CHAPTER 1

PROBABILITY AND STATICS

OBJECTIVE QUESTIONS

1. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(a) $\frac{16}{25}$ (b) $\left(\frac{9}{10}\right)^3$

(c) $\frac{27}{75}$ (d) $\frac{18}{25}$

2. The probability that it will rain today is 0.5. the probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow?

(a) 0.3 (b) 0.25

(c) 0.4 (d) 0.35

3. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is

(a) 1/6 (b) 3/8

(c) 1/8 (d) 1/2

4. The probability that two friends share the same birth-month is

(a) 1/6 (b) 1/12

(c) 1/144 (d) 1/24

5. Consider two events E_1 and E_2 such that $P(E_1) = \frac{1}{2}$, $P(E_2)=\frac{1}{3}$ and $P(E_1 \cap E_2)=\frac{1}{5}$. Which of the following statements is true?

(a) $P(E_1 \cup E_2)=\frac{2}{3}$

(b) E_1 and E_2 are independent

(c) E_1 and E_2 are not independent

(d) $P(E_1/E_2) = 4/5$

6. E_1 and E_2 are events in a probability space satisfying the following constrains $P(E_1) = P(E_2)$: $P(E_1 \cap E_2)=1$; E_1 & E_2 are independent then $P(E_1)$ is

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

7. In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

(a) 90 and 9 (b) 9 and 90

(c) 81 and 9 (d) 9 and 81

8. Four fair coins are tossed simultaneously, the probability that atleast one heads and atleast one tails turn up is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{15}{16}$

9. A regression model is used to express a variable Y as a function of another variable X. This implies that

(a) There is a causal relationship between Y and X

(b) A value of X may be used to estimate a value of Y

(c) Values of X exactly determine values of Y

(d) There is no causal relationship between Y and X

10. Let P(E) denote the probability of an event E. Given P(A) = 1, P(B) = $\frac{1}{2}$ the values of P(A/B) and P(B/A) respectively are

(a) $\frac{1}{4}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{4}$

(c) $\frac{1}{2}, 1$ (d) $1, \frac{1}{2}$

11. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(a) 100% (b) 50%

(c) 49% (d) None

12. If a fair coin is tossed 4 times, what is the probability that two heads and two tails will result?

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{5}{8}$ (d) $\frac{3}{4}$

13. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 student have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data sturctures and computer organization, 15 students have taken all the three courses. how many students have not taken any of the three courses?

(a) 15 (b) 20

(c) 25 (d) 35

14. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is

(a) 0.240 (b) 0.200

(c) 0.040 (d) 0.008

15. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the card is NOT replaced?

(a) 1/26 (b) 1/52

(c) 1/169 (d) 1/221

16. The following data about the flow of liquid was observed in a continuous chemical process plant

Flow rate (liters/sec)	7.5 to 7.7	7.7 to 7.9	7.9 to 8.1	8.1 to 8.3	8.3 to 8.5	8.5 to 8.7
Frequency	1	5	35	17	12	10

Mean flow rate of the liquid is

(a) 8.00 litres/sec (b) 8.06 litres/sec

(c) 8.16 litres/sec (d) 8.26 litres/sec

17. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

18. If P and Q are two random events, then which of the following is true?

(a) Independence of P and Q implies that Probability $(P \cap Q) = 0$

(b) Proability $(P \cap Q) \geq$ Probability(P) + Proability(Q)

(c) If P and Q are mutually exclusive then they must be independent

(d) Probability $(P \cap Q) \leq$ Probability(P)

19. Two dice are thrown simultaneously. the probability that the sum of numbers on both exceeds 8 is

(a) $\frac{4}{36}$ (b) $\frac{7}{36}$

(c) $\frac{9}{36}$ (d) $\frac{10}{36}$

20. A lot has 10% defective items. 10 items are chosen randomly from this lot. The proabability that exactly 2 of the chosen items are defective is

(a) 0.0036 (b) 0.1937

(c) 0.2234 (d) 0.3874

21. A single die is thrown two times. What is the probability that the sum is neither 8 nor 9?

(a) $\frac{1}{9}$ (b) $\frac{5}{36}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

22. The probability that there are 53 sundays in a randomly chosen leap year is

(a) $\frac{1}{7}$ (b) $\frac{1}{14}$

(c) $\frac{1}{28}$ (d) $\frac{2}{7}$

23. The life of a bulb (in hours) is a random variable with an exponential distribution $f(t) = \alpha e^{-\alpha t}$, $0 \le t \le \infty$. The probability that its value lies between 100 and 200 hours is

(a) $e^{-100\alpha} - e^{-200\alpha}$ (b) $e^{-100} - e^{-200}$

(c) $e^{-100\alpha} + e^{-200\alpha}$ (d) $e^{-200\alpha} - e^{-100\alpha}$

24. Using given data points tabulated below, a straight line passing through the origin is fitted using least squares method. The slope of the line is

x	1	2	3
y	1.5	2.2	2.7

(a) 0.9 (b) 1

(c) 1.1 (d) 1.5

25. Assume that the duration in minutes of a telephone conversation follows the exponential distribution $f(x) = \frac{1}{5}e^{-x/5}, x \ge 0$. The probability that the conversation will exceed five minutes is

(a) $\frac{1}{e}$ (b) $1-\frac{1}{e}$

(c) $\frac{1}{e^2}$ (d) $1-\frac{1}{e^2}$

26. Two cards are drawn at random in succession with replacement from a deck of 52 well shuffled cards proability of getting both 'Aces' is

(a) $\frac{1}{169}$ (b) $\frac{2}{169}$

(c) $\frac{1}{13}$ (d) $\frac{2}{13}$

27. Random variable X takes on the values 1, 2 or 3 with probabilities $\frac{2+5P}{5}, \frac{1+3P}{5}$ and $\frac{1.5+2P}{5}$ respectively. The values of P and E(X) are respectivley

(a) 0.05, 1.87 (b) 1.90, 5.87

(c) 0.05, 1.10 (d) 0.25, 1.40

28. If X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} k(5x-2x^2); & 0 \le x \le 2 \\ 0; & \text{Otherwise} \end{cases}$$

Then $P(x > 1)$ is

(a) 3/14 (b) 4/5

(c) 14/17 (d)17/28

29. An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the paper is

(a) 0.5 (b) 0.18

(c) 0.12 (d) 0.06

30. X is uniformly distributed random variable that take values between 0 and 1. The value of $E(X^3)$ will be

(a) 0 (b) 1/8

(c) 1/4 (d) 1/2

31. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be

(a) 16/3 (b) 6

(c) 36 (d) 256/9

32. Consider a gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be

(a) 0 (b) 0.5

(c) 1 (d) 10 σ

33. For a random variable $x(-\infty < x < \infty)$ following normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. Then the probability of x lying between 90 and 110 *i.e.* $P(90 \leq x \leq 110)$ is equal to

(a) $1 - 2\alpha$ (b) $1 - \alpha$

(c) $1 - \alpha/2$ (d) 2α

34. In a game, two players X and Y toss a coin alternately. Whosever gets a 'head' first, wins the game and the game is terminated. Assuming that player X starts the game the probability of player X winning the game is

(a) 1/3 (b) 1/3

(c) 2/3 (d) 3/4

35. Three values of x and y are to be fitted in a straight line in the form $y = a + bx$ by the method of least squares. Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$, $\Sigma xy = 46$, the values of a and b are respectively

(a) 2, 3 (b) 1, 2

(c) 2, 1 (d) 3, 2

36. A discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance to X as 1.5. Which of the following statements is true?

K	1	2	3	4	5
P(X = K)	0.1	0.2	0.4	0.2	0.1

(a) Both the student and the teacher are right

(b) Both the student and the teacher are wrong

(c) The student is wrong but the teacher is right

(d) The student is right but the teacher is wrong

37. A screening test is carried out to detect a certain disease. It is found that 12% of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting positive report is 0.01, the probability that a person tested gets an incorrect report is

(a) 0.0027 (b) 0.0173

(c) 0.1497 (d) 0.2100

38. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

(a) pq + (1 − p) (1 −q)

(b) (1 − q) p

(c) (1 − p) q

(d) pq

39. If a random variable X satisfies the Poisson's distribution with a mean value of 2, then the probability that $X \geq 2$ is

(a) $2e^{-2}$ (b) $1 - 2e^{-2}$

(c) $3e^{-2}$ (d) $1 - 3e^{-2}$

40. The box 1 contains chips numbered 3, 6, 9, 12 and 15. The box 2 contains chips numbered 6, 11, 16, 21 and 26. Two chips, one from each box are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is

(a) $\frac{6}{25}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{19}{25}$

41. If the difference between the expectation of the square of a random variable $[E(X^2)]$ and the square of the expectation of the random variable $[E(X)]^2$ is denoted by R, then,

(a) $R = 0$ (b) $R < 0$

(c) $R \geq 0$ (d) $R > 0$

42. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Out of X's shock absorbers, 96% are reliable. Out of Y's shock absorbers, 72% are reliable. The probability that a randomly choosen shock absorber, which is found to reliable, is made by Y is

(a) 0.288 (b) 0.334

(c) 0.667 (d) 0.720

43. Let X be a normal random variable with mean 1 and variance 4. The probability $P(X < 0)$ is

(a) 0.5

(b) Greater than zero and less than 0.5

(c) Greater than 0.5 and less than 1.0

(d) 1.0

44. The probability that a student knows the correct answer to a multiple choice question is 2/3. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{5}{9}$ (d) $\frac{8}{9}$

45. Find the value of λ such that the function f(x) is a valid probability density function

$$f(x) = \begin{cases} \lambda(x-1)(2-x) & ; 1 \le x \le 2 \\ 0 & ; \text{otherwise} \end{cases}$$ ....

46. In any given year, the probability of an earthquake greater than magnitude 6 occurring in the Garhwal Himalayas is 0.04. The average time between successive occurrences of such earthquakes is .... years.

47. Given that x is a random variable in the range $[0, \infty]$ with a probability density function $\frac{e^{-x/2}}{K}$, the value of the constant K is ....

48. Consider a dice with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is ....

49. Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is X/1269 The value of X is ....

50. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 it is ....

51. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6, respectively. The mean value and the variance of the number of defective pieces produced by the machine in a day, respectively, are

(a) 1 and 1/3 (b) 1/3 and 1

(c) 1 and 4/3 (d) 1/3 and 4/3

52. The probability density function on the interval [a, 1] is given by $\frac{1}{x^2}$ and outside this interval the value of the function is zero. The value of a is .....

53. Consider the following experiment.

Step-1 : Flip a fair coin twice.

Step-2 : If the outcomes are (TAILS, HEADS) then output is Y and stop.

Step-3 : If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output is N and stop.

Step-4 : If the outcomes are (TAILS, TAILS), then go to Step-1.

The probability that the output of the experiment is Y is (up to two decimal places) ....

54. Suppose that a shop has an equal number of LED bulbs of two different types. the probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is ....

55. If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X + 2)^2]$ equals ....

56. The probability density function of a continuous random variable distributed uniformly between x and y (for y > x) is

(a) $\frac{1}{x-y}$ (b) $x - y$

(c) $y - x$ (d) $\frac{1}{y-x}$

57. Solution of recurrence relation,

$a_{n+1} = 4a_n - 9n,\ a_1 = 8,\ n \geq 1$ is

(a) $2^n + 3^n + 1$ (b) $2^n + 3^n + 3$

(b) $(3)2^n + 3^n - 1$ (d) $4^n + 3n + 1$

58. Solution of recurrence relation

$u_n = 2u_{n-1} + 1,\ n \geq 1,\ u_0 = 1$ is

(a) $2^n - 1$ (b) $2^{n+1} - 1$

(b) $2^n + 1$ (d) 2^{n+1}

59. If $a_n = n + (-1)^n a_{n-1},\ a_0 = 1$ then a_n is

(a) 1 (b) 4

(c) 5 (d) 8

60. If $a_n = -8a_{n-1} - 15a_{n-2};\ n \geq 2$ such that $a_0 = 0$, $a_1 = 2$ then the solution is

(a) $a_n = (-3)^n - (-5)^n$

(b) $a_n = n(-3)^n - n(-5)^n$

(c) $a_n = n(-3)^n - (-5)^n$

(d) $a_n = (-5)^n - (-3)^n$

61. What is the characteristic equation for Fibonacci sequence, 0, 1, 1, 2, 3, 5, 8, 13, ...

(a) $r^2 - r - 1 = 0$ (b) $r^2 + r - 1 = 0$

(c) $2r^2 - r - 1 = 0$ (d) $2r^2 + r - 1 = 0$

62. Consider recurrence relation,

$a_n = 2_n + a_{n-1};\ a_1 = 4$ then $a_{10} = ?$

(a) 96 (b) 112

(c) 132 (d) 108

63. The recurrence relation corresponding to solution,

$$a_n = \left(\frac{21}{4}\right)2^n - \left(\frac{5}{4}\right)6^n$$

(a) $a_n = 10a_{n-1} - 15a_{n-2}$

(b) $a_n = 8a_{n-1} - 12a_{n-2}$

(c) $a_n = 15a_{n-1} - 10a_{n-2}$

(d) $a_n = 12a_{n-1} - 8a_{n-2}$

64. What is recurrence relation for 1, 6, 31, 156, 780, ...

(a) $a_n = 3a_{n-1} + 3$

(b) $a_n = 5a_{n-1} + 1$

(c) $a_n = 4a_{n-1} + n$

(d) $a_n = 2a_{n-1} + n$

65. What is the generating function for sequence, 1, 3, 7, 15, 31, 63, ...

(a) $\frac{1}{1-x-x^2}$ (b) $\frac{1}{1-3x+2x^2}$

(c) $\frac{1}{1-2x-x^2}$ (d) $\frac{1}{1-x-2x^2}$

66. What is the multiplication of sequences 1, 2, 3, 4, ... with 1, 3, 5, 7, 11, ...

(a) 1, 3, 7, 9, ... (b) 1, 5, 14, 30, ...

(c) 1, 4, 7, 9, ... (d) 4, 8, 9, 14, ...

67. Let 'A' is generating function for the sequence 4, 7, 13, 22, ...

Then determine generating function for the sequence of differences between terms in terms of 'A'.

(a) $\frac{(1-x)A}{x} - \frac{4}{x}$ (b) $(1-x)A - \frac{4}{x^3}$

(c) $Ax - \frac{4}{x}$ (d) $(1-x)A - x^2$

68. In how many ways 5 Indians, 4 Americans and 7 pakistanis can be seated in a row such that all persons of same nationality will sit together.

(a) 5! 4! 7! (b) 5! 11!

(c) 3! 5! 4! 7! (d) (3) 5! 4! 7!

69. In how many ways the letters of the word 'ALLAHABAD' such that both L do not occur together (both L are indistinguishable)

(a) 1860 (b) 1680

(c) 3360 (d) 5880

70. In a party, every person shakes hands with the every other person. If there are 105 hands shakes then number of persons in the party are

(a) 15 (b) 14

(c) 21 (d) 25

71. The number of positive integers which can be formed by using any number of digits from 0, 1, 2, 3, 4, 5 without repetition are

(a) 1200 (b) 1500

(c) 1600 (d) 1630

72. Number of ways in which letters of the word 'KERRY' can be arranged, are

(a) 5! (b) $\frac{5!}{2!}$

(c) 5!2! (d) 2!

73. Number of ways in which letters of the word 'CHINESE' can be arranged such that all the vowels lie together, are

(a) 7! (b) 5!

(c) 5! 3! (d) $3 \times 5!$

74. Number of ways in which letters of the word 'VOWEL' can be arranged so that all the vowels lie at odd numbered places, are

(a) $3 \times 2! \times 3!$ (b) 2! 3!

(c) $2! \times 5!$ (d) 5!

75. In how many ways, a committee of 5 members can be formed from a group of 6 men and 4 women such that committee consist of at least one women?

(a) 246 (b) 340

(c) 290 (d) 315

76. In how many ways a student can bring 4 pen from his home such that he have red, black and green pens at home?

(a) 30 (b) 15

(c) 25 (d) 45

77. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$

(c) $\sqrt{6}$ (d) 6

78. The standard deviation of the following frequency distribution is

X	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard deviation.

(a) 1.38 (b) 1.42

(c) 1.45 (d) 1.60

79. The mean of five observations is 4 and their variance is 5.2. If three of these observations are 1,2 and 6 , then the other two are

(a) 2 and 9 (b) 3 and 8

(c) 4 and 7 (d) 5 and 6

80. The mean deviation about the median of the following distribution is

Marks obtained	10	11	12	14	15
Number of students	2	3	8	3	4

(a) 1 (b) 1.25

(c) 1.5 (d) 1.75

81. If a variable takes the discrete values $\alpha - 4$, $\alpha-\frac{7}{2}, \alpha-\frac{5}{2}, \alpha-3, \alpha-2, \alpha+\frac{1}{2}, \alpha-\frac{1}{2}, \alpha+5, (\alpha>0)$

then the median is

(a) $\alpha-\frac{5}{4}$ (b) $\alpha-\frac{1}{2}$

(c) $\alpha-2$ (d) $\alpha+\frac{5}{4}$

82. If the mean of the numbers 27 + x, 31 + x, 89 + x, 107 + x, 156 + x is 82, then the mean of 130 + x, 126 + x, 68 + x, 50 + x, 1 + x is

(a) 75 (b) 157

(c) 82 (d) 80

83. A rifleman is firing at a distant target and hence, has only 10% chances of hitting it. Find the number of rounds, he must fire in order to have more than 50% chances of hitting it at least once.

(a) 6 (b) 9

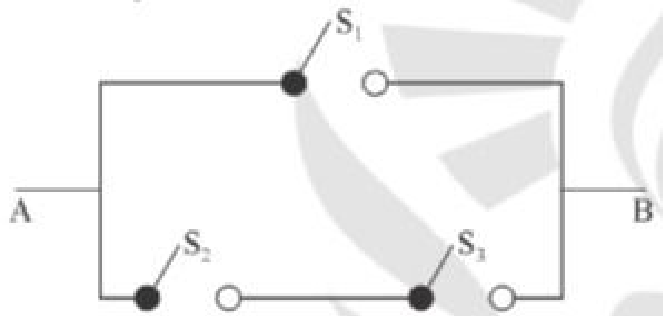
(c) 3.5 (d) 7

84. A bag contains n+1 coins. It is known that one of these coins shows heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is 7/12, then find the value of n.

(a) 7 (b) 5

(c) 2.5 (d) 10

85. Trical system has open-closed switches S_1, S_2 and S_3 as shown.



The switches operate independently of one another and the current will flow from A to B either if S_1 is closed or if both S_2 and S_3 are closed. If $P(S_1) = P(S_2) = P(S_3) = 1/2$, then find the probability that the circuit will work.

(a) $\frac{5}{8}$ (b) $\frac{1}{8}$

(c) $\frac{8}{5}$ (d) $\frac{8}{3}$

86. The odds against a certain event are 5:2 and the odds in favor of other event, independent of the former are 6:5. Then find the probability that at least one of the events will happen.

(a) 26/77 (b) 1/77

(c) 52/77 (d) 13/77

87. Three persons work independently on a problem. If the probabilities that they will solve it are 1/3,1/4 and 1/5, then find the probability that none can solve it.

(a) 7/5 (b) 3/5

(c) 11/5 (d) 2/5

88. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first, second and third cards are jack, queen and king, respectively?

(a) $\frac{1}{13}\times\frac{4}{51}\times\frac{2}{25}$ (b) $\frac{1}{13}\times\frac{3}{51}\times\frac{2}{25}$

(c) $\frac{1}{13}\times\frac{4}{51}\times\frac{1}{25}$ (d) $\frac{1}{13}\times\frac{4}{51}$

89. A box contains 10 mangoes out of which 4 are rotten. Two mangoes are taken out together. If one of them is found to be good, then find the probability that the other is also good.

(a) $\frac{1}{13}$ (b) $\frac{5}{13}$

(c) $\frac{3}{13}$ (d) $\frac{5}{169}$

90. If $P(A) = 0.8, P(B) = 0.5$, and $P(B/A) = 0.4$, find

(i) $P(A \cap B)$

(ii) $P(A/B)$

(iii) $P(A \cup B)$

(a) 0.32, 0.64, 0.98 (b) 0.30, 0.60, 0.90

(c) 0.35, 0.65, 0.68 (d) 0.04, 0.064, 0.098

91. A tain disease, when it is in effective in detecting a 150 yields a false positive result for 0.5% of the healthy person tested (that is, if a healthy person is tested, then, with probability 0.005 , the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

(a) $\frac{11}{133}$ (b) $\frac{1}{133}$

(c) $\frac{22}{133}$ (d) $\frac{133}{22}$

92. In an entrance test, there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then find the probability that he was guessing.

(a) $\frac{1}{35}$ (b) $\frac{1}{39}$

(c) $\frac{1}{73}$ (d) $\frac{1}{37}$

93. Each of the n turns contains 4 white and 6 black balls. The $(n+1)^{th}$ turn contains 5 white and 5 black balls. One of the n+1 turns is chosen at random and two balls are drawn from it without replacement. Both the balls turn out to be black. If the probability that the $(n+1)^{th}$ turn was chosen to draw the balls is 1/16, then find the value of n.

(a) 8 (b) 1/10

(c) 20 (d) 10

94. Die A has 4 red and 2 white faces, whereas die B has 2 red and 4 white faces. A coin is flipped once. If it shows a head, the game continues by throwing die A; if it shows tail, then die B is to be used. If the probability that die A is used is 32/33 when it is given that red turns up every time in firstn throws, then find the value of n.

(a) 5 (b) 7

(c) 25 (d) 0.5

95. If A and B are events such that $P(A' \cup B') = 3/4$, $P(A' \cap B') = 1/4$ and $P(A) = 1/3$, then find the value of $P(A' \cap B)$

(a) $\frac{1}{12}$ (b) $\frac{5}{12}$

(c) $\frac{5}{144}$ (d) $\frac{25}{12}$

96. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, then find the probability that it is rusted or is a nail.

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{11}{16}$ (d) $\frac{3}{16}$

97. The probability that at least one of the events A and B occurs is 0.6 . If A and B occur simultaneously with probability 0.2 , then find $P(\overline{A}) + P(\overline{B})$?

(a) 1.2 (b) 2.1

(c) 1/12 (d) 1/21

98. A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that there are at least two balls of each color?

(a) 125 (b) 425

(c) 325 (d) 625

99. Find the number of permutations of all the letters of the word MATHEMATICS which starts with consonants only.

(a) $\frac{5\times10!}{2!\times2!\times2!}$ (b) $\frac{1\times10!}{2!\times2!\times2!}$

(c) $\frac{9\times10!}{2!\times2!\times2!}$ (d) $\frac{7\times10!}{2!\times2!\times2!}$

100. Find the sum of all the numbers that can be formed with the digits 2,3,4,5 taken all at a time.

(a) 93324 (b) 93124

(c) 93524 (d) None of these

101. A six letters word is formed using the letters of the word LOGARITHM with or without repetition. Find the number of words that contain exactly three different letters.

(a) 90 (b) 360

(c) 45360 (d) all of these

102. How many different signals can be given using any number of flags from 5 flags of different colors?

(a) 325 (b) 125

(c) 225 (d) 425

103. Find the number of three-digit numbers which are divisible by 5 and have distinct digits.

(a) 72 (b) 136

(c) 172 (d) Both (a) and (b)

❑❑❑

Answer Key

1. ***Ans: (d)***

Required probability $= \dfrac{8\times9\times9}{900} = \dfrac{18}{25}$

2. ***Ans: (c)***

Given $P(E_1) = 0.5,\ P(E_2) = 0.6$

$$P(E_1 \cup E_2) = 0.7$$

Required probability

$$P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2) = 0.4$$

3. ***Ans: (b)***

Probability of getting an odd number when a die is rolled $= \dfrac{3}{6} = \dfrac{1}{2}$.

Required probability $= {}^3C_1 \times \dfrac{1}{2} \times \left(\dfrac{1}{2}\right)^2 = \dfrac{3}{8}$

4. ***Ans: (b)***

One of two persons may born in any month. Then the probability that the second person also may born in the same month is 1/12

$\therefore$ Required probability $P(E) = 1/12$

5. ***Ans: (c)***

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{19}{30} \neq \frac{2}{3}$$

$\therefore \quad P(E_1 \cap E_2) \neq P(E_1)P(E_2)$

Hence the events are dependent.

$$P(E_1 / E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{3}{5} \neq \frac{4}{5}$$

6. ***Ans: (d)***

$$P(E_1 \cap E_2) = P(E_1)\,P(E_2) = 1$$

$\because \quad P(E_1) = P(E_2)$

$\therefore \quad P(E_2) = 1$

7. ***Ans: (a)***

$p = 0.1,\ n = 900,\ q = 1 - p = 0.9$

Mean $= np = 90$

S.D. $= \sigma = \sqrt{npq} = 9$

8. ***Ans: (c)***

$$n(S) = 16$$

Probability of all heads or all tails appearing

$$= \frac{2}{16} = \frac{1}{8}$$

$\therefore$ Required probability

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

9. ***Ans: (b)***

$$Y = aX + b$$

10. ***Ans: (d)***

$$P(A) = 1,\ P(B) = 1/2$$

$$P(A \cap B) = P(A)P(B) = 1/2$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = 1$$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{1}{2}$$

11. ***Ans: (c)***

The probability that first screw drawn is not defective is 7/10.

The probability that second screw drawn is also not defective is 7/10.

Required probability $= \dfrac{7}{10} \times \dfrac{7}{10} = \dfrac{49}{100}$

12. ***Ans:(a)***

Required probability

$$= P\ (X = 2) = {}^4C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

13. ***Ans: (c)***

Given $p(P) = \dfrac{125}{200},\ p(D) = \dfrac{85}{200},\ p(C) = \dfrac{65}{200}$

$$p(P \cap D) = \frac{50}{200},\ p(P \cap C) = \frac{35}{200},$$

$$p(D\cap C)=\frac{30}{200},\ p(P\cap D\cap C)=\frac{15}{200}$$

$$p(P\cup D\cup C)=p(P)+p(D)+p(C)-p(P\cap D)$$

$$-p(D\cap C)-p(P\cup C)+p(P\cap D\cap C)=\frac{7}{8}$$

$$\Rightarrow\ p(\bar{P}\cap\bar{D}\cap\bar{C})=1-\frac{7}{8}=\frac{1}{8}$$

No. of students, who have not taken any of the three courses

$$=\frac{1}{8}\times 200=25$$

14. ***Ans: (c)***

$P(G_i)=0.2$;

Where, $i = 1, 2, 3$

$$P(G_2\cap G_3/G_1)=P(G_2\cap G_3\cap G_1)/P(G_1)$$

$$=\frac{0.2\times 0.2\times 0.2}{0.2}=0.04$$

15. ***Ans: (d)***

Probability of drawing two king cards without replacement

$$=\frac{1}{13}\times\frac{3}{51}=\frac{1}{221}$$

16. ***Ans: (c)***

Mid value:	7.6	7.8	8.0	8.2	8.4	8.6
Freq :	1	5	35	17	12	10

$$\text{Mean}=\frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$=\frac{652.8}{80}=8.16\ \text{litres/sec.}$$

17. ***Ans: (b)***

3 balls of different colors can be drawn in 6 ways

$$\text{Required probability}=6\times\left(\frac{10}{60}\right)\left(\frac{20}{60}\right)\left(\frac{30}{60}\right)=\frac{1}{6}$$

18. ***Ans: (d)***

Verifying the each options

(a) $P(P\cap Q)=P(P)\,P(Q)$

Where P and Q are independent events

(b) $P(P\cap Q)=P(P)+P(Q)-P(P\cup Q)$

$\Rightarrow P(P\cap Q)\le P(P)+P(Q)$

(c) If $P(P\cap Q)=0$ then it does not imply that $P(P\cap Q)=P(P)P(Q)$

(d) $P(P\cap Q)\le P(P)$

19. ***Ans: (d)***

$n(S) = 6\times 6 = 36$

E = {(3, 6), (4, 5), (4, 6) (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

Required probability = n(E) / n(S) = 10/36

20. ***Ans: (b)***

Given $p = 0.1,\ q = 0.9,\ n = 10$

Required probability = $P(X = 2)$

$= {}^{10}C_2\,(0.1)^2\,(0.9)^8$

$= 0.1937$

21. ***Ans: (d)***

Let E be the event of getting the sum 8 or 9

$\Rightarrow$ $n(E) = 9$

$$p(E)=\frac{9}{36}=\frac{1}{4}$$

Required probability = $1 - p(E) = \frac{3}{4}$

22. ***Ans: (d)***

Leap year = 366 days

= (52 × 7) + 2 days sample space

= {S M, M T, T W, W Th, Th F, F Sat, Sat S}

Required probability = $\frac{2}{7}$

23. ***Ans: (a)***

$P(100 < x < 200)$

$$=\int_{100}^{200} f(t)dt=\int_{100}^{200}\alpha e^{-\alpha t}dt=e^{-100\alpha}-e^{-200\alpha}$$

24. ***Ans: (b)***

Let the required line is $y = bx$ ...(1)

($\because$ line passing through the origin)

Then the normal equation of (1) is given by

$$\Sigma xy = b\Sigma x^2$$

$$\Rightarrow\quad b=\frac{\Sigma xy}{\Sigma x^2}=\frac{14}{14}=1$$

x	y	xy	x^2
1	1.5	1.5	1
2	2.2	4.4	4
3	2.7	8.1	9
$\Sigma x = 6$	$\Sigma y = 6.4$	$\Sigma xy = 14$	$\Sigma x^2 = 14$

25. ***Ans: (a)***

$$P(5 < x < \infty) = \int_5^\infty f(x)dx = \int_5^\infty \frac{1}{5}e^{-x/5}dx = \frac{1}{e}$$

26. ***Ans: (a)***

Required probability

$$= P(E) = \frac{^4C_1}{^{52}C_1} \times \frac{^4C_1}{^{52}C_1} = \frac{1}{169}$$

27. ***Ans: (a)***

Sum of probabilities = 1

$$\frac{2+5P}{5} + \frac{1+3P}{5} + \frac{1.5+2P}{5} = 1$$

$\Rightarrow$ $4.5 + 10P = 5$

$\Rightarrow$ $P = 0.05$

$$\text{Mean} = 1\left(\frac{2+5P}{5}\right) + 2\left(\frac{1+3P}{5}\right) + 3\left(\frac{1.5+2P}{5}\right)$$

$$= \frac{8.5+0.85}{5} = 1.87$$

28. ***Ans: (d)***

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_0^2 k(5x-2x^2)dx = 1$$

$$\Rightarrow \quad k = \frac{3}{14}$$

$$P(X > 1) = \int_1^\infty f(x)dx$$

$$= \int_1^2 \frac{3}{14}(5x-2x^2)dx$$

$$= \frac{17}{28}$$

29. ***Ans: (c)***

Given that $P(1) = 0.3$, $P(2) = 0.2$, $P(1/2) = 0.6$

Using conditional probablity, the probability that a student is failed in paper 1 given that he is failed in paper 2 is given by

$$P(1/2) = \frac{P(1 \cap 2)}{P(2)}$$

$$\Rightarrow \quad 0.6 = \frac{P(1 \cap 2)}{0.2}$$

Required probability = $P(1 \cap 2) = 0.12$

30. ***Ans: (c)***

For uniform distribution,

$$f(x) = \frac{1}{b-a}; \quad a < x < b$$

Here, $f(x) = \frac{1}{1-0} = 1$

$$E(X^3) = \int_0^1 x^3 f(x)dx = 1/4$$

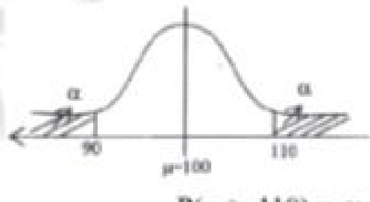
31. ***Ans: (a)***

$$Var(X) = \frac{(b-a)^2}{12} \text{ for } a < x < b = \frac{16}{3}$$

32. ***Ans: (b)***

Since the curve is symmetric.

33. ***Ans: (a)***



$$P(x \geq 110) = \alpha$$

$$\Rightarrow \quad P(x \leq 90) = \alpha$$

$$\Rightarrow \quad P(90 \leq x \leq 110) = 1 - 2\alpha$$

34. ***Ans: (c)***

Let p be the probability of getting head

$\therefore$ $P(E) = p + q^2p + q^4p + \ldots\ldots$

$= p[1 + q^2 + q^4 + \ldots\ldots]$

$$= \frac{p}{1-q^2} = \frac{2}{3}$$

35. ***Ans: (d)***

Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$, $\Sigma xy = 46$ and three values of x and y to fit the straight line

Let $y = a + bx$

Then the normal equations are

$$\Sigma y = na + b\Sigma x$$

and $$\Sigma xy = a\Sigma x + b\Sigma x^2$$

where n = number of points

$\therefore$ $$21 = 3a + 6b \quad ...(1)$$

$$46 = 6a + 14b \quad ...(2)$$

From equation (1) and (2)

$$a = 3 \text{ and } b = 2$$

36. ***Ans: (b)***

Given $\mu = 3.5$,

$\sigma^2 = 1.5$,

Actually, $\mu = \Sigma xP(x) = 3.0$

$$\sigma^2 = \Sigma x^2P(x) - [\Sigma xP(x)]^2$$

$$= 10.6 - 9 = 1.6$$

37. ***Ans: (c)***

Probability of getting positive report = 0.01

and probability of getting negative report = 0.99

Required probability = probability of getting incorrect report when test is positive or negative

$$= (0.01)(0.12) + (0.99)(0.15)$$

$$= \frac{12}{10000} + \frac{99 \times 15}{10000} = 0.1497$$

38. ***Ans: (a)***

Probability of faulty assembly of any computer = p

Probability that testing process gives the correct result = q

Required probability

= probability of faulty assembly when it is tested correct or probability of faulty assembly when it is tested incorrect

= pq + (1 – p)(1 – q)

39. ***Ans: (d)***

Given mean of poisson distribution is 2 *i.e.*, λ = 2

Required probability $P(X \geq 2)$

$$= 1 - P(X = 0) + P(X = 1)$$

$$= 1 - \left[\frac{\lambda^0 e^{-2}}{1} + \frac{\lambda e^{-2}}{1}\right]$$

$$= 1 - e^{-2}[1 + 2] = 1 - 3e^{-2}$$

40. ***Ans: (d)***

$$n(S) = {}^5C_1 \times {}^5C_1 = 25$$

Let E be the event of picking one chip from each box such that product of numbers on chips is even number.

$\therefore$ $$n(E) = (2 \times 5) - (3 \times 3) = 19$$

$\therefore$ Required probability $= \frac{19}{25}$

41. ***Ans: (c)***

$(R = \text{var}(X) = E(X^2) - [E(X)]^2 \geq 0)$

Variance can never be negative.

42. ***Ans: (b)***

$P(X) = 0.6 \qquad P(Y) = 0.4$

$$P\left(\frac{R}{X}\right) = 0.96 \qquad P\left(\frac{R}{Y}\right) = 0.72$$

$$P\left(\frac{Y}{R}\right) = \frac{P(Y \cap R)}{P(R)}$$

$$P\left(\frac{Y}{R}\right) = \frac{P(Y)P\left(\frac{R}{Y}\right)}{P(X)P\left(\frac{R}{X}\right) + P(Y)P\left(\frac{R}{Y}\right)}$$

$$P\left(\frac{Y}{R}\right) = \frac{(0.4)(0.72)}{(0.6)(0.96) + (0.4)(0.72)}$$

$$P\left(\frac{Y}{R}\right) = \frac{0.288}{0.576 + 0.288}$$

$$P\left(\frac{Y}{R}\right) = \frac{0.288}{0.864} = 0.334$$

43. ***Ans: (b)***

$$\mu = 1;\ \sigma^2 = 4 \Rightarrow \lambda = 2$$

$$P(X < 0) = P\left(Z < \frac{-u}{\sigma}\right) = P\left(Z < -\frac{1}{2}\right)$$

$$= P(Z < -0.5)$$

$$P(X < 0) = 0.5 - P(0 < Z < 5)$$

Greater than zero & less than 0.5

44. ***Ans: (d)***

The probability of the student answering the question correctly = $\frac{2}{3}$

The probability of answering correctly by guessing

$$= \frac{1}{4}$$

$$\text{Required probability} = \frac{\frac{2}{3}}{\frac{2}{3}+\frac{1}{3}\times\frac{1}{4}} = \frac{8}{9}$$

45. ***Ans. 6***

$$\int_1^2 f(x)\,dx = 1$$

$$\Rightarrow \lambda\int_1^2(-x^2+3x-2)dx = 1$$

$$\Rightarrow \lambda\left[\frac{-x^3}{3}+\frac{3x^2}{2}-2x\right]_1^2 = 1$$

$$\Rightarrow \lambda\left[\frac{27-26}{6}\right] = 1 \Rightarrow \frac{\lambda}{6} = 1$$

$$\therefore \quad \lambda = 6$$

46. ***Ans. 25***

$$P = 0.04 = \frac{4}{100}$$

Hence, for 1 earthquake = $\frac{100}{4}$ years = 25 years

47. ***Ans. 2***

$$\int_0^{\infty}\frac{e^{-x/2}}{K} = 1$$

$$\frac{1}{K}\left[\frac{e^{-x/2}}{(-1/2)}\right]_0^{\infty} = 1$$

$$\Rightarrow \quad \frac{-2}{K}(0-1) = 1$$

$$\Rightarrow \quad K = 2$$

48. ***Ans. 1/7***

$$P(n) = kn; \text{ where } n = 1 \text{ to } 6$$

we know $\sum_x P(x) = 1$

$$\Rightarrow \quad K[1+2+3+4+5+6] = 1$$

$$\Rightarrow \quad K = \frac{1}{21}$$

$\therefore$ required probability is $P(n=3) = 3K = \frac{1}{7}$

49. ***Ans. 10***

22 occured in following ways

6 6 6 4 $\rightarrow$ 4 ways

6 6 5 5 $\rightarrow$ 6 ways

Required probability

$$= \frac{6+4}{1296} = \frac{10}{1296}$$

$$\Rightarrow \quad X = 10$$

50. ***Ans. 0.259 to 0.261***

Let A = divisible by 2,

B = divisible by 3

and C = divisible by 5, then

$n(a) = 50$,

$n(b) = 33$, $n(c) = 20$

$n(A\cap B) = 16$, $n(B\cap C) = 6$,

$n(A\cap C) = 10$, $n(A\cap B\cap C) = 3$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)$$

$$-P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

$$= 74/100$$

$\therefore$ Required probability is

$$P(\bar{A}\cap\bar{B}\cap\bar{C}) = 1 - P(A\cup B\cup C) = 0.26.$$

51. ***Ans. (a)***

Let 'x' be no. of defective pieces.

x	0	1	2
P(x)	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

$$\text{mean } (\mu) = E(x) = \Sigma x P(x)$$

$$= \left(0\times\frac{1}{6}\right)+\left(1\times\frac{2}{3}\right)+\left(2\times\frac{1}{6}\right)$$

$$= 0+\frac{2}{3}+\frac{1}{3} = 1$$

$$E(x^2) = \Sigma x^2 P(x)$$

$$= \left(0\times\frac{1}{6}\right)+\left(1\times\frac{2}{3}\right)+\left(4\times\frac{1}{6}\right)$$

$$= 0+\frac{2}{3}+\frac{2}{3} = \frac{4}{3}$$

$$\text{Variance} = V(x) = E(x^2) - \{E(x)\}^2$$

$$= \frac{4}{3} - 1 = \frac{1}{3}$$

52. ***Ans. 0.5***

Given, $f(x) = \begin{cases} \frac{1}{x^2}; & a \le x \le 1 \\ 0; & \text{otherwise} \end{cases}$

We know that,

$$\int_0^1 f(x)dx = 1$$

$$\Rightarrow \int_a^1 \frac{1}{x^2} dx = 1$$

$$\Rightarrow \left(\frac{-1}{x}\right)_a^1 = 1$$

$$\Rightarrow \frac{1}{a} - 1 = 1$$

$$\Rightarrow a = 0.5$$

53. ***Ans. 0.33***

From the given steps we can observe that probabilities of Y are

$$\frac{1}{4}, \left(\frac{1}{4}\right)\left(\frac{1}{4}\right), \left(\frac{1}{4}\right)^2 \frac{1}{4}, \ldots$$

Required probability

$$= \frac{1}{4} + \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\left(\frac{1}{4}\right)^2 \times \frac{1}{4}\right) + \ldots$$

$$= \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \ldots$$

$$= \frac{1}{4}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \ldots\right)$$

$$= \frac{1}{4}\left(\frac{1}{1-1/4}\right) = \frac{1}{4} \times \frac{4}{3}$$

$$= \frac{1}{3} = 0.33$$

54. ***Ans. 0.55***

E_1 → Event of selecting type-I bulb

E_2 → Event of selecting type-II bulb

A → Event of selecting a bulb lasts more than 100 hours

Given, $P(E_1) = 0.5$

$$P\left(\frac{A}{E_1}\right) = 0.7$$

$$P(E_2) = 0.5$$

$$P\left(\frac{A}{E_2}\right) = 0.4$$

Required probability,

$$P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= 0.5 \times 0.7 + 0.5 \times 0.4$$

$$= 0.55$$

55. ***Ans. 54***

For Poisson distribution,

$$E(X) = \text{Mean} = \lambda = 5$$

$$E(X^2) = \lambda^2 + \lambda$$

Now,

$$E[(X+2)^2] = E(X^2 + 4X + 4)$$

$$= E(X^2) + 4E(X) + 4$$

$$= [(\lambda^2 + \lambda) + 4\lambda + 4]$$

Where $\lambda = 5$

$$= 54$$

56. ***Ans. (d)***

57. ***Ans. (d)***

$$a_1 = 8 \quad \ldots(i)$$

$$a_{n+1} = 4a_n - 9n$$

$$\Rightarrow a_2 = 4a_1 - 9$$

$$= (4 \times 8) - 9 = 23 \quad \ldots(ii)$$

Only option (d) satisfy equation (i) and (ii).

58. ***Ans.* (b)**

$$u_n = 2u_{n-1} + 1$$

$$u_0 = 1 \quad \ldots(i)$$

$$u_1 = 2u_0 + 1 = 3 \quad \ldots(ii)$$

Only option (b) satisfy equation (i) and (ii).

59. ***Ans. (a)***

$$a_n = n + (-1)^n a_{n-1}$$

$$a_0 = 1$$

$$a_1 = 1 - a_0 = 0$$

$$a_2 = 2 + a_1 = 2$$

$$a_3 = 3 - a_2 = 1$$

$$a_4 = 4 + a_3 = 5$$

60. ***Ans. (a)***

$$a_n = -8a_{n-1} - 15a_{n-2}$$

$$a_0 = 0, \; a_1 = 2$$

$a_2 = -8_{a^*} - 15a_0 = -16$

Only option (a) satisfy these values.

61. ***Ans. (a)***

Recursive relation for Fibonacci sequence is,

$$a_n = a_{n-1} + a_{n-2}$$

put $a_n = r^n$

$$r^n - r^{n-1} - r^{n-2} = 0$$

$$r^{n-2}(r^2 - r - 1) = 0$$

$\Rightarrow$ Characteristic equation is,

$$r^2 - r - 1 = 0$$

62. ***Ans. (b)***

$$a_n - a_{n-1} = 2n$$

$$a_2 - a_1 = 4 \quad \text{...(i)}$$

$$a_3 - a_2 = 6 \quad \text{...(ii)}$$

$$a_n - a_{n-1} = 2n \quad \text{...(n)}$$

Adding (i), (ii), (iii), ... (n)

We get, $a_n - a_1 = 4 + 6 + 8 + ... + 2n$

$a_n - a_1 = (2 + 4 + 6 + 8 + ... + 2n) - 2$

$$a_n - a_1 = 2(1 + 2 + 3 + ... + n) - 2$$

$$a_n - 4 = \frac{2n(n+1)}{2} - 2$$

$$a_n = n(n + 1) + 2$$

Hence, $a_{10} = 10(10 + 1) + 2 = 112$

63. ***Ans. (b)***

$$a_n = \left(\frac{21}{4}\right)2^n - \left(\frac{5}{4}\right)6^n$$

$$a_0 = \left(\frac{21}{4}\right) - \left(\frac{5}{4}\right) = 4$$

$$a_1 = \frac{21}{2} - \frac{15}{2} = 3$$

$$a_2 = 21 - 45 = -24$$

By putting n = 2 in options, only option (b) satisfy.

64. ***Ans. (b)***

1, 6, 31, 156, 780, ...

Hence, $a_0 = 1,\ a_1 = 6,\ a_2 = 31,$ ....

Only option (b) satisfy these values.

65. ***Ans. (b)***

Recurrence relation can be written as

$$a_n = 2a_{n-1} + 1;\ n \geq 1$$

Such that $a_0 = 1$

Now, $a_n - 2a_{n-1} - 1 = 0;\ n \geq 1$

$$\sum_{n=1}^{\infty}(a_n - 2a_{n-1} - 1)x^n = 0$$

$$\sum_{n=1}^{\infty}a_n x^n - 2\sum_{n=1}^{\infty}a_{n-1}x^n - \sum_{n=1}^{\infty}x^n = 0 \quad \text{...(i)}$$

Generating function is,

$$f(x) = \sum_{n=0}^{\infty}a_n x^n = a_0 + a_1 x + a_2 x^2 + ...$$

Hence from equation (i),

$$[f(x) - a_0] - 2xf(x) - [x + x^2 + x^3 + ...] = 0$$

$$f(x) - 1 - 2xf(x) - x(1 + x + x^2 + ...) = 0$$

$$f(x)(1 - 2x) - 1 - \frac{x}{1-x} = 0$$

$$f(x)(1 - 2x) = \frac{1}{1-x}$$

$$f(x) = \frac{1}{(1-x)(1-2x)}$$

$$= \frac{1}{1 - 3x + 2x^2}$$

Alternate Method : Use long division method and check each option as we discussed in the class.

66. ***Ans. (b)***

Let generating function of 1st sequence

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1x + a_2x^2 + ...$$

Let generating function of 2^{nd} sequence is,

$$g(x) = \sum_{n=0}^{\infty} b_n x^n = b_0 + b_1x + b_2x^2 + ...$$

Now, $f(x) \cdot g(x) = (a_0 + a_1x + a_2x^2 + ...)$
$(b_0 + b_1x + b_2x^2 + ...)$
$= (a_0b_0) + (a_0b_1 + a_1b_0)x +$
$(a_0b_2 + a_1b_1 + a_2b_0)x^2 + ...$

put $a_0 = 1,\ a_1 = 2,\ a_2 = 3$
$b_0 = 1,\ b_1 = 3,\ b_2 = 5$

we get

$f(x)\,g(x) = 1 + 5x + 14x^2 + ...$

Hence, resultant sequence is,

1, 5, 14, ...

67. ***Ans. (a)***

Sequence is, 4, 7, 13, 22, ...

The sequence of difference between terms is, 3, 6, 9, ...

Let 'G' is the generating function the resultant sequence.

Then, $G = 3 + 6x + 9x^2 + ...$ ...(i)

According to question

A = Generating function of given sequence.

$\Rightarrow$ $A = 4 + 7x + 13x^2 + 22x^3 + ...$

Now, $xA = 4x + 7x^2 + 13x^3 + 22x^4 + ...$

$A - xA = 4 + 3x + 6x^2 + 9x^3 + ...$

$\Rightarrow 3x + 6x^2 + 9x^3 + ... = (1 - x)A - 4$

$$\Rightarrow 3 + 6x + 9x^2 + ... = \frac{(1-x)A - 4}{x}$$

$$= \frac{(1-x)A}{x} - \frac{4}{x}$$

68. ***Ans. (c)***

$\underline{I\ I\ I\ I\ I}\ \ \underline{A\ A\ A\ A}\ \ \underline{P\ P\ P\ P\ P\ P\ P}$

Required arrangements = $3! \times 5! \times 4! \times 7!$

69. ***Ans. (d)***

Total possible ways without any restriction

$$= \frac{9!}{4!2!} = 7560$$

Number of ways in which both 'L' occur together

$$= \frac{8!}{4!} = 1680$$

Required answer = 7560 − 1680
= 5880

70. ***Ans. (a)***

If there are 'n' number of persons in the party then total number of handshakes = ${}^nC_2$

According to question,

${}^nC_2 = 105$

$$\Rightarrow \frac{n(n-1)}{2} = 105$$

$n^2 - n - 210 = 0$

$(n - 15)(n + 14) = 0$

$n = 15, -14$

but $n \neq$ negetive

$\Rightarrow$ $n = 15$

71. ***Ans. (d)***

No. of ways to have one-digit integers

$= {}^5C_1 = 5$ (0 can not be selected).

No. of ways to have 2-digit integers

$= {}^5C_1 \times {}^5C_1 = 25$

(Because for 1^{st} digit, we can select any integer except '0' and for 2^{nd} digit we can select any integer from remaining 5 integers)

Simmilarly,

For 3-digit integers

$= {}^5C_1 \times {}^5C_1 \times {}^4C_1$

$= 100$

For 4-digit integers

$= {}^5C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1$

$= 300$

For 5-digit integers

$= {}^5C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1$

$= 600$

For 6-digit integers

$= {}^5C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 600$

$\Rightarrow$ Total number of ways

$= 5 + 25 + 100 + 300 + 600 + 600$

$= 1630$

72. ***Ans. (b)***

73. ***Ans. (d)***

$$5! \times \frac{3!}{2!} = 3 \times 5!$$

74. We need to select 2 odd places from 3 odd places to fill the vowels E and O.

Hence, number of ways

$= {}^3C_2 \times 2! \times 3!$

$= 3 \times 2! \times 3!$

75. ***Ans. (a)***

Required answer

= (Total number of ways)

− (Numbers of ways without women)

$= {}^{10}C_5 - {}^6C_5$

$= \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} - 6$

$= 252 - 6 = 246$

76. ***Ans. (b)***

Number of ways $= {}^{n+r-1}C_r$

$= {}^{3+4-1}C_4 = {}^6C_4 = 15$

77. ***Ans. (a)***

Given, data are 6,5,9,13,12,8, and 10

x_i	x_i^2
6	36
5	25
9	81
13	169
12	144
8	64
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 619$

$$S.D. = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{7\times619-3969}{49}}$$

$$= \sqrt{\frac{4333-3969}{49}}$$

$$= \sqrt{\frac{364}{49}} = \sqrt{\frac{52}{7}}$$

78. ***Ans. (a)***

x_i	f_i	$d_i = x_i - 4$	$f_i d_i$	$f_i d_i^2$
2	4	-2	-8	16
3	9	-1	-9	9
4	16	0	0	0
5	14	1	14	14
6	11	2	22	44
7	6	3	18	54
Total	60		$\Sigma f_i d_i = 37$	$\Sigma f_i d_i^2 = 137$

$$\therefore \quad S.D. = \sqrt{\frac{\Sigma f_i d_i^2}{n} - \left(\frac{\Sigma f_i d_i}{n}\right)^2}$$

$$= \sqrt{\frac{137}{60} - \left(\frac{37}{60}\right)^2}$$

$$= \sqrt{2.2833 - (0.616)^2}$$

$$= \sqrt{2.2833 - 0.3794}$$

$$= \sqrt{1.9037} = 1.38$$

79. ***Ans. (c)***

Let the two unknown items be x and y, then

$$\text{Mean} = 4 \Rightarrow \frac{1+2+6+x+y}{5} = 4$$

$\Rightarrow \quad x + y = 11$

and $\quad$ Variance = 5.2

$$\Rightarrow \frac{1^2+2^2+6^2+x^2+y^2}{5} - (\text{Mean})^2 = 5.2$$

$$41 + x^2 + y^2 = 5\,[5.2 + (4)^2]$$

$$41 + x^2 + y^2 = 106$$

$$x^2 + y^2 = 65$$

Solving (1) and (2) for x and y, we get

$$x = 4, y = 7 \text{ or } x = 7, y = 4$$

80. ***Ans. (b)***

Marks Obtained (x_i)	f_i	c·f	$d_i = \lvert x_i - M_e \rvert$	$f_i d_i$
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
Total	$\Sigma f_i = 20$			$\Sigma f_i d_i = 25$

Now, $$M_e = \left(\frac{20+1}{2}\right)^{th} \text{ item}$$

$$= \left(\frac{21}{2}\right) = (10.5)^{th} \text{ item}$$

$$\therefore \quad M_e = 12$$

$$\therefore \quad \text{M.D.} = \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{25}{20} = 1.25$$

81. ***Ans. (a)***

Arrange the data as follows :

$$\alpha - \frac{7}{2}, \alpha - 3, \alpha - \frac{5}{2}, \alpha - 2, \alpha - \frac{1}{2}, \alpha + \frac{1}{2}, \alpha + 4, \alpha + 5$$

Median

$$= \frac{1}{2} \text{ [value of } 4^{th} \text{ item + value of } 5^{th} \text{ item]}$$

$$\therefore \quad \text{Median} = \frac{\alpha - 2 + \alpha - \frac{1}{2}}{2}$$

$$= \frac{2\alpha - \frac{5}{2}}{2} = \alpha - \frac{5}{4}$$

82. ***Ans. (a)***

Given

$$82 = \frac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$$

$$\Rightarrow \quad 82 \times 5 = 410 + 5x$$

$$\Rightarrow \quad 410 - 410 = 5x$$

$$\Rightarrow \quad x = 0$$

Therefore, the required mean is

$$\bar{x} = \frac{130+x+126+x+68+x+50+x+1+x}{5}$$

$$= \frac{375+5x}{5} = \frac{375+0}{5} = \frac{375}{5} = 75$$

83. ***Ans. (d)***

Let a rifleman fires n number of rounds.

Probability of hitting the target,

$$p = \frac{1}{10}$$

$\therefore$ Probability of not hitting the target,

$$q = 1 - \frac{1}{10} = \frac{9}{10}$$

$\therefore$ Probability of hitting the target at least once

$$= 1 - \left(\frac{9}{10}\right)^n$$

Given that $1 - \left(\frac{9}{10}\right)^n > \frac{1}{2}$

$$\therefore \quad \left(\frac{9}{10}\right)^n < \frac{1}{2}$$

So, the least value of n is 7.

84. ***Ans. (b)***

Let E_1 denote an event when a coin with two heads is selected and E_2 an event when a fair coin is selected. Let A be the event when the toss results in head. Then,

$$P(E_1) = 1/(n+1),$$

$$P(E_2) = n/(n+1),$$

$$P(A/E_1) = 1$$

and $$P(A/E_2) = 1/2.$$

Using total probability theorem, we have

or $$\frac{7}{12} = \frac{1}{n \times 1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$$

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

or $$12 + 6n = 7n + 7$$

or $$n = 5$$

85. ***Ans. (a)***

$$P(S_1) = P(S_2) = P(S_3) = \frac{1}{2}$$

Let E be event that "the current will flow".

$$P(E) = P((S_2 \cap S_3) \cup S_1))$$

$$= P(S_2 \cap S_3) + P(S_3) - P(S_1 \cap S_2 \cap S_3)$$

$$= P(S_2)P(S_3) + P(S_3) - P(S_1)P(S_2)P(S_3)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{8}$$

86. ***Ans. (c)***

P{ First event does not happen }

$$= \frac{5}{5+2} = \frac{5}{7}$$

P{Second event does not happen}

$$= \frac{5}{5+6} = \frac{5}{11}$$

$\therefore$ P{Both the events fail to happen}

$$= \frac{5}{7} \times \frac{5}{11} = \frac{25}{77}$$

Therefore, the probability that at least one of the events will happen is

1–P (none of two happens)

$$= 1 - \frac{25}{77} = \frac{52}{77}$$

87. ***Ans. (d)***

Let three persons be A,B and C.

Clearly, the probabilities of solving the problem by A,B and C are independant.

Given that

$$P(A) = 1/3,$$

$$P(B) = 1/4$$

and $$P(C) = 1/5.$$

$\therefore$ P(none can solve the problem)

$$= P(A' \cap B' \cap C')$$

$$= P(A')P(B')P(C')$$

$$= (1-P(A))(1-P(B))(1-P(C))$$

$$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)$$

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$$

88. ***Ans. (a)***

Let events J be first drawn card is jack, Q be second drawn card is queen and K be third drawn card is king.

We have to find the value of $P(J \cap Q \cap K)$

Now,

$$P(J \cap Q \cap K) = P(J) \times P(Q/J) \times P(K/(J \cap Q))$$

$$P(J) = \frac{4}{52} = \frac{1}{13}$$

P(Q/J)= P(drawing queen card if jack card has been drawn)

$$= \frac{4}{51}$$

$P(K/J \cap Q)$ = P{drawing king card if jack and queen cards have been drawn}

So,

$$\frac{40}{50} = \frac{2}{25}$$

$$(P(J \cap Q \cap K) = P(J) \times P(Q/J) \times P(K(J \cap Q))$$

$$= \frac{1}{13} \times \frac{4}{51} \times \frac{2}{25}$$

89. ***Ans. (b)***

Let A be the event that the first mango is good, and B be the event that the second one is good. Then, required probability is

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Now, probability that both mangoes are good is

$$P(A \cap B) = \frac{^6C_2}{^{10}C_2}$$

Probability that first mango is good is

$$P(A) = \frac{^6C_2}{^{10}C_2} + \frac{^6C_1 \times {^4C_1}}{^{10}C_2}$$

Hence $$P(B/A) = \frac{^6C_2}{^6C_2 + {^6C_1} \times {^4C_1}}$$

$$= \frac{15}{15+24} = \frac{5}{13}$$

90. ***Ans. (a)***

It is given that

$P(A) = 0.8,$

$P(B) = 0.5,$

and $P(B/A) = 0.4$

(i) $P(B/A) = 0.4$

$\therefore$ $$\frac{P(A \cap B)}{P(A)} = 0.4$$

or $$\frac{P(A \cap B)}{0.8} = 0.4$$

$$P(A \cap B) = 0.32$$

(ii) $$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

or $$P(A/B) = \frac{0.32}{0.5} = 0.64$$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or $P(A \cup B) = 0.8 + 0.5 - 0.32$

$= 0.98.$

91. ***Ans. (c)***

Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Since E_1 and E_2 are events complimentary to each other, we have

$$P(E_2) = 1 - P(E_1) = 1 - 0.001$$

$$= 0.999$$

Let A be the event that the blood test result is positive.

$P(A/E_1)$ = P (result is positive given the person has disease)

$= 99\%$

$= 0.99$

$P(A/E_2)$ = P(result is positive given that the person has no disease)

$= 0.5\%$

$= 0.005$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1/A)$.

By using Bayes' theorem, we obtain

$$P(E_1/A) = \frac{p(E_1)P(A/E_1)}{p(E_1)p(A/E_1) + p(E_2)p(A/E_2)}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

$$= \frac{0.00099}{0.005985} = \frac{22}{133}$$

92. Ans. (d)

We define the following events:

A_1 : He knows the answer

A_2 : He does not know the answer

E : He gets the correct answer

Then,

$$P(A_1) = 9/10,$$

$$P(A_2) = 1 - 9/10 = 1/10,$$

$$P(E/A_1) = 1, P(E/A_2) = 1/4.$$

Therefore, the required probability is

$$P(A_2/E) = \frac{P(A_2)P(E/A_2)}{P(A_1)P(E/A_1)+P(A_2)P(E/A_2)}$$

$$= \frac{\frac{1}{10}\times\frac{1}{4}}{\frac{9}{10}\times 1+\frac{1}{10}\times\frac{1}{4}} = \frac{1}{37}$$

93. Ans. (d)

Let E_1 denote the event that one of the first n turns is chosen and E_2 denote the event that (n+1)th turn is selected. Adenotes the event that two balls drawn are black. Then,

$$P(E_1) = n/(n+1),$$

$$P(E_2) = 1/(n+1),$$

$$P(A/E_1) = {}^{6}C_2 / {}^{10}C_2 = 1/3$$

and $$P(A/E_2) = {}^{5}C_2 / {}^{10}C_2 = 2/9$$

Using Bayes' theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$\frac{1}{16} = \frac{\left(\frac{1}{n+1}\right)\frac{2}{9}}{\left(\frac{n}{n+1}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{n+1}\right)\left(\frac{2}{9}\right)}$$

$$\frac{1}{16} = \frac{2}{3n+2}$$

$$n = 10$$

94. Ans. (a)

Let R be the event that a red face appears in each of the first n throws.

E_1 : Die A is used when head has already fallen

E_2 : Die B is used when tail has already fallen

$$\therefore\ P\left(\frac{R}{E_1}\right) = \left(\frac{2}{3}\right)^n \text{ and } P\left(\frac{R}{E_2}\right) = \left(\frac{1}{3}\right)^n$$

As per the given condition,

$$\frac{P(E_1)P(R/E_1)}{P(E_1)P(R/E_1)+P(E_2)P(R/E_2)} = \frac{32}{33}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}\right)^n}{\frac{1}{2}\left(\frac{2}{3}\right)^n+\frac{1}{2}\left(\frac{1}{3}\right)^n} = \frac{32}{33}$$

$$\frac{2^n}{2^n+1} = \frac{32}{33}$$

$$n = 5$$

95. Ans. (b)

$$P(A'\cup B') = \frac{3}{4}$$

$$\Rightarrow\ P(A\cap B)') = \frac{3}{4}$$

$$\Rightarrow\ P(A\cap B) = \frac{1}{4}$$

$$P(A'\cap B') = \frac{1}{4}$$

$$\Rightarrow\ P(A\cup B) = \frac{3}{4}$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$$

$$\Rightarrow \quad P(B) = \frac{3}{4} - \frac{1}{12} = \frac{2}{3}$$

Now, $P(A' \cap B) = P(B) - P(A \cap B)$

$$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

96. ***Ans. (c)***

The total number of nails and nuts is

$6+10 = 16.$

$$P(R) = \frac{1}{2} \text{ (R stands for rusted)}$$

$$P(N) = \frac{6}{16} \text{ (N stands for nails)}$$

$P(R \cap N) = \dfrac{3}{16}$ [$\therefore$ 3 nails are rusted out of 6 nails]

$$P(R \cup N) = P(R) + P(N) - P(R \cap N)$$

$$= \frac{1}{2} + \frac{6}{16} - \frac{3}{16}$$

$$= \frac{8+6-3}{16}$$

$$= \frac{11}{16}$$

97. ***Ans. (a)***

It is given that

$$P(A \cup B) = 0.6 \text{ and } P(A \cap B) = 0.2.$$

Therefore,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = P(A) + P(B) - 0.2$$

$$P(A) + P(B) = 0.8$$

$$1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$$

$$P(\bar{A}) + P(\bar{B}) = 1.2$$

98. ***Ans. (b)***

The selection of 6 balls, consisting of at least two balls of each color from 5 red and 6 white balls can be made in the following ways :

Red balls (5)	White balls (6)	Number of ways
2	4	${}^5C_2 \times {}^6C_4 = 150$
3	3	${}^5C_3 \times {}^6C_3 = 200$
4	2	${}^5C_4 \times {}^6C_2 = 75$
	Total	425

99. ***Ans. (d)***

We have letters (MM),(AA),(TT),H,E,I,C,S

Number of words starting with M or T

$$= \frac{10!}{2! \times 2!}$$

($\because$ Two pairs of identical letters are available)

Number of words starting with T or C or S

$$= \frac{10!}{2! \times 2! \times 2!}$$

($\because$ Three pairs of identical letters are available)

Therefore, total number of words

$$= 2 \times \frac{10!}{2! \times 2!} + 3 \times \frac{10!}{2! \times 2! \times 2!}$$

$$= \frac{7 \times 10!}{2! \times 2! \times 2!}$$

100. ***Ans. (a)***

The total number of numbers formed with the digits 2, 3, 4,5 taken all at a time is equal to the number of arrangements of 4 digits, taken all at a time, i.e., ${}^4P_4 = 4! = 24$.

To find the sum of these 24 numbers, we have to find the sum of the digits at unit's, ten's, hundred's, and thousand's places in all these numbers.

Consider the digits in the unit's places in all these numbers.

If 2 is the digit in unit's place, the remaining three places can be filled in 3! ways or we can say 2 occurs in unit's place 3!(= 6) times. Similarly, each digit occurs 6 times.

So, the total sum of the digits in the unit's place in all these numbers is $(2+3+4+5)\times 3!=84$.

Similarly, the sum of digits is 84 in ten's, hundred's, and thousand's places.

Hence, the sum of all the numbers is

$84\,(10^0 + 10^1 + 10^2 + 10^3) = 93324$

101. ***Ans. (d)***

We have letters L, O, G, A, R, I, T, H, M.

Words contain exactly three different letters.

Three letters can be selected in $^9C_3$ ways.

Now we have following cases for the occurrence of these three letters.

Case I: Occurrence of letters is 4,1,1

The letter which is occurring four times can be selected in $^3C_1$ ways.

Then letters can be arranged in $\frac{6!}{4!}$ ways.

So, number of words in this case are

$$^3C_1 \times \frac{6!}{4!} = 90$$

Case II: Occurrence of letters is 3,2,1

The letter which is occurring three times can be selected in $^3C_1$ ways.

The letter which is occurring two times can be selected in $^2C_1$ ways. Then letters can be arranged in $\frac{6!}{3!2!}$ ways.

So, number of words in this case are

$$^3C_1 \times {^2C_1} \times \frac{6!}{3!2!} = 360$$

Case III: Occurrence of letters is 2, 2, 2

Since each letter is occurring twice, number of words are

$$\frac{6!}{2!2!2!} = 90$$

So, total number of words

$$= {^9C_3} \times (90 + 360 + 90)$$

$$= 84 \times 540$$

$$= 45360$$

102. ***Ans. (a)***

The signals can be made by using one or more flags at a time.

The total number of signals when r flags are used at a time from 5 flags is equal to the number of permutations of 5 , taking r at a time, i.e., ^5P_r

Since r can take the values 1,2,3,4,5, by the fundamental principle of addition, the total number of signals is

$$^5P_1 + {^5P_2} + {^5P_3} + {^5P_4} + {^5P_5}$$

$$= 5+(5\times 4)+(5\times 4\times 3)+(5\times 4\times 3\times 2)+(5\times 4\times 3\times 2\times 1)$$

$$= 5+20+60+120+120$$

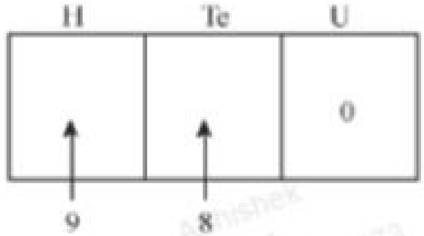
$$= 325$$

103. ***Ans. (d)***

A number is divisible by 5 if digit at unit place is either ' 0 ' or ' 5 '.

So, we have following two cases:

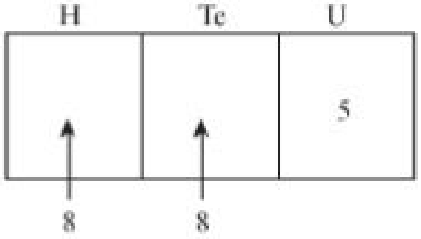
Case I : Digit ' 0 ' at unit place



In this case hundred's place, can be filled in 9 ways and tens place can be filled in 8 ways.

So, number of numbers $=9\times 8=72$ ways.

Case II : Digit ' 5 ' at unit place



In this case hundred place can be filled in 8 ways (as zero cannot be used).

Tens place can be filled in 8 ways (as zero can be used)

So, number of numbers

$= 8 \times 8 = 64$ ways.

Therefore, total number of numbers

$= 72 + 64 = 136$