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Assignment 2

Abdul Munam Aziz
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Q1- $C \to DI$; $C \to D$, $C \to I$
$EC \to AB$; $EC \to A$, $EC \to B$

- $A \to C$; $A^+ = A$, No C keep
- $AB \to C$; $AB^+ = ABC$ $(A \to C)$. Includes C . Redundant
- $C \to D$; $C^+ = CI$ $(C \to I)$. No D. keep
- $C \to I$; $C^+ = CI$, Includes I, redundant
- $CD \to I$; $CD^+ = CDI$ $(C \to I)$, includes I, redundant
- $EC \to A$; $EC^+ = ECDI$ $(C \to D, C \to I)$, No A, keep
- $EC \to B$; $EC^+ = ECDI$, No B, keep
- $EI \to C$; $EI^+ = EI$, No C , keep

x ——— x

- $EC \to A$
- $E^+ = E$ No A
- $C^+ = CDI$ (No A, keep)
- Both E and C needed

- $EC \to B$
- $E^+ = E$ No B
- $C^+ = CDI$ No B
- Both needed

- $EI \to C$
- $E^+ = E$ No C
~~$C^+ = CDI$~~ · $I^+ = I$ No C
- Both needed.

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· Minimal cover= {A→C, C→D, C→I, EC→A, EC→B, EI→C}

· A^+ = ACDI (A→C, C→D, C→I) - missing E
· E^+ = E - missing A, B, C, D, I
· None work alone

· EC
· EC→A; EC^+ = ECA
· EC→B; EC^+ = ECAB
· A→C; present
· C→D, C→I; EC^+ = ECABDI (all attributes)

E^+=E, C^+= CDI (minimal)

· EI
· EI→C; EI^+ = EIC
· C→D; EI^+ = EICD
· Add A; $EICA^+$ = EICAD (A→C), then EICADB (EC→A, EC→B)

EI^+ = ABCDEI (all attributes)
E^+ = E, I^+ = I (Minimal

AE

- $A \to C$; $AE^+ = AEC$
- $C \to D$, $C \to I$; $AE^+ = AECDI$
- $EC \to A$, $EC \to B$: $AE^+ = AECDIB$ (all attributes)

$A^+ = ACDI$, $E^+ = E$. Minimal

- AD
- $A \to C$; $AD^+ = ADC$
- ~~$C \to D$, $C \to I$: A~~
- $C \to D$, $C \to I$: $AD^+ = ADCI$ - missing E, B

- $AC^+ = ACDI$ ($A \to C$, $C \to D$, $C \to I$) — incomplete
- $BD^+ = BD$

→ no other combinations work without E (since B requires EC)

→ Triplets are not minimal, hence not considered (" or are having incomplete closures or no paths for relations)

eg: AEC

~~AEC^+~~ $A \to C$; $AEC^+ = AEC$

$C \to D$; $AEC^+ = AECD$

$C \to I$; $AEC^+ = AECDI$

$EC \to A, B$: $AEC^+ = AECDIB$ (all attributes)

$AE^+ = ACDEIB$ $\quad A^+ = ACDI$ $\quad E^+ = E$, AEC is a superkey but not minimal.

$EC^+ = ACDEIB$ $\quad C^+ = CDI$

C.K = EC, EI, AE

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· $ABC^+ = A \to C \to D, I$ missing E

Q2- $F1 = \{A \to C, AB \to C, C \to DG, CD \to G, EC \to AB, EG \to C\}$

$F2 = \{A \to C, C \to D, C \to G, EC \to A, EC \to B, EG \to C\}$

Checking if $F2 \subseteq F1^+$

· $A \to C$ is in F1

· $C \to D$; From $C \to DG \Rightarrow C \to D$

· $C \to G$; From $C \to DG \Rightarrow C \to G$

· $EC \to A$; From $EC \to AB \Rightarrow EC \to A$

· $EC \to B$; $EC \to AB \Rightarrow EC \to B$

· $EG \to C$; lies in F1

· Checking for $F1 \subseteq F2^+$

· $A \to C$ is in F2

· $AB \to C$; $A \to C$ is in F2 ; B doesn't affect outcome $\Rightarrow$ redundant

· $C \to DG$; $C \to D$, $C \to G$ are in F2

· $CD \to G$; $C \to G$ in F2 $\Rightarrow CD \to G$ is redundant

· $EC \to AB$; $EC \to A$, $EC \to B$ in F2 $\Rightarrow EC \to AB$

· $EG \to C$ in F2

· All FDs in F1 are implied by F2, and vice versa

· $F1 \equiv F2$ because each FD in F1 can be derived from F2 and vice versa.

Q3- $ABC \to BDE$ $\Rightarrow$ $ABC \to B$, ~~D, E~~, $ABC \to D$, $ABC \to E$

· $ABC \to G$

·

New set = $\{D \to E, ABC \to B, ABC \to D, ABC \to E, B \to G, A \to C, ABC \to G\}$

· $ABC \to B$; ABC^+ without it : $A \to C$, $B \to G$ gives ABCG. B is present. Redundant

· $ABC \to D$: ABC^+ : $A \to C$, $B \to G$ give ABCG. No D. Keep

· $ABC \to E$: ABC^+ : $A \to C$; $B \to G$, ~~D→E~~, $ABC \to D$ giving ABCDG, then $D \to E$. E is present. redundant

· $ABC \to G$ → ABC^+ without it : $A \to C$, $B \to G$ gives ABCG redundant

Final set = $\{D \to E, ABC \to D, B \to G, A \to C\}$ → Minimal cover

~~AB+~~ · Testing AB ✓

$AB^+ = AB$

· $A \to C$; $AB^+ = ABC$

· $B \to G$; $AB^+ = ABCG$

· $ABC \to D$; $AB^+ = ABCGD$

· $D \to E$: $AB^+ = ABCGDE$

· $A^+ = AC$ $(A \to C)$, then ACG ($B \to G$ needs B). B, D, E missing

· $B^+ = BG$ $(B \to G)$. Missing A, C, D, E

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AD

$AD^+ = AD$

- $A \to C$: $AD^+ = ADC$
- $D \to E$: $AD^+ = ADCE$
- $ABC \to D$: Need B
- $B \to G$: Need B

$AD^+ = ADCE$ ✗

AC

- $A \to C$; Has C
- $B \to G$; Needs B
- $ABC \to D$; 〃

$AC^+ = AC$ ✗

- BC

- $B \to G$: $BC^+ = BCG$
- $A \to C$: Need A
- $ABC \to D$: 〃

$BC^+ = BCG$ ✗

- BD

- $B \to G$: $BD^+ = BDG$
- $D \to E$: $BD^+ = BDGE$
- $A \to C$: Need A

$BD^+ = BDEG$ ✗

$A^+ = ACG$, $B^+ = BG$, $C^+ = C$, $D^+ = DE$ ✗

ABD

- $A \to C$: $ABD^+ = ABCD$
- $B \to G$: $ABD^+ = ABCDG$
- $D \to E$: $ABD^+ = ABCDEG$

Not ~~minal~~ minimal (AB alone works)

~~Ca~~ C.K = AB

Minimal cover= $F = \{ D \to E, ABC \to D, B \to G, A \to C \}$

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Q4-

CD's minimal = C^+: ABCE

CD

- $CD \to B$; $CD^+ = CDB$
- $C \to AB$; $CD^+ = CDAB$
- $A \to E$; $CD^+ = CDABE$ (all attributes)

- $C^+ = CAB$ ($C \to AB$), then $CABE$ ($A \to E$).
- $D^+ = DE$ ($D \to E$)

- BD

- $BD \to C$: $BD^+ = BDC$
- $C \to AB$: $BD^+ = BDCAB$
- $A \to E$: $BD^+ = BDCABE$

$B^+ = B$

$D^+ = DE$

→ Minimal

- AD
- $A \to E$; $AD^+ = ADE$
- $D \to E$; Already has E

$AD^+ = ADE$ ×

- BC
- $C \to AB$; $BC^+ = BCAB$
- $A \to E$; $BC^+ = BCABE$

×

- AB
- $A \to E$; $AB^+ = ABE$
- $BD \to C$; Need D

×

$A^+ = AE$ ($A \to E$)

$B^+ = B$

$D^+ = DE$ ($D \to E$)

$E^+ = E$

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· C^+

· $C \to AB$; $C^+ = CAB$

· $A \to E$; $C^+ = CABE$

· No FD adds D

$C^+ = CABE$

optional

ABD

· $A \to C$; $ABD^+ = ABDC$

· $B \to G$; $ABD^+ = ABCDG$

· $D \to E$; $ABD^+ = ABCDEG$

(not minimal for all other triplets) ✗

Candidate keys: CD, BD

Q5-

AB

· $AB \to C$; $AB^+ = ABC$

· $AC \to D$; $AB^+ = ABCD$

→ $A^+ = A$

$B^+ = B$

minimal and candidate ✓

AC

· $AC \to D$; $AC^+ = ACD$

· $CD \to B$; $AC^+ = ACDB$

$A^+ = A$

$C^+ = C$

minimal and candidate

AD

· $AD \to B$; $AD^+ = ADB$

· $AB \to C$; $AD^+ = ADBC$ ✓

$A^+ = A$

$D^+ = D$ minimal and candidate

BC

· $BC^+ = BC$

· No FDs apply directly

$AB \to C$, needs A,

$CD \to B$ needs D)

✗ A, D missing

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BD

- No FDs directly apply

$BD^+ = BD$

missing A, C ×

CD

- $CD \to B$; $CD^+ = CDB$
- No FD adds A
(AB → C needs A)

- $CD^+ = CDB$, A missing ×

- $A^+ = A$, $B^+ = B$, $C^+ = C$, $D^+ = D$

- $ABC^+ = ABCD$ (AB → C / AC → D) Not minimal, and similar for other triplets (AB or AC suffices)

candidate keys = AB, AC, AD

Q6 -

AE

- $A \to C$; $AE^+ = AEC$
- $C \to BD$; $AE^+ = AECBD$
- $D \to A$; Already Present

- $A^+ = AC$ (A → C) , then ACBD (C → BD) missing E
- $E^+ = E$ missing A, B, C, D

minimal, and ~~can~~ C.K

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CE

- $C \to BD$; $CE^+ = CEBD$
- $D \to A$; $CE^+ = CEBDA$

- $C^+ = CBD$ ($C \to BD$). Missing A, E
- $E^+ = E$
- minimal and C.K
  =

DE

- $D \to A$; $DE^+ = DEA$
- $A \to C$; $DE^+ = DEAC$
- $C \to BD$; $DE^+ = DEACBD$

- $D^+ = DA$ ($D \to A$), then DAC ($A \to C$), then $DACBD$ ($C \to BD$). Missing E
- $E^+ = E$
- minimal and C.K

AB

- $A \to C$; $AB^+ = ABC$
- $C \to BD$; $AB^+ = ABCBD$

  $AB^+ = ABCD$, E missing

  X

AC

- $C \to BD$; $AC^+ = ACBD$
- $D \to A$; already has A

  E missing

  X

AD

- $A \to C$; $AD^+ = ADC$
- $C \to BD$; $AD^+ = ABDC$, E missing

  X

BC

- $C \to BD$; $BC^+ = BCD$
- $D \to A$; $BC^+ = ABCD$, E missing
  ×

- BD
- $D \to A$: $BD^+ = BDA$
- $A \to C$; $BD^+ = BADC$
- $C \to BD$; $BD^+ = BACD$, E missing
  ×

$A^+ = ABCD$, $B^+ = B$, $C^+ = CBD$ ($C \to BD$), $D^+ = DABCD$ ($D \to A$, $A \to C$, $C \to BD$)
$E^+ = E$

C.K = AE, CE, DE

- Triplet keys are not in minimal order hence no C.K present in ~~them~~ these combinations

Q7-

- $ABC \to CDEG$ becomes $ABC \to C$, $ABC \to D$, $ABC \to E$, $ABC \to G$

~~New set = {~~

- $ABC \to C$: ABC^+ without it: $A \to B$ gives ABC, $C \to E$ gives ABCE, $D \to G$ (No D yet).
  $ABC^+ = ABCE$, ~~e is~~ C is already present redundant.

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- $ABC \to D$; ABC^+ without it: $A \to B$ gives ABC, $C \to B$ gives $ABCE$, No D. keep

- $ABC \to E$; ABC^+ without it: $A \to B$ gives ABC, $C \to E$ gives $ABCE$. E present, redundant

- $ABC \to G$: ABC^+ : $A \to B$ gives ABC, $C \to E$ gives $ABCE$, No G ($D \to G$ needs D). keep

- $C \to E$: $C^+ = C$, No E, keep
- $A \to B$: $A^+ = A$, No B, keep
- $D \to G$: $D^+ = D$, No G, keep

$\underline{ABC \to D \text{ test}}$

- $AB^+ = AB$ $(A \to B)$, No D
- $AC^+ = ACB$ $(A \to B, C \to E)$, no D
- $BC^+ = BCE$ $(C \to E)$, not D
- All 3 (A, B, C) are needed (no smaller subset derives D)

$\underline{ABC \to G \text{ test}}$

- $ABC \to G$
- $AB^+ = AB$, no G
- $AC^+ = ACB$, no G, $(A \to B)$
- $BC^+ = BCE$, no G, $(C \to E)$

Minimal cover= { ~~AB~~ $ABC \to D$, $ABC \to G$, $C \to E$, $A \to B$, ~~$D \to G$~~ $D \to G$ }

$A^+ = A \to B$ gives $\underline{AB}$

$C^+ = CE$ $(C \to E)$ None work alone

$D^+ = DG$ $(D \to G)$

$\underline{AC}$

- $A \to B$; $AC^+ = ACB$
- $C \to E$; $AC^+ = ACBE$
- $ABC \to D$; $AC^+ = ACBED$
- $D \to G$: $AC^+ = ACBEDG$ (all attributes)

$A^+ = AB$

$C^+ = CE$ (minimal and C.K)

$\underline{AD}$

- $A \to B$; $AD^+ = ADB$
- $D \to G$; $AD^+ = ADBG$
- $ABC \to D$; Need C

missing E, C ✗

$\underline{CD}$

- $C \to E$; $CD^+ = CDE$
- $D \to G$; $CD^+ = CDEG$
- $ABC \to D$; Need A

missing A, B ✗

- BC^+ : B, $C \to E$ gives BCE A, D, G missing
- All pairs except AC miss these attributes, Triplets like ACD (super set of AC) work but aren't minimal.

C.K = $\underline{AC}$ (only)

Q8-

Checking $G \subseteq F^+$

- $A \to BC$; $A \to BC$ gives ABC . Matches ($A \to BC$ is in F)
- $B \to D$; $B^+ = BD$ ($B \to D$) . $B \to D$ is in F
- $C \to E$; $C^+ = CE$ ($C \to E$) . $C \to E$ present in F
- $BD \to E$; $BD^+ = BDE$ ($D \to E$) $\nearrow B \to D$ (BDD already present) . $D \to E$ present in F . Matches
- $A \to D$; $A^+ = ABCDE$ ($A \to BC$, $B \to D$, $D \to E$) , ~~includes~~ D All present in F; matches

All FDs in G are implied by F

checking if $F \subseteq G^+$

- $A \to BC$; $A^+ = ABC$ ($A \to BC$) . $A \to BC$ is in G
- $B \to D$; $B^+ = BD$ ($B \to D$) : $B \to D$ is in G
- $C \to E$; $C^+ = CE$ ($C \to E$) : $C \to E$ is in G
- $D \to E$; $D^+ = D$ (no direct FD with D derives E) not implied by F

· $BD \to E$ ×

· $A \to D$, $A \to BC$ ×

- ~~F and G are no~~ $F \not\equiv G$ (not equivalent as $D \to E$ does not allow ~~∅~~ F to be implied by ~~F~~ G

Q9- a) $R1(A, C, D)$ $R2(A,B,C)$ $R3(D,E)$

· $AC^+ = ACB$ $(AC \to B)$ - missing D, E

· $ACD^+ = ACDBE$ $(AC \to B, D \to E,)$ All attributes

· Minimality: $A^+ = A$, $C^+ = C$, $D^+ = DE$, $AC^+ = ACB$

ACD is minimal

C.K = ~~ACB~~ ACD → only 1 chosen for now

→ Decomposing R1 into R1

$R1(A,C,D)$, $R' = R2(A,B,C) \bowtie R3(D,E)$

common Attributes = $R1 \cap R' = R1 \cap (R2 \cap R3)$

$= ACD \cap (ABC \cap DE) = ACD \cap D = \cancel{A}D$

· $R1(A,C;D)$: $D^+ = DE$ (From F), but only D is in R1. Missing A, and C. Not a superkey

· $R' = R2 \cap R3$

$AC \to B$ (in R2), $D \to E$ (in R3)

· $D^+ = DE$ A,B,C missing., not a superkey for R'

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→ Decomposing R' into R2 and R3

- $R2 \cap R3 = ABC \cap DE = \phi$ Using Chase test

- AC → B
Row1, Row2
A = a1, C = c1 vs c2

A	B	C	D	E
a1	b2	c1	d1	e3
a1	b2	c2	d2	e2
a3	b3	c3	d3	e3

- D → E
- D = d1 vs d3, no match
- D = d2 vs d3, no match
- Final row must show consistency

- ~~Dec~~ Lossless, because (R1 ⋈ R2) with AC as key then ⋈ R3 with D → E preserves all tuples

b) R1 (A, B, D), R2 (A, B, C) R3 (D, E)

- Decomposing R into R1 and (R2, R3)
- R1 (A, B, D), R' = R2 (A, B, C) ⋈ R3 (D, E)
- $R1 \cap R' = ABD \cap (ABC \cap DE) = ABD \cap D = D$
- $D^+ = DE$, not a S.k for R1 (A, B, D) or R'(ABCDE)

· Decomposing R':

· $R2 \cap R3 = \emptyset$

Chase test

A	B	C	D	E
a1	b1	c2	d1	e3
a2	b2	c2	d2	e2
a3	b3	c3	d3	e3

· $AC \to B$
no match across rows

· $D \to B$, no D match initially

· $R1 \cap R2 = \cancel{AB}$ $\cancel{AB^+}$ AB : a1b1 vs a2b2. No FD forces equality unless a1=a2, b1=b2

· Join has introduced supirious tuples (a1b1c2d1e3)

→ Decomposition is lossy

· $R1 \cap R2 = AB$, $AB^+ = AB$ (no FD). Not a superkey for R1 or R2

· No dependancy preserves the join