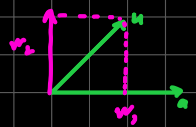
3.3

- u and v are orthogonal if $u \cdot v = 0$
- $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$ → point-normal eq

→ $ax+by+c=0$

$n = (a,b)$

- Projection theorem: $u = w_1 + w_2$



w_1 is a scalar multiple of a

w_2 is orthogonal to a

- $\|\text{proj}_a u\| = \dfrac{|u \cdot a|}{\|a\|}$

→ $w_1 = \text{proj}_a u = \dfrac{u \cdot a}{\|a\|^2} \cdot a$

$w_2 = u - \text{proj}_a u = u - \dfrac{u \cdot a}{\|a\|^2} \cdot a$

The planes

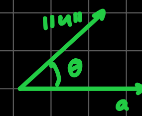
$$x + 2y - 2z = 3 \quad \text{and} \quad 2x + 4y - 4z = 7$$

are parallel since their normals, $(1, 2, -2)$ and $(2, 4, -4)$, are parallel vectors. Find the distance between these planes.

Solution To find the distance D between the planes, we can select an arbitrary point in one of the planes and compute its distance to the other plane. By setting $y = z = 0$ in the equation $x + 2y - 2z = 3$, we obtain the point $P_0(3, 0, 0)$ in this plane. From (19), the distance between P_0 and the plane $2x + 4y - 4z = 7$ is

$$D = \frac{|2(3) + 4(0) + (-4)(0) - 7|}{\sqrt{2^2 + 4^2 + (-4)^2}} = \frac{1}{6}$$

$\|\text{proj}_a u\| = \|u\| \cos\theta$



- Pythagoras theorem: $\|u+v\|^2 = \|u\|^2 + \|v\|^2$

- Distance b/w planes: $D = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}$

Q3- $P(-1,3,-2)$, $n(-2,1,-1)$

→ $-2(x+1) + (y-3) - (z+2) = 0$ ✓

Q4- $P(1,1,4)$, $n = (1,9,8)$

→ $(x-1) + 9(y-1) + 8(z-4) = 0$ ✓

Q5- $P(2,0,0)$ $n = (0,0,2)$

→ $2z = 0$ ✓

Q6- $P(0,0,0)$; $n(1,2,3)$

→ $x + 2y + 3z = 0$ ✓

Q7- $4x - y + 2z = 5$, $7x - 3y + 4z = 8$

$n_1 = (4,-1,2)$, $n_2 = (7,-3,4)$

neither of the normal vectors are scalar multiples of each other, so not parallel ✓

Q12- $x - 2y + 3z = 4$, $-2x + 5y + 4z = -1$

$(1,-2,3) \cdot (-2,5,4)$

$= -2 - 10 + 12$

$= \underline{\underline{0}}$

$\underline{\underline{\text{perpendicular}}}$

Q13 (a) $u = (1,-2)$, $a = (-4,-3)$ ✓

$\|\text{proj}_a u\| = \dfrac{|(1,-2) \cdot (-4,-3)|}{\sqrt{(-4)^2 + (-3)^2}} = \dfrac{|-4+6|}{5} = \dfrac{2}{5}$

Q10- $(-4,1,2) \cdot (x,y,z) = 0$, $(8,-2,-4) \cdot (x,y,z) = 0$

$n_1 = (-4,1,2)$, $n_2 = (8,-2,-4)$

$(8,-2,-4) = -2(-4,1,2)$

$\underline{\underline{\text{parallel}}}$ ✓

Q11- $3x - y + z - 4 = 0$, $x + 2z + 1 = 0$

$n_1 = (3,-1,1)$, $n_2 = (1,0,2)$

$n_1 \cdot n_2 = (3,-1,1) \cdot (1,0,2)$

$= 3 + 0 + 2 = 5$

$5 \neq 0$, $\underline{\underline{\text{not perp}}}$

Q14- (b) $u = (3,-2,6)$, $a = (1,2,-7)$

$$\text{proj}_a u = \frac{|(3,-2,6)\cdot(1,2,-7)|}{\sqrt{1^2+2^2+(-7)^2}}$$

$$= \frac{|3-4-42|}{3\sqrt{6}} = \frac{43}{3\sqrt{6}} = \frac{43\sqrt{6}}{18}$$

Q15- $u = (6,2)$, $a = (3,-9)$

$$\text{proj}_a u = \frac{(6,2)\cdot(3,-9)}{3^2+(-9)^2}(3,-9)$$

$$= \frac{18-18}{9+81}(3,-9)$$

$$= \underline{\underline{0}}$$

$$u - \text{proj}_a u = (6,2) - 0 = (6,2)$$

Q20- $u = (5,0,-3,7)$, $a = (2,1,-1,-1)$

$$\text{proj}_a u = \frac{(5,0,-3,7)\cdot(2,1,-1,-1)}{2^2+1^2+1+1}(2,1,-1,-1)$$

$$= \frac{10+3-7}{7}(2,1,-1,-1)$$

$$= \frac{6}{7}(2,1,-1,-1)$$

$$u - \text{proj}_a u = (5,0,-3,7) - \left(\frac{12}{7},\frac{6}{7},-\frac{6}{7},-\frac{6}{7}\right)$$

$$= \frac{23}{7},-\frac{6}{7},-\frac{15}{7},\frac{55}{7})$$

Q21- $(-3,1)$; $4x+3y+4=0$

$$\frac{|4(-3)+3(1)+4|}{\sqrt{4^2+3^2}} = 1$$

Q25- $(3,1,-2)$; $x+2y-2z-4=0$

$$d = \frac{|3+2(1)-2(-2)-4|}{\sqrt{1^2+2^2+(-2)^2}}$$

$$= \frac{5}{3}$$

Q27- $2x-y-z=5$, $-4x+2y+2z=12$

$P: (0,0,-5)$

$$d = \frac{|-4(0)+2(0)+2(-5)-12|}{\sqrt{4^2+2^2+2^2}}$$

$$= \frac{11\sqrt{6}}{6}$$

Q28- $2x-y+z=1$, $2x-y+z=-1$

$P: (0,0,1)$

$$d = \frac{|2(0)-0+1+1|}{\sqrt{2^2+(-1)^2+1^2}} = \frac{\sqrt{6}}{3}$$

Q29- $u = (1,0,1)$, $v = (0,1,1)$

$b = (x,y,z)$

$x+z=0$ -①
$y+z=0$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$y+z=0$, $x+z=0$
$y=-z$, $x=-z$

$z=t$, $y=-t$, $x=-t$

$$\|b\| = \sqrt{t^2+(-t)^2+(-t)^2} = \sqrt{3t^2} = t\sqrt{3}$$

$t\sqrt{3} = 1$
$t = \pm\frac{1}{\sqrt{3}}$

$\rightarrow \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$\rightarrow \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$