

Ex-7.2

(i) $\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$

(ii) $\mathcal{L}^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n; n=1,2,3,\dots$

(iii) $\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

(iv) $\mathcal{L}^{-1}\left(\frac{k}{s+k^2}\right) = \sin kt$

(v) $\mathcal{L}^{-1}\left(\frac{s}{s+k^2}\right) = \cos kt$

(vi) $\mathcal{L}^{-1}\left(\frac{k}{s-k^2}\right) = \sinh kt$

(vii) $\mathcal{L}^{-1}\left(\frac{s}{s-k^2}\right) = \cosh kt$

Ex-1(a)

$$\mathcal{L}^{-1}\left(\frac{1}{s^5}\right) = ?$$

Sol:-

$$\mathcal{L}^{-1}\left(\frac{1}{s^5}\right) = \frac{1}{4!} \mathcal{L}^{-1}\left(\frac{4!}{s^5}\right) = \frac{1}{24} t^4 \quad \text{Ans}$$

Ex-1(b) $\mathcal{L}^{-1}\left(\frac{1}{s^2+7}\right) = ?$

Sol:- $\mathcal{L}^{-1}\left(\frac{1}{s^2+7}\right) = \frac{1}{\sqrt{7}} \mathcal{L}^{-1}\left(\frac{\sqrt{7}}{s^2+(\sqrt{7})^2}\right) = \frac{1}{\sqrt{7}} \sin \sqrt{7}t. \quad \text{Ans}$

Ex-2 $\mathcal{L}^{-1}\left(\frac{-2s+6}{s^2+4}\right) = ?$

Sol:- $\mathcal{L}^{-1}\left(\frac{-2s+6}{s^2+4}\right) = \mathcal{L}^{-1}\left(\frac{-2s}{s^2+4} + \frac{6}{s^2+4}\right)$

$$= -2 \mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) + \frac{6}{2} \mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right)$$

$$= -2 \cos 2t + 3 \sin 2t \quad \text{Ans}$$

Ex. 3

$$\mathcal{L}^{-1}\left(\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right) = ?$$

$$\text{Sol: } \frac{s^2+6s+9}{(s-1)(s-2)(s+4)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s+4}$$

$$\frac{s^2+6s+9}{(s-1)(s-2)(s+4)} = \frac{-16/s}{s-1} + \frac{25/6}{s-2} + \frac{1/30}{s+4}$$

$$\mathcal{L}^{-1}\left(\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right) = -\frac{16}{s} \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + \frac{25}{6} \mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + \frac{1}{30} \mathcal{L}^{-1}\left(\frac{1}{s+4}\right)$$

$$= -\frac{16}{5} e^t + \frac{25}{6} e^{2t} + \frac{1}{30} e^{-4t} \quad \text{Ans}$$

Ex. 4

$$\frac{dy}{dt} + 3y = 13 \sin 2t ; \quad y(0) = 6$$

$$\mathcal{L}\left(\frac{dy}{dt}\right) + 3\mathcal{L}(y) = 13\mathcal{L}(\sin 2t)$$

$$sY(s) - 6 + 3Y(s) = 13\left(\frac{2}{s^2+4}\right)$$

$$(s+3)Y(s) = 6 + 26$$

$$Y(s) = \frac{6}{s+3} + \frac{26}{(s+3)(s^2+4)} = \frac{6s^2+50}{(s+3)(s^2+4)} \quad (*)$$

$$\frac{6s^2+50}{(s+3)(s^2+4)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4}$$

$$= \frac{8}{s+3} + \frac{-2s+6}{s^2+4}$$

From (*)

$$Y(s) = \cancel{\frac{8}{s+3}} + \frac{8}{s+3} + \frac{-2s+6}{s^2+4}$$

$$y(t) = 8\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) + 8\mathcal{L}^{-1}\left(\frac{-s}{s^2+4}\right) + 3\mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right)$$

$$y(t) = 8e^{-3t} - 2\cos t + 2\sin 2t \quad \text{Ans}$$

Ex. 5 $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^{-4t}; \quad y(0) = 1; \quad y'(0) = 5$

$$\mathcal{L}\left(\frac{d^2y}{dx^2}\right) - 3\mathcal{L}\left(\frac{dy}{dx}\right) + 2\mathcal{L}(y) = \mathcal{L}(e^{-4t})$$

$$[s^2 Y(s) - s y(0) - y'(0)] - 3[s Y(s) - y(0)] + 2Y(s) = \frac{1}{s+4}$$

$$(s^2 Y(s) - s - 5) - 3(s Y(s) - 1) + 2Y(s) = \frac{1}{s+4}$$

$$(s^2 - 3s + 2) Y(s) = s + 2 + \frac{1}{s+4}$$

$$Y(s) = \frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)} \quad (*)$$

$$Y(s) = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s+4}$$



$$y(t) = \mathcal{L}^{-1}(Y(s)) = -\frac{16}{5} e^t + \frac{25}{6} e^{2t} + \frac{1}{30} e^{-4t} \quad \text{Ans}$$