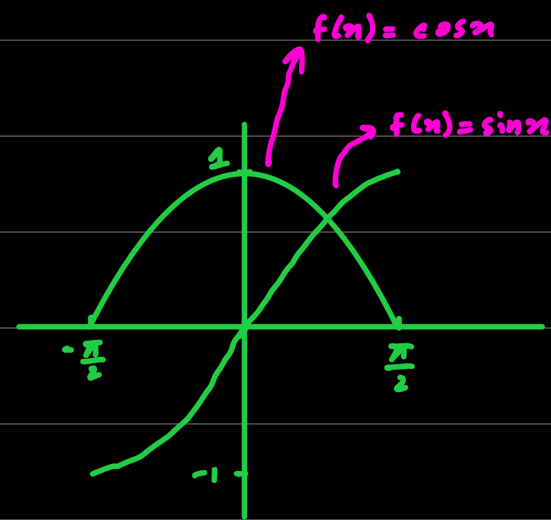
Application Of Derivatives

(4.1 – Extreme values of function on Closed Intervals)

- **Absolute Maximum:** $f(x) \le f(c) \quad \forall x$ in D
- **Absolute Minimum:** $f(x) \ge f(c) \quad \forall x$ in D

- Max & min values are extreme values
- Abs. max or min are also global max or min.

Q- $f(x) = \cos x$, $f(x) = \sin x$ $\quad D = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$f(x) = \cos x$:

Abs max $= 1 \quad (x = 0)$

Abs min $= 0$

$\left(x = -\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \sin x$:

Abs max $= 1 \quad \left(x = \frac{\pi}{2}\right)$

Abs min $= -1 \quad \left(x = -\frac{\pi}{2}\right)$

- Functions can have several extreme values
- Function might not have a max or min if domain is unbounded or fails to contain an endpoint

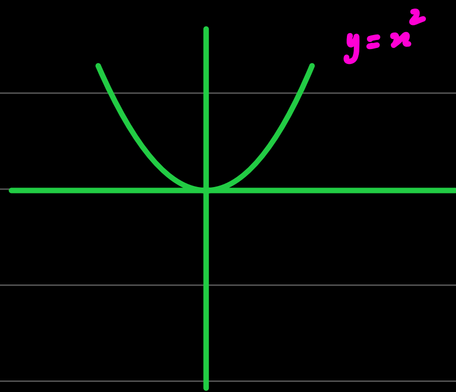
Q- $y = x^2$

$(-\infty, \infty)$ → No absolute max. Min $= 0$, $x = 0$

$[0,2]$ → Abs max $= 4$, $x = 2$ | Abs min $= 0$, $x = 0$

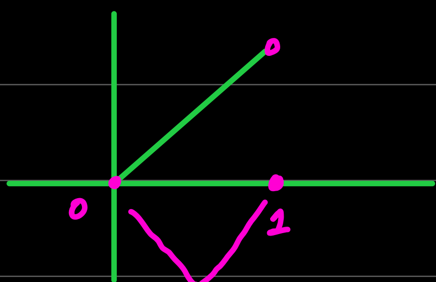
$(0,2]$ → No abs min, Abs max $= 4$, $x = 0$

$(0,2)$ → No abs min or max



- Extreme Value theorem: If f is cont., then f contains both an absolute maximum, minimum in $[a,b]$ for closed interval

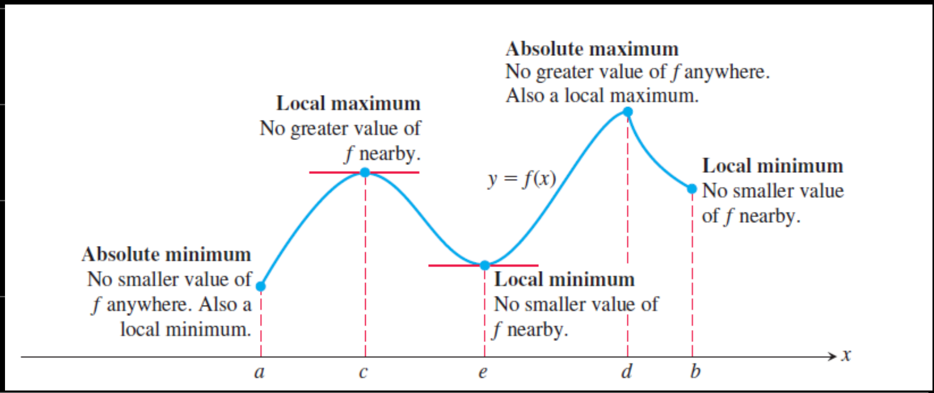
Q- $y = \begin{cases} x, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$



- Local max: $f(x) \le f(c)$ $\forall x$ in D
- Local min: $f(x) \ge f(c)$

Also accomodates absolute max/min

- Local extrema are also called relative extrema



Finding Extrema

- First Derivative Theorem for local extreme values: $f'(c) = 0$

↳ Only places where f can have extreme values:

1. interior pts where f' is 0
2. interior pts where f is undefined
3. endpoints of domain of f

- Critical pts: An interior pt of the domain of 'f' where f' is 0 or undefined

Note: Extreme values are obtained on end-points or where $f' = 0$

- A function may have a critical pt without having local extreme values

e.g: $y = x^3$, $y = x^{\frac{1}{3}}$. These do not have extreme values , but have critical pt.

Finding Absolute Extrema of a cont. Function on a finite closed Interval

1. Find all critical pts
2. Evaluate f at all critical and end points
3. Take the largest & smallest of these values