234513

No. of Continuation Sheets attached ______	Sheet No. ....

PUNJAB COLLEGES

Name ______ Roll No.: ______

Class: ______ Section (if any): ______

Subject: ______ Date: ______

Student's Signature: ....

Invigilator's Name: .... Signature: ....

AWARD LIST

(To be filled by the Examiner)

Q. No.	01 (MCQs)	02	03	04	05	06	07	08	09	10	Total Marks
Marks Obtained											

Examiner's Name: .... Signature: ....

Please start writing from here

* Fourier Series of $f(x)$ over $(-P, P)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{P} + b_n \sin \frac{n\pi x}{P} \right)$$

where $a_0 = \frac{1}{P} \int_{-P}^{P} f(x)\, dx$

$$a_n = \frac{1}{P} \int_{-P}^{P} f(x) \cos \frac{n\pi x}{P}\, dx$$

$$b_n = \frac{1}{P} \int_{-P}^{P} f(x) \cdot \sin \frac{n\pi x}{P}\, dx$$

* Ex. Expand $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 \le x < \pi \end{cases}$

in a fourier Series.

Sol:- $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi} \int_{-\pi}^{0} 0 . dx + \int_{0}^{\pi} (\pi - x)\, dx$

$$= \frac{1}{\pi} \left(\pi x - \frac{x^2}{2} \right)_0^{\pi} = \pi/2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{\pi}\int_{0}^{\pi}(\pi - x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(\pi - x)\cdot\frac{\sin nx}{n}\Big|_0^{\pi} - \frac{1}{n}\int \sin nx\,dx\right]$$

$$= -\frac{1}{n\pi}\,\frac{\cos nx}{n}\Big|_0^{\pi} = -\frac{1}{n\pi}\left[\frac{\cos n\pi}{n} - \frac{\cos(0)}{n}\right]$$

$$= -\frac{1}{n\pi}\left[\frac{(-1)^n}{n} - \frac{1}{n}\right]$$

$$= +\frac{1}{n^2\pi}\left(1 - (-1)^n\right) = \frac{1-(-1)^n}{n^2\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(\pi - x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi}(\pi - x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(\pi - x)\left(-\frac{\cos nx}{n}\right)\Big|_0^{\pi} + \int_0^{\pi}\frac{\cos nx}{n}(-1)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(0 + \frac{\pi}{n}\right) - \frac{1}{n}\int_0^{\pi}\cos nx\,dx\right.$$

$$= \frac{1}{n} - \frac{1}{n}\left|\frac{\sin nx}{n}\right|_0^{\pi} = \frac{1}{n} - 0 = \frac{1}{n}$$

Hence $f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{1-(-1)^n}{n^2\pi}\cdot\cos nx + \frac{1}{n}\sin nx\right\}$

* Fourier Cosine Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{p} x$$

$$a_0 = \frac{2}{p} \int_0^p f(x)\, dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos \frac{n\pi}{p} x \, dx$$

* Fourier Sine Series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{p} x$$

$$b_n = \frac{2}{p} \int_0^p f(x) \cdot \sin \frac{n\pi}{p} x \, dx$$

Ex. $f(x) = x \; ; -2 < x < 2$

Expand the above odd function in a fourier series.

$$b_n = \int_0^2 x \cdot \sin \frac{n\pi}{2} x \, dx = \frac{4(-1)^{n+1}}{n\pi}$$

Hence $f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi}{2} x.$

Ex. $f(x) = x^2 ; 0 < x < L$

Expand in a cosine Series.

$$a_0 = \frac{2}{L}\int_0^L x^2\,dx = \frac{2}{L}\left|\frac{x^3}{3}\right|_0^L$$

$$= \frac{2}{L}\left(\frac{L^3}{3}\right) = \frac{2L^2}{3}$$

$$a_n = \int_0^L x^2 \cos\frac{n\pi}{L}x\,dx = \frac{4L^2(-1)^n}{n^2\pi^2}$$

Hence $f(x) = \frac{L^2}{3} + \frac{4L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi}{L}x$