Dated: Wednesday 22nd Nov.'23

LECTURE #

7.2 NATURAL LOGARITHMS -

→ The natural logarithm is the function given by,

$$\ln x = \int_1^x \frac{1}{t}\,dt \quad , \quad x > 0$$

→ The number 'e' is the number in the domain of the natural logarithm that satisfies

$$\ln(e) = \int_1^e \frac{1}{t}\,dt = 1$$

* $$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx} \quad , \quad u > 0$$

→ EXAMPLE #1

Find derivatives of:

(a) $\ln 2x$
(b) $\ln(x^2+3)$
(c) $\ln|x|$

Dated:

SOLUTION

(a) $\frac{d}{dx}\ln 2x = \frac{1}{2x}\frac{d}{dx}(2x) = \frac{1}{\cancel{2}x}\times\cancel{2} = \frac{1}{x}, \; x>0$

(b) $\frac{d}{dx}\ln(x^2+3) = \frac{1}{x^2+3}\cdot\frac{d}{dx}(x^2+3)$

$= \frac{1}{x^2+3}\times 2x$

$= \frac{2x}{x^2+3}$

(c) $\frac{d}{dx}\ln|x| = \frac{1}{|x|}\times\frac{d}{dx}(|x|)$

* $|x| = \sqrt{x^2}$

$\Rightarrow \frac{d}{dx}|x| = \frac{1}{\cancel{2}}(x^2)^{-\frac{1}{2}}\times\cancel{2}x$

$= \frac{x}{(x^2)^{1/2}}$

$= \frac{x}{|x|}$

$\Rightarrow \frac{d}{dx}\ln|x| = \frac{1}{|x|}\times\frac{x}{|x|}$

$= \frac{\cancel{x}}{x^{\cancel{2}}} = 1/x, \; x>0.$

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→ ALGEBRAIC PROPERTIES OF THE NATURAL LOGARITHM

For any numbers $b>0$ and $x>0$, the natural logarithm satisfies the following rules:

1) PRODUCT RULE:

$\ln bx = \ln b + \ln x$

2) QUOTIENT RULE–

$\ln b/x = \ln b - \ln x$

3) RECIPROCAL RULE–

$\ln 1/x = -\ln x$

4) POWER RULE:

$\ln x^r = r \ln x$

→ EXAMPLE#2

(a) $\ln 4 + \ln \sin x = \ln(4 \sin x)$ (PRODUCT RULE)

(b) $\ln \dfrac{x+1}{2x-3} = \ln(x+1) - \ln(2x-3)$ (QUOTIENT RULE)

(c) $\ln 1/8 = -\ln 8$ (RECIPROCAL RULE)

(d) $= -\ln 2^3$

$= -3\ln 2$ (POWER RULE)

ted:

→ THE GRAPH AND RANGE OF $\ln x$:

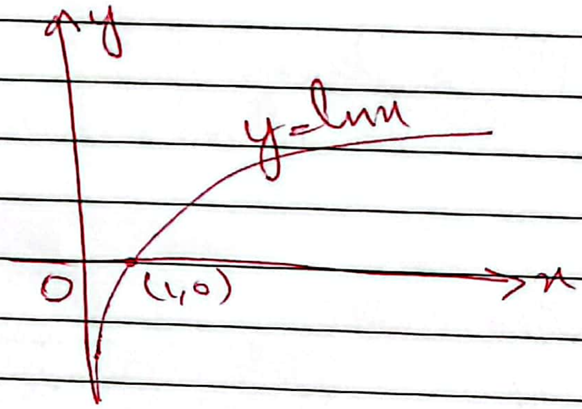
The derivative $\frac{d}{dx}(\ln x)$ is positive for $x>0$

⟹ $\ln x$ is an increasing function of 'x'.

$$\frac{d}{dx}(\ln x) = 1/x$$

$$\frac{d^2}{dx^2}(\ln x) = -1/x^2 \text{ is negative}$$

⟹ The graph of $\ln x$ is CONCAVE DOWN



→ The domain of $\ln x$ is the set of positive real numbers. The range is the entire real line.

Dated:

→ If 'u' is a differentiable function that is never zero, then

$$\int \frac{1}{u}\,du = \ln|u| + c$$

→ EXAMPLE#3

Evaluate $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta}\,d\theta$

SOLUTION

Let, $u = 3+2\sin\theta$

$\Rightarrow du = 2\cos\theta\,d\theta$

→ When $\theta = -\pi/2$, $u = 3+2\sin(-\pi/2) = 1$

→ When $\theta = \pi/2$, $u = 3+2\sin(\pi/2) = 5$

$$\Rightarrow \int_1^5 \frac{2}{u}\,du$$

$$= 2\ln|u|\Big]_1^5$$

$$= 2\ln|5| - 2\ln|1|$$

$$= 2\ln|5|$$

ted:

→ INTEGRALS (TRIGNOMETRIC FUNCTIONS) –

* $\int \tan u\,du = \ln|\sec u| + c$
* $\int \sec u\,du = \ln|\sec u + \tan u| + c$
* $\int \cot u\,du = \ln|\sin u| + c$
* $\int \csc u\,du = -\ln|\csc u + \cot u| + c$

→ EXAMPLE # 4

Evaluate $\int_0^{\pi/6} \tan 2x\,dx$

SOLUTION

Let $u = 2x$, $u(0) = 2(0) = 0$

$\Rightarrow du = 2dx$, $u(\pi/6) = 2\pi/6 = \pi/3$

$\Rightarrow dx = \frac{du}{2}$

$$\int_0^{\pi/3} \tan u \frac{du}{2}$$

$$= \frac{1}{2}\int_0^{\pi/3} \tan u\,du$$

$$= \frac{1}{2} \ln|\sec u| \Big]_0^{\pi/3}$$

$$= \frac{1}{2}(\ln 2 - \ln 1) = \frac{1}{2}\ln 2.$$

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→ EXAMPLE #5

find dy/dx if $y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}$, $x>1$.

SOLUTION

$$\ln y = \ln \frac{(x^2+1)(x+3)^{1/2}}{x-1}$$

$$= \ln\left((x^2+1)(x+3)^{1/2}\right) - \ln(x-1)$$

(QUOTIENT RULE)

$$= \ln(x^2+1) + \ln(x+3)^{1/2} - \ln(x-1)$$

(PRODUCT RULE)

$$\ln y = \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \ln(x-1)$$

(POWER RULE)

* Taking derivatives of both sides w.r.t 'x',

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2+1} \times 2x + \frac{1}{2} \times \frac{1}{x+3} - \frac{1}{x-1}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{1}{x-1}\right)$$

$$\rightarrow \frac{dy}{dx} = \frac{(x^2+1)(x+3)^{1/2}}{x-1}\left(\frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{1}{x-1}\right)$$