Functions

(1.1)

- Relation b/w 2 variables.
- can be represented through ① Equation, ② Graph, ③ set / numerically
④ verbally like Area of circle depends on radius

Domain: set of all possible input values

Range: // // // output //

Note: $(0,\infty)$ → All positive real numbers

$[0,\infty)$ → all non-negative real numbers

Q- $y = x^2$; $x > 0$

Domain: All positive real numbers / $(0,\infty)$

Range: $(0,\infty)$ / All positive real numbers

Domain & Ranges

- **Natural Domain** → no restriction on input values : $y = x^2$, $x \in R$
- **Restrictive Domain** → restriction on input values : $y = \frac{1}{x}$; $x \neq 0$

Q- $y = x^2$; $x \geq 2$

$D_f = [2, \infty)$ // can also describe in words

$R_f = [4, \infty)$

Q- $g(x) = \sqrt{x^2 - 3x}$

$g(x) = \sqrt{x(x-3)}$

$D = (-\infty, 0] \cup [3, \infty)$

Natural Domains & Ranges

$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$

Q- $\frac{4}{3-t}$

$D = (-\infty, 3) \cup (3, \infty)$ // $(-\infty, \infty) - [3]$

$R = (-\infty, \infty) - [0]$

Q- $f(t) = \frac{t}{|t|}$

→ $\frac{t}{t}$, $\frac{t}{-t}$

$1, -1$

Note: Graphs of ordinary functions must be known; x^2, x^3, $\sqrt{x}$, trig, logs

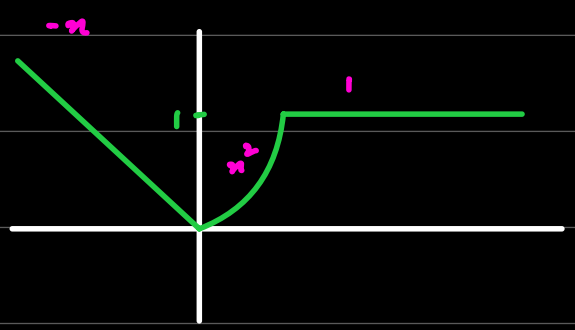
→ $y = x^4, x^5$ (more flat)

like x^2 x^3

Piecewise Defined Functions

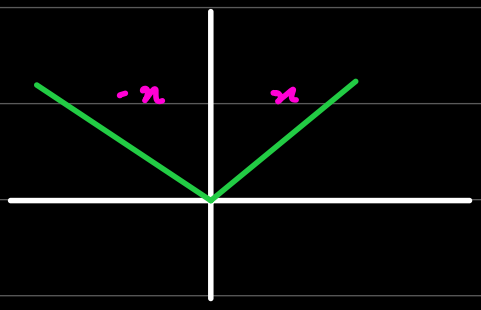
· Function which is defined in pieces by using different formulaes on different parts of its domain.

Q- $f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$

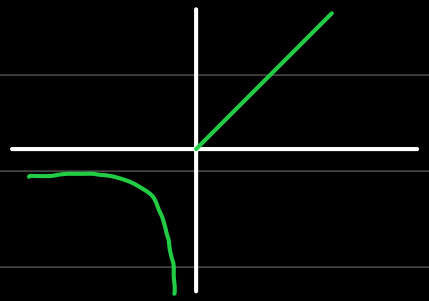


* Use vertical line test to check if function exists.

Q- $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$



Q- $g(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & x \geq 0 \end{cases}$



Types of Functions

- **Floor Functions:** denoted by $\lfloor x \rfloor$ takes a real number and rounds it down to the nearest integer that is less than or equal to the original number. e.g $\lfloor 3.7 \rfloor = 3$, $\lfloor -0.1 \rfloor = -1$, $\lfloor 3 \rfloor = 3$

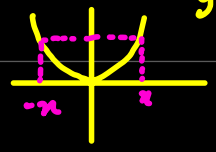
- **Ceiling Functions:** rounds the numbers upto the nearest integer that is greater than or equal to the original number. e.g: $\lceil 3.2 \rceil = 4$, $\lceil -2.1 \rceil = -2$

- **Increasing Functions:** $f(x_2) > f(x_1)$, whenever $x_1 < x_2$

- **Decreasing Function:** $f(x_2) < f(x_1)$, whenever $x_1 < x_2$

Even Functions

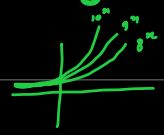
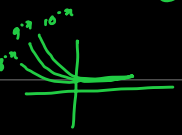
- $f(-x) = f(x)$
- e.g: $y = x^2, x^4$, $\cos x$, $\sec x$
- Graph is symmetric about y-axis
- $(x,y) \rightarrow (-x,y)$

→ 

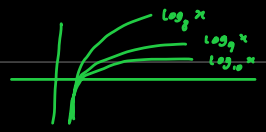
Odd Functions

- $f(-x) = -f(x)$
- e.g: $y = x, x^3$, $\sin x$, $\operatorname{cosec} x$, $\tan x$, $\cot x$
- graph is symmetric about origin (rotation of 180°)
- $(x,y) \rightarrow (-x,-y)$

* see zainematics graphs notes

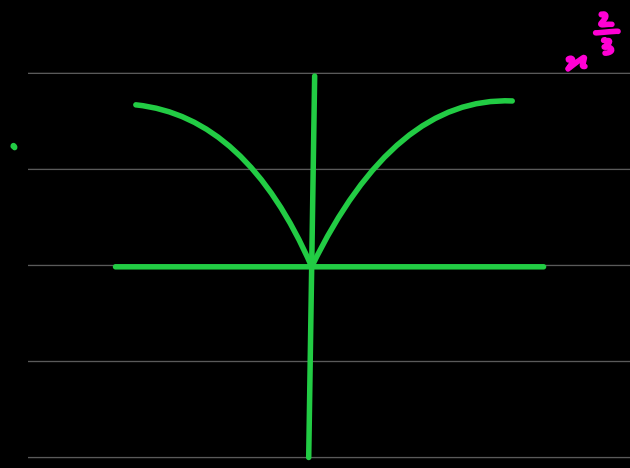
- Linear Functions: $f(x) = ax + b$
- Identity Functions: $y = x$, $y = c$
- Power Functions: $f(x) = x^a$; a = +ve / -ve integer or fractions $\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{2}{3}$
- Rational Functions: $f(x) = \frac{P(x)}{Q(x)}$; $Q(x) \neq 0$
- Algebraic Functions: function formed by adding, subtracting and dividing 2 polynomials
- Exponential Functions: a^x graphs  , 

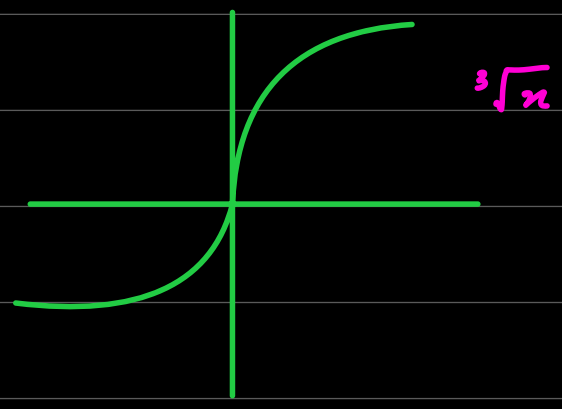
- **Logarithmic Function:**

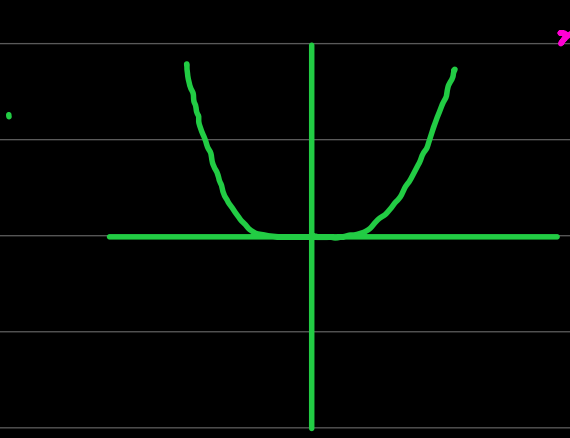


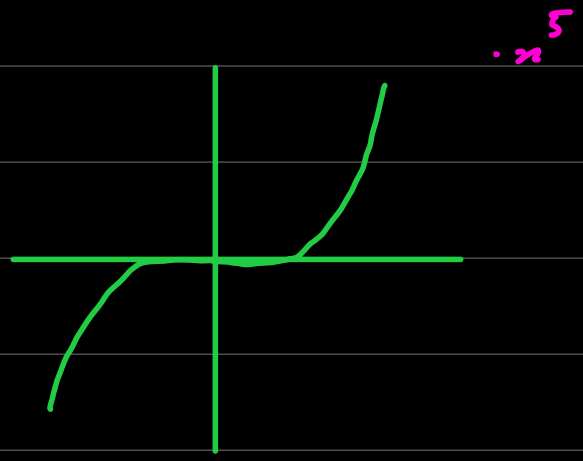
- **Trignometric functions:** $\sin x$, $\cos x$, $\tan x$, $\operatorname{cosec} x$, $\sec x$, $\cot x$

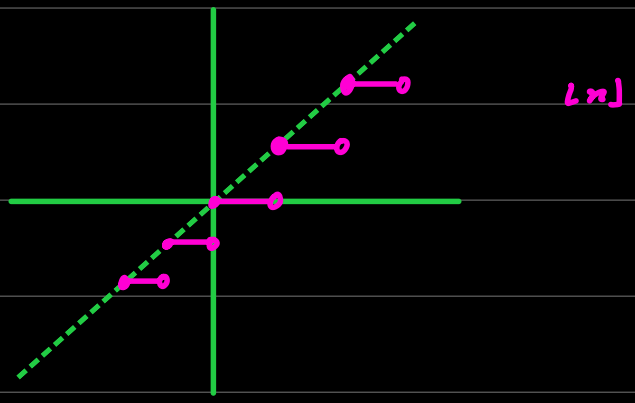
Graphs

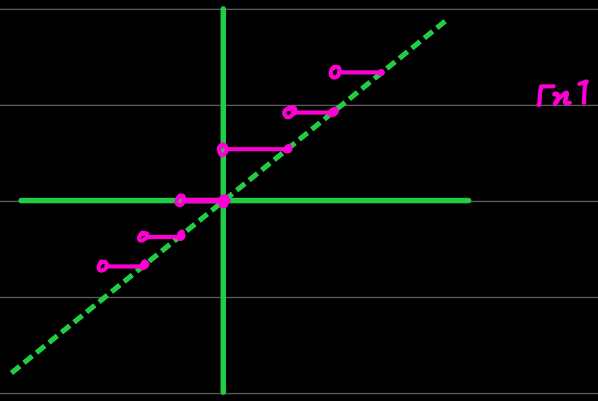







x^5


$\lfloor x \rfloor$


$\lceil x \rceil$

1.2

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $(cf)(x) = cf(x)$ (c ↳ constant)
- $(f/g)(x) = \frac{f(x)}{g(x)}$; $g(x) \neq 0$

- For above resulting functions **Domain: $D(f(x)) \cap D(g(x))$** and **Range: $R(f(x)) \cap R(g(x))$**

except for case $(f/g)(x)$ where $g(x) = 0$

Composite Functions

- $(f \circ g)(x) = f(g(x))$
- $(g \circ f)(x) = g(f(x))$

Note: Find domain & Range through Domain of the function that is being subbed in.

e.g: $f(x) = x^2$, $g(x) = \sqrt{x}$

$(f \circ g)(x) = (\sqrt{x})^2 = x$

Domain → $[0, \infty)$ not $(-\infty, \infty)$

Shifting Graphs

- $f(x) \pm k$ (shifts k units $\overset{+}{\text{upwards}}$ or $\overset{-}{\text{downwards}}$)
- $f(x \pm k)$ (shifts k units $\overset{+}{\text{left}}$ or $\overset{-}{\text{right}}$)

Scaling Graphs

- $af(x)$: (stretch parallel to y-axis with factor a) $a>1$ stretch vertically
  $a<1$ compress vertically
- $f(ax)$: (stretch parallel to x-axis with factor $\frac{1}{a}$) $a>1$ compress Horizontally
  $a<1$ stretch Horizontally

Reflection

- $-f(x)$: Reflection along x-axis
- $f(-x)$: Reflection along y-axis