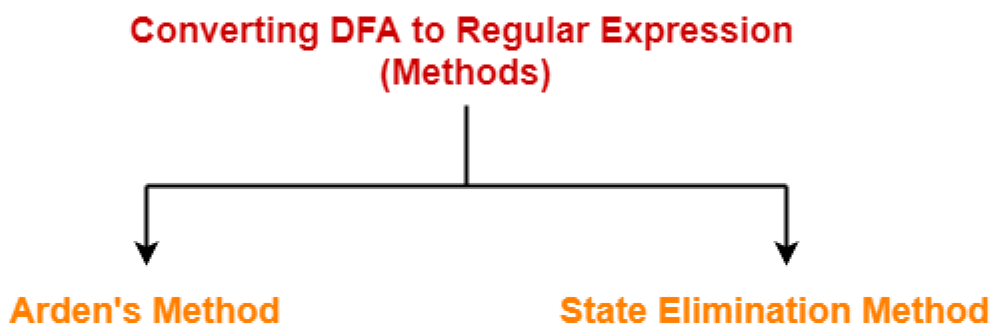
The two popular methods for converting a DFA to its regular expression are-



1. Arden's Method
2. State Elimination Method

In this article, we will discuss State Elimination Method.

State Elimination Method-

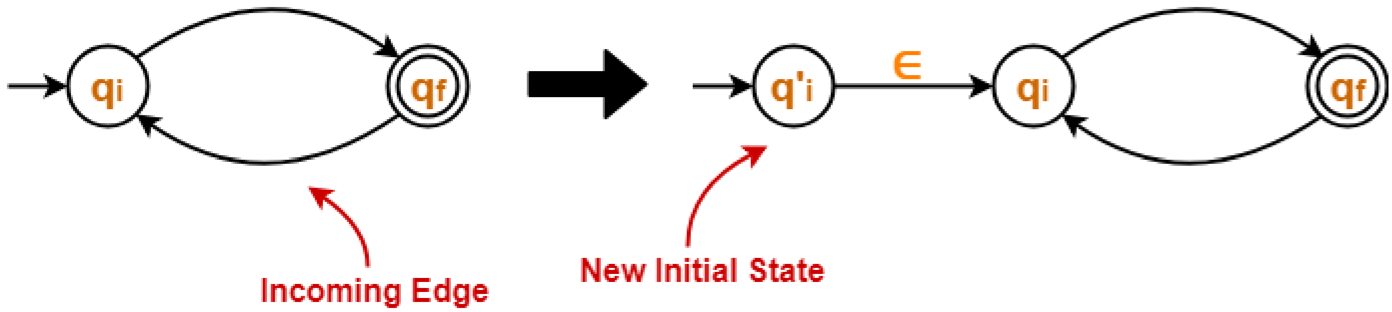
This method involves the following steps in finding the regular expression for any given DFA-

Step-01:

Thumb Rule
The initial state of the DFA must not have any incoming edge.

- If there exists any incoming edge to the initial state, then create a new initial state having no incoming edge to it.

Example-

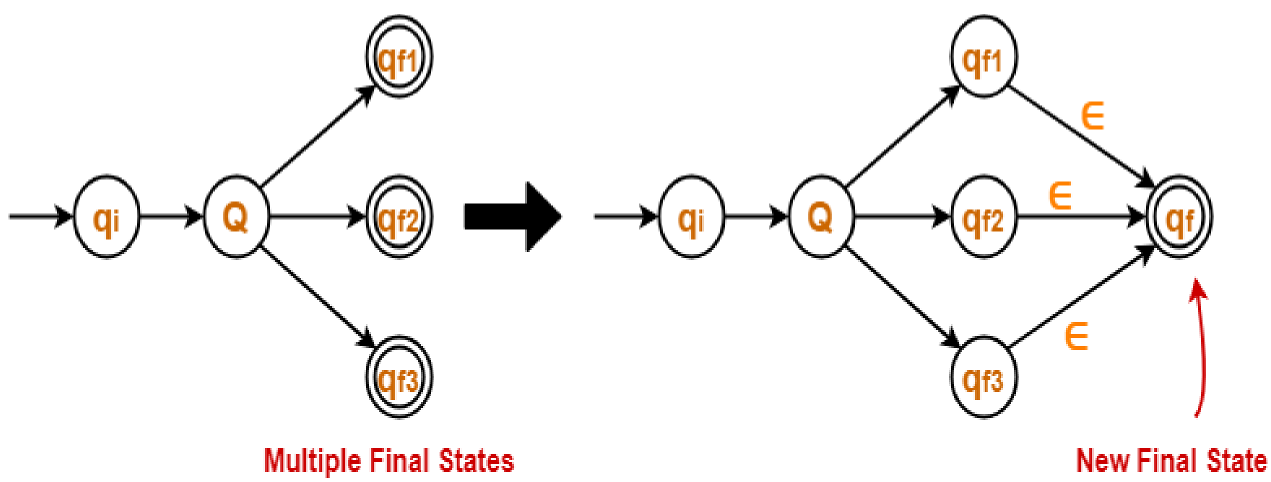


Step-02:

Thumb Rule
There must exist only one final state in the DFA.

If there exists multiple final states in the DFA, then convert all the final states into non-final states and create a new single final state.

Example-

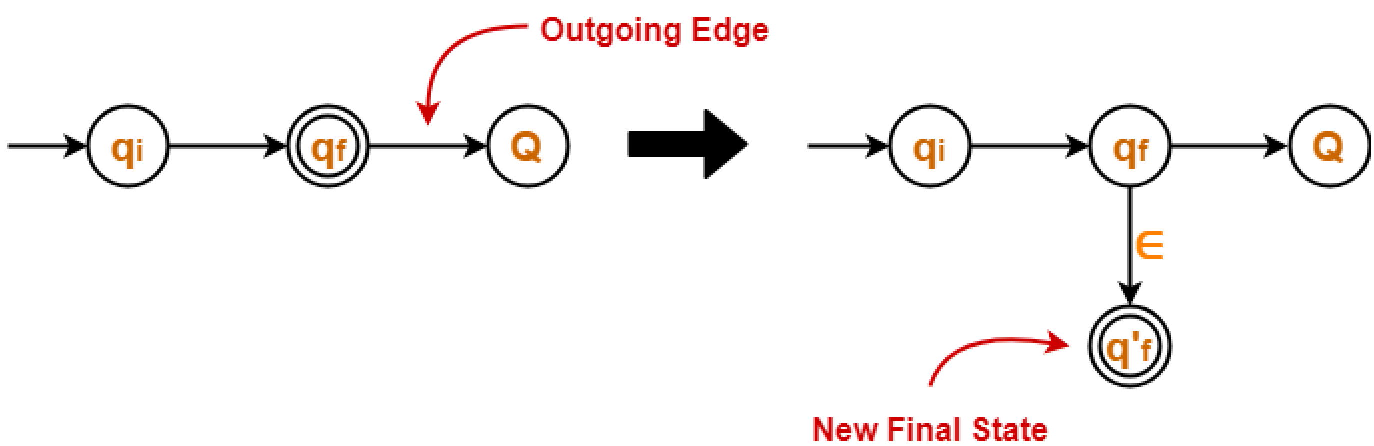


Step-03:

Thumb Rule
The final state of the DFA must not have any outgoing edge.

- If there exists any outgoing edge from the final state, then create a new final state having no outgoing edge from it.

Example-



Step-04:

- Eliminate all the intermediate states one by one.
- These states may be eliminated in any order.

In the end,

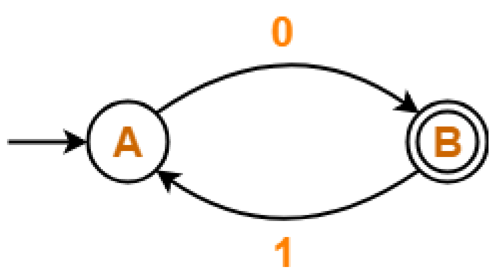
- Only an initial state going to the final state will be left.
- The cost of this transition is the required regular expression.

NOTE
The state elimination method can be applied to any finite automata.
(NFA, ∈-NFA, DFA etc)

PRACTICE PROBLEMS BASED ON CONVERTING DFA TO REGULAR EXPRESSION-

Problem-01:

Find regular expression for the following DFA-

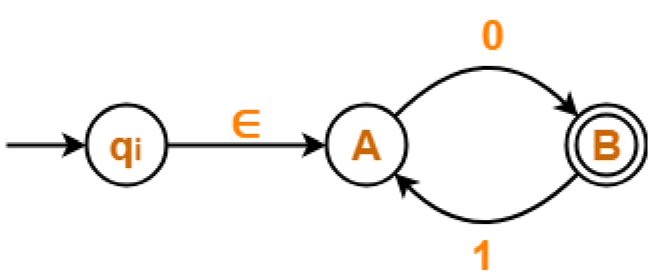


Solution-

Step-01:

- Initial state A has an incoming edge.
- So, we create a new initial state q_i.

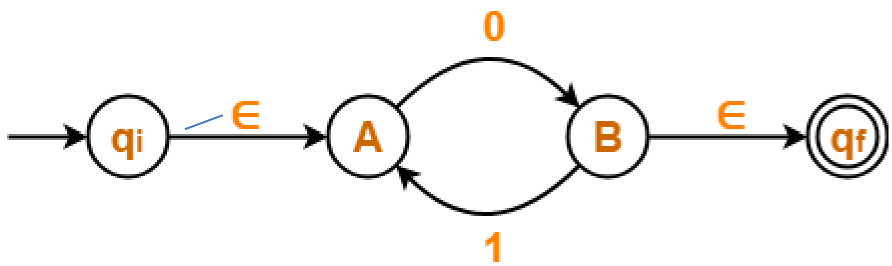
The resulting DFA is-



Step-02:

- Final state B has an outgoing edge.
- So, we create a new final state q_f

The resulting DFA is-



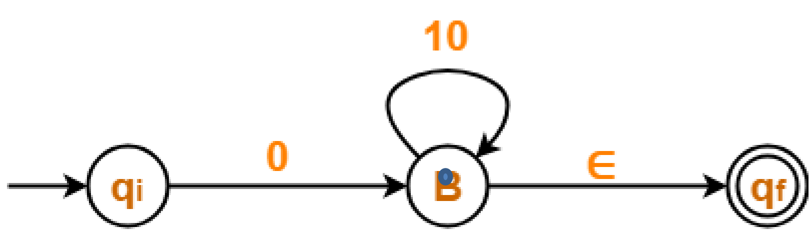
Step-03:

Now, we start eliminating the intermediate states.

First, let us eliminate state A.

- There is a path going from state q_i to state B via state A.
- So, after eliminating state A, we put a direct path from state q_i to state B having cost ∈.0 = 0
- There is a loop on state B using state A.
- So, after eliminating state A, we put a direct loop on state B having cost 1.0 = 10.

Eliminating state A, we get-



Step-04:

Now, let us eliminate state B.

- There is a path going from state q_i to state q_f via state B.
- So, after eliminating state B, we put a direct path from state q_i to state q_f having cost 0.(10)*.∈ = 0(10)*

Eliminating state B, we get-

From here,

Regular Expression = 0(10)*

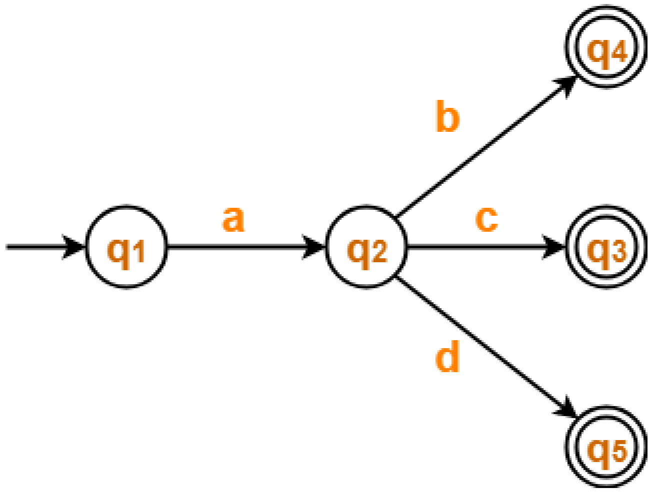
NOTE-

In the above question,

- If we first eliminate state B and then state A, then regular expression would be = (01)*0.
- This is also the same and correct.

Problem-02:

Find regular expression for the following DFA-

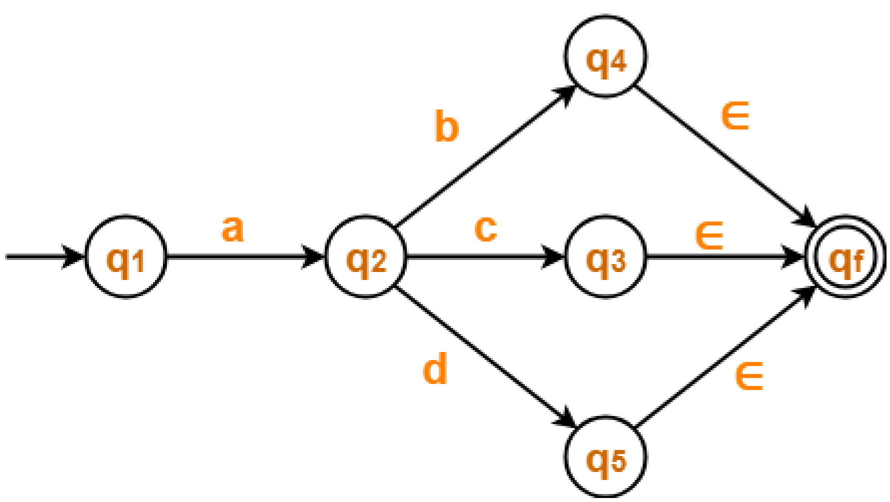


Solution-

Step-01:

- There exist multiple final states.
- So, we convert them into a single final state.

The resulting DFA is-

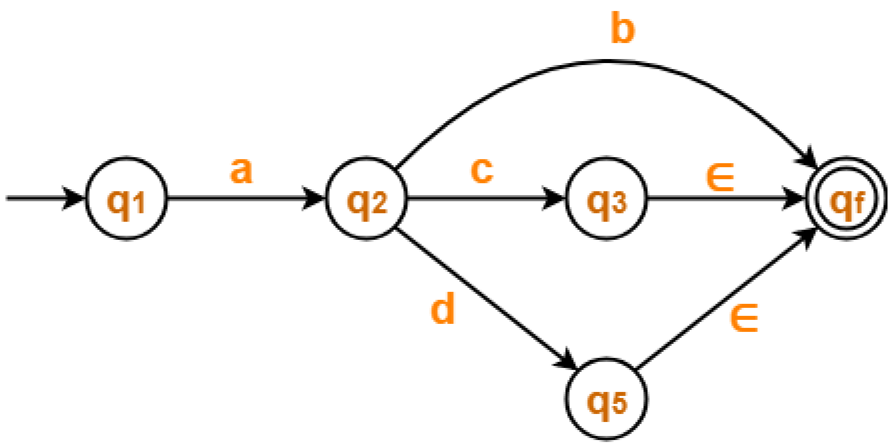


Step-02:

Now, we start eliminating the intermediate states.

First, let us eliminate state q_4.

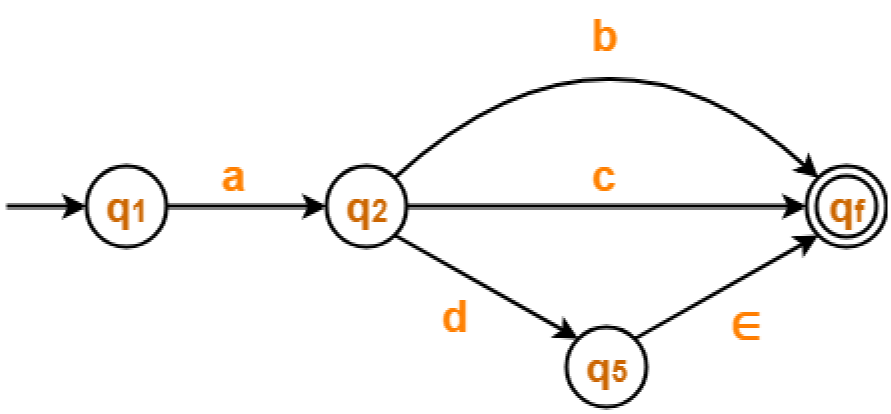
- There is a path going from state q_2 to state q_f via state q_4.
- So, after eliminating state q_4 , we put a direct path from state q_2 to state q_f having cost $b.\in = b$.



Step-03:

Now, let us eliminate state q_3.

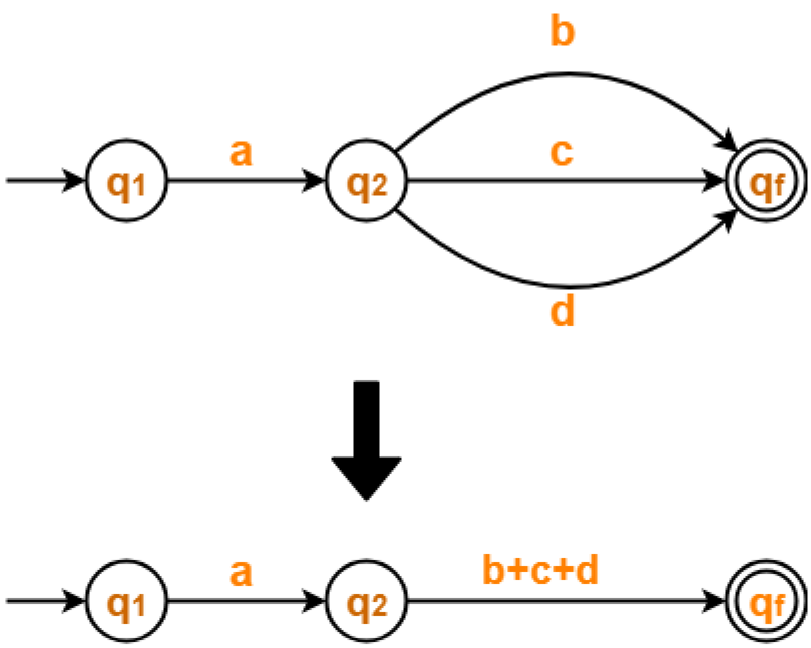
- There is a path going from state q_2 to state q_f via state q_3.
- So, after eliminating state q_3 , we put a direct path from state q_2 to state q_f having cost $c.\in = c$.



Step-04:

Now, let us eliminate state q_5.

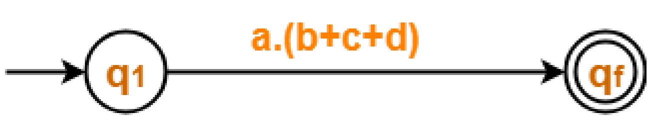
- There is a path going from state q_2 to state q_f via state q_5.
- So, after eliminating state q_5 , we put a direct path from state q_2 to state q_f having cost d.∈ = d.



Step-05:

Now, let us eliminate state q_2.

- There is a path going from state q_1 to state q_f via state q_2.
- So, after eliminating state q_2 , we put a direct path from state q_1 to state q_f having cost a.(b+c+d).

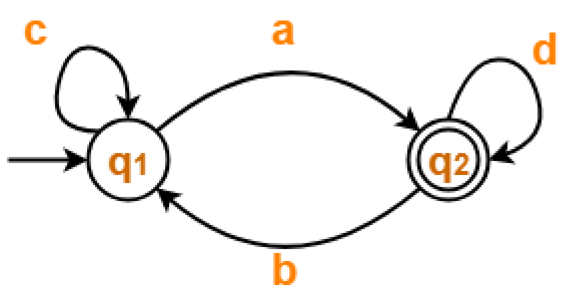


From here,

Regular Expression = a(b+c+d)

Problem-03:

Find regular expression for the following DFA-

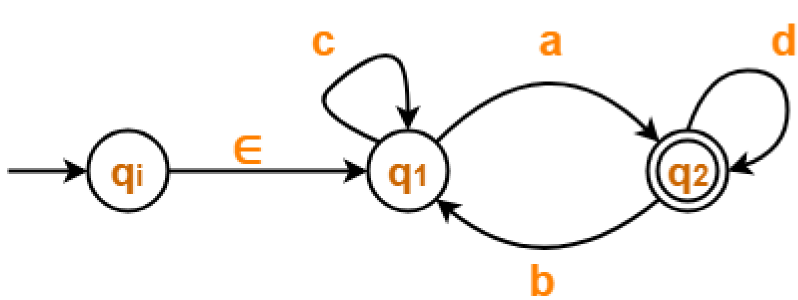


Solution-

Step-01:

- Initial state q_1 has an incoming edge.
- So, we create a new initial state q_i.

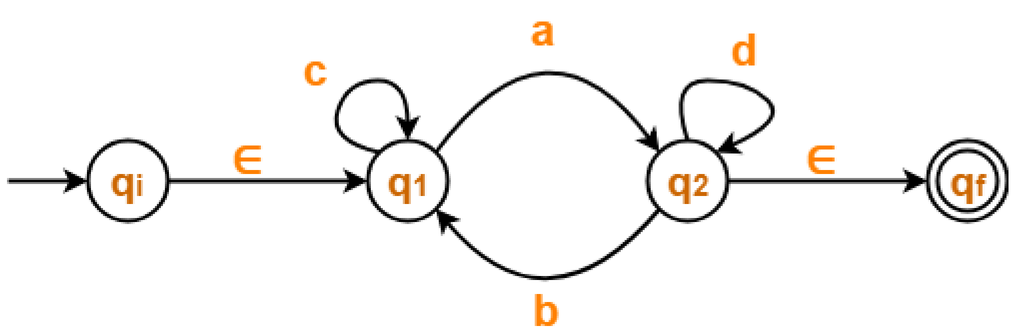
The resulting DFA is-



Step-02:

- Final state q_2 has an outgoing edge.
- So, we create a new final state q_f.

The resulting DFA is-



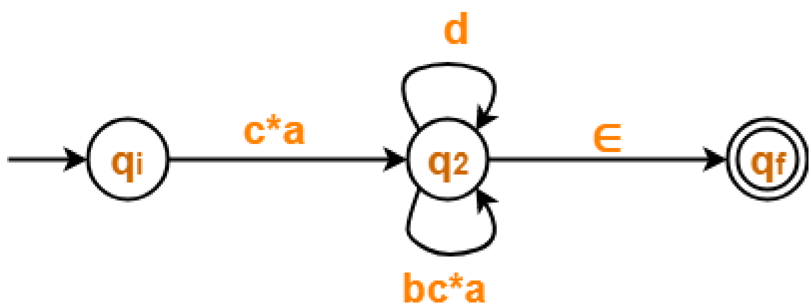
Step-03:

Now, we start eliminating the intermediate states.

First, let us eliminate state q_1.

- There is a path going from state q_i to state q_2 via state q_1 .
- So, after eliminating state q_1, we put a direct path from state q_i to state q_2 having cost ∈.c*.a = c*a
- There is a loop on state q_2 using state q_1 .
- So, after eliminating state q_1 , we put a direct loop on state q_2 having cost b.c*.a = bc*a

Eliminating state q_1, we get-



Step-04:

Now, let us eliminate state q_2.

- There is a path going from state q_i to state q_f via state q_2 .
- So, after eliminating state q_2, we put a direct path from state q_i to state q_f having cost c*a(d+bc*a)*∈ = c*a(d+bc*a)*

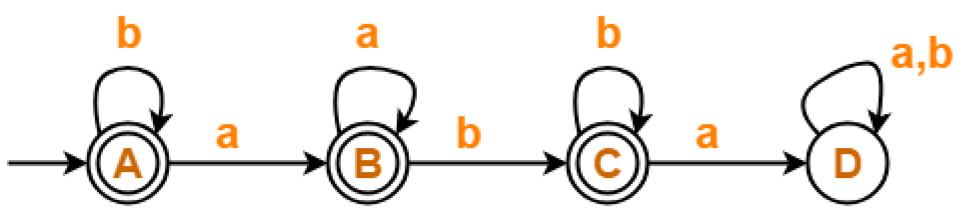
Eliminating state q_2, we get-

From here,

Regular Expression = c*a(d+bc*a)*

Problem-04:

Find regular expression for the following DFA-

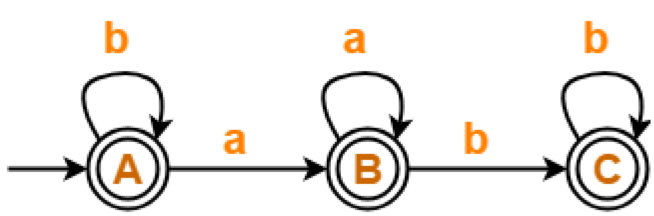


Solution-

Step-01:

- State D is a dead state as it does not reach to any final state.
- So, we eliminate state D and its associated edges.

The resulting DFA is-

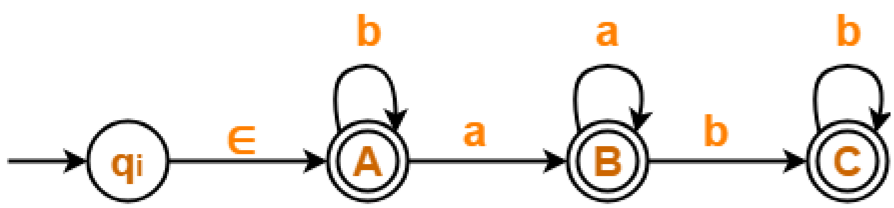


Step-02:

- Initial state A has an incoming edge (self loop).

- So, we create a new initial state q_i.

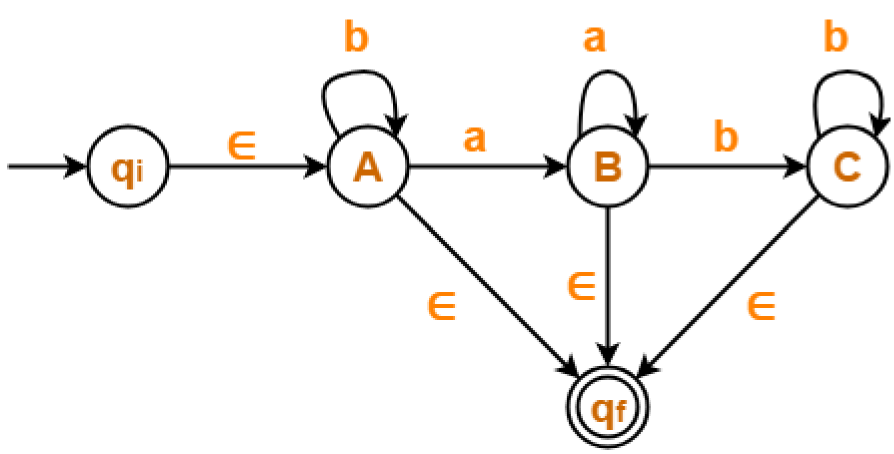
The resulting DFA is-



Step-03:

- There exist multiple final states.
- So, we convert them into a single final state.

The resulting DFA is-



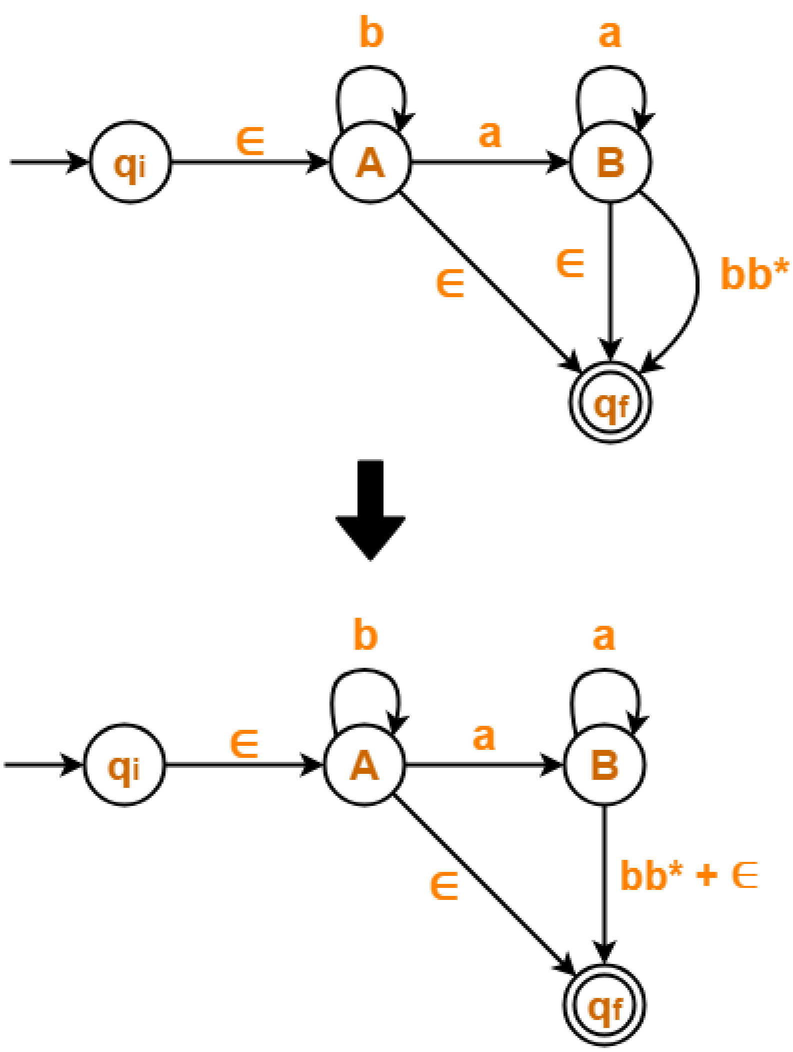
Step-04:

Now, we start eliminating the intermediate states.

First, let us eliminate state C.

- There is a path going from state B to state q_f via state C.
- So, after eliminating state C, we put a direct path from state B to state q_f having cost $b.b^*.\in = bb^*$

Eliminating state C, we get-

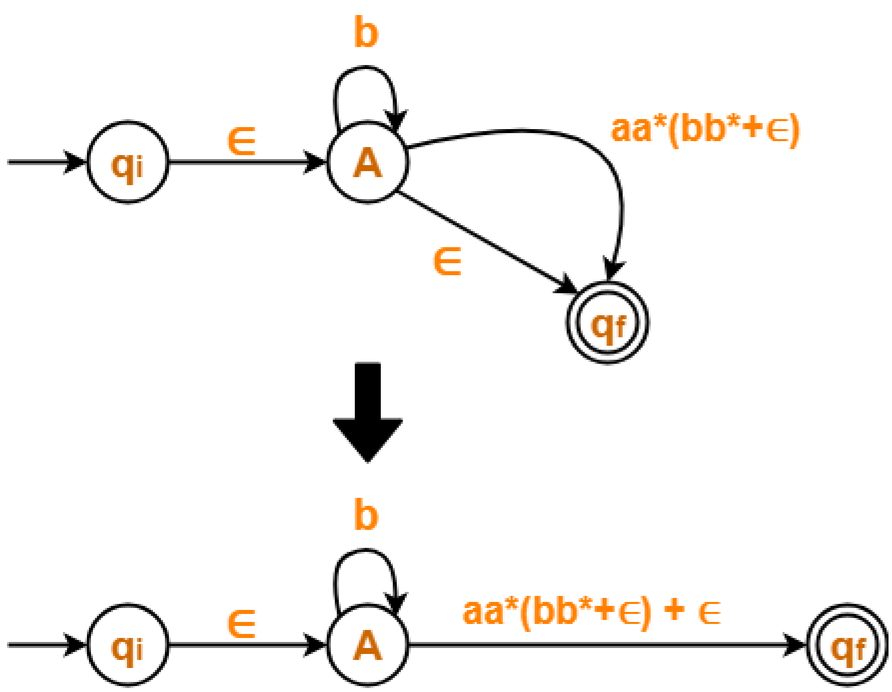


Step-05:

Now, let us eliminate state B.

- There is a path going from state A to state q_f via state B.
- So, after eliminating state B, we put a direct path from state A to state q_f having cost a.a*.(bb*+∈) = aa*(bb*+∈)

Eliminating state B, we get-

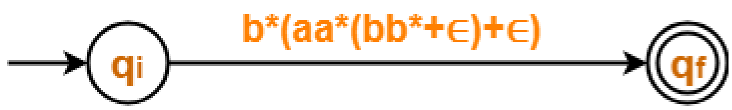


Step-06:

Now, let us eliminate state A.

- There is a path going from state q_i to state q_f via state A.
- So, after eliminating state A, we put a direct path from state q_i to state q_f having cost ∈.b*.(aa*(bb*+∈)+∈) = b*(aa*(bb*+∈)+∈)

Eliminating state A, we get-



From here,

Regular Expression = b*(aa*(bb*+∈)+∈)

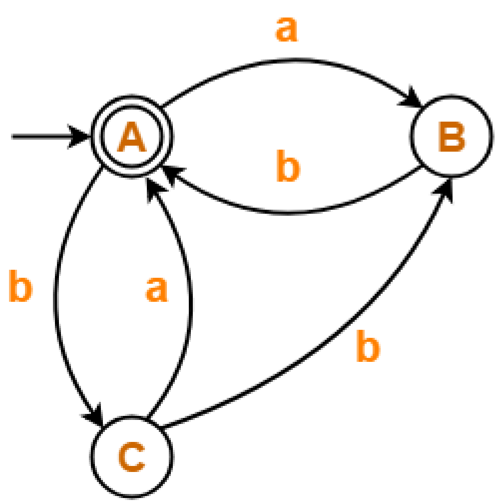
We know, bb* + ∈ = b*

So, we can also write-

Regular Expression = b*(aa*b*+∈)

Problem-05:

Find regular expression for the following DFA-

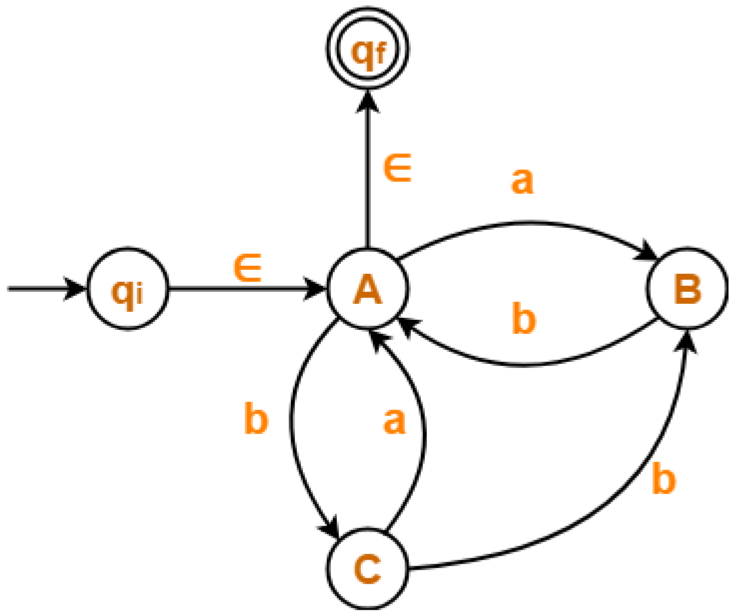


Solution-

Step-01:

- Since initial state A has an incoming edge, so we create a new initial state q_i.
- Since final state A has an outgoing edge, so we create a new final state q_f.

The resulting DFA is-



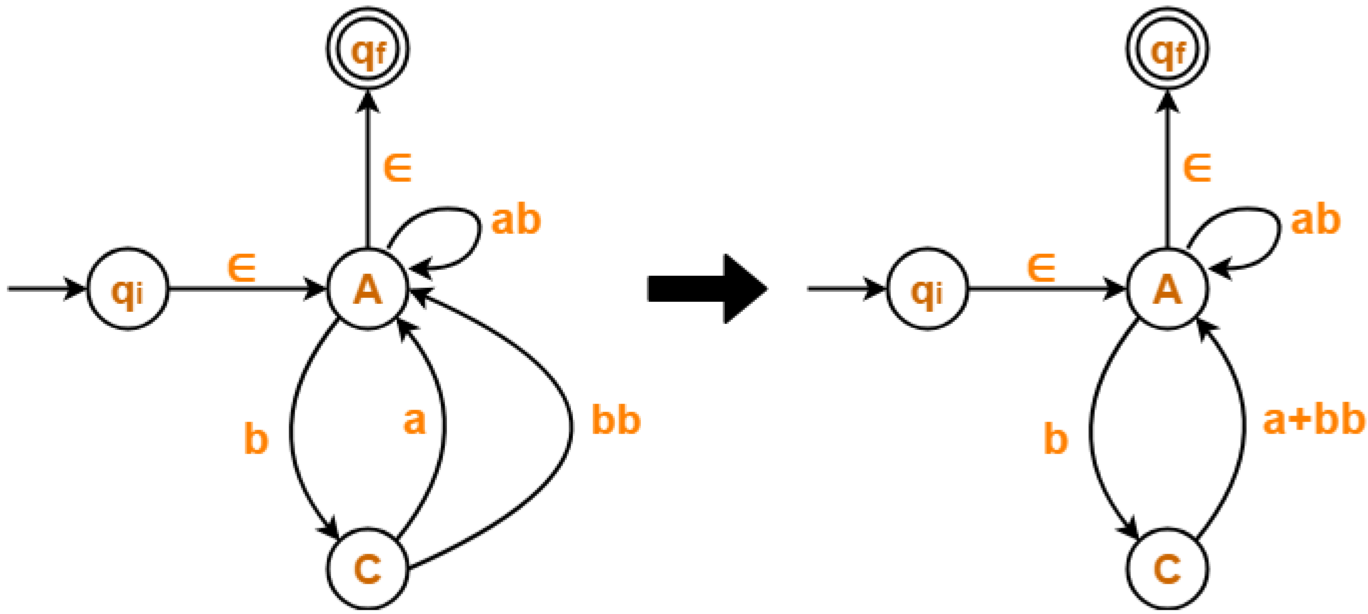
Step-02:

Now, we start eliminating the intermediate states.

First, let us eliminate state B.

- There is a path going from state C to state A via state B.
- So, after eliminating state B, we put a direct path from state C to state A having cost b.b = bb.
- There is a loop on state A using state B.
- So, after eliminating state B, we put a direct loop on state A having cost a.b = ab.

Eliminating state B, we get-

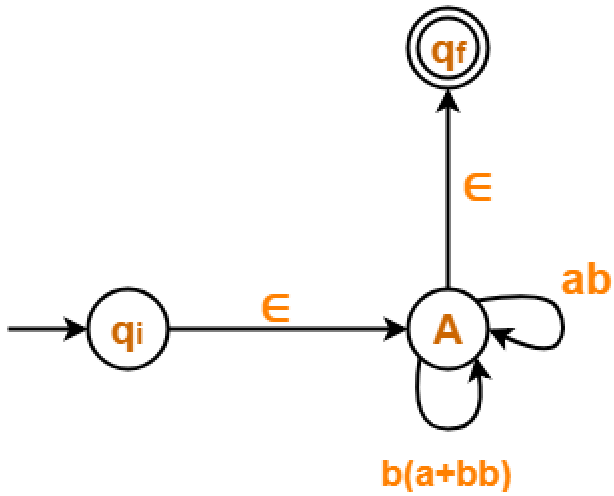


Step-03:

Now, let us eliminate state C.

- There is a loop on state A using state C.
- So, after eliminating state C, we put a direct loop on state A having cost b.(a+bb) = b(a+bb)

Eliminating state C, we get-

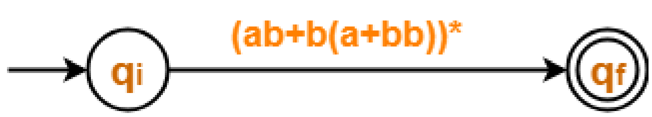


Step-04:

Now, let us eliminate state A.

- There is a path going from state q_i to state q_f via state A.
- So, after eliminating state A, we put a direct path from state q_i to state q_f having cost $\in.(ab + b(a+bb))^*\in = (ab + b(a+bb))^*$

Eliminating state A, we get-

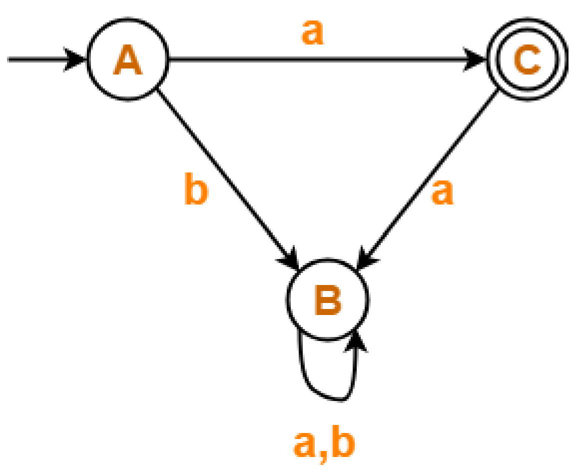


From here,

Regular Expression = (ab + b(a+bb))*

Problem-06:

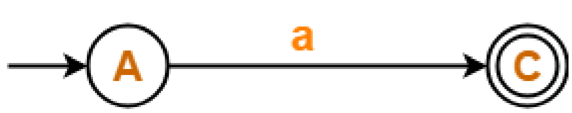
Find regular expression for the following DFA-



Solution-

- State B is a dead state as it does not reach to the final state.
- So, we eliminate state B and its associated edges.

The resulting DFA is-

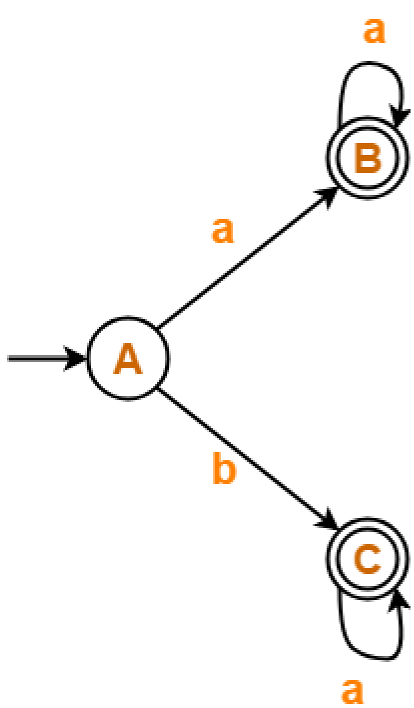


From here,

Regular Expression = a

Problem-07:

Find regular expression for the following DFA-

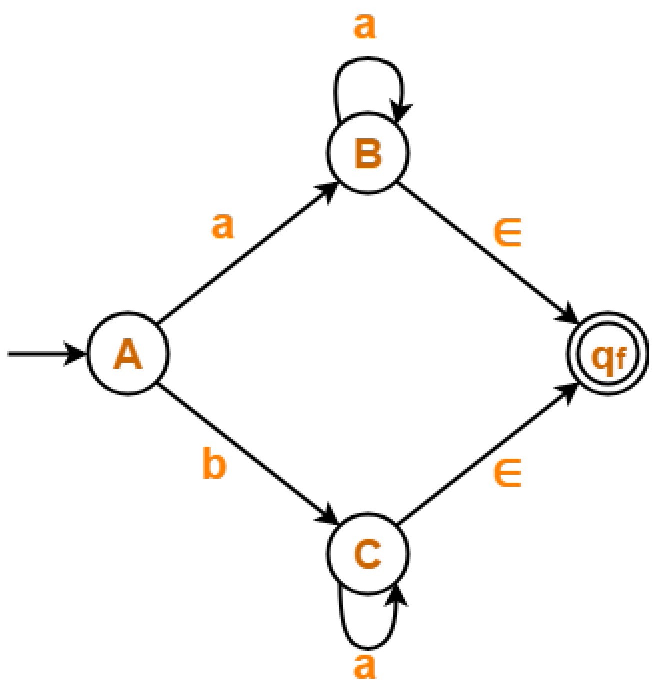


Solution-

Step-01:

- There exist multiple final states.
- So, we create a new single final state.

The resulting DFA is-



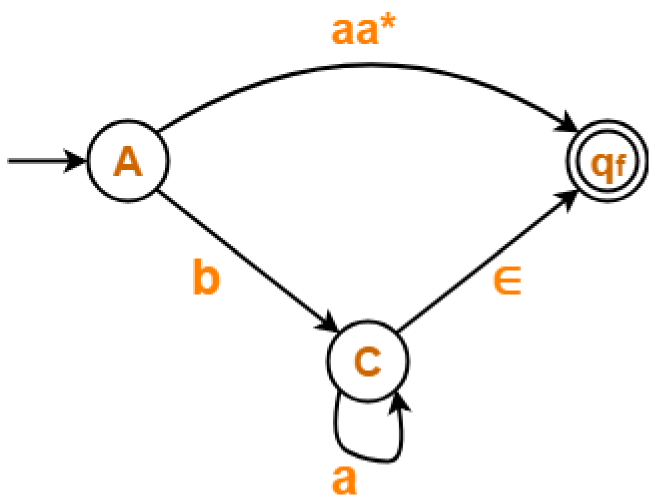
Step-02:

Now, we start eliminating the intermediate states.

First, let us eliminate state B.

- There is a path going from state A to state q_f via state B.
- So, after eliminating state B, we put a direct path from state A to state q_f having cost $a.a^*.\in = aa^*$.

Eliminating state B, we get-

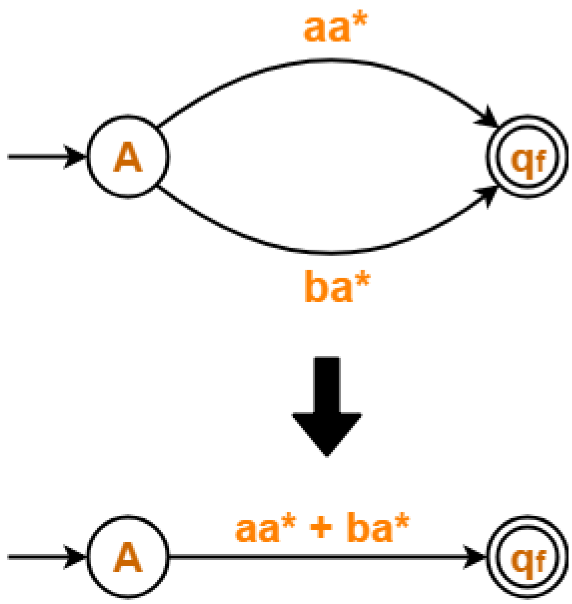


Step-03:

Now, let us eliminate state C.

- There is a path going from state A to state q_f via state C.
- So, after eliminating state C, we put a direct path from state A to state q_f having cost b.a*.∈ = ba*.

Eliminating state C, we get-



From here,

Regular Expression = aa* + ba*