

Chapter # 15

Oscillations

Exercise Problems: 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13.

1) $t = 0.25s$

$X = 36cm$

(a) $T = ?$

$t = T/2$

$T = 2t$

$T = 2(0.25)$

$T = 0.5s$

(b) $f = ?$

$f = \frac{1}{T} = \frac{1}{0.5}$

$f = 2Hz$

(c) $X_m = ?$

$X_m = X/2 = 36/2$

$X_m = 18cm$

2) $m = 0.12kg$

$x_m = 8.5cm = 0.085m$

$t = 0.2s$

(a) maximum force is related to the acceleration amplitude. By Newton's

second Law.

$$F_{\max} = m a_m$$

$$a_m = \omega^2 x_m$$

$$F_m = m \omega^2 x_m$$

$$\omega = 2\pi f$$

$$f = \frac{1}{T} = \frac{1}{0.2}$$

$$\boxed{f = 5 \text{ Hz}}$$

$$\omega = 2\pi(5)$$

$$\boxed{\omega = 10\pi \text{ rad/s}}$$

Now,

$$F_m = (0.12 \text{ kg})(10\pi \text{ rad/s})^2 (0.085 \text{ m})$$

$$\boxed{F_m = 10 \text{ N}}$$

$$(b) \quad \omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 = \frac{k}{m}$$

$$k = \omega^2 m$$

$$= (10\pi \text{ rad/s})^2 (0.12 \text{ kg})$$

$$\boxed{k = 1.2 \times 10^2 \text{ N/m}}$$

3) $x_m = 2.2 \text{ cm} = 0.022 \text{ m}$

$f = 6.60 \text{ Hz}$

$a_m = ?$

$a_m = \omega^2 x_m$

$\omega = 2\pi f = 2\pi(6.60)$

$\omega = 41.46$

$a_m = (41.46)^2 (0.022)$

$a_m = 37.8 \text{ m/s}^2$

4) Frequency of four springs is.

$f = 3 \text{ Hz}$

mass of car

$m_{\text{car}} = 1450 \text{ kg}$

mass of each five passengers that added. = 73

(a) $\omega = 2\pi f = 2\pi(3)$

$\omega = 6\pi \text{ rad/s}$

$k = ?$

Each spring is considered to support one fourth of the mass of car

$\omega = \sqrt{\frac{k}{m_{\text{car}}/4}}$

$$K = \frac{1}{4} m_{\text{car}} \omega^2$$

$$K = \frac{1}{4} (1450) (6\pi)^2$$

$$K = 1.29 \times 10^5 \text{ N/m}$$

(b) New mass being supported by the four springs is:

$$m_{\text{total}} = [1450 + 5(73)] = 1815$$

$$\omega_{\text{new}} = \sqrt{\frac{K}{m_{\text{total}}/4}}$$

$$\omega_{\text{new}} = 2\pi f_{\text{new}}$$

$$f_{\text{new}} = \frac{\omega_{\text{new}}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{K}{m_{\text{total}}/4}}$$

$$f_{\text{new}} = \frac{1}{2\pi} \sqrt{\frac{2.68 \cdot 1.29 \times 10^5}{1815/4}}$$

$$f_{\text{new}} = 2.68 \text{ Hz}$$

5) Displacement $D = 20 \text{ mm} = 0.001 \text{ m}$

$$x_m = ? , v_m = ? , a_m = ?$$

$$f = 120 \text{ Hz}$$

(a) The amplitude x_m is half the range of the displacement D .

$$x_m = D/2 = 0.001/2$$

$$x_m = 1.0 \times 10^{-3} \text{ m}$$

(b) $v_m = \omega x_m = (2\pi f) x_m$

$$= 2\pi (120) (1.0 \times 10^{-3})$$

$$v_m = 0.75 \text{ m/s}$$

(c) $a_m = \omega^2 x_m = (2\pi f)^2 x_m$

$$= (2\pi (120))^2 1.0 \times 10^{-3}$$

$$a_m = 5.7 \times 10^2 \text{ m/s}^2$$

6) $m = 1.00 \times 10^{-20} \text{ kg}$

$$t = 1.00 \times 10^{-5} \text{ s}$$

$$v_m = 1.00 \times 10^3 \text{ m/s}$$

$$\omega = ?$$

$$x_m = ?$$

$$(a) \quad \omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{1.00 \times 10^{-5}}$$

$$\boxed{\omega = 6.28 \times 10^5 \text{ rad/s}}$$

$$(b) \quad V_m = \omega X_m$$

$$X_m = \frac{V_m}{\omega} = \frac{1.00 \times 10^3 \text{ m/s}}{6.28 \times 10^5 \text{ rad/s}}$$

$$\boxed{X_m = 1.59 \times 10^{-3} \text{ m}}$$

$$7) \quad X_m = 1 \mu\text{m} = 1 \times 10^{-6} \text{ m}$$

(a) magnitude a of the diaphragm's acceleration equal to g .

$$a = \omega^2 X_m$$

$$g = \omega^2 X_m$$

$$\omega = \sqrt{g/X_m}$$

$$\omega = 2\pi f \Rightarrow f = \frac{1}{2\pi} \sqrt{g/X_m} = \frac{1}{2\pi} \sqrt{\frac{9.8}{1 \times 10^{-6}}}$$

$$\boxed{f = 498 \text{ Hz}}$$

(b) As we can see. $f \propto \sqrt{g}$

For frequencies greater than 498 Hz, the a exceeds g for some part of the motion.

9) $x(t) = (6.0\text{m})(\cos((3\pi)t + \pi/3))$

$t = 2.0\text{s}$

(a) $x = ?$

$$x(2.0) = (6.0)\cos(3\pi(2.0) + \pi/3)$$

$$\boxed{x = 3.0\text{m}}$$

(b) $v = ?$

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin((3\pi)t + \pi/3)$$

$$v(2.0) = -3\pi(6.0)\sin(3\pi(2.0) + \pi/3)$$

$$\boxed{v = -49\text{ m/s}}$$

(c) $a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos(3\pi(t) + \pi/3)$

$$a(2.0) = -(3\pi)^2(6.0)\cos(3\pi(2.0) + \pi/3)$$

$$\boxed{a = -2.7 \times 10^2\text{ m/s}^2}$$

(d) The phase of the motion is:

$$(3\pi(t) + \pi/3)$$

At $t = 2\text{s}$.

$$3\pi(2) + \pi/3 \approx \boxed{20\text{ rad}}$$

$$(e) \quad \omega = 3\pi$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{3\pi}{2\pi}$$

$$\boxed{f = 1.5 \text{ Hz}}$$

$$(f) \quad T = \frac{1}{f} = \frac{1}{1.5}$$

$$\boxed{T = 0.67 \text{ s}}$$

$$10) \quad t = 0.75 \text{ s}$$

$$(a) \quad T = ?$$

The problem describes the time taken to execute one cycle of the motion. The period is.

$$\boxed{T = 0.75 \text{ s}}$$

$$(b) \quad f = ?$$

$$f = \frac{1}{T} = \frac{1}{0.75}$$

$$\boxed{f = 1.3 \text{ Hz}}$$

$$(c) \quad \omega = 2\pi f = 2\pi(1.3)$$

$$\boxed{\omega = 8.4 \text{ rad/s}}$$

11) Two identical springs constant \Leftarrow

$$K = 7580$$

$$m = 0.245 \text{ kg}$$

$$f = ?$$

$$F = -2Kx$$

$$ma = -2Kx$$

$$m \frac{d^2x}{dt^2} = -2Kx$$

$$m \frac{d^2x}{dt^2} + 2Kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{2K}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

$$\omega^2 = 2K/m$$

$$\omega = \sqrt{\frac{2K}{m}}$$

$$2\pi f = \sqrt{\frac{2K}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{2K}{m}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{2(7580)}{0.245}}$$

$$f = 39.6 \text{ Hz}$$

13) $m = 0.500 \text{ kg}$

$$x_m = 35 \text{ cm} = 0.35 \text{ m}$$

$$t = 0.500 \text{ s}$$

(a) $T = ?$

The motion repeats every 0.5s
so period must be

$$T = 0.5 \text{ s}$$

(b) $f = ?$

$$f = \frac{1}{T} = \frac{1}{0.5}$$

$$f = 2.0 \text{ Hz}$$

(c) $\omega = 2\pi f = 2\pi(2)$

$$\omega = 12.6 \text{ rad/s}$$

(d) $k = ?$

$$k = m\omega^2 = (0.5 \text{ kg})(12.6)^2$$

$$k = 79.0 \text{ Nm}$$

(e) $v_m = ?$

$$v_m = \omega x_m = (79)(0.35)$$

$$v_m = 4.40 \text{ m/s}$$

(f) $F_m = kx_m = (79)(0.35)$

$$F_m = 27.6 \text{ N}$$