Halliday & Resnick's

Fundamentals of Physics

Extended Edition Wiley

Chapter 21

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Electric Charge

21.2 Electric Charge:

Charges with the same electrical sign repel each other, and charges with opposite electrical signs attract each other.

Fig. 21-1 Static cling, an electrical phenomenon that accompanies dry weather, causes these pieces of paper to stick to one another and to the plastic comb, and your clothing to stick to your body. *(Fundamental Photographs)*

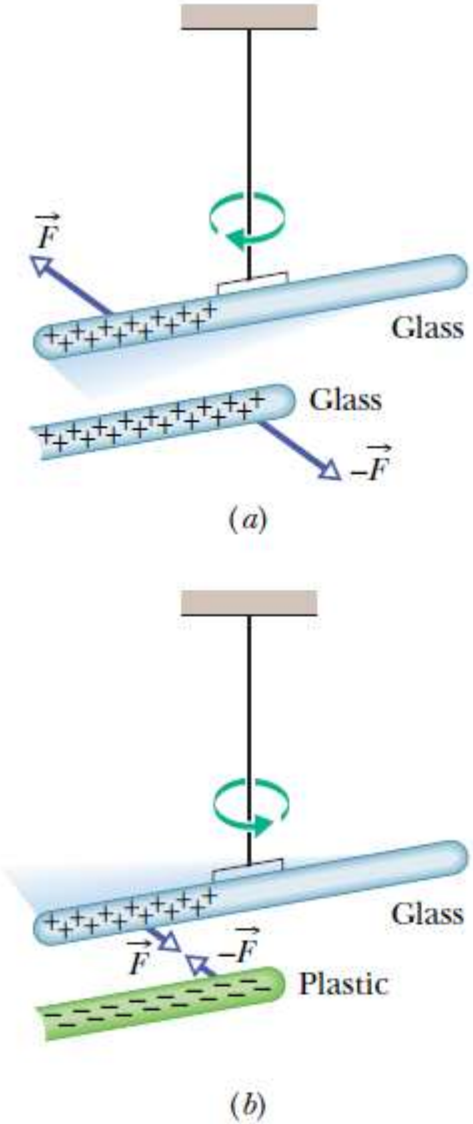


Fig. 21-2 (*a*) Two charged rods of the same sign repel each other. (*b*) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

21.3 Conductors and Insulators

Conductors are materials through which charge can move freely; examples include metals (such as *copper in common lamp wire), the human body, and tap water*.

Nonconductors—also called insulators—are materials through which charge cannot move freely; examples include *rubber, plastic, glass, and chemically pure water.*

Semiconductors are materials that are intermediate between conductors and insulators; examples include *silicon and germanium in computer chips*.

Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance.

The properties of conductors and insulators are due to the structure and electrical nature of atoms.

Atoms consist of positively charged ***protons***, negatively charged ***electrons***, and electrically neutral ***neutrons***. The protons and neutrons are packed tightly together in a central nucleus.

When atoms of a conductor come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (*positive ions).*We call the mobile electrons ***conduction electrons.***

There are few (if any) free electrons in a nonconductor.

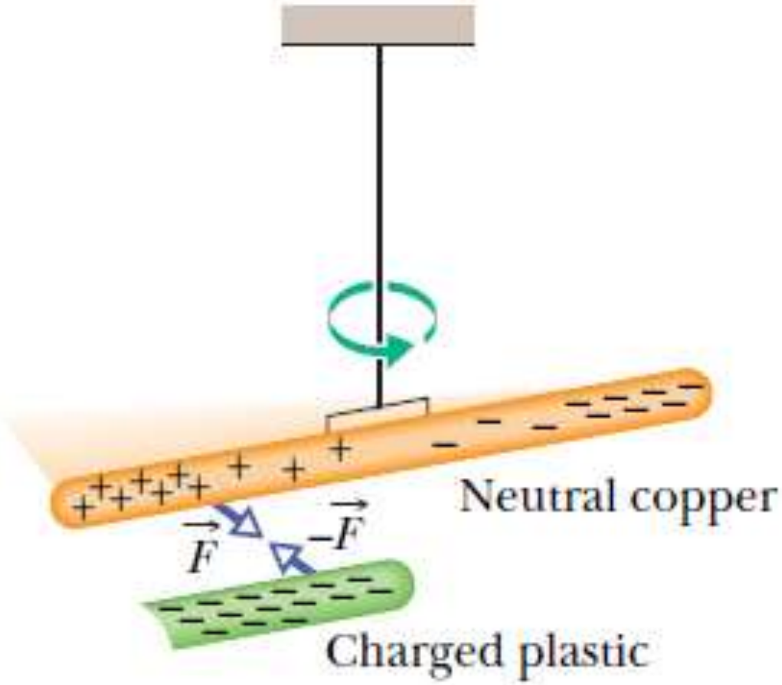


Fig. 21-4 A neutral copper rod is electrically isolated from its surroundings by being suspended on a nonconducting thread. Either end of the copper rod will be attracted by a charged rod. Here, conduction electrons in the copper rod are repelled to the far end of that rod by the negative charge on the plastic rod. Then that negative charge attracts the remaining positive charge on the near end of the copper rod, rotating the copper rod to bring that near end closer to the plastic rod.

21.4 Coulomb's Law

This force of repulsion or attraction due to the charge properties of objects is called an **electrostatic force.**

The equation giving the force for charged particles is called **Coulomb's law:**

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{\mathrm{r}} \qquad \text{(Coulomb's law)},$$

where particle 1 has charge q_1 and particle 2 has charge q_2, and F is the force on particle 1.
Here iis a unit vector along an axis extending through the two particles, r is the distance between them, $\hat{\mathrm{r}}$ nd k is a constant.

The SI unit of charge is the ***coulomb.***

The constant $k = \dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}.$

The quantity ε_0 is called the ***permittivity constant***

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}.$$

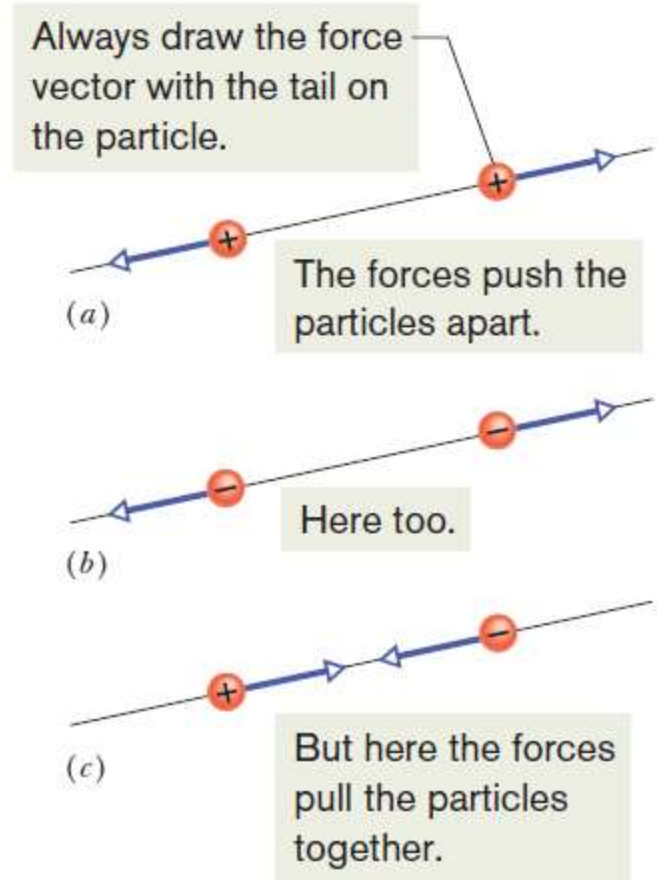


Fig. 21-6 Two charged particles repel each other if they have the same sign of charge, either (*a*) both positive or (*b*) both negative. (*c*) They attract each other if they have opposite signs of charge.

Current is the rate *dq/dt* at which charge moves past a point or through a region

$$i = \frac{dq}{dt} \qquad \text{(electric current)},$$

in which *i* is the current (in amperes) and *dq* (in coulombs) is the amount of charge moving past a point or through a region in time *dt* (in seconds).

Therefore,

$$1\,\mathrm{C} = (1\,\mathrm{A})(1\,\mathrm{s}).$$

21.4 Coulomb's Law

If there are *n* charged particles, they interact independently in pairs, and the force on any one of them, say particle 1, is given by the vector sum

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}$$

in which, $\mathbf{F}_{1,4}$ is the force acting on particle 1 due to the presence of particle 4, etc.

As with gravitational force law, the shell theorem has analogs in electrostatics:

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center.

- If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

Example, The net force due to two other particles:

(a) Figure 21-8*a* shows two positively charged particles fixed in place on an x axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?

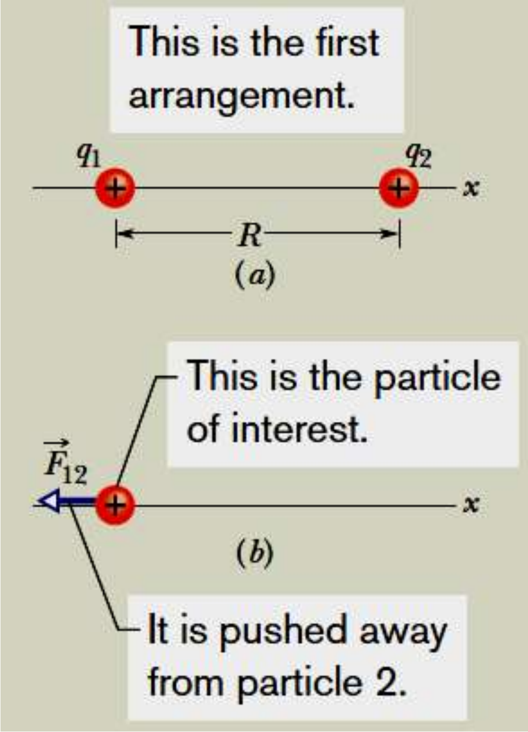


Fig. 21-8 (*a*) Two charged particles of charges q_1 and q_2 *are fixed in* place on an *x axis*. (*b*) *The free-body* diagram for particle 1, showing the electrostatic force on it from particle 2.

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})$$

$$\times \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{(0.0200\ \mathrm{m})^2}$$

$$= 1.15 \times 10^{-24}\ \mathrm{N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$1.15 \times 10^{-24}\ \mathrm{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}. \qquad \text{(Answer)}$$

Example, The net force due to two other particles, cont.:

(b) Figure 21-8*c* is identical to Fig. 21-8*a* except that particle 3 now lies on the *x* axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?

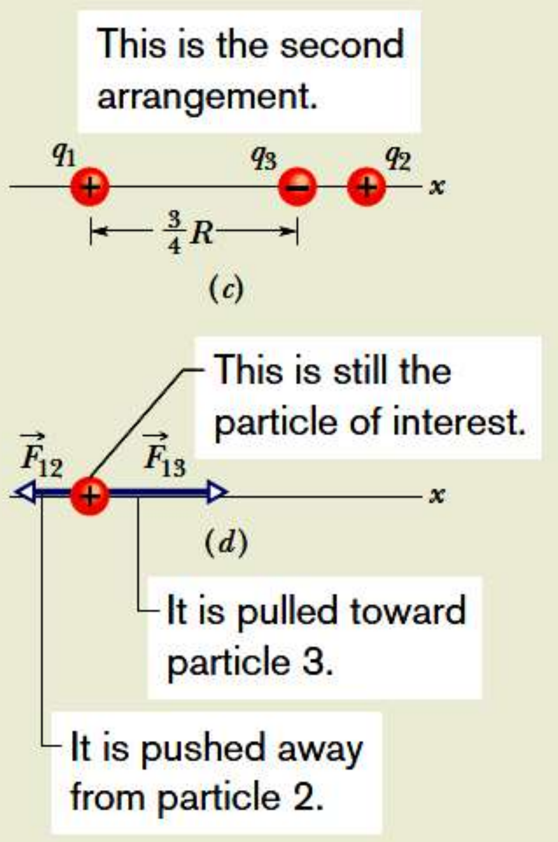


Fig. 21-8 (c) Particle 3 included. (*d*) *Free-body diagram* for particle 1.

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$
$$= 2.05 \times 10^{-24}\ \text{N}.$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}.$$

The net force $\vec{F}_{1,\text{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}} + (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}$$
$$= (9.00 \times 10^{-25}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,\text{net}}$ has the following magnitude and direction (relative to the positive direction of the *x* axis):

$$9.00 \times 10^{-25}\ \text{N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

Example, The net force due to two other particles, cont.:

(c) Figure 21-8*e* is identical to Fig. 21-8*a* except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?

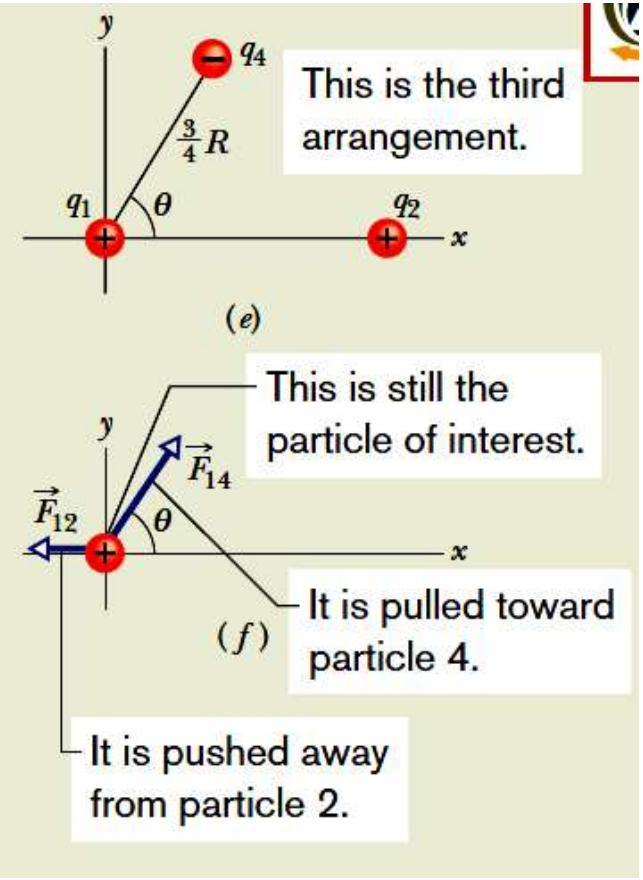


Fig. 21-8 (*e*) Particle 4 included. (*f*) Freebody diagram for particle 1.

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$
$$= 2.05 \times 10^{-24}\ \text{N}.$$

$$F_{1,\text{net},x} = F_{12,x} + F_{14,x} = F_{12} + F_{14}\cos 60°$$
$$= -1.15 \times 10^{-24}\ \text{N} + (2.05 \times 10^{-24}\ \text{N})(\cos 60°)$$
$$= -1.25 \times 10^{-25}\ \text{N}.$$

The sum of the y components gives us

$$F_{1,\text{net},y} = F_{12,y} + F_{14,y} = 0 + F_{14}\sin 60°$$
$$= (2.05 \times 10^{-24}\ \text{N})(\sin 60°)$$
$$= 1.78 \times 10^{-24}\ \text{N}.$$

The net force $\vec{F}_{1,\text{net}}$ has the magnitude

$$F_{1,\text{net}} = \sqrt{F_{1,\text{net},x}^2 + F_{1,\text{net},y}^2} = 1.78 \times 10^{-24}\ \text{N}. \qquad \text{(Answer)}$$

To find the direction of $\vec{F}_{1,\text{net}}$, we take

$$\theta = \tan^{-1}\frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0°.$$

However, this is an unreasonable result because $\vec{F}_{1,\text{net}}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct θ, we add 180°, obtaining

$$-86.0° + 180° = 94.0°. \qquad \text{(Answer)}$$

Example, Equilibrium of two forces:

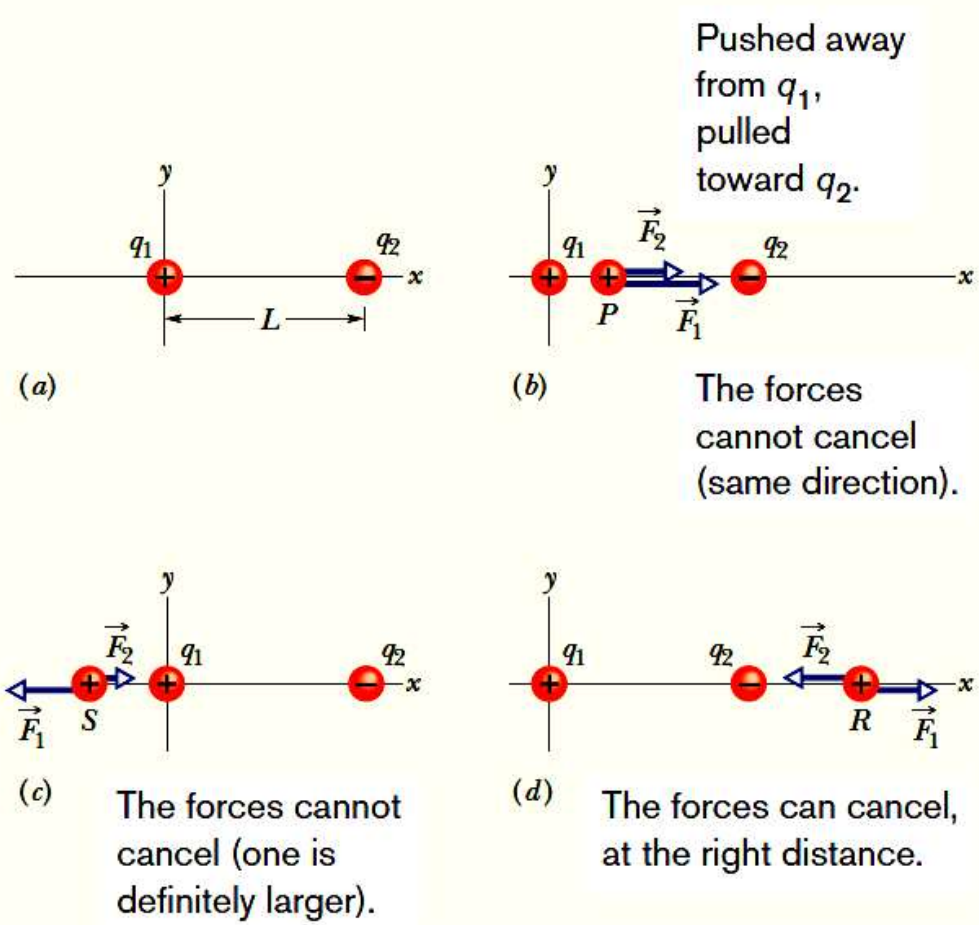
Figure 21-9*a* shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}.$$

$$\left(\frac{x-L}{x}\right)^2 = \frac{1}{4}.$$

$$\frac{x-L}{x} = \frac{1}{2},$$

$$x = 2L.$$



The equilibrium at x=2Lis unstable; that is, if the proton is displaced leftward from point *R*, then F_1 *and* F_2 both increase but F_2 increases more (because q_2 is closer than q_1), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both F_1 and F_2 decrease but F_2 decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

Example, Charge Sharing:

In Fig. 21-10*a*, two identical, electrically isolated conducting spheres *A* and *B* are separated by a (center-to-center) distance *a* that is large compared to the spheres. Sphere *A* has a positive charge of $+Q$, and sphere *B* is electrically neutral. Initially, there is no electrostatic force between the spheres. (Assume that there is no induced charge on the spheres because of their large separation.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

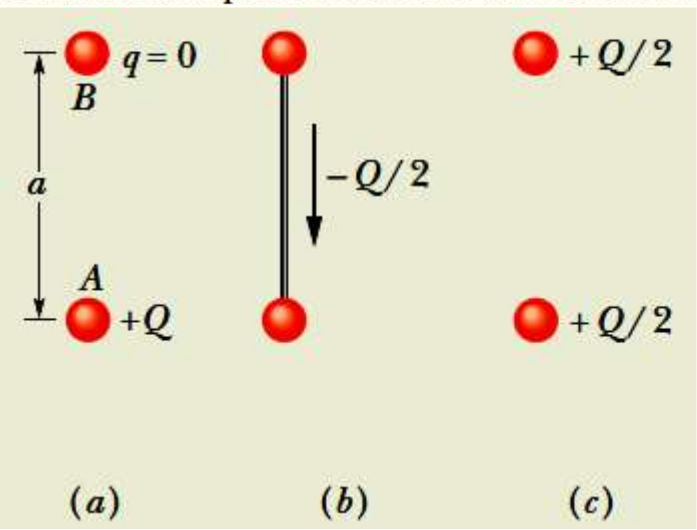


Fig. 21-10 *Two small conducting spheres A and B. (a) To start, sphere A is charged positively. (b) Negative charge is transferred from B to A through a connecting wire. (c) Both spheres are then charged* positively.

(1) Since the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount). (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

Reasoning: When the spheres are wired together, the (negative) conduction electrons on B move away from one another (along the wire to positively charged A— Fig. 21-10b.)

As B loses negative charge, it becomes positively charged, and as A gains negative charge, it becomes less positively charged. The transfer of charge stops when the charge on B has increased to Q/2 and the charge on A has decreased to Q/2, which occurs when Q/2 has shifted from B to A.

The spheres, now positively charged, repel each other.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\varepsilon_0}\left(\frac{Q}{a}\right)^2.$$

Example, Charge Sharing, cont.:

(b) Next, suppose sphere A is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

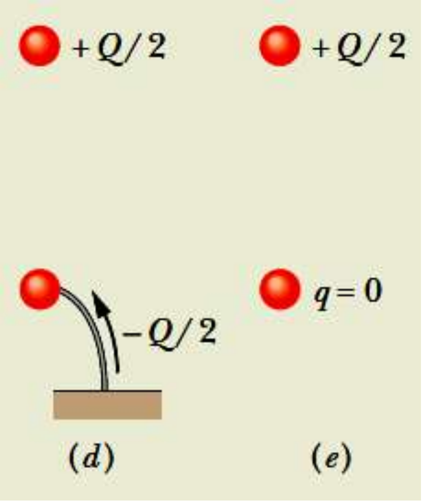


Fig. 21-10 (*d*) *Negative charge is transferred through a grounding wire* to sphere *A*. *(e) Sphere A is then neutral.*

Reasoning: When we provide a conducting path between a charged object and the ground (which is a huge conductor), we neutralize the object.

Were sphere *A* negatively charged, the mutual repulsion between the excess electrons would cause them to move from the sphere to the ground.

However, because sphere *A* is positively charged, electrons with a total charge of *Q/2* move from the ground up onto the sphere (Fig. 21-10*d*), leaving the sphere with a charge of 0 (Fig. 21-10*e*). Thus, there is (again) no electrostatic force between the two spheres.

21.5 Charge is Quantized

- Since the days of Benjamin Franklin, our understanding of of the nature of electricity has changed from being a type of ‘continuous fluid’ to a collection of smaller charged particles. The total charge was found to always be a multiple of a certain **elementary charge**, “e”:

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots,$$

The value of this elementary charge is one of the fundamental constants of nature, and it is the magnitude of the charge of both the proton and the electron. The value of “e” is:

$$e = 1.602 \times 10^{-19}\ \mathrm{C}.$$

21.5 Charge is Quantized

Table 21-1

The Charges of Three Particles

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

Elementary particles either carry no charge, or carry a single elementary charge. When a physical quantity such as charge can have only discrete values, rather than any value, we say the quantity is **quantized.** It is possible, For example, to find a particle that has no charge at all, or a charge of +10e, or -6e, but not a particle with a charge of, say, 3.57e.

21.5 Charge is Quantized

Many descriptions of electric charge use terms that might lead you to the conclusion that charge is a substance. Phrases like:

"Charge on a sphere"
"Charge transferred"
"Charge carried on the electron"

However, charge is a ***property*** of **particles,** one of many properties, such as mass.

Example, Mutual Electric Repulsion in a Nucleus:

The nucleus in an iron atom has a radius of about 4.0×10^{-15} m and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by 4.0×10^{-15} m?

KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

Calculation: Table 21-1 tells us that the charge of a proton is $+e$. Thus, Eq. 21-4 gives us

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$= \frac{(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.602 \times 10^{-19}\ \mathrm{C})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 14\ \mathrm{N}. \qquad \text{(Answer)}$$

No explosion: This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force to be acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force.

(b) What is the magnitude of the gravitational force between those same two protons?

KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-2).

Calculation: With m_p ($= 1.67 \times 10^{-27}$ kg) representing the mass of a proton, Eq. 21-2 gives us

$$F = G\frac{m_p^2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(1.67 \times 10^{-27}\ \mathrm{kg})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 1.2 \times 10^{-35}\ \mathrm{N}. \qquad \text{(Answer)}$$

21.6 Charge is Conserved

If one rubs a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body during the process.

This hypothesis of **conservation of charge** has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles.

Example 1: ***Radioactive decay of nuclei***, in which a nucleus transforms into (becomes) a different type of nucleus.

A uranium-238 nucleus (^{238}U) transforms into a thorium- 234 nucleus (^{234}Th) by emitting an *alpha particle*. An alpha particle has the same makeup as a helium-4 nucleus, it has the symbol ^{4}He. Here the net charge is 238.

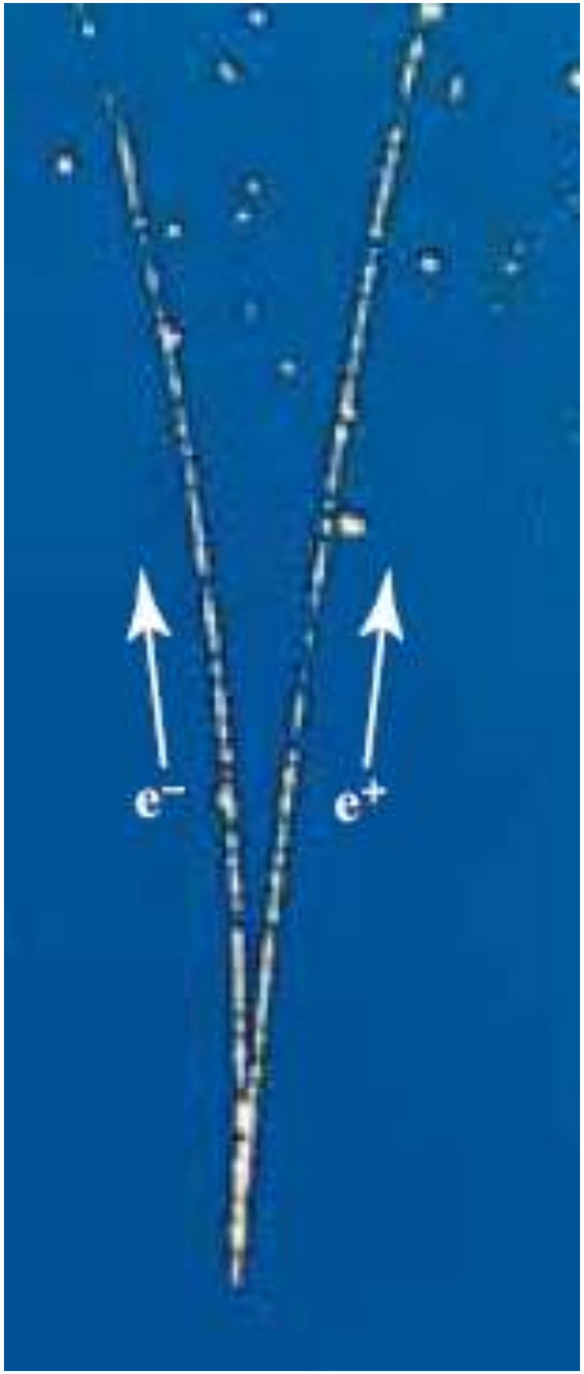
$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + {}^{4}\mathrm{He},$$

Example 2: An electron e (charge *-e)* and its antiparticle, the positron e (charge +*e)*, undergo an annihilation process, transforming into two gamma rays (high-energy light):. Here the net charge is zero.

$$\mathrm{e}^- + \mathrm{e}^+ \rightarrow \gamma + \gamma \qquad \text{(annihilation).}$$

Example 3: A gamma ray transforms into an electron and a positron. Here the net charge is again zero.

$$\gamma \rightarrow \mathrm{e}^- + \mathrm{e}^+ \qquad \text{(pair production).}$$



A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom.
Being electrically neutral, the gamma ray did not generate a telltale trail of bubbles along its path, as the electron and positron did.
(Courtesy Lawrence Berkeley Laboratory)

In the annihilation process, the net charge of the system is zero both before and after the event. Charge is conserved.

In pair production, the converse of annihilation, charge is also conserved.

Photograph shows such a pair-production event that occurred in a bubble chamber.

A gamma ray entered the chamber from the bottom and at one point transformed into an electron and a positron.

Because those new particles were charged and moving, each left a trail of tiny bubbles.

The trails were curved because a magnetic field had been set up in the chamber.)