3.4

- $x = x_0 + tv$; line is parallel to v
  - x_0: a point on line
  - v: vector parallel to line L
  - t: parameter

We showed in Example 6 of Section 1.2 that the general solution of the homogeneous linear system

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 \\ 0 & 0 & 5 & 10 & 0 & 15 \\ 2 & 6 & 0 & 8 & 4 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

is

$$x_1 = -3r - 4s - 2t, \quad x_2 = r, \quad x_3 = -2s, \quad x_4 = s, \quad x_5 = t, \quad x_6 = 0$$

which we can rewrite in vector form as

$$\mathbf{x} = (-3r - 4s - 2t, r, -2s, s, t, 0)$$

According to Theorem 3.4.3, the vector $\mathbf{x}$ must be orthogonal to each of the row vectors

$$\mathbf{r}_1 = (1, 3, -2, 0, 2, 0)$$
$$\mathbf{r}_2 = (2, 6, -5, -2, 4, -3)$$
$$\mathbf{r}_3 = (0, 0, 5, 10, 0, 15)$$
$$\mathbf{r}_4 = (2, 6, 0, 8, 4, 18)$$

We will confirm that $\mathbf{x}$ is orthogonal to $\mathbf{r}_1$, and leave it for you to verify that $\mathbf{x}$ is orthogonal to the other three row vectors as well. The dot product of $\mathbf{r}_1$ and $\mathbf{x}$ is

$$\mathbf{r}_1 \cdot \mathbf{x} = 1(-3r - 4s - 2t) + 3(r) + (-2)(-2s) + 0(s) + 2(t) + 0(0) = 0$$

which establishes the orthogonality.

→ $x = tv$ → line passes through origin

→ $x = x_0 + t_1 v_1 + t_2 v_2$; plane is parallel to non-collinear vectors v_1, v_2

- $x = x_0 + t(x_1 - x_0)$; The line containing x_1 and x_0 is parallel to the vector $v = x_1 - x_0$

  OR

  $x = (1-t)x_0 + tx_1$

- $x = x_0 + t(x_1 - x_0)$ $0<t<1$

  OR

  $x = (1-t)x_0 + tx_1$ $0<t<1$

  } defined as a line segment

Q1- $P(-4,1)$; $v = (0,-8)$

$x = x_0 + tv$

$x = (-4,1) + t(0,-8)$ - vector eq

$x = (-4, -8t)$ parametric

$x = -4,\ y = -8t$

Q4- $P = (-9,3,4)$, $v = (-1,6,0)$

$x = (-9,3,4) + t(-1,6,0)$

$x = (-9-t, 3+6t, 4)$

$x = -9-t$, $y = 3+6t$, $z = 4$

Q5- $x = (3-5t, -6-t)$

$x = (3,-6) + t(-5,-1)$

$v = (-5,-1)$, $P = (3,-6)$

Q6- $(x,y,z) = (4t, 7, 4+3t)$

$(x,y,z) = (0,7,4) + t(4,0,3)$

$v = (4,0,3)$

$P = (0,7,4)$

Q7- $x = (1-t)(4,6) + t(-2,0)$

$= (4-4t, 6-6t) + (-2t, 0)$

$= (4-6t, 6-6t)$

$= (4,6) + t(-6,-6)$

$P = (4,6)$, $v = (-6,-6)$

Q13- $v = (-2,3)$ → find by inspection

$x = t(a,b)$

$x = t(3,2)$

$x = 3t,\ y = 2t$

Q14- $v = (1,-4)$

$(4,1)$

$1 \cdot a - 4b = 0$

$x = t(4,1)$

$x = 4t$, $y = t$

Q15- $v = (4,0,-5)$

$x = t_1(5,0,4) + t_2(0,1,0)$

Q12- $P(0,5,-4)$, $v_1 = (0,0,-5)$, $v_2 = (1,-3,-2)$

$x = (0,5,-4) + t_1(0,0,5) + t_2(1,-3,-2)$

Q17- $x_1 + x_2 + x_3 = 0$

$2x_1 + 2x_2 + 2x_3 = 0$

$3x_1 + 3x_2 + 3x_3 = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \\ 3 & 3 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2 3

$x_2 = s$, $x_3 = t$

$x_1 + x_2 + x_3 = 0$

$x_1 = -s - t$ — 1

$x = (-s-t, s, t)$

$x \cdot r_1 = \cancel{-s} \cancel{-t} \cancel{+s} \cancel{+t} = 0$

$x \cdot r_2 = 0$

$x \cdot r_3 = 0$

Q19- $x_1 + 5x_2 + x_3 + 2x_4 - x_5 = 0$

$x_1 - 2x_2 - x_3 + 3x_4 + 2x_5 = 0$

$$\begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 1 & -2 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 0 & -7 & -2 & 1 & 3 & 0 \end{bmatrix} \quad R_2 - R_1$$

$$= \begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 0 & 1 & \frac{2}{7} & -\frac{1}{7} & -\frac{3}{7} & 0 \end{bmatrix} \quad -\frac{1}{7}R_2$$

$$= \begin{bmatrix} 1 & 0 & -\frac{3}{7} & \frac{19}{7} & \frac{8}{7} & 0 \\ 0 & 1 & \frac{2}{7} & -\frac{1}{7} & -\frac{3}{7} & 0 \end{bmatrix} \quad R_1 - 5R_2$$

$x_2 = -\frac{2}{7}x_3 + \frac{1}{7}x_4 + \frac{3}{7}x_5$

$x_3 = s,\ x_4 = t,\ x_5 = z$

$x_1 = \frac{3}{7}s - \frac{19}{7}t + -\frac{8}{7}z$

$x = \left(\frac{3}{7}s - \frac{19}{7}t \cdot \frac{8}{7}z,\ -\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}z,\ s, t, z\right)$