Notes DS

- Proposition, Propositional variables

- Conjunction AND $\wedge$
- Disjunction OR $\vee$
- Negation NOT $\neg$ / ~

- $p \longrightarrow q$

Sufficient condition — Necessary condition

consequence

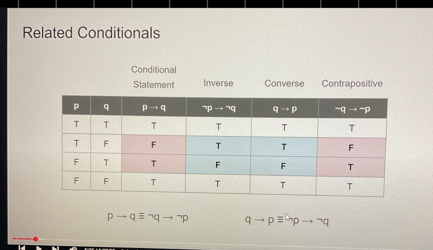
Premise

conclusion

hypothesis

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

- If A then B
  $A \to B$
- If A only then B
  $B \to A$
- A unless B
  $\neg B \to A$
- A if B
  $A \to B$
- A only if B
  $A \to B$



Related Conditionals

p	q	Conditional Statement p → q	Inverse ¬p → ¬q	Converse q → p	Contrapositive ~q → ~p
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

$p \to q \equiv \neg q \to \neg p$ $q \to p \equiv \neg p \to \neg q$

- $\neg p \to \neg q$: Inverse
- $\neg q \to \neg p$: Contrapositive
- $q \to p$: Converse

- Inverse = Converse
- Conditional = Contrapositive

- either A or B : XOR $\oplus$
  * Always check context

- $q \to p$: p is a necessary condition for q
  $p \to q$: p is sufficient for q
- $= (q \to p) \wedge \neg(p \to q)$
- $= \neg(p \to q)$ $\to$ p is sufficient but not necessary condition for q

- $p \leftrightarrow q$ $\to$ p is sufficient and not necessary condition for q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- $p \to q \not\equiv q \to p$
- $(p \to q) \to r \not\equiv p \to (q \to r)$

- $p \to q = \neg(p \wedge \neg q)$
- $= \neg p \vee q$

- $F \vee p \equiv p$; $T \wedge p \equiv p$
- $T \vee p \equiv T$; $F \wedge p \equiv F$

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$; $\neg(p \wedge q) \equiv \neg p \vee \neg q$

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$; $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

- Tautology: Always True $\neg p \vee p$
- Contradiction: Always False $p \wedge \neg p$
- Contingency: Maybe True / False

Predicates:

- Propositional Funcs
- Multiple parameters
- Domain of each parameter of predicate must be defined

- Quantifiers : Existential / Universal

$\exists x$: there exists
$\forall x$: For all

- $\neg x \, P(x) \equiv \forall x \, \neg P(x)$
- $\neg \forall x \, P(x) \equiv \exists x \, \neg P(x)$

- $\forall x \, (R(x) \rightarrow G(x))$: Universal Quantification
- $\exists x \, (R(x) \wedge G(x))$: Existential Quantification

- $\exists x \, \forall y \, E(x,y) \not\equiv \forall y \, \exists x \, E(x,y)$

- $\forall x \, P(x) \equiv T$
- $\exists x \, P(x) \equiv F$

$\Big]$ Empty domains