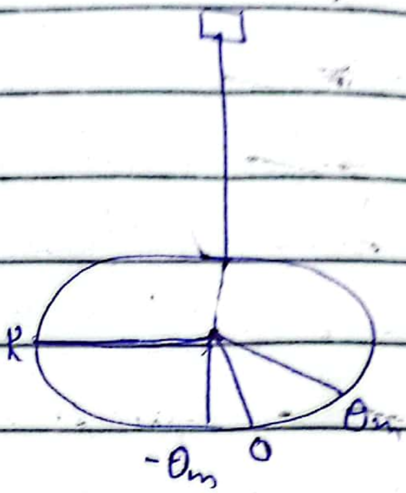
Angular SHM: (Torsional Pendulum)



$\tau = -K\theta$

$I\alpha = -K\theta$

$$I\frac{d^2\theta}{dt^2} = -K\theta$$

$$I\frac{d^2\theta}{dt^2} + K\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{K}{I}\right)\theta = 0$$

Compare with eq. of SHM.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

from above

$$\omega^2 = \frac{K}{I}$$

$$\boxed{\omega = \sqrt{\frac{K}{I}}}$$

∴ Time Period

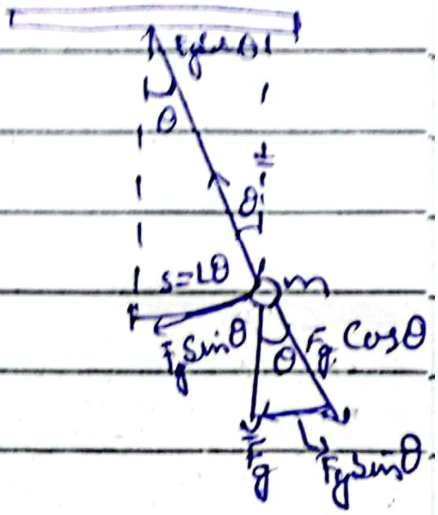
$$\omega = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{k}}}$$

Simple Pendulum:



$$\tau = r \perp F$$

$$\tau = -L\,(F_g \sin\theta)$$

$$\tau = -Lmg\sin\theta$$

$$\tau = I\alpha \quad \therefore \quad \alpha = \frac{d^2\theta}{dt^2}$$

$$I\alpha = -Lmg\sin\theta$$

$$I\frac{d^2\theta}{dt^2} = -Lmg\theta \qquad \sin\theta \approx \theta$$

$$I\frac{d^2\theta}{dt^2} + mgL\theta = 0$$

$$\frac{d^2\theta}{dt} + \left(\frac{mgL}{I}\right)\theta = 0$$

Compare with SHM Eq.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{mgL}{mL^2}\right)\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{g}{L}\right)\theta = 0$$

Now compare

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

$$\omega^2 = \frac{g}{L}$$

$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

Time Period,

$$T = \frac{2\pi}{\omega}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$