7.1 Make-up work

Separation and Independence

(1) Give an example of a two-dimensional dataset for which the data are linearly separable, but not linearly independent.

```
[[1, 0],
 [2, 1],
 [3, 1]]
```

(2) Can you find a dataset which is linearly independent but not linearly separable?

```
[[1, 0, 1],
 [2, 1, 0],
 [4, 1, 1]]
```

*labels are in the last column

Regression and the mean

Show that ordinary least squares regression fitted to a dataset (x_n, y_n), n=1,...,N will go through the mean of both the x and y datapoints.

When equalizing the derivative of the OLS to 0 in $y = ax + b$ we get

$$\begin{cases} \dfrac{\partial E}{\partial a} = -2\sum_{i=1}^{N}(y_i - ax_i - b)x_i = 0 \\ \\ \dfrac{\partial E}{\partial b} = -2\sum_{i=1}^{N}(y_i - ax_i - b) = 0 \end{cases}$$

$$\begin{cases} a < x^2 > +b < x > - < xy >= 0 \\ a < x > +b- < y >= 0 \end{cases}$$, where <x> is the mean of x

We can see that in the second equation, $a < x > +b$ is the value of the regression when we plug in mean of x, and it is equal to mean of y. Thus, this line $ax + b$ crossed the mean $(< x >, < y >)$

The logistic sigmoid

The logistic sigmoid function is defined as σ (x) = e^x /(1 + e^x).

(1) What is the inverse function, σ^(−1)(x)?

$$y = \frac{1}{1 + e^{-x}} \Rightarrow x = ln(\frac{y}{1 - y})$$

(2) Show that the derivative is σ (x)(1- σ (x)).

Let $a = 1 + e^{-x}$

$$(\frac{1}{a})' = -\frac{1}{a^2}a' = -\frac{1}{(1 + e^{-x})^2} * (-e^{-x}) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

$$\sigma(x)(1 - \sigma(x)) = \frac{1}{1 + e^{-x}} * \frac{e^{-x}}{1 + e^{-x}} = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Proved.