Assignment 4

Description

This assignment is a problem set consisting of two parts. In **Part A**, you will revisit some of the problems from the classroom activities. In **Part B**, you will apply the concepts and techniques discussed in class to solve novel problems.

Grading

This assignment has a significant weight (3x), so make sure to allot your time accordingly. Each exercise will be graded on the tagged LO. Additionally, you will receive a single holistic score on the LO **#mathtools**, for appropriate usage of SageMath and other ancillary mathematical techniques, and a single holistic score on the HC **#professionalism**.
Include an appendix in which you highlight 2 to 5 specific HC applications. For each highlighted use, give the HC hashtag and a 1 to 2 sentence explanation that demonstrates the strength of your application. The total length of this appendix should be no more than 200 words.

Collaboration

Math is a lot more fun with friends. We encourage you to collaborate and to discuss the problem sets with each other. However, it is key that you are able to apply the skills and concepts you learn in class and explain your approach on your own. For this reason, you should not be sharing **any** written work with each other outside of class and are required to write up all solutions independently.

Submission

Consider these options for presenting your work neatly. You may choose to type your solutions using LaTeX, Google Documents or other software. You may also write on a graphic tablet and submit a PDF of your work. Hand-written solutions on paper are also acceptable, so long as they are scanned/photographed clearly and are fully legible. There exist scanning apps that will allow you to take photos of pages and create a PDF out of them (Office Lens, for example).

- Low scores will be assigned for illegible work and/or images of poor quality.
- All graphs should be included in your PDF file, with a relevant caption.
- The common conventions for writing an essay will apply. For example, if you use external resources, make sure to cite them using proper formatting.

Part A

A.1 *Heartwarming* (**#Diffapplication**) *Adapted from Thomas's "Calculus"*

In the late 1860s, Adolf Fick, a professor of physiology in the Faculty of Medicine in Würzberg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is probably about 7 L/min. At rest it is likely to be a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min. Your cardiac output can be calculated with the formula

$$y = \frac{Q}{D},$$

where Q is the number of milliliters of CO_2 you exhale in a minute and D is the difference between the CO_2 concentration (ml/L) in the blood pumped to the lungs and the CO_2 concentration in the blood returning from the lungs. With $Q = 233$ ml/min and $D = 97 - 56 = 41$ ml/L, $y = 233$ ml/min/ 41 ml/L ≈ 5.68 L/min,

fairly close to the 6 L/min that most people have at basal (resting) conditions. (Data courtesy of J. Kenneth Herd, M.D., Quillan College of Medicine.)

Suppose that when $Q = 233$ and $D = 41$, we also know that D is decreasing at the rate of 2 units a minute but that Q remains unchanged. What is happening to the cardiac output?

A.2 *Least Squares* (**#Diffapplication**) *Adapted from Stewart's "Calculus"*

Suppose that a scientist has reason to believe that two quantities x and y are related linearly, that is $y = mx + b$, at least approximately, for some values of m and b. The scientist performs an experiment and collects data in the form of points (x_1, y_1), (x_2, y_2), ... ,(x_n, y_n), and then plots these points. The points don't lie exactly on a straight line, so the scientist wants to find constants m and b so that the line $y = mx + b$ "fits" the points as well as possible.

Let $d_i = y_i - (mx_i + b)$ be the vertical deviation of the point (x_i, y_i) from the line. The **method of least squares** determines m and b so as to minimize $\sum_{i=1}^{n} d_i^2$, the sum of the squares of these vertical deviations. Show that, according to this method, the line of best fit is obtained when m and b satisfy the following system of equations:

$$m\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i$$
$$m\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

Part B

B.1 *Airplane!* (**#Diffapplication**)

Airlines want to maximize profits with every flight. Since cargo is expensive to fly, airlines are now charging baggage fees. With baggage fees, customers are less likely to take big bags with them, giving less of a profit to the airline. Let's model a specific example:

(a) Airlines noticed that if they charge \$20 for each bag, then in a given flight, 50 bags will be checked. They also noticed that for every \$2 increase in the baggage fee, 2 fewer bags will be checked in. How can we model the revenue due to checked bags as a function of the number of price increases x? What are the constraints on the value of x?

(b) There are also some costs associated with carrying bags. This specific company noticed that the cost of checking N bags is equal to

$$C(N) = \frac{N^2}{20} + 7N$$

Write a function that describes the profit from checked bags in terms of the number of price increases x.

(c) Find the number of price increases x that will maximize this company's profits. What will be the optimal baggage fee? How many bags will be taken on the flight?

B.2 *Allele Alliteration* (**#Diffapplication**) *Adapted from Stewart's "Calculus"*

Three alleles (alternative versions of a gene) A,B, and O determine the four blood types A (AA or AO), B (BB or BO), O (OO), and AB. The Hardy-Weinberg Law states that the proportion of individuals in a population who carry two different allelles is

$$P = 2pq + 2pr + 2rq$$

where p, q, and r are the proportions of A, B, and O in the population. Use the fact that $p + q + r = 1$ to determine the maximal value P can take.