

### 3.1

Q1- (a)  $(4-1, 1-5) = (3, -4)$  ✓  
 (b)  $(0-2, 0-3, 4-0) = (-2, -3, 4)$  ✓

Q2 (a)  $(-5, 0)$  ✓  
 (b)  $(-3, 4, 0)$  ✓

Q3 (a)  $\overrightarrow{P_1 P_2} = (-1, 3)$  ✓  
 (b)  $= (-3, 6, 1)$  ✓

Q4 (a)  $\overrightarrow{P_1 P_2} = (2, -3)$  ✓  
 (b)  $= (-1, 6, 1)$  ✓

\* Q5 (a) A (1, 1), B (n<sub>1</sub>, n<sub>2</sub>)

$(n_1 - 1, n_2 - 1) = (1, 2)$

$n_1 = 1+1=2, n_2 = 2+1=3$

T.P = (2, 3) ✓

(b) B (-1, -1, 2),  $\vec{u} = (1, 1, 3)$

A = (n, y, z)

$(-1-n, -1-y, 2-z) = (1, 1, 3)$

$-1-n=1, -1-y=1, 2-z=3$   
 $n=-2, y=-2, z=-1$

I.P = (-2, -2, -1) ✓

Q6 (a)  $(2-n, -y) = (1, 2)$   
 $2-n=1, -y=2$   
 $n=1, y=-2$

T.P = (1, -2) ✓

(b)  $(n, y-2, z) = (1, 1, 3)$

$n=1, y-2=1, z=3$   
 $y=3$

T.P = (1, 3, 3) ✓

\* Q7- (a)  $\vec{OA} - \vec{OP} = (4, -2, -1)$   
 $(3, 0, -5) - (n, y, z) = (4, -2, -1)$

$3-n=4, -y=-2, -z-5=-1$   
 $n=-1, y=2, z=-4$

$\vec{OP} = (-1, 2, -4)$

(b)  $\vec{OA} - \vec{OP} = -(4, -2, -1)$

$(3, 0, -5) - (n, y, z) = (-4, 2, 1)$

$3-n=-4, -y=2, -z-5=1$   
 $n=7, y=-2, z=-6$

$\vec{OP} = (7, -2, -6)$

Q8 (a)  $\vec{OA} - \vec{OP} = (6, 7, -3)$   
 $\vec{OA} = (6, 7, -3) + (-1, 3, -5)$   
 $= (5, 10, -8)$  ✓

(b)  $\vec{OQ} = (-6, -7, 3) + (-1, 3, -5)$   
 $= (-7, -4, -2)$  ✓

Q9- (b)  $(0, 5) = 3(4, -1)$   
 $= (0, 5) - (12, -3)$   
 $= (-12, 8)$  ✓

c)  $2u - 10w$   
 $(8, -2) - (-30, -30)$   
 $= (38, 28)$  ✓

Q12 d)  $\frac{1}{2} [(7, 1, -4, -2, 3) - 5(0, 4, -1, 1, 2) + 2(1, 2, -3, 5, 0)] + (0, 4, -1, 1, 2)$   
 $= \frac{1}{2} [(7, 1, -4, -2, 3) - (0, 20, -5, 5, 10) + (2, 4, -6, 10, 0)] + (0, 4, -1, 1, 2)$   
 $= \frac{1}{2} (-9, -15, -5, 3, -7) + (0, 4, -1, 1, 2)$   
 $= \left(\frac{9}{2}, -\frac{15}{2}, -\frac{5}{2}, \frac{3}{2}, -\frac{7}{2}\right) + (0, 4, -1, 1, 2)$   
 $= \left(\frac{9}{2}, -\frac{7}{2}, -\frac{7}{2}, \frac{5}{2}, -\frac{3}{2}\right)$  ✓

Q14-  $u = (1, 2, -3, 5, 0)$ ,  $v = (0, 4, -1, 1, 2)$ ,  $w = (7, 1, -4, -2, 3)$

$2u - v + w = 7u + w$   
 $(2, 4, -6, 10, 0) + (0, -4, 1, -1, -2) + w = 7u + (7, 1, -4, -2, 3)$   
 $w + (2, 0, -5, 9, -2) = 7u + (7, 1, -4, -2, 3)$   
 $6w = (-5, -1, -1, 11, -5)$

$n = \left(-\frac{5}{6}, -\frac{1}{6}, \frac{1}{6}, \frac{11}{6}, -\frac{5}{6}\right)$  ✓

Q13-  $u = (-3, 2, 1, 0)$ ,  $v = (4, 7, -3, 2)$ ,  $w = (5, -2, 8, 1)$

$3u + v - 2w = 3n + 2w$

$3n = 3u + v - 4w$

$3n = (-9, 6, 3, 0) + (4, 7, -3, 2) - (20, -8, 32, 4)$

$3n = (-25, 21, -32, -2)$

$n = \left(-\frac{25}{3}, 7, -\frac{32}{3}, -\frac{2}{3}\right)$  ✓

Q15- (a) Not parallel as  $v = ku$  not satisfied

(b) Parallel ✓

(c) Parallel ✓  $u = kv \Rightarrow u = 0v = 0$

Q16-  $u = (4, -1)$

(a)  $u = kv$

$(4, -1) = k(8t, -2)$

$$\begin{aligned} 8t &= 4 & -2k &= 1 \\ t &= \frac{1}{2} & k &= -\frac{1}{2} \\ \therefore t &= 1 & & \end{aligned}$$

$v = ku$

$(8t, -2) = k(4, -1)$

$t = 1$  only

$$\begin{aligned} 8t &= 4 & -k &= -2 \\ t &= 1 & k &= 2 \\ \therefore t &= & & \end{aligned}$$

(b)  $u = (4, -1)$

$$(8t, 2t)$$

$$u = kv$$

$$(4, -1) = k(8t, 2t)$$

$$8t + k = 4, \quad 2kt = -1$$

$$8t + \left(-\frac{1}{2}k\right) = 4 \quad k = -\frac{1}{2}t$$

$$-4 \neq 4$$

(c)  $(1, t^2)$

$$u = av$$

$$(4, -1) = a(1, t^2)$$

$$a = 4, \quad at^2 = -1$$

$$t^2 = -\frac{1}{4}, \quad \text{not possible}$$

$$v = bu$$

$$(1, t^2) = b(4, -1)$$

$$4b = 1, \quad t^2 = -b$$

$$b = \frac{1}{4}, \quad t^2 = -\frac{1}{4}; \quad \text{not possible}$$

$\rightarrow$  not parallel

Linear Combinations:  $w = k_1 v_1 + k_2 v_2 + \dots + k_r v_r$

$$v = bu$$

$$(8t, 2t) = b(4, -1)$$

$$8t = 4b, \quad 2t = -b$$

$$8t = 4(-2t) \quad b = -2t$$

$$8t = -8t$$

$$16t = 0$$

$$t = 0$$

parallel if  $t = 0$

$=$

Q17-  $u = (1, -1, 3, 5), \quad v = (2, 1, 0, -3) \rightarrow$  use Gauss-Jordan elimination method

$$au + bv = (1, -4, 9, 18)$$

$$a(1, -1, 3, 5) + b(2, 1, 0, -3) = (1, -4, 9, 18) \quad \text{for show } a's \\ 21-22$$

$$(a+2b, b-a, 3a, 5a-3b) = (1, -4, 9, 18)$$

$$3a = 9 \quad b-a = -4$$

$$a=3 \quad b-3=-4$$

$$\underline{= \quad b=-1}$$

$$\underline{= \quad b=1}$$

\* Q23-

(a) mid-point =  $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$

$$\vec{PQ} = \vec{OP} - \vec{OQ}$$

$$\frac{1}{2}\vec{PA} + \vec{OP} = \vec{OQ}$$

(b)  $\vec{PQ} = (5, -7, 3)$

$$\frac{3}{4}\vec{PA} = \left(\frac{15}{4}, -\frac{21}{4}, \frac{9}{4}\right)$$

$$\vec{OQ} = \vec{OP} + \frac{3}{4}\vec{PA}$$

$$= (2, 3, -2) + \left(\frac{15}{4}, -\frac{21}{4}, \frac{9}{4}\right)$$

$$= \left(\frac{23}{4}, -\frac{9}{4}, \frac{1}{4}\right) \checkmark$$

