UNIT-XI

ALGORITHM

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CHAPTER 1

ASYMPTOTIC ANALYSIS OF ALGORITHM

OBJECTIVE QUESTIONS

1. What is time complexity of following code :

```
int a = 0 ;
for (i = 0 ; i < N ; i + +) {
    for (j = N ; j > i ; j – –) {
        a = a + i + j ;
    }
}
```

(a) 0 (N) (b) 0(N*log(N))

(c) 0(N*Sprt (N)) (d) 0(N*N)

2. What is time complexity of following code

```
int a = 0 , i = N ;
while (i > 0) {
    a + = i ;
    i / = 2 ;
}
```

(a) 0 (N) (b) $0(\sqrt{N})$

(c) 0 (N/2) (d) 0(log N)

3. What is time and space comlexity of following code

```
int a = 0 , b = 0 ;
for (i = 0 ;i < N ; i + + ) {
    a = a + rand ( ) ;
}
for (j = 0 ; j < M ; j + + ) {
    b = b + rand ( ) ;
}
```

(a) 0(N*M) time, 0(1) space

(b) 0(N+M) time, 0(N+M) space

(c) 0(N+M) time, 0(1) space

(d) 0(N*M) time, 0(N+M) space

4. What is time complexity of following code :

```
int i, j, k = 0 ;
for (i = n/2 ; i ≤ n ; j = j*2)
    for (j = 2 ; j < = n ; j = j*2)
        K = K + n/2 ;
    }
}
```

(a) 0(n) (b) 0(n logn)

(c) $0(n^2)$ (d) $0(n^2 \log n)$

5. Which of the following case does not exist in complexity theory?

(a) Best case (b) Worst case

(c) Average case (d) Null case

6. The complexity of linear search algorithm is

(a) 0(n) (b) 0(logn)

(c) $0(n^2)$ (d) 0(nlogn)

7. The complexity of binary search algorithm is

(a) 0(n) (b) 0(logn)

(c) $0(n^2)$ (d) 0(nlogn)

8. The complexity of Fibonacci series is

(a) $0(2^n)$ (b) 0(logn)

(c) $0(n^2)$ (d) 0(nlogn)

9. Worst case occur in quick sort when

(a) Pivot is the median of the array

(b) Pivot is the smallest element

(c) Pivot is the middle element

(d) None f the mentioned

10. If for an algorithm time complexity is given by $0(\log_2 n)$ then complexity will be

(a) Constant (b) Polynomial

(c) Exponential (d) Logarithmic

11. An algorithm that requires ... operations to complete its task on n data elements is said to have a linear runtime.

(a) $n^3 + 9$ (b) $3n^2 + 3n + 2$

(c) $2n + 1$ (d) 9

12. The complexity of adding two matrices of order m*n is

(a) m+n (b) mn

(c) max(m, n) (d) min (m, n)

13. An algorithm is made up of two modules M1 and M2. If order of M1 is f(n) & M2 is g(n) then order of algorithm is?

(a) g(n) + g(n) (b) f(n) × g(n)

(c) Max(f(n), g(n)) (d) Min(f/n), g(n))

14. An algorithm may have how many input quantities

(a) One or more (b) Zero or more

(c) Two or more (d) None of the above

15. The two main resources that we consider for an algorithm are

(a) Memory space and processor time

(b) Space complexity and time complexity

(c) Input and output properties

(d) None of the above

16. Program with highest run-time complexity is

(a) Tower of hanoi

(b) Fibonacci series

(c) Prime number series

(d) None of the above

17. The concept of order (Big 0) is important because

(a) It can be used to decide the best algorithm that solves a given problem

(b) It determines the maximum size of a problem that can be solved in a given system, in a given amount of time

(c) Both a and b

(d) None

18. Solution of the recurrence $T(n) = 3T(n/2) + n^2$ is

(a) $\theta(n)$ (b) $\theta(\log n)$

(c) $\theta(n^2)$ (d) $\theta(n \log n)$

19. Solution of recurrence $T(n) = 4T(n/2) + n^2$ is

(a) $\theta(n)$ (b) $\theta(\log n)$

(c) $\theta(n^2)$ (d) $\theta(n^2 \log n)$

20. Average successful search time for sequential search on 'n' items is

(a) n/2 (b) (n–1)/2

(c) (n+2)/2 (d) None of these

21. Upper bound for $f(n) = n^2 + 1$ is

(a) $C = 2$ and $n_0 = 1$

(b) $C = 1$ and $n_0 = 1$

(c) $C = 1$ and $n_0 = 2$

(d) None of these

22. Solution for recurrence

$T(n) = 2T(n/2) + n/\log n$ is

(a) $\theta(n)$ (b) $\theta(2^n)$

(c) $\theta(n^2)$ (d) $\theta(n \log \log n)$

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Answer Key

1. *Ans: (d)*

The above code runs total no. of times

$= N + (N-1) + (N-2) + \ldots + 1 + 0$

$= N*(N+1)/2$

$= 1/2*N^2 + 1/2*N$

$= 0(N^2)$ times

2. *Ans: (d)*

3. *Ans: (c)*

4. *Ans: (b)*

5. *Ans: (d)*

6. *Ans: (a)*

7. *Ans: (b)*

8. *Ans: (a)*

9. *Ans: (b)*

This happens when pivot is the smallest or the largest element. Then one of the partitions is empty and we repeat recursively the procedure of (N–1) elements.

10. *Ans: (d)*

Complexity will be logarithmic

11. *Ans: (c)*

12. *Ans: (b)*

13. *Ans: (d)*

14. *Ans: (b)*

15. *Ans: (a)*

16. *Ans: (a)*

17. *Ans: (c)*

18. *Ans: (c)*

19. *Ans: (d)*

20. *Ans: (c)*

21. *Ans: (a)*

22. *Ans: (d)*

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CHAPTER 2

RECURRENCE RELATION

OBJECTIVE QUESTIONS

1. What is the value of following recurrence

$T(n) = T(n/4) + T(n/2) + cn^2$

$T(1) = C$

$T(0) = 0$

Where C is positive constant

(a) $0(n^3)$ (b) $0(n^2)$

(c) $0(n^2\log n)$ (d) $0(n\log n)$

2. What is the value of following recurrence

$T(n) = 5T(n/5) + \sqrt{n}$, $T(1) = 1$, $T(0) = 0$

(a) $\theta(n)$ (b) $\theta(n^2)$

(c) $\theta(\sqrt{n})$ (d) $\theta(n\log n)$

3. The running time of an algorithm is represented by following recurrence relation

if $n <= 3$ then $T(n) = n$

else $T(n) = T(n/3) + cn$

Which one of the following represents the time complexity of the algorithm?

(a) $\theta(n)$ (b) $\theta(n\log n)$

(c) $\theta(n^2)$ (d) $\theta(n^2\log n)$

4. $T(n) = 3T(n/2 + 47) + 2n^2 + 10n - \frac{1}{2}$

T(n) will be

(a) $0(n^2)$ (b) $0(n^{3/2})$

(c) $0(n\log n)$ (d) None

5. Let T(n) be the function defined by

$T(1) = 1$, $T(n) = 2T([n/2]) + \sqrt{n}$ for $n \geq 2$.

Which of the following statement is true?

(a) $T(n) = 0(\sqrt{n})$ (b) $T(n) = 0(n)$

(c) $T(n) = 0(\log n)$ (d) None of these

6. The recurrence relation

$T(1) = 2$

$T(n) = 3T(n/4) + n$

has the solution T(n) equal to

(a) $0(n)$ (b) $0(\log n)$

(c) $0(n^{3/4})$ (d) None of these

7. A polynomial p(x) is such that

$p(0) = 5$, $p(1) = 4$, $p(2) = 9$ and $p(3) = 20$

The minimum degree it can have is

(a) 1 (b) 2

(c) 3 (d) 4

8. $\sum_{1 \leq k \leq n} O(k)$, where O(n) stands for order n is.

(a) $0(2^n)$ (b) $0(\log n)$

(c) $0(n^2)$ (d) $0(n\log n)$

9. Time complexity of an algorithm T(n), where n is the input size is given by

$T(n) = T(n-1) + 1/n$, if $n > 1$

$T(n) = 1$, otherwise

The order of this algorithm is

(a) $\log n$ (b) n

(c) n^2 (d) n^n

10. The concept of order (Big O) is important because

(a) It can be used to decide the best algorithm that solves a given problem

(b) It determines the maximum size of a problem that can be solved in a given amount of time

(c) It is the lower bound of the growth rate of algorithm

(d) Both (a) and (b)

11. What is the time complexity of the following recursive function?

```
int DoSomething (int n) {
if (n <= 2)
return 1 ;
else
return (DoSomething (floor (sqrt(n))) + n);
}
```

(a) $\Theta(n^2)$

(b) $\Theta(n \log_2 n)$

(c) $\Theta(\log_2 n)$

(d) $\Theta(\log_2 \log_2 n)$

12. Consider the following functions:

$f(n) = 2^n$

$g(n) = n!$

$h(n) = n^{\log n}$

Which of the following statements about the asymptotic behaviour of f(n), g(n), and h(n) is true?

(a) $f(n) = O(g(n)); g(n) = O(h(n))$

(b) $f(n) = \Omega(g(n)); g(n) = O(h(n))$

(c) $g(n) = O(f(n)); h(n) = O(f(n))$

(d) $h(n) = O(f(n)); g(n) = \Omega(f(n))$

13. The running time of an algorithm is represented by the following recurrence relation.

$$T(n) = \begin{cases} n & n \le 3 \\ T\left(\frac{n}{3}\right) + cn & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm?

(a) $\theta(n)$ (b) $\theta(n \log n)$

(c) $\theta(n^2)$ (d) $\theta(n^2 \log n)$

14. Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is ALWAYS TRUE?

(a) $A(n) = \Omega(W(n))$ (b) $A(n) = \Theta(W(n))$

(c) $A(n) = O(W(n))$ (d) $A(n) = o(W(n))$

15. What is time complexity of the recurrence relation using master theorem, $(T(n) = 7T\left(\frac{n}{3}\right) + n^2$?

(a) $\theta(n^2)$ (b) $\theta(n)$

(c) $\theta(n^3)$ (d) $\theta(\log n)$

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Answer Key

1. *Ans: (b)*
2. *Ans: (a)*
3. *Ans: (a)*
4. *Ans: (a)*

   Apply master theorem, it is case-2 of master theorem $T(n) = 0(n^2)$
5. *Ans: (b)*
6. *Ans: (a)*
7. *Ans: (b)*
8. *Ans: (b)*
9. *Ans: (a)*
10. *Ans: (a)*
11. *Ans: (d)*
12. *Ans: (d)*
13. *Ans: (a)*
14. *Ans: (c)*
15. *Ans: (a)*

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CHAPTER 3

DIVINE AND CONQUER

OBJECTIVE QUESTIONS

1. The complexity of comparison based sorting algorithms is.

(a) θ (n log n) (b) θ(n)

(c) θ (n 2) (d) θ (n A n)

2. When s be a sorted array of n integers, and t(n) denote the time taken for the most efficient algorithm to determine if there are two elements with sum less than 1000 in s, then which of the following statements is true?

(a) t(n) is 0 (1)

(b) $n \le t(n) \le n \log_2 n$

(c) $n \log_2 n \le t(n) < \left(\frac{n}{2}\right)$

(d) $t(n) = \left(\frac{n}{2}\right)$

3. Which of the following statements is true?

I. As the number of the entries in a hash table increases, the number of collisions increases.

II. Recursive programs are efficient.

III. The worst case complexity for Quicksort is $O(n^2)$

IV. Binary search using a linear linked list is efficient.

(a) I and II (b) II and III

(c) I and IV (d) I and III

4. The algorithm design technique used in the quick sort algorithm is.

(a) Dynamic programming

(b) Backtracking

(c) Divide and conquer

(d) Greedy method

5. A machine needs a minimum of 100 sec to sort 1000 names by quick sort. The minimum time needed to sort 100 names will be approximately.

(a) 50.2 sec (b) 6.7 sec

(c) 72.7 sec (d) 11.2 sec

6. A machine took 200 sec to sort 200 names, using bubble sort. In 800 sec, it can approximately sort.

(a) 400 names (b) 800 names

(c) 750 names (d) 900 names

7. For merging two sorted lists of sizes m and n into a sorted list of size m + n, we require comparisons of.

(a) O (m) (b) O (n)

(c) O (m + n) (d) O (log n + log n)

8. The way a card game player arranges his cards as he picks them up one by one, is an example of

(a) bubble sort

(b) selection sort

(c) insertion sort

(d) merge sort

9. A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. The roots is stored at X[1]. For a node stored at X[i], the left child, if any, is stored in X[2i] and the right child, if any, in X[2i + 1]. To be able to store any binary tree on n vertices, the minimum size of X should be.

(a) $\log_2 n$ (b) n

(c) 2n + 1 (d) $2^n - 1$

10. Which one of the following in place sorting algorithms needs the minimum number of swaps?

(a) Quick sort (b) Insertion sort

(c) Selection sort (d) Heap sort

11. An element in an array X is called a leader if it is greater than all elements to the right of it in X. The best algorithm to find all leaders in an array.

(a) Solves it in linear time using a left to right pass of the array

(b) Solves in linear time using a right to left pass of the array

(c) Solves it using divide and conquer in time $\theta(n \log n)$

(d) Solves it in time $\theta(n^2)$

12. The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is.

(a) 2^h (b) $2^{h-1} - 1$

(c) $2^{h+1} - 1$ (d) 2^{h+1}

13. The maximum number of binary trees that can be formed with three unlabeled nodes is.

(a) 1 (b) 5

(c) 4 (d) 3

14. Which of the following sorting algorithms has the lowest worst-case complexity?

(a) Merge sort (b) Bubble sort

(c) Quick sort (d) Selection sort

15. A complete n-ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If L = 41, and I = 10, then what is the value of n?

(a) 3 (b) 4

(c) 5 (d) 6

16. Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into two sub-lists each of which contains at least one-fifth of the elements. Let T(n) be the number of comparisons required to sort n elements. Then

(a) $T(n) \le 2T\left(\frac{n}{5}\right) + n$

(b) $T(n) \le T\left(\frac{n}{5}\right) + T\left(\frac{4n}{5}\right) + n$

(c) $T(n) \le 2T\left(\frac{4n}{5}\right) + n$

(d) $T(n) \le 2T\left(\frac{n}{2}\right) + n$

17. In quick sort, for sorting n elements, the $(n/4)^{th}$ smallest element is selected as pivot using an O(n) time algorithm. What is the worst case time complexity of the quick sort?

(a) $\theta(n)$

(b) $\theta(n \log n)$

(c) $\theta(n^2)$

(d) $\theta(n^2 \log n)$

18. The worst case running time to search for an element in a balanced binary search tree with $n2^n$ elements is.

(a) $\Theta(n \log n)$

(b) $\Theta(n2^n)$

(c) $\Theta(n)$

(d) $\Theta(\log n)$

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Answer Key

1. *Ans. (a)*

2. *Ans. (a)*

3. *Ans. (d)*

4. *Ans. (c)*

5. *Ans. (b)*

In the best case quick sort algorithm makes nlogn comparisons.

So, $1000 \times \log(1000) = 9000$ comparisons, which takes 100 s.

To sort 100 names a minimum of 100 (log 100) = 600 comparisons are needed.

This takes $100 \times \frac{600}{9000} = 6.7$ sec.

6. *Ans. (a)*

For sorting 200 names, bubble sort makes

$200 \times \frac{199}{2} = 19900$ comparisons.

So, time needed for 1 comparison is 200 sec (approximately).

In 800 sec it can make 80,000 comparisons.

We have to find n, such that

$$n\frac{(n-1)}{2} = 80{,}000.$$

Solving, we get $n \approx 400$.

7. *Ans. (c)*

Each comparison will append one item to the existing merge list. So, in worst case, one needs m + n – 1 comparisons which is of order m + n.

8. *Ans. (c)*

9. *Ans. (d)*

10. *Ans. (c)*

11. *Ans. (c)*

Let x contains n elements divide it by n/2 repeatedly so the recurrence become

$T(1) = 1$

$T(n) = 2T(n/2) + 1$

$T(n) = O(n \log n)$

12. *Ans. (c)*

13. *Ans. (b)*

14. *Ans. (a)*

Merge sort has lowest worst – case complexity, i.e. O(n log n), whereas all remaining three has $O(n^2)$.

15. *Ans. (c)*

16. *Ans. (b)*

17. *Ans. (b)*

Average complexity of quick sort is θ (n log n).

18. *Ans. (c)*

The worst case running time of search in BST is log n where n is total number of elements in the tree. The total number of elements are n. 2^n thus search R. T. will be long (n. 2^n).

$\log n.\ 2n = \log + \log 2^n$

$= \log n + n \log 2$

$= \theta(n)$

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CHAPTER 4

TREE AND GRAPH REPRESENTATION

OBJECTIVE QUESTIONS

1. In a binary max heap containing n numbers, the smallest element can be found in time.

(a) O (n) (b) O (log n)

(c) O (log log n) (d) O (1)

2. We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. Total time required for this is.

(a) $\Theta(\log n)$ (b) $\Theta(n)$

(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

3. What is full binary tree?

(a) Each node has exactly zero or two children

(b) Each node has exactly two children

(c) All the leaves are at the same level

(d) Each made has exactly one or two children

4. What is complete binary tree?

(a) Each node has exactly zero or two children

(b) A binary tree, which is completely filled with the possible exception of the bottom level, which is filled from right to left

(c) A binary tree which is completely filled with possible exception of bottom level, which is filled from left to right

(d) None

5. In a full binary tree if number of internal node is I, then number of leaves L are?

(a) L = 2I (b) L = I + 1

(c) L = I – 1 (d) L = 2I – 1

6. In a full binary tree if there are L leaves, then total number of nodes N are?

(a) N = 2L (b) N = L + 1

(c) N = L – 1 (d) N = 2L – 1

7. With what data structure priority queue can be implemented?

(a) Array (b) List

(c) Heap (d) All of these

8. Which of the following is an application of priority queue?

(a) Huffman codes

(b) Interrupt handling in operating system

(c) Undo operation in text editors

(d) Bayesian spam filter

9. An array consist of n elements. Time complexity of building a heap will be in order of

(a) $0(n^2 \log n)$ (b) 0(nlogn)

(c) $0(n^2)$ (d) 0(nlog logn)

10. Heap can be used as

(a) Priority queue

(b) Stack

(c) A decreasing order array

(d) None

11. For an undirected graph G with n vertices and e edges, the sum of the degree of each vertex is

(a) ne (b) 2n

(c) 2e (d) e^n

12. Graph traversal is different from tree traversal because

(a) Tree is not connected

(b) Graph may have loops

(c) Tree have root

(d) None

13. The number of comparisons done by sequential search is

(a) N/2 + 1 (b) (N + 1)/2

(c) (N – 1)/2 (d) (N – 2)/2

14. Which of the following is not in place sorting algorithm

(a) Selection sort (b) Heap sort

(c) Quick sort (d) Merge sort

15. Time complexity of heap sort in worst case is

(a) 0(logn) (b) 0(n)

(c) 0(nlogn) (d) $0(n^2)$

16. Which of the following statement is correct about binary tree

(a) Every binary tree is either complete or full

(b) Every complete binary tree is also a full binary tree

(c) Every full binary tree is also a complete binary tree

(d) A binary tree can not be both complete and full

17. Suppose we have numbers between 1 and 1000 in a binary search tree and want to search for the number 363. Which of the following could not be the sequence of the node examined?

(a) 2, 252, 401, 398, 330, 344, 397, 363

(b) 924, 220, 911, 244, 898, 258, 362, 363

(c) 925, 202, 911, 240, 912, 245, 258, 363

(d) 2, 399, 387, 219, 266, 382, 381, 278, 363

18. Suppose a complete binary tree has height h > 0. The minimum number of leaf nodes possible in term of h is?

(a) $2^h - 1$ (b) $2^{h-1} + 1$

(c) 2^{h-1} (d) $2^h + 1$

19. The worst case complexity of deleting any arbitrary node value element from heap is

(a) 0(logn) (b) 0(n)

(c) 0(nlogn) (d) $0(n^2)$

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Answer Key

1. *Ans. (a)*
2. *Ans. (a)*
3. *Ans. (a)*
4. *Ans. (c)*
5. *Ans. (b)*
6. *Ans. (d)*
7. *Ans. (d)*
8. *Ans. (c)*

   Undo operation is achieved using a stack.
9. *Ans. (b)*

   The total time taken will be N times the complexity of adding a single element to the heap. And a single element takes log N time, so this is equal to N logn.
10. *Ans. (a)*

    The property of heap is that the value of root must be either greater or less than both of its children, makes it work like a priority queue.
11. *Ans. (c)*
12. *Ans. (c)*
13. *Ans. (b)*
14. *Ans. (d)*
15. *Ans. (c)*
16. *Ans. (c)*
17. *Ans. (c)*
18. *Ans. (c)*
19. *Ans. (a)*

    The total possible operation in delecting the existing node and relocating new position to all its connected nodes will be equal to height of the heap.

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CHAPTER 5

GREEDY METHOD

OBJECTIVE QUESTIONS

Common Data Q. 1-2

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the wight of the edge {ij}.

$$W=\begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

1. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

(a) 7 (b) 8

(c) 9 (d) 10

2. What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

(a) 7 (b) 8

(c) 9 (d) 10

3. Which statement is true about Krushkal's algorithm?

(a) It is a greedy algorithm for the minimum spanning tree problem

(b) It constructs spanning tree by selecting edges in increasing order of their weights

(c) It does not accept creation of cycles in spanning tree

(d) All of the above

4. Algorithm which solves the all-pair shortest path problem is.

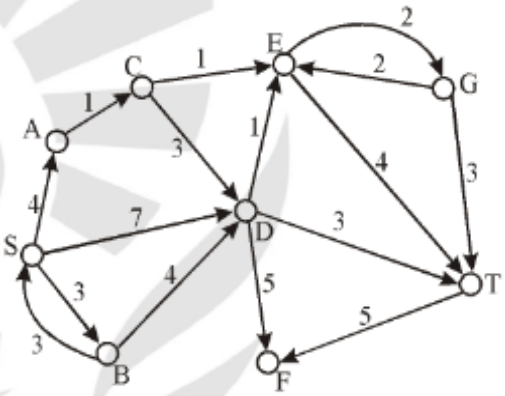
(a) Dijkstra's algorithm

(b) Floyd's algorithm

(c) Prim's algorithm

(d) Warshall's algorithm

5. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



(a) SDT (b) SBDT

(c) SACDT (d) SACET

6. Consider a weighted complete graph G on the vertex set $(v_1, v_2, \ldots, v_n)$ such that the weight of the edge (v_i, v_j) is $2|i-j|$. The weight of a minimum spanning tree of G is.

(a) n – 1 (b) 2n – 2

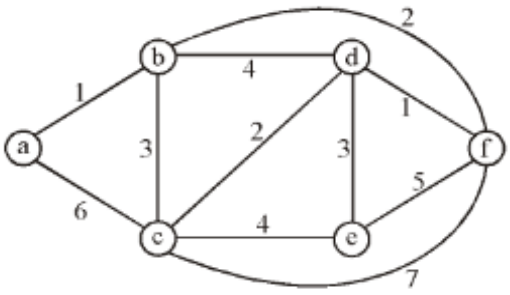
(c) $\left(\frac{n}{2}\right)$ (d) n^2

7. To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is.

(a) Queue (b) Stack

(c) Heap (d) B-Tree

8. Consider the following graph:



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Krushkal's algorithm?

(a) (a – b), (d – f), (b – f), (d – c), (d – e)

(b) (a – b), (d – f), (d – c), (b – f), (d – e)

(c) (d – f), (a – b), (d – c), (b – f), (d – e)

(d) (d – f), (a – b), (b – f), (d – e), (d – c)

9. Let T be a depth first search tree in a undirected graph G. Vertices u and v are leaves of this tree T. The degrees of both u and v in G are at least 2. Which one of the following statements is true?

(a) There must exist a vertex w adjacent to both u and v in G.

(b) There must exist a vertex w whose removal disconnects u and v in G.

(c) There must exist a cycle in G containing u and v

(d) There must exist a cycle in G containing u and all its neighbours in G.

10. In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of time complexity, by

(a) Dijkstra's algorithm starting from S.

(b) Warshall's algorithm

(c) performing a DFS starting from S

(d) performing a BFS starting from S

11. Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w.

Which of the following is FALSE?

(a) There is a minimum spanning tree containing e.

(b) If e is not in a minimum spanning tree T, then in the cycle fromed by adding e to T, all edges have the same weight.

(c) Every minimum spanning tree has an edge of weight w

(d) e is present in every minimum spanning tree

12. The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity?

(a) $\Theta(n)$ (b) $\Theta(m)$

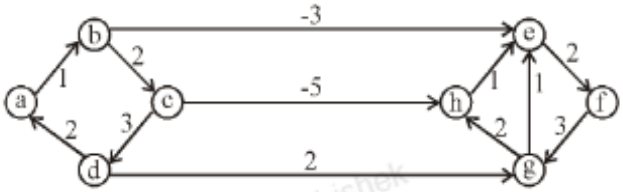
(c) $\Theta(m + n)$ (d) $\Theta(mn)$

13. The Breadth First Search algorithm has been implemented using the queue data structure, One possible order of visiting the nodes of the following graph is.

(a) MNOPQR (b) NQMPOR

(c) QMNPRO (d) QMNPOR

14. Dijkstra's single source shortest path algorithm when run from vertex a in the below graph, computes the correct shortest path distance to



(a) only vertex a

(b) only vertices a, e, f, g, h

(c) only vertices a, b, c, d

(d) all the vertices

15. Which of the following statement (s) is/are correct regarding Bellman-Ford shortest path algorithm?

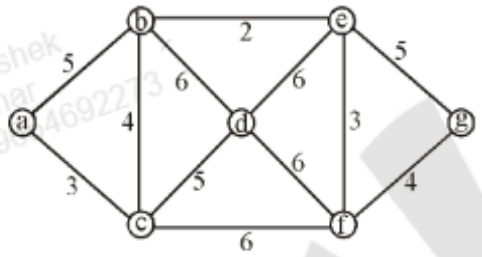
P. A always finds a negative weighted cycle, if one exists.

Q. Finds whether any negative weighted cycle is reachable from the source.

(a) P only (b) Q only

(c) both P and Q (d) neither P nor Q

16. Consider the following graph:



Which one of the following is NOT the sequence of edges added to the minimum spannign tree using Kruskal's algorithm?

(a) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)

(b) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)

(c) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)

(d) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

17. Dijkstra's algorithm bears similarity to which of the following for computing minimum spanning tree?

(a) Breadth first search

(b) Prim's algorithm

(c) Both (a) and (b)

(d) None of these

18. To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is

(a) Queue (b) Stack

(c) Heap (d) B-Tree

19. Let C_1, C_2, C_3, C_4, represent coins

C_1 = 25 paisa C_2 = 10 paisa

C_3 = 5 paisa C_4 = 1 paisa

To represent 48 paisa, what is minimum number of coins used using greedy approach?

(a) 6 (b) 7

(c) 8 (d) 9

Linked Answer Q.20-21

Suppose the letters a, b, c, d, e, f have probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}$, respectively.

20. Which of the following is the Huffman code for the letters a, b, c, d, e, f ?

(a) 0, 10, 110, 1110, 11110, 11111

(b) 11, 10, 011, 010, 001, 000

(c) 11, 10, 01, 001, 0001, 0000

(d) 110, 100, 010, 000, 001, 111

21. What is the average length of the correct answer to Q. 20?

(a) 3 (b) 2.1875

(c) 2.25 (d) 1.9375

❑❑❑

Answer Key

1. *Ans. (d)*
2. *Ans. (b)*
3. *Ans. (d)*
4. *Ans. (b)*
5. *Ans. (d)*
6. *Ans. (b)*
7. *Ans. (c)*
8. *Ans. (d)*

   Ordering the weights, the sequence obtained is

   1, 1, 2, 2, 3, 3, 4, 4, 5, 6, 7

   Now, in order to obtain minimum spanning tree using Kruskal's algorithm, we have to add edges with weights in increasing order such that weight of the spanning tree is minimum.

   In option (d), weights of the edges are 1, 1, 2, 3, 2 which contradicts Kruskal's algorithm.

9. *Ans. (d)*
10. *Ans. (d)*

    In case of unweighted, undirected graphs, BFS gives the most time efficient computation for shortest path. It is guaranteed to find first shortest path.

11. *Ans. (d)*

    e is need not to be present in every spanning tree since there may be edges (in cycle formed by adding e), which has same weight as e has.

12. *Ans. (c)*

    The most efficient algorithm for finding the number of connected components (articulation point) in an undirected graph on n vertices and n edges using depth-first search takes O(m + n) time. Assume n ≤ m.

13. *Ans. (c)*
14. *Ans. (d)*
15. *Ans. (b)*

    In Bellman Ford shortest path algorithm, we choose a node first and then we find any negative weighted cycle is reachable from source.

16. *Ans. (d)*
17. *Ans. (c)*
18. *Ans. (a)*
19. *Ans. (a)*
20. *Ans. (a)*

    Characters having higher probability will be needec lesses bits so number of bits needed are 1, 2, 3, 4, 5, 5.

21. *Ans. (d)*

    Average length

    = Σ bits required × probability

    $$= \frac{1}{2}\times 1 + 2\times\frac{1}{4} + 3\times\frac{1}{8} + 4\times\frac{1}{16} + 5\times\frac{1}{32} + 5\times\frac{1}{32}$$

    = 1.9375

❍❍❍

CHAPTER 6

DYNAMIC PROGRAMMING

OBJECTIVE QUESTIONS

Common Data Q. 1

A sub-sequence of a given sequences is just the given sequence with some elements (possibly none or all) left out. We are given two sequences X[m] and Y[n] of lengths m and n, respectively, with indexes of X and Y starting from 0.

1. We wish to find the length of the longest common sub-sequence (LCS) of X[m] and Y[n] as l(m, n), where an incomplet recursive definition for the function l(i, j) to compute the length of the LCS of X[m] and Y[n] is given below:

l(i, j) = 0, if either i = 0 or j = 0

= expr l, if i, j < 0 and X [i – 1] = Y [j – 1]

= expr 2, if i, j > 0 and X [i – 1] ≠ Y [j – 1]

Which one of the following options is correct?

(a) expr 1 ≡ l (i – 1, j) + l

(b) expr 1 ≡ l (i, j – 1)

(c) expr 2 ≡ max (l (i – 1, j), l (i, j – 1))

(d) expr 2 ≡ max (l(i – 1, j – 1), l (i, j))

2. Which of the following standard algorithm is not dynamic programming based

(a) Bellman-ford algorithm for single source shortest path

(b) Floyd warshall for all pair shortest paths

(c) 0-1 Knapsack problem

(d) Prim's minimum spanning tree

3. We use dynamic programming approach when

(a) We need an optimal solution

(b) The solution has optimal substructure

(c) The given problem can be reduced to a 3-SAT problem

(d) Its faster than greedy

4. Consider the following two subsequences :

X = < B, C, D, C, A, B, C > and

Y = < C, A, D, B, C, B >

The length of longest common subsequence of X and Y is

(a) 5 (b) 3

(c) 4 (d) 2

5. Optimal solution can be created for a problem by constructing optimal solutions for its subproblems, the problem possesses

(a) Overlapping subproblem

(b) Optimal substructure

(c) Memorization

(d) Greedy

6. If a problem can be solved by combining optimal solutions to non-overlapping problems, the strategy is called

(a) Dynamic programming

(b) Greedy

(c) Divide and conquer

(d) Recursion

7. A greedy algorithm can be used to solve all the dynamic programming problems

(a) True

(b) False

8. Which of the following problem is not solved using dynamic programming?

(a) 0/1 Knapsack problem

(b) Matrix chain multiplication problem

(c) Edit distance problem

(d) Fractional knapsack problem

9. You are given a knapsack that can carry a maximum weight of 60. There are 4 items with weight {20, 30, 40, 70} and values {70, 80, 90, 200}. What is maximum value of items you can carry using the knapsack?

(a) 160 (b) 200

(c) 170 (d) 90

10. What is time complexity of Brute force algorithm used to solve knapsack problem?

(a) 0(n) (b) 0(n!)

(c) $0(2^n)$ (d) $0(n^3)$

11. Consider the matrices P, Q and R which are 10×20, 20×30 and 30×40 matrices respectively. What is the minimum no. of multiplication required to multiply the three matrices.

(a) 18000 (b) 12000

(c) 24000 (d) 32000

12. Consider the strings "PQRSTPQRS" and "PRATPBRQRPS" what is length of longest common subsequence?

(a) 9 (b) 8

(c) 7 (d) 6

13. Which of the following is the longest common subsequence between the strings "hbcfgmnapq" and "cbhgrstnmq"?

(a) hgmq (b) cfnq

(c) bfmq (d) None

14. Which algorithm design method can be used when solution to the problem can be viewed as the result of a sequence of decisions.

(a) Dynamic programming

(b) Backtracking

(c) Branch and bound

(d) Greedy method

15. How many distinct BST's can be constructed with 3 distinct keys?

(a) 4 (b) 5

(c) 6 (d) 9

❑❑❑

Answer Key

1. *Ans. (c)*
2. *Ans. (d)*

   Prim's minimum spanning tree is greedy algorithm
3. *Ans. (b)*
4. *Ans. (c)*
5. *Ans. (b)*
6. *Ans. (c)*
7. *Ans. (b)*
8. *Ans. (d)*
9. *Ans. (a)*

   This can be achived by choosing the items 1 and 3 that have a total weight of 60.
10. *Ans. (c)*
11. *Ans. (a)*

    The minimum no. of multiplications are 18000. This is the case when the matrices are parenthesized as (P*Q)*R
12. *Ans. (c)*
13. *Ans. (d)*
14. *Ans. (a)*
15. *Ans. (b)*

❑❑❑

CHAPTER 7

SOME IMPORTANT ALGORITHMS

OBJECTIVE QUESTIONS

1. Breadth-first traversal (BFS) is a method to traverse.

(a) all successors of a visited node before any successors of any of those successors.

(b) a single path of the graph as far as it can go

(c) graph using shortest path

(d) none of these

2. Identify the correct statements about DFS traversal.

I. It can be used to determine whether a graph is acyclic or not.

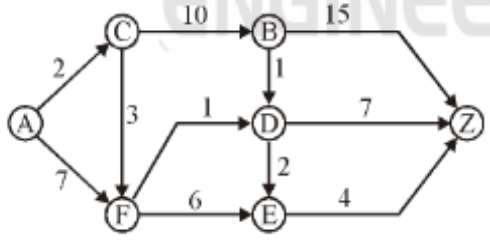
II. It identify the connected component of an undirected graph.

III. Traverses a single path of the graph until it visits a node with no successor.

(a) I and III (b) II and III

(c) I and II (d) I, II and III

3. Using breadth first search for a shortest path from A to Z, the order in which nodes are considered best for the path is.



(Note that these are node orders, not full paths.)

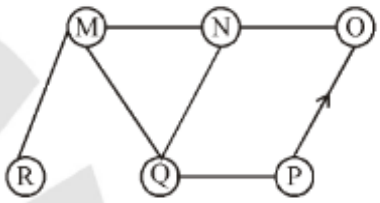
(a) A < C < F < D < E

(b) A < C < E < B

(c) A< C < F < E < B

(d) A < C < D < F

4. The BFS algorithm has been implemented using queue data structure. One possible order of visiting the nodes of the following graph is



(a) MNOPQR

(b) NQMPOR

(c) QMNPRO

(d) QMNPOR

5. Given two vertices in a graph s and t, which of the two traversals (BFS and DFS) can be used to find if there is a path from s to t?

(a) Only BFS

(b) Only DFS

(c) Both BFS & DFS

(d) Neither BFS nor DFS

6. Depth first search is equivalent to which of the traversal in binary trees?

(a) Pre-order Traversal

(b) Post order Traversal

(c) Level-order Traversal

(d) In order Traversal

7. The worst case running time to search for an element in a binary search tree with $n2^n$ elements is

(a) $\theta(n \log n)$ (b) $\theta(n2^n)$

(c) $\theta(n)$ (d) $\theta(\log n)$

8. What is maximum height of any AVL-tree with 7 modes? Assume that the height of a tree with a single node is 0.

(a) 2 (b) 3

(c) 4 (d) 5

9. Which of the following is not a stable sorting algorithm in its typical implementation?

(a) Insertion sort (b) Merge sort

(c) Quick sort (d) Bubble sort

10. What is best time complexity of bubble sort?

(a) N^2 (b) N logn

(c) N (d) $N(logn)^2$

11. You have to sort 1GB of data with only 100 MB of available main memory. Which sorting technique will be most appropriate?

(a) Heap sort (b) Merge sort

(c) Quick sort (d) Insertion sort

12. Which of the following sorting algorithm has the lowest worst-case complexity?

(a) Merge sort (b) Bubble sort

(c) Quick sort (d) Selection sort

13. What is the correct order of the following algorithm with respect to their time complexity in best case?

(a) Merge sort > Quick sort > Insertion sort > Selection sort

(b) Insertion sort > Quick sort > Merge sort > Selection sort

(c) Merge sort > Selection sort > Quick sort > Insertion sort

(d) Merge sort > Quick sort > Selection sort > Insertion sort

14. Which is the best algorithm to use for elements in array that are more than 1 million in general?

(a) Merge sort

(b) Bubble sort

(c) Quick sort

(d) Insertion sort

15. A sorting technique is called stable if

(a) It takes 0(nlogn) time

(b) It maintains the relative order of occurence of non-distinct elements

(c) It uses divide and conquer paradigm

(d) It takes 0(n) space

16. Which one of the following in place sorting algorithm needs minimum number of swaps?

(a) Quick sort

(b) Insertion sort

(c) Selection sort

(d) Heap sort

❑❑❑

Answer Key

1. *Ans: (a)*
2. *Ans: (a)*
3. *Ans: (a)*
4. *Ans: (c)*
5. *Ans: (c)*
6. *Ans: (a)*
7. *Ans: (c)*
8. *Ans: (b)*
9. *Ans: (c)*
10. *Ans: (c)*
11. *Ans: (b)*
12. *Ans: (a)*
13. *Ans: (b)*
14. *Ans: (c)*
15. *Ans: (b)*
16. *Ans: (c)*

❍❍❍

CHAPTER 8

NP COMPLETENESS

OBJECTIVE QUESTIONS

1. Which of the following is correct?
   (a) 3-Sat ≤p CNF-Sat
   (b) At the language level, 3-Satisfiable ≤p CNF-Satisfiable
   (c) Both (a) and (b)
   (d) None of these

2. If π_A be a problem that belongs to the class NP. Then which one of the following is TRUE?
   (a) there is no polynomial time algorithm for π_A.
   (b) If π_A can be solved deterministrically in polynomial time, then P = NP.
   (c) If π_A is NP - hard, then it is NP - complete
   (d) π_A may be undecidable.

3. The recurrence relation capturing the optimal execution time of the Towers of Hanoi problem with n discs is.
   (a) T(n) = 2T(n – 2) + 2
   (b) T(n) = 2T(n – 1) + n
   (c) T(n) = 2T (n/2) + 1
   (d) T(n) = 2T (n – 1) + 1

4. Assuming P ≠ NP, which of the following is TRUE?
   (a) NP - complete = NP
   (b) NP- complete ∩ P = ∅
   (c) NP-hard = NP
   (d) P = NP-complete

5. The problem 3-SAT and 2-SAT are
   (a) Both in P
   (b) Both NP complete
   (c) NP complete and P respectively
   (d) Undecidable and P respectively

6. The hardest NP problem can be
   (a) NP complete (b) NP hard
   (c) P (d) None

7. Travelling salesman problem belongs to which class?
   (a) P (b) NP
   (c) Linear (d) None

8. A problem which is both ___ and __ is said to be NP complete
   (a) NP, P (b) NP, NP hard
   (c) P, NP complete (d) None

9. Which of the following can be used to define NP complexity class?
   (a) Verifier (b) Polynomial time
   (c) Both (a) and (b) (d) None

10. Which of the following are not in NP?
    (a) All problems of P
    (b) Boolean satistiability problem
    (c) Integer factorization problem
    (d) None

11. Which of the following is optimization problem
    (a) Travelling salesman problem
    (b) Hill climbing problem
    (c) 3-SAT problem
    (d) Hamiltonian cycle

12. Assuming P! = NP, which of the following is true?
    A. NP complete = NP
    B. NP complete ∩ P = ϕ
    C. NP – hard = NP
    D. P = NP complete
    (a) A (b) B
    (c) C (d) D

❑❑❑

Answer Key

1. *Ans: (c)*
2. *Ans: (c)*
3. *Ans: (d)*
4. *Ans: (b)*

   If P! = NP, then it implies that no NP-Complete problem can be solved in polynomial time which implies that the set P and the set NPC are disjoint.
5. *Ans: (c)*
6. *Ans: (a)*
7. *Ans: (b)*
8. *Ans: (a)*
9. *Ans: (c)*
10. *Ans: (d)*
11. *Ans: (b)*
12. *Ans: (b)*

❑❑❑

CHAPTER 9

HASHING

OBJECTIVE QUESTIONS

1. What is a hash table?
   (a) A structure that maps values to keys
   (b) A structure that maps keys to values
   (c) A structure used for storage
   (d) A structure used to implement stack and queue
2. If several elements are competing for the same bucket in the hash table, what is it called?
   (a) Diffusion
   (b) Replication
   (c) Collision
   (d) None of the mentioned
3. What is direct addressing?
   (a) Distinct array position for every possible key
   (b) Fewer array positions than keys
   (c) Fewer keys than array positions
   (d) None of the mentioned
4. What is the search complexity in direct addressing?
   (a) O(n) (b) O(logn)
   (c) O(nlogn) (d) O(1)
5. What is a hash function?
   (a) A function has allocated memory to keys
   (b) A function that computes the location of the key in the array
   (c) A function that creates an array
   (d) None of the mentioned
6. What can be the techniques to avoid collision?
   (a) Make the hash function appear random
   (b) Use the chaining method
   (c) Use uniform hashing
   (d) All of the mentioned
7. What is the load factor?
   (a) Average array size
   (b) Average key size
   (c) Average chain length
   (d) None of the mentioned
8. What is simple uniform hashing?
   (a) Every element has equal probability of hashing into any of the slots
   (b) A weighted probabilistic method is used to hash elements into the slots
   (c) All of the mentioned
   (d) None of the mentioned
9. In simple uniform hashing, what is the search complexity?
   (a) O(n) (b) O(logn)
   (c) O(nlogn) (d) O(1)
10. In simple chaining, what data structure is appropriate?
    (a) Singly linked list
    (b) Doubly linked list
    (c) Circular linked list
    (d) Binary trees
11. Which of the following schemes does quadratic probing come under?
    (a) rehashing
    (b) extended hashing
    (c) separate chaining
    (d) open addressing
12. Quadratic probing overcomes primary collision.
    (a) True (b) False

13. Which among the following is the best technique to handle collision?

(a) Quadratic probing

(b) Linear probing

(c) Double hashing

(d) Separate chaining

14. For the given hash table, in what location will the element 58 be hashed using quadratic probing?

0	49
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

(a) 1 (b) 2

(c) 7 (d) 6

15. A good hash approach is to derive the hash value that is expected to be dependent of any patterns that might exist in the data.

(a) True (b) False

16. What is the hash function used in the division method?

(a) h(k) = k/m

(b) h(k) = k mod m

(c) h(k) = m/k

(d) h(k) = m mod k

17. Using division method, in a given hash table of size 157, the key of value 172 be placed at position _____

(a) 19 (b) 72

(c) 15 (d) 17

18. What is the average retrieval time when n keys hash to the same slot?

(a) Theta(n) (b) Theta(n2)

(c) Theta(nlog n) (d) Big-Oh(n2)

19. Collisions can be reduced by choosing a hash function randomly in a way that is independent of the keys that are actually to be stored.

(a) True (b) False

20. A good hash approach is to derive the hash value that is expected to be dependent of any patterns that might exist in the data.

(a) True (b) False

21. What is the hash function used in the division method?

(a) h(k) = k/m

(b) h(k) = k mod m

(c) h(k) = m/k

(d) h(k) = m mod k

22. Which scheme provides good performance?

(a) open addressing

(b) universal hashing

(c) hashing by division

(d) hashing by multiplication

❑❑❑

Answer Key

1. *Ans. (b)*
2. *Ans. (c)*
3. *Ans. (a)*
4. *Ans. (d)*
5. *Ans. (b)*
6. *Ans. (d)*
7. *Ans. (c)*
8. *Ans. (a)*
9. *Ans. (d)*
10. *Ans. (b)*
11. *Ans. (d)*
12. *Ans. (a)*
13. *Ans. (a)*
14. *Ans. (b)*
15. *Ans. (b)*
16. *Ans. (b)*
17. *Ans. (c)*
18. *Ans. (a)*
19. *Ans. (a)*
20. *Ans. (b)*
21. *Ans. (b)*
22. *Ans. (b)*

❑❑❑