- $\det(A) = \det(A^T)$; A is square matrix, and below
- $\det(B) = k\det(A)$; when B results from multiplication of k with A
- $\det(B) = -\det(A)$; row interchanged
- → other operations $\det(A) = \det(B)$
- If kE, then $\det(E) = k$
  ↳ for only one row
- A, having 2 proportional rows/columns has $\det(A) = 0$

- For Row reduction, follow det rules

- Take out common instead of multiplying

Q4- $A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 2 & -3 \\ -1 & 1 & 5 \end{bmatrix} \longrightarrow \left[\begin{array}{ccc} 4 & 2 & -1 \\ 0 & 2 & -3 \\ -1 & 1 & 5 \end{array}\right] \begin{array}{cc} 4 & 2 \\ 0 & 2 \\ -1 & 1 \end{array}$

$|A| = 4\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -1 \\ 2 & -3 \end{vmatrix}$

$= 4(13) + 4$

$= 52 + 4 = \underline{\underline{56}}$

$= (40+6) - (2-12)$

$= 46 + 10 = \underline{\underline{56}}$

$A^T = \begin{bmatrix} 4 & 0 & -1 \\ 2 & 2 & 1 \\ -1 & -3 & 5 \end{bmatrix}$

$|A^T| = 4\begin{vmatrix} 2 & 1 \\ -3 & 5 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix}$

$= 52 + 4 = \underline{\underline{56}}$

Q7- $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= -1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

Q23- $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)\ (c-a)(c-b)$

$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ a^2 & b^2 & c^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & c^2 - \cancel{a^2} - b^2 + \cancel{a^2} \end{vmatrix}$

$= (c^2-b^2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & 1 \end{vmatrix}$

$= (c^2-b^2) \cdot \begin{vmatrix} 1 & 1 \\ 0 & b-a \end{vmatrix}$

$= c^2-b^2\ (b-a)$ ✓

$= (b-a)(c-b)(c+b)$