

Inequalities

Algebraic Properties

① $a < b \Rightarrow a + c < b + c$

② $a < b \Rightarrow a - c < b - c$

③ $a < b$ and $c > 0 \Rightarrow ac < bc$










④ $a < b$ and $c < 0 \Rightarrow ac > bc$

★ ⑤ $a > 0 \Rightarrow \frac{1}{a} > 0$

★ ⑥ If 'a' and 'b' both are positive or both are negative, then $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

e.g: $-2 < 1$ $1 < 2$, $-2 < -1$
 $-\frac{1}{2} > 1$ $1 > \frac{1}{2}$ $-\frac{1}{2} > -1$
X

Types of Intervals

Notation	Set description	Type	Picture
(a, b)	$\{x \mid a < x < b\}$	Open	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed	
$[a, b)$	$\{x \mid a \leq x < b\}$	Half-open	
$(a, b]$	$\{x \mid a < x \leq b\}$	Half-open	
(a, ∞)	$\{x \mid x > a\}$	Open	
$[a, \infty)$	$\{x \mid x \geq a\}$	Closed	
$(-\infty, b)$	$\{x \mid x < b\}$	Open	
$(-\infty, b]$	$\{x \mid x \leq b\}$	Closed	
$(-\infty, \infty)$	\mathbb{R} (set of all real numbers)	Both open and closed	

Absolute Value Properties

① $|-a| = |a|$

② $|ab| = |a||b|$

③ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

④ $\sqrt{a^2} = |a|$

⑤ $|a+b| \leq |a| + |b|$ (Triangle Inequality)

↳ case 1: 'a' and 'b' have same signs

$$|a+b| = |a| + |b|$$

↳ case 2: 'a' and 'b' have different signs

$$|a+b| < |a| + |b|$$

Absolute Value & Intervals

① $|x| = a \iff x = \pm a$

② $|x| < a \iff -a < x < a$ → This is a positive integer

③ $|x| \leq a \iff -a \leq x \leq a$

④ $|x| > a \iff x > a \text{ or } x < -a$

⑤ $|x| \geq a \iff x \geq a \text{ or } x \leq -a$