Halliday & Resnick's

Fundamentals of Physics

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Jearl Walker

Electric Fields

Chapter 22

22.2 The Electric Field:

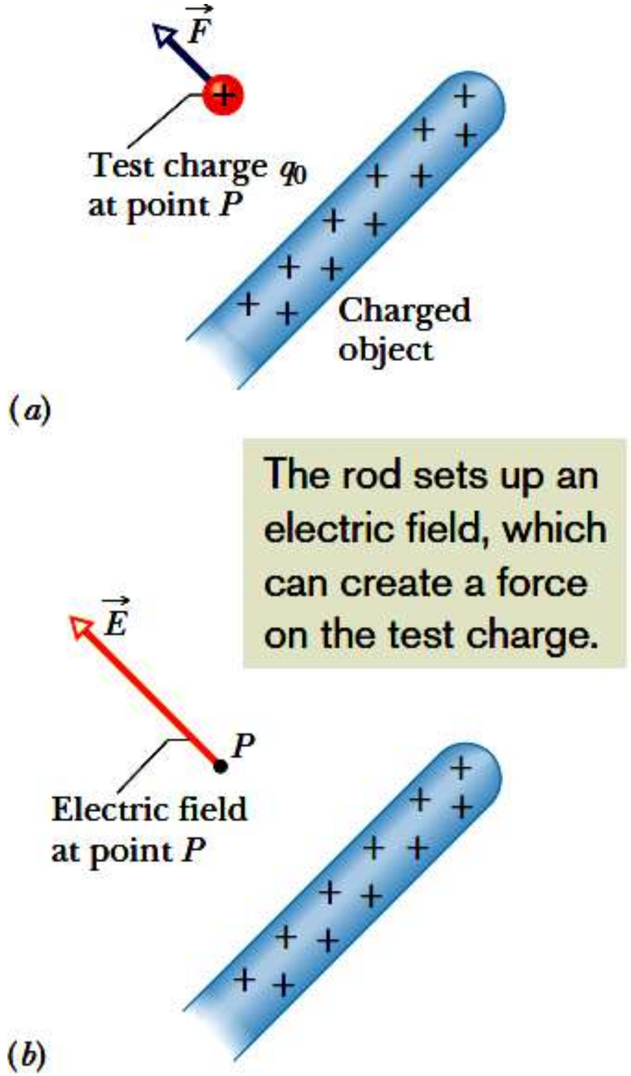


Fig. 22-1 (*a*) A positive test charge q_0 placed at point *P* near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (*b*) The electric field $\vec{E}$ at point *P* produced by the charged object.

The Electric Field is a *vector field*.

The electric field, ***E***, consists of a distribution of vectors, one for each point in the region around a charged object, such as a charged rod.

We can define the electric field at some point near the charged object, such as point *P* in Fig. 22-1a, as follows:

•A positive test charge q_0, placed at the point will experience an electrostatic force, **F**.

•The electric field at point *P* due to the charged object is defined as the electric field, **E**, at that point:

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).}$$

The SI unit for the electric field is the newton per coulomb (N/C).

22.2 The Electric Field:

Table 22-1

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

22.3 Electric Field Lines:

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

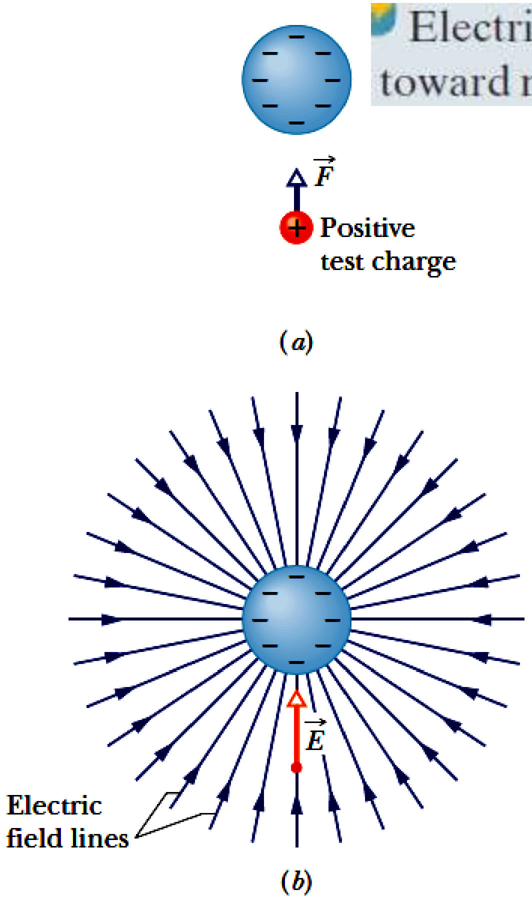


Fig. 22-2 (*a*) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

- At any point, the direction of a straight field line or the direction of the tangent to a curved field line gives the direction of at that point.

- The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of ***E***.

Thus, ***E*** is large where field lines are close together and small where they are far apart.

22.3 Electric Field Lines:

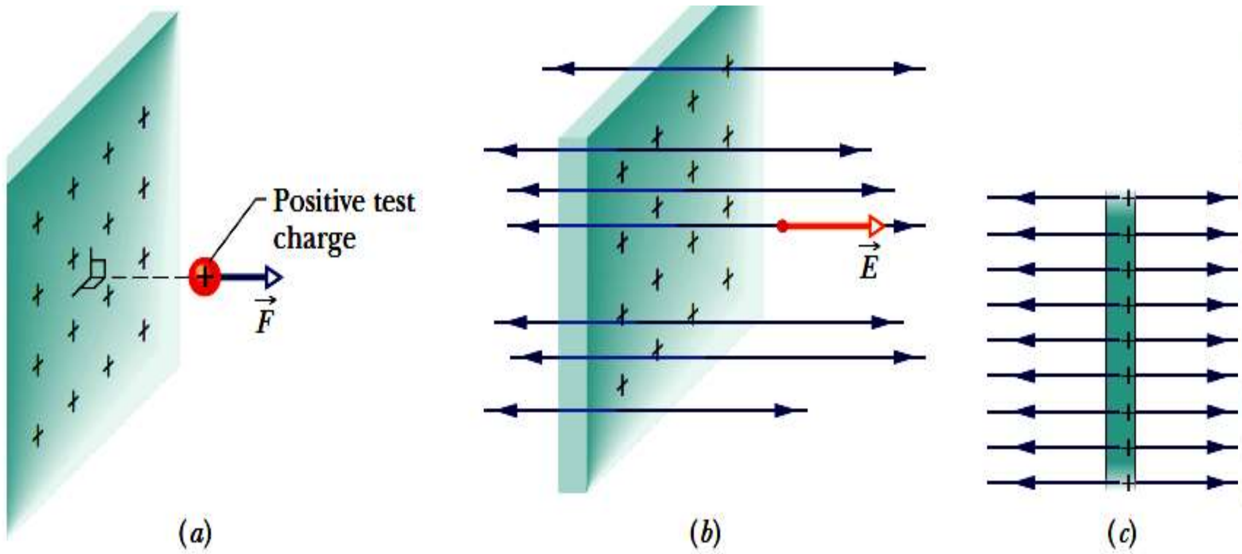
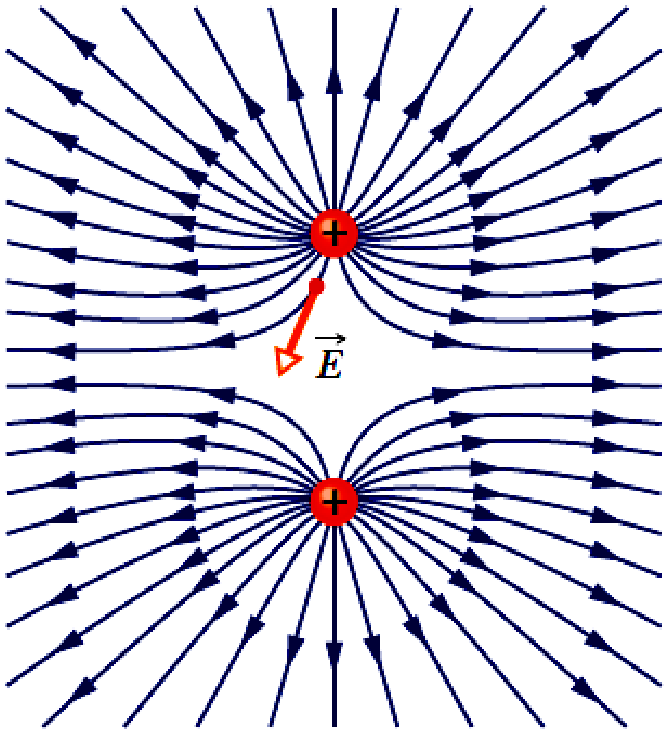


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) The electric field vector at one point is shown; note that it is tangent to the field line through that point.



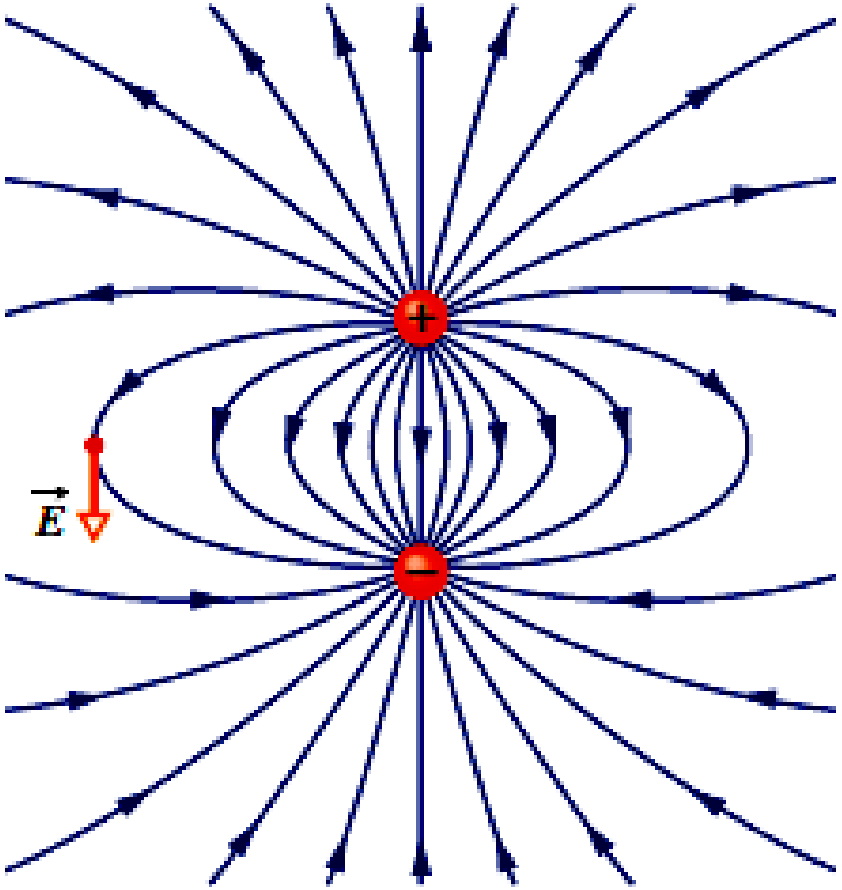


Fig. 22-5 Field lines for a positive point charge and a nearby negative point charge that are equal in magnitude. The charges attract each other. The pattern of field lines and the electric field it represents have rotational symmetry about an axis passing through both charges in the plane of the page. The electric field vector at one point is shown; the vector is tangent to the field line through the point.

22.4 The Electric Field due to a Point:

To find the electric field due to a point charge q (or charged particle) at any point a distance r from the point charge, we put a positive test charge q_0 at that point.

The direction of E is directly away from the point charge if q is positive, and directly toward the point charge if q is negative. The electric field vector is:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}} \qquad \text{(point charge).}$$

The net, or resultant, electric field due to more than one point charge can be found by the superposition principle. If we place a positive test charge q_0 near n point charges $q_1, q_2, \ldots, q_n$, then, the net force, $\mathbf{F_o}$, from the n point charges acting on the test charge is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

The net electric field at the position of the test charge is

$$\begin{aligned}\vec{E} &= \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.\end{aligned}$$

Example, The net electric field due to three charges:

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

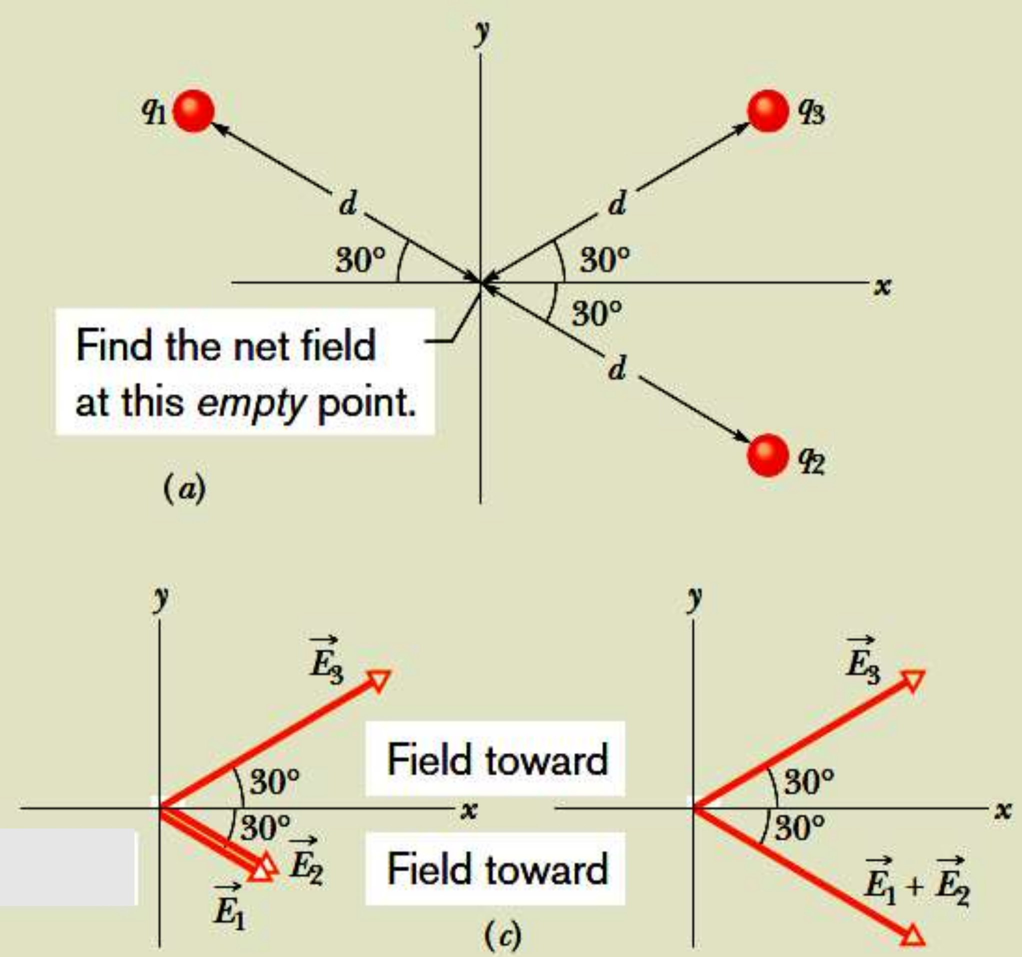


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

From the symmetry of Fig. 22-7c, we realize that the equal y components of our two vectors cancel and the equal x components add.

Thus, the net electric field at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30°$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

22.5 The Electric Field due to an Electric Dipole:

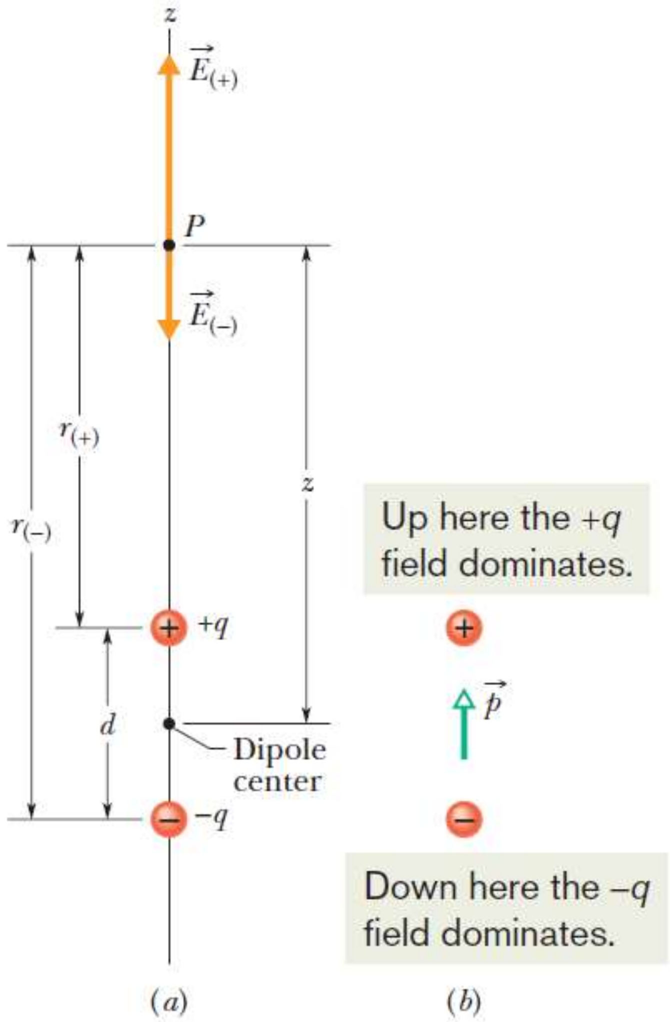
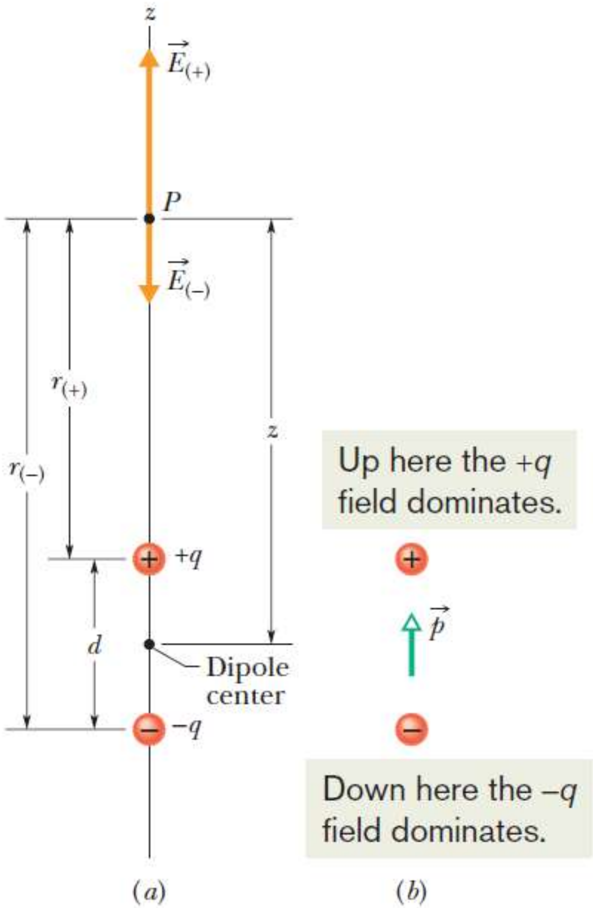


Fig. 22-8 (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

22.5 The Electric Field due to an Electric Dipole:

From symmetry, the electric field E at point P—and also the fields E_+ and E_- due to the separate charges that make up the dipole—must lie along the dipole axis, which we have taken to be a z axis. From the superposition principle for electric fields, the magnitude E of the electric field at P is



$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2_{(+)}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2_{(-)}}$$

$$= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}.$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right).$$

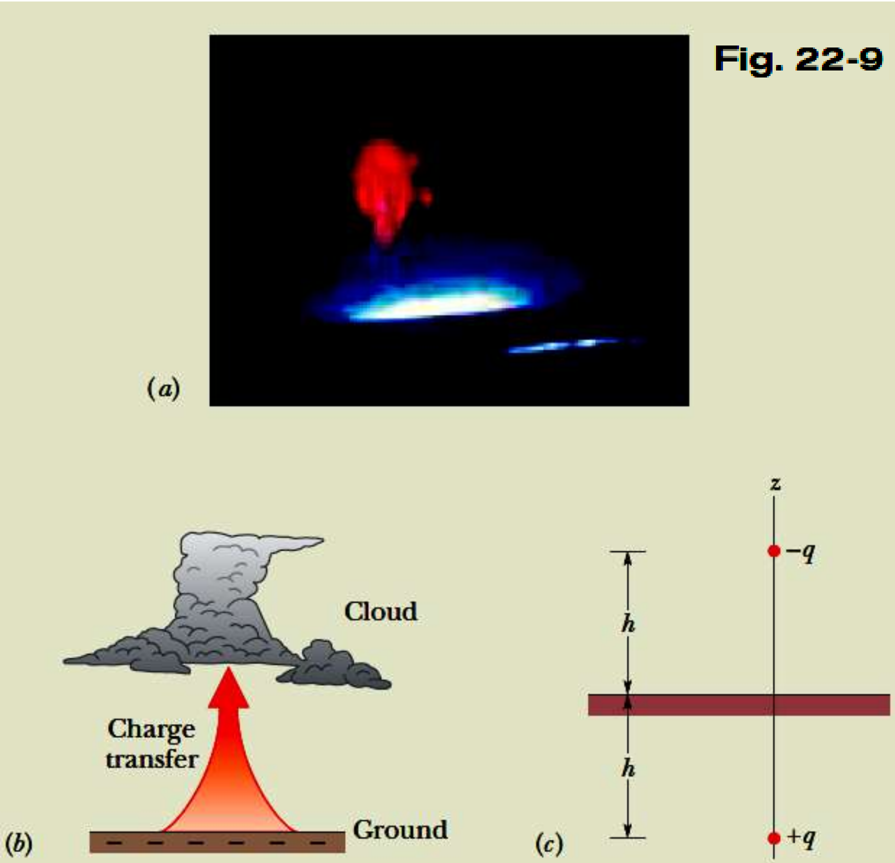
$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}.$$

$$d/2z \ll 1 \longrightarrow E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

The product qd, which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the ***electric dipole moment*** of the dipole.

$$\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).}}$$

Example, Electric Dipole and Atmospheric Sprites:



Sprites (Fig. 22-9*a)* are huge flashes that occur far above a large thunderstorm. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge -q from the ground to the base of the clouds (Fig. 22-9*b)*.

We can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge -q at cloud height h and charge $+q$ at below-ground depth h *(Fig.* 22-9*c)*. If $q = 200\ C$ and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30$ km somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9*c*. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\ \text{C})(2)(6.0\times 10^3\ \text{m})}{(30\times 10^3\ \text{m})^3}$$

$$= 1.6\times 10^3\ \text{N/C}. \qquad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times 10^2\ \text{N/C}. \qquad \text{(Answer)}$$

22.6 The Electric Field due to a Continuous Charge:

When we deal with continuous charge distributions, it is most convenient to express the charge on an object as a *charge density rather than as a total charge.* For a line of charge, for example, we would report the *linear charge density* (or charge per unit length) λ, whose SI unit is the coulomb per meter.

Table 22-2 shows the other charge densities we shall be using.

Table 22-2

Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

22.8: A Point Charge in an Electric Field

The electrostatic force F acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

$$\boxed{\vec{F} = q\vec{E},}$$

When a charged particle, of charge *q*, is in an electric field, ***E*,** set up by other stationary or slowly moving charges, an electrostatic force, ***F*,** acts on the charged particle as given by the above equation.

22.8: A Point Charge in an Electric Field: Measuring the Elementary Charge

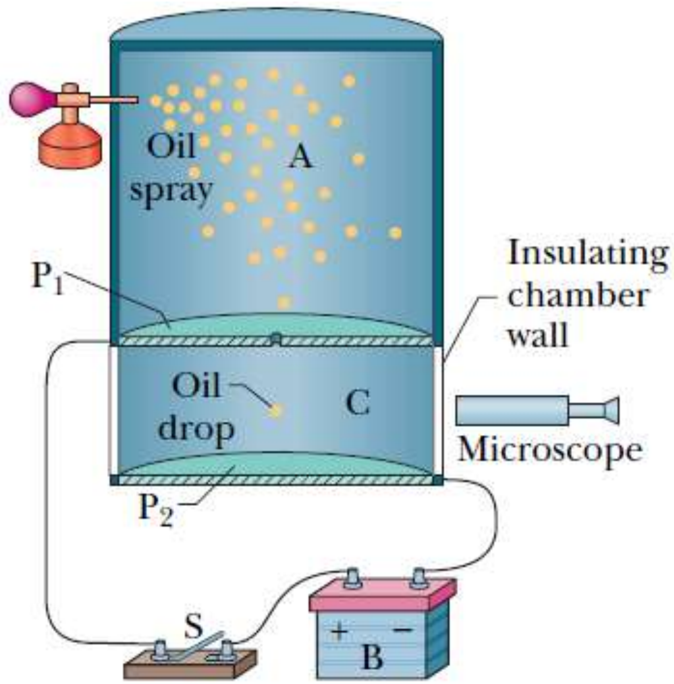


Fig. 22-14 The Millikan oil-drop apparatus for measuring the elementary charge *e*. When a charged oil drop drifted into chamber C through the hole in plate P_1, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

Ink-Jet Printing

The need for high-quality, high-speed printing has caused a search for an alternative to impact printing, such as occurs in a standard typewriter.
Building up letters by **squirting tiny drops of ink** at the paper is one such alternative.

Figure shows a **negatively charged drop moving between two conducting deflecting plates, between which a uniform, downward-directed electric field has been set up.**

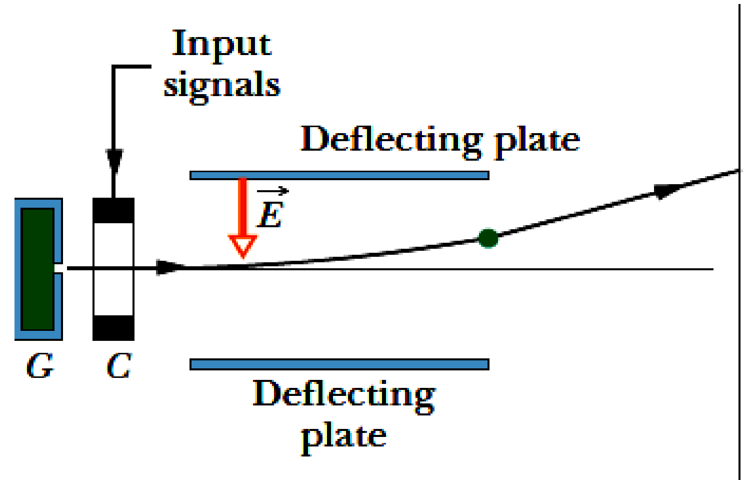


Fig. 22-15 Ink-jet printer. Drops shot from generator G receive a charge in charging unit C. An input signal from a computer controls the charge and thus the effect of field $\vec{E}$ where the drop lands on the paper.

The drop is deflected upward according to Eq. F=qE and then strikes the paper at a position that is determined by the magnitudes of and the charge q of the drop.

In practice, E is held constant and the position of the drop is determined by the charge q delivered to the drop in the charging unit, through which the drop must pass before entering the deflecting system. The charging unit, in turn, is activated by electronic signals that encode the material to be printed.

Electrical Breakdown and Sparking

If the magnitude of an **electric field in air exceeds a certain critical value Ec**, the air undergoes electrical breakdown, a process whereby the **field removes electrons from the atoms in the air.**

The air then begins to conduct electric current because the freed electrons are propelled into motion by the field.

As they move, they collide with any atoms in their path, causing those atoms to emit light.

We can see the paths, commonly called **sparks,** taken by the freed electrons because of that emitted light.

Figure 22-18 shows sparks above charged metal wires where the electric fields due to the wires cause electrical breakdown of the air.

Figure 22-18 The metal wires are so charged that the electric fields they produce in the surrounding space cause the air there to undergo electrical breakdown.

Example, Motion of a Charged Particle in an Electric Field

Figure 22-17 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

KEY IDEA

The drop is negatively charged and the electric field is directed *downward*. From Eq. 22-28, a constant electrostatic force of magnitude QE acts *upward* on the charged drop. Thus, as the drop travels parallel to the x axis at constant speed v_x, it accelerates upward with some constant acceleration a_y.

Calculations: Applying Newton's second law ($F = ma$) for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \qquad (22\text{-}30)$$

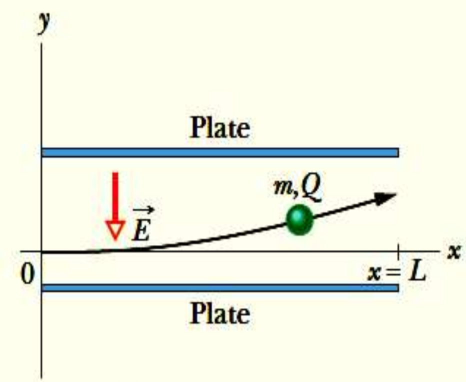


Fig. 22-17 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \tfrac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t, \qquad (22\text{-}31)$$

respectively. Eliminating t between these two equations and substituting Eq. 22-30 for a_y, we find

$$y = \frac{QEL^2}{2mv_x^2}$$

$$= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2}$$

$$= 6.4 \times 10^{-4}\ \text{m}$$

$$= 0.64\ \text{mm}. \qquad \text{(Answer)}$$