6.1

- $\langle u, v \rangle = u \cdot v$
- $\|v\| = \sqrt{\langle v, v \rangle}$
- $d(u, v)$
  $= \|u - v\|$
  $= \sqrt{\langle u-v, u-v \rangle}$

An ***inner product*** on a real vector space V is a function that associates a real number $\langle \mathbf{u}, \mathbf{v} \rangle$ with each pair of vectors in V in such a way that the following axioms are satisfied for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in V and all scalars k.

1. $\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{v}, \mathbf{u} \rangle$ **[Symmetry axiom]**
2. $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{w} \rangle + \langle \mathbf{v}, \mathbf{w} \rangle$ **[Additivity axiom]**
3. $\langle k\mathbf{u}, \mathbf{v} \rangle = k\langle \mathbf{u}, \mathbf{v} \rangle$ **[Homogeneity axiom]**
4. $\langle \mathbf{v}, \mathbf{v} \rangle \geq 0$ and $\langle \mathbf{v}, \mathbf{v} \rangle = 0$ if and only if $\mathbf{v} = \mathbf{0}$ **[Positivity axiom]**

A real vector space with an inner product is called a ***real inner product space***.

- $\|u\| = 1$, set of points satisfying this is called the unit sphere
- Matrices:
  $\langle u, v \rangle = Au \cdot Av$
  $= v^T A^T A u$