Waves ch #16

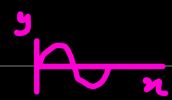
- $$\underbrace{y = h(x,t)}$$

transverse displacement of any oscillating element as a function h of the time t.

→ $$\underbrace{y(x,t)}_{\text{displacement}} = \underbrace{y_m}_{\text{Amplitude}} \overbrace{\sin\left(\overbrace{kx - wt}^{\text{phase}}\right)}^{\text{oscillating term}}$$

k = Angular wave number $(rad\ m^{-1})$

x = Horizontal Distance



- y_m always a +ve quantity

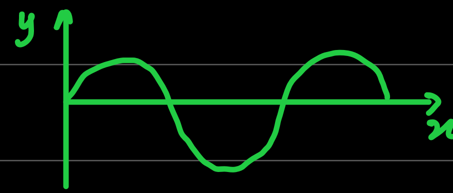
w = Angular Frequency $(rad\ s^{-1})$

t = time

- **Amplitude:** Magnitude of the maximum displacement of particles from equilibrium position
- **Phase:** Arguement of sine function $kx - wt$
- **Wavelength:** Distance b/w 2 cosecutive points on the wave which are in phase.

Derivation Using wavelength

- $y(x_1, t) = y_m \sin(kx_1 - \omega t)$ — 1



→ $t=0$ → $y(x,0) = y_m \sin(kx)$

Similary after 1 wavelength → $y(x+\lambda, 0) = y_m \sin(kx + k\lambda)$

$\therefore$ $\boxed{k\lambda = 2\pi \qquad k = \frac{2\pi}{\lambda}}$

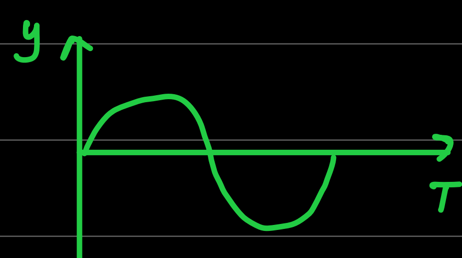
Derivation of Time period

- $y(0,t) = y_m \sin(-\omega t)$

$= -y_m \sin(\omega t)$

$= -y_m \sin(\omega t + \omega T)$

$\therefore \omega T = 2\pi$



- $y(x,t) = y_m \sin(kx - \omega t + \phi)$

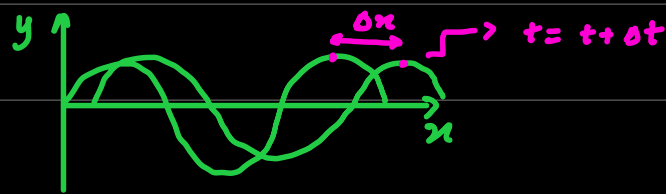
+ve ϕ shifts graph to left and -ve ϕ shifts graph to right

$$\boxed{T = \frac{2\pi}{\omega}}$$

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$

Speed of Wave

$$v = \frac{\Delta x}{\Delta t}$$



$kx - \omega t = \text{constant}$

$$\frac{d}{dt}(kx - \omega t)$$

$$k\frac{dx}{dt} - \omega\frac{dt}{dt} = 0$$

$vk - \omega = 0$

$$\boxed{v = f\lambda}$$

$$v = \frac{\omega}{k} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = \frac{\lambda}{\frac{1}{f}} = f\lambda$$

Q- When wave travels in opposite direction

$kx + wt =$ constant , b/c x decreases with time

→ follow through from above

× ———————— ×

- Similarly, $y(x,t) = y_m \sin(kx \pm wt)$

- $y = y_m \sin(kx - wt)$; x constant

$$\frac{\partial y}{\partial t} = -w y_m \cos(kx - wt)$$

$$\boxed{v = -w y_m \cos(kx - wt)}$$

$$\frac{\partial v}{\partial t} = -w^2 y_m \sin(kx - wt)$$

$$\boxed{a = -w^2 y_m \sin(kx - wt)}$$

- Exercise: 3, 4, 5, 7, 9, 10