Random Variable

- Random variable: Function that associates a real number with each element in the sample space
- Discrete random variable : Possible outcomes are countable
- continuous random variable : Possible outcomes are not countable
- Probability function / " mass function / " distribution : set of ordered pairs $(x, f(x))$; $f(x) = P(X=x)$
- Cummulative Distribution Function: $F(x) = P(X \le x) = \sum_{t \le x} f(t)$; $-\infty < x < \infty$

- For Discrete random variable

- $f(x) \geq 0$
- $\sum_{x} f(x) = 1$

- $P(X=x) = f(x)$

Expectation & Variance

- $E(X) = \sum x \cdot p(x)$
- $var(x) = E(x^2) - [E(x)]^2$

Q- **Example 3.9:** If a car agency sells 50% of its inventory of a certain foreign car equipped with side airbags, find a formula for the probability distribution of the number of cars with side airbags among the next 4 cars sold by the agency.

Sample points: _ _ _ _ $2 \times 2 \times 2 \times 2 = 16$

$p = 0.5$, $q = 0.5$

$\rightarrow {}^{n}C_{r} (0.5)^{r} (0.5)^{n-r}$

$$\left.\begin{array}{l} P(X=1) = {}^{4}C_{1} (0.5)^{1} (0.5)^{3} \\ P(X=2) = {}^{4}C_{2} (0.5)^{2} (0.5)^{2} \end{array}\right] \rightarrow {}^{4}C_{x} (0.5)^{4}$$

$f(x) = P(X=x) = \frac{1}{16}\binom{4}{x}$; $x = 0, 1, 2, 3, 4$

x	$P(X=x)$
0	1/16
1	1/4
2	3/8
3	1/4
4	1/16

Cummulative Distribution Function

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1/16 & , 0 \le x < 1 \\ 5/16 & , 1 \le x < 2 \\ 11/16 & , 2 \le x < 3 \\ 15/16 & , 3 \le x < 4 \\ 1 & , x \geq 4 \end{cases}$$

$F(0) = f(0) = 1/16$

$F(1) = f(0) + f(1) = \frac{1}{16} + \frac{1}{4} = \frac{5}{16}$

$F(2) = F(1) + f(2) = \frac{5}{16} + \frac{3}{8} = \frac{11}{16}$

$F(3) = F(2) + f(3) = \frac{11}{16} + \frac{1}{4} = \frac{15}{16}$

$F(4) = F(3) + f(4) = \frac{15}{16} + \frac{1}{16} = 1$

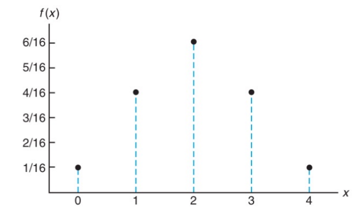
Figure 3.1: Probability mass function plot.

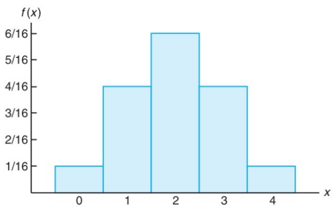
Figure 3.2: Probability histogram.

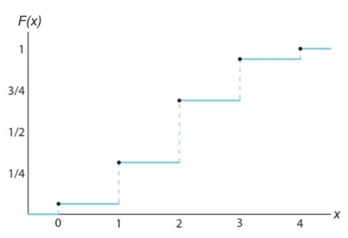
Figure 3.3: Discrete cumulative distribution function.

Q- A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find probability distribution for the number of defectives

$P(X=0) = \dfrac{{}^3C_0 \times {}^{17}C_2}{{}^{20}C_2} = \dfrac{68}{95}$

$P(X=1) = \dfrac{{}^3C_1 \times {}^{17}C_1}{{}^{20}C_2} = \dfrac{51}{190}$

$P(X=2) = \dfrac{\binom{3}{2} \cdot \binom{17}{0}}{\binom{20}{2}} = \dfrac{3}{190}$

x	0	1	2
$f(x)$	$\frac{68}{95}$	$\frac{51}{190}$	$\frac{3}{190}$

Q- Num of cars $= x$, that pass through washing station b/w 4-5 PM on any working day. Prob Distribution:-

x	4	5	6	7	8	9
$P(x)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$
$g(x).p(x)$	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{11}{4}$	$\frac{13}{4}$	$\frac{5}{2}$	$\frac{17}{6}$
$g(x)^2.p(x)$	$\frac{49}{12}$	$\frac{27}{4}$	$\frac{121}{4}$	$\frac{169}{4}$	$\frac{75}{2}$	$\frac{289}{6}$

- Let $g(x) = 2x-1$, represent the amount of money in dollars paid to attendant by the manager. Find the attendant's expected earning, standard deviation

- $E(g(x)) = \frac{7}{12} + \frac{3}{4} + \frac{11}{4} + \frac{13}{4} + \frac{5}{2} + \frac{17}{6} = \$\, \frac{38}{3} \approx \$12.6666$

- $\sum g(x)^2.p(x) = \frac{49}{12} + \frac{27}{4} + \frac{121}{4} + \frac{169}{4} + \frac{75}{2} + \frac{289}{6} = 169$

- $s = \sqrt{169 - \left(\frac{38}{3}\right)^2} = \$\, \frac{\sqrt{77}}{3}$

Q- 3 cards drawn in succession from deck of 52 playing cards. Find prob distribution for spades. Find expected value of spades, find its standard deviation

13- spades

$P(X=0) = \dfrac{{}^{13}C_0 \times {}^{39}C_3}{{}^{52}C_3} = \dfrac{703}{1700}$

$P(X=1) = \dfrac{{}^{13}C_1 \times {}^{39}C_2}{{}^{52}C_3} = \dfrac{741}{1700}$

$P(X=2) = \dfrac{\binom{13}{2} \cdot \binom{39}{1}}{\binom{52}{3}} = \dfrac{117}{850}$

$P(X=3) = \dfrac{\binom{13}{3} \cdot \binom{39}{0}}{\binom{52}{3}} = \dfrac{11}{850}$

x^2	0	1	4	9
x	0	1	2	3
$f(x)$	$\frac{703}{1700}$	$\frac{741}{1700}$	$\frac{117}{850}$	$\frac{11}{850}$
$x.p(x)$	0	$\frac{741}{1700}$	$\frac{234}{1700}$	$\frac{33}{850}$
$x^2.p(x)$	0	$\frac{741}{1700}$	$\frac{468}{1700}$	$\frac{99}{850}$

$E(x) = 0 + \frac{741+234}{1700} + \frac{33}{850} = \frac{1041}{1700}$

$s = \sqrt{\left(\frac{741+468}{1700} + \frac{99}{850}\right) - \left(\frac{1041}{1700}\right)^2} = 0.579827$