Chapter 29

Magnetic Fields due to Currents

29.2: Magnetic Field due to a Long Straight Wire:

The magnitude of the magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire)}.$$

The magnetic field vector at any point is tangent to a circle.

$$\mu_0 = 4\pi \times 10^{-7}\,\text{T·m/A} \approx 1.26 \times 10^{-6}\,\text{T·m/A}.$$

In electromagnetism, **permeability** is the measure of the ability of a material to support the formation of a **magnetic** field within itself. Hence, it is the degree of magnetization that a material obtains in response to an applied **magnetic** field.

Wire with current into the page
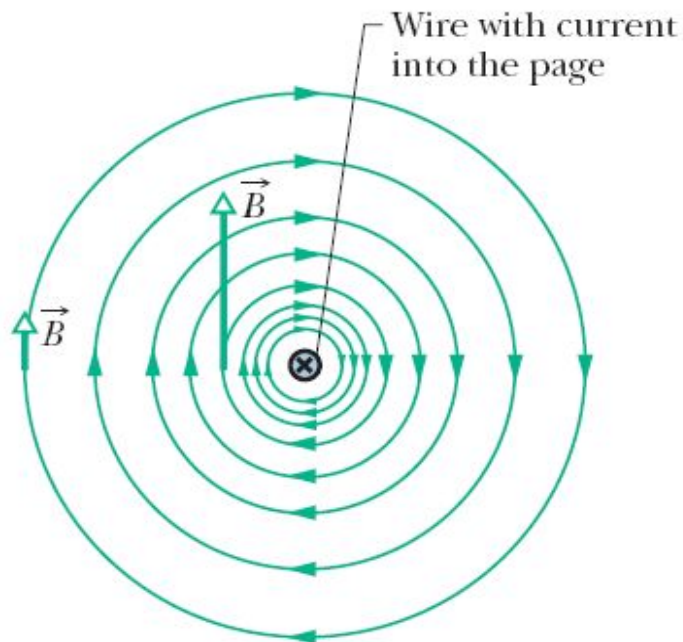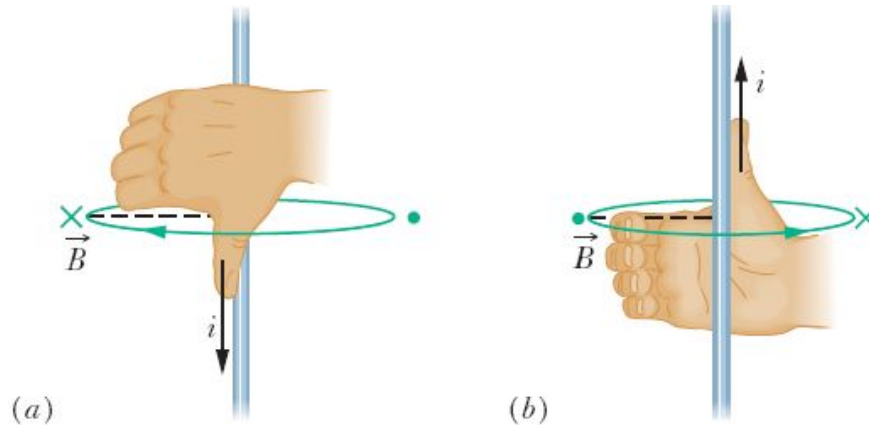
$\vec{B}$

$\vec{B}$

Fig. 29-2 The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

Fig. 29-3 Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current. *(Courtesy Education Development Center)*

29.2: Magnetic Field due to a Long Straight Wire:

> *Right-hand rule:* Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.

(a)

(b)

Fig. 29-4 A right-hand rule gives the direction of the magnetic field due to a current in a wire. (*a*) The magnetic field **B** at any point to the left of the wire is perpendicular to the dashed radial line and directed into the page, in the direction of the fingertips, as indicated by the x. (*b*) *If the* current is reversed, at any point to the left is still perpendicular to the dashed radial line but now is directed out of the page, as indicated by the dot.

The line integral $\oint \vec{B} \cdot d\vec{s}$ of the magnetic field $\vec{B}$ along any closed path is equal to the total current enclosed inside the path multiplied by μ_0.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law)}.$$

The **ds** is the current-length element vector along the direction of an Amperian loop.

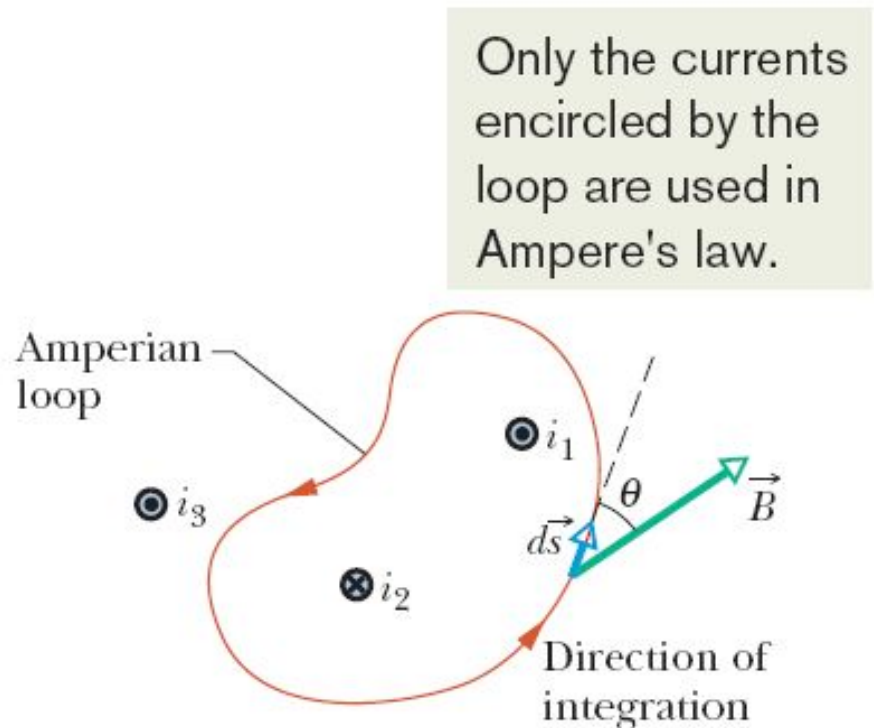Only the currents encircled by the loop are used in Ampere's law.

Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

Determination of all B.ds. The closed path is divided into n elements and take their sums.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

Direction of integration

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration.

A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.
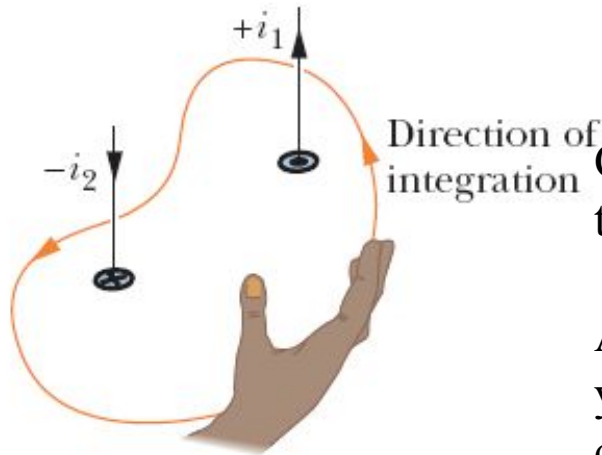
Fig. 29-12 A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-11.

All currents inside the loop **parallel** to the thumb are counted as **positive**.

All currents inside the loop **antiparallel** to the thumb are counted as **negative**.

All currents outside the loop are not counted.

In this example : $i_{enc} = i_1 - i_2$.

29.4: Ampere's Law, Magnetic Field Outside a Long Straight Wire Carrying Current:

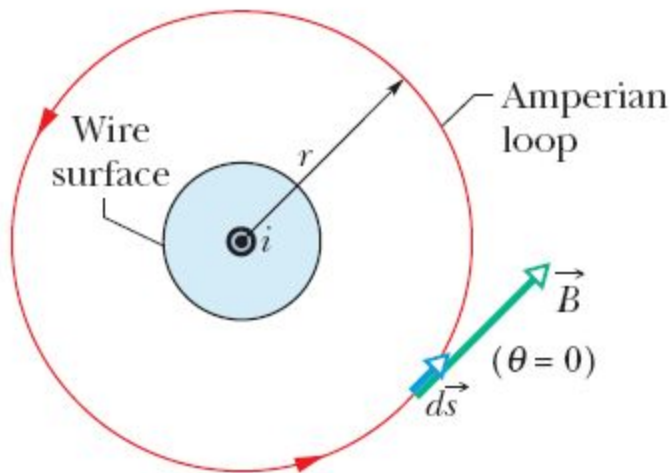All of the current is encircled and thus all is used in Ampere's law.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \theta \, ds = B \oint ds = B(2\pi r).$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{(outside straight wire)}.$$

Fig. 29-13 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

29.4: Ampere's Law, Magnetic Field Inside a Long Straight Wire Carrying Current:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

$$i_{enc} = i \, \frac{\pi r^2}{\pi R^2}.$$

$$B(2\pi r) = \mu_0 i \, \frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r \qquad \text{(inside straight wire).}$$

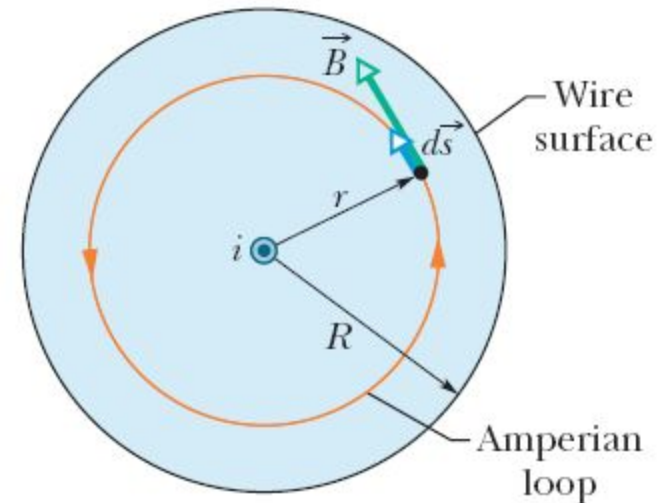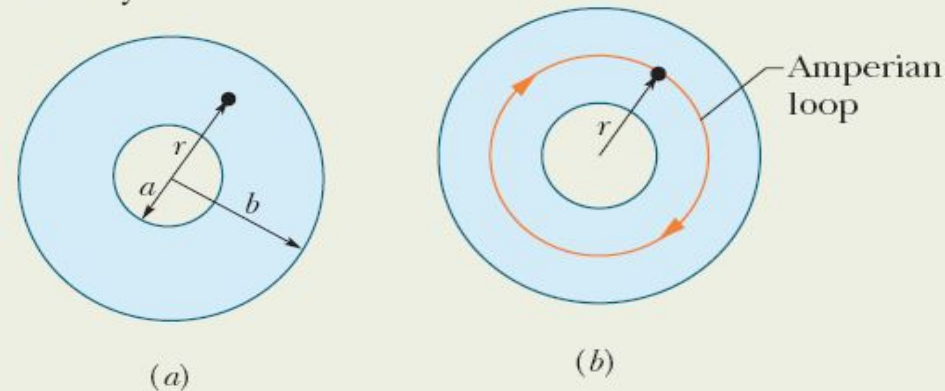Only the current encircled by the loop is used in Ampere's law.

Fig. 29-14 Using Ampere's law to find the magnetic field that a current i produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Example, Ampere's Law to find the magnetic field inside a long cylinder of current.

Figure 29-15a shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-15a, which is at radius $r = 3.0$ cm from the central axis of the cylinder?

(a)　　　　　　　　　　　(b)

Calculations: We write the integral as

$$i_{enc} = \int J \, dA = \int_a^r cr^2 (2\pi r \, dr)$$

$$= 2\pi c \int_a^r r^3 \, dr = 2\pi c \left[\frac{r^4}{4} \right]_a^r$$

$$= \frac{\pi c (r^4 - a^4)}{2}.$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2} (r^4 - a^4).$$

Solving for B and substituting known data yield

$$B = -\frac{\mu_0 c}{4r} (r^4 - a^4)$$

$$= -\frac{(4\pi \times 10^{-7} \, \text{T·m/A})(3.0 \times 10^6 \, \text{A/m}^4)}{4(0.030 \, \text{m})}$$

$$\times [(0.030 \, \text{m})^4 - (0.020 \, \text{m})^4]$$

$$= -2.0 \times 10^{-5} \, \text{T}.$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5} \, \text{T} \qquad \text{(Answer)}$$

Note: Here direction of Amperian loop is in clockwise direction, therefore the enclosed current direction will be taken negative.

29.3: Force Between Two Parallel Wires:

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

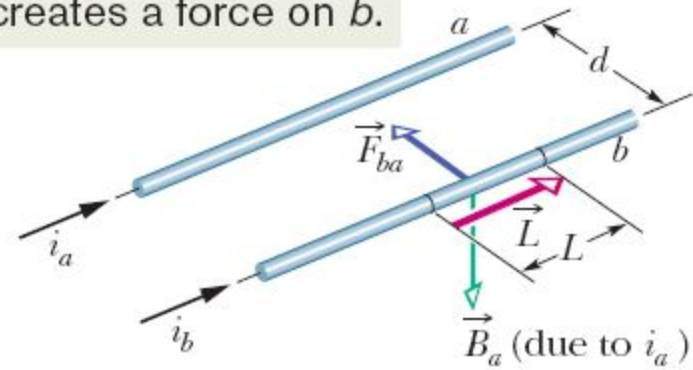The field due to a at the position of b creates a force on b.

Fig. 29-9 Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire b produced by the current in wire a. $\vec{F}_{ba}$ is the resulting force acting on wire b because it carries current in $\vec{B}_a$.

To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Parallel currents attract each other, and antiparallel currents repel each other.

The Solenoid

The solenoid is a long, tightly wound helical wire coil in which the coil length is much larger than the coil diameter. Viewing the solenoid as a collection of single circular loops, one can see that the magnetic field inside is approximately uniform.
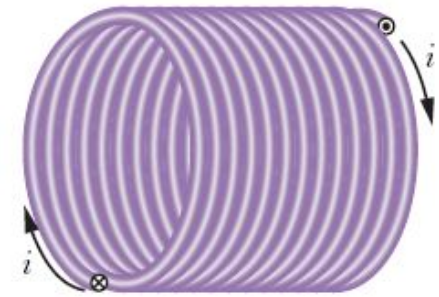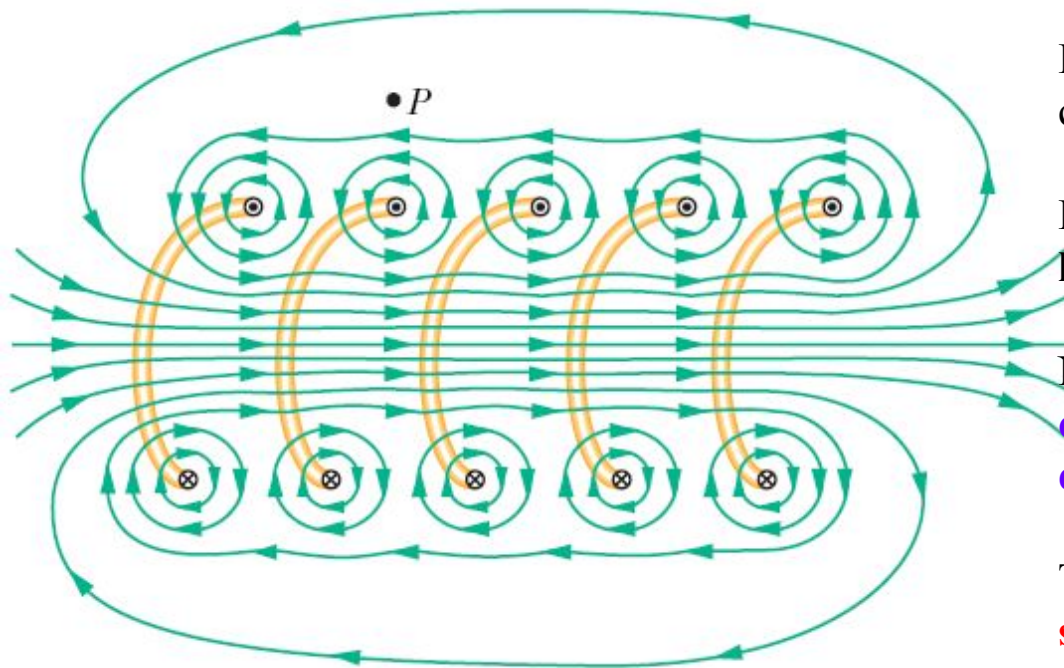

Fig. 29-16 A solenoid carrying current i.

Fig. 29-17 A vertical cross section through the central axis of a "stretched-out" solenoid.

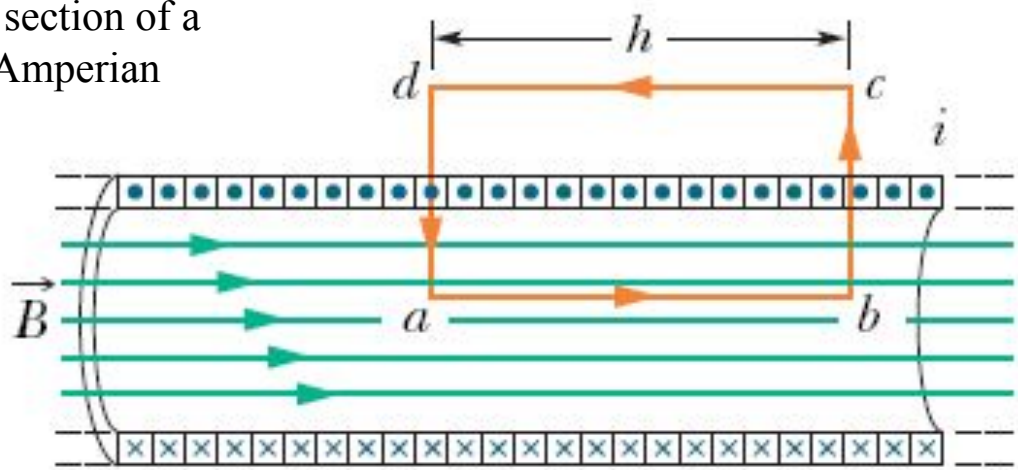**Each turn produces circular magnetic field lines near itself.**

Near the **solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis.**

The **closely spaced field lines there indicate a strong magnetic field**.

Outside the solenoid the field lines are widely spaced; the field there is very weak.

29.5: Solenoids:

Fig. 29-19 Application of Ampere's law to a section of a long ideal solenoid carrying a current i. The Amperian loop is the rectangle $abcda$.

We will use the Amperian loop $abcd$. It is a rectangle with its long side parallel to the solenoid axis. One long side (ab) is inside the solenoid, while the other (cd) is outside:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}$$

$$\int_a^b \vec{B} \cdot d\vec{s} = \int_a^b B\, ds \cos 0 = B \int_a^b ds = Bh \qquad \int_b^c \vec{B} \cdot d\vec{s} = \int_c^d \vec{B} \cdot d\vec{s} = \int_d^a \vec{B} \cdot d\vec{s} = 0$$

$$\rightarrow \oint \vec{B} \cdot d\vec{s} = Bh \qquad \text{The enclosed current } i_{enc} = nhi.$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \rightarrow Bh = \mu_0 nhi \rightarrow B = \mu_0 ni$$

Here n be the number of turns per unit length of the solenoid
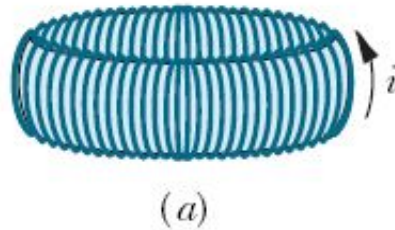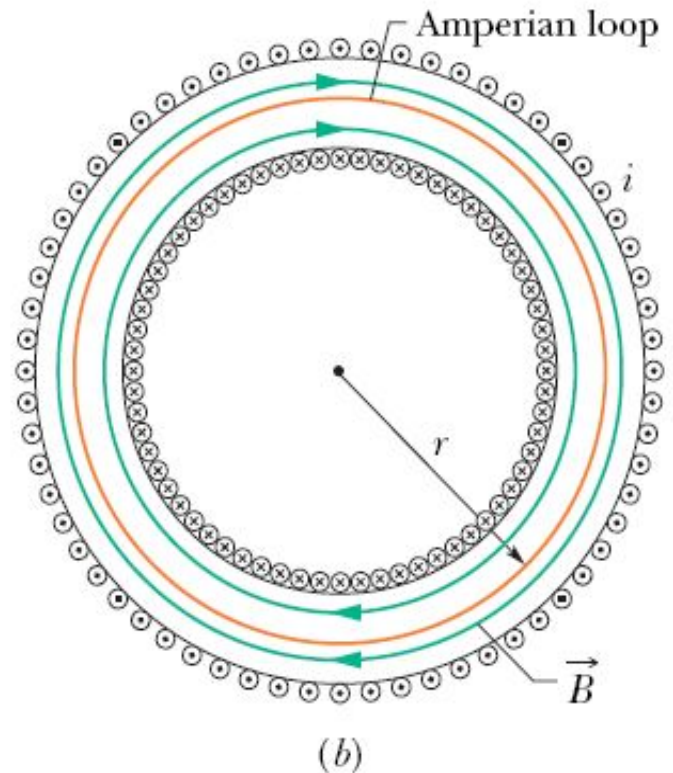
29.5: Magnetic Field of a Toroid:

Fig. 29-20 (a) A toroid carrying a current i. (b) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.

$$(B)(2\pi r) = \mu_0 i N,$$

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \qquad \text{(toroid).}$$

A Toroid which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet.

where i is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop in the form of a circle) and N is the total number of turns.

Example, The field inside a solenoid:

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in = (4\pi \times 10^{-7}\,\text{T·m/A})(5.57\,\text{A})\frac{5 \times 850\,\text{turns}}{1.23\,\text{m}}$$

$$= 2.42 \times 10^{-2}\,\text{T} = 24.2\,\text{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.

P-1 : A Surveyor is using a magnetic compass 6.1m below a power line in which there is a steady current of 100A. (a) what is the magnetic field at the site of the compass due to the power line? (b) will this field interfere seriously with the Compass reading?

The horizontal Component of Earth Magnetic field at the site 20μT?

$i = 100A$, $\imath = 6.1m$

(a) $\vec{B} = ?$ at the site of Compass due to power line?

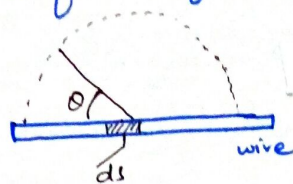(b) Will this field interfere seriously with the Compass Needle=?

horizontal Component of $B = 20\mu T$ at the site is

$$B = \frac{\mu_0 \cdot i}{2\pi \imath}$$

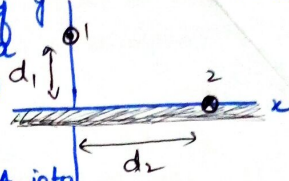$$B = \frac{4\pi \times 10^{-7} \times 100}{2 \times 3.14 \times 6.1} = 3.3 \, \mu T$$

Earth $B = 50 \mu T$ but we'll use $B = 20\mu T$ (given)

(b) $B = 3.3 \mu T$ is $\frac{1}{6}^{th}$ of $\bar{B} = 20\mu T$

Therefore it May affect the Compass Reading.

Force b/w Two parallel Currents:

P-35: Fig 29-63 shows wire 1 in cross section; the wire is long and straight, carries a current of 4mA out of the page, and is at distance $d_1 = 2.40$ cm from a surface. wire 2, which is parallel to wire 1 and also long, is at horizontal distance $d_2 = 5$ cm from wire 1 and carries a current of 6.80mA into the page. What is the x-component of The Magnetic force per unit length on wire 2 due to wire 1?

$i_1 = 4$mA (current in long straight wire) out of the page
$d_1 = 2.4$ cm (wire 1)
$d_2 = 5$ cm (wire 2)
$i_2 = 6.8$mA (into the page)

$F_x/L = ?$ (on the wire 2 due to wire 1)

Formula: $\quad F = \dfrac{\mu_0\, i_a\, i_b}{2\pi d}$, $\quad \sqrt{d_1^2 + d_2^2} = d$

$$\cos\theta = \dfrac{d_2}{\sqrt{d_1^2 + d_2^2}}$$

So,

$$\dfrac{F_x}{L} = \dfrac{\mu_0\, i_1\, i_2}{2\pi d}\cos\theta$$

$$= \dfrac{\mu_0\, i_1\, i_2}{2\pi d} \times \dfrac{d_2}{\sqrt{d_1^2 + d_2^2}}$$

$$\dfrac{F_x}{L} = \dfrac{\mu_0\, i_1\, i_2}{2\pi \sqrt{d_1^2 + d_2^2}} \times \dfrac{d_2}{\sqrt{d_1^2 + d_2^2}}$$

$$\dfrac{F_x}{L} = \dfrac{\mu_0\, i_1\, i_2\, d_2}{2\pi (d_1^2 + d_2^2)} = \dfrac{4\pi\times 10^{-7} \times 4\times 10^{-3} \times 6.8\times 10^{-3} \times 5\times 10^{-2}}{2\pi \times \left[(2.4\times 10^{-2})^2 + (5\times 10^{-2})^2 \right]} = 8.84\times 10^{-11}\ \text{N/}$$

(b) $\cancel{\oint} \oint \vec{B}\cdot d\vec{s} = \mu_0\, i_{enc}$

$$= \mu_0(-5-5-3A)$$

$$= -1.6\times 10^{-5}\ \text{Tm}$$

P-43: Figure 29-67 shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2$ cm carrying uniform current 170 A. What is the Magnitude of the current's Magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?

$a = 2$ cm of long cylindrical conductor

$i = 170 A$

$|B| = ?$ in radial distance (a) $r = 0$

(b) $r = 1$ cm

(c) 2 cm (wire surface)

(d) 4 cm $= r$

Fig 29-67

Ampere's law for B.
outside wire:

$$B = \frac{\mu_0 i}{2\pi r}$$

Inside wire: $\quad B = \frac{\mu_0 i}{2\pi R^2} r \qquad R = a$
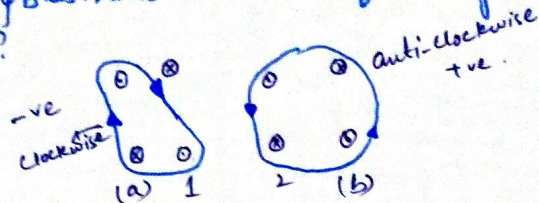
$r = 0$

(a) $\quad B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r = 0$

(b) $\quad B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r = \frac{4\pi \times 10^{-7} \times 170 \times 1 \times 10^{-2}}{2\pi \times (2 \times 10^{-2})^2}$

$$B = 8.5 \times 10^{-4} T$$

(c) $\quad r = a \, ; \quad B = \frac{\mu_0 i r}{2\pi r^2} = \frac{\mu_0 i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 170}{2\pi \times (2 \times 10^{-2})}$

$$\boxed{B = 1.7 \times 10^{-3} T}$$

P-45: Each of the eight conductors in Fig 29-69 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot \vec{ds}$. What is the value of the integral for (a) Path 1 and (b) Path 2?

8 conductors carry $i = 2A$

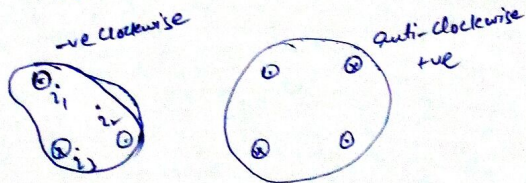Two paths indicated for $\oint \vec{B} \cdot \vec{ds}$

What is the value of the Integral (a) Path 1

(b) Path 2

(a) path is in clockwise (current into the page $\rightarrow$ +ve

R·H·R (Ampère's law) and ⊙ current out of the page $\rightarrow$ −ve

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 (i_2)$$

$$= -\mu_0 i_2 = -4\pi \times 10^{-7} (2)$$

$$\oint_a \vec{B} \cdot \vec{ds} = -2.5 \times 10^{-6} \, Tm$$

i_1 and i_3 will cancel out

$i = i_2 = i_3 = 2A$

(b) $\oint_b \vec{B} \cdot \vec{ds} = 0$

two currents are out of the page and two currents are into the page.

P-51: A 200-turn solenoid having length of 25cm and a diameter of 10cm carries a current of 0.29A. Calculate the Magnitude of the Magnetic field $\vec{B}$ inside the solenoid.

$N = 200$ (Solenoid)

length $= 25$cm and diameter $= 10$cm ; $i = 0.29$A

$B = ?$ (inside the Solenoid)

$$B = \mu_0 i n = \mu_0 i \left(\frac{N}{l}\right) = 4\pi \times 10^{-7} \times 0.29 \times \left(\frac{200}{2.5 \times 10^{-2}}\right)$$

$$\boxed{B = 3 \times 10^{-4} \, T}$$

P-52: A solenoid 1.30m long and 2.60cm in diameter carries a current of 18A. The Magnetic field inside the Solenoid is 23.0 mT. Find the length of the wire forming the Solenoid?

length of Solenoid $= 1.3$m

diameter $= 2.6$cm ; i.e. $r = 1.3$cm

$i = 18$A ; $B = 23$mT

length of the wire forming the Solenoid $= ?$

$$B = \mu_0 i n = \mu_0 i \frac{N}{l} \quad ; \quad B = \mu_0 i \frac{N}{l}$$

$$\frac{Bl}{\mu_0 i} = N \quad —(1)$$

$(2\pi r) N$

loop $\times$ No. of loops (turns) $= 2\pi r N$

from eq ①

$$= 2\pi r \frac{Bl}{\mu_0 i}$$

$$= \frac{2\pi \times 1.3 \times 10^{-2} \times 23 \times 10^{3} \times 1.3}{4\pi \times 10^{-7} \times 18} = \frac{3.88 \times 10^{-9}}{3.6 \times 10^{-6}}$$

$$\simeq 108m \simeq 107.7 \, m$$

Chapter 29:

Exercise problems:

1, 3, 35, 36, 45, 43, 49, 51, 52, 53

Sample problems:

29.03, 29.04