Coconut

Chapter 15

OSCILLATIONS

ϕ also called epoch.

$(\omega t+\phi) \rightarrow$ phase

$\phi \rightarrow$ phase constant

$x_m =$ amplitude

$x = x_m \cos(\omega t + \phi)$

$\frac{dx}{dt} = v = -\omega x_m \sin(\omega t + \phi)$

$\frac{dv}{dt} = a = -\omega^2 x_m \cos(\omega t + \phi)$

$v(t) = -\omega x(t)$

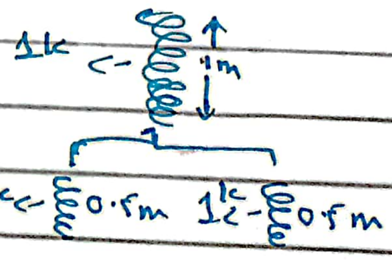
$a(t) = -\omega^2 x(t)$

$\omega = \sqrt{\frac{k}{m}}$, $\omega = 2\pi f$

$2\pi f = \sqrt{\frac{k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

One spring with 'k' = 2 springs of each '2k' in series



$\Rightarrow$ cutting the spring in half will double the spring constant.

$\frac{v(t)}{x(t)} = \frac{-\omega x_m \sin\phi}{x_m \cos\phi} = -\omega \tan\phi$

$F = -kx$, $F_{max} = m\, a_{max}$, $a_m = -\omega^2 x_m$

Angular Frequency

$$x(t) = x(t+T)$$

cosine repeats when argument repeats by 2π

$$\omega t = \omega t$$

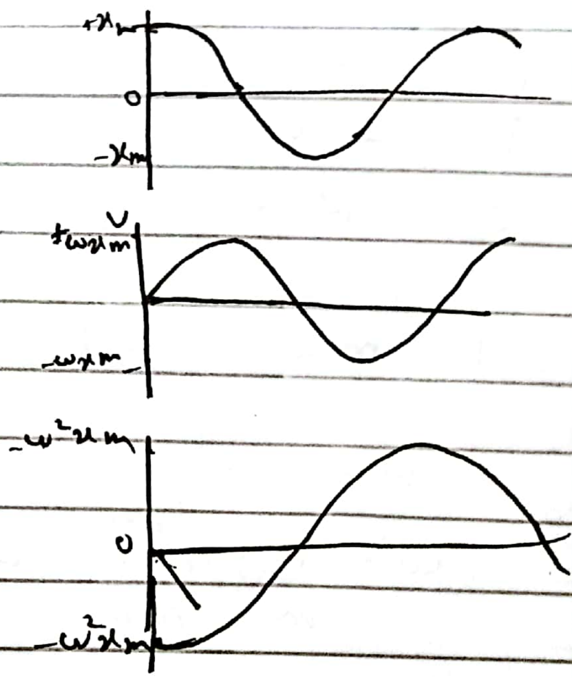
$$\omega(t+T) = \omega t + 2\pi$$

$$\omega t + \omega T = \omega t + 2\pi$$

$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Velocity



$$x(t) = x_m \cos(\omega t + \phi)$$

Taking derivative

$$v(t) = \frac{d\,x(t)}{dt} = -x_m \omega \sin(\omega t + \phi)$$

Acceleration

> Phase shift = $\pi/2$

$$a(t) = \frac{d\,v(t)}{t} = -\omega^2 x_m \cos(\omega t + \phi)$$

$$a(t) = -\omega^2 x(t)$$

SHM EQUATION — Force Law

$F = ma = m(-\omega^2 x) = -(m\omega^2)x$

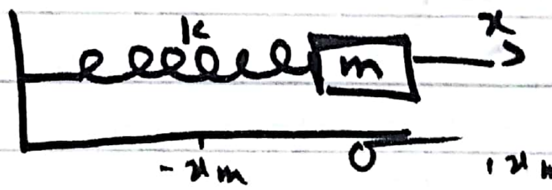
mass and ω are constant

$F = -Kx$

$K = m\omega^2$

$\omega^2 = \frac{K}{m} \Rightarrow \omega = \sqrt{\frac{K}{m}}$

MASS - SPRING SYSTEM



$F \propto x$

$F \propto -x(t)$

$F = -Kx(t)$ (Hooke's Law) ①

$F = ma = m(-\omega^2 x(t))$ - ②

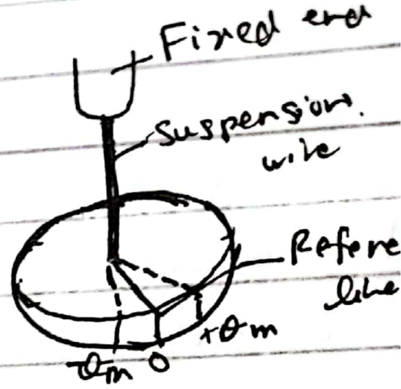
$K = m\omega^2$

$\omega^2 = \frac{K}{m} \Rightarrow \sqrt{\frac{K}{m}} = \omega$

As $T = \frac{2\pi}{\omega}$ $\sqrt{\frac{m}{K}}$

$T = 2\pi\sqrt{\frac{m}{K}}$

TORSIONAL PENDULUM / ANGULAR SHO



$F = -kx$

Relplace $F \to \tau$ (Restoring torque)

$x \to \theta$

$\tau = -\kappa\theta(t)$

κ is Greek $\to$ kappa

↳ called "Torsional Constant"

Depends

↳ length of wire

↳ Diameter of wire

↳ material of suspension wire

As $T = 2\pi\sqrt{\frac{m}{k}}$

Replace $k \to \kappa$; $m \to I$ (rotational Inertia)

$T = 2\pi\sqrt{\frac{I}{\kappa}}$ (Torsional Pendulum)

We know $\tau = I\alpha$

$I\alpha = -\kappa\theta(t) \Rightarrow \alpha = -\kappa\theta(t)/I$ — ①

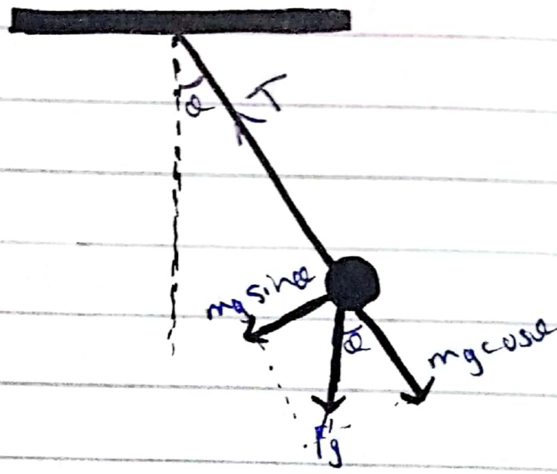
also $\alpha = -\omega^2\theta(t)$ — ②

Comparing

$\omega^2 = \frac{\kappa}{I} \Rightarrow \omega = \sqrt{\frac{\kappa}{I}}$

$I = \frac{1}{2}MR^2$ for disk with rotational axis in middle of disk

SIMPLE PENDULUM



$mg\cos\theta$ cancels T

$F_r = -mg\sin\theta$

$\tau = -Lmg\sin\theta$

As $\tau = I\alpha$

$I\alpha = -Lmg\sin\theta$

$$\alpha = -\frac{L\cdot mg\sin\theta}{I}$$

For small Angle $\sin\theta \approx \theta$

$$\alpha = -\frac{Lmg}{I}\theta$$

$$\alpha = -\omega^2 x(t)$$

$$\omega^2 = \frac{Lmg}{I}$$

$$\omega = \sqrt{\frac{Lmg}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{Lmg}}$$

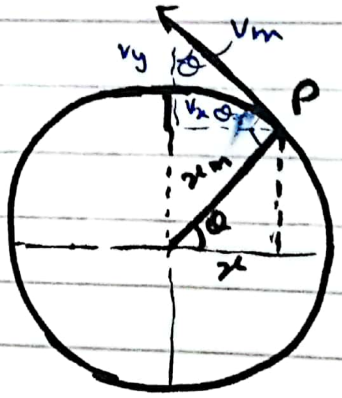
$I = mL^2$

$$T = 2\pi\sqrt{\frac{mL^2}{Lmg}} = 2\pi\sqrt{\frac{l}{g}}$$

SHM is projection of uniform circular motion on diameter of circle in which the circular motion occurs.

UNIFORM CIRCULAR MOTION AND SHM

using Projection method



Displacement

$$\cos\theta = \frac{x}{x_m}$$

$$v = \frac{d}{t}$$

$$\omega = \theta/t$$

$$\theta = \omega t$$

$$x(t) = x_m \cos\theta$$

$$x(t) = x_m \cos(\omega t + \phi)$$

Velocity

$$\sin\theta = \frac{V(t)}{V_m}$$, V is directed towards $-x$-axis

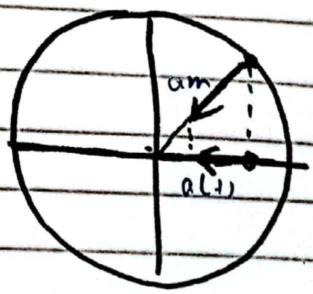
$$\sin\theta = -V(t)/V_m$$

$$V(t) = -V_m \sin\theta$$

we know, $V_m = \omega X_m$ and $\theta = \omega t$

$$V(t) = -\omega x_m \sin(\omega t + \phi)$$

Acceleration



$$\cos\theta = -\frac{a(t)}{a_m}$$ ($-$ve x axis)

$$a(t) = -a_m \cos\theta$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

Electric Field Due to Electric Dipole

Electric Dipole

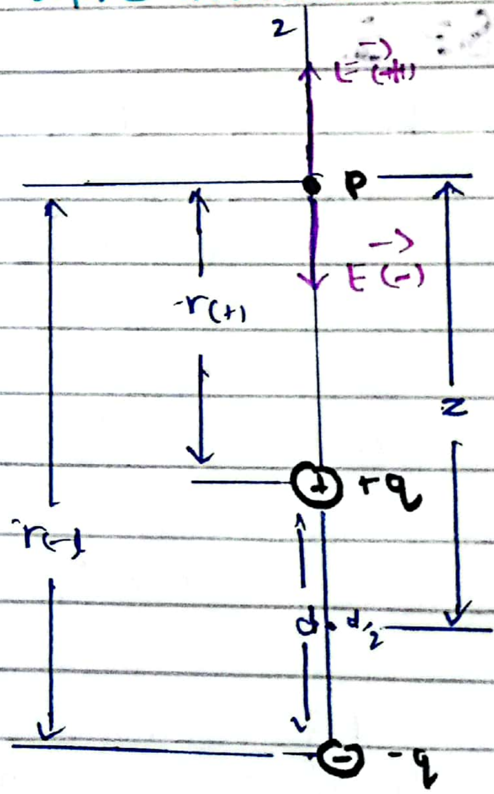
Combination of two charges with equal magnitude and opposite sign placed at distance "d" apart is called an electric dipole.

Electric Dipole Moment

The product of either charge and the displacement between them is called dipole moment

$\vec{p} = q\vec{d}$

From -ve to the charge.



CASE-I

$E_{net} = E_{(+)} - E_{(-)}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-d/2)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(z+d/2)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(z-d/2)^2} - \frac{1}{(z+d/2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(z+d/2)^2 - (z-d/2)^2}{[(z-d/2)(z+d/2)]^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2zd}{[z^2-(d/2)^2]^2}\right]$$

But $z >> d/2$

$$\Rightarrow \frac{q}{4\pi\varepsilon_0}\left[\frac{2zd}{z^4}\right]$$

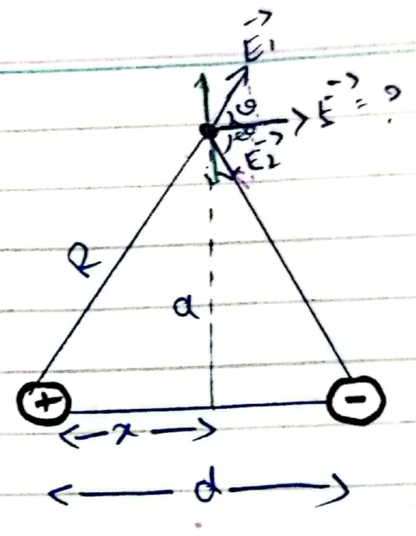
As $p = q \times d \Rightarrow d = p/q$

$$= \frac{q}{4\pi\varepsilon_0} \frac{2}{z^3} \cdot \frac{p}{q}$$

$E_{net} =$

$$\frac{p}{2\pi\varepsilon_0 z^3}$$

CASE - II



$$E = k\frac{Q}{R^2} \quad \text{(General Equation)}$$

$$E_1 = k\frac{q}{R^2} \quad , \quad E_2 = k\frac{q}{R^2}$$

$$E_{1x} = \frac{kq}{R^2}\cos\theta$$

$$\cos\theta = \frac{x}{R} \quad , \quad R = \sqrt{x^2+a^2}$$

$$E_{1x} = K\frac{q}{R^2}\cos\theta = \frac{kq}{x^2+a^2}\cdot\frac{x}{R} = \frac{k\,q\,x}{(x^2+a^2)(\sqrt{x^2+a^2})}$$

$$= \frac{k\,q\,x}{(x^2+a^2)^{3/2}}$$

Similarly, $E_{2x} = \dfrac{kqx}{(x^2+a^2)^{3/2}}$

$$E_{net} = E_{1x} + E_{2x}$$

$$= \frac{2kqx}{(x^2+a^2)^{3/2}} \quad \text{But } x = d/2 \Rightarrow \frac{2kq}{((d/2)^2+a^2)^{3/2}}\cdot\frac{d}{2}$$

$$= \frac{kqd}{(a^2)^{3/2}} \quad (a >> d/2)$$

$$= \frac{kq}{a^3}\frac{p}{q}$$

$$\boxed{E_{net} = \frac{p}{4\pi\varepsilon_0 a^3}}$$

Coconut :)

Chapter 23 Gauss' Law

Electric Flux

$d\phi = \vec{E} \cdot d\vec{A}$ ($d\vec{A}$ is patch vector area)

$\phi = \int \vec{E} \cdot d\vec{A}$ (total flux)

$\phi = \oint \vec{E} \cdot d\vec{A}$ (net flux in closed surface (Gauss law))

$\phi = (\vec{E} \cos\theta) A$

Gauss' Law And Coulombs Law

Net flux ϕ through a closed surface $\phi = \frac{q}{\varepsilon_0}$

q is charge enclosed by surface,

$\phi = \frac{Q}{\varepsilon_0}$ ($Q = q_1 + q_2 + \dots q_n$)

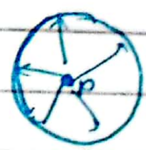
$\Rightarrow q = \phi \varepsilon_0$

$\therefore \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$ (Gauss Law)

When gaussian surface is spherical, circular

-> E is constant

$\varepsilon_0 E \oint dA = q$



Sum of patch areas of sphere = $4\pi r^2$

$\varepsilon_0 E (4\pi r^2) = q$

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

Charged Isolated Conductor

The excess charge on isolated conductor will move entirely to the surface of the conductor. No excess charge will be found within the body of the conductor.

Surface charge Density $= \sigma = \frac{Q}{A} = \frac{q}{A}$

$E = \frac{q}{4\pi r^2 \varepsilon_0} = \frac{q}{A\varepsilon_0}$, $E = \frac{\sigma}{\varepsilon_0}$ (For sphere only)

OR

$\sigma = \frac{q}{A} \Rightarrow q = \sigma A$

also $\varepsilon_0 E \oint dA = q \Rightarrow \varepsilon_0 E A = \sigma A$

$E = \frac{\sigma}{\varepsilon_0}$

APPLICATIONS OF GAUSS LAW

Cylindrical Symmetry

→ Positively charged Wire/Rod of Infinite Length.

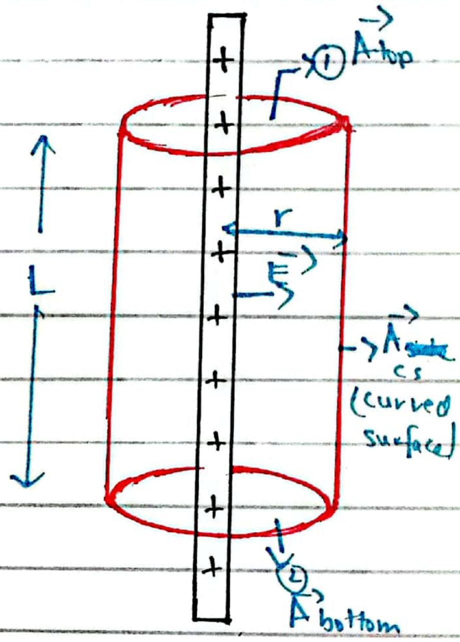
$\phi_{total} = \phi_{cs} + \phi_{top} + \phi_{bottom}$

$= \cancel{E.A\cos(0) + E.A\cos(90°)} + E.A\cos(90°)$

$= EA_{cs}$

Curved surface area of cylinder $= 2\pi rL$

$\phi_{total} = E(2\pi rL)$



Linear charge density $= \lambda = \frac{q}{L}$

$\Rightarrow Q_{inside} = \lambda \times L$

Using Gauss' Law

$\phi = \frac{Q_{inside}}{\varepsilon_0}$

$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$

$$\boxed{E = \frac{\lambda}{2\pi r\varepsilon_0}}$$

→ Magnitude of E. field at a distance r, direction is ⊥ to line of charge

Alternate

$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{enc}$

$0 + EA_{cs} + 0 = \frac{q_{enc}}{\varepsilon_0}$

$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi r\varepsilon_0}$

PLANER SYMMETRY

Nonconducting Sheet

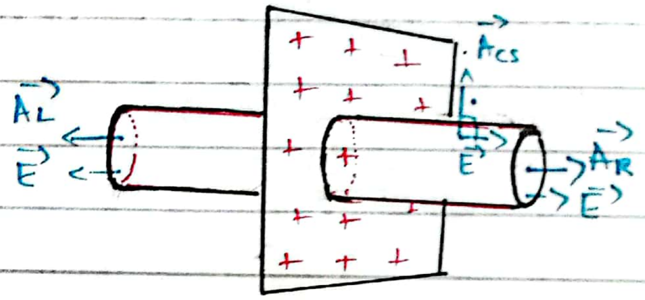
→ Infinite Sheet of Charge

$\phi_{total} = \phi_R + \phi_L + \phi_{cs}$

$= EA_R\cos 0° + EA_L\cos 0° + EA_{cs}\cos 90°$

$= EA + EA$

$\phi_{total} = 2EA$



Gauss Law, $\phi = \frac{q}{\varepsilon_0}$

$2EA = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{2A\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$ $\quad (q/A = \sigma)$

Conducting Sheet

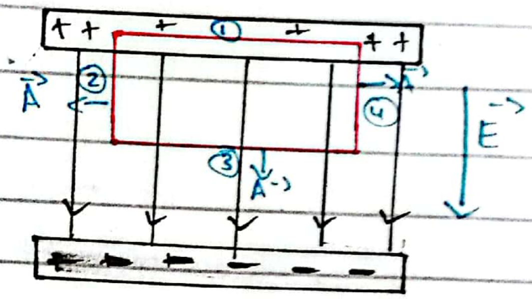
→ Two parallel sheets / Parallel Plate Conductor

$\phi_{total} = \phi_1 + \phi_2 + \phi_3 + \phi_4$

→ E inside conductor is always zero

$= 0 + EA\cos 90° + EA\cos 0° + EA\cos 90°$

$\phi_{total} = EA$



Gauss' Law

$\phi = \frac{q}{\varepsilon_0}$

$EA = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{A\varepsilon_0}, \quad \frac{Q}{A} = \sigma$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

~~Note - All Charge are~~
• Coconut :)

SPHERICAL SYMMETRY

-> 'r' is radius of gaussian surface
-> 'R' is radius of spherical object/surface / Shell
-> Excess charge is always uniformly distributed on shell surface

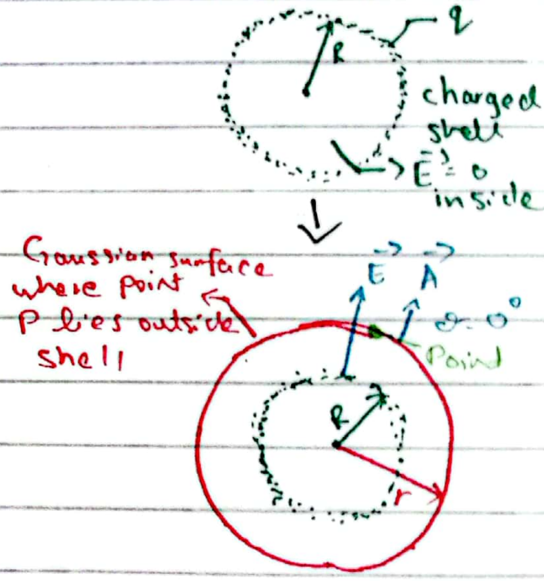
~~Field Inside a Solid Sphere/Shell~~

SHELL THEOREM

The charges are in layers of ~~shell~~ shell, not a solid sphere.

CASE-I Point is Outside the $\boxed{\text{Shell}}$

-> Draw circular gaussian surface outside the shell, equidistant from the shell
-> $r > R$



Gauss Law,

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

Electric field intensity is constant because of spherical shape

At every Point, angle between $\vec{E}$ and patch element $\theta = 0°$

$$\phi = \oint E\, d\cos 0° = \oint E\, dA = E \oint dA = E \oint (4\pi r^2)$$

Gauss Law, $\varepsilon_0 \phi = q$

$$\varepsilon_0 [E(4\pi r^2)] = q$$

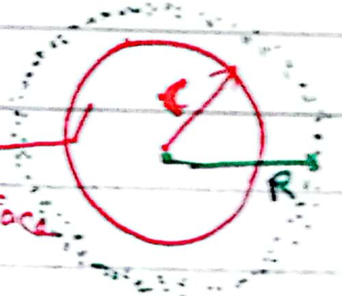
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{kq}{r^2}$$

(Also the expression of $\vec{E}$ of point charge)

-> Charged shell behaves as point charge for all points outside the shell

CASE-II Point P is Inside the $\boxed{\text{Shell}}$

-> Draw gaussian surface inside the shell equidistant from all sides.



-> $r < R$

Gauss Law

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

But no charge is enclosed by the gaussian surface

$$\therefore q_{enc} = 0$$

$$\varepsilon_0 \oint \vec{E} \cdot dA = 0$$

$$\vec{E} \oint d\vec{A} = 0$$

~~Either~~ As area can never be zero, $\oint d\vec{A} \neq A$

$$\therefore E = 0$$

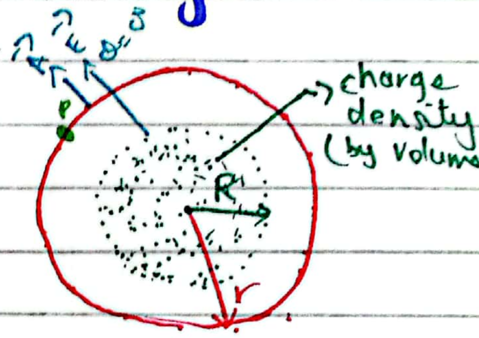
-> Electric field inside the shell of charge is zero and any charge placed inside shell will experience no Force.

ELECTRIC FIELD DUE TO $\boxed{\text{SPHERICAL SHELL}}$ OF CHARGE

The charges are in solid spherical shell, not just a layer

CASE-I Point is Outside Spherical Charge

Draw gaussian surface outside charged sphere



$r > R$

Since charge is uniformly distributed,

$$\rho = \frac{\text{Total charge}}{\text{volume}} = \frac{q}{V}$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} \qquad \text{(Take } R \text{, not } r\text{)}$$

$$q = \rho\left[\frac{4}{3}\pi R^3\right]$$

Gauss' Law

$$\phi = \oint \vec{E}\cdot d\vec{A}$$

$\theta = 0°$ (At every point)

$$= \oint E\, dA \cos 0° = \oint E\, dA = E\oint dA = E \oint 4\pi r^2$$

Gauss Law

$$\varepsilon_0 \phi = q$$

$$\varepsilon_0 [E(4\pi r^2)] = q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \Rightarrow \frac{1}{4\pi\varepsilon_0 r^2}\cdot\frac{\rho 4\pi R^3}{3} = \frac{\rho}{3\varepsilon_0}\cdot\frac{R^3}{r^2}$$

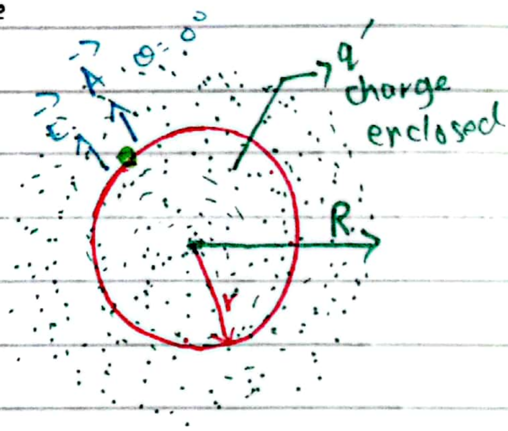
-> Spherical charge Distribution at distance 'r' has same value as if charges were concentrated at centre of sphere

CS CamScanner

CASE-II Point is Inside Sphere of Charges

Draw gaussian surface inside charged sphere

$r < R$



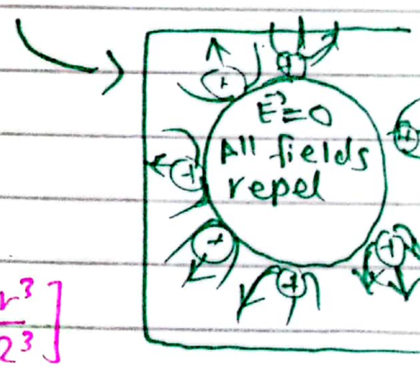
Charge is uniformly distributed

$$\rho = \frac{\text{Total charge}}{\text{Volume}} = \frac{q}{V}$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

$$q = \rho\left[\frac{4}{3}\pi R^3\right] \quad — ①$$

- Charges outside gaussian surface does not set $\vec{E}$ inside surface
- Only enclosed charges setup q field $\vec{E}$



$\Rightarrow$ $q' = \rho\left[\frac{4}{3}\pi r^3\right]$ ② (Take r, not R)

Dividing both ① and ②

$$\frac{q'}{q} = \frac{\rho\left[\frac{4}{3}\pi R^3\right]}{\rho\left[\frac{4}{3}\pi r^3\right]} \Rightarrow \frac{q'}{q} = \frac{r^3}{R^3} \Rightarrow q' = q\left[\frac{r^3}{R^3}\right]$$

Gauss' Law, ~~$\varepsilon_0 \phi = q$~~ $\phi = \oint \vec{E}\, d\vec{A} = E\oint dA = E(4\pi r^2)$ $(\theta = 0°)$

$$\varepsilon_0 \phi = q$$

$$\varepsilon_0\left[E(4\pi r^2)\right] = q'$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}\cdot\left[\frac{r^3}{R^3}\right]q, \quad E = \frac{q}{4\pi\varepsilon_0}\cdot\frac{r}{R^3}$$

$\rightarrow$ When $r = 0 \Rightarrow E = 0$

$\rightarrow$ When $r = R \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ ($\vec{E}$ max)

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Ch 28 Magnetic Field

$$\vec{F}_B = q\,\vec{V} \times \vec{B}$$

$$F_B = |q|\, v B \sin\alpha$$

Unit of Magnetic Field = 1 tesla = 1T = $\frac{N}{A \cdot m}$ (SI-unit)

★ $1\,T = 10^4\,G$ (gauss)

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2eV}{m}}, \quad J = \frac{I}{A}, \quad J = ne\,V_d$$

(ne is carrier charge density)
(n = number of charges per unit volume)

$$n = \frac{BI}{Vle}, \quad l = \frac{A}{d}$$

$V = vBd$
(Potential) (velocity)

★ F_B is deflecting Force, does not do work
F_E does work

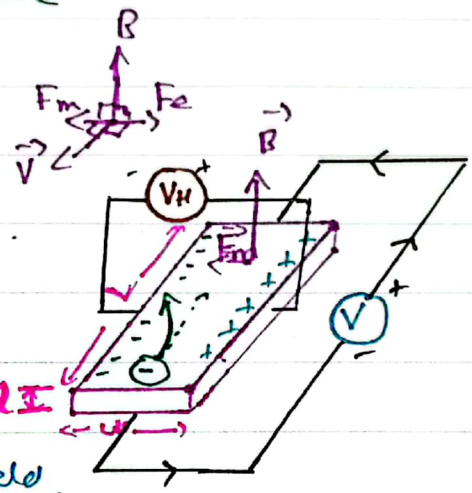
$$F = qE, \quad F = qvB\sin\theta, \quad F = ILB\sin\theta, \quad F = \frac{mv^2}{r}, \quad F = mg$$

$$V = \frac{IB}{lne}, \quad n = \frac{IB}{Vle}, \quad E = V_d B$$

★ $1\,eV = 1.6 \times 10^{-19}\,J$

THE HALL EFFECT

Hall Effect is production of Potential Difference (Hall voltage) across current carrying conductor (in presence of magnetic field), perpendicular to both current and magnetic field.



In Simple Words :) : Every current is automatically associated with magnetic field.
" When potential Diff is applied across length of conductor, hall voltage is produced across width of conductor."

$\Rightarrow \quad F_E = F_B$

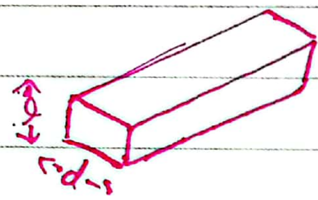
$eE = eV_dB \quad \because (qE = qvB)$

$E = V_d B \quad -①$

$\Rightarrow \quad J = neV_d$

$V_d = \dfrac{J}{ne} = \dfrac{I}{Ane}$

$V_d = \dfrac{I}{ldne} \quad \because (A = l \times d)$



l = thickness
d = width

Also $E = \dfrac{V}{d}$

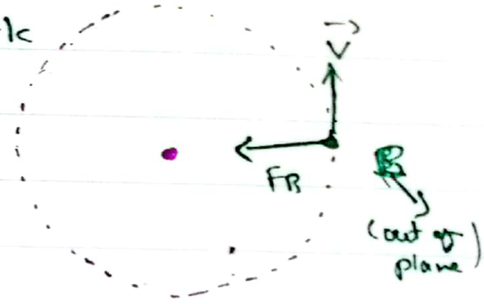
★ n = number density of charge carriers (number per unit volume)

$\Rightarrow ① \quad \dfrac{V}{d} = \dfrac{I}{lnde} B$

$V = \dfrac{IB}{lne} \Rightarrow n = \dfrac{IB}{Vle}$

Circulating Charged Particle

* $\vec{F_B}$ is ~~...~~ only deflecting force, does zero work
* $\vec{F_e}$ does work on particle



$$F_B = F_c \quad \text{(centripetal Force)}$$

Radius

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

Time Period

$$T = \frac{\text{circumference}}{\text{velocity}} = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right)$$

$$T = \frac{2\pi m}{qB}$$

Frequency

$$f = 1/T = \frac{qB}{2\pi m}$$

Angular Frequency

$$\omega = 2\pi f$$

$$\omega = 2\pi\left(\frac{qB}{2\pi m}\right)$$

$$\omega = \frac{qB}{m}$$

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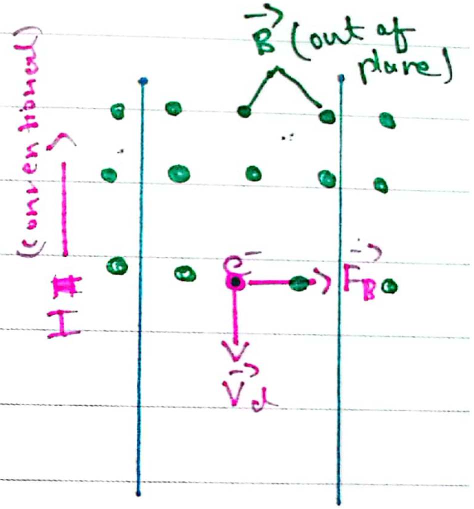
Magnetic Force on Current Carrying Wire

We know $F = q V_d B \sin\theta$ — (1)

$\Rightarrow q = It = I \dfrac{L}{V_d}$



(1) $\Rightarrow$ $F = I \dfrac{L}{V_d} V_d B \sin\theta$

$F = ILB \sin\theta$

$\theta = 90°$

$F = ILB$

$$\boxed{\vec{F}_B = I \vec{L} \times \vec{B}}$$

* If wire is bend, we can break in small pieces and differentiate

$d\vec{F}_B = I\, d\vec{L} \times \vec{B}$

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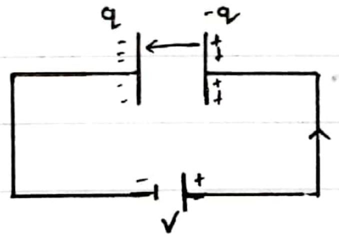
Ch# 5 CAPACITANCE

-> Ability to store charge

$Q \propto V$ (Q is charge on one plate, don't sum both charges)

$Q = CV$

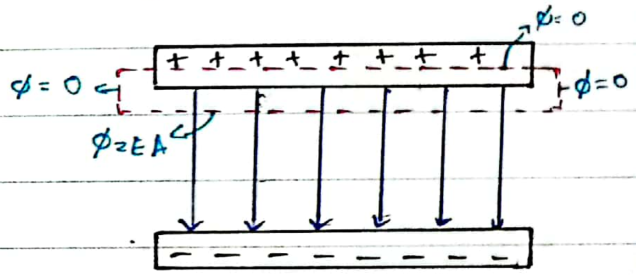
$C = \frac{Q}{V}$ (Farad = F = CV^{-1})



Parallel Plate Capacitor

-> First we find Potential Difference by integrating $\vec{E}$ with ds (path length)

-> To find $\vec{E}$, take gaussian surface



$$\oint \vec{E} d\vec{A} = q$$

$$\Rightarrow E = \frac{q}{A\varepsilon_0} \quad \text{(Derived in ch 23)}$$

$$\Rightarrow q = EA\varepsilon_0$$

-> Now calculating Potential Difference

-> Integration path will be -ve to the plate

$$\Delta V = V_f - V_i$$

$$V_f - V_i = -\int_{+}^{-} \vec{E} \cdot d\vec{s}$$

$$V = \int^{+} E \cdot ds = E \int^{+} E \cdot ds = E|ds|_i^f$$

$$\boxed{V = Ed}$$

$$C = \frac{Q}{V} = \frac{EA\varepsilon_0}{Ed}$$

$$C = \frac{A\varepsilon_0}{d}$$

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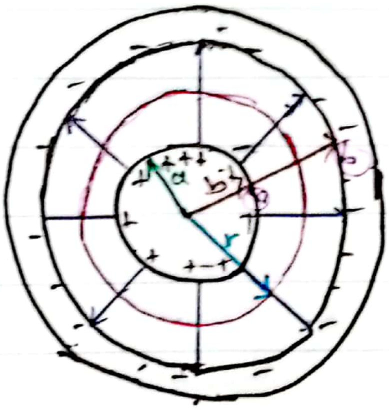
Cylindrical Capacitance

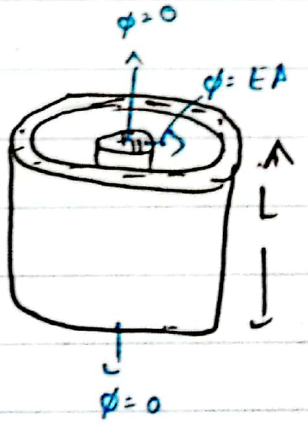
$\cancel{\phi} \; q = \varepsilon_0 A E$ (Derived in ch 23)

$q = \varepsilon_0 E (2\pi r \underbrace{L}_{\text{height}})$ area of gaussian surface



$$E = \frac{q}{2\pi \varepsilon_0 L r}$$

$$V = \int_{-}^{+} E\,ds = \frac{-q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r}$$



$$V = \frac{\cancel{-}q}{2\pi\varepsilon_0 L}\,\Big|\ln r\Big|_b^a = \frac{q}{2\pi\varepsilon_0 L}\,\Big|\ln r\Big|_a^b \quad (ds = -dr)$$

$$V = \frac{q}{2\pi\varepsilon_0 L}\left[\ln(b) - \ln(a)\right]$$

$$V = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow C = \frac{q}{V} = q \times \frac{2\pi\varepsilon_0 L}{q \times \ln(b/a)}$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$

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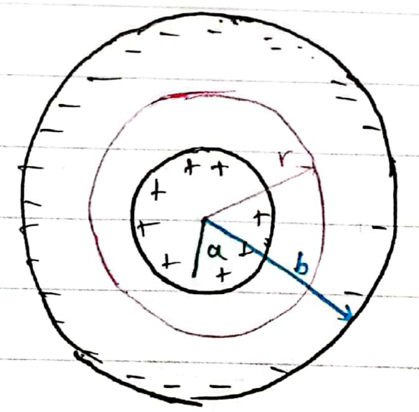
Spherical Capacitor

$q = \varepsilon_0 E A$

$q = \varepsilon_0 E (\underbrace{4\pi r^2}_{\text{surface area of gaussian surface}})$



$V = \int_{-}^{+} E\,ds = -\int_{-}^{+} E\,dr = \int_{+}^{-} E\,dr \quad (ds = -dr)$

$$V = \int_a^b \frac{q}{4\pi r^2 \varepsilon_0}\,dr = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}\,dr$$

$$V = \frac{q}{4\pi\varepsilon_0}\left|-\frac{1}{r}\right|_a^b = -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{a}\right] = -\frac{q}{4\pi\varepsilon_0}\left(\frac{a-b}{ab}\right)$$

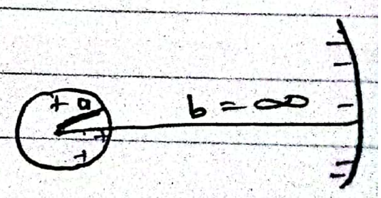
$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

$$C = \frac{q}{V} = q \times \frac{4\pi\varepsilon_0}{q}\left(\frac{ab}{b-a}\right)$$

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

Isolated Sphere

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b\to\infty}} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - 0} = 4\pi\varepsilon_0 a$$

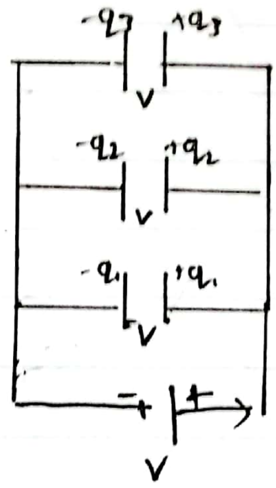


$a = R$

$C = 4\pi\varepsilon_0 R$

Capacitors in Parallel

-> Capacitance increases
-> Charge is distributed across capacitors
-> Voltage at each capacitor remains same



$q_1 = C_1 V$, $q_2 = C_2 V$, $q_3 = C_3 V$

$Q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3) V$

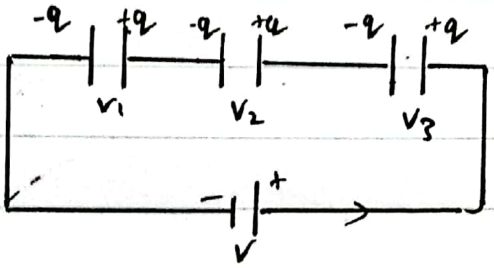
$Q = C_{eq} V$

$C_{eq} = \frac{Q}{V}$

$C_{eq} = C_1 + C_2 + C_3 + \dots C_n$

Capacitance in Series

-> Capacitance Decreases
-> Charge remains same
-> Voltage is distributed



$V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$, $V_3 = \frac{q}{C_3}$

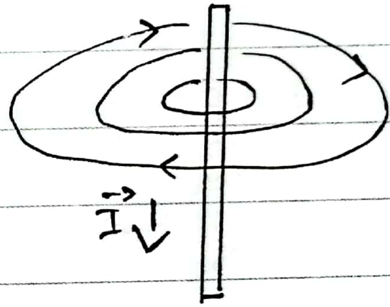
$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$\frac{V}{q} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots + \frac{1}{C_n}$

Ch 29 Magnetic Fields due to Currents

Magnetic Field Outside a Current Carrying Wire



According to Amperes Law

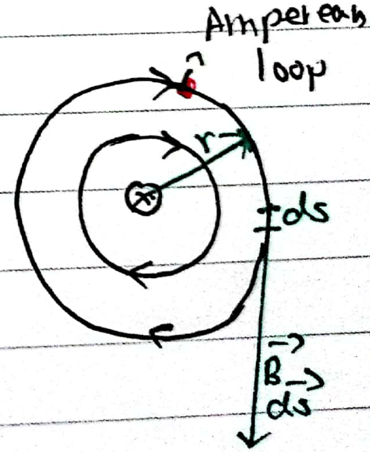
$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I_{enclosed}$$

ds = circumference (small part)

$$B \int ds = \mu_0 I_{enc} \cdot \cos\theta$$

$\theta = 0°$, $\int ds = 2\pi r$

$B(2\pi r) = \mu_0 I$ (All current is enclosed in loop)



$$B = \frac{\mu_0 I}{2\pi r}$$

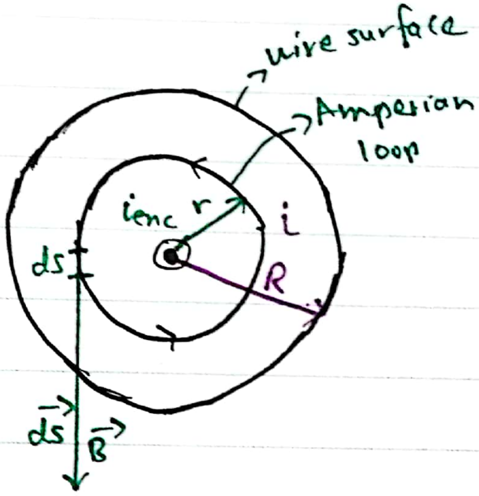
($\mu_0 = 4\pi \times 10^{-7} NA^{-2}$ or Tm/A)

Magnetic Field Inside a Current Carrying Wire

According to Amperes Law

$$\oint \vec{B}.\vec{ds} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I_{enc}$$



Now,

$$\frac{I_{enc}}{I_t} = \frac{\pi r^2}{\pi R^2}$$ (It= total current)

$$I_{enc} = \frac{I_t r^2}{R^2}$$

$$\Rightarrow B(2\pi r) = \mu_0 \frac{I_t r^2}{R^2}$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right). r$$

In Solenoids

Amperes law

$\oint \vec{B}\cdot\vec{ds} = \mu_0 I_{enc}$

$\oint \vec{B}\cdot\vec{ds} =$

$$\int_a^b B.ds\cos\theta + \int_b^c B.ds\cos\theta + \int_c^d B.ds\cos\theta + \int_d^a B ds\cos\theta$$

$$= \int_a^b B ds\cos 0^\circ + \int_b^c B ds\cos 90^\circ + \int_c^d 0 + \int_d^a B ds\cos 90^\circ$$

$$= B\int_a^b ds + 0 + 0$$

$\oint \vec{B}\cdot\vec{ds} = Bh$

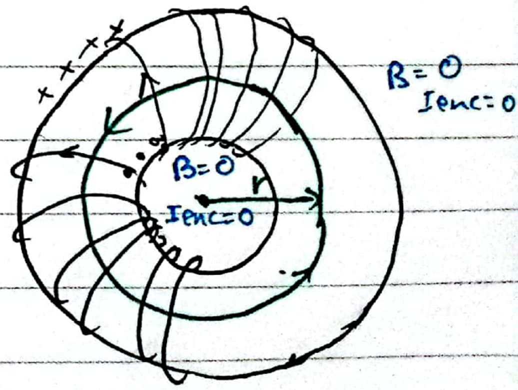
Now, $I_{enc} = nhI$ ($n = \frac{\text{number of turns}}{\text{total Length}}$, h = length)

$\Rightarrow$ $\cancel{Bd}$ $Bh = \mu_0 I_{enc}$

$Bh = \mu_0 n h I$

$\cancel{B}$ $B = \mu_0 n I$

Toroids



$\oint B.ds = \mu_0 I_{enc}$

$B\oint ds = \mu_0 I_{enc}$

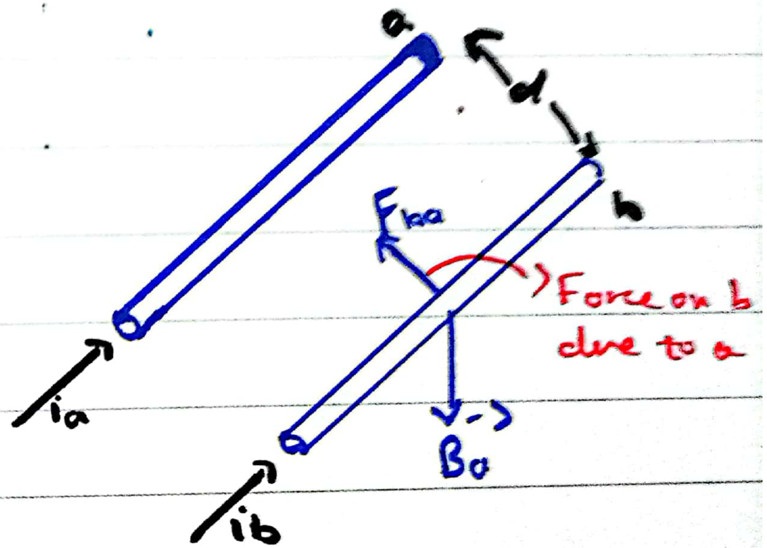
$B(2\pi r) = \mu_0 NI$

$(I_{enc} = NI)$ N = Number of turns

$$B = \frac{\mu_0 NI}{2\pi r}$$

$\boxed{2\pi r = \text{total length} \quad \text{so } \frac{N}{2\pi r} = n}$

Force Between Two Parallel Wires



Field of a ($\vec{B_a}$) produces force on b ($\vec{F_{ba}}$)

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

$$F_{ba} = i_b \vec{L} \times \vec{B_a}$$

$$= \frac{\mu_0 L i_a i_b}{2\pi d}$$

$$F_{ba} = \frac{\mu_0 L I_a I_b}{2\pi d}$$