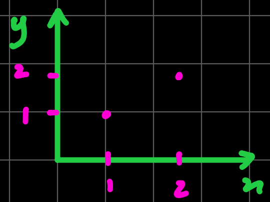
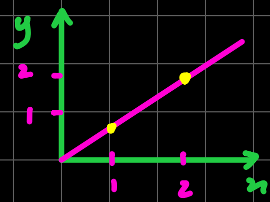
Discrete Mathematics (1.1)

- Study of Discrete objects — discrete means distinct or not connected
- Description of branches of mathematics that have a common property - discrete and not continuous



$y = x \quad y \in N \quad x \in N$

- Take on certain values within the range (discrete)



$y = x \quad y \in N \quad x \in N$

- Takes on all values within the range (continuous)

- Logic → construct valid arguments
- once mathematical statement becomes true it becomes Theorem

- Proposition is a declarative sentence that can be true or false but not both.

- $1+1=2$
- $x+1=2$

→ Adam is good at playing football and this time he is representing his college at national level.

p= Adam good at playing football
q= representing college

$p \wedge q$

operators

- **Negation:** $\neg p$ → If is not the case that or opposite. | NOT operation

p	$\neg p$
T	F
F	T

- **conjunction:** $p \wedge q$ is true only when p = True and q = True | AND operation

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Note: 'but', 'and' is same

Q- 12 is divisible by 3 and / but 3 is a prime number

- **Disjunction:** $p \vee q$ is True when p=True or q=True or both are True | OR operation / Inclusive or

Q- To get a job, experience with c++ <u>or</u> java is mandatory → p or q or both

inclusive or

- **Exclusive or / XOR :** $p \oplus q$

Q- when you buy a car, you get $2500 cashback <u>or</u> accessories worth $2500

exclusive or

→ p or q but not both

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- coffee or tea comes with dinner Ex
- You can pay using us dollars or euros Ex
- Dinner for two includes two items from column A or three items from column B Ex
- A password must be at least three digits or eight characters long In
- To take discrete mathematics you must have taken calculus or a course in CS In

- Conditional / Implication: "if p then q" is denoted by $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

] q dominant

→ T

- p is called hypothesis/premise, and q is called conclusion

1st yeh then yeh

Q- If you $\underbrace{\text{try hard}}_{p}$, then you will $\underbrace{\text{succeed}}_{q}$] argument

- case 1: You tried hard, and you succeed T $p=T$ $q=T$
- case 2: " " " , and you failed F $p=T$ $q=F$
- case 3: " didn't try hard, and you succeeded T $p=F$ $q=T$
- case 4: " didn't try hard, and you failed T $p=F$ $q=F$

If $p=F$, then we can not move to q, and we can not make proposition False

Not False = True

Q- If you have $\underbrace{\text{connection with seniors}}_{p}$, then you will $\underbrace{\text{get promoted}}_{q}$.

- Note: If p is false, then it doesn't matter what will be the truth value of q.

  $p \to q$ is always true

- If p then q
- p implies q
- q when p
- q whenever p
- q follows from p

=

- p only if q ①
- q is necessary for p
- p is sufficient for q
- q unless $\neg p$

] Very imp

- p only if q is not equivalent to if q then p

Q- I will stay at home only if I'm sick ①

= If I'll stay at home, then I'm sick

q is necessary for p

Q- Good Food is necessary to keep us alive

- Other factors to consider

→ we can only say q is necessary for p, then we can only gurrantee that when q is false, then p is definitely false

- p is sufficient for q

Q- It is sufficient for you to travel by car in order to reach your destination on time

→ other factors

- Truth of A gurrantees truth of B, but we can't gurrantee falsity of B from falsity of A.
- when p is True, q is True

- why p is not necessary for q, why q is not sufficient for p

- TT of conditional → comparison

- q unless $\neg p$

- if p then q

  ↓

  q is true when p is true

  ↓

  q is true except when p is false

  ↓

  q is true unless p is false

- Converse : $q \to p$
- Contrapositive: $\neg q \to \neg p$
- Inverse: $\neg p \to \neg q$

- Implies/condition: $p \to q$

Q- If it rains today (p), then I will stay at home (q)

- Converse: I will stay at home if it rains today
- Contrapositive: If I will not stay at home, then it does not rain today
- Inverse: If it does not rain today, then I will not stay at home.

- Implication & contrapositive are equivalent
- converse and inverse are equivalent
- Neither converse, nor inverse are equivalent to implication

Bi-conditional Operator

- $p \leftrightarrow q$ = p if and only if q / $p \leftrightarrow q$ is True whenever the truth values of p and q are same

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p only if q = if p then q and p if q = if q then p

$$(p \to q) \wedge (q \to p) \equiv p \leftrightarrow q$$