

There are 18 mathematics majors and 325 computer science majors at a college.

- a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
- b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

A multiple-choice test contains 10 questions. There are four possible answers for each question.

- a) In how many ways can a student answer the questions on the test if the student answers every question?
- b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

How many bit strings of length ten both begin and end with a 1?

How many bit strings are there of length six or less, not counting the empty string?

How many bit strings with length not exceeding n , where n is a positive integer, consist entirely of 1s, not counting the empty string?

How many bit strings of length n , where n is a positive integer, start and end with 1s?

How many strings of five ASCII characters contain the character @ ("at" sign) at least once? [Note: There are 128 different ASCII characters.]

How many strings of three decimal digits

a) do not contain the same digit three times? _____

b) begin with an odd digit? _____

c) have exactly two digits that are 4s? _____

How many strings of four decimal digits

a) do not contain the same digit twice? _____

b) end with an even digit? _____

c) have exactly three digits that are 9s? _____

A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

How many license plates can be made using either two or three uppercase English letters followed by either two or three digits?

How many strings of eight uppercase English letters are there

a) if letters can be repeated?

b) if no letter can be repeated?

c) that start with X, if letters can be repeated?

- d) that start with X, if no letter can be repeated?
- e) that start and end with X, if letters can be repeated?
- f) that start with the letters BO (in that order), if letters can be repeated?
- g) that start and end with the letters BO (in that order), if letters can be repeated?
- h) that start or end with the letters BO (in that order), if letters can be repeated?

How many strings of eight English letters are there

- a) that contain no vowels, if letters can be repeated?
- b) that contain no vowels, if letters cannot be repeated?
- c) that start with a vowel, if letters can be repeated?
- d) that start with a vowel, if letters cannot be repeated?
- e) that contain at least one vowel, if letters can be repeated?
- f) that contain exactly one vowel, if letters can be repeated?
- g) that start with X and contain at least one vowel, if letters can be repeated?
- h) that start and end with X and contain at least one vowel, if letters can be repeated?

How many different functions are there from a set with 10 elements to sets with the following numbers of elements?

- a) 2 _____
- b) 3 _____
- c) 4 _____

d) 5 _____

How many one-to-one functions are there from a set with five elements to sets with the following number of elements?

a) 4 _____

b) 5 _____

c) 6 _____

d) 7 _____

How many functions are there from the set $\{1, 2, \dots, n\}$, where n is a positive integer, to the set $\{0, 1\}$?

How many functions are there from the set $\{1, 2, \dots, n\}$, where n is a positive integer, to the set $\{0, 1\}$

a) that are one-to-one? _____

b) that assign 0 to both 1 and n ? _____

c) that assign 1 to exactly one of the positive integers less than n ?

How many subsets of a set with 100 elements have more than one element?

How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

- a) the bride must be in the picture?
- b) both the bride and groom must be in the picture?
- c) exactly one of the bride and the groom is in the picture?

In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if

- a) the bride must be next to the groom?
- b) the bride is not next to the groom?
- c) the bride is positioned somewhere to the left of the groom?

How many bit strings of length seven either begin with two 0s or end with three 1s?

How many bit strings of length 10 either begin with three 0s or end with two 0s?

Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?

Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters *, >, <, !, +, and =.

- a) How many different passwords are available for this computer system?
- b) How many of these passwords contain at least one occurrence of at least one of the six special characters?
- c) Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes one nanosecond for a hacker to check each possible password.

The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? (Note that the name of a variable may contain fewer than eight characters.)

Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s.

Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

- a) How many socks must he take out to be sure that he has at least two socks of the same color?
- b) How many socks must he take out to be sure that he has at least two black socks?

A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

- a) How many balls must she select to be sure of having at least three balls of the same color?
- b) How many balls must she select to be sure of having at least three blue balls?

Show that if f is a function from S to T , where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 in S such that $f(s_1) = f(s_2)$, or in other words, f is not one-to-one.

What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?

Show that every sequence of $n^2 + 1$ distinct real numbers contains a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing.

Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends.

How many permutations of $\{a, b, c, d, e, f, g\}$ end with a ?

There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

How many bit strings of length 10 contain

a) exactly four 1s? _____

b) at most four 1s? _____

c) at least four 1s? _____

d) an equal number of 0s and 1s? _____

A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

How many permutations of the letters $ABCDEFGH$ contain

a) the string BCD ? _____

b) the string $CFGA$? _____

c) the strings BA and GF ? _____

d) the strings ABC and DE ? _____

e) the strings ABC and CDE ? _____

f) the strings CBA and BED ? _____

Using the binomial theorem, find the expansion of $(x + y)^5$

Find the coefficient of x^5y^8 in $(x + y)^{13}$.

How many terms are there in the expansion of $(x + y)^{100}$ after like terms are collected?

What is the coefficient of x^7 in $(1 + x)^{11}$?

What is the coefficient of x^9 in $(2 - x)^{19}$?

What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

Give a **combinatorial proof** to show that

$$2^n = \sum_{k=0}^n \binom{n}{k}$$

Show that if n is a nonnegative integer, then

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2$$

Hint: Use Vandermonde's identity.

Count the number of paths in the xy plane between the origin $(0,0)$ and point (m,n) , where m and n are nonnegative integers, such that each path is made up of a series of steps, where each step is a move one unit to the right or a move one unit upward. (No moves to the left or downward are allowed.)

Two such paths from $(0,0)$ to $(5,3)$ are illustrated here.

