Lec-9

Ex # 2.1

1

(1) Find average rate of change?

$f(x) = x^3 + 1$

a) $[2, 3]$

Formula

Average rate of change

$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Sol: $\frac{\Delta y}{\Delta x} = \frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \frac{f(3) - f(2)}{3 - 2}$

$f(3) = 3^3 + 1 = 28$

$f(2) = 2^3 + 1 = 9$

$\Rightarrow \frac{\Delta f}{\Delta x} = \frac{28 - 9}{3 - 2} = \frac{19}{1} = \boxed{19}$ Ans

(3) $h(t) = \cot t$

(b) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

Sol: $\frac{\Delta h}{\Delta t} = \frac{h(t_2) - h(t_1)}{t_2 - t_1} = \frac{h\left(\frac{\pi}{2}\right) - h\left(\frac{\pi}{6}\right)}{\frac{\pi}{2} - \frac{\pi}{6}}$

$h\left(\frac{\pi}{2}\right) = \cot\frac{\pi}{2} = 0$

$h\left(\frac{\pi}{6}\right) = \cot\frac{\pi}{6} = \sqrt{3}$

$\Rightarrow \frac{\Delta h}{\Delta t} = \frac{0 - \sqrt{3}}{\frac{\pi}{2} - \frac{\pi}{6}} = \frac{-\sqrt{3}}{\frac{3\pi - \pi}{6}} = \frac{-\sqrt{3}}{\frac{2\pi}{6}} = \frac{-6\sqrt{3}}{2\pi}$

$= \boxed{\frac{-3\sqrt{3}}{\pi}}$ Ans

CamScanner

2

(6) $P(\theta) = \theta^3 - 4\theta^2 + 5\theta$; $[1, 2]$

Sol:

$$\frac{\Delta P}{\Delta \theta} = \frac{P(\theta_2) - P(\theta_1)}{\theta_2 - \theta_1} = \frac{P(2) - P(1)}{2-1}$$

$$P(2) = 2^3 - 4(2^2) + 5(2)$$
$$= 8 - 16 + 10 = 2$$

$$P(1) = 1^3 - 4(1^2) + 5(1)$$
$$= 1 - 4 + 5 = 2$$

$$\Rightarrow \frac{\Delta P}{\Delta \theta} = \frac{2-2}{2-1} = \boxed{0} \quad \text{Ans}$$

(9) Find slope of curve. And eq. of tangent line?

$y = x^2 - 2x - 3$, $P(+2, -3)$

Formulae

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_1+h) - f(x_1)}{h}$$

$$y - y_1 = m(x - x_1)$$

Sol:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_1+h) - f(x_1)}{h}$$

$$m = \frac{\left[(2+h)^2 - 2(2+h) - 3\right] - \left[(2)^2 - 2(2) - 3\right]}{h}$$

$$= \frac{\{\cancel{4} + h^2 + 4h\} - \cancel{4} - 2h - \cancel{3} - \cancel{4} + \cancel{4} + \cancel{3}}{h}$$

$$= \frac{h^2 + 2h}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}}$$

$$= h + 2$$

As $h \to 0$

$$\boxed{m = 2} \quad \text{Ans}$$

$$y - y_1 = m(x - x_1)$$
$$y + 3 = 2(x - 2)$$
$$y = -3 + 2x - 4$$
$$\boxed{y = 2x - 7} \quad \text{Ans}$$

15. $y = \frac{1}{x}$; $P\left(-2, -\frac{1}{2}\right)$

Sol:

Slope $m = \frac{\Delta y}{\Delta x} = \frac{f(x_1 + h) - f(x_1)}{h}$

$$= \frac{\frac{1}{-2+h} - \frac{1}{-2}}{h}$$

$$= \frac{\frac{1}{-2+h} + \frac{1}{-2}}{h}$$

$$= \frac{1}{h}\left[\frac{2 + (-2+h)}{2(-2+h)}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{2} - \cancel{2} + h}{-4 + 2h}\right]$$

$$= \frac{1}{\cancel{h}}\left[\frac{\cancel{h}}{-4 + 2h}\right] = \frac{1}{-4 + 2h}.$$

As $h \to 0$

$$m = \frac{1}{-4+0} = \boxed{-\frac{1}{4}} \quad \text{Ans}$$

Eq of tangent line

$$y - y_1 = m(x - x_1)$$
$$y = -\frac{1}{2} - \frac{1}{4}(x + 2)$$
$$= -\frac{1}{2} - \frac{1}{4}x - \frac{1}{2} = \boxed{-\frac{1}{4}x - 1} \quad \text{Ans}$$

(18) $y = \sqrt{7-x}$, $P(-2,3)$

Sol: **Slope**

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_1+h) - f(x_1)}{h}$$

$$= \frac{\sqrt{7-(-2+h)} - \sqrt{7-(-2)}}{h}$$

$$= \frac{\sqrt{7+2-h} - \sqrt{7+2}}{h}$$

$$= \frac{\sqrt{9-h} - \sqrt{9}}{h}$$

$$= \frac{\sqrt{9-h} - 3}{h}$$

$$= \frac{\sqrt{9-h} - 3}{h} \times \frac{\sqrt{9-h} + 3}{\sqrt{9-h} + 3}$$

$$= \frac{(\sqrt{9-h})^2 - (3)^2}{h(\sqrt{9-h}+3)}$$

$$= \frac{\cancel{9} - h - \cancel{9}}{h(\sqrt{9-h}+3)}$$

$$= \frac{-1}{\sqrt{9-h}+3}$$

As $h \to 0$

$$= \frac{-1}{\sqrt{9}+3} = \frac{-1}{3+3} = \boxed{\frac{-1}{6}} \text{ Ans}$$

Eq of tangent line

$$y - y_1 = m(x - x_1)$$

$$y = 3 - \frac{1}{6}(x+2)$$

$$y = 3 - \frac{1}{6}x - \frac{2}{6} = -\frac{1}{6}x + 3 - \frac{1}{3}$$

$$\boxed{y = -\frac{1}{6}x + \frac{8}{3}} \text{ Ans}$$

Ex 3.1

(5) Find eq. of tangent line at point. Sketch

$y = 4 - x^2 \quad ; \quad (\underset{x_0}{-1}, \underset{y_0}{3})$

Sol:

$y = f(x) = 4 - x^2.$

$f(x_0+h) = 4 - (x_0+h)^2$

$= 4 - (-1+h)^2$

$= 4 - 1 - h^2 + 2h$

$\Rightarrow f(x_0+h) = 3 - h^2 + 2h.$

$f(x_0) = 4 - x_0^2$

$= 4 - (-1)^2 = 4 - 1 = 3.$

$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$ — slope of tangent line.

$= \lim_{h \to 0} \frac{\cancel{3} - h^2 + 2h - \cancel{3}}{h}$

$= \lim_{h \to 0} \frac{-h^2 + 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(-h+2)}{\cancel{h}}$

$\boxed{m = 2}$

Eq. of tangent line

$y - y_0 = m(x - x_0)$

$y - 3 = 2(x+1)$

$y = 3 + 2x + 2$

$\boxed{y = 2x + 5}$

Formulae

$\frac{y - y_0}{x - x_0} = m$

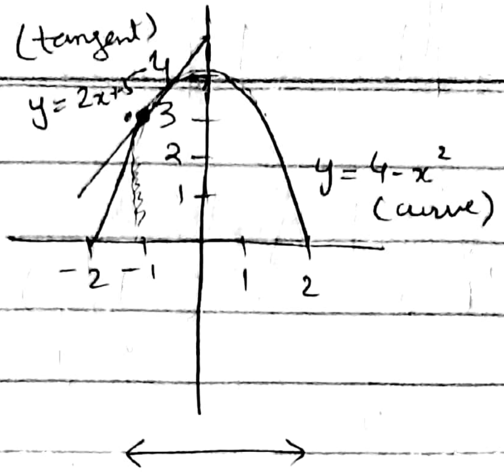
~~$\Rightarrow y - y_0 = m(x - x_0)$~~

$y - y_0 = m(x - x_0)$

(Eq. of tangent line),

(1) $m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$

6



7 $y = 2\sqrt{x}$; $(1, 2)$

Sol: $f(x_0+h) = 2\sqrt{x_0+h} = 2\sqrt{1+h}$

$f(x_0) = 2\sqrt{x_0} = 2\sqrt{1} = 2$

$$m = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{2\sqrt{1+h} - 2}{h}$$

$$= \lim_{h\to 0} \frac{2\sqrt{1+h} - 2}{h} \times \frac{2\sqrt{1+h} + 2}{2\sqrt{1+h} + 2}$$

$$= \lim_{h\to 0} \frac{(2\sqrt{1+h})^2 - 2^2}{h(2\sqrt{1+h} + 2)}$$

$$= \lim_{h\to 0} \frac{4(1+h) - 4}{2h\sqrt{1+h} + 2h}$$

$$= \lim_{h\to 0} \frac{\cancel{4} + 4h - \cancel{4}}{2h[\sqrt{1+h} + 1]} = \lim_{h\to 0} \frac{\cancel{4}^{2}\cancel{h}}{\cancel{2}\cancel{h}[\sqrt{1+h} + 1]}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{1+h} + 1}$$

$$= \frac{2}{\sqrt{1} + 1} = \frac{2}{2} = \boxed{1}$$

Ans

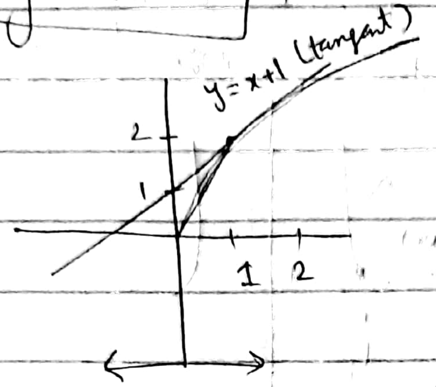
Eq. of tangent line

$$y - y_0 = m(x - x_0)$$

$$y - 2 = 1(x - 1)$$

$$y = 2 + x - 1$$

$$\boxed{y = x + 1}$$



$y = 2\sqrt{x}$
(curve)

⑧ $y = \frac{1}{x^2}$; $(-1, 1)$

Sol:

$$f(x_0+h) = \frac{1}{(x_0+h)^2} = \frac{1}{(-1+h)^2} = \frac{1}{1+h^2-2h}$$

$$f(x_0) = \frac{1}{x_0^2} = \frac{1}{(-1)^2} = 1$$

$$\therefore m = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{1+h^2-2h} - 1}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{1} - \cancel{1} - h^2 + 2h}{1+h^2-2h}}{h}$$

$$= \lim_{h\to 0} \frac{2h - h^2}{h(1+h^2-2h)} = \lim_{h\to 0} \frac{\cancel{h}(2-h)}{\cancel{h}(1+h^2-2h)}$$

$$= \frac{2-0}{1+0-0} = \boxed{2}$$

Ans

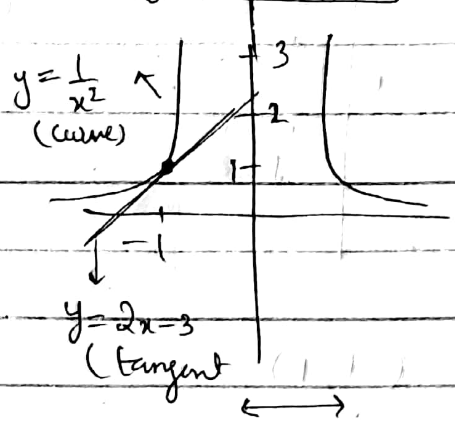
Eq. of tangent line is:

$$y - y_0 = m(x - x_0)$$

$$y - 1 = 2(x+1)$$

$$y = +1 + 2x + 2$$

$$\boxed{y = 2x + 3}$$



(13) Find the slope of function. Also find eq for tangent line?

Q: $g(x) = \dfrac{x}{x-2}$; $(3, 3)$

Sol:

$$f(x_0 + h) = \frac{x_0 + h}{x_0 + h - 2}$$

$$= \frac{3+h}{3+h-2} = \frac{3+h}{1+h}$$

$$f(x_0) = \frac{x_0}{x_0 - 2} = \frac{3}{3-2} = 3$$

$$\Rightarrow \quad m = \lim_{h \to 0} \frac{\frac{3+h}{1+h} - \frac{3}{1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3+h-3(1+h)}{1+h}}{h} = \lim_{h \to 0} \frac{\cancel{3} + h - \cancel{3} - 3h}{h(1+h)}$$

$$= \lim_{h\to 0} \frac{-2\cancel{K}}{\cancel{K}(1+h)} = \lim_{h\to 0} \frac{-2}{1+h}$$

$$= \frac{-2}{1+0} = \boxed{-2} \quad \text{Ans.}$$

Now eq for tangent line,

$$y - y_0 = m(x - x_0)$$

$$y - 3 = -2(x - 3)$$

$$y = 3 - 2x + 6$$

$$\boxed{y = 9 - 2x} \quad \text{Ans}$$

(16) $h(t) = t^3 + 3t \quad ; \quad (1, 4)$.

Sol: $f(x_0 + h) = f(t_0 + h) = (t_0 + h)^3 + 3(t_0 + h)$

$$= (1+h)^3 + 3(1+h)$$

$$= 1 + h^3 + 3h(1+h) + 3 + 3h.$$

$$\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$= 1 + h^3 + 3h + 3h^2 + 3 + 3h.$$

$$= h^3 + 3h^2 + 6h + 4$$

$$f(x_0) = f(t_0) = t_0^3 + 3t_0$$

$$= (1) + 3(1)$$

$$= 1 + 3 = 4.$$

$$m = \lim_{h\to 0} \frac{f(t_0+h) - f(t_0)}{h} = \lim_{h\to 0} \frac{h^3 + 3h^2 + 6h + 4 - 4}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h^2 + 3h + 6)}{\cancel{h}}$$

$$= 0 + 3(0) + 6 = \boxed{6} \quad \text{Ans}$$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$y - 4 = 6(x - 1)$$

$$y = 4 + 6x - 6 = \boxed{6x - 2.} \quad \text{Ans.}$$

(22) Find slope of curve at point?

$$y = \frac{x-1}{x+1}, \quad x = 0$$

Sol: $f(x+h) = \dfrac{x+h-1}{x+h+1}$

$$= \frac{0+h-1}{0+h+1} = \frac{h-1}{h+1}$$

$$f(x) = \frac{x-1}{x+1} = \frac{0-1}{0+1} = \frac{-1}{1} = -1$$

$$m = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{h-1}{h+1} - (-1)}{h} = \lim_{h\to 0} \frac{h-1-[(-1)(h+1)]}{h(h+1)}$$

$$= \lim_{h\to 0} \frac{h-1+(h+1)}{h(h+1)}$$

$$m = \lim_{h \to 0} \frac{h - \cancel{1} + h + \cancel{1}}{h(h+1)}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}(h+1)}$$

$$= \lim_{h \to 0} \frac{2}{h+1}$$

$$= \frac{2}{0+1} = \boxed{2}$$ Ans.

25 Ex: At what point do the graph of function have horizontal tangent line.

$$f(x) = x^2 + 4x - 1$$

Sol: $$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - 1 - x^2 - 4x + 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + h^2 + 2xh + \cancel{4x} + 4h - \cancel{1} - \cancel{x^2} - \cancel{4x} + \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2xh + 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h + 2x + 4)}{\cancel{h}}$$

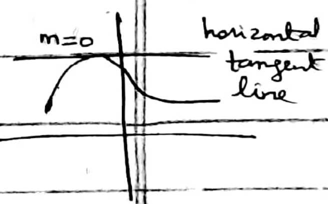
$$= 0 + 2x + 4 = 2x + 4$$

$$m = 2x + 4 \Rightarrow 0 = 2x + 4$$

$$2x = -4$$

$$\boxed{x = -2}$$

Concept



Horizontal tangent line is parallel to x-axis. Here slope is zero because there is no change in tangent line so y-axis is constant.

Put $x = -2$ in given eq. $f(x) = x^2 + 4x - 1$.

$$y = f(x) = (-2)^2 + 4(-2) - 1$$
$$= 4 - 8 - 1$$
$$y = -5$$

Hence $(-2, -5)$ is the point on the graph where there is horizontal tangent line.

$\longleftrightarrow$

27) Find eqns of all lines having slope $m = -1$ that are tangent to the curve $y = \frac{1}{x-1}$.

Sol: Here

$$m = -1, \quad y = \frac{1}{x-1}$$

$$\Rightarrow m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$m = \lim_{h \to 0} \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$$

$$m = \lim_{h \to 0} \frac{x-1-(x+h-1)}{h(x-1)(x+h-1)}$$

$$m = \lim_{h \to 0} \frac{\cancel{x} - \cancel{1} - \cancel{x} - h + \cancel{1}}{h(x-1)(x+h-1)}$$

$$m = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{-1}{(x-1)(x+h-1)} = \frac{-1}{(x-1)(x-1)} = \frac{-1}{(x-1)^2}$$

P.T.O

$$m = \frac{-1}{(x-1)^2}$$

$$-1 = \frac{-1}{(x-1)^2}$$

$$-(x-1)^2 = -1.$$

$$(x-1)^2 = 1.$$

$$\sqrt{(x-1)^2} = \sqrt{1}$$

$$x-1 = \pm 1$$

$$x = 1+1 \quad , \quad x = 1-1$$

$$\boxed{x = 2} \quad , \quad \boxed{x=0}$$

Put $x=2$ in $y = f(x) = \frac{1}{x-1}$

$$f(2) = \frac{1}{2-1} = 1 \qquad (2,1)$$

Put $x=0$ in $f(x) = \frac{1}{x-1}$

$$f(0) = \frac{1}{0-1} = -1 \qquad (0,-1)$$

For $(2,1) \Rightarrow$ $y - y_0 = m(x - x_0)$

$$y - 1 = -1(x-2)$$

$$y = 1 - (x-2)$$

$$= 1 - x + 2$$

$$y = 3 - x = \boxed{-(x-3)}$$ Ans

For $(0,-1) \Rightarrow y - y_0 = m(x - x_0)$

$$y + 1 = -1(x-0)$$

$$y = -1 - 1(x) = -1 - x$$

$$\boxed{y = -(x+1)}$$ Ans

30 Find rate of change?

Speed of rocket: At 't' sec after liftoff, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing 10 sec after liftoff?

Sol: Given that

$$f(t) = 3t^2 \text{ ft}$$
$$t = 10 \text{ sec}$$

$$\text{Rate of change} = \lim_{h\to 0} \frac{f(t+h) - f(t)}{h}$$

$$f(t+h) = 3(t+h)^2$$
$$= 3(10+h)^2 = 3(100+h^2+20h)$$
$$= 300 + 3h^2 + 60h.$$

$$f(t) = 3t^2 = 3(10)^2 = 3(100) = 300$$

$$\text{Rate of change} = \lim_{h\to 0} \frac{\cancel{300} + 3h^2 + 60h - \cancel{300}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3h+60)}{\cancel{h}} = \lim_{h\to 0} 3h+60$$

$$= 3(0) + 60 = \boxed{60 \text{ ft/sec}} \text{ Ans.}$$

32 **Ball's Changing Volume:** What is the rate of change of the volume of a ball $\left(V = \frac{4}{3}\pi r^3\right)$ w.r.t radius when the radius is $r = 2$?

Sol: Given that

$$f(r) = \frac{4}{3}\pi r^3$$

$$r = 2.$$

$$\text{Rate of change} = \lim_{h \to 0} \frac{f(r+h) - f(r)}{h}.$$

$$f(r+h) = \frac{4}{3}\pi(r+h)^3$$

$$= \frac{4}{3}\pi\left[r^3 + h^3 + 3r^2h + 3rh^2\right]$$

$$= \frac{4}{3}\pi\left[2^3 + h^3 + 3(2)^2h + 3(2)h^2\right]$$

$$f(r+h) = \frac{4}{3}\pi\left(8 + h^3 + 12h + 6h^2\right)$$

$$f(r) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3$$

$$= \frac{4}{3}\pi(8) = \frac{32}{3}\pi$$

$$\Rightarrow \text{Rate of change} = \lim_{h \to 0} \frac{\cancel{\frac{32\pi}{3}} + \frac{4\pi h^3}{3} + 16\pi h + 8\pi h^2 - \cancel{\frac{32\pi}{3}}}{h}$$

$$= \lim_{h \to 0} \frac{4\pi h^3 + 48\pi h + 24\pi h^2}{3h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4\pi h^2 + 48\pi + 24\pi h)}{3\cancel{h}}$$

$$= \frac{4\pi(0) + 48\pi + 24\pi(0)}{3} = \frac{48\pi}{3} = \boxed{16\pi}$$

Ans.

Ex 3.2

(2) Using definition, calculate derivative of function and find values of the derivatives as specified.

$F(x) = (x-1)^2 + 1 \quad ; \quad F'(-1) \; ; \; F'(0) \; ; \; F'(2)$

Sol:

$$\frac{dy}{dx} = F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

$$F'(x) \lim_{h \to 0} \frac{(x+h-1)^2 + 1 - [(x-1)^2 + 1]}{h}$$

$$\lim_{h \to 0} = \frac{(x)^2 + (h)^2 + (-1)^2 + 2(x)(h) + 2(h)(-1) + 2(x)(-1) + 1 - (x^2 - 2x + 1 + 1)}{h}$$

$$\lim_{h \to 0} = \frac{\cancel{x^2} + h^2 + \cancel{1} + 2xh - 2h - \cancel{2x} + \cancel{1} - \cancel{x^2} + \cancel{2x} - \cancel{1} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2xh - 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(h + 2x - 2)}{\cancel{h}}$$

$$= 0 + 2x - 2 = \boxed{2x - 2}$$

$$F'(x) = 2x - 2$$

$F'(-1) = 2(-1) - 2 = -2 - 2 = \boxed{-4}$

$F'(0) = 2(0) - 2 = 0 - 2 = \boxed{-2}$

$F'(2) = 2(2) - 2 = 4 - 2 = \boxed{2}$

$\longleftrightarrow$

$$k(z) = \frac{1-z}{2z}, \quad k'(-1);\ k'(1);\ k'(\sqrt{2})$$

(4)

Sol:

$$\frac{dy}{dx} = k'(z) = \lim_{h\to 0} \frac{k(z+h) - k(z)}{h}$$

$$k'(z) = \lim_{h\to 0} \frac{\frac{1-(z+h)}{2(z+h)} - \left(\frac{1-z}{2z}\right)}{h}$$

$$= \lim_{h\to 0} \frac{z(1-(z+h)) - (1-z)(z+h)}{2z(z+h)\cdot h}$$

$$= \lim_{h\to 0} \frac{z(1-z-h) - (z+h-z^2-zh)}{2hz(z+h)}$$

$$= \lim_{h\to 0} \frac{\cancel{z} - \cancel{z^2} - \cancel{zh} - \cancel{z} - \cancel{h} + \cancel{z^2} + \cancel{zh}}{2\cancel{h}z(z+h)}$$

$$= \lim_{h\to 0} \frac{-1}{2z(z+h)} = \frac{-1}{2z(z+0)}$$

$$\boxed{k'(z) = \frac{-1}{2z^2}}$$

$$k'(-1) = \frac{-1}{2(-1)^2} = \boxed{\frac{-1}{2}}$$

$$k'(1) = \frac{-1}{2(1)^2} = \boxed{\frac{-1}{2}}$$

$$k'(\sqrt{2}) = \frac{-1}{2(\sqrt{2})^2} = \boxed{\frac{-1}{4}}$$

6) $r(s) = \sqrt{2s+1}$, $r'(0), r'(1), r'(\frac{1}{2})$

Sol: $\frac{dy}{dx} = r'(s) = \lim_{h\to 0} \frac{r(s+h) - r(s)}{h}$

$$r'(s) = \lim_{h\to 0} \frac{\sqrt{2(s+h)+1} - \sqrt{2s+1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2s+2h+1} - \sqrt{2s+1}}{h} \times \frac{\sqrt{2s+2h+1} + \sqrt{2s+1}}{\sqrt{2s+2h+1} + \sqrt{2s+1}}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2s+2h+1}\right)^2 - \left(\sqrt{2s+1}\right)^2}{h\left[\sqrt{2s+2h+1} + \sqrt{2s+1}\right]}$$

$$= \lim_{h\to 0} \frac{\cancel{2s} + 2\cancel{h} + \cancel{1} - \cancel{2s} - \cancel{1}}{\cancel{h}\left[\sqrt{2s+2h+1} + \sqrt{2s+1}\right]}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2s+2h+1} + \sqrt{2s+1}}$$

$$= \frac{2}{\sqrt{2s+0+1} + \sqrt{2s+1}} = \frac{2}{\sqrt{2s+1} + \sqrt{2s+1}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{2s+1}} = \boxed{\frac{1}{\sqrt{2s+1}}}$$

$$r'(0) = \frac{1}{\sqrt{2(0)+1}} = \boxed{1}$$

$$r'(1) = \frac{1}{\sqrt{2(1)+1}} = \frac{1}{\sqrt{2+1}} = \boxed{\frac{1}{\sqrt{3}}}$$

$$r'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2\left(\frac{1}{2}\right)+1}} = \frac{1}{\sqrt{1+1}} = \boxed{\frac{1}{\sqrt{2}}}$$

Q. Find indicated derivatives? 19

10) $\frac{dv}{dt}$ if $v = t - \frac{1}{t}$

Sol:
$$\frac{dv}{dt} = \lim_{h\to 0} \frac{v(t+h) - v(t)}{h}$$

$$= \lim_{h\to 0} \frac{(t+h) - \frac{1}{t+h} - \left(t - \frac{1}{t}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(t+h)^2 - 1}{t+h} - \left(\frac{t^2-1}{t}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{t[(t+h)^2 - 1] - (t+h)(t^2-1)}{t(t+h)}}{h}$$

$$= \lim_{h\to 0} \frac{t[(t^2+2th+h^2)-1] - (t^3 - t + t^2h - h)}{ht(t+h)}$$

$$= \lim_{h\to 0} \frac{\cancel{t^3} + 2t^2h + th^2 - \cancel{t} - \cancel{t^3} + \cancel{t} - t^2h + h}{ht(t+h)}$$

$$= \lim_{h\to 0} \frac{t^2h + th^2 + h}{ht(t+h)} = \lim_{h\to 0} \frac{\cancel{h}(t^2 + th + 1)}{\cancel{h}t(t+h)}$$

$$= \lim_{h\to 0} \frac{t^2 + th + 1}{t(t+h)} = \frac{t^2 + 0 + 1}{t(t+0)}$$

$$= \frac{t^2+1}{t^2} = \frac{t^2}{t^2} + \frac{1}{t^2}$$

$$= \boxed{1 + \frac{1}{t^2}}$$ Ans

(11) $\frac{dp}{dq}$ if $p = q^{3/2}$

Sol: $\frac{dp}{dq} = \lim_{h \to 0} \frac{p(q+h) - p(q)}{h}$

$= \lim_{h \to 0} \frac{(q+h)^{3/2} - q^{3/2}}{h}$

$= \lim_{h \to 0} \frac{(q+h)(q+h)^{1/2} - q \cdot q^{1/2}}{h}$ (for rationalization purpose)

$= \lim_{h \to 0} \frac{q[(q+h)^{1/2} - q^{1/2}]}{h} + \frac{k[(q+h)^{1/2}]}{k}$

$= \lim_{h \to 0} \left[\frac{q[(q+h)^{1/2} - q^{1/2}]}{h} \times \frac{(q+h)^{1/2} + q^{1/2}}{(q+h)^{1/2} + q^{1/2}} \right] + (q+h)^{1/2}$

$= \lim_{h \to 0} \left[\frac{q[\{(q+h)^{1/2}\}^2 - \{(q^{1/2})\}^2]}{h[(q+h)^{1/2} + q^{1/2}]} \right] + (q+h)^{1/2}$

$= \lim_{h \to 0} \left[\frac{q(q+h-q)}{h[(q+h)^{1/2} + q^{1/2}]} \right] + (q+h)^{1/2}$

$= \lim_{h \to 0} \frac{qk}{k[(q+h)^{1/2} + q^{1/2}]} + (q+h)^{1/2}$

$= \lim_{h \to 0} \frac{q}{(q+h)^{1/2} + q^{1/2}} + (q+h)^{1/2}$

$= \frac{q}{q^{1/2} + q^{1/2}} + q^{1/2}$

$$\frac{dp}{dq} = \frac{q}{2q^{1/2}} + q^{1/2}$$

$$= \frac{q^{1-1/2}}{2} + q^{1/2}$$

$$= \frac{q^{1/2}}{2} + q^{1/2} = \frac{q^{1/2} + 2q^{1/2}}{2}$$

$$= \boxed{\frac{3}{2} q^{1/2}}$$ Ans

$\longleftrightarrow$

(13) Differentiate the function and find slope of tangent line at given value of indp. variable?

$$f(x) = x + \frac{9}{x} \quad , \quad x = -3$$

Tip: You can solve this by definition or directly to find derivative

Sol:

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(x + \frac{9}{x}\right)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{9}{x}\right)$$

$$f'(x) = 1 + 9\left(-\frac{1}{x^2}\right) = \boxed{1 - \frac{9}{x^2}}$$

At $x = -3$

$$f'(-3) = 1 - \frac{9}{(-3)^2} = 1 - \frac{9}{9} = 1 - 1 = \boxed{0}$$ Ans

$\longleftrightarrow$

16. $y = \dfrac{x+3}{1-x}$, $x = -2$.

Sol: $\dfrac{d}{dx}(y) = \dfrac{d}{dx}\left(\dfrac{x+3}{1-x}\right)$.

$$= \frac{(1-x)\frac{d}{dx}(x+3) - (x+3)\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$= \frac{(1-x)(1) - (x+3)(-1)}{(1-x)^2}$$

$$= \frac{1-\cancel{x}+\cancel{x}+3}{(1-x)^2}$$

$$y'(x) = \boxed{\frac{4}{(1-x)^2}}.$$

$$y'(-2) = \frac{4}{[1-(-2)]^2} = \frac{4}{(1+2)^2} = \boxed{\frac{4}{9}}$$ Ans

17. Differentiate function. Find eq. of tangent line at indicated point?

$y = f(x) = \dfrac{8}{\sqrt{x-2}}$; $(x,y) = (6,4)$

Sol: $m = f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\dfrac{8}{\sqrt{x+h-2}} - \dfrac{8}{\sqrt{x-2}}}{h}$$

$$= \lim_{h\to 0} \frac{8\sqrt{x-2} - 8\sqrt{x+h-2}}{h(\sqrt{x-2}\,\sqrt{x+h-2})}$$

P.T.O.

$$f'(x) = \lim_{h\to 0} \frac{8\left(\sqrt{x-2} - \sqrt{x+h-2}\right)}{h\left(\sqrt{x-2}\sqrt{x+h-2}\right)}$$

$$= \lim_{h\to 0} \frac{8\sqrt{x-2} - \sqrt{x+h-2}}{h\left(\sqrt{x-2}\sqrt{x+h-2}\right)} \times \frac{\sqrt{x-2} + \sqrt{x+h-2}}{\sqrt{x-2} + \sqrt{x+h-2}}$$

$$= \lim_{h\to 0} \frac{8\left(\sqrt{x-2}\right)^2 - \left(\sqrt{x+h-2}\right)^2}{h\left(\sqrt{x-2}\sqrt{x+h-2}\right)\left(\sqrt{x-2} + \sqrt{x+h-2}\right)}$$

$$= \lim_{h\to 0} \frac{8\left[\cancel{x} - \cancel{2} - \cancel{x} - h + \cancel{2}\right]}{\cancel{h}\left(\sqrt{x-2}\sqrt{x+h-2}\right)\left(\sqrt{x-2} + \sqrt{x+h-2}\right)}$$

$$= \lim_{h\to 0} \frac{-8}{\left(\sqrt{x-2}\right)\left(\sqrt{x-2}\right)\left(\sqrt{x-2} + \sqrt{x-2}\right)}$$

$$= \lim_{h\to 0} \frac{-\cancel{8}^{\,4}}{\left(\sqrt{x-2}\right)^2 \cancel{2}\sqrt{x-2}}$$

$$= \lim_{h\to 0} \frac{-4}{(x-2)\left(\sqrt{x-2}\right)}$$

At $x = 6$

$$f'(6) = \lim_{h\to 0} \frac{-4}{(6-2)\sqrt{6-2}} = \frac{-\cancel{4}}{4(2)}$$

$$= -\frac{1}{2}$$

$$m = \boxed{f'(6) = -\frac{1}{2}}$$

$\Rightarrow y - y_0 = m(x - x_0) \Rightarrow y - 4 = -\frac{1}{2}(x-6)$

$$y = 4 + \frac{x}{2} + 3$$

$$\boxed{y = -\frac{1}{2}x + 7}$$

(22) Find the values of derivative?

$$\left.\frac{dw}{dz}\right|_{z=4} \quad \text{if } w = z + \sqrt{z}$$

<u>Sol</u>:

$$\frac{dw}{dz} = \frac{d}{dz}\left(z+\sqrt{z}\right)$$

$$= \frac{d}{dz}(z) + \frac{d}{dz}(\sqrt{z})$$

$$= 1 + \frac{1}{2} z^{1/2 - 1} \frac{d}{dz}(z)$$

$$= 1 + \frac{1}{2} z^{-1/2} (1)$$

$$= 1 + \frac{1}{2\sqrt{z}}$$

At $z = 4$

$$\left.\frac{dw}{dz}\right|_{z=4} = \frac{1}{2\sqrt{z}} + 1$$

$$= \frac{1}{2\sqrt{4}} + 1 = \frac{1}{2(2)} + 1$$

$$= \frac{1}{4} + 1 = \frac{1+4}{4}$$

$$= \boxed{\frac{5}{4}}$$ Ans

25 Use Alternative Formula for derivatives

Formula $\boxed{f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}}$

$$g(x) = \frac{x}{x-1}.$$

Sol: $f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$

$$f(z) = \frac{z}{z-1}$$

$$f(x) = \frac{x}{x-1}$$

$$f'(x) = \lim_{z \to x} \frac{\frac{z}{z-1} - \frac{x}{x-1}}{z - x}$$

$$= \lim_{z \to x} \frac{z(x-1) - x(z-1)}{(z-1)(x-1)(z-x)}$$

$$= \lim_{z \to x} \frac{\cancel{zx} - z - \cancel{xz} + x}{(z-1)(x-1)(z-x)}$$

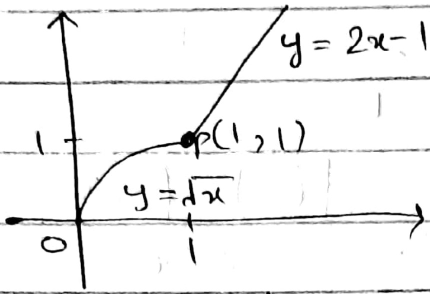
$$= \lim_{z \to x} \frac{x - z}{(z-1)(x-1)(z-x)} = \lim_{z \to x} \frac{-\cancel{(z-x)}}{(z-1)(x-1)\cancel{(z-x)}}$$

$$= \lim_{z \to x} \frac{-1}{(z-1)(x-1)}$$

$$= \frac{-1}{(x-1)(x-1)}$$

$$g'(x) = f'(x) = \boxed{\frac{-1}{(x-1)^2}}$$ Ans

39. Compute right hand and left hand derivatives to show that the functions are not differentiable at point P.



Sol:

$$y = \sqrt{x}$$

$$y = (x)^{1/2}$$

L.H.D: $\dfrac{dy}{dx} = \dfrac{1}{2} x^{1/2 - 1}$

$$= \frac{1}{2} x^{-1/2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}}}$$

Put P(1, 1)

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{2\sqrt{1}}$$

$$= \frac{1}{2}.$$

$$y = 2x - 1.$$

R.H.D: $\boxed{\dfrac{dy}{dx} = 2}$

As L.H.D $\neq$ R.H.D

Not differentiable

42. Determine if piecewise defined function is differentiable at the origin.

$$f(x) = \begin{cases} x^{2/3}, & x \geq 0 \\ x^{1/3}, & x < 0 \end{cases}$$

Sol: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^{1/3}$

$= (0)^{1/3} = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^{2/3}$

$= (0)^{2/3} = 0$

Because limits exist so

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\Rightarrow f'(x) = \lim_{x \to 0^-} x^{1/3} = \lim_{x \to 0} \frac{1}{3} x^{1/3 - 1}$

$= \lim_{x \to 0} \frac{1}{3} x^{-2/3}$

$= \frac{1}{3} (0)^{-2/3} = \infty$

$f'(x) = \lim_{x \to 0^+} x^{2/3} = \lim_{x \to 0^+} \frac{2}{3} x^{2/3 - 1}$

$= \lim_{x \to 0^+} \frac{2}{3} x^{-1/3}$

$= \frac{2}{3} (0)^{-1/3} = \infty$

Function is not differentiable at origin.

$\longleftrightarrow$

Ex 3.3

Find first and second derivatives?

(7) $w = 3z^{-2} - \frac{1}{z}$

Sol:

$w = 3z^{-2} - z^{-1}$

$\frac{dw}{dz} = w' = 3(-2z^{-3}) - (-z^{-2})$

$\boxed{w' = -6z^{-3} + z^{-2}}$

$w'' = -6(-3z^{-4}) - 2z^{-3}$

$= 18z^{-4} - 2z^{-3}$

$\boxed{w'' = \frac{18}{z^4} - \frac{2}{z^3}}$

(11) $r = \frac{1}{3s^2} - \frac{5}{2s} \Rightarrow r = \frac{1}{3}s^{-2} - \frac{5}{2}s^{-1}$

Sol: $\frac{dr}{ds} = \frac{1}{3}(-2s^{-3}) - \frac{5}{2}(-s^{-2})$

$\boxed{r' = -\frac{2}{3}s^{-3} + \frac{5}{2}s^{-2}}$

$r'' = -\frac{2}{\cancel{3}}(-\cancel{3}s^{-4}) + \frac{5}{\cancel{2}}(-\cancel{2}s^{-3})$

$= 2s^{-4} - 5s^{-3} = \boxed{\frac{2}{s^4} - \frac{5}{s^3}}$ Ans

9

(5) Find y' (a) By applying product rule
(b) by multiplying factors to produce a sum of Simpler terms to differentiate

$$y=(x^2+1)\left(x+5+\frac{1}{x}\right).$$

Sol:

(a) By Product rule,

$$y' = (x^2+1)\frac{d}{dx}\left(x+5+\frac{1}{x}\right) + \left(x+5+\frac{1}{x}\right)\frac{d}{dx}(x^2+1)$$

$$= (x^2+1)\left(1-\frac{1}{x^2}\right)+\left(x+5+\frac{1}{x}\right)(2x)$$

$$= x^2-\cancel{1}+\cancel{1}-\frac{1}{x^2}+2x^2+10x+2.$$

$$\boxed{y' = 3x^2+10x+2-\frac{1}{x^2}}$$ Ans

(b) By multiplying to produce sum of Simpler terms

$$y = x^3+5x^2+x+x+5+\frac{1}{x}$$

$$y = x^3+5x^2+2x+5+x^{-1}$$

Diff w.r.t 'x'

$$y' = 3x^2+10x+2-x^{-2}$$

$$\boxed{y' = 3x^2+10x+2-\frac{1}{x^2}}$$ Ans

30

(19) Find derivative of the function.

$$g(x) = \frac{x^2-4}{x+0.5}$$

Sol: Using quotient rule,

$$g'(x) = \frac{(x+0.5)\frac{d}{dx}(x^2-4) - (x^2-4)\frac{d}{dx}(x+0.5)}{(x+0.5)^2}$$

$$g'(x) = \frac{(x+0.5)(2x) - (x^2-4)(1)}{(x+0.5)^2}$$

$$= \frac{2x^2 + x - x^2 + 4}{(x+0.5)^2}$$

$$\boxed{g'(x) = \frac{x^2 + x + 4}{(x+0.5)^2}}$$

(21) $v = (1-t)(1+t^2)^{-1}$

Sol: By quotient rule

$$v' = 1-t/(1+t^2)$$

$$v' = \frac{(1+t^2)(-1) - (1-t)(2t)}{(1+t^2)^2}$$

$$= \frac{-1 - t^2 - 2t + 2t^2}{(1+t^2)^2}$$

$$\boxed{v' = \frac{t^2 - 2t - 1}{(1+t^2)^2}}$$ Ans

By product rule

$$v' = (1-t)[-(1+t^2)^{-2}(2t)] + (1+t^2)^{-1}(-1)$$

$$= 1-t\left[\frac{-2t}{(1+t^2)^2}\right] \frac{-1}{1+t^2}$$

$$= \frac{-2t}{(1+t^2)^2} + \frac{2t^2}{(1+t^2)^2} - \frac{1}{1+t^2}$$

$$= \frac{-2t + 2t^2 - (1+t^2)}{(1+t^2)^2}$$

$$= \frac{-2t + 2t^2 - 1 - t^2}{(1+t^2)^2}$$

$$\boxed{v' = \frac{t^2 - 2t - 1}{(1+t^2)^2}}$$ Ans

27. $y = \dfrac{1}{(x^2-1)(x^2+x+1)}$

Sol: $y = \dfrac{1}{x^4+x^3+\cancel{x^2}-\cancel{x^2}-x-1}$

$= \dfrac{1}{x^4+x^3-x-1}$

$y' = \dfrac{-1}{(x^4+x^3-x-1)^2}\dfrac{d}{dx}(x^4+x^3-x-1)$

$\left(\because \dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}\dfrac{d}{dx}(x)\right)$

$y' = \dfrac{-(4x^3+3x^2-1)}{(x^4+x^3-x-1)^2}$

$\boxed{y' = \dfrac{-4x^3-3x^2+1}{(x^4+x^3-x-1)^2}}$ Ans

30. Find derivative of all orders of the function

$y = \dfrac{x^5}{120}$

Sol: $y' = \dfrac{1}{120}(5x^4) = \dfrac{1}{24}x^4$

$y'' = \dfrac{1}{24}(4x^3) = \dfrac{1}{6}x^3$

$y''' = \dfrac{1}{6}(3x^2) = \dfrac{1}{2}x^2$

$y^4 = \dfrac{1}{2}(2x) = x$

$y^5 = 1$

$\boxed{y^{(n)} = 0 \quad \forall\ n \geq 6}$ Ans

(37) Find 1^{st} and 2^{nd} derivatives of the function

$$w = \left(\frac{1+3z}{3z}\right)(3-z)$$

Sol:

$$w = \frac{3 - z + 9z - 3z^2}{3z}$$

$$= \frac{3+8z-3z^2}{3z}$$

$$= \frac{\cancel{3}}{\cancel{3}z} + \frac{8\cancel{z}}{3\cancel{z}} - \frac{\cancel{3}z^{\cancel{2}}}{\cancel{3}\cancel{z}}$$

$$= \frac{1}{z} + \frac{8}{3} - z = z^{-1} + \frac{8}{3} - z$$

Diff w.r.t 'z'

$$\frac{dw}{dz} = w' = -z^{-2} - 1$$

$$w'' = -(-2z^{-3}) - 0$$

$$= 2z^{-3} = \boxed{\frac{2}{z^3}}$$ Ans

(42) Statement from book.

a) Find eq. for horizontal tangent line (I)

Find eqns for the lines that are perpendicular to these tangent lines (II)

Sol: $y = x^3 - 3x - 2$

$m = \boxed{y' = 3x^2 - 3}$ → slope

$m = 3x^2 - 3$

(P.T.O)

$$3x^2 - 3 = 0$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$\boxed{x = \pm 1}$$

At $x = 1$

$$y = 1^3 - 3(1) - 2$$

$$y = 1 - 3 - 2 = -4$$

$$\boxed{y = -4}$$

At $x = -1$

$$y = (-1)^3 - 3(-1) - 2$$

$$= -1 + 3 - 2 = 0$$

$$\boxed{y = 0}$$

b) Smallest slope of curve ?

Tip

Check that point of x ~~where the slope~~ ~~At what~~ ~~point~~ of x is small.

- We take $\boxed{x=0}$

$$\boxed{m = 3x^2 - 3}$$

At $x = 0$

$$3x^2 - 3 = m$$

$$3(0) - 3 = m$$

$$\boxed{m_1 = -3}$$

$$\boxed{m_2 = \frac{1}{3}}$$ → Normal line or perpendicular tangent's slope.

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{1}{3}(x - 0)$$

$$\boxed{y = \frac{1}{3}x - 2}$$

We know that $x_1 = 0$

then $f(x_1) = 0 - 0 - 2$

$y = -2$

Point $(0, -2)$

Ex 3.4

Statement from book:

Given $s = f(t)$

a) Find displacement and average velocity?

b) Find speed and acceleration

c) When, if ever, during interval does the body change direction?

1. $s = t^2 - 3t + 2$; $0 \leq t \leq 2$

Sol: a) $\Delta s = ?$, $V_{av} = ?$

$$\Delta s = f(t+\Delta t) - f(t) = f(t_2) - f(t_1)$$

$$\Delta s = f(2) - f(0)$$

$$= [2^2 - 3(2) + 2] - [0^2 - 3(0) + 2]$$

$$= 4 - 6 + \cancel{2} - \cancel{2}$$

$$\Rightarrow \boxed{\Delta s = -2 \text{ m}}$$

~~$V_{av} = \frac{ds}{dt} = \frac{d}{dt}(t^2 - 3t + 2) = 2t - 3$~~

$$\Rightarrow V_{av} = \frac{\Delta s}{\Delta t} = \frac{-2\text{ m}}{2\text{ s}} = -1\text{ m/s} = \boxed{-1\text{ ms}^{-1}}$$

b) $|v(t)| = ?$, $a(t) = ?$

$$\Rightarrow V = \frac{ds}{dt} = \frac{d}{dt}(t^2 - 3t + 2) = 2t - 3$$

$$|v(0)| = |2(0) - 3| = |-3| = 3\text{ ms}^{-1}$$

$$|v(2)| = |2(2) - 3| = |4 - 3| = 1\text{ ms}^{-1}$$

- Speed has no position
- velocity changes its position.

(P.T.O)

$\Rightarrow a(t) = \frac{dV}{dt} = \frac{d^2s}{dt^2} = 2$

$a(0) = 2\,ms^{-2}$

$a(2) = 2\,ms^{-2}$

c) $V(t) = ?$

$2t - 3 = 0 \Rightarrow 2t = 3$

$\boxed{t = \frac{3}{2}}$ → change in the position

$\boxed{V(t) = 0 \text{ in order to check change in the direction of body}}$

Interval : $0 < t < \frac{3}{2}$

$\frac{3}{2} < t < 2$

(4) $S = \frac{t^4}{4} - t^3 + t^2$, $0 \leq t \leq 3$

Sol: a) $\Delta S = ?$, $V_{av} = ?$

$\Delta S = f(t_2) - f(t_1) = f(3) - f(0)$

$= \left[\frac{3^4}{4} - 3^3 + 3^2\right] - [0 - 0 + 0]$

$= \frac{81}{4} - 27 + 9 = \frac{81 - 108 - 36}{4}$

$\boxed{\Delta S = \frac{9}{4}\,m}$

$V_{av} = \frac{\Delta S}{\Delta t} = \frac{9/4}{3-0} = \frac{\cancel{9}^{3}}{4(\cancel{3})} = \boxed{\frac{3}{4}\,ms^{-1}}$

(P.T.O)

b) $|v(t)| = ?$ $\quad a(t) = ?$

$$v(t) = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{t^4}{4} - t^3 + t^2\right]$$

$$= \frac{4t^3}{4} - 3t^2 + 2t$$

$$v(t) = t^3 - 3t^2 + 2t$$

$$|v(t)| = |t^3 - 3t^2 + 2t|$$

$$|v(0)| = |0 - 0 + 0| = \boxed{0\ ms^{-1}}$$

$$|v(3)| = |3^3 - 3(3)^2 + 2(3)| = |27 - 27 + 6|$$

$$= |6| = \boxed{6\ ms^{-1}}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 3t^2 - 6t + 2$$

$$a(0) = 0 - 0 + 2 = \boxed{2\ ms^{-2}}$$

$$a(3) = 3(3)^2 - 6(3) + 2$$

$$= 3(9) - 18 + 2 = 27 - 18 + 2$$

$$= \boxed{11\ ms^{-2}}$$

c) $v(t) = 0$

$$t^3 - 3t^2 + 2t = 0$$

$$t(t^2 - 3t + 2) = 0$$

$\boxed{t = 0}$, $\quad t^2 - 3t + 2 = 0$

$$t^2 - 2t - t + 2 = 0$$

$$t(t-2) - 1(t-2) = 0$$

$$(t-1)(t-2) = 0$$

$$\boxed{t = 1}, \quad \boxed{t = 2}$$

Interval: $0 < t < 1$ $\quad$ Body changes its position

$1 < t < 2$ $\quad$ at $t = 1$ and $t = 2$.

$2 < t < 3$.

5) $S = \frac{25}{t^2} - \frac{5}{t}$, $1 \leq t \leq 5$

Sol: $\Delta s = f(t_2) - f(t_1)$

$= f(5) - f(1)$

$= \left[\frac{25}{5^2} - \frac{5}{5}\right] - \left[\frac{25}{1^2} - \frac{5}{1}\right]$

$= 0 - [25 - 5] = \boxed{-20\ m}$

$V_{av} = \frac{\Delta s}{\Delta t} = \frac{-20}{4} = \boxed{-5\ ms^{-1}}$

b) $|v(t)| = ?$ $a = ?$

$V(t) = \frac{ds}{dt} = -\frac{25(2)}{t^3} + \frac{5}{t^2}$

$\boxed{V(t) = -\frac{50}{t^3} + \frac{5}{t^2}}$

$|v(t)| = \left|-\frac{50}{t^3} + \frac{5}{t^2}\right|$

$|v(1)| = |-50 + 5| = |-45| = \boxed{45\ ms^{-1}}$

$|v(5)| = \left|\frac{-50}{5^3} + \frac{5}{5^2}\right| = \left|-\frac{2}{5} + \frac{1}{5}\right| = \boxed{\frac{1}{5}\ ms^{-1}}$

$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \frac{50(3)}{t^4} - \frac{5(2)}{t^3}$

$= 150t^{-4} - 10t^{-3}$

$a(1) = 150 - 10 = 140\ ms^{-2}$

$a(5) = 150(5)^{-4} - 10(5)^{-3}$

$= \frac{150}{625} - \frac{10}{125} = \frac{6}{25} - \frac{2}{25} = \boxed{\frac{4}{25}\ ms^{-2}}$

c) $V = 0$

$$-\frac{50}{t^3} + \frac{5}{t^2} = 0$$

$$\frac{-50 + 5t}{t^3} = 0 \Rightarrow -50 + 5t = 0$$

$$5t = 50$$

$$\boxed{t = 10s} \rightarrow \text{no change in body's position}$$

Position does not change.

$\longleftrightarrow$

EX 3.5

6) Find $\frac{dy}{dx}$?

$$y = x^2 \cot x - \frac{1}{x^2}$$

Sol:

$$y' = x^2 \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(x^2) - \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= x^2(-\operatorname{cosec}^2 x) + \cot x(2x) - \left(\frac{-2}{x^3}\right)$$

$$\boxed{y' = -x^2 \operatorname{Cosec}^2 x + 2x\cot x + \frac{2}{x^3}}$$ Ans

$\longleftrightarrow$

8) $g(x) = \frac{\cos x}{\sin^2 x}$

Sol:

$$g(x) = \frac{1}{\sin x} \frac{\cos x}{\sin x} = \operatorname{cosec} x \cot x.$$

$$g'(x) = \operatorname{cosec} x \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(\operatorname{cosec} x)$$

$$g'(x) = \operatorname{cosec} x\left(-\operatorname{cosec}^2 x\right) + \cot x\left(-\operatorname{cosec} x \cot x\right)$$

$$= \operatorname{cosec}^3 x - \cot^2 x \operatorname{cosec} x$$

$$\boxed{g'(x) = \operatorname{cosec} x\left(\operatorname{cosec}^2 x - \cot^2 x\right)}$$ Ans

(14) $y = \dfrac{\cos x}{x} + \dfrac{x}{\cos x}$

Sol:

$$y' = \frac{x(-\sin x) - \cos x(1)}{x^2} + \frac{\cos x(1) - x(-\sin x)}{(\cos x)^2}$$

$$\boxed{y' = \frac{-x\sin x - \cos x}{x^2} + \frac{\cos x + x\sin x}{(\cos x)^2}}$$ Ans

(17) $f(x) = x^3 \sin x \cos x$.

Sol:

$$f'(x) = x^3 \sin x \frac{d}{dx}(\cos x) + x^3 \cos x \frac{d}{dx}(\sin x) + \sin x \cos x \frac{d}{dx}(x^3)$$

Formula

$$\frac{d}{dx}(u \cdot v \cdot t) = u \cdot v \cdot \frac{d}{dx}(t) + u \cdot t \frac{d}{dx}(v) + v \cdot t \cdot \frac{d}{dx}(u)$$

$$= x^3 \sin x(-\sin x) + x^3 \cos x(\cos x) + \sin x \cos x(3x^2)$$

$$\boxed{f'(x) = -x^3 \sin^2 x + x^3 \cos^2 x + 3x^2 \sin x \cos x}$$ Ans

(~~P.T.O~~)

22 Find $\frac{ds}{dt} = ?$

$S = \frac{\sin t}{1-\cos t}$

Sol: $S' = \frac{(1-\cos t)(\cos t) - \sin t(\sin t)}{(1-\cos t)^2}$

$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2}$

$= \cancel{\frac{(1-\cos t)\cos t - \sin^2 t}{(1-\cos t)^2}}$ $\frac{\cos t - (\cos^2 t + \sin^2 t)}{(1-\cos t)^2}$

$= \frac{\cos t - 1}{(1-\cos t)^2} = \frac{-(1-\cos t)}{(1-\cos t)^{\cancel{2}}}$

$= \boxed{\frac{-1}{1-\cos t} \text{ or } \frac{1}{\cos t - 1}}$

Ans

$\longleftrightarrow$

23 Find $\frac{dr}{d\theta} = ?$ $r = 4 - \theta^2 \sin\theta.$

Sol: $r' = 0 - [\theta^2(\cos\theta) + \sin\theta(2\theta)]$

$= -\theta^2\cos\theta - 2\theta\sin\theta.$

$\boxed{r' = -\theta(\theta\cos\theta + 2\sin\theta)}$

$\longleftrightarrow$

(31) To find $\frac{dp}{dq} = ?$

$$P = \frac{q \sin q}{q^2 - 1}$$

Sol:

$$p' = \frac{(q^2-1)[q(\cos q) + \sin q] - q \sin q (2q)}{(q^2-1)^2}$$

$$= \frac{(q^2-1)(q \cos q + \sin q) - 2q^2 \sin q}{(q^2-1)^2}$$

$$= \frac{q \cos q (q^2-1) + q^2 \sin q - \sin q - 2q^2 \sin q}{(q^2-1)^2}$$

$$= \frac{q \cos q (q^2-1) + q^2 \sin q - \sin q}{(q^2-1)^2}$$

$$\boxed{p' = \frac{q \cos q (q^2-1) + \sin q (q^2 - 1)}{(q^2-1)^2}}$$

Ans

(35) Graph curve over given interval together with tangent lines at the given values of x.

$y = \sin x \quad , -3\pi/2 \le x \le 2\pi$

$x = -\pi, 0, 3\pi/2.$

Sol:

$y' = \cos x$

At $x = -\pi$

~~Slope of tangent at $x - \pi$,~~

$y' = \cos(-\pi) = -1 \; m_1$

Slope of tangent at $x = 0$,

$y' = \cos(0) = 1 \; m_2$

Slope of tangent at $x = 3\pi/2$

$y' = \cos\left(\frac{3\pi}{2}\right) = 0 \; m_3$

42

At $x = -\pi$

$y = \sin(-\pi) = 0 \qquad (\overset{x_1}{-\pi}, \overset{y_1}{0})$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x + \pi)$

$\boxed{y = -x - \pi} \rightarrow$ Eq of tangent line

At $x = 0$

$y = \sin(0) = 0 \qquad (\overset{x_1}{0}, \overset{y_1}{0})$

$y - y_1 = m(x - x_1)$

$y = 1(x - 0)$

$\boxed{y = x} \rightarrow$ Eq of tangent line

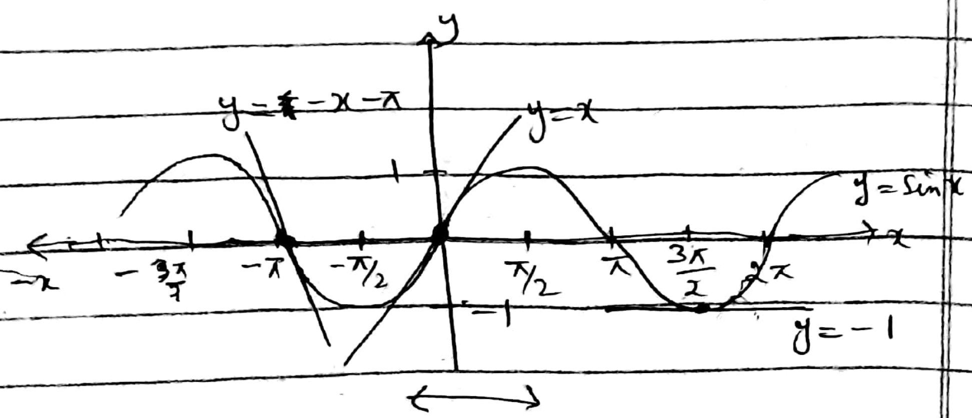
At $x = \frac{3\pi}{2}$

$y = \sin\left(\frac{3\pi}{2}\right) = -1 \qquad \left(\overset{x_1}{\frac{3\pi}{2}}, \overset{y_1}{-1}\right)$

$y - y_1 = m(x - x_1)$

$y + 1 = 0$

$\boxed{y = -1} \rightarrow$ Eq of tangent line



39 Do graphs of function have any horizontal tangent lines in interval $0 \le x \le 2\pi$. If so where? If not, why not?

$y = x + \sin x$

Sol: $\boxed{y' = 1 + \cos x}$

To Check Horizontal Tangent line,

$1 + \cos x = 0$ ($\because m = 0$ for horizontal tangent line)

$\cos x = -1$

At $\boxed{x = \pi.}$

Yes, at $x = \pi$, it exists

$\longleftrightarrow$

44 $y = \dfrac{\cos x}{3 - 4\sin x}$

Sol: $y' = \dfrac{(3-4\sin x)(-\sin x) - \cos x(-4\cos x)}{(3-4\sin x)^2}$

$= \dfrac{-3\sin x + 4\sin^2 x + 4\cos^2 x}{(3-4\sin x)^2}$

$= \dfrac{-3\sin x + 4(\sin^2 x + \cos^2 x)}{(3-4\sin x)^2} = \dfrac{-3\sin x + 4}{(3-4\sin x)^2}$

$\Rightarrow 0 = \dfrac{-3\sin x + 4}{(3-4\sin x)^2}$

$4 - 3\sin x = 0$

$4 = 3\sin x \Rightarrow \boxed{\sin x = \dfrac{4}{3}}$

No, it does not exist.

$\longleftrightarrow$

44.

49. Find the limits

$$\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$$

Sol:

$$= \sin\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \sin(0) = \boxed{0} \text{ Ans}$$

50. $$\lim_{x \to -\pi/6} \sqrt{1 + \cos(\pi \operatorname{cosec} x)}$$

Sol:

$$= \sqrt{1 + \cos\left(\pi \operatorname{cosec}\left(-\frac{\pi}{6}\right)\right)}$$

$$= \sqrt{1 + \cos(\pi(-2))}$$

$$= \sqrt{1 + \cos(-2\pi)}$$

$$= \sqrt{1+1} = \boxed{\sqrt{2}} \text{ Ans}$$

Ex 3.6

1. Given $y = f(u)$
and $u = g(x)$
Find $\frac{dy}{dx} = ?$

Formula: $\boxed{\frac{dy}{dx} = f'(g(x))\,g'(x)}$

$y = 6u - 9$, $u = \frac{1}{2}x^4$.

Sol:

Here $y = f(u) = 6u - 9$

$u = g(x) = \frac{1}{2}x^4$

By Chain Rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(g(x))\,g'(x)$

$= 6 \cdot \frac{\cancel{4}^{2}x^3}{\cancel{2}}$

$\boxed{\frac{dy}{dx} = 12x^3}$ Ans

5. $y = \sqrt{u}$, $u = \sin x$.

Sol: $y = f(u) = \sqrt{u}$, $u = g(x) = \sin x$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \cos x$

$= \frac{\cos x}{2\sqrt{u}}$

$\boxed{\frac{dy}{dx} = \frac{\cos x}{2\sqrt{\sin x}}}$ Ans

(8) $y = -\sec u \;,\; u = \frac{1}{x} + 7x$

Sol: $y = f(u) = -\sec u$

$u = g(x) = \frac{1}{x} + 7x$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$= \left(-\sec u \tan u\right)\left(-\frac{1}{x^2} + 7\right)$$

$$\boxed{\frac{dy}{dx} = -\sec\left(\frac{1}{x} + 7x\right)\cdot\tan\left(\frac{1}{x} + 7x\right)\left(-\frac{1}{x^2} + 7\right)}$$

Ans

(9) Write function in form $y = f(u)$ and $u = g(x)$. Then Find $\frac{dy}{dx}$?

$y = (2x+1)^5$.

Sol: Let

$u = g(x) = 2x+1$

$y = f(u) = u^5$.

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = f'(g(x))\cdot g'(x)$$

$$\Rightarrow \frac{dy}{dx} = 5u^4 \cdot 2$$

$$= 5(2x+1)^4 \cdot 2$$

$$\boxed{\frac{dy}{dx} = 10(2x+1)^4}$$ Ans

12) $y = \left(\dfrac{\sqrt{x}}{2} - 1\right)^{-10}$.

<u>Sol:</u>

$\cancel{y = f(u) = (u)^{-9}}$

$u = g(x) = \dfrac{\sqrt{x}}{2} - 1$

$y = f(u) = u^{-10}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$= -10u^{-11}\left(\frac{1}{2}\left(\frac{1}{2}x^{-1/2}\right)\right)$

$= -10u^{-11}\left(\frac{1}{4}x^{-1/2}\right)$

$\boxed{\dfrac{dy}{dx} = \dfrac{-5}{2\sqrt{x}}\left(\dfrac{\sqrt{x}}{2} - 1\right)^{-11}}$ Ans

15) $y = \sec(\tan x)$.

<u>Sol:</u> $u = g(x) = \tan x$.

$y = f(u) = \sec u$.

$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= \sec u \tan u \cdot \sec^2 x$

$\boxed{\dfrac{dy}{dx} = \sec(\tan x)\tan(\tan x)\sec^2 x}$ Ans

(21) Find derivatives of functions

$$s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t.$$

<u>Sol:</u>

$$\frac{ds}{dt} = \frac{ds}{du}\cdot\frac{du}{dt}$$

$$= \frac{4}{3\pi}\cos 3t\cdot\frac{d}{dt}(3t) + \frac{4}{5\pi}(-\sin 5t)\frac{d}{dt}(5t)$$

$$= \cancel{3}\cdot\frac{4}{\cancel{3}\pi}\cos 3t + 5\cdot\frac{4}{\cancel{5}\pi}(-\sin 5t)$$

$$= \frac{4}{\pi}\cos 3t - \frac{4}{\pi}\sin 5t$$

$$= \frac{4}{\pi}\left[\cos 3t - \sin 5t\right]$$

$$\boxed{\frac{ds}{dt} = \frac{4}{\pi}(\cos 3t - \sin 5t)}$$ Ans

49

(40) $q(t) = \cot\left(\frac{\sin t}{t}\right)$

Sol:

$$\frac{dq}{dt} = -\operatorname{cosec}^2\left(\frac{\sin t}{t}\right) \cdot \frac{d}{dt}\left(\frac{\sin t}{t}\right)$$

$$= -\operatorname{cosec}^2\left(\frac{\sin t}{t}\right)\left[\frac{t\cos t - \sin t}{t^2}\right]$$

$$\boxed{q'(t) = -\operatorname{cosec}^2\left(\frac{\sin t}{t}\right)\left(\frac{t\cos t - \sin t}{t^2}\right)}$$

(58) Find $\frac{dy}{dt}$

$$y = \sqrt{3t + \sqrt{2 + \sqrt{1-t}}}$$

Sol: $\frac{dy}{dt} = \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}} \cdot \frac{d}{dt}\sqrt{3t+\sqrt{2+\sqrt{1-t}}}$

$$= \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}}\left[\frac{1}{2}\left(3t+\sqrt{2+\sqrt{1-t}}\right)^{-1/2}(3)\right]$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}} \frac{d}{dt}\left(3t+\sqrt{2+\sqrt{1-t}}\right)$$

$$= \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}}\left[3 + \frac{d}{dt}\sqrt{2+\sqrt{1-t}}\right]$$

$$= \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}}\left[3 + \frac{1}{2\sqrt{2+\sqrt{1-t}}}\frac{d}{dt}\left(2+\sqrt{1-t}\right)\right]$$

$$= \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}}\left[3 + \frac{1}{2\sqrt{2+\sqrt{1-t}}}\left(\frac{1}{2\sqrt{1-t}}(-1)\right)\right]$$

$$\boxed{\frac{dy}{dt} = \frac{1}{2\sqrt{3t+\sqrt{2+\sqrt{1-t}}}}\left[3 + \frac{-1}{4\sqrt{2+\sqrt{1-t}}\sqrt{1-t}}\right]}$$ Ans

61 Find y''.

$$y = \frac{1}{9} \cot(3x-1)$$

Sol:

$$y' = \frac{-1}{9} \operatorname{cosec}^2(3x-1)\,(3)$$

$$= \frac{-1}{3} \operatorname{cosec}^2(3x-1)$$

$$y'' = -\frac{1}{3}\left[2 \operatorname{cosec}(3x-1) \cdot [\operatorname{cosec}(3x-1)]'\right]$$

$$= \frac{-2}{3} \operatorname{cosec}(3x-1)\left[-\operatorname{cosec}(3x-1)\cot(3x-1)\,(3)\right]$$

$$y'' = 2\operatorname{cosec}^2(3x-1)\cot(3x-1)$$

68 Find the value of $(f \circ g)'$ at given value of x.

$$f(u) = 1 - \frac{1}{u}, \quad u = g(x) = \frac{1}{1-x}, \quad x = -1$$

Sol: $(f \circ g)'(x) = f'(g(x))\, g'(x)$

$$g'(x) = -1\,(1-x)^{-2}(-1)$$

$$= \frac{1}{(1-x)^2} \Rightarrow g'(-1) = \frac{1}{4}$$

$$g(-1) = \frac{1}{2}$$

$$f'(u) = -\left(-\frac{1}{u^2}\right) = \frac{1}{u^2} \Rightarrow f'(g(-1)) = f'\left(\frac{1}{2}\right) = 4$$

$(f \circ g)'(x) = f'(g(x))\, g'(x)$

$(f \circ g)'(-1) = f'(g(-1)) \cdot g'(-1)$

$= 4 \cdot \frac{1}{4} = \boxed{1}$

$\longleftrightarrow$

Topic 3.7 Implicit Differentiation

Explicit Diff: Used for functions where you have a clear expression for y in terms of x.

Implicit Diff:
- Used when the relationship b/t x and y is not explicitly defined.
- Useful when dealing with eqns involving multiple variables
- Implicit diff often involves using chain rule.

Rules To Solve (Pg 150)

1. Diff b/s of eqn w.r.t 'x' treating 'y' as diff func. of x
2. Collect the terms of $\frac{dy}{dx}$ on one side of eqn and solve for $\frac{dy}{dx}$.

Example 2 + 3 (from book) → Pg 150.

Ex 3.7

(1) Find $\frac{dy}{dx}$. (use implicit diff.)

$x^2y + xy^2 = 6.$

Sol:
$$\frac{d}{dx}(x^2y + xy^2) = \frac{d}{dx}(6).$$

$$x^2\frac{dy}{dx} + y(2x) + x\left[2y\frac{dy}{dx}\right] + y^2(1) = 0$$

$$(x^2 + 2xy)\frac{dy}{dx} + 2xy + y^2 = 0$$

$$(x^2+2xy)\frac{dy}{dx} = -2xy - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2+2xy}}$$

(7) $y^2 = \frac{x-1}{x+1}$

Sol:
$$2y\frac{dy}{dx} = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$\cancel{2}y\frac{dy}{dx} = \frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2} = \frac{\cancel{2}}{(x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{y(x+1)^2}}$$

(12) $x^4 + \sin y = x^3 y^2$

Sol: $4x^3 + \cos y \, y' = 3x^2 y^2 + x^3(2y y')$

$$[\cos y - 2x^3 y] y' = 3x^2 y^2 - 4x^3$$

$$\boxed{y' = \frac{3x^2 y^2 - 4x^3}{\cos y - 2x^3 y}}$$

(17) Find $\frac{dr}{d\theta}$

$\sin(r\theta) = \frac{1}{2}$

Sol: $\cos(r\theta) \frac{d}{d\theta}(r\theta) = \frac{d}{d\theta}\left(\frac{1}{2}\right)$

$$\cos(r\theta)[r + \theta r'] = 0$$

$$\cos(r\theta) r + \theta \cos(r\theta) r' = 0$$

$$r' = -\frac{r \cancel{\cos} r\theta}{\theta \cancel{\cos} r\theta}$$

$$\boxed{r' = -\frac{r}{\theta}}$$

20 $\frac{dy}{dx} = ?$ $\frac{d^2y}{dx^2} = ?$

$$x^{2/3} + y^{2/3} = 1$$

Sol: $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$

$$y' = \frac{-\cancel{2/3}\,x^{-1/3}}{\cancel{2/3}\,y^{-1/3}}$$

$$= -\frac{y^{1/3}}{x^{1/3}} = \boxed{-\left(\frac{y}{x}\right)^{1/3}}$$

$$y'' = -\frac{d}{dx}\left(-\frac{y^{1/3}}{x^{1/3}}\right)$$

$$= -\left[\frac{x^{1/3}\left(\frac{1}{3}y^{-2/3}y'\right) - y^{1/3}\left(\frac{1}{3}x^{-2/3}\right)}{x^{2/3}}\right]$$

$$= -\left[\frac{x^{1/3}\left(\frac{1}{3}y^{-2/3}\cdot\left(-\frac{y^{1/3}}{x^{1/3}}\right)\right) - y^{1/3}\left(\frac{1}{3}x^{-2/3}\right)}{x^{2/3}}\right]$$

$$y'' = \frac{y^{1/3}}{3x^{4/3}} + \frac{1}{3y^{1/3}x^{2/3}}$$

29 Find slope of curve at given points

$y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$

Sol: $2yy' + 2x = 4y^3 y' - 2$

$$y' = \frac{-2 - 2x}{2y - 4y^3}$$

$$= \frac{\cancel{2}(-1-x)}{\cancel{2}(y-2y^3)} = \frac{-1-x}{y-2y^3}$$

$$y'\Big|_{(-2,1)} = \frac{-1+2}{1-2} = \boxed{-1}$$

$$y'\Big|_{(-2,-1)} = \frac{-1+2}{-1-2(-1)} = \boxed{1}$$