3.5

- $u \times v = \left(\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right)$

Relationships Involving Cross Product and Dot Product

If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in 3-space, then

(a)	$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 0$	[$\mathbf{u}\times\mathbf{v}$ *is orthogonal to* $\mathbf{u}$]
(b)	$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 0$	[$\mathbf{u}\times\mathbf{v}$ *is orthogonal to* $\mathbf{v}$]
(c)	$\lVert\mathbf{u}\times\mathbf{v}\rVert^2 = \lVert\mathbf{u}\rVert^2\lVert\mathbf{v}\rVert^2 - (\mathbf{u}\cdot\mathbf{v})^2$	[*Lagrange's identity*]
(d)	$\mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = (\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{u}\cdot\mathbf{v})\mathbf{w}$	[*vector triple product*]
(e)	$(\mathbf{u}\times\mathbf{v})\times\mathbf{w} = (\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{v}\cdot\mathbf{w})\mathbf{u}$	[*vector triple product*]

Properties of Cross Product

If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are any vectors in 3-space and k is any scalar, then:

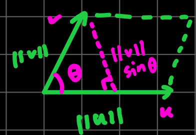
(a) $\mathbf{u}\times\mathbf{v} = -(\mathbf{v}\times\mathbf{u})$
(b) $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = (\mathbf{u}\times\mathbf{v}) + (\mathbf{u}\times\mathbf{w})$
(c) $(\mathbf{u}+\mathbf{v})\times\mathbf{w} = (\mathbf{u}\times\mathbf{w}) + (\mathbf{v}\times\mathbf{w})$
(d) $k(\mathbf{u}\times\mathbf{v}) = (k\mathbf{u})\times\mathbf{v} = \mathbf{u}\times(k\mathbf{v})$
(e) $\mathbf{u}\times\mathbf{0} = \mathbf{0}\times\mathbf{u} = \mathbf{0}$
(f) $\mathbf{u}\times\mathbf{u} = \mathbf{0}$

$$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{u}\cdot\left(\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix}\mathbf{i} - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix}\mathbf{j} + \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}\mathbf{k}\right)$$
$$= \begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix}u_1 - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix}u_2 + \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}u_3$$
$$= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3\end{vmatrix}$$

- $u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}\hat{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}\hat{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}\hat{k}$

- $\lVert u \times v \rVert = \lVert u \rVert \lVert v \rVert \sin\theta$
- $\lVert u \times v \rVert$ = Area of Parallelogram

→ In 3d, it is vol. of p



- $u, v, w \rightarrow u\cdot(v\times w)$ scalar triple product

- If $u\cdot(v\times w) = 0$, then all vectors have the same initial point and lie in the same plane.

- $\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$ = Vol. of Parallelipiped

Q1- (a) $w = (2,6,7)$ $v = (0,2,-3)$

$$\begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ 2 & 6 & 7 \end{vmatrix}$$

14+18

$= 38\hat{i} - 6\hat{j} - 4\hat{k}$

$= (32,-6,-4)$

Q9- $u = (1,-1,2)$, $v = (0,3,1)$

$$u\times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

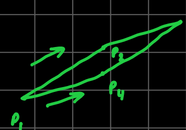
$= -7i - j + 3k$

$\lVert u\times v \rVert = \sqrt{(-7)^2 + (-1)^2 + 3^2} = \sqrt{59}$ unit^2

Q11- $P_1(1,2)$, $P_2(4,4)$, $P_3(7,5)$, $P_4(4,3)$

$\vec{P_1P_2} = (3,2)$

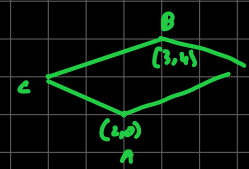
$\vec{P_1P_4} = (3,1)$



$\vec{P_1P_2} \times \vec{P_1P_4} = \begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix} = 3-6 = 3\hat{k}$

Area= 3

Q13- $A(2,0)$, $B(3,4)$, $C(-1,2)$



$\vec{CA} = (3,-2)$

$\vec{CB} = (4,2)$

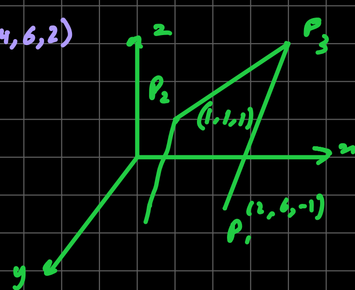
$\vec{CA}\times\vec{CB} = \begin{vmatrix} 3 & -2 \\ 4 & 2 \end{vmatrix}$

$= 14\hat{k}$

Area of ▱ $= \sqrt{14^2} = 14$

Area of Δ = 7

Q 15- $P_1(2,6,-1)$, $P_2(1,1,1)$, $P_3(4,6,2)$



$\vec{P_3P_2} = (-3,-5,-1)$

$\vec{P_3P_1} = (-2,0,-3)$

$\lVert \vec{P_3P_2}\times\vec{P_3P_1} \rVert = \begin{vmatrix} i & j & k \\ -3 & -5 & -1 \\ -2 & 0 & -3 \end{vmatrix}$

$= 15i - 7j - 10k$

Area of Δ $= \frac{1}{2}\sqrt{15^2 + (-7)^2 + (-10)^2}$

$= \frac{\sqrt{374}}{2}$