# Study guide

The purpose of this session is to learn about applying Monte Carlo simulations to pricing call and put options in stock markets. The readings cover basic random walk theory, which is used as a simple model for stock price movements, and introduce options at a very basic level. You will create some random walk simulations for your pre-class work assignment. We build on such a random walk simulation to estimate the fair price of options in class.

In order to be prepared for class, you need to be able to do the following.

- Understand what call and put options are and how they behave with respect to the strike price and the underlying price of the stock. What happens if the strike price is above or below the underlying stock price, respectively? The answer depends on whether you have a call or a put option.
- Understand why we use a Gaussian random walk to model stock prices. What are the assumptions and limitations of a Gaussian random walk as a model of stock prices? How do we use Gaussian random variables, which can be positive or negative, to model stock prices, which are always positive?

## Random walks

There are 3 types of random walks in one dimension that you should understand for today. All 3 types also generalize to more than one dimension (see your pre-class work), but for modeling stock prices we will use 1-dimensional random walks only.

- **Discrete random walks** Randomly take 1 step to the left or 1 step to the right — either with the same probability or with different probabilities for going left or right if it is a biased random walk. This type of random walk results in a binomial distribution over the final position of the walk after $N$ steps.
- **Continuous random walks** Add a Gaussian distributed random number to the current position at each step. If it is an unbiased random walk, the mean of the Gaussian is 0. The standard deviation of the Gaussian determines the step size of the random walk. This type of random walk results in a Gaussian distribution over the final position of the walk after $N$ steps (because the sum of $N$ normally distributed random numbers is also normally distributed).
- **Continuous multiplicative random walks** This is the one used for stock market prices. At each step, multiply the current price by (1 + a Gaussian distributed random number). The mean of the Gaussian determines the expected growth of the price (positive or negative). The standard deviation of the Gaussian determines the volatility of (fluctuations in) the stock price. This type of random walk results (to a very good approximation) in a log-normal distribution over the final price of the stock after $N$ steps.

## Options

- If you buy a **call option** with expiry date $D$ and strike price $K$, you can (but don't have to) **buy** the underlying stock at price $K$ at the predetermined future date. So, as the stock price gets close to or goes **above** $K$, your call option increases in value — since you get to buy the stock at price $K$ rather than buying at the real, higher price of the stock.
- If you buy a **put option** with expiry date $D$ and strike price $K$, you can (but don't have to) **sell** the underlying stock at price $K$ at the predetermined future date. So, as the stock price gets close to or goes **below** $K$, your put option increases in value — since you get to sell the stock at price $K$ rather than selling at the real, lower price of the stock.
- If we could see the future and we knew what the future price, $F$, of a stock would be, we know that a call option would be worth $\max(0, F - K)$ in the future. A call option can never have a negative value (since we don't have to buy the stock in the future) and it will have a positive value if the price of the stock is greater than the strike price.
    - If we knew the future price of the stock and therefore the future value of the call option, we could calculate the present value of the call option. Future and present values are not the same — see the section below for why.
    - We can't see the future, but we can simulate it based on a model. In class, we will use the random walk pricing model to simulate possible future prices for a stock and use those future prices to compute the expected present value of a call option.
- The famous Black–Scholes model was developed precisely to price options. It gives a formula for the price, whereas we will be using a Monte Carlo simulation to estimate the price. The [assumptions behind the Black–Scholes pricing model] are the same as the assumptions behind our stock price random walk. What makes the simulation very powerful, though, is that it is much easier to change the assumptions of the pricing model and calculate the effect on stock or option prices, than it would be to derive a new set of equations to replace the Black–Scholes formula for the new set of assumptions. (Merton and Scholes got the Nobel Prize in Economics for completing the mathematical derivation, so it's not easy.)

## Other important finance concepts

- Money now is worth more than money in the future. There are all sorts of technical, social, and philosophical reasons for this, but we will stick to the following simple argument.
- If you had ₹100 (Indian rupees) right now and could earn risk-free interest of 7.5% per annum on your money, ₹100 right now is worth ₹107.50 in 1 year without taking any risk. We use this risk-free rate of return to convert between present values (₹100) and future values (₹107.50). (The current 10-year Indian government bond yield is about 7.5%. Long-term bond yields vary by country, so your risk-free rate of return depends on the country you are in, and how stable its government and economy are.)
- If we know something has a present value of $x$, then its future value is $x\,(1+r)^t$, where $r$ is the risk-free rate of return and $t$ is the number of years between the present and the future.
- If we know something has a future value of $x$, then its present value is $x\,(1+r)^{-t}$, where $r$ is the risk-free rate of return and $t$ is the number of years between the present and the future.