4.1

4, 7, 9, 11

1. If **u** and **v** are objects in V, then $\mathbf{u}+\mathbf{v}$ is in V.
2. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$
3. $\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w}$
4. There exists an object in V, called the ***zero vector***, that is denoted by $\mathbf{0}$ and has the property that $\mathbf{0}+\mathbf{u}=\mathbf{u}+\mathbf{0}=\mathbf{u}$ for all **u** in V.
5. For each **u** in V, there is an object $-\mathbf{u}$ in V, called a ***negative*** of **u**, such that $\mathbf{u}+(-\mathbf{u})=(-\mathbf{u})+\mathbf{u}=\mathbf{0}$.
6. If k is any scalar and **u** is any object in V, then $k\mathbf{u}$ is in V.
7. $k(\mathbf{u}+\mathbf{v})=k\mathbf{u}+k\mathbf{v}$
8. $(k+m)\mathbf{u}=k\mathbf{u}+m\mathbf{u}$
9. $k(m\mathbf{u})=(km)(\mathbf{u})$
10. $1\mathbf{u}=\mathbf{u}$

Q1- (a) $u+v=(u_1+v_1, u_2+v_2)$, $ku=(0, ku_2)$

$u+v=(2,6)$
$ku=(0,6)$

d) $k(u+v) = ku+kv$

$k(u_1+v_1, u_2+v_2)$

$= (0, ku_2+kv_2)$

$= (0, k(u_2+v_2))$

$= (0, ku_2)+(0, kv_2)$

$= k(u_1,u_2)+k(v_1,v_2)$ — ①

* Represent back in original form

e) $1.u=u$

$1.(u_1,u_2)=(0,u_2) \neq (u_1,u_2)$

Q2- $u+v=(u_1+v_1+1, u_2+v_2+1)$, $ku=(ku_1, ku_2)$

(a) $u=(0,4)$, $v=(1,-3)$, $k=2$

$u+v=(2,2)$
$ku=(0,8)$

*

(b) $(0,0)+(u_1,u_2)$

$=(u_1+1, u_2+1) \neq (u_1,u_2)$, hence $(0,0)$ is not the zero vector

*

(c) $(-1,-1)+(u_1,u_2)$

$=(u_1,u_2)$; $(-1,-1)$ is a zero vector

d) $-u=(-u_1,-u_2)$

$(u_1,u_2)+(-u_1,-u_2)$

$=(u_1-u_1+1, u_2-u_2+1)=(1,1)=0$

e) $k(u+v)$

$=k(u_1+v_1+1, u_2+v_2+1)$

$=(ku_1+kv_1+k, ku_2+kv_2+k)$

$ku+kv=(ku_1,ku_2)+(kv_1,kv_2)$

$=(ku_1+kv_1+1, ku_2+kv_2+1)$

$k(u+v)\neq ku+kv$ — ⑦

$(k+m)u = ku+mu$ — ⑧

$(k+m)(u_1,u_2)$

$((k+m)u_1, (k+m)u_2)$

$(ku_1+ku_2)+(mu_1,mu_2)$

$=(ku_1+mu_1+1, ku_2+mu_2+1)$

$(ku_1+mu_1, ku_2+mu_2) \neq (ku_1+mu_1+1, ku_2+mu_2+1)$