Q#1 (a).

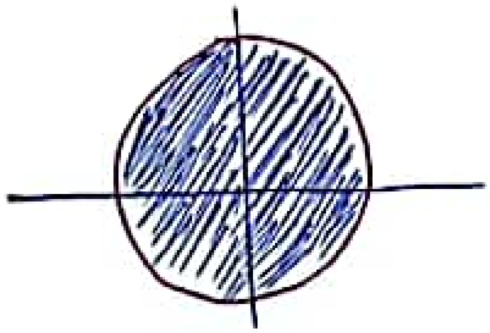
Sol:

(i) Domain $D = \{(x, y) : x^2 + y^2 < 16\} \longrightarrow ②$

(ii) Range $[\frac{1}{4}, \infty) \longrightarrow ①$

(iii) The domain is open because every pt. in $D$ is an interior point. $\longrightarrow ②$

(iv). $D$ is bounded because it lies inside a disk of finite radius. $\longrightarrow ①$



Q#1 (b).

Sol:

$\lim_{(x,y)\to(0,0)} f(x, y)$ by direct substitution gives the indeterminate form $0/0$.

Along the line $y = Kx$, $K \neq 1$ the function has value

$$f(x,y)\Big|_{y=Kx} = \frac{x^2 - Kx}{x - Kx} = \frac{x - K}{1 - K} \longrightarrow ②$$

$$\lim_{(x,y)\to(0,0)} f(x,y)\Big|_{y=Kx} = \lim_{(x,y)\to(0,0)} \frac{x - K}{1 - K} = \frac{-K}{1-K}$$

This limit varies with different values of $K$. $\longrightarrow ②$

e.g. for $K=0$, the limit is $0$, for $K=-1$, the limit is $1/2$.

Hence, $f(x, y)$ has no limit as $(x, y) \to (0, 0)$.

Q#2. Sol:

By the chain rule, we have

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u} \longrightarrow (a)$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} \longrightarrow (b)$$

So, the calculations we require are

$$\frac{\partial w}{\partial x} = \frac{2x}{x^2+y^2+z^2}, \quad \frac{\partial w}{\partial y} = \frac{2y}{x^2+y^2+z^2}, \quad \frac{\partial w}{\partial z} = \frac{\partial z}{x^2+y^2+z^2},$$

→ 2

$$\frac{\partial x}{\partial u} = e^v \sin u + u e^v \cos u, \quad \frac{\partial y}{\partial u} = e^v \cos u - u e^v \sin u, \quad \frac{\partial z}{\partial u} = e^v,$$

→ 2

$$\frac{\partial x}{\partial v} = u e^v \sin u, \quad \frac{\partial y}{\partial v} = u e^v \cos u, \quad \frac{\partial z}{\partial v} = u e^v$$

→ 2

By ~~subs~~ using actual values of x and y and putting these partial derivatives in, eq (a) and eq (b), becomes

$$\frac{\partial w}{\partial u} = \frac{2}{u}, \quad \frac{\partial w}{\partial v} = 2$$

→ 2 → 2

Q#3. Sol:

Given a temperature $T(x,y,z) = 2xy - yz$, $P(1,-1,1)$

$$\frac{\partial T}{\partial x} = 2y = 2(-1) = -2$$

$$\frac{\partial T}{\partial y} = 2x - z = 1, \quad \frac{\partial T}{\partial z} = -y = 1$$

and $\vec{\nabla} T = -2\hat{i} + 1\hat{j} + 1\hat{k}$ → 5 and $|T| = \sqrt{2^2+1^2+1^2} = \sqrt{6} = 2.44$ → 2

The maximum derivative is 2.44. So, No possible rate of temperature change is greater than this value 2.44, ∴ 3 is refused

So, there is No direction u in which the rate of change of temperature at P(1,-1,1) is $-3°C/ft$.

→ 3

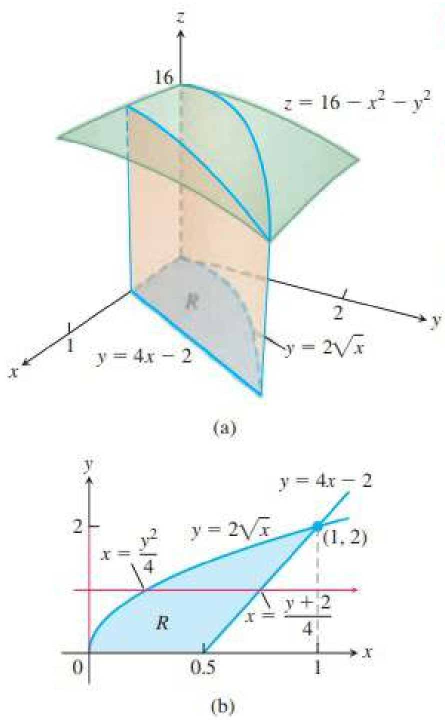



**FIGURE 15.18** (a) The solid "wedgelike" region whose volume is found in Example 4. (b) The region of integration $R$ showing the order $dx\,dy$.

**EXAMPLE 4** Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region $R$ bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the $x$-axis.

**Solution** Figure 15.18a shows the surface and the "wedgelike" solid whose volume we want to calculate. Figure 15.18b shows the region of integration in the $xy$-plane. If we integrate in the order $dy\,dx$ (first with respect to $y$ and then with respect to $x$), two integrations will be required because $y$ varies from $y = 0$ to $y = 2\sqrt{x}$ for $0 \le x \le 0.5$, and then varies from $y = 4x - 2$ to $y = 2\sqrt{x}$ for $0.5 \le x \le 1$. So we choose to integrate in the order $dx\,dy$, which requires only one double integral whose limits of integration are indicated in Figure 15.18b. The volume is then calculated as the iterated integral:

$$\iint_R (16 - x^2 - y^2)\, dA$$

$$= \int_0^2 \int_{y^2/4}^{(y+2)/4} (16 - x^2 - y^2)\, dx\, dy \longrightarrow 5$$

$$= \int_0^2 \left[16x - \frac{x^3}{3} - xy^2\right]_{x=y^2/4}^{x=(y+2)/4} dx \longrightarrow 2$$

$$= \int_0^2 \left[4(y+2) - \frac{(y+2)^3}{3\cdot 64} - \frac{(y+2)y^2}{4} - 4y^2 + \frac{y^6}{3\cdot 64} + \frac{y^4}{4}\right] dy \longrightarrow 3$$

$$= \left[\frac{191y}{24} + \frac{63y^2}{32} - \frac{145y^3}{96} - \frac{49y^4}{768} + \frac{y^5}{20} + \frac{y^7}{1344}\right]_0^2 = \frac{20803}{1680} \approx 12.4. \longrightarrow 5 \quad \blacksquare$$