Chapter : 15

Mass-spring :-

$f \propto -x$

$f = -kx$

$\because f = ma$

$ma = -kx$

$a = \frac{-k}{m} x$

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$a \propto -m$

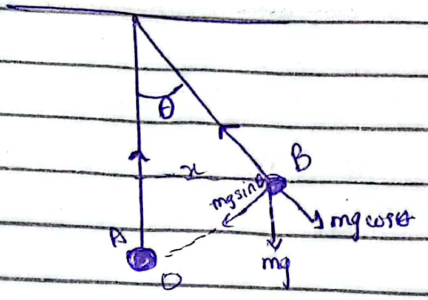
$a = -\omega^2 k m$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

$T = 2\pi \sqrt{\frac{m}{k}}$

Simple Pendulum :-



$\tau = I\alpha \rightarrow (1)$

$\tau = f \times L$

$\tau = -mg \sin\theta L \rightarrow (2)$

Compare (1) and (2)

$-mg \sin\theta l = I\alpha$

for smaller angle

$\sin\theta \approx \theta$

$mgl\theta = I\alpha$

$a = -\frac{mgL}{I}\theta$

$\downarrow \quad \downarrow \quad \swarrow$

$\therefore a = -\omega^2 x$

Alternative :

$f = -mg \sin\theta$

$\sin\theta \approx \theta$

for smaller angle,

$f = -mg\theta$

$\because f = ma$

$a = -g\theta$

$\theta = \frac{\text{Arc length}}{l}$

$\therefore \omega^2 = \frac{mgL}{I}$

$\omega = \sqrt{\frac{mgL}{I}} \rightarrow (3)$

We know that

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$.

putting in (3)

$T = 2\pi \sqrt{\frac{I}{mgL}}$

for rod.

$I = mL^2$.

$T = 2\pi \sqrt{\frac{mL^2}{mgL}}$

$$\boxed{T = 2\pi \sqrt{\frac{L}{g}}}$$

$\Rightarrow AB = OB = x$.

$\theta = \frac{x}{l}$.

$a = -g\, x/l$.

$k = \frac{g}{l}$ ($a \rightarrow$ constant)

compare $a = -\omega^2 x$ to $a = -g\frac{x}{l}$.

we have.

$\omega = \sqrt{\frac{g}{l}}$

we know

$T = \frac{2\pi}{\omega}$.

$$\boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

Torsional Pendulum:

Torque is directly proportional to angle.

$\tau \propto -\theta$

$\tau = -k\theta$ (torsional constant)

$I\alpha = -k\theta$

$I(-\omega^2\theta) = -k\theta$

$I\omega^2 = k$

$\omega^2 = \frac{k}{I}$

$\omega = \sqrt{\frac{k}{I}}$ $\quad \therefore \omega = \frac{2\pi}{T}$

$\frac{2\pi}{T} = \sqrt{\frac{k}{I}}$

$$\boxed{T = 2\pi\sqrt{\frac{I}{k}}}$$

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formulas.

$\rightarrow v = r\omega$

$\rightarrow a = r\alpha$

$\rightarrow a_c = \frac{v^2}{r}$

$\rightarrow \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$

$\omega = 2\pi \cdot \frac{1}{T}$

$\rightarrow \omega = 2\pi f$

$x = x_0 \sin\omega t$

$x = x_m \cos(\omega t + \phi)$

$\omega = \sqrt{\frac{k}{m}}$

$\rightarrow v = \omega x_m$

$\rightarrow a = \omega^2 x_m$

$\rightarrow f = m\omega^2 x_m$

$\rightarrow \omega = 2\pi f$