Halliday & Resnick's
Fundamentals of Physics
Jearl Walker

Chapter 26

Current and Resistance

26.2: Electric Current:

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current.
If there is to be an electric current through a given surface, there must be a net flow of charge through that surface. Two examples are given.

1. The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of 106 m/s. If you pass a hypothetical plane through such a wire, conduction electrons pass through it in both directions at the rate of many billions per second—but there is no *net* transport of charge and thus *no current through the wire.*

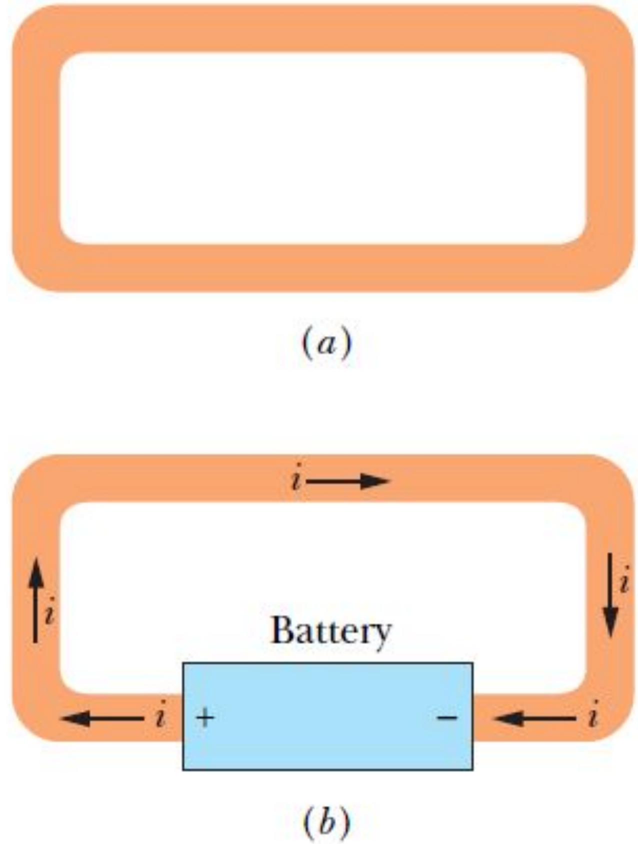
However, if you connect the ends of the wire to a battery, you slightly bias the flow in one direction, with the result that there now is a *net* transport of charge and thus an electric current through the wire.

2. The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million coulombs per second.

There is no net transport of charge, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.

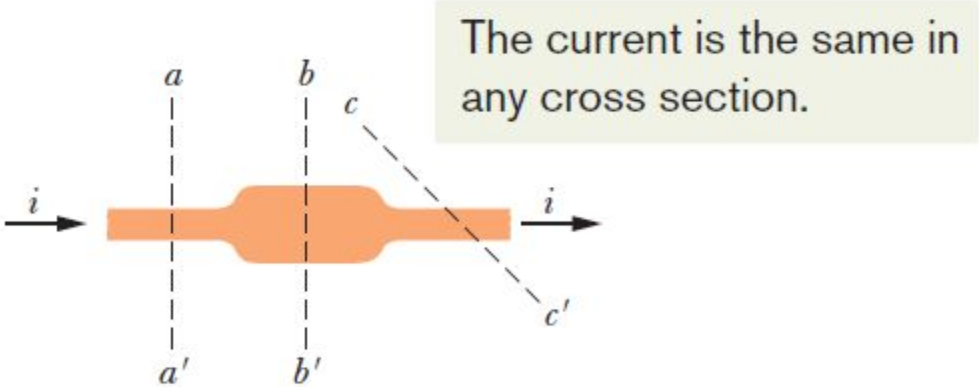
26.2: Electric Current:

Fig. 26-1 (*a*) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (*b*) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current *i*.



26.2: Electric Current:

Fig. 26-2 The current i through the conductor has the same value at planes aa', bb', and cc'.



The figure shows a section of a conductor, part of a conducting loop in which current has been established. If charge dq passes through a hypothetical plane (such as *aa'*) in time dt, then the current i through that plane is defined as:

$$i = \frac{dq}{dt} \qquad \text{(definition of current).}$$

The charge that passes through the plane in a time interval extending from 0 to t is:

$$q = \int dq = \int_0^t i\, dt$$

Under steady-state conditions, the current is the same for planes *aa', bb',* and *cc'* and for all planes that pass completely through the conductor, no matter what their location or orientation.

The SI unit for current is the coulomb per second, or the ampere (A):

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

26.2: Electric Current, Conservation of Charge, and Direction of Current:

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

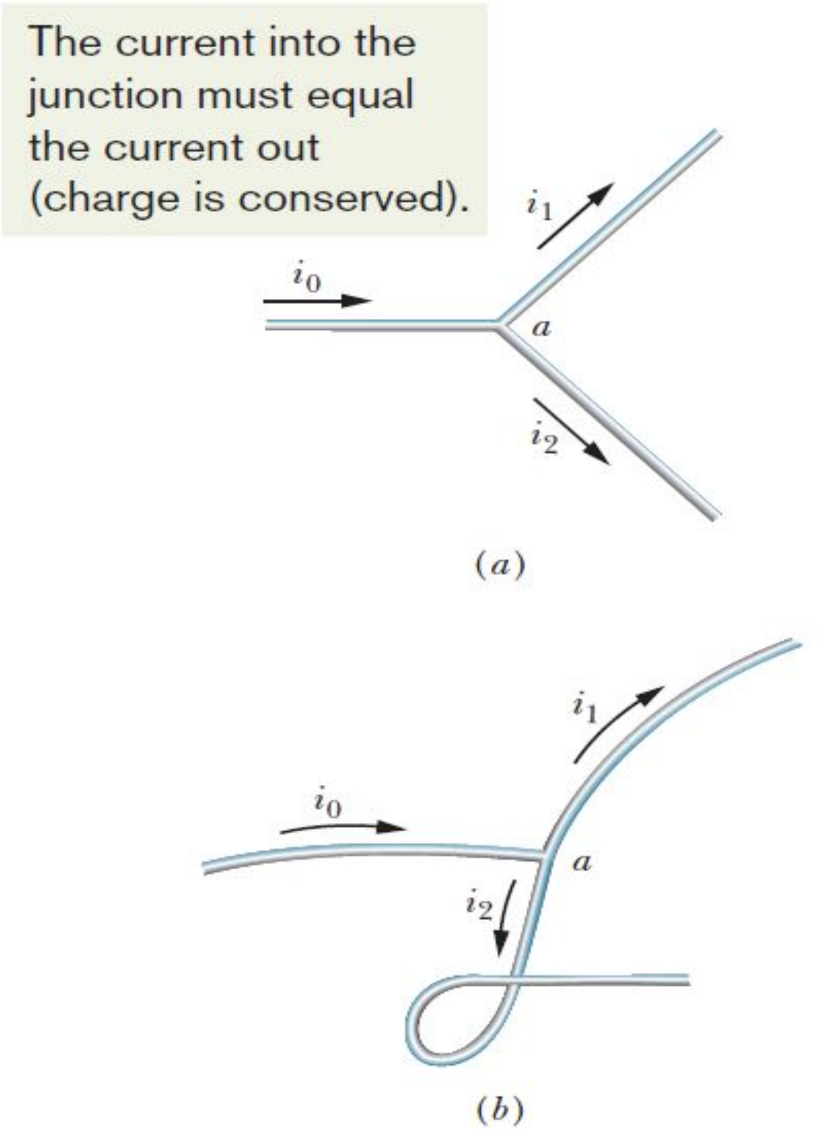


Fig. 26-3 The relation $i_0 = i_1 + i_2$ is true at junction a no matter what th orientation in space of the three wires Currents are scalars, not vectors.

26.3: Current Density:

The magnitude of **current density, *J*,** is equal to the current per unit area through any element of cross section. It has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

$$i = \int \vec{J} \cdot d\vec{A}.$$

If the current is uniform across the surface and parallel to ***dA***, then ***J*** is also uniform and parallel to ***dA***.

$$i = \int J\, dA = J \int dA = JA$$

$$J = \frac{i}{A},$$

Here, *A* is the total area of the surface.

The SI unit for current density is the ampere per square meter (A/m^2).

26.3: Current Density:

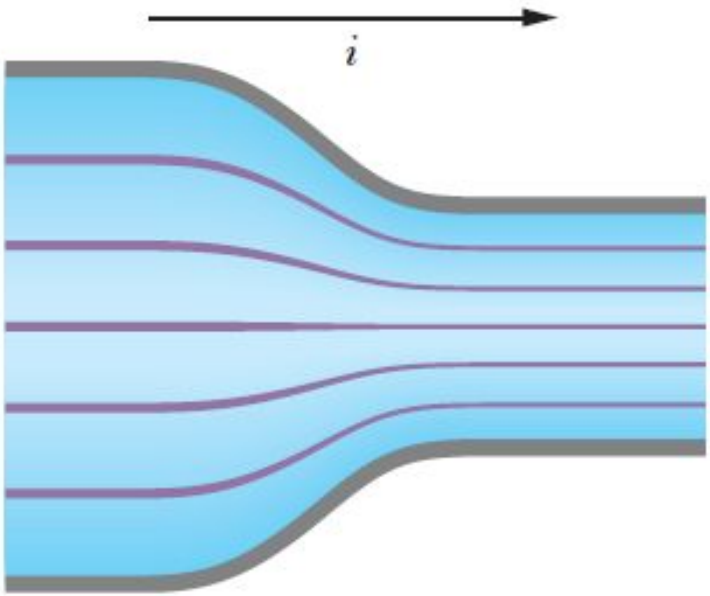


Fig. 26-4 Streamlines representing current density in the flow of charge through a constricted conductor.

Figure 26-4 shows how current density can be represented with a similar set of lines, which we can call *streamlines*.

The current, which is toward the right, makes a transition from the wider conductor at the left to the narrower conductor at the right.

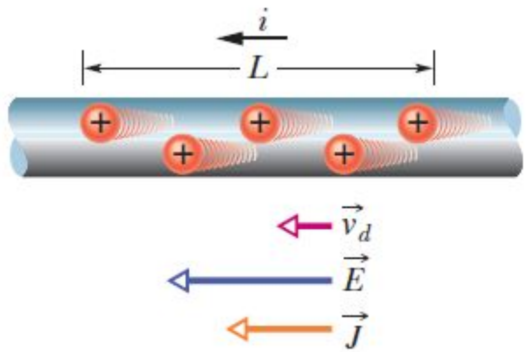
Since charge is conserved during the transition, the amount of charge and thus the amount of current cannot change.

However, the current density changes—it is greater in the narrower conductor.

26.3: Current Density, Drift Speed:

Current is said to be due to positive charges that are propelled by the electric field.

Fig. 26-5 Positive charge carriers drift at speed v_d in the direction of the applied electric field $\vec{E}$. By convention, the direction of the current density $\vec{J}$ and the sense of the current arrow are drawn in that same direction.



When a conductor has a current passing through it, the electrons move randomly, but they tend to *drift* with a **drift speed** $\boldsymbol{v_d}$ in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared with the speeds in the random motion.

In the figure, the equivalent drift of positive charge carriers is in the direction of the applied electric field, ***E***. If we assume that these charge carriers all move with the same drift speed v_d and that the current density ***J*** is uniform across the wire's cross-sectional area *A,* then the number of charge carriers in a length *L* of the wire is *nAL*. Here *n* is the number of carriers per unit volume.

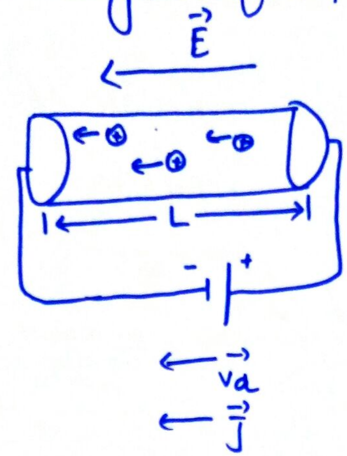
The total charge of the carriers in the length *L,* each with charge *e,* is then $q = (nAL)e.$

The total charge moves through any cross section of the wire in the time interval $t = \frac{L}{v_d}.$

$$\Rightarrow \quad i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d. \quad \Rightarrow \quad v_d = \frac{i}{nAe} = \frac{J}{ne}$$

$$\Rightarrow \quad \boxed{\vec{J} = (ne)\vec{v}_d.}$$

Current Density, Drift Speed:



Volume = AL

So,

$$i = \frac{q}{t} \quad -(1)$$

So, Total charge will be

$$q = Ne$$

$$q = nALe \quad -(a)$$

And

$$s = vt$$

$$L = V_d t$$

$$t = L/V_d \quad -(b)$$

$\because$ Charge Quantization

$$q = Ne$$

$\because$ Electron density

$$n = \frac{N}{V}$$

$$N = nV$$

$$N = nAL$$

Substituting (a) and (b) in eq (1)

$$i = \frac{nA\cancel{L}e}{\cancel{L}/V_d} = nAeV_d \Rightarrow i = nAeV_d$$

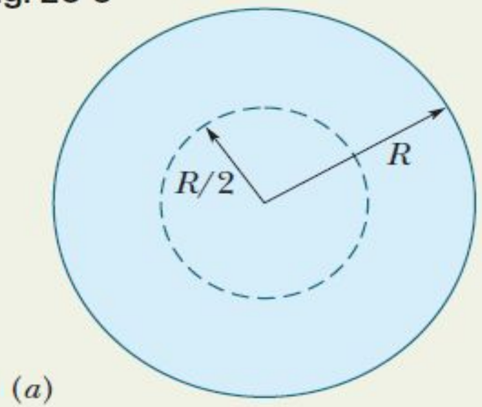
So,

$$\frac{i}{A} = neV_d \Rightarrow J = (ne)V_d$$

Example, Current Density, Uniform and Nonuniform:

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m². What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6a)?

Fig. 26-6



Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2.$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2)$$

$$= 1.9 \text{ A.} \qquad \text{(Answer)}$$

d Nonuniform,

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m^4 and r is in meters. What now is the current through the same outer portion of the wire?

Calculations: The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\,dA \cos 0 = J\,dA.$$

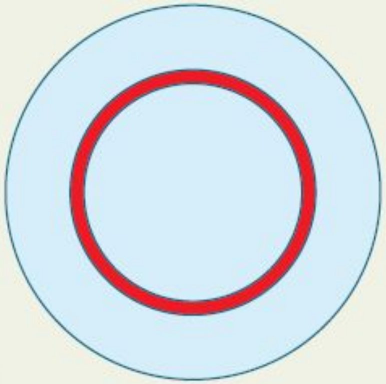
$$\begin{aligned}
i &= \int \vec{J} \cdot d\vec{A} = \int J\,dA \\
&= \int_{R/2}^{R} ar^2\, 2\pi r\,dr = 2\pi a \int_{R/2}^{R} r^3\,dr \\
&= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4 \\
&= \frac{15}{32}\pi(3.0 \times 10^{11}\ \text{A/m}^4)(0.0020\ \text{m})^4 = 7.1\ \text{A}.
\end{aligned}$$

(Answer)

Calculations: The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\,dA \cos 0 = J\,dA.$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r\,dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig. 26-6b). We can then integrate



(b)

Fig. 26-6

26.4: Resistance and Resistivity:

We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = \frac{V}{i} \qquad \text{(definition of } R\text{)}.$$

The SI unit for resistance that follows from Eq. 26-8 is the volt per ampere. This has a special name, the **ohm** (symbol Ω):

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere}$$
$$= 1 \text{ V/A}.$$

In a circuit diagram, we represent a resistor and a resistance with the symbol -\/\/\/-.

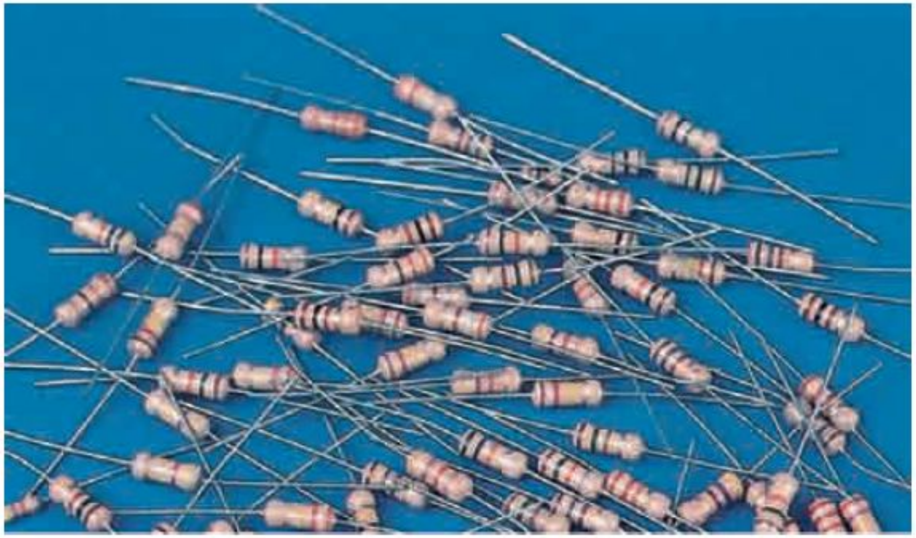


Fig. 26-7 An assortment of resistors. The circular bands are color-coding marks that identify the value of the resistance. *(The Image Works)*

26.4: Resistance and Resistivity:

The resistivity, ρ, of a resistor is defined as:

$$\rho = \frac{E}{J} \rightarrow \vec{E} = \rho\vec{J}.$$

The SI unit for ρ is $\Omega.m$.

The conductivity σ of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho} \rightarrow \vec{J} = \sigma\vec{E}.$$

Table 26-1

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

26.4: Resistance and Resistivity, Calculating Resistance from Resistivity:

Resistance is a property of an object. Resistivity is a property of a material.

$$E = V/L \quad \text{and} \quad J = i/A.$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}.$$

$$\boxed{R = \rho \frac{L}{A}.}$$

Current is driven by a potential difference.

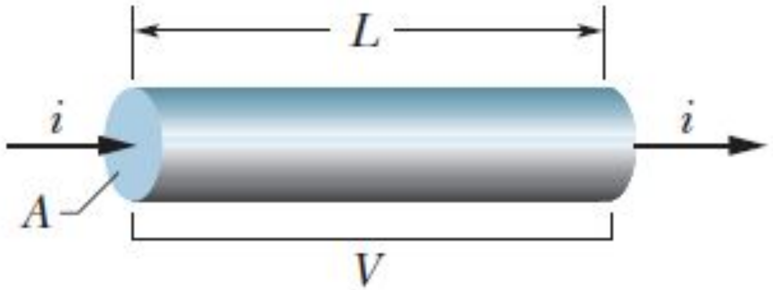


Fig. 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

If the streamlines representing the current density are uniform throughout the wire, the electric field, E, and the current density, J, will be constant for all points within the wire.

26.4: Resistance and Resistivity, Variation with Temperature:

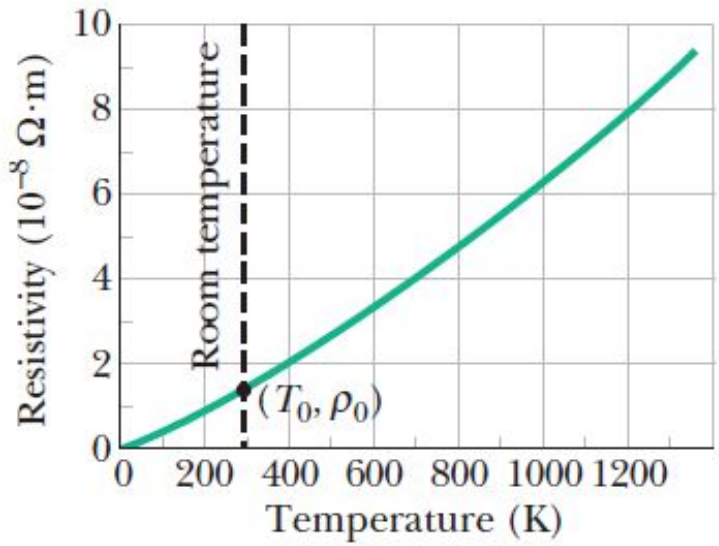


Fig. 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.

Resistivity can depend on temperature.

The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range.

For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\boxed{\rho - \rho_0 = \rho_0 \alpha (T - T_0).}$$

Example, A material has resistivity, a block of the material has a resistance.:

A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm × 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm × 15 cm)?

KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A, according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15$ cm $= 0.15$ m and

$$A = (1.2\ \text{cm})^2 = 1.44 \times 10^{-4}\ \text{m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(0.15\ \text{m})}{1.44 \times 10^{-4}\ \text{m}^2} 3$$

$$= 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega. \qquad \text{(Answer)}$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2\ \text{cm})(15\ \text{cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(1.2 \times 10^{-2}\ \text{m})}{1.80 \times 10^{-3}\ \text{m}^2}$$

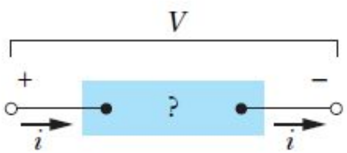
$$= 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega. \qquad \text{(Answer)}$$

26.5: Ohm's Law:

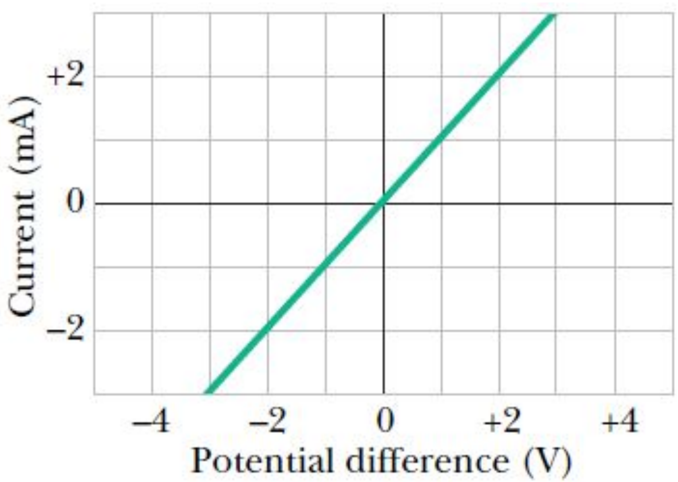
Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.



(*a*)



(*b*)

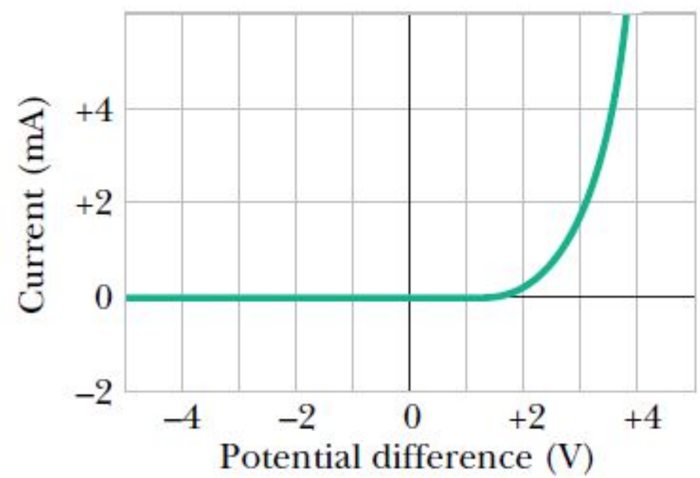


Fig. 26-11 (*a*) A potential difference V is applied to the terminals of a device, establishing a current i. (*b*) A plot of current i versus applied potential difference V when the device is a 1000 Ω resistor. (*c*) A plot when the device is a semiconducting *pn* junction diode.

P-1 : During the 4min a 5A current is set up in a wire how many (a) Coulumbs and (b) electrons pass through any cross-section across the wire's width?

$t = 4\text{min} = 4\times60 = 240\,\text{sec}$

current setup in the wire $= i = 5A$

(a) $q = i\times t$

As $i = \frac{q}{t}$; So,

$$\boxed{q = 5\times240 = 1.2\times10^{3}\,C}$$

(b) $q = ne \Rightarrow n = \frac{q}{e} = \frac{1.2\times10^{3}}{1.6\times10^{-19}}$

$$\boxed{n = 7.5\times10^{21}}$$

P-8 : A small but measureable current of $1.2\times10^{-10}A$ exists in a copper wire whose diammeter is 2.5mm. The Number of charge carriers per unit volume is $8.49\times10^{28}m^{-3}$ Assuming the current is uniform, calculate the (a) current density and (b) electron drift speed.

$i = 1.2\times10^{-10}A$ (Copper wire)

wire's diameter $= 2.5mm$

$q/\text{Vol.} = 8.49\times10^{28}/m^{3} = n$

Assumption current is uniform

(a) $\vec{J} = \frac{i}{A}$; $A = \pi r^{2} = \pi\left(\frac{d}{2}\right)^{2} = \frac{\pi d^{2}}{4}$

$A = \frac{3.14\times(2.5\times10^{-3})^{2}}{4}$

$A = 4.9\times10^{-6}$

$= \frac{1.2\times10^{-10}}{5\times10^{-6}} \Rightarrow$

$$\boxed{J = 2.4\times10^{-5}\,A/m^{2}}$$

(b) $\vec{J} = nev_d \Rightarrow V_d = \frac{J}{ne} = \frac{2.4\times10^{-5}}{8.4\times10^{28}\times1.6\times10^{-19}} =$

$$\boxed{V_d = 1.8\times10^{-15}\,m/s}$$

P-13: How long does it take electrons to get from a car battery to the starting Motor? Assume the current is 300A and the electrons travel through a copper wire with cross-sectional area $0.21 cm^2$ and length 0.85m. The number of charge per carriers per unit volume is $8.49 \times 10^{28} m^{-3}$.

$t = ?$

$i = 300 A$

$A = 0.21 cm^2$ (copper wire's cross-sectional Area)

$n = 8.49 \times 10^{28} /m^3$

length = 0.85m

$t = \frac{d}{v}$ where d is length of the wire so,

$$t = \frac{L}{V_d} = \frac{L}{i/Ane} = \frac{LAne}{i} \quad \text{— (1)}$$

As from the formula's $\Rightarrow \vec{J} = ne V_d$

$J = \frac{i}{A}$ So above become

$$\frac{i}{A} = ne V_d$$

$$V_d = \frac{i}{neA}$$

Now using eq(1)

$$t = \frac{LAne}{i} = \frac{0.85 \times 0.21 \times 10^{-4} \times 10^{-2} \times 1.6 \times 10^{-19} \times 8.49 \times 10^{28}}{300}$$

$t = 808 sec$

$\boxed{t = 13.47 \text{ min}}$

P-17: A wire of Nichrom (a nickel-chromium-iron alloy commonly used in heating elements) is 1m long and 1mm² in cross-sectional area. It carries a current of 4A when a 2V potential difference is applied b/w its ends. Calculate the conductivity σ of Nichrome.

length = 1m of Nichrome

Area = A = 1mm²

$i = 4A$ and V (potential) = 2 Volts

$\sigma = ?$ of Nichrome (conductivity)

$\sigma = \frac{1}{\rho}$ $R = \rho L/A$

$\sigma = \frac{1}{RA/L}$ $RA = \rho L$

$\rho = \frac{RA}{L}$

$\sigma = \frac{L}{RA}$ as $R = \frac{V}{i}$ (ohm's law)

$\sigma = \frac{L}{V/i \cdot A} = \frac{Li}{VA} = \frac{1 \times 4}{2 \times 1 \times (10^{-3})^2}$

$\boxed{\sigma = 2 \times 10^6 / \Omega . m}$

P-18: A wire 4m long and 6mm in diammeter has (4)
a resistance of 15.0 mΩ. A potential difference of 23V is applied b/w the ends. (a) What is the current in the wire? (b) What is the Magnitude of current density? (c) Calculate the resistivity of the wire material. (d) identify the Material using 26-1, identify the Material.

length of the wire = 4m

diameter = 6mm.

Resistance R = 15 mΩ

V = 23 Volts (potential difference)

(a) $i = ?$ (in the wire)

(b) $J = ?$

(c) $\rho = ?$ Resistivity of the Material

(d) identify Material Table 26-1

(a) $\Rightarrow i = \frac{V}{R} = \frac{23}{15\times10^{-3}} = 1.53\times10^{3}A$

(b) $J = \frac{i}{A} = \frac{i}{\pi r^2} = \frac{i}{\pi d^2/4} = \frac{4i}{\pi d^2}$

$= \frac{4\times1.53\times10^{-3}}{3.14\times(6\times10^{-3})^2} = 5.41\times10^{7} A/m^2$

(c) $\rho = \frac{RA}{L} = \frac{15\times10^{-3}\times\pi d^2/4}{4m}$

$\rho = \frac{15\times10^{-3}\times3.14\times(6\times10^{-3})^2}{4\times4}$

$\rho = 10.6\times10^{-8}\,\Omega m$

(d) using table Platinium.

P-29: A potential difference of 3nV is setup across a 2cm length of copper wire that has a radius of 2.00mm. How much charge drifts through a cross section in 3.00ms?

P.D = 3nV

length of the copper wire = 2 cm

q = ? (charge drift) = ?

t = 3ms

using $q = it$

$i = ? \Rightarrow i = \frac{V}{R}$

$R = \frac{\rho L}{A} = \frac{1.69\times10^{-8}\times0.02}{\pi(r^2)}$ $r = 2\times10^{-3}$ m

$R = 2.7\times10^{-5}\,\Omega$

$i = \frac{3\times10^{-9}}{2.7\times10^{-5}} = 1.115\times10^{-4}$ A

$q = 1.115\times10^{-4}\times3\times10^{-3} = 3.35\times10^{-7}$ C

P-23: When 115V is applied across a wire that is 10m long and has a 0.30mm radius, the Magnitude of the current density is 1.4×10^{8} A/m². Find the 'ρ' of the wire?

The area is

$A = \pi r^2 = (3.1416)\times(0.30\times10^{-3})^2 = 2.83\times10^{-7}\,m^2$

$i = JA = (1.4\times10^{8})\times(2.83\times10^{-7})$

$i = 39.584$A

The Resistance

$R = V/i = 115/39.58 = 2.9052$ ohm

$\cancel{R = \rho/LA}$ $R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L}$

$\rho = \frac{2.9052\times2.83\times10^{-7}}{10} = 8.2\times10^{-8}$ ohm.m

P.31:
An electrical cable consist of 125 strands of fine wire, each having $2.65\,\mu\Omega$ resistance. The same potential difference is applied b/w the ends of all the strands and result in a total current 0.750A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is resistance of the cable?

125 strands of electric cable, each wire is having

$R = 2.65\,\mu\Omega$

Total current = 0.750A

(a) $i = ?$ current in each strand

(b) What is applied P.D = ?

(c) Resistance of the cable = $R = ?$

(a) $i = \frac{V}{R}$ ⇒ But V is not given

Thus $i = \frac{0.750}{125} = 6\,mA = \frac{\text{Total } i}{\text{No. of Strands}}$

(b) $V = iR = 6\times10^{-3} \times 2.65\times10^{-6} = 1.59\times10^{-8}\,V$

(c) $R = \frac{2.65\times10^{-6}}{125} = 2.12\times10^{-8}\,\Omega$

Chapter 26: Current and resistance

Exercise problems:

1, 2, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 23, 29, 31, 32

Sample problems:

26.02, 26.04