x = 1, z = 0$ and $z = (2 - x)/2$.
Evaluate the triple integral with order $dx\, dy\, dz$.

Question#3:

D is bounded by the planes $x = 0, x = 2, z = -y$ and by $z = y^2/2$.
Evaluate the triple integral with orders $dy\, dz\, dx$ and $dz\, dy\, dx$ to verify that you obtain the same volume either way.

Question#4:

D is bounded by the planes $x = 2, y = 1, z = 0$ and $z = 2x + 4y - 4$.
Evaluate the triple integral with orders $dz\, dy\, dx$ and $dx\, dy\, dz$ to verify that you obtain the same volume either way.

Question#5:

D is bounded by the plane $z = 2y$ and by $y = 4 - x^2$.
Evaluate the triple integral with order $dz\, dy\, dx$.

Question#6:

D is bounded by the coordinate planes and by $z = 1 - y/3$ and $z = 1 - x$.
Evaluate the triple integral with order $dx\, dy\, dz$.

Question#7:

Converting to a polar integral Integrate $f(x, y) = [\ln(x^2 + y^2)]/\sqrt{x^2 + y^2}$ over the region $1 \le x^2 + y^2 \le e$.

Question#8:
Converting to polar integrals

a. The usual way to evaluate the improper integral
$I = \int_0^\infty e^{-x^2}\, dx$ is first to calculate its square:

$$I^2 = \left(\int_0^\infty e^{-x^2}\, dx\right)\left(\int_0^\infty e^{-y^2}\, dy\right) = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy.$$

Evaluate the last integral using polar coordinates and solve the resulting equation for I.

b. Evaluate

$$\lim_{x\to\infty} \operatorname{erf}(x) = \lim_{x\to\infty} \int_0^x \frac{2e^{-t^2}}{\sqrt{\pi}}\, dt.$$

Question#9:

Existence Integrate the function $f(x, y) = 1/(1 - x^2 - y^2)$ over the disk $x^2 + y^2 \leq 3/4$. Does the integral of $f(x, y)$ over the disk $x^2 + y^2 \leq 1$ exist? Give reasons for your answer.

Question#10:
First sketch the region and then find the volume of the region enclosed by the $y = x^2$ from $x = -1$ to $x = 1$ and $y + z = 1$.

Question#11:
First sketch the region and then find the volume of the region in the first octant bounded by the coordinate planes and the planes $x + z = 1$ and $y + 2z = 2$.

Question#12:
First sketch the region and then find the volume of the region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x/2)$, $0 \leq x \leq 1$.

Question#13:
Set up the limits of integration for evaluating the triple integral of a function $F(x, y, z)$ over the tetrahedron D with vertices $(0, 0, 0), (1, 1, 0), (0, 1, 0), and\ (0, 1, 1)$. Use the order of integration $dy\, dz\, dx$ and then $dz\, dy\, dx$.

Question#14:

Evaluate the integrals by changing the order of integration in an appropriate way.

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\, dx\, dy\, dz$$

Question#15:
Finding an upper limit of an iterated integral Solve for a:

$$\int_0^1 \int_0^{4-a-x^2} \int_a^{4-x^2-y} dz\, dy\, dx = \frac{4}{15}.$$

Question#16:

In Exercises ___ set up the iterated integral for evaluating $\iiint_D f(r, \theta, z)\, dz\, r\, dr\, d\theta$ over the given region D.

(a) D is the right circular cylinder whose base is the circle $r = 2 \sin \theta$ in the xy-plane and whose top lies in the plane $z = 4 - y$.

(b) D is the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = 2 - y$.

(c) D is the right circular cylinder whose base is the circle $r = 3 \cos \theta$ and whose top lies in the plane $z = 5 - x$.

(d) D is the prism whose base is the triangle in the xy-plane bounded by the y-axis and the lines $y = x$ and $y = 1$ and whose top lies in the plane $z = 2 - x$.

Question#17:

Cylinder and cones Find the volume of the solid cut from the thick-walled cylinder $1 \le x^2 + y^2 \le 2$ by the cones $z = \pm\sqrt{x^2 + y^2}$.

Question#18:

Cylinder and sphere Find the volume of the region cut from the solid cylinder $x^2 + y^2 \le 1$ by the sphere $x^2 + y^2 + z^2 = 4$.

Question#19:

Find the average value of the function $f(r, \theta, z) = r$ over the solid ball bounded by the sphere $r^2 + z^2 = 1$. (This is the sphere $x^2 + y^2 + z^2 = 1$.)

Question#20:

Vertical planes in cylindrical coordinates

a. Show that planes perpendicular to the x-axis have equations of the form $r = a \sec \theta$ in cylindrical coordinates.

Question#21:

Evaluate $\int_C x\, ds$, where C is

a. the straight-line segment $x = t$, $y = t/2$, from $(0, 0)$ to $(4, 2)$.

b. the parabolic curve $x = t$, $y = t^2$, from $(0, 0)$ to $(2, 4)$.

Question#22:

Find the line integral of $f(x, y) = ye^{x^2}$ along the curve $\mathbf{r}(t) = 4t\mathbf{i} - 3t\mathbf{j}$, $-1 \le t \le 2$.

Question#23:

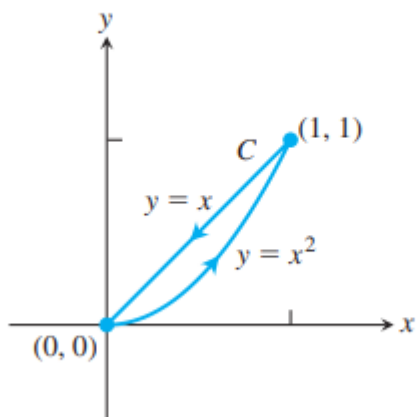Evaluate $\int_C \dfrac{x^2}{y^{4/3}}\, ds$, where C is the curve $x = t^2$, $y = t^3$, for $1 \le t \le 2$.

Question#24:

Mass of wire with variable density Find the mass of a thin wire lying along the curve $r(t) = \sqrt{2}t\mathbf{i} + \sqrt{2}t\mathbf{j} + (4 - t^2)\mathbf{k}$, $0 \leq t \leq 1$, if the density is **(a)** $\delta = 3t$ and **(b)** $\delta = 1$.

Question#25:

Evaluate $\int_C (x + \sqrt{y})\, ds$ where C is given in the accompanying figure.

Question#26:

Write a detail note on the use of line integrals in computer vision, a field within computer science that deals with enabling computers to gain high-level understanding from digital images or videos.

Note: Utilize insights from open-source AI resources to enrich your explanation.