Vectors

Properties

- $\hat{i}.\hat{i}=1$, $\hat{j}.\hat{j}=1$, $\hat{k}.\hat{k}=1$ (Similar dot product $=0$)
- $\hat{i}.\hat{j}=0$, $\hat{j}.\hat{k}=0$, $\hat{k}.\hat{i}=0$ (Unsimilar dot product $=0$)

- $\hat{i}\times\hat{i}=0$, $\hat{j}\times\hat{j}=0$, $\hat{k}\times\hat{k}=0$ (Similar cross-product)
- $\hat{i}\times\hat{k}=\hat{j}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{i}\times\hat{k}=\hat{j}$ (Unsimilar cross-product)

- $\vec{A}\times\vec{B}=0$ → then vectors are parallel / antiparallel
- $\vec{A}\cdot\vec{B}=0$ → then vectors are perpendicular

- $\vec{A}\cdot\vec{B}=\vec{A}\times\vec{B}$ → when $\theta=45^\circ$

Q- If $r_1 = 3i-2j+k$, $r_2 = 2i-4j-3k$ and $r_3 = -i+2j+2k$. Find $|2r_1 - 3r_2 - 5r_3|$

$2(3i-2j+k) - 3(2i-4j-3k) - 5(-i+2j+2k)$

$= \cancel{6}i - 4j + 2k - \cancel{6}i + 12j + 9k + 5i - 10j - 10k$

$= 5i - 2j + k$

mag. $= \sqrt{5^2 + (-2)^2 + 1^2}$

$= \sqrt{25+4+1} = \sqrt{30}$

Q- $\vec{A} = 3i-j-4k$, $\vec{B} = -2i+4j-3k$, $\vec{C} = i+2j-k$. Find $|3A - 2B + 4C|$

$3(3i-j-4k) - 2(-2i+4j-3k) + 4(i+2j-k)$

$= 9i - 3j - 12k + 4i - \cancel{8}j + 6k + 4i + \cancel{8}j - 4k$

$= 17i - 3j - 10k$

mag. $= \sqrt{17^2 + (-3)^2 + (-10)^2}$

$= \sqrt{289 + 9 + 100}$

Dot Product

* Dot product always gives a scalar quantity

* cross product always gives a vector quantity

· $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

→ $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$

Q- Find the angle b/w $\vec{A} = 2i + 2j - k$ and $\vec{B} = 6i - 3j + 2k$

$$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|A||B|} \right)$$

$$\vec{A} \cdot \vec{B} = (2i + 2j - k) \cdot (6i - 3j + 2k)$$

$$= 12 - 6 - 2$$

$$= 4$$

$$|\vec{A}||\vec{B}| = \sqrt{2^2 + 2^2 + (-1)^2} \cdot \sqrt{6^2 + (-3)^2 + 2^2}$$

$$= \sqrt{4+4+1} \cdot \sqrt{36 + 9 + 4}$$

$$= 3 \cdot 7 = 21$$

$$\theta = \cos^{-1}\left(\frac{4}{21} \right)$$

Q- The value of a, so that $\vec{A} = 2i + aj + k$, $\vec{B} = 6i - 3j + 2k$ are perpendicular

$(2i + aj + k) \cdot (6i - 3j + 2k) = 0$

If perpendicular use dot-product

$12 - 3a + 2 = 0$

If parallel use cross-product

$3a = 14$

$|\vec{A} \times \vec{B}|$ max when perpendicular

$a = \frac{14}{3}$

Q- The value of a, so that $\vec{A} = 3i + 2j - 6k$, $\vec{B} = 4i - 3j + ak$ are perpendicular

$(3i + 2j - 6k) \cdot (4i - 3j + ak) = 0$

$12 - 6 - 6a = 0$

$6a = 6$

$a = 1$

Cross - Product

- $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \times \hat{n}$ where $\hat{n} = \hat{i}, \hat{j}, \hat{k}$, θ is the smaller of the two angles

clockwise -ve, anticlockwise +ve, using RHL

Q- If $\vec{A} = 2i - 3j - k$ and $\vec{B} = i + 4j - 2k$. Find the cross-product magnitude

$$\vec{A} \times \vec{B} = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} i & j & k \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix} \end{array}$$

$$= i(6+4) - j(-4+1) + k(8+3)$$

$$= 10i + 3j + 11k$$

Additional

- Vector addition is assosciative. e.g: $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$
- Vector addition is commutative e.g: $\vec{a}+\vec{b} = \vec{b}+\vec{a}$
- Negative sign reverses vector direction. $b\uparrow\downarrow -b$ e.g: $\vec{b}+(-\vec{b}) = 0$ (Null vector form)
- Only vectors of the same kind can be added
- The process of finding components of vector is called resolving the vector

- Note: always take angle from +ve x-axis. If going clockwise, place -ve sign with angle.
- $\vec{a}_x = a\cos\theta$, $\vec{a}_y = a\sin\theta$
- $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$; $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$
- (a_x, a_y, a_z) ; where a_x, a_y, a_z are only scalar components
- i, j, k form is also called unit vector form
- Sample Problems: 3.02, 3.03, 3.04, 3.05, 3.07
- Exercise Problems: 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 34, 35, 36, 37, 38, 39, 40

Note: Always give vector in unit vector form with magnitude and with direction from +ve x-axis counter clockwise.