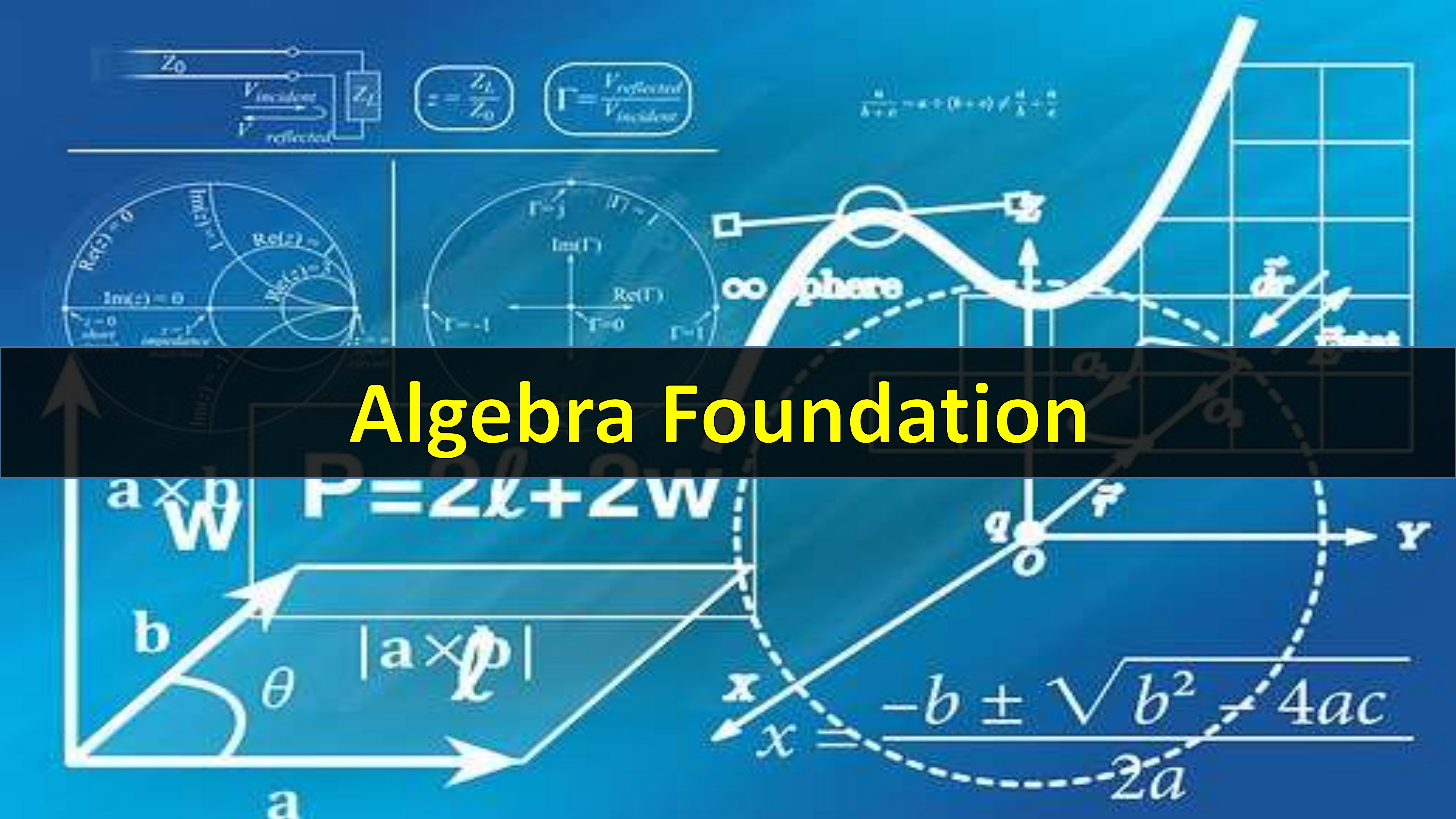
Math And Statistics For Data Science

By Eng. Mohammed Marwan Shahin

Outlines

- **Algebra Foundation** - Covers Algebraic Equations, Quadratic Equations and Functions
- **Calculus** - First order and second order derivatives, Partial Derivatives, Gradients
- **Linear Algebra** - Vectors and their Operations, Matrix and its operations, Vector Transformation using Matrices.
- **Probability** - Probability Basics, Conditional Probability, Random Variables and Random processes
- **Descriptive & Inferential Statistics**

Algebra Foundation



Balancing

Al-Kitāb al-mukhtaṣar fī hīsāb al-ğabr wa'l-muqābala

calculation

-al-Kwarizmi

820 AD

Algebra : Means Restoration or Completion

What is Algebra?

Variables

Constants

Equations

Left Side = Right Side

Equation

$$3X + 4 = 10$$

$$3X + 4 - 4 = 10 - 4$$

$$\frac{3X}{3} = \frac{6}{3}$$

$$X = 2$$

Equation

$$3X + 4 = 10$$

$$3(2) + 4 = 10$$

$$6 + 4 = 10$$

Distributive Property

$$3\,(X + 2) = 12$$

$$3X + 6 = 12$$

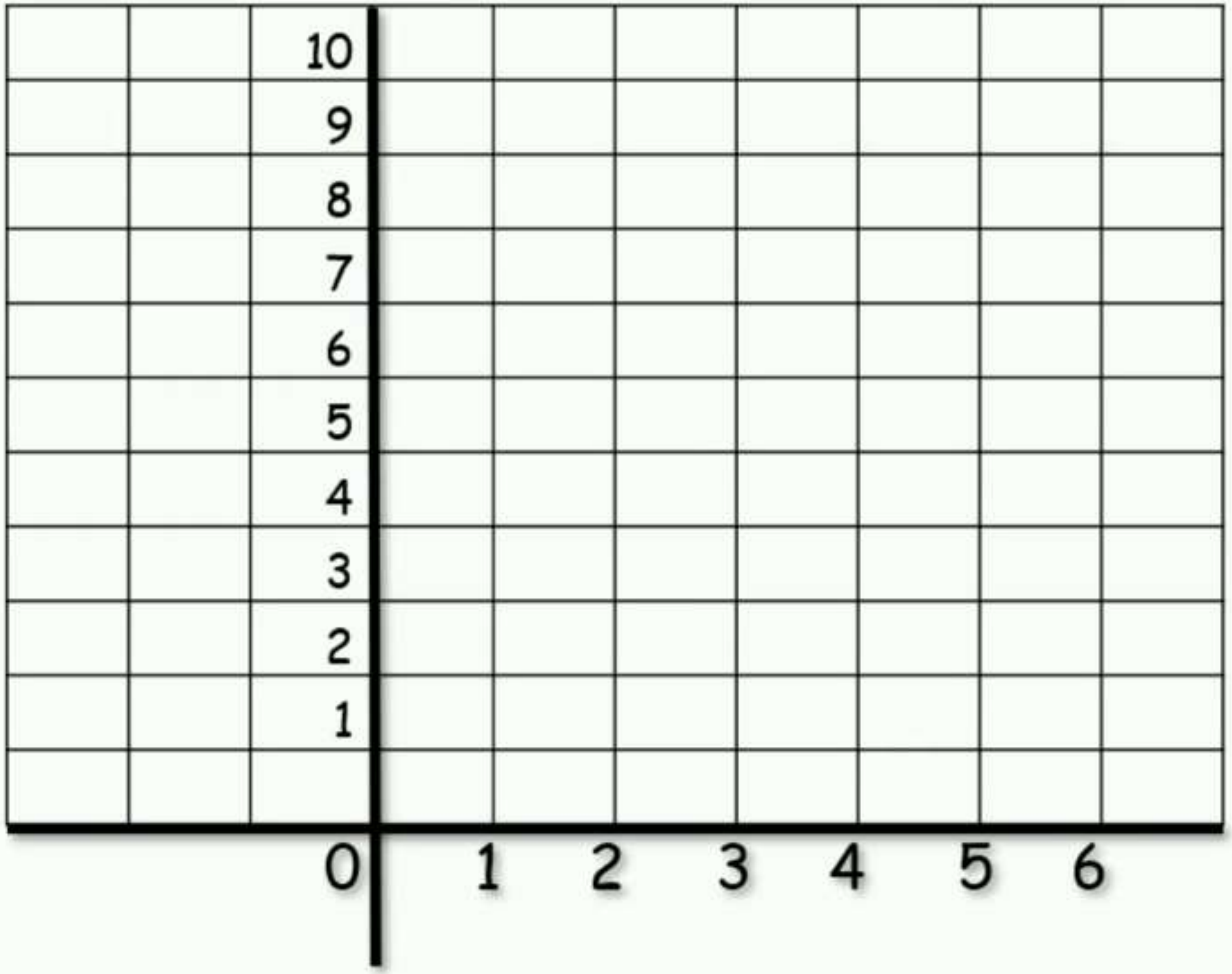
Variable

$$Age = Grade + 5$$

$$y = x + 5$$

Linear Equation

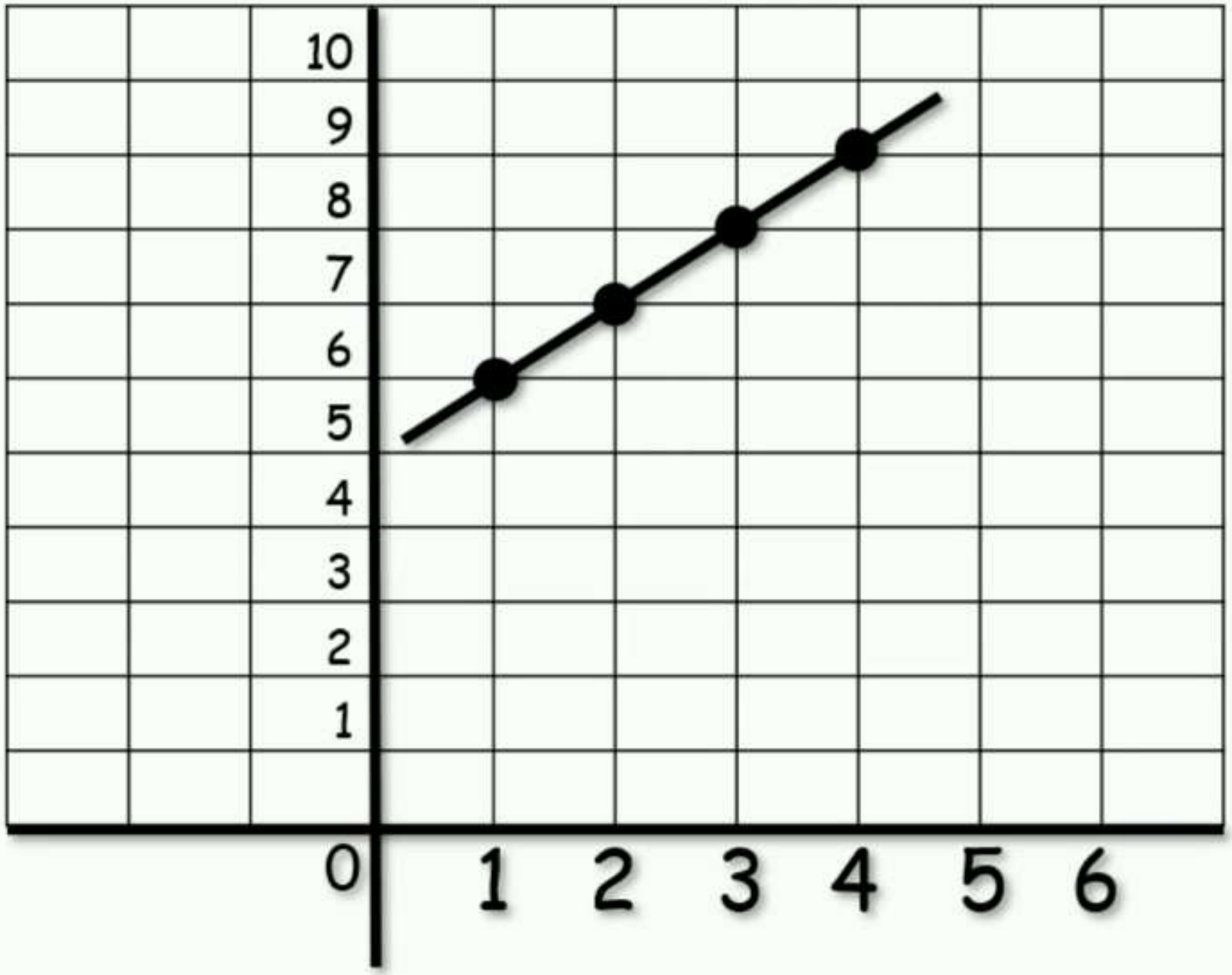
Linear Equation



$$y = x + 5$$

x	y
1	6
2	7
3	8
4	9

Linear Equation



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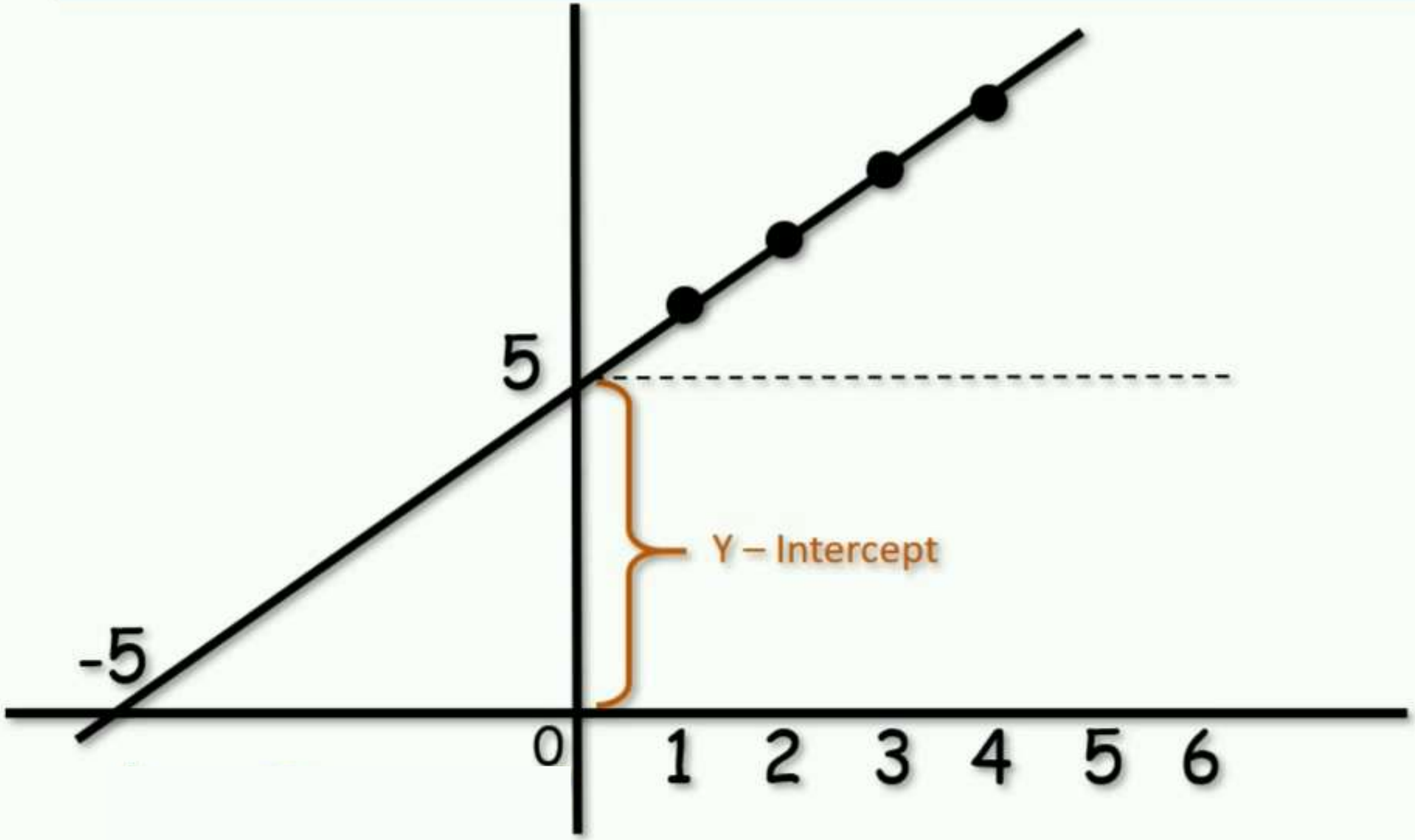
Linear Equation



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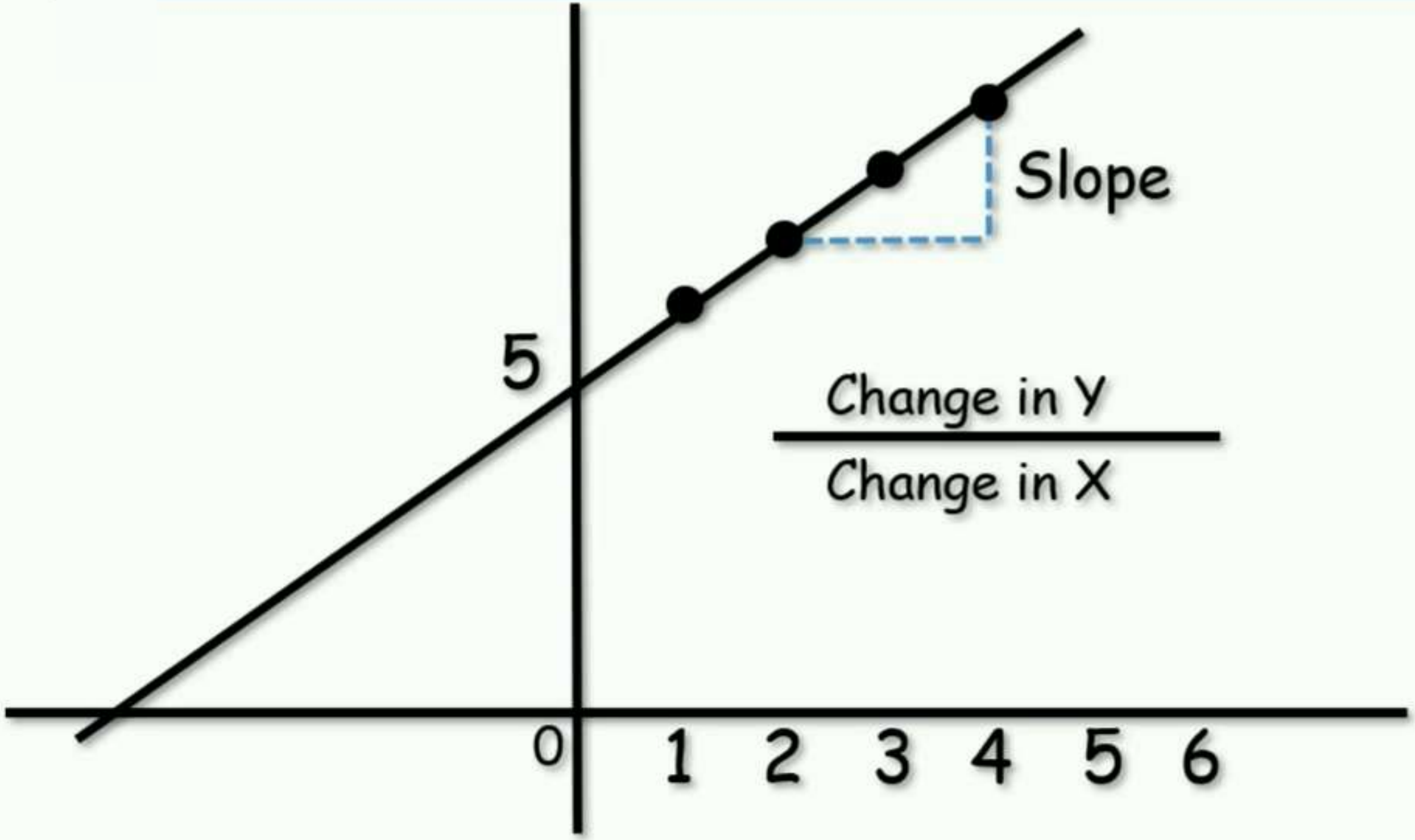
Linear Equation



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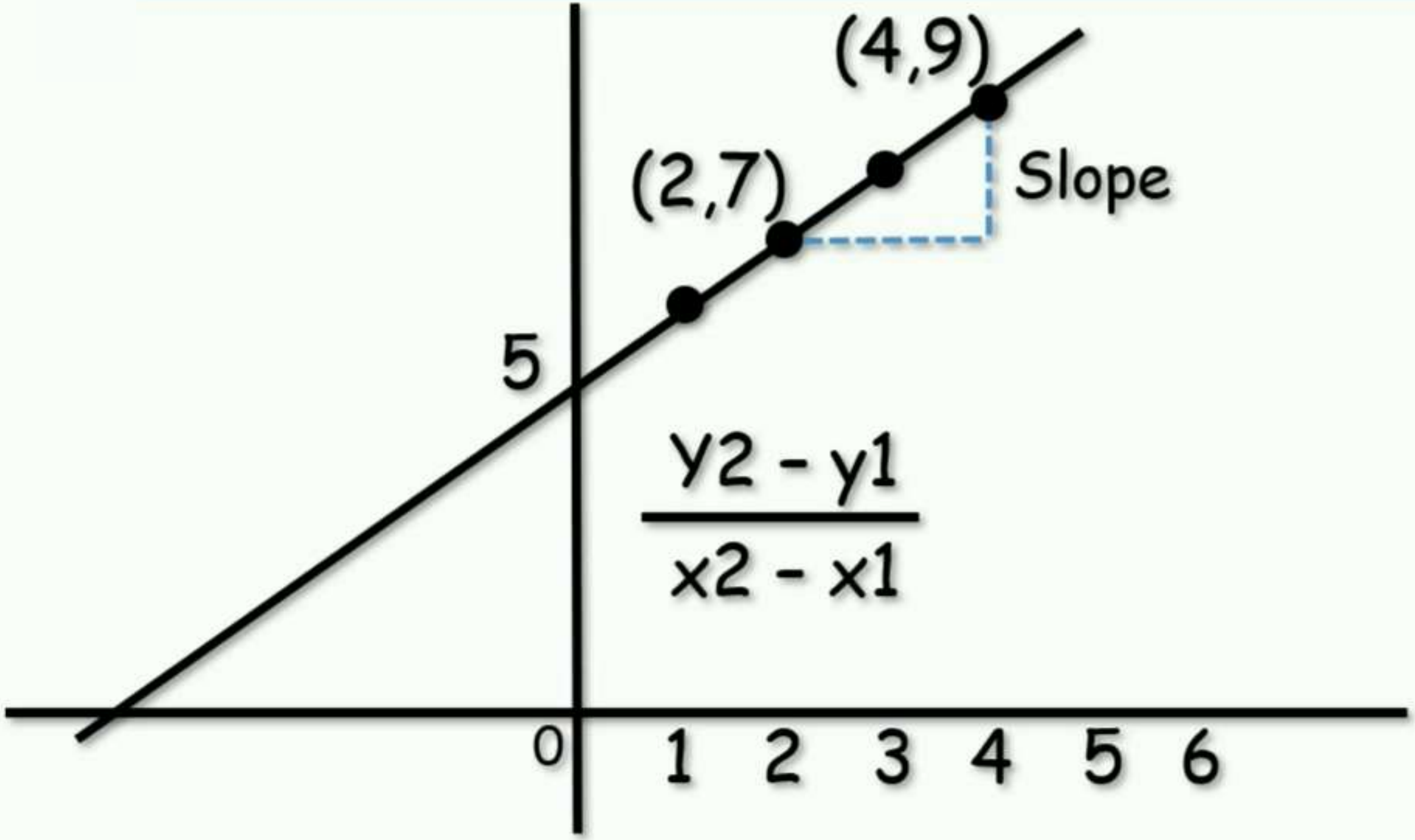
2	7
3	8
4	9

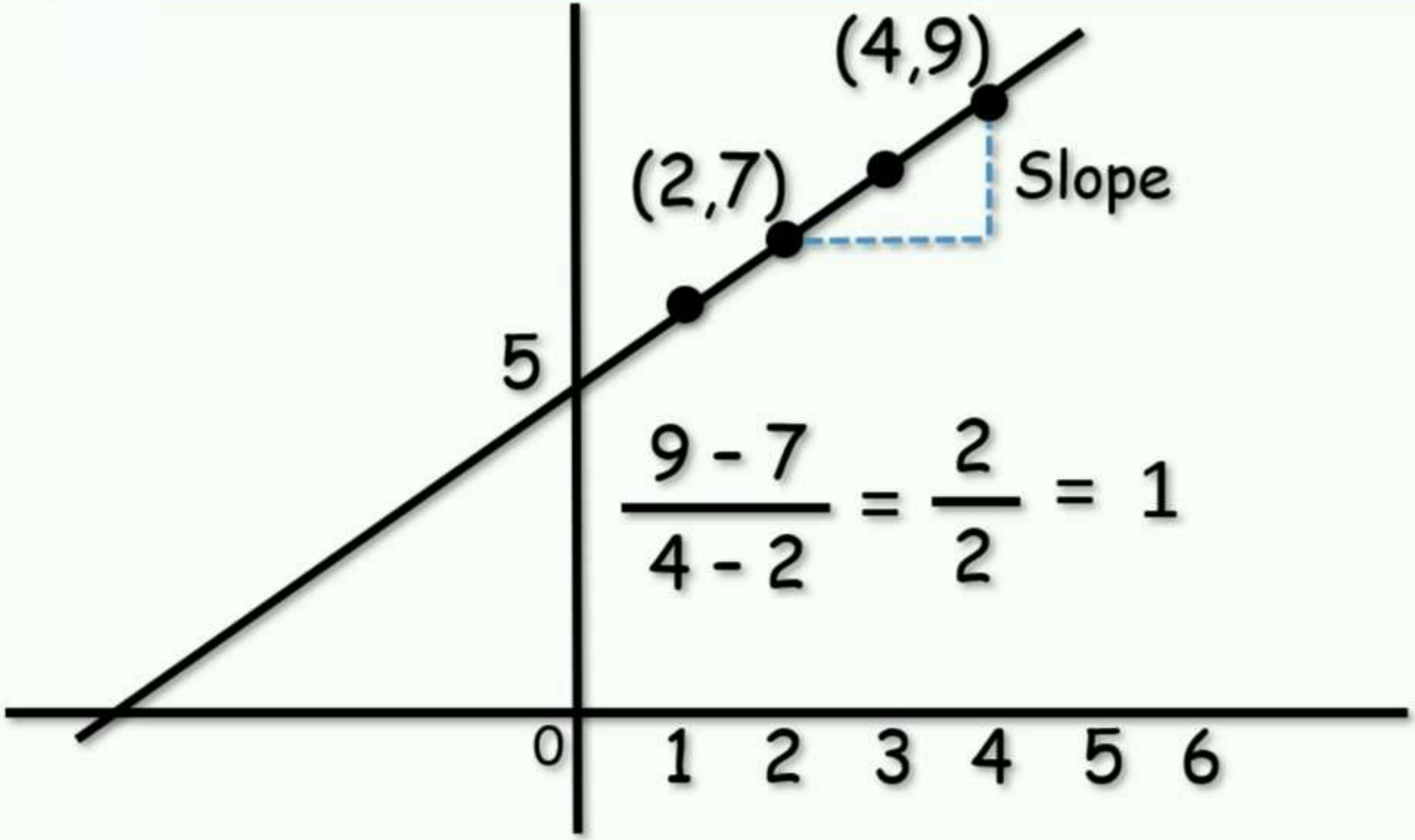
Linear Equation



$$y = x + 5$$

x	y
1	6
2	7
3	8
4	9

Linear Equation



$$y = x + 5$$

x	y
1	6
2	7
3	8
4	9

Algebra Subjects

- Exponents
- Logarithm
- Polynomial
- Factoring
- Quadratic Equations

Exponents

- How many times we should multiply a number by itself?

Index or exponent or power

$$4^2 = 4 \times 4$$

Base

$$4^3 = \underline{4} \times \underline{4} \times \underline{4}$$

Exponents

$4^{-3} =$

$4^{0} =$

Exponents

$$4^{-3} = 1 \div (4 \times 4 \times 4)$$

$$4^{0} = 1$$

Exponents Arithmetic

$$x^3 * x^2 =$$

Exponents Arithmetic

$$X^3 * X^2 = X^{3+2} = X^5$$

$$X * X * X \quad * \quad X * X$$

Exponents Arithmetic

$$x^3 \times x^2 = x^{3+2} = x^5$$

$$x^3 \div x^2 =$$

Exponents Arithmetic

$$X^3 \times X^2 = X^{3+2} = X^5$$

$$X^3 \div X^2 = X^{3-2} = X^1$$

$$\frac{X * X * X}{X * X}$$

In this lecture...

- Exponents
- Logarithm
- Polynomial
- Factoring
- Quadratic Equations

Logarithm

$$4^{3} = 64$$

$$4^{?} = 64 \Rightarrow \log_{4}(64) = \boxed{}$$

Logarithm

$$4^3 = 64$$

$$4^? = 64 \quad \Rightarrow \quad \log_4(64) = 3$$

Logarithm

$$\log_2(4) = ?$$

$$2^{?} = 4$$

In this lecture...

- Exponents
- Logarithm
- Polynomial
- Factoring
- Quadratic Equations

Polynomial

- Poly → Many
- Nomial → Terms

$$2x^2 + 3x + 5$$

exponents variables constants

terms

Polynomial Arithmetic

- Poly → Many
- Nomial → Terms

$$
\begin{array}{r}
2x^2 + 3x + 5 \\
+ \quad 3x^2 - 2x + 3 \\
\hline
\end{array}
$$

Polynomial Arithmetic

- Poly → Many
- Nomial → Terms

$$
\begin{array}{r}
2x^2 + 3x + 5 \\
+ \quad 3x^2 - 2x + 3 \\
\hline
5x^2 + 1x + 8
\end{array}
$$

Polynomial Arithmetic

$$
\begin{array}{rr}
 & 2x^2 + 3x + 5 \\
\times & 3x^2 - 2x + 3 \\
\hline
\end{array}
$$

Polynomial Arithmetic

$$
\begin{array}{lcccccc}
 & 2x^2 & + & 3x & + 5 & & \\
\times & 3x^2 & - & 2x & + 3 & & \\
\hline
 & 6x^4 & - & 4x^3 & + 6x^2 & & \\
 & & & 9x^3 & - 6x^2 & + 9x & \\
 & & & & 15x^2 & - 10x & + 15
\end{array}
$$

In this lecture...

- Exponents
- Logarithm
- Polynomial
- Factoring
- Quadratic Equations

Factoring

- What can I multiply with what to get the required equation or number?

$$\underbrace{5 \times 3}_{\text{Factors}} = 15$$

Factoring

- What can I multiply with what to get the required equation or number?

$$3x + 9 \Rightarrow 3\,(x + 3)$$

Factoring

- What can I multiply with what to get the required equation or number?

$$2x^3 - 8x \rightarrow$$

Factoring

- What can I multiply with what to get the required equation or number?

$$2x^3 - 8x \Rightarrow 2(x^3 - 4x) \Rightarrow 2x\boxed{(x^2 - 4)}$$

Difference of Squares

Difference of Squares

$$(a^2 - b^2) =$$

$$(x^2 - 4) =$$

Difference of Squares

$$(a^2 - b^2) = (a + b)(a - b)$$

$$(x^2 - 4) =$$

Difference of Squares

$$(a^2 - b^2) = (a + b)(a - b)$$

$$(x^2 - 4) = (x^2 - 2^2) = (x + 2)(x - 2)$$

Factoring

- What can I multiply with what to get the required equation or number?

$$2x^3 - 8x$$

Factoring

- What can I multiply with what to get the required equation or number?

$$2x^3 - 8x \Rightarrow 2x(x+2)(x-2)$$

In this lecture...

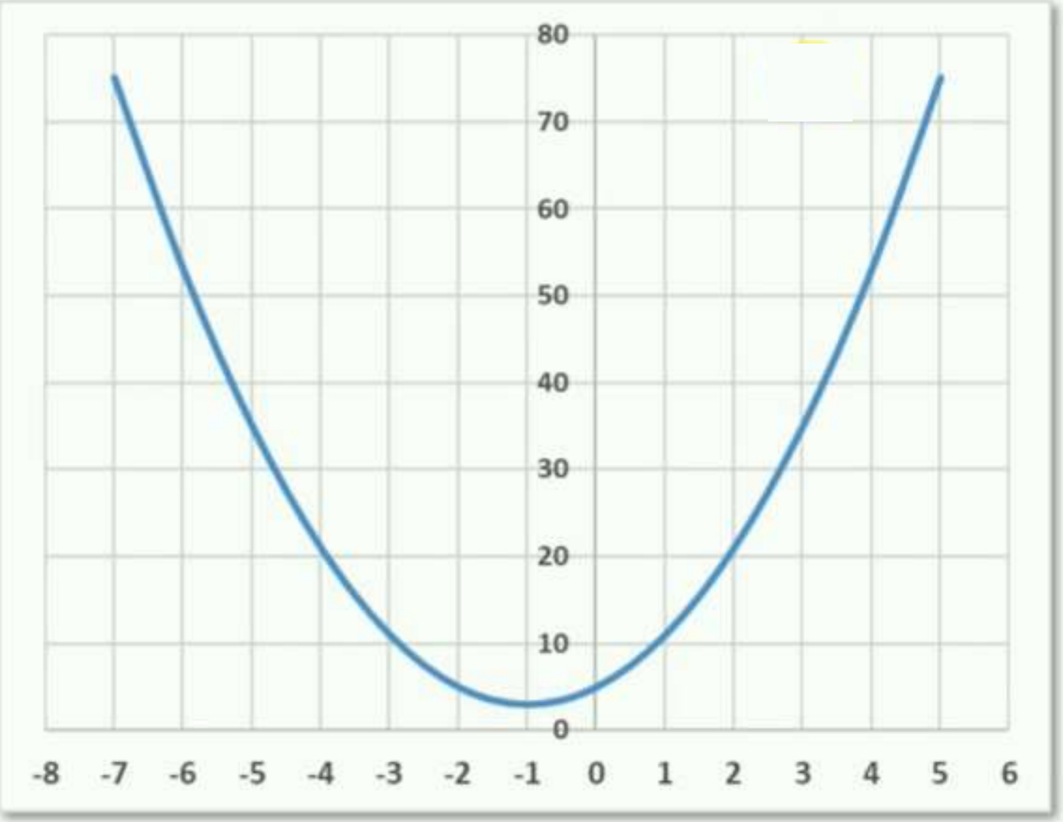
- Exponents
- Logarithm
- Polynomial
- Factoring
- Quadratic Equations

Quadratic Equation

Special type of polynomial with “Quad” or Square.

$$ax^2 \quad + \quad bx \quad + c$$

$$2x^2 \quad + \quad 4x \quad + 5$$



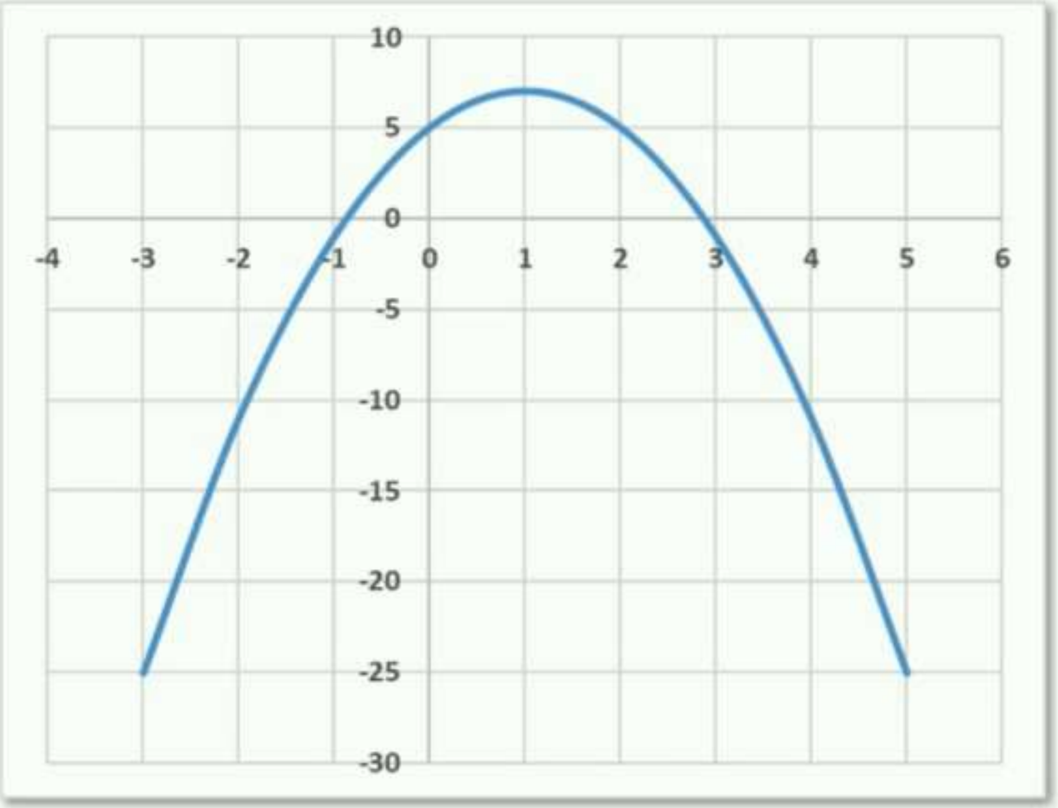
Quadratic equation

$-2x^2 + 4x + 5$

$2x^2 + 4x + 5$

Quadratic equation

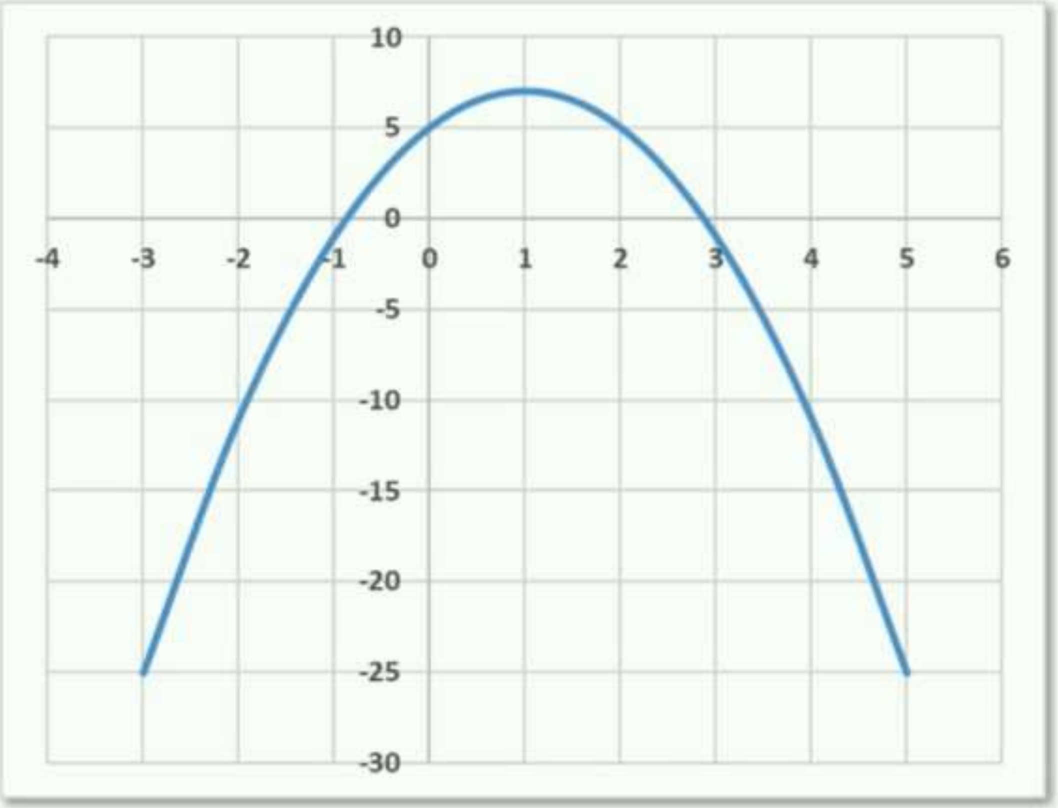
$-2x^2 + 4x + 5$



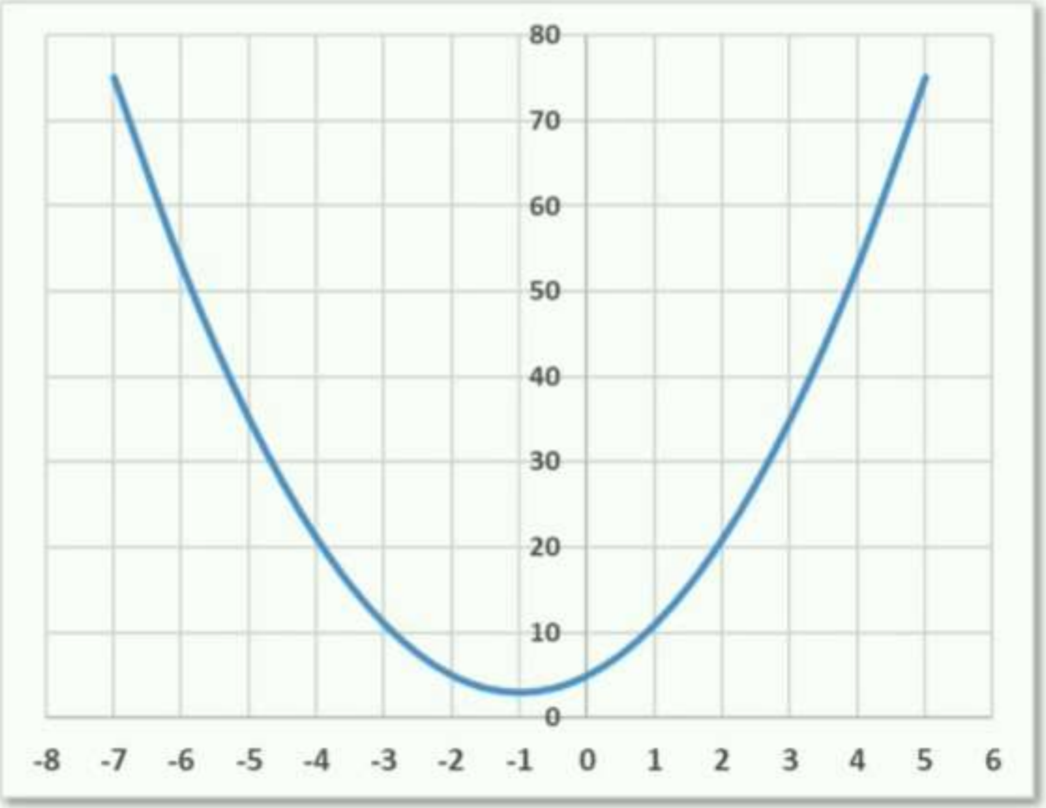
$2x^2 + 4x + 5$

Quadratic equation

$$-2x^2 + 4x + 5$$



$$2x^2 + 4x + 5$$



Common Equations

$$(x + a)^2 =$$

$$(x - a)^2 =$$

$$(x + a) * (x - a) =$$

Common Equations

$$(x + a)^2 = x^2 + 2xa + a^2$$

$$(x - a)^2 = x^2 - 2xa + a^2$$

$$(x + a) * (x - a) = x^2 - a^2$$

What is a Function?

Input → function → Output

Process the input

X → Square → X^2

$$f(x) = x^2$$

What is a Function?

$$\underset{\text{function}}{f}(\underset{\text{input}}{x}) = \underset{\text{output}}{x^2}$$

What is a Function?

$$f(x) = x^2$$

$$f(x) = x + 5$$

$$f(x) = 4x - 9$$

Age as a function of grade?

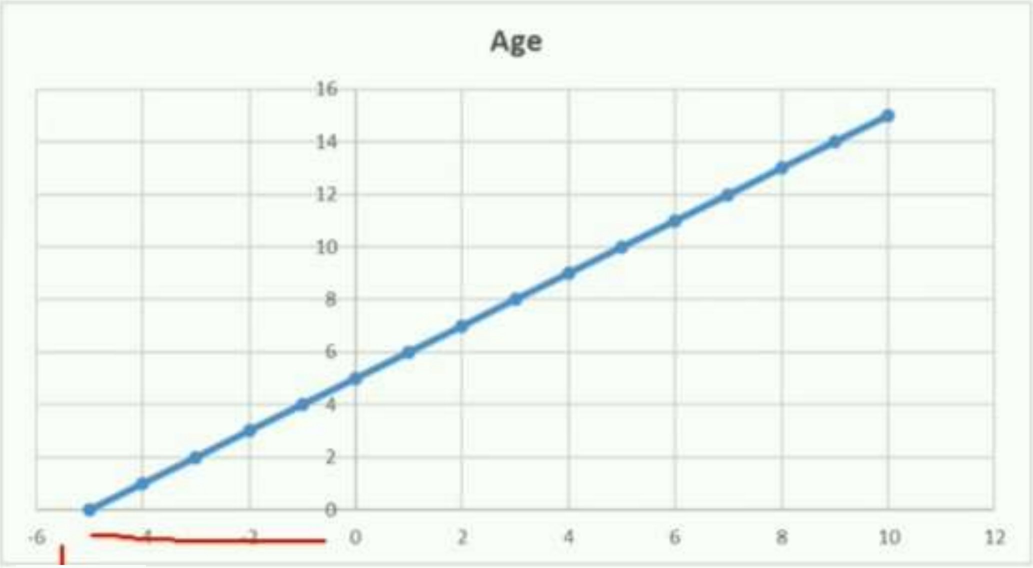
$$f(x) = x + 5$$

Grade	Age
1	1 + 5 = 6
2	2 + 5 = 7
3	3 + 5 = 8
4	4 + 5 = 9
5	5 + 5 = 10

Allowed values → 1 to 10

Age as a function of grade?

$$f(x) = x + 5$$

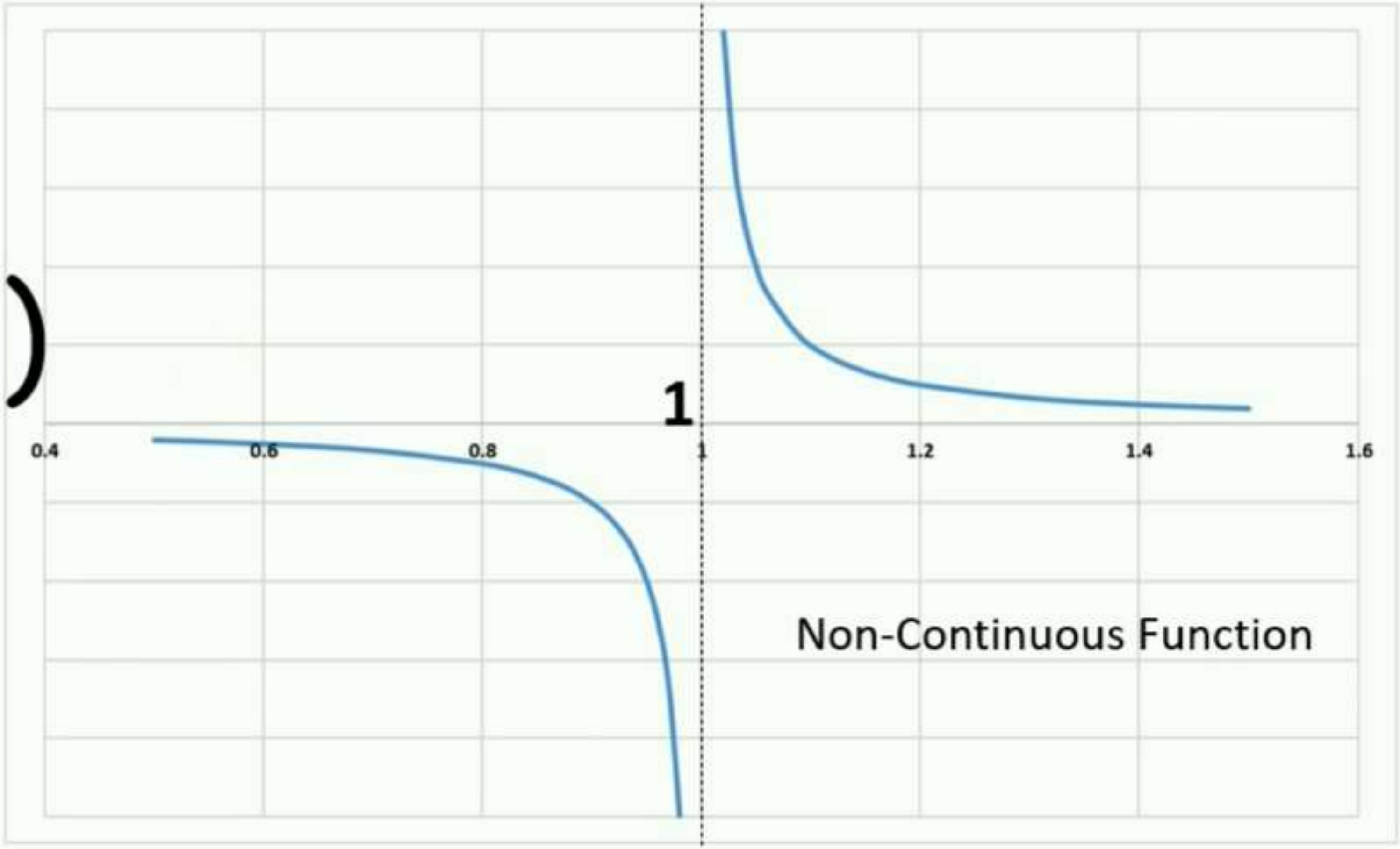


Grade	Age
1	1 + 5 = 6
2	2 + 5 = 7
3	3 + 5 = 8
4	4 + 5 = 9
5	5 + 5 = 10

Allowed values → 1 to 10

Continuous Functions

$$f(x) = 1/(x - 1)$$



It's not a continuous function : if it's continuous function we should draw it without lifting the pen.

Quiz

Question 1:

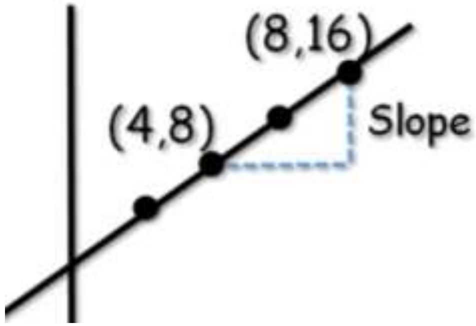
- **What will be the value of "x" when we solve the following equation?**

  $2x + 4 = 18$

- ○ 7
- ○ 11
- ○ 9
- ○ 6

Question 2:

- **What will be the slope of the line in this plot**



- 4
- 1
- 2
- 8

Question 3:

- **What will be the value of the following equation?**

$$\log_2 (8) =$$

- ○ 3
- ○ 4
- ○ 2
- ○ None of the above.

Question 4:

$$f(x) = 1/x$$

- **The following function is NOT a continuous function. True or False?**

True

False

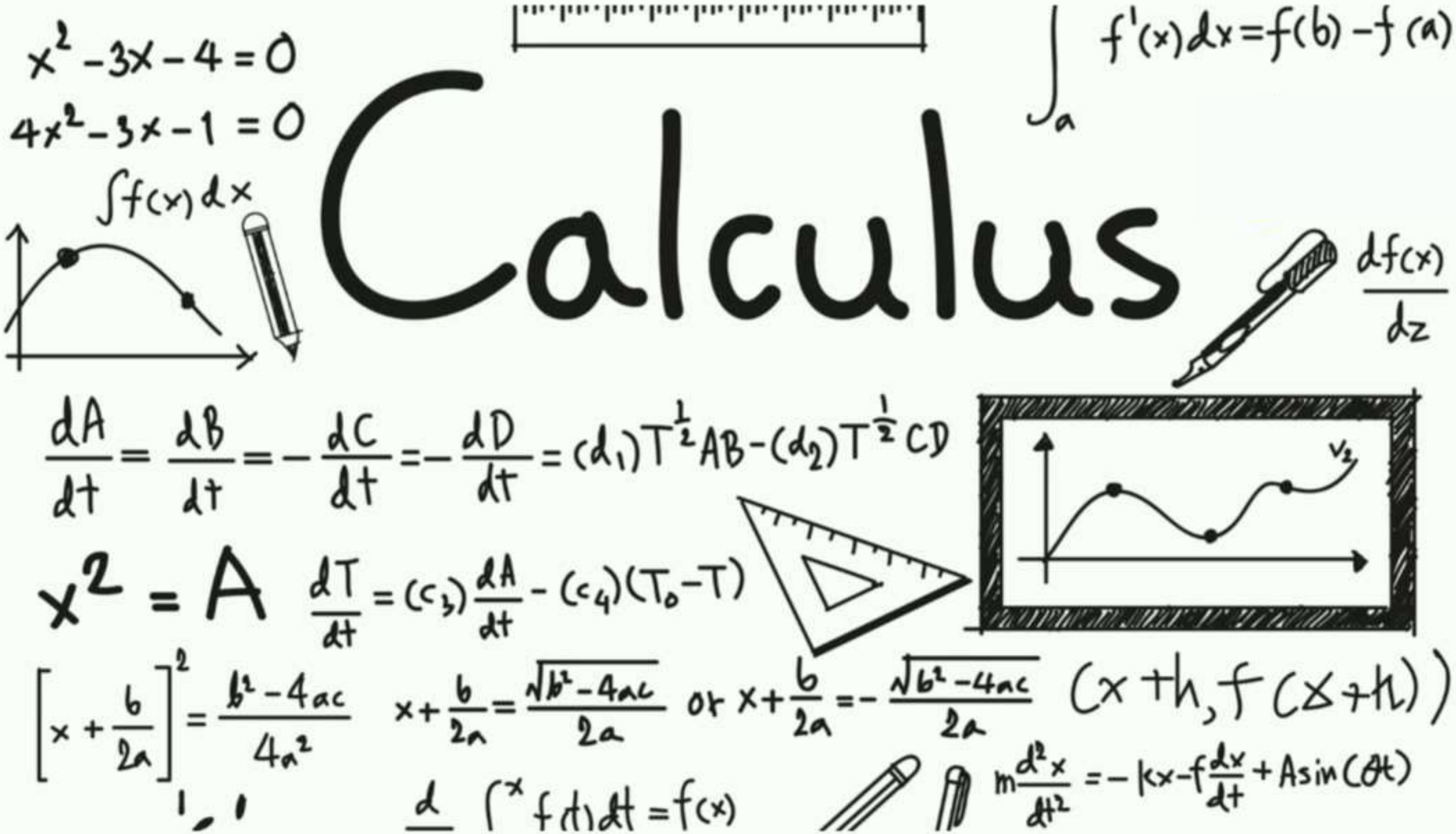
Question 4:

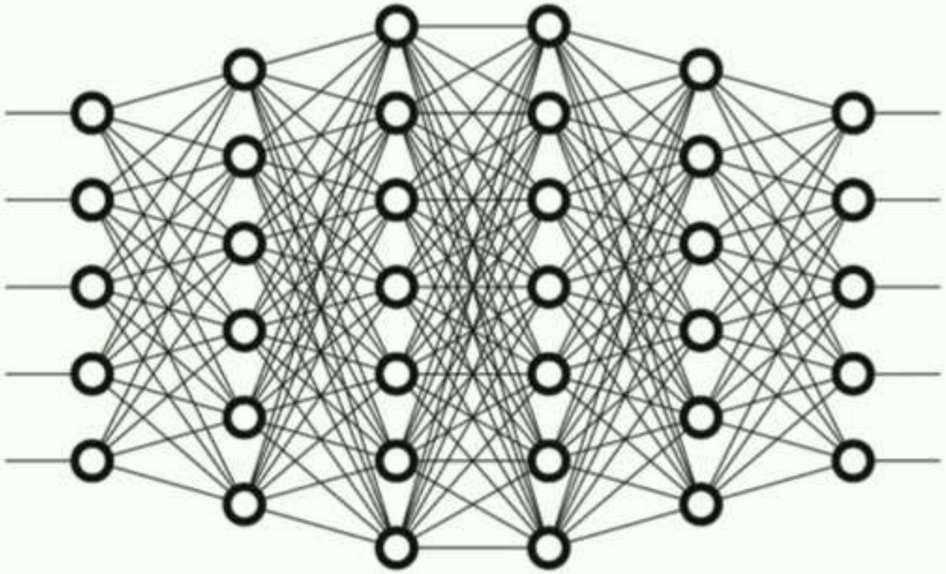
$$f(x) = 1/x$$

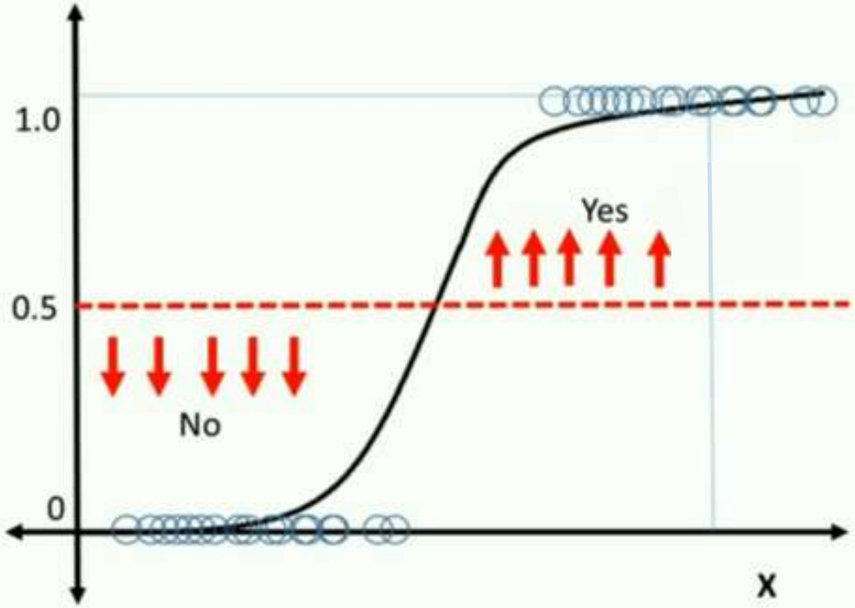
- **The following function is NOT a continuous function. True or False?**

True

False


Calculus
x² - 3x - 4 = 0
4x² - 3x - 1 = 0
∫f(x) dx
∫a f'(x) dx = f(b) - f(a)
df(x)/dz
dA/dt = dB/dt = - dC/dt = - dD/dt = (d1)T^½ AB - (d2)T^½ CD
x² = A
dT/dt = (c3) dA/dt - (c4)(T0 - T)
[x + b/2a]² = (b² - 4ac)/4a²
x + b/2a = √(b² - 4ac)/2a or x + b/2a = - √(b² - 4ac)/2a
(x + h, f(x + h))
m d²x/dt² = - kx - f dx/dt + Asin(θt)
d ∫x f(t) dt = f(x)
v2





$$\frac{dy}{dx} = 24x^3 - 6x^2 - 24x + 1$$

$$\frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$

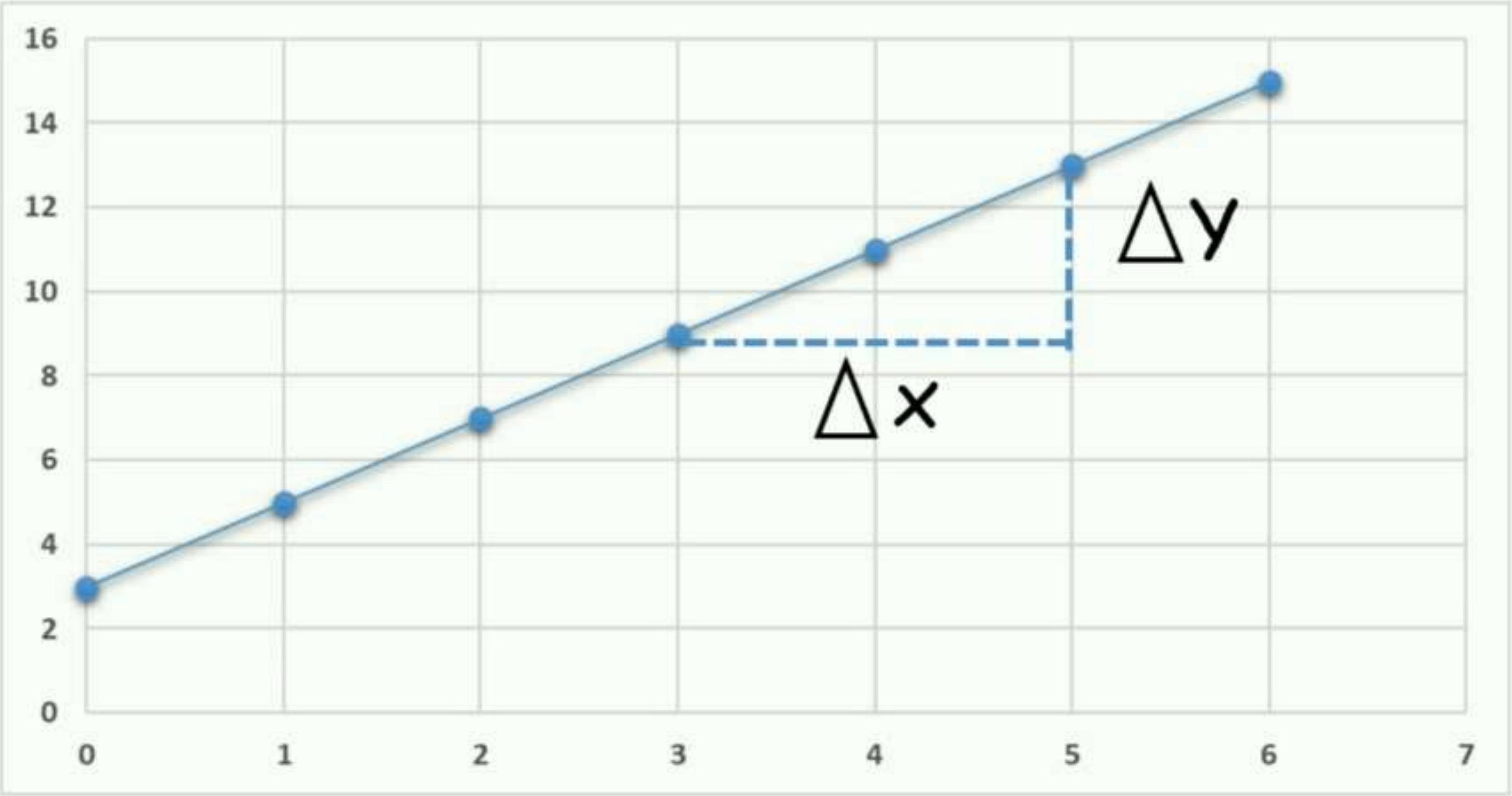
$$\frac{\partial(f(x, y))}{\partial x} = 2X$$

Rate of Change

$$y = 2x + 3$$

Rate of Change

$$= \frac{\Delta y}{\Delta x}$$



Rate of Change

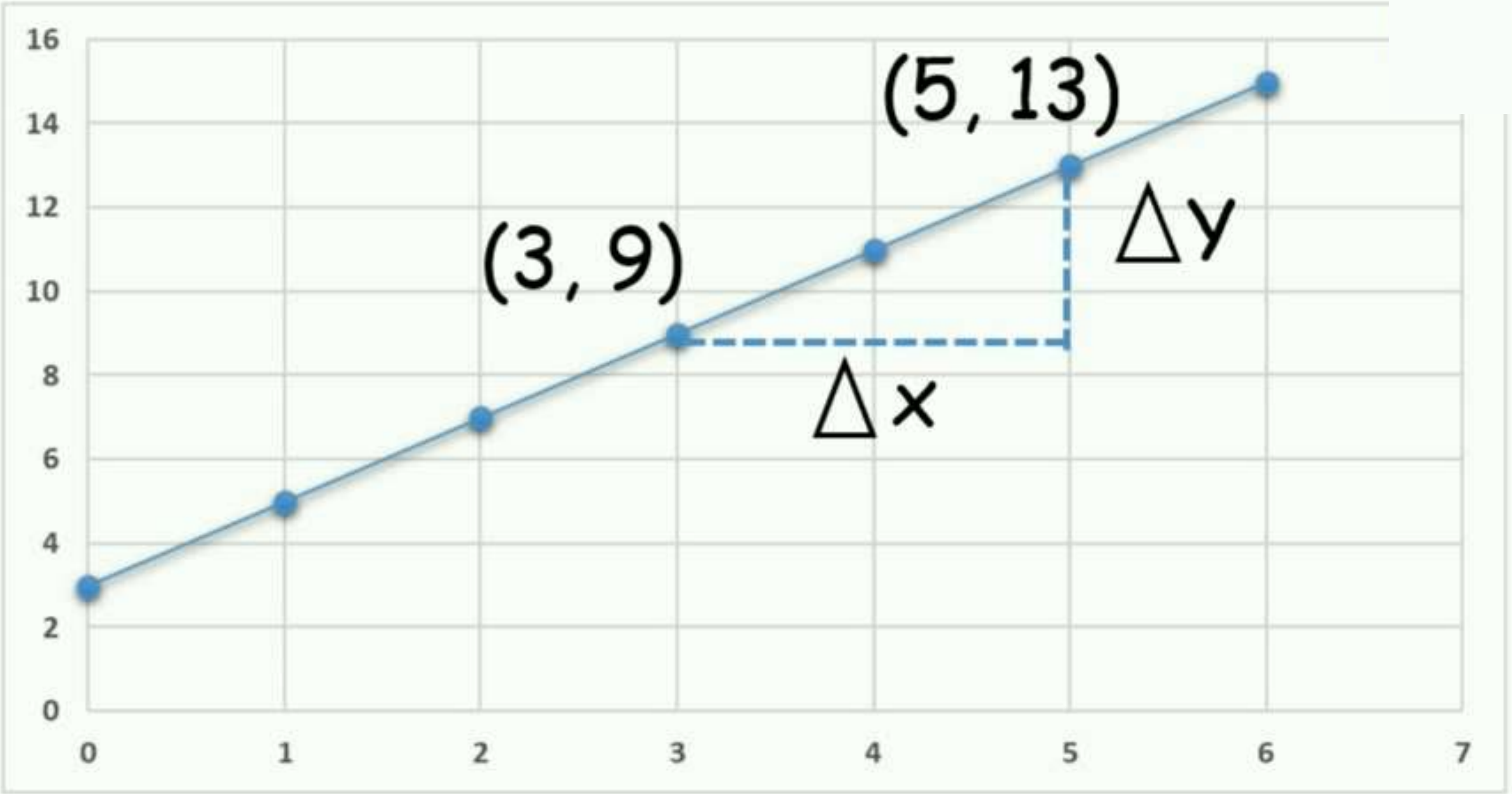
$y = 2x + 3$

Rate of Change

$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{13 - 9}{5 - 3}$$

$$= 2$$

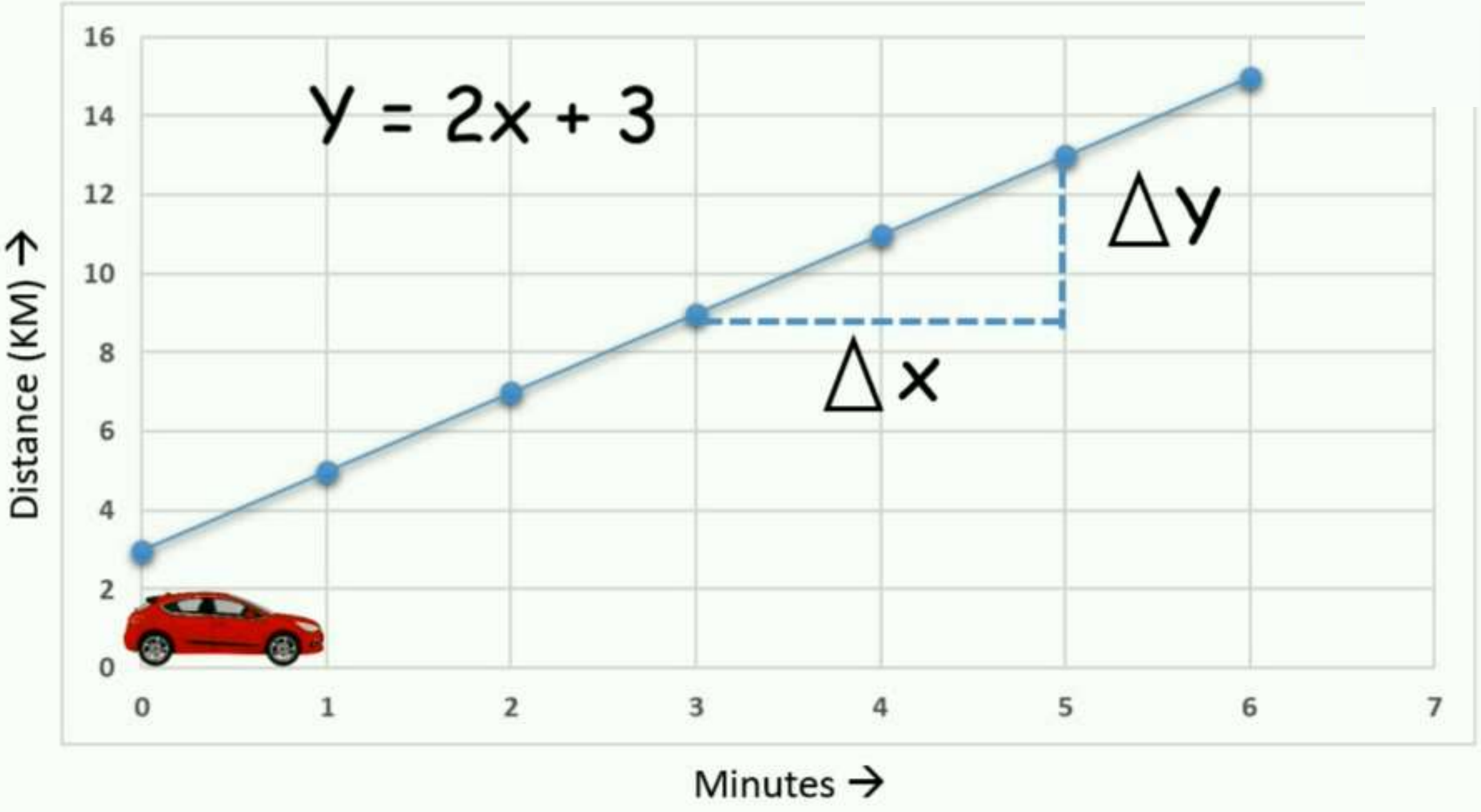


Rate of Change

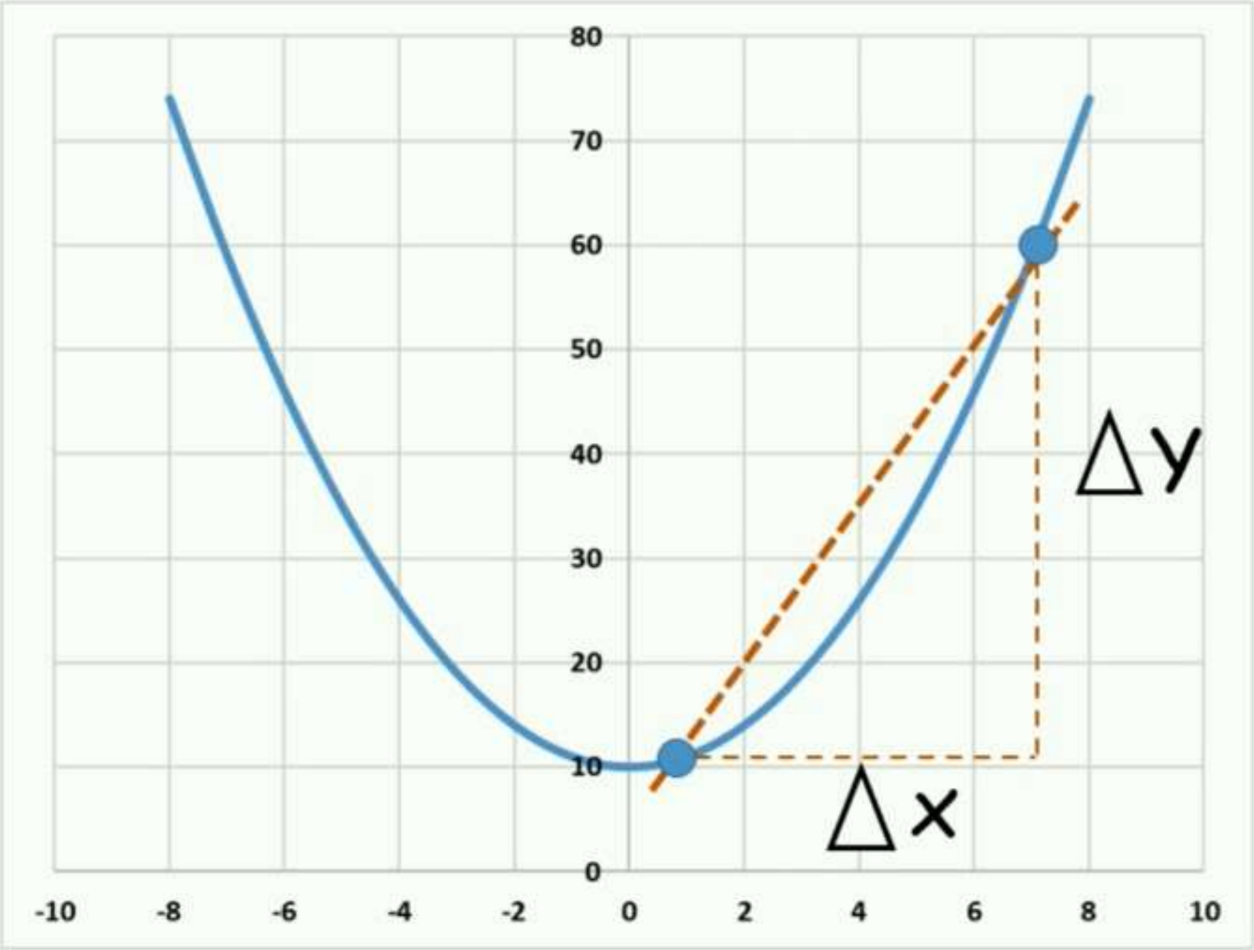
Rate of Change

$$\frac{\Delta \text{ Distance}}{\Delta \text{ Time}}$$

$= \quad$ 2KM/minute



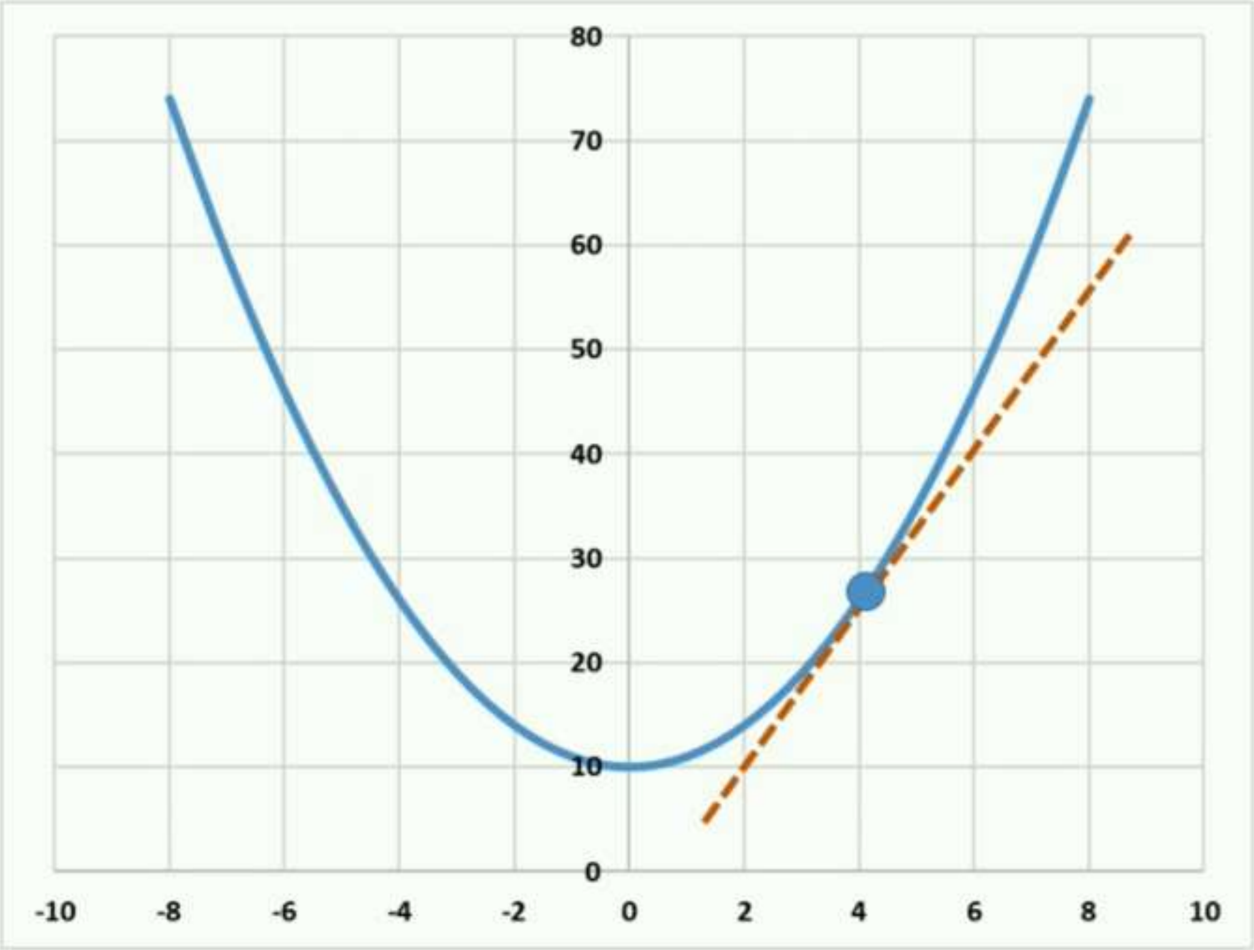
Slope between two points



Average Slope

$$= \frac{\Delta y}{\Delta x}$$

Slope at "The Point" or Gradient

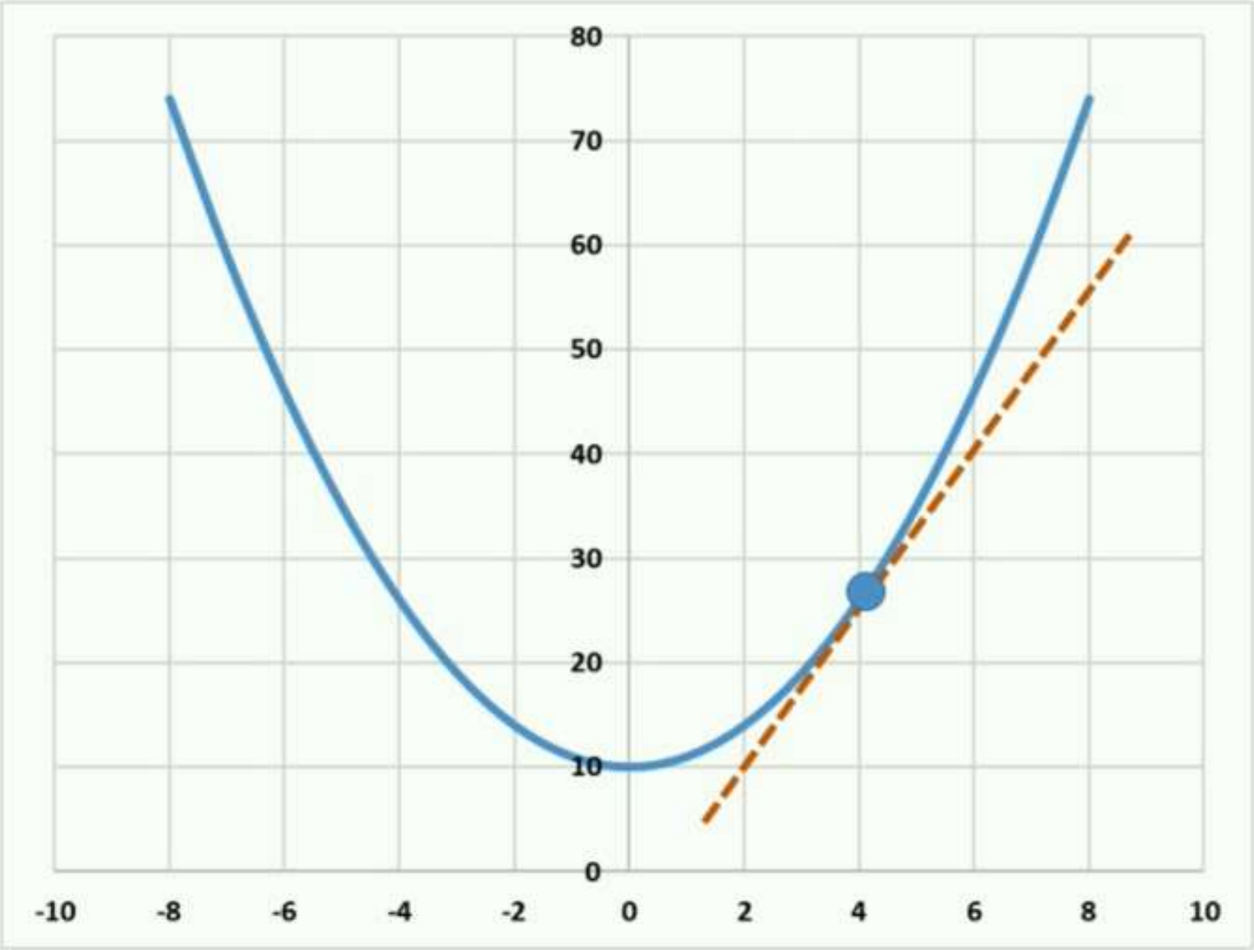


$\Delta y = 0$

$\Delta X = 0$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \text{undefined}$$

Slope at "The Point" or Gradient

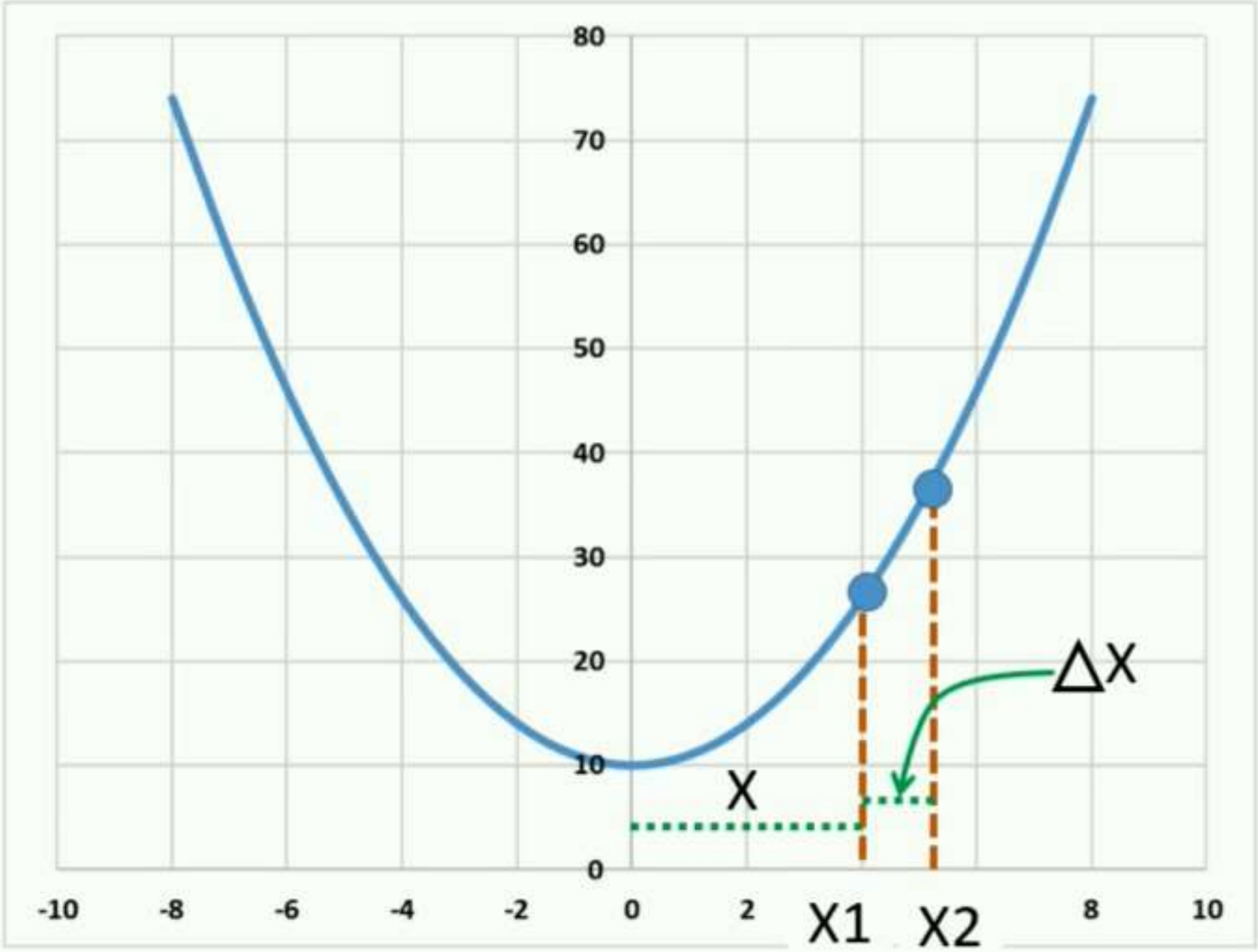


$\Delta y = 0$

$\Delta X = 0$

$Y = f(x) = x^2 + 10$

Slope at the point – Rate of Change



$$Y = f(x) = x^2 + 10$$

$$\Delta Y = y_2 - y_1 \qquad \Delta X = x_2 - x_1$$

$$x_1 = x; \qquad x_2 = x + \Delta x$$

$$Y_2 = f(x_2) = f(x + \Delta x)$$

$$= (x + \Delta x)^2 + 10$$

Slope at the point – Rate of Change

$$Y_2 = f(x_2) = f(x + \Delta x)$$

$$= (x + \Delta x)^2 + 10$$

$$= (x^2 + 2x^*\Delta x + \Delta x^2) + 10$$

Slope at the point – Rate of Change

$$Y_2 = f(x_2) = f(x + \Delta x)$$

$$= (x + \Delta x)^2 + 10$$

$$= (x^2 + 2x^*\Delta x + \Delta x^2) + 10$$

$$\Delta Y = y_2 - y_1 = \underbrace{\boxed{(x^2 + 2x^*\Delta x + \Delta x^2) + 10}}_{y_2} - \underbrace{\boxed{(x^2 + 10)}}_{y_1}$$

Slope at the point – Rate of Change

$$Y_2 = f(x_2) = f(x + \Delta x)$$

$$= (x + \Delta x)^2 + 10$$

$$= (x^2 + 2x*\Delta x + \Delta x^2) + 10$$

$$\Delta Y = y_2 - y_1 = \underbrace{\boxed{(\cancel{x^2} + 2x*\Delta x + \Delta x^2) + \cancel{10}}}_{} - \boxed{(\cancel{x^2} + \cancel{10})}$$

with y_2 above the first box and y_1 above the second box.

Slope at the point – Rate of Change

$$\Delta Y = y_2 - y_1 = \overbrace{\boxed{(\cancel{x^2} + 2x^*\Delta x + \Delta x^2) + \cancel{10}}}^{y_2} \; - \; \overbrace{\boxed{(\cancel{x^2} + \cancel{10})}}^{y_1}$$

$$\Delta Y = 2x^*\Delta x + \Delta x^2$$

$$= \Delta x \;(2x + \Delta x)$$

Distributive Property
3 (X + 2) = 12
3X + 6 = 12

Slope at the point – Rate of Change

$$\Delta Y = \Delta x \ (2x + \Delta x)$$

$$\text{Average Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta x \ (2x + \Delta x)}{\Delta x}$$

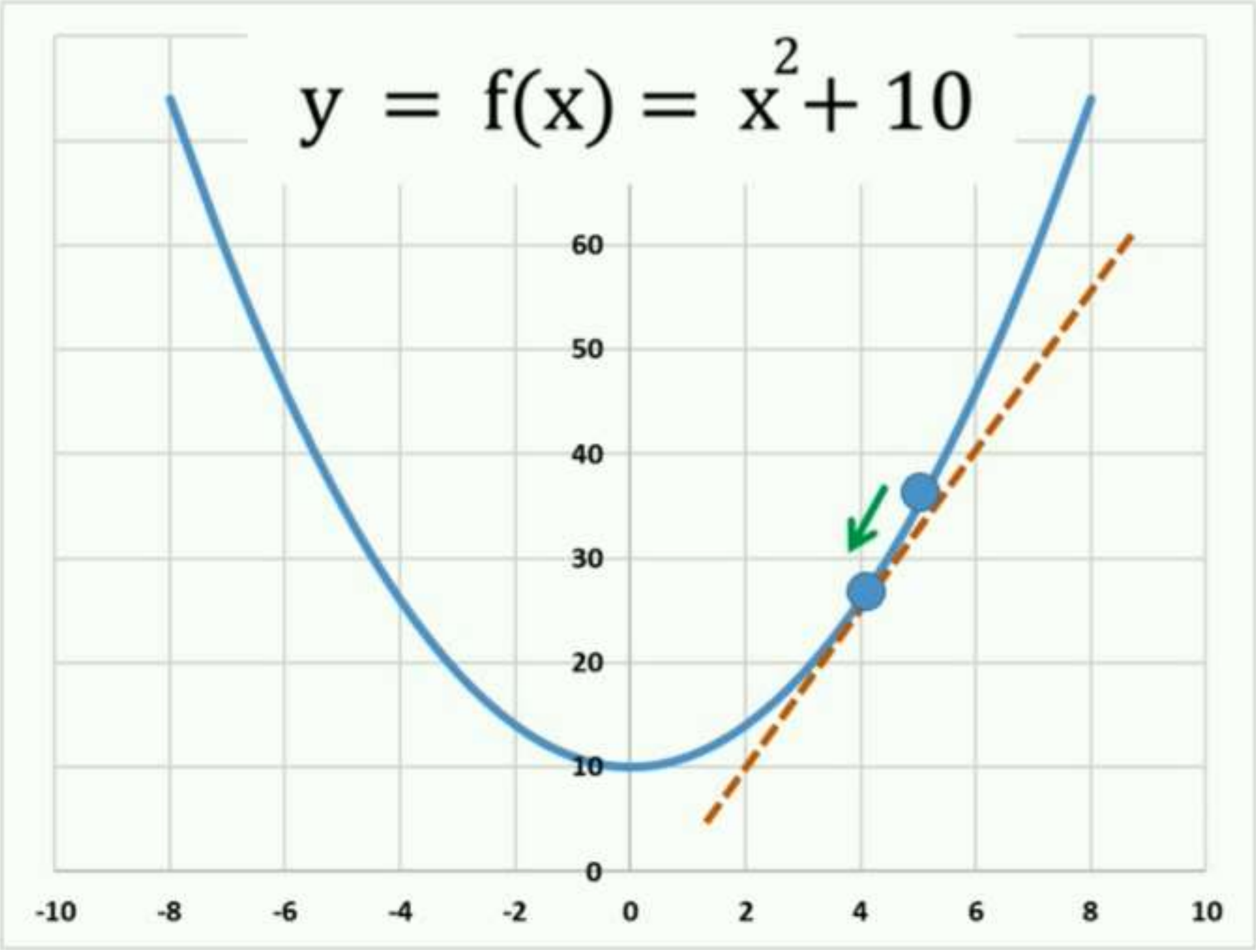
Slope at the point – Rate of Change

$$\Delta Y = \Delta x \; (2x + \Delta x)$$

$$\text{Average Slope} = \frac{\Delta y}{\Delta x} = \frac{\cancel{\Delta x} \; (2x + \Delta x)}{\cancel{\Delta x}}$$

$$\text{Average Slope} = 2x + \Delta x$$

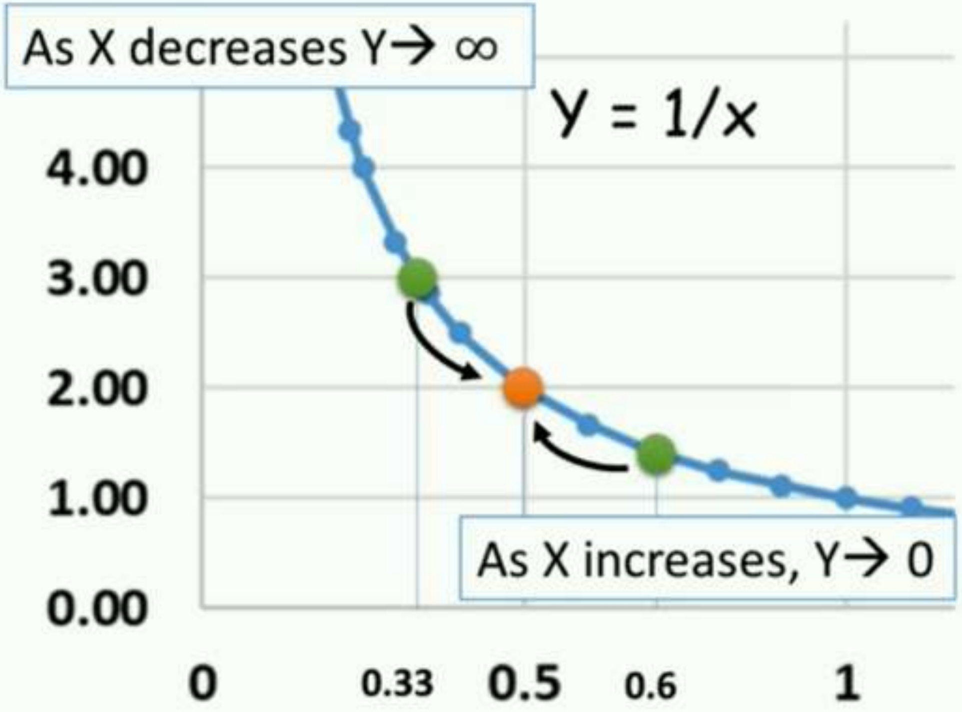
Derivative



$$\text{Slope} = 2x + \Delta x$$

$$\Delta x \rightarrow 0$$

Limits



$$\lim_{x \to 0.5} \frac{1}{x} = 2$$

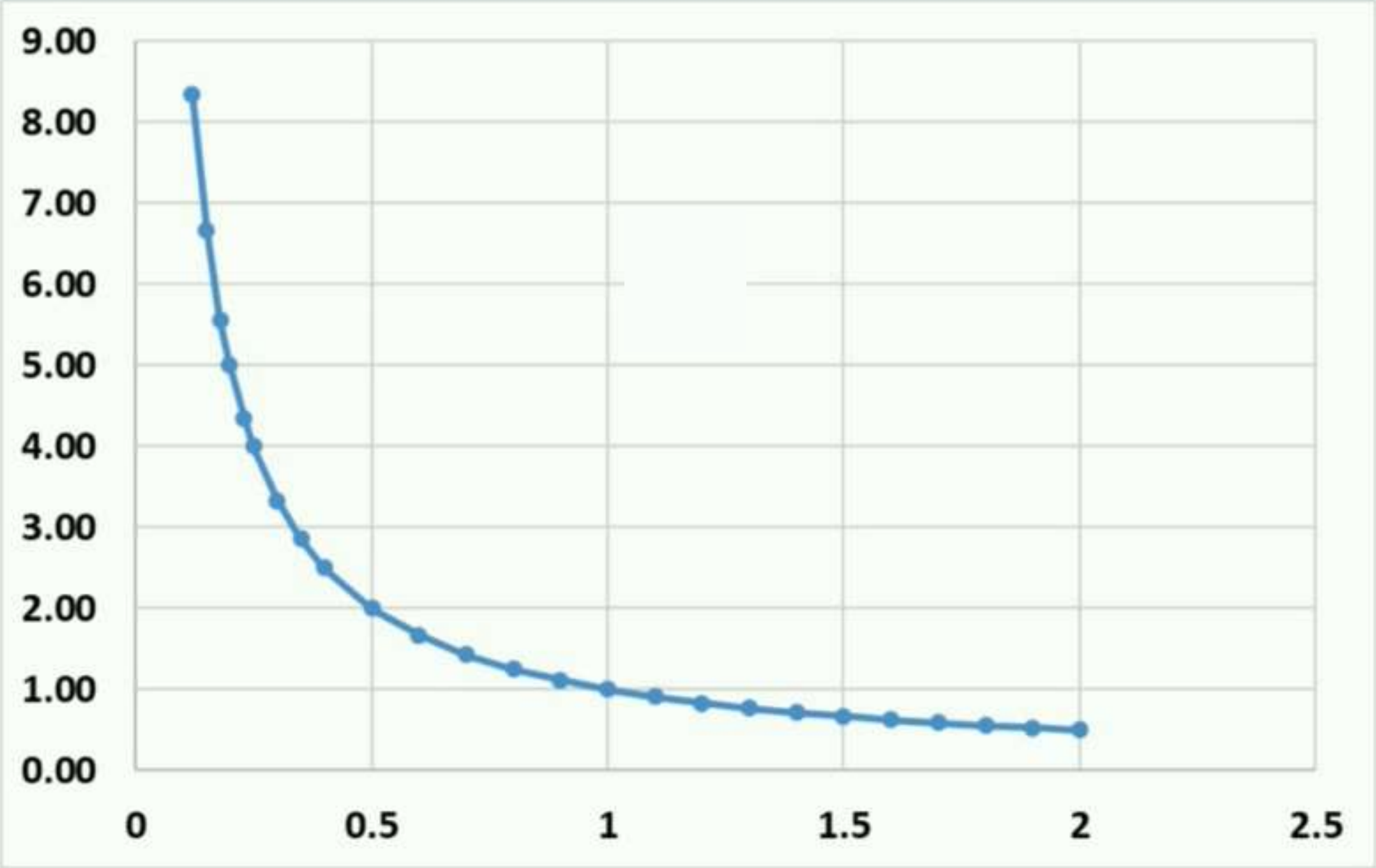
$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to 0} \frac{1}{x} = \infty$$

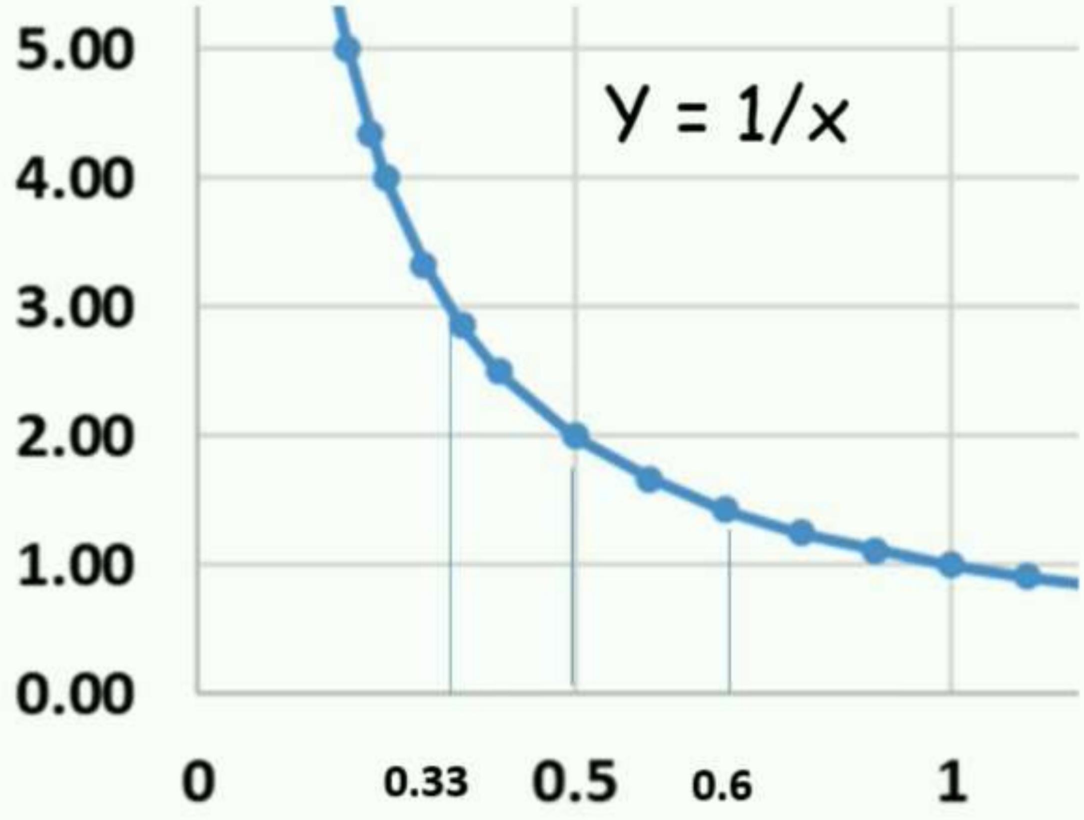
Limits

$Y = 1/x$

$X \neq 0$

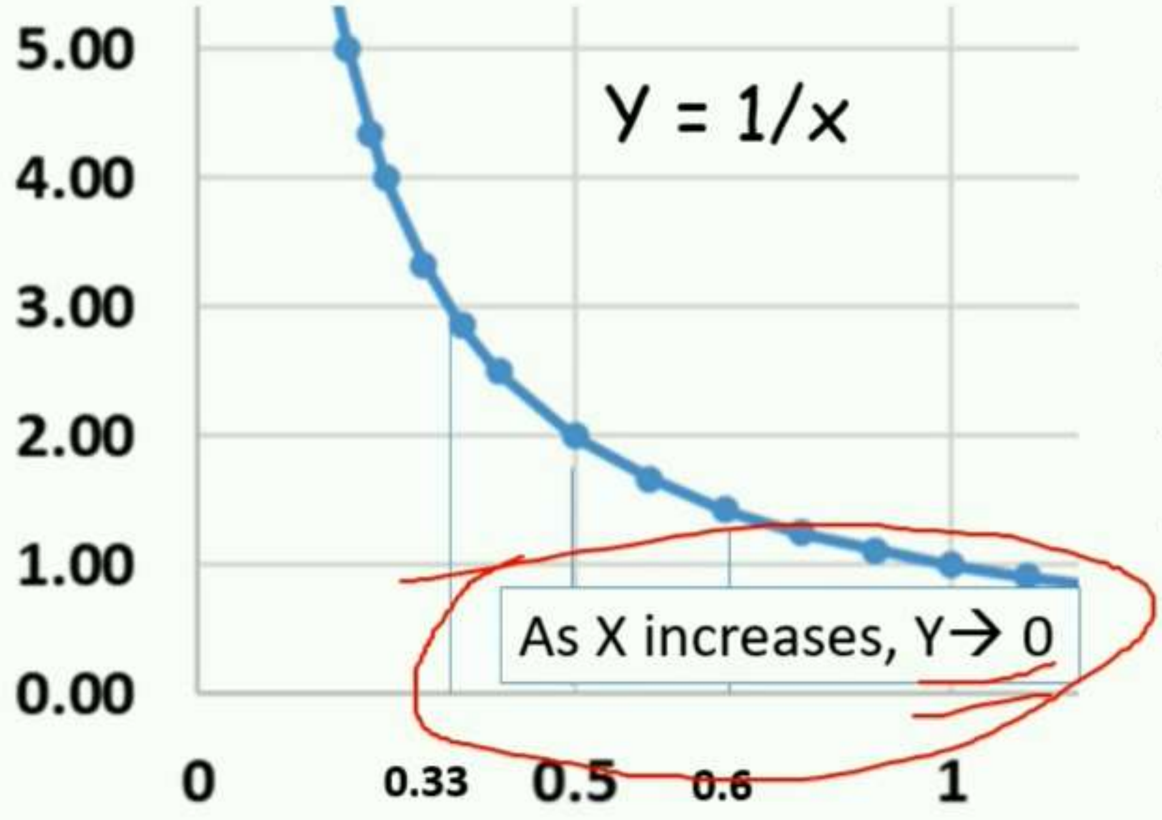


Limits



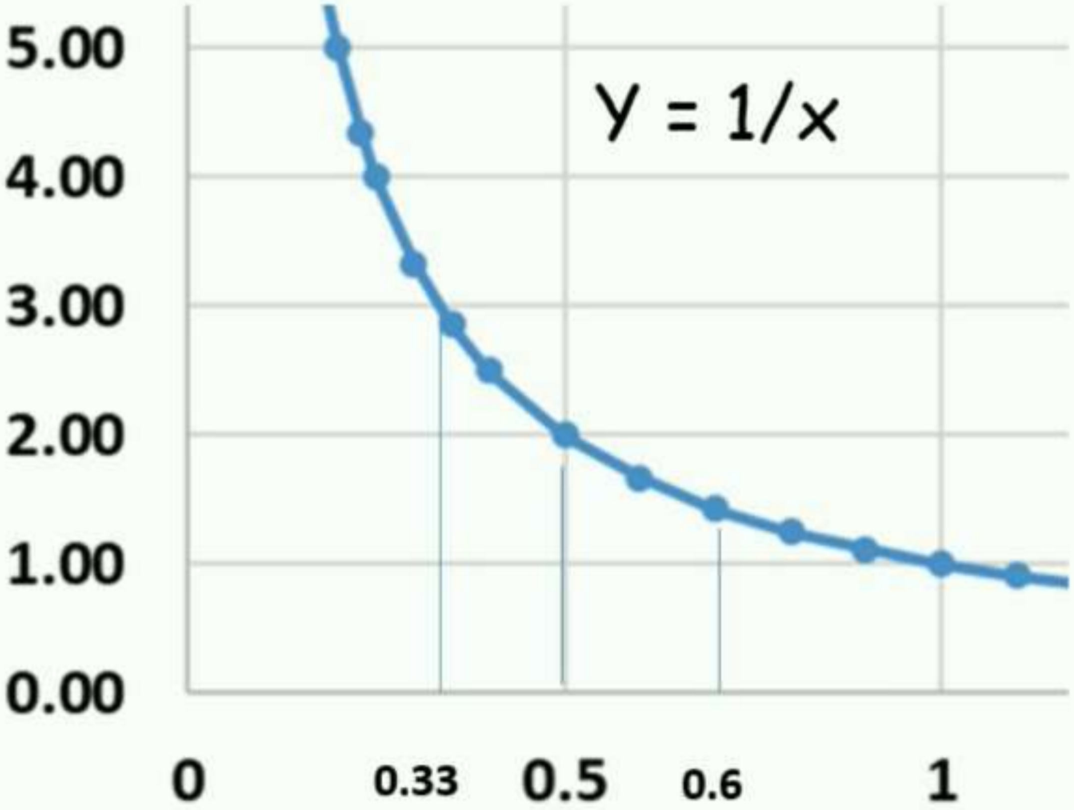
X	y
1	1
10	0.1
100	0.01
1000	0.001
10,000	0.0001
1,000,000	0.000001

Limits



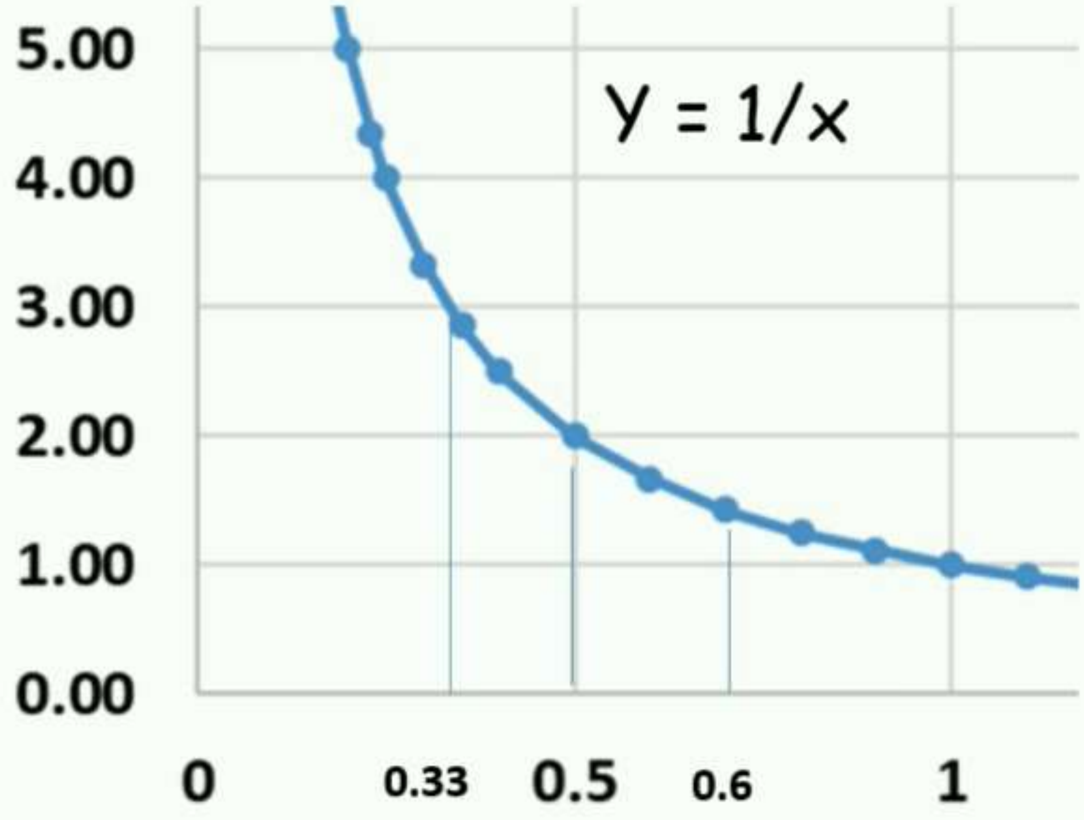
X	Y
1	1
10	0.1
100	0.01
1000	0.001
10,000	0.0001
1,000,000	0.000001

Limits



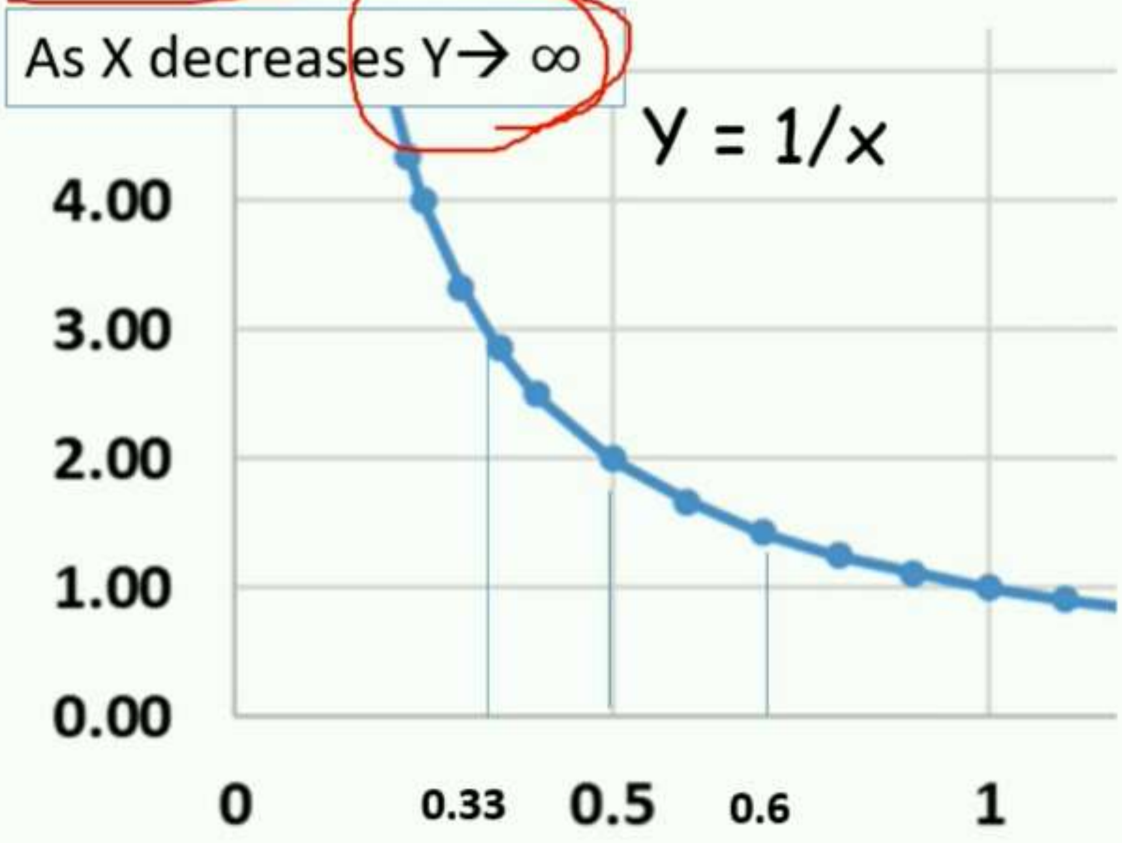
X	y
1	1
0.5	2
0.01	100

Limits



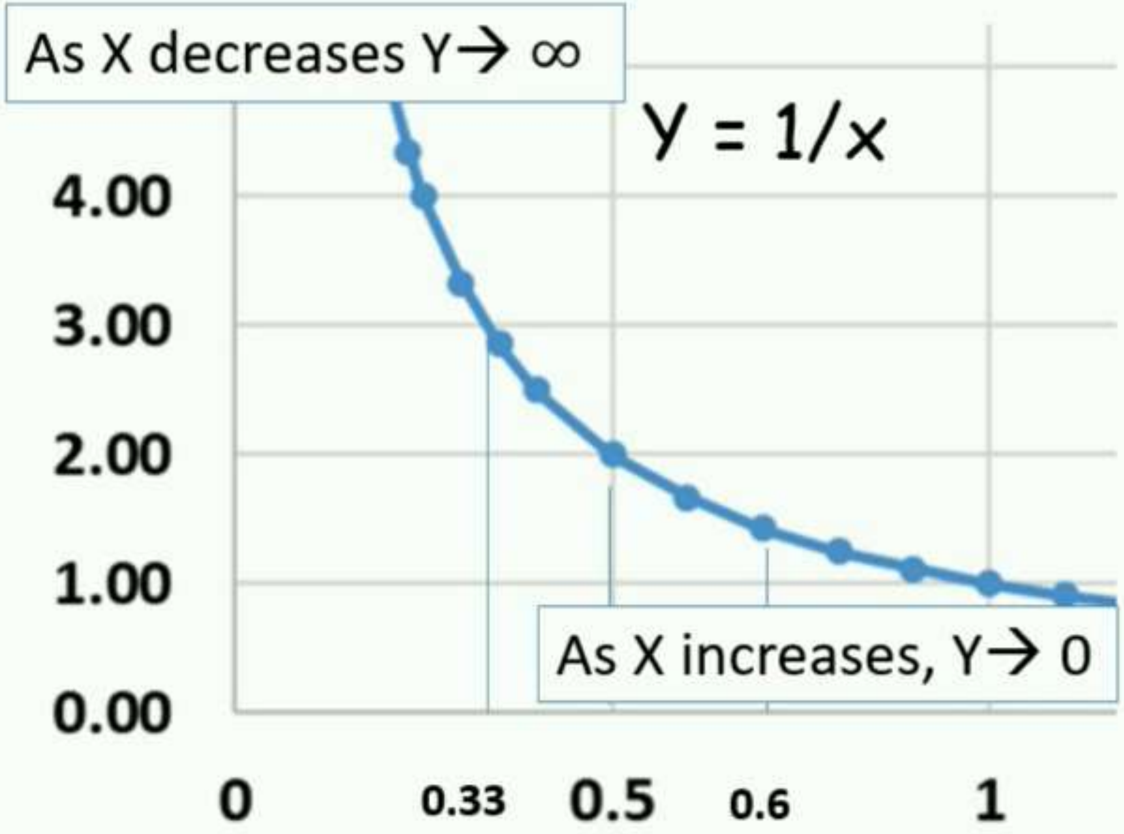
X	Y
1	1
0.5	2
0.01	100
0.001	1000
0.0001	10,000
0.000001	1,000,000

Limits



X	Y
1	1
0.5	2
0.01	100
0.001	1000
0.0001	10,000
0.000001	1,000,000

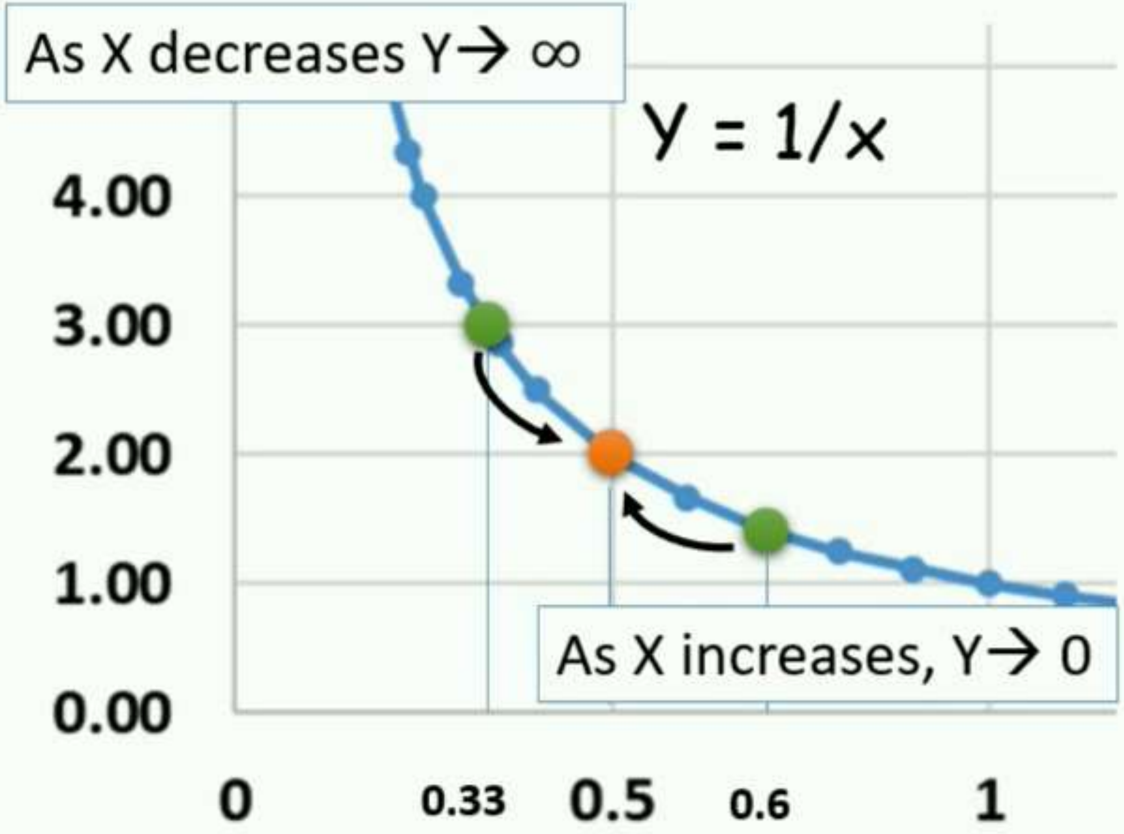
Limits



X	y
1	1
10	0.1
100	0.01
1000	0.001
10,000	0.0001
1,000,000	0.000001

X	y
1	1
0.5	2
0.01	100
0.001	1000
0.0001	10,000
0.000001	1,000,000

Limits

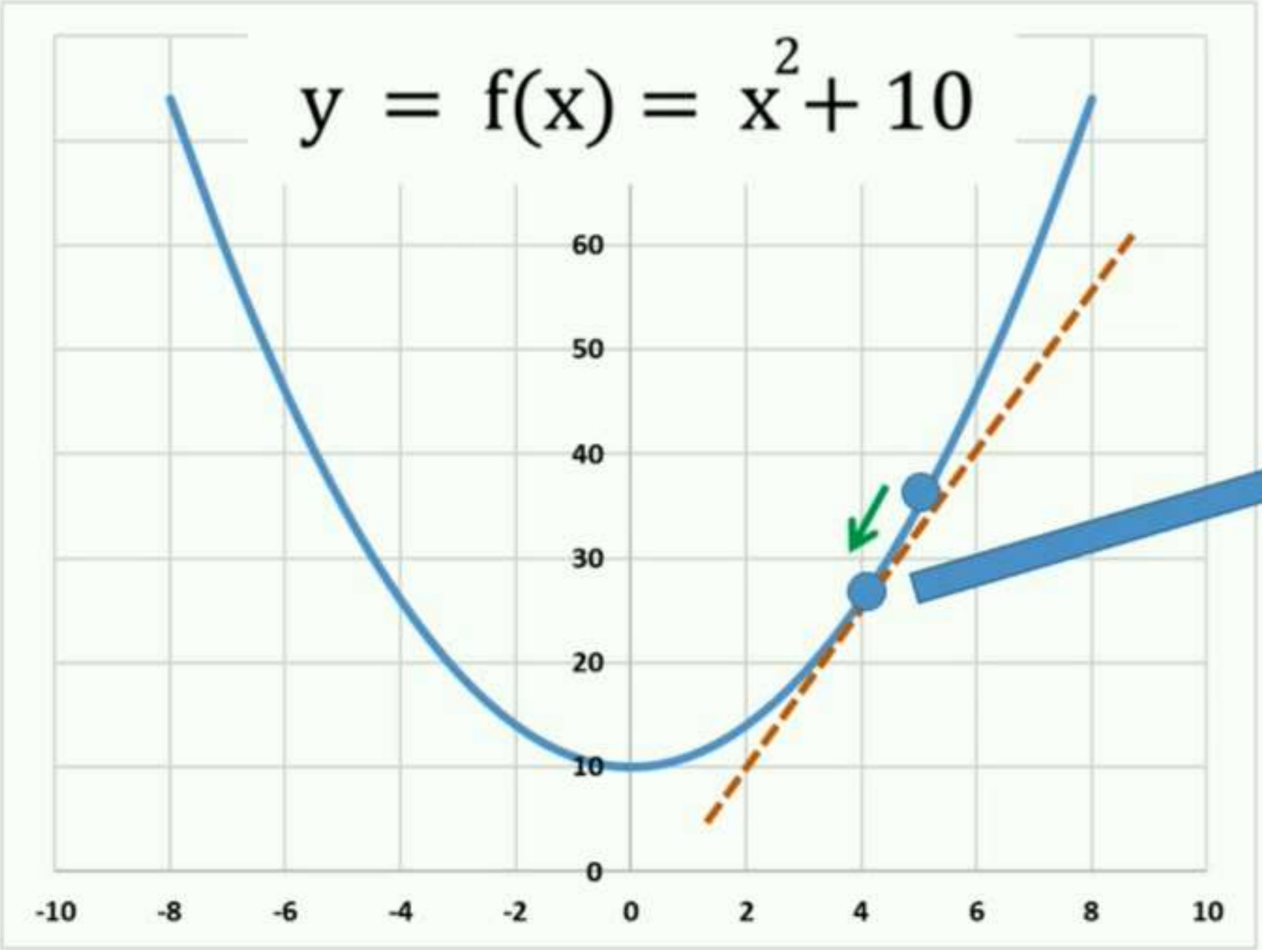


$$\lim_{x \to 0.5} \frac{1}{x} = 2$$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to 0} \frac{1}{x} = \infty$$

Derivative

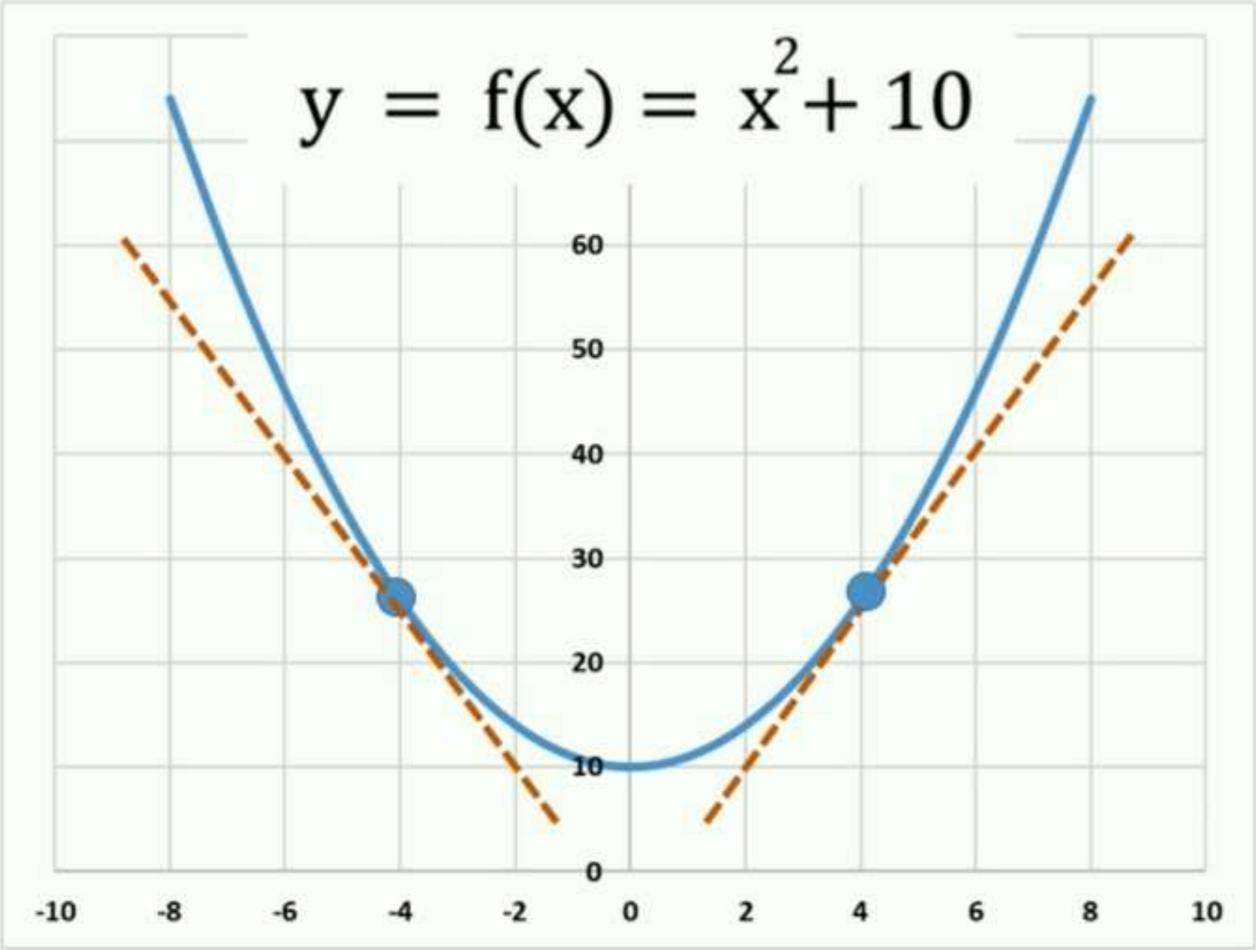


$$\text{Slope} = 2x + \Delta x$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} (2x + \underset{0}{\Delta x})$$

$$\frac{dy}{dx} = 2x$$

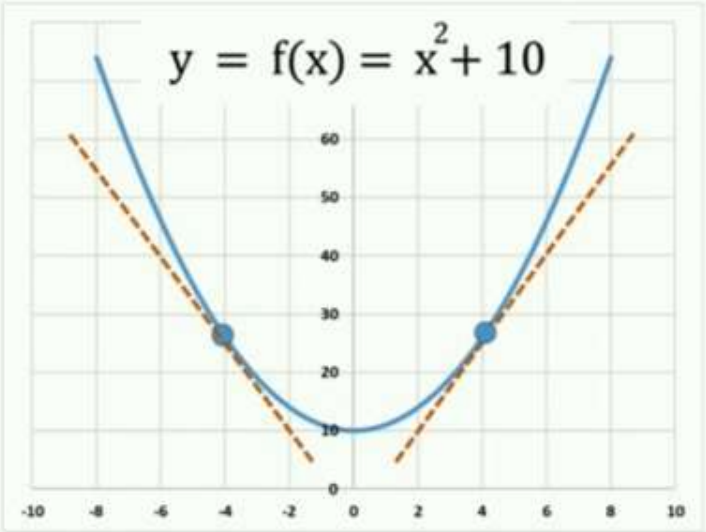
Derivative



$$\frac{dy}{dx} = 2x$$

1. $x = 4; \quad \text{slope} = 8$
2. $x = -4; \quad \text{slope} = -8$

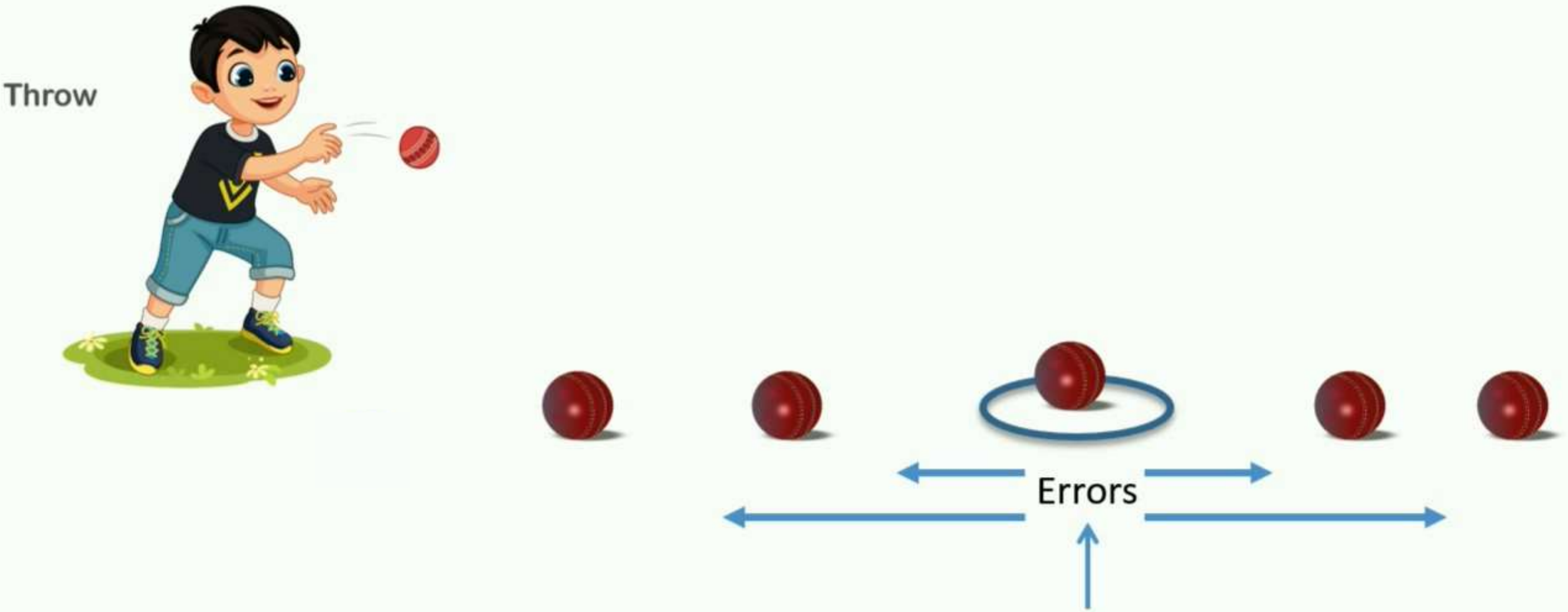
Why to know the “Slope at the Point”?



$$\frac{dy}{dx} = 2x$$

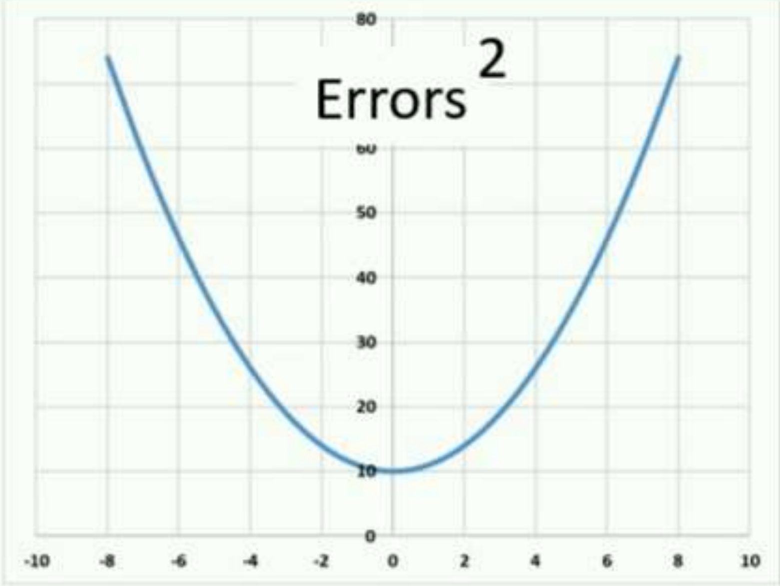
1. $x = 4;$ slope $= 8$
2. $x = -4;$ slope $= -8$

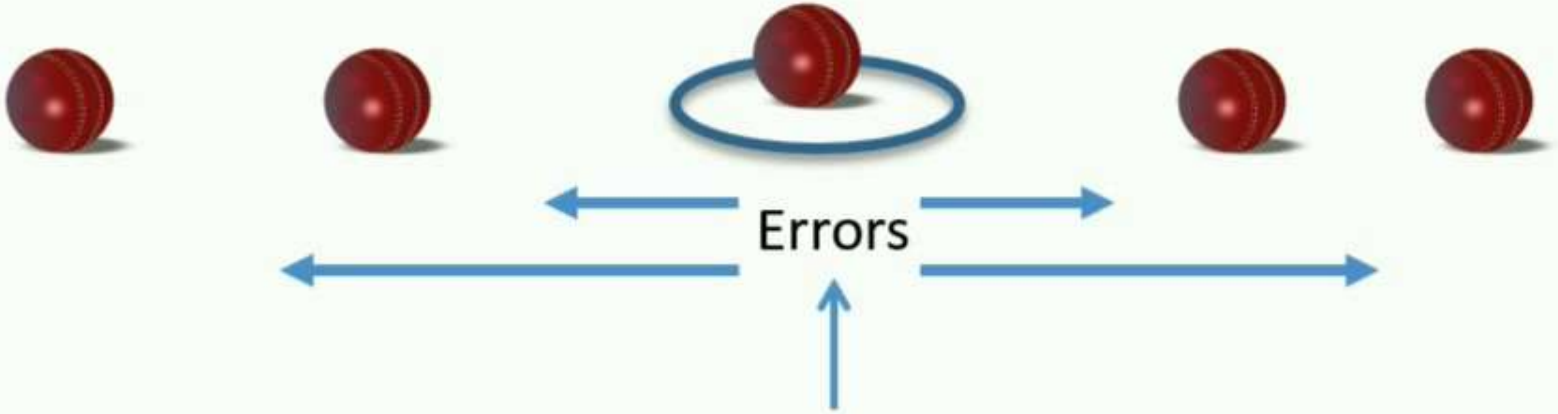
Why to know the “Slope at the Point”?



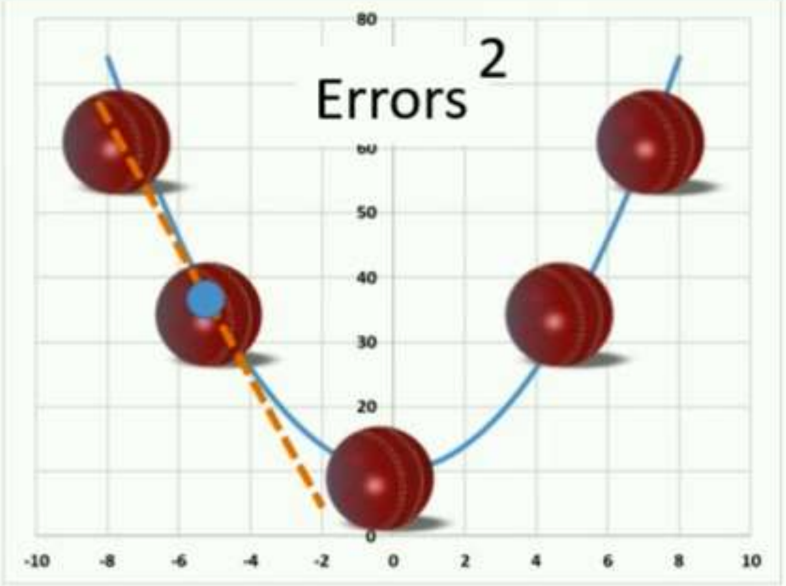
Why to know the "Slope at the Point"?

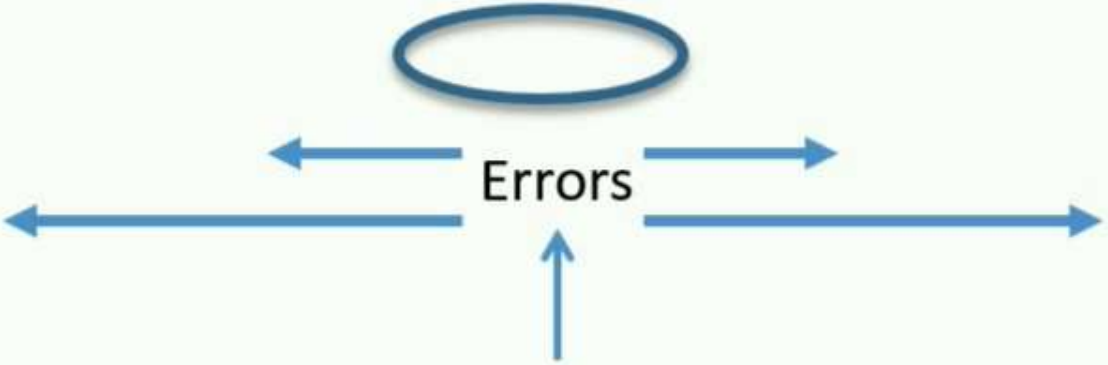
Throw





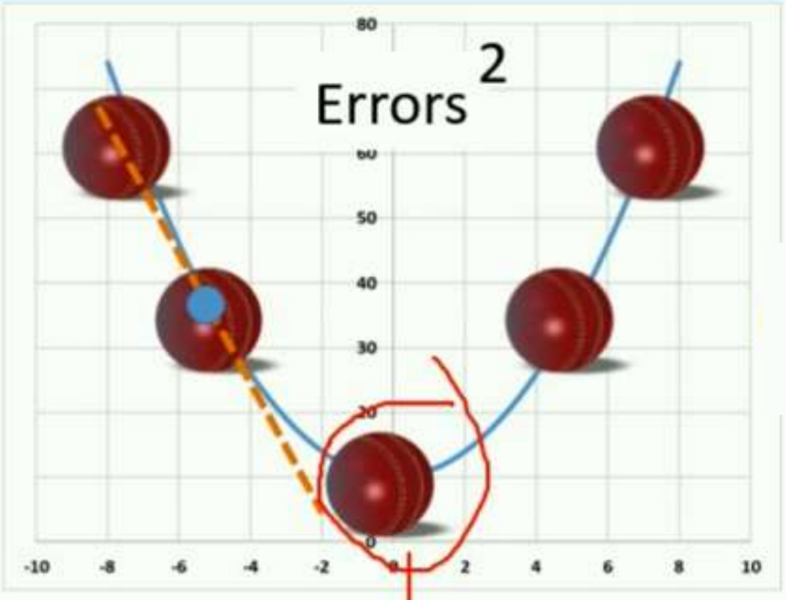
Why to know the "Slope at the Point"?



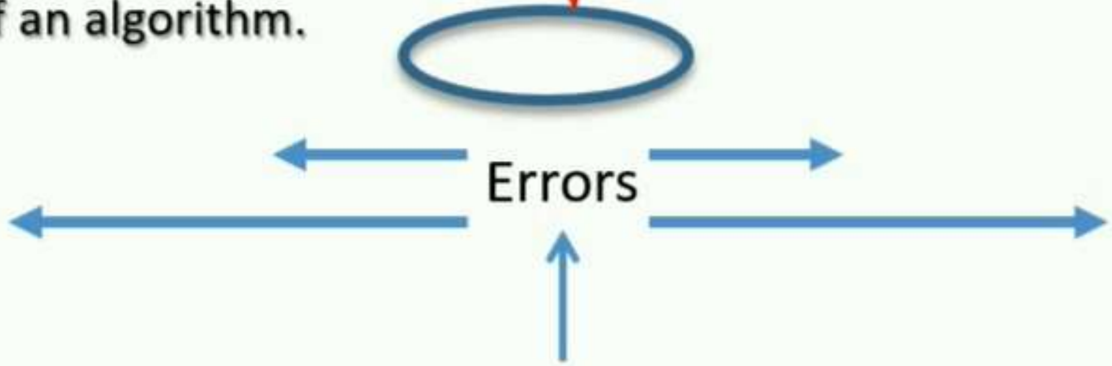


Why to know the "Slope at the Point"?

Throw



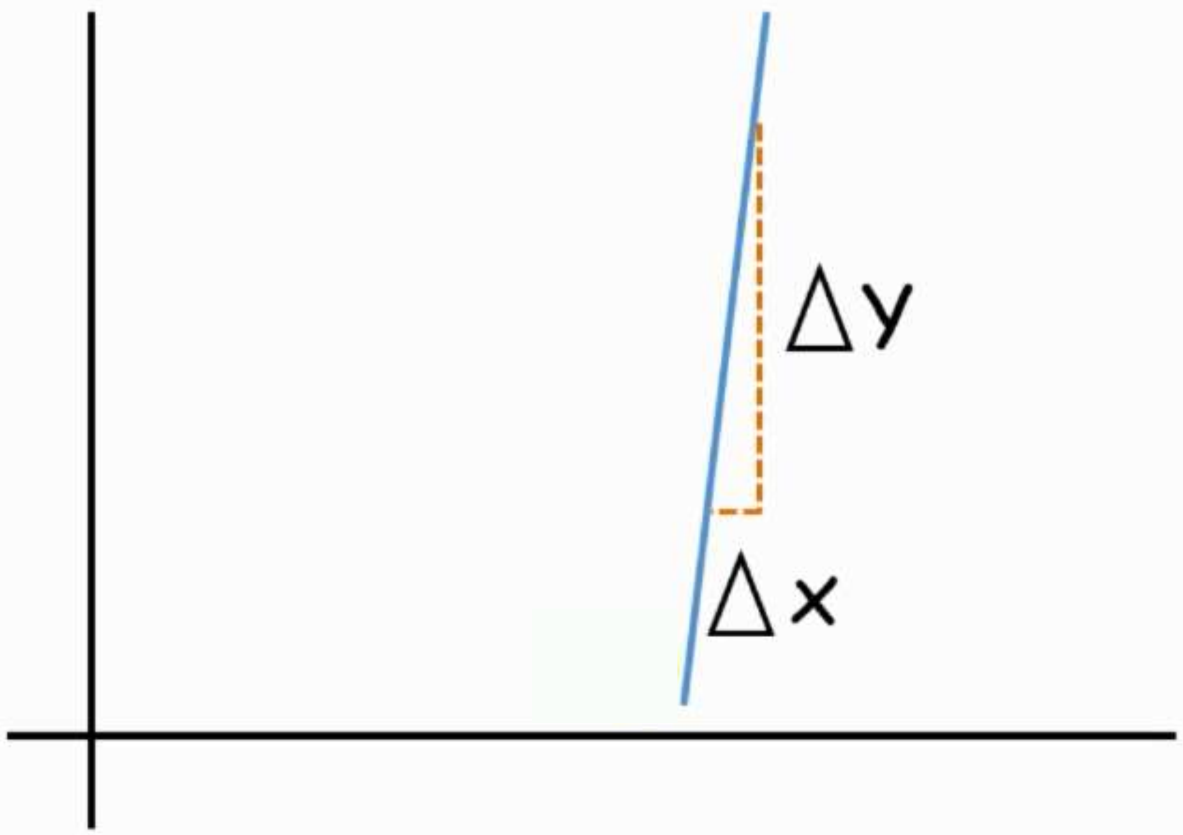
Optimization or Minimization of errors to get best accuracy of an algorithm.



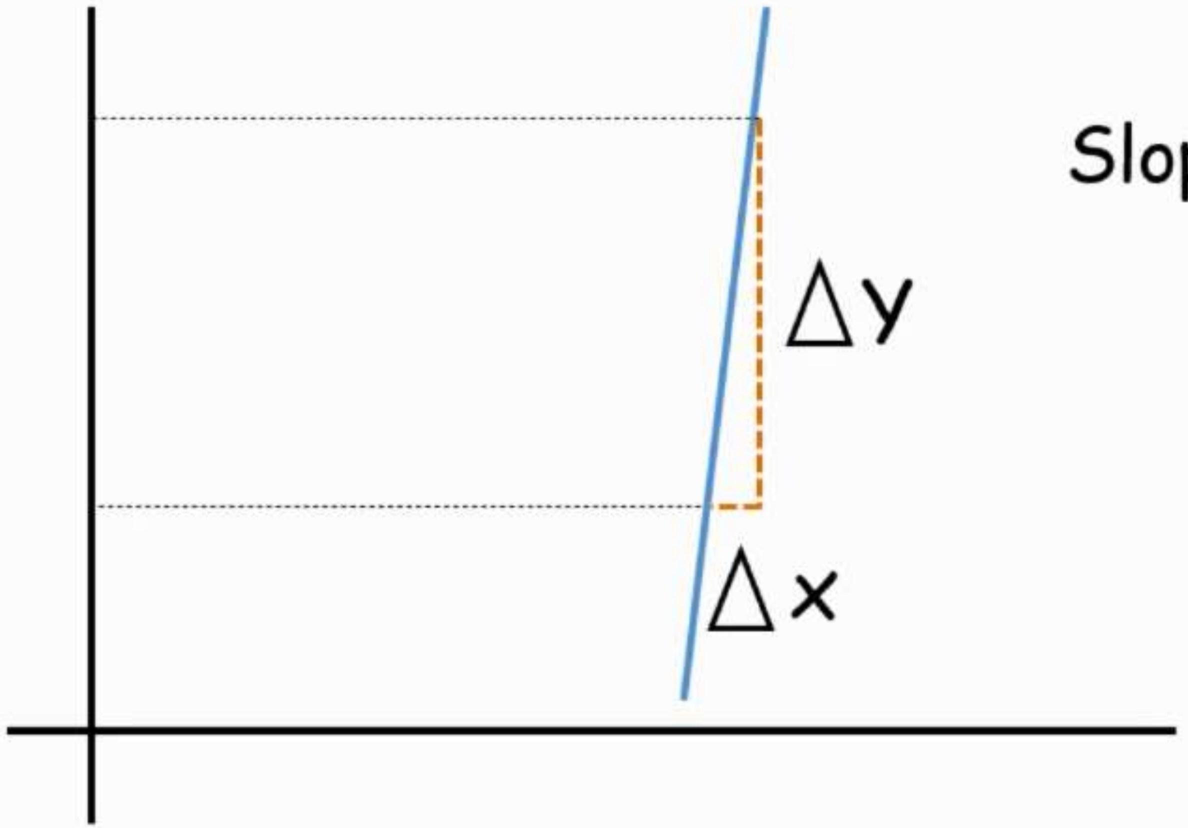
Derivative rules

- Derivative of a vertical line
- Derivative of a horizontal line
- Differentiability for various functions
- Power rule of derivative

Derivative Rules



Derivative Rules



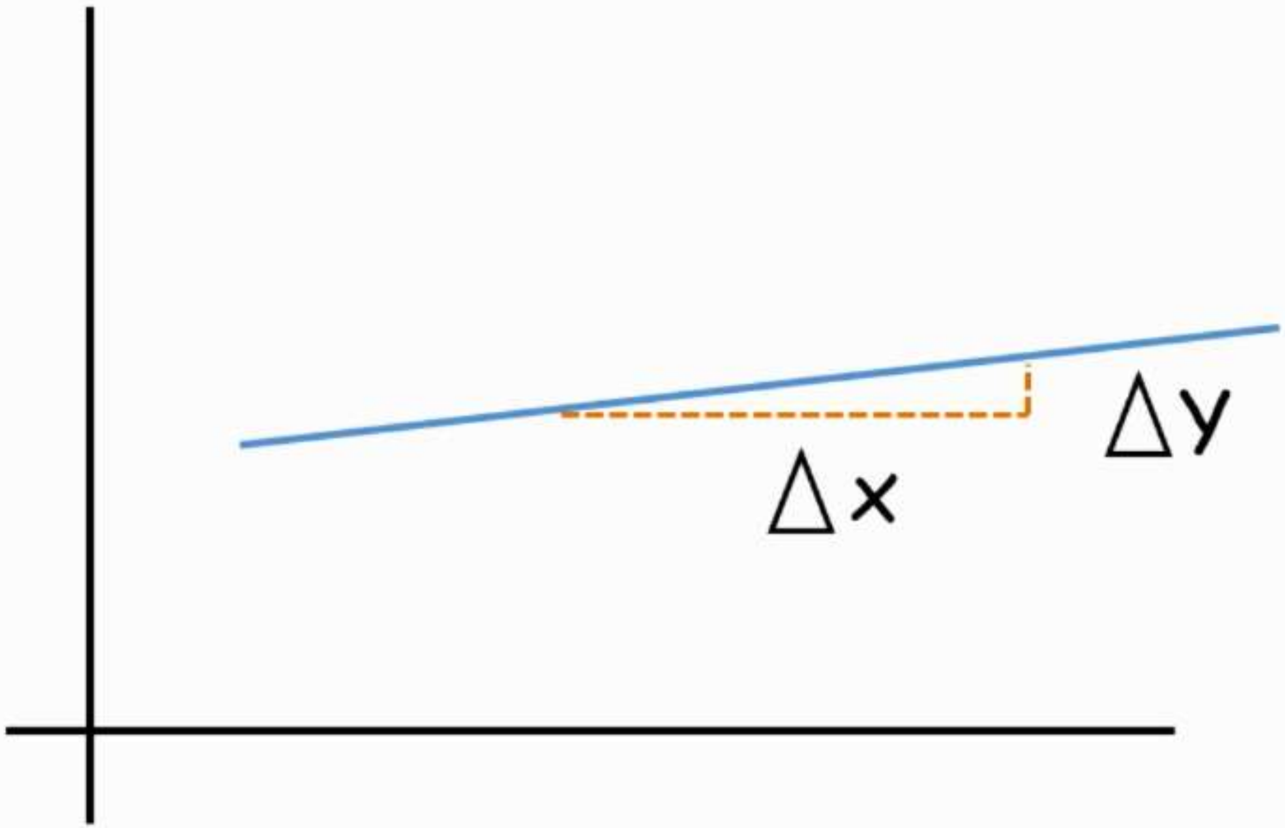
$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

Derivative Rules

$$\Delta x = 0$$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \infty \text{ or Undefined}$$

Derivative Rules

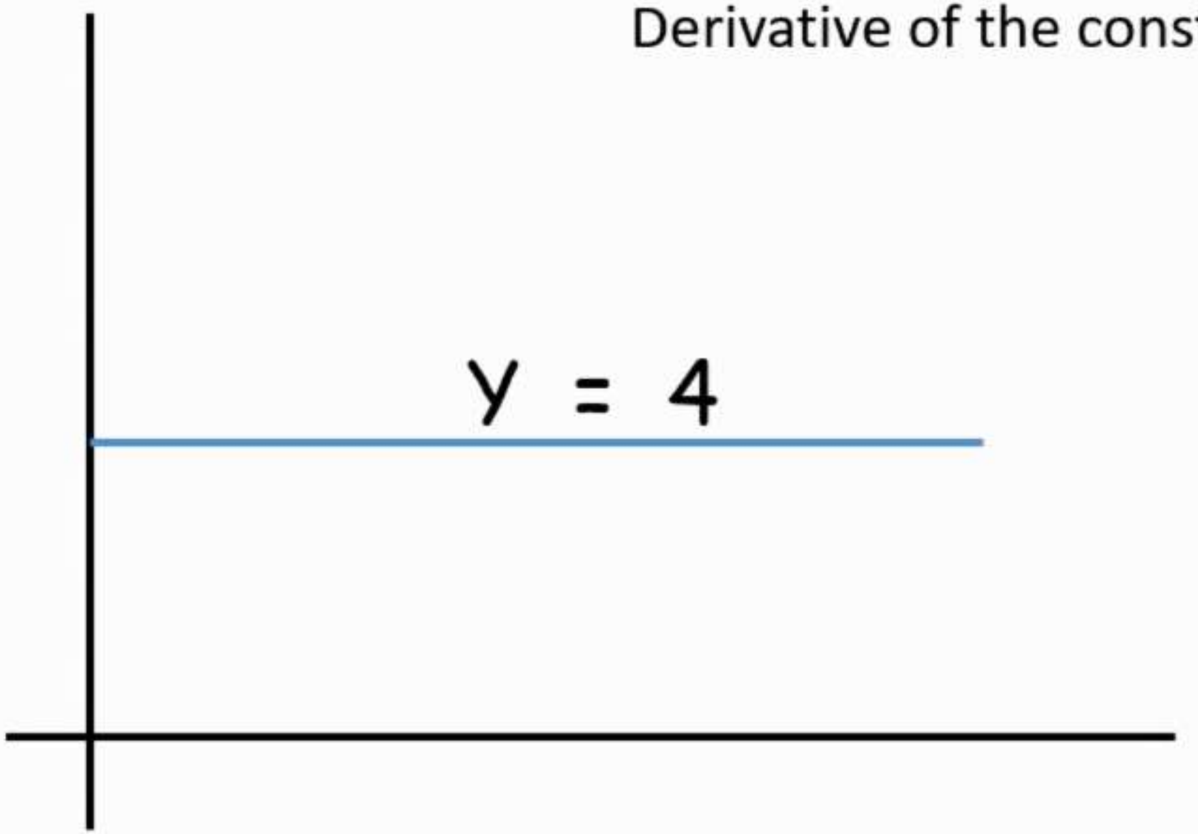


$$\text{Slope} = \frac{\Delta y}{\Delta x} \downarrow$$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = 0$$

Derivative Rules – Constant

Derivative of the constant is ZERO.



$$\Delta y = 0$$

$$\frac{dy}{dx} = 0$$

$$\frac{d(4)}{dx} = 0$$

Power Rule of Derivative

$$y = f(x) = x^2 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 2x$$

$$y = f(x) = x^3 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 3x^2$$

Power Rule of Derivative

$$y = f(x) = x^2 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = \boxed{}$$

$$y = f(x) = x^3 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = \boxed{}$$

Power Rule of Derivative

$$y = f(x) = x^2 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 2x$$

$$y = f(x) = x^3 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 3x^2$$

Power Rule of Derivative

$$y = f(x) = ax^n \quad \Rightarrow \quad \frac{dy}{dx} = a*n\ x^{n-1}$$

Power Rule of Derivative

$$y = f(x) = x^2 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 2x$$

(Original Index – 1)

$$y = f(x) = x^3 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 3x^2$$

Remove constant

(Original Index * Original Coefficient)

Power Rule of Derivative

(Original Index – 1)

$$y = f(x) = x^3 + 10 \Rightarrow \frac{dy}{dx} = 3x^2$$

Remove constant

(Original Index * Original Coefficient)

$$y = f(x) = 2x^3 + 4x^2 - \underline{7x} + 9 \Rightarrow \frac{dy}{dx} = \boxed{}$$

Power Rule of Derivative

$$y = f(x) = x^3 + 10 \Rightarrow \frac{dy}{dx} = 3x^2$$

(Original Index – 1)

Remove constant

(Original Index * Original Coefficient)

$$y = f(x) = 2x^3 + 4x^2 - \underline{7x} + 9 \Rightarrow \frac{dy}{dx} = 6x^2 + 8x^1 - 7x^0$$

Second Order Derivative

$$\frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d^2 y}{dx^2}$$

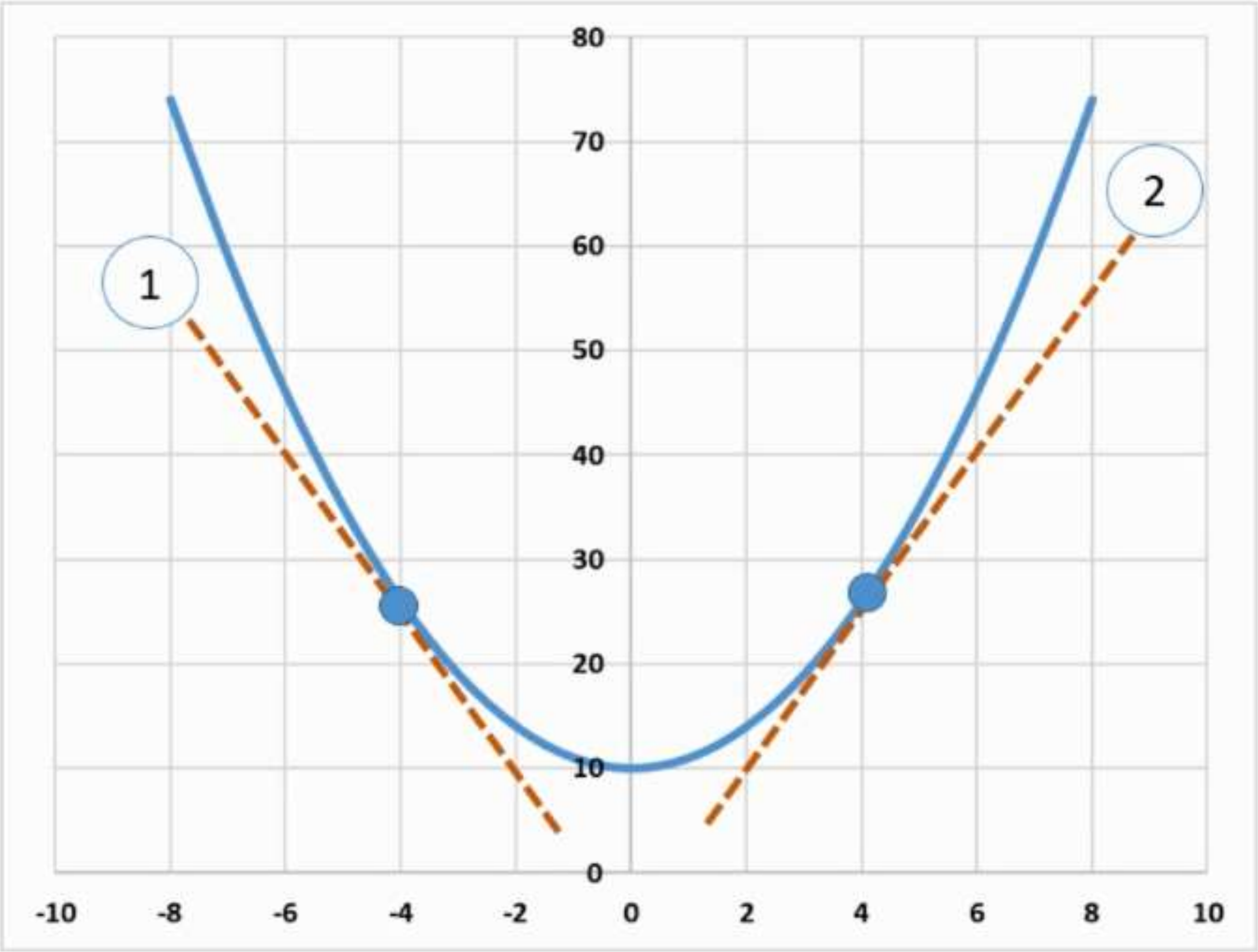
Derivative for directions

$$y = f(x) = x^2 + 10$$

$$\frac{dy}{dx} = 2x$$

1 $\frac{dy}{dx} = -8$

2 $\frac{dy}{dx} = +8$



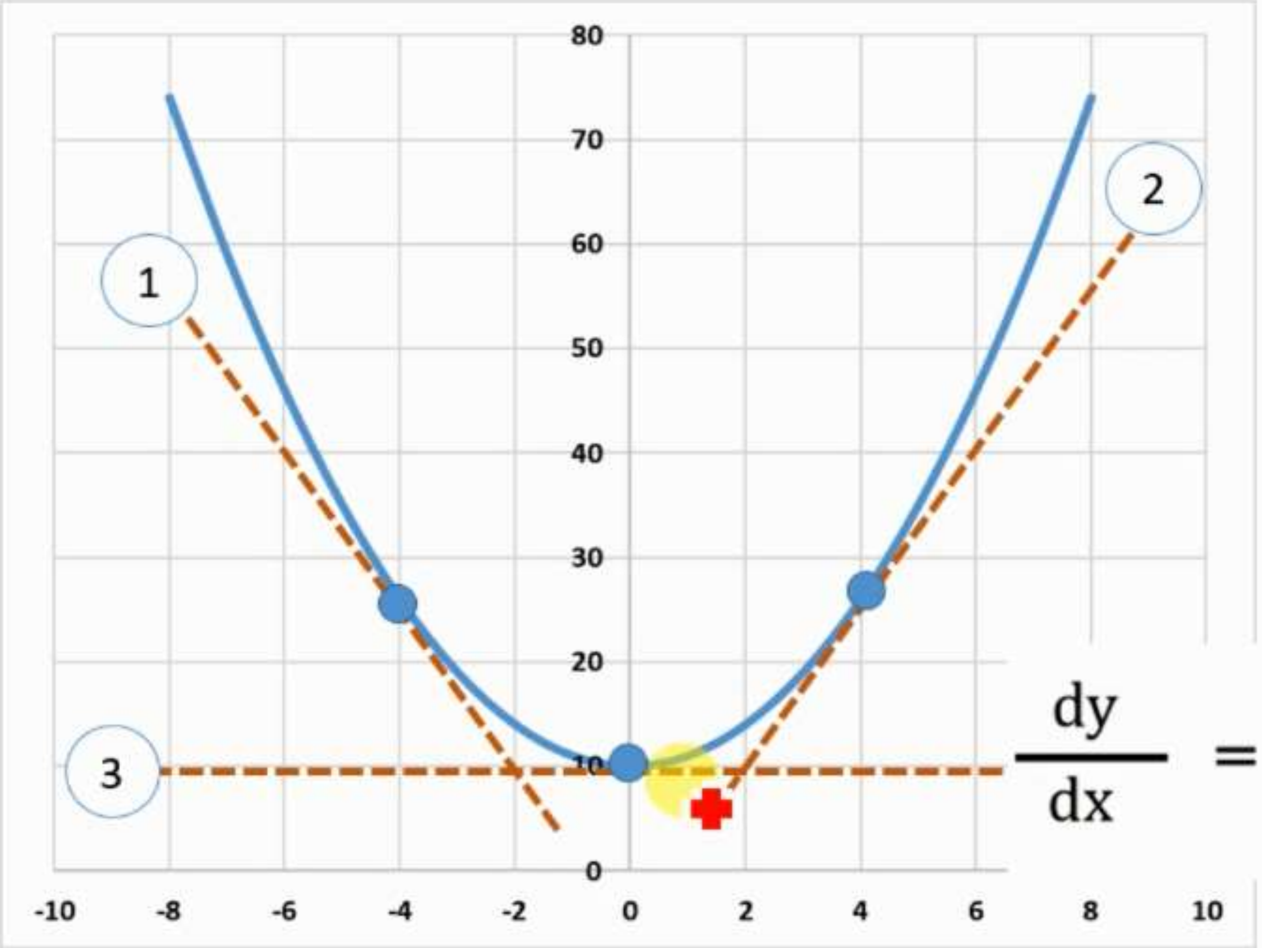
Derivative for directions

$$y = f(x) = x^2 + 10$$

$$\frac{dy}{dx} = 2x$$

1 $\frac{dy}{dx} = -8$

2 $\frac{dy}{dx} = +8$



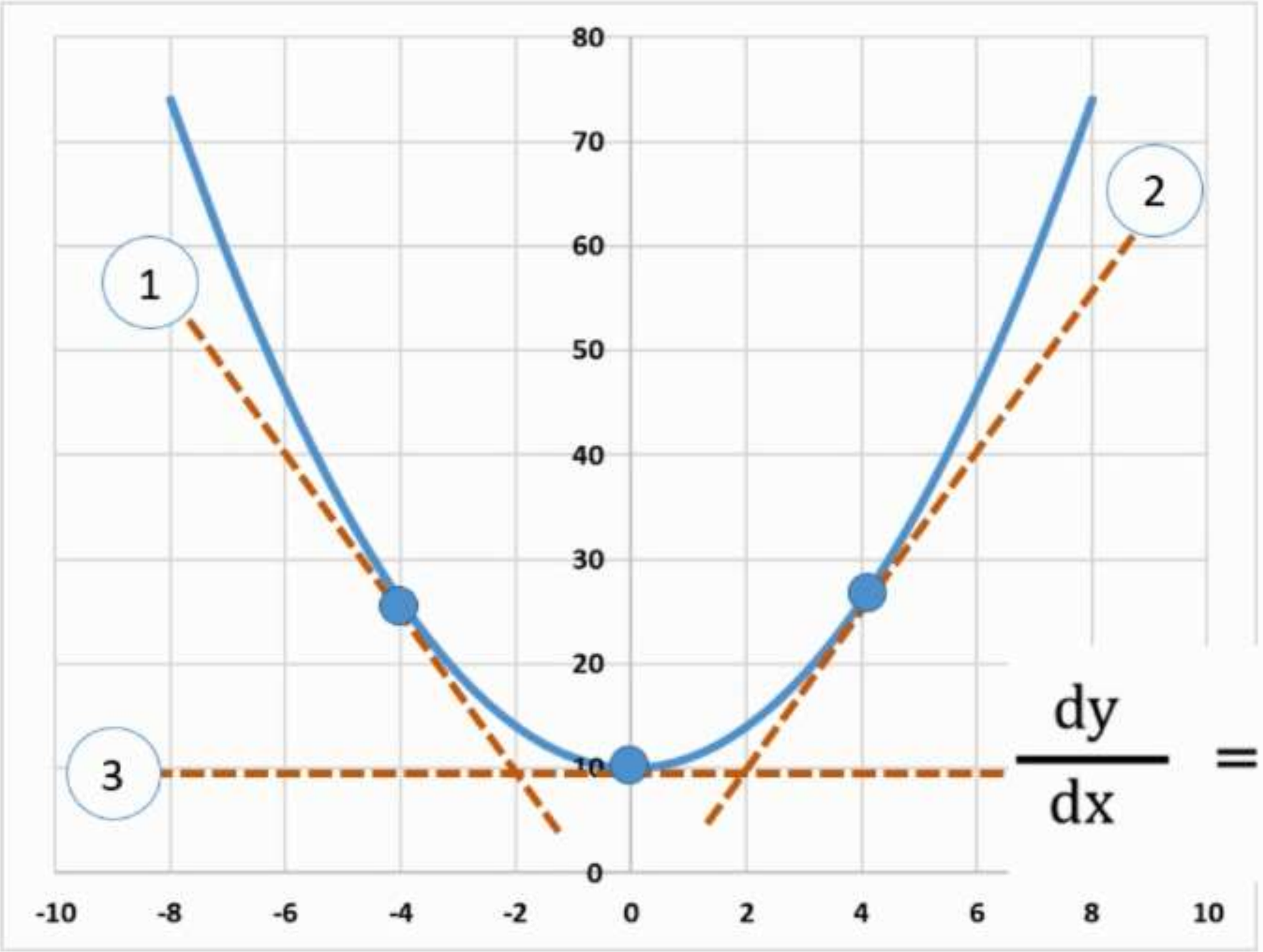
$$\frac{dy}{dx} = 0$$

Derivative Direction

$$y = f(x) = x^2 + 10$$

$$y = f(x) = 0 + 10$$

$$y = f(x) = 10$$



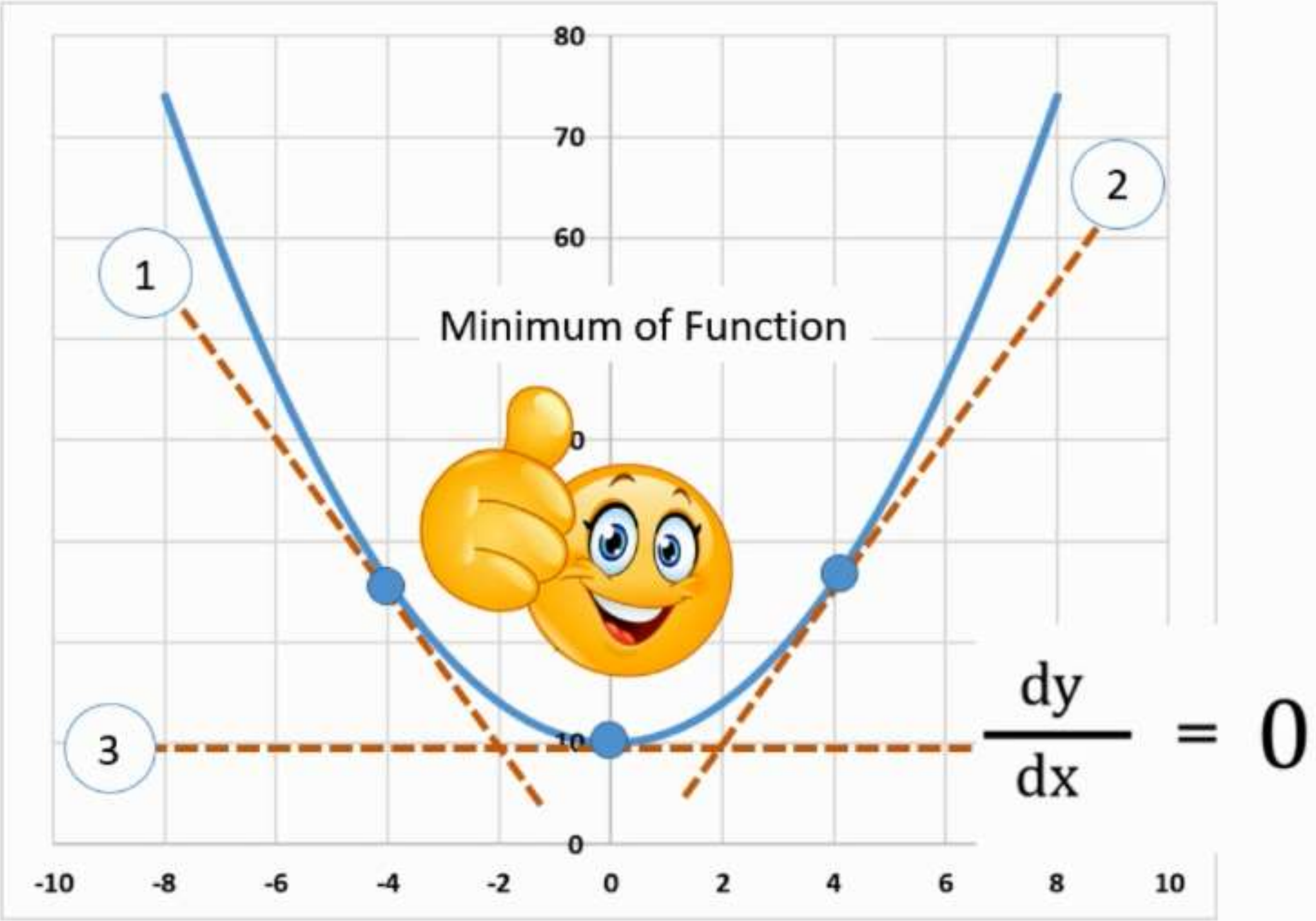
$$\frac{dy}{dx} = 0$$

Derivative Direction

$$y = f(x) = x^2 + 10$$

$$y = f(x) = 0 + 10$$

$$y = f(x) = 10$$

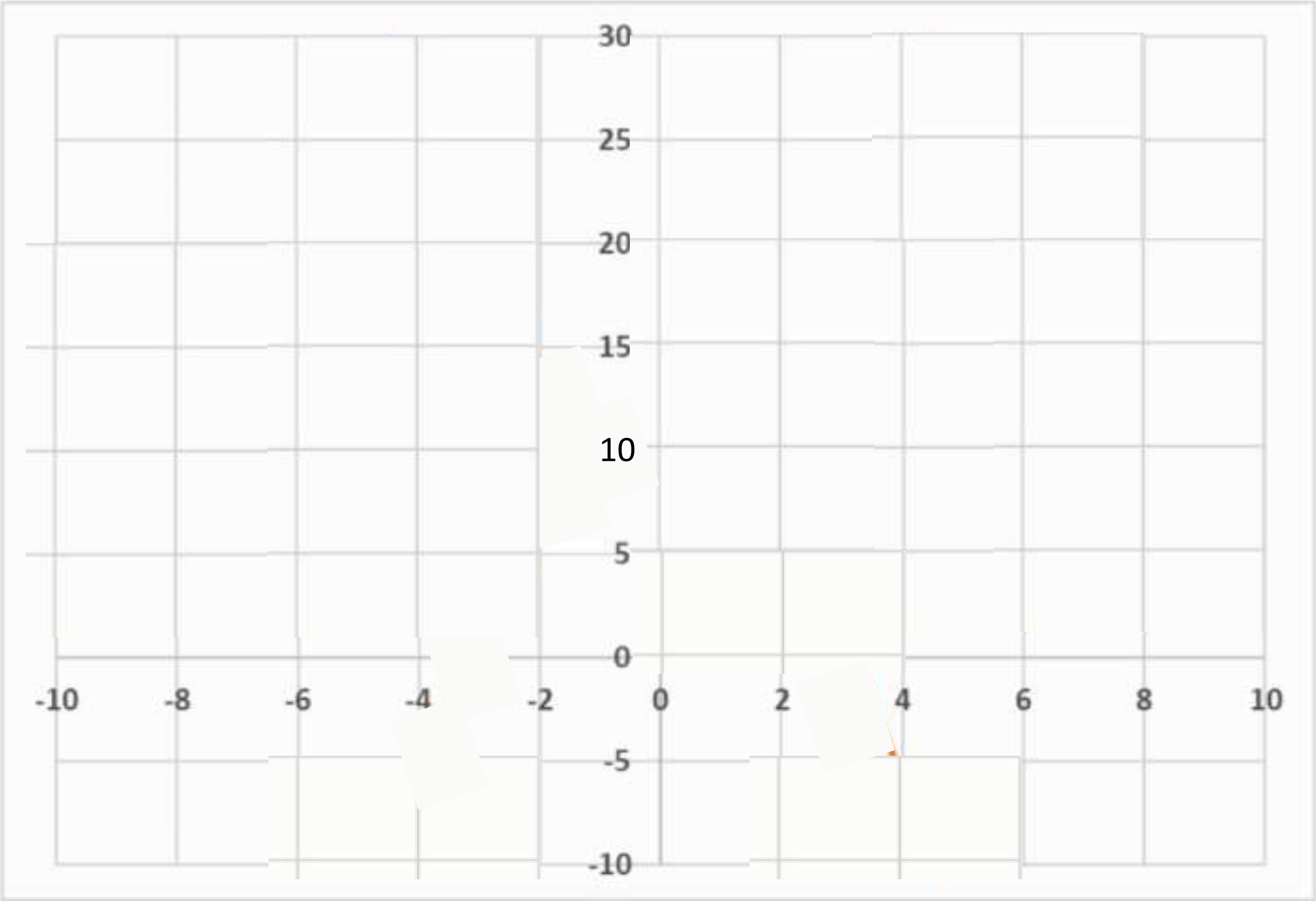


Maxima or Minima?

$$y = f(x) = x^2 + 10$$

?

$$y = f(x) = -x^2 + 10$$

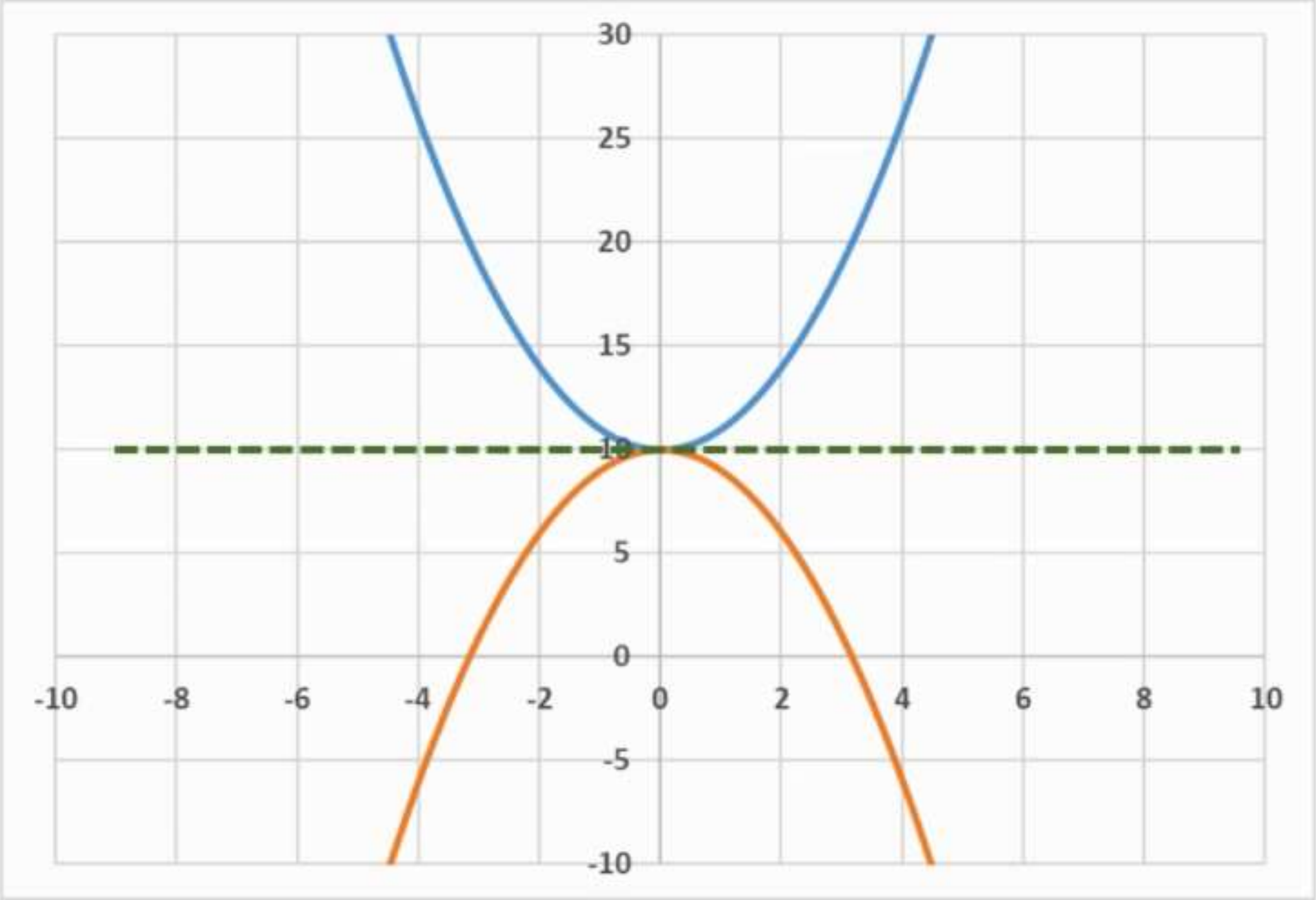


Maxima or Minima?

$$y = f(x) = x^2 + 10$$

?

$$y = f(x) = -x^2 + 10$$



Second Order Derivative

$$\frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d^2y}{dx^2}$$

Second Order Derivative

$$y = f(x) = x^2 + 10 \rightarrow \frac{dy}{dx} = \boxed{} \rightarrow \frac{d^2y}{dx^2} = \boxed{}$$

$$y = f(x) = -x^2 + 10 \rightarrow \frac{dy}{dx} = \boxed{} \rightarrow \frac{d^2y}{dx^2} = \boxed{}$$

Second Order Derivative

$$y = f(x) = x^2 + 10 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow \frac{d^2y}{dx^2} = 2$$

$$y = f(x) = -x^2 + 10 \Rightarrow \frac{dy}{dx} = -2x \Rightarrow \frac{d^2y}{dx^2} = -2$$

Rules for Maxima and Minima

Second Derivative < 0 → Local Maxima

Second Derivative > 0 → Local Minima

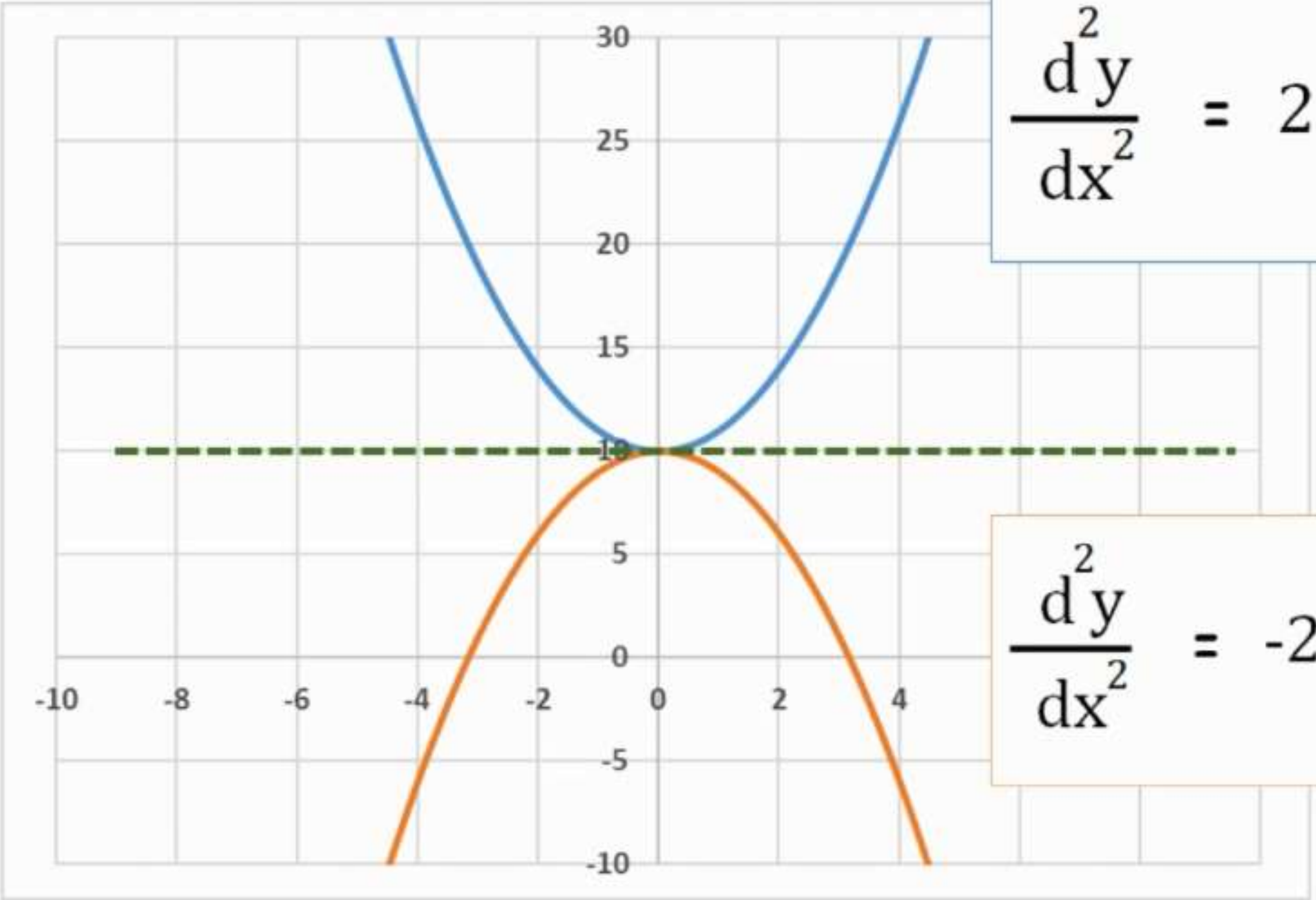
Maxima or Minima?

$$y = f(x) = x^2 + 10$$

Minima at y = 10

$$y = f(x) = -x^2 + 10$$

Maxima at y = 10



$$\frac{d^2y}{dx^2} = 2$$

$$\frac{d^2y}{dx^2} = -2$$

Maxima or Minima?

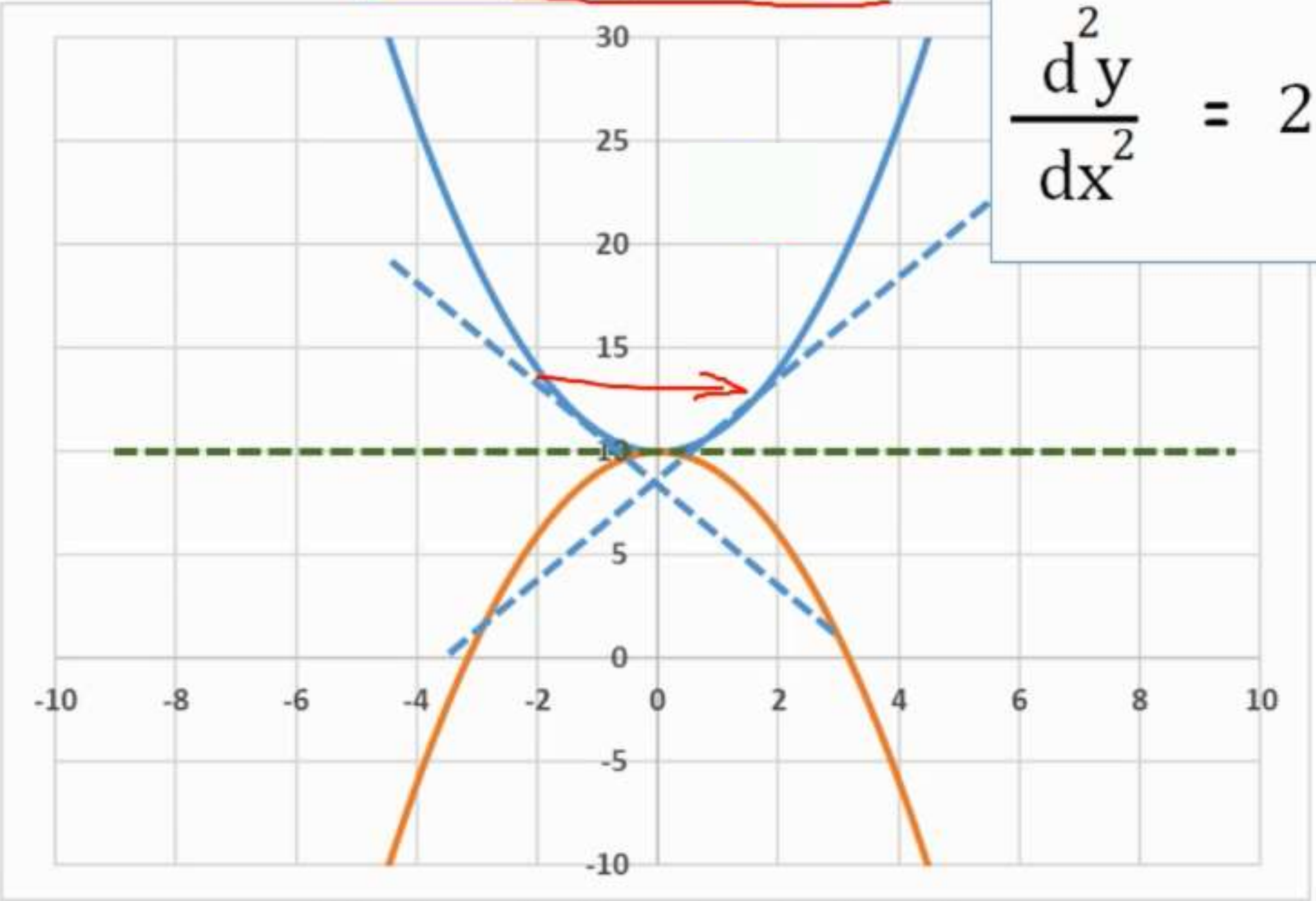
$$y = f(x) = x^2 + 10$$

Minima at y = 10

$$y = f(x) = -x^2 + 10$$

Maxima at y = 10

Shift from Negative slope to Positive

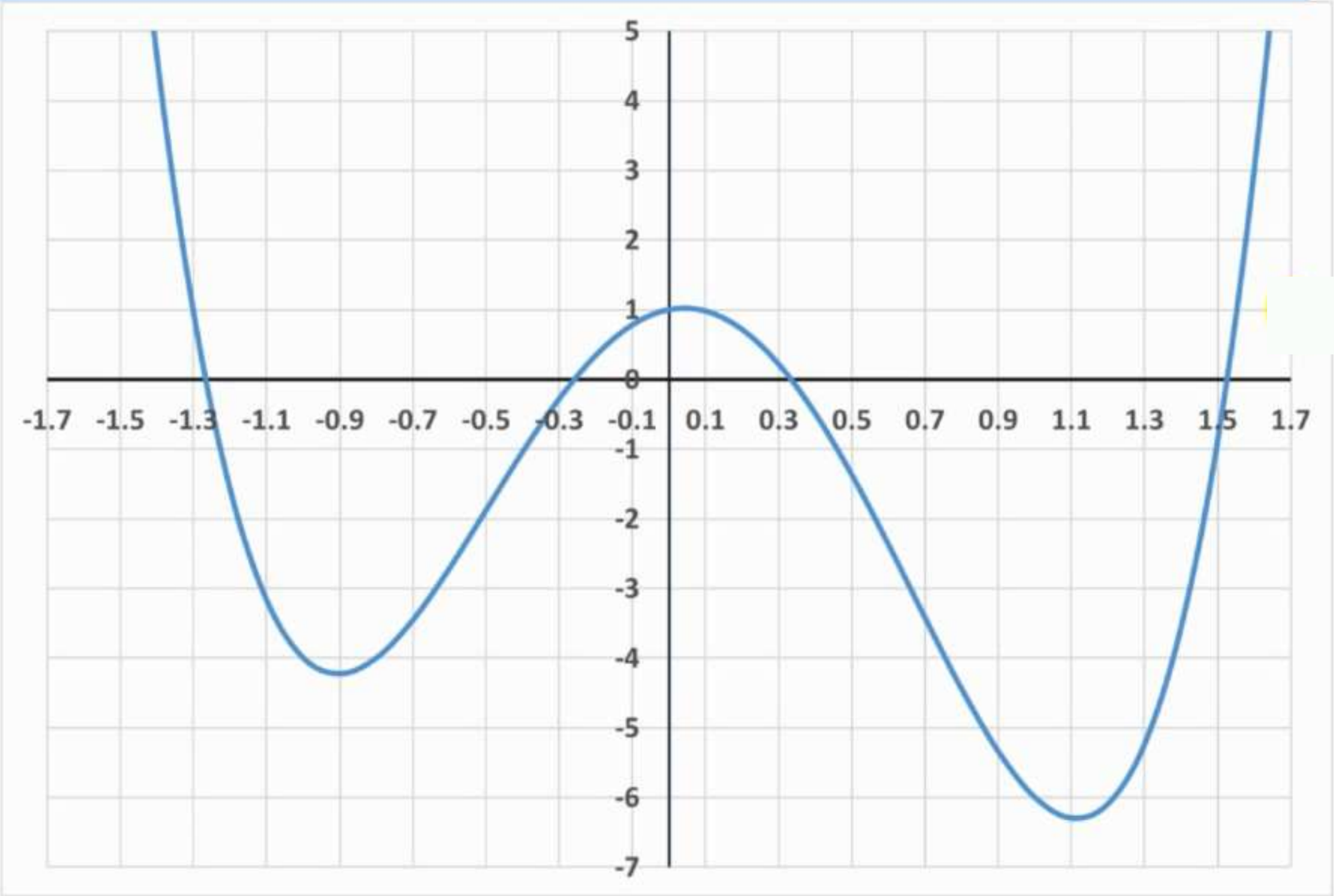


$$\frac{d^2y}{dx^2} = 2$$

Shift from Positive slope to Negative

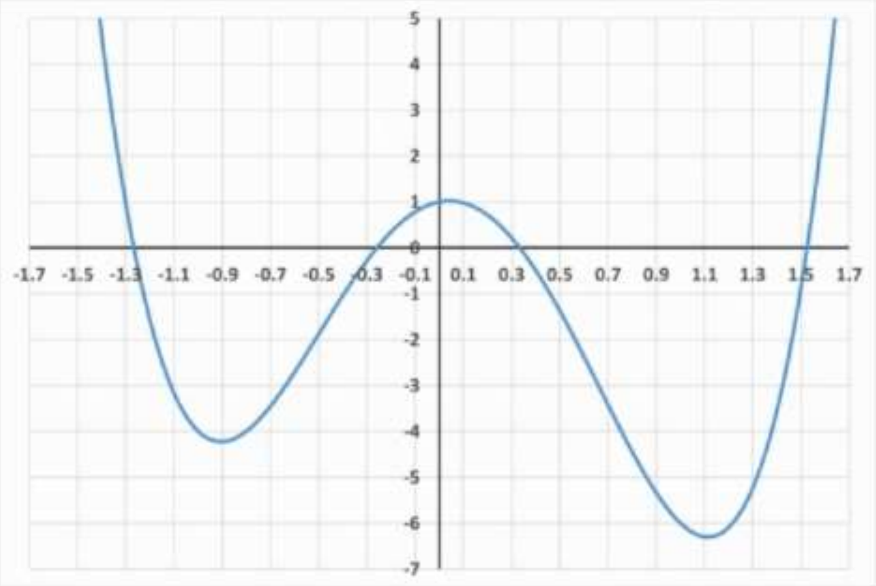
Derivative for Maxima and Minima

$$y = 6x^4 - 2x^3 - 12x^2 + x + 1$$



Derivative for Maxima and Minima

$$y = 6x^4 - 2x^3 - 12x^2 + x + 1$$



Step 1 – Get the first Derivative

Step 2 – Get the Second Derivative

Step 3 – Identify points where slope is zero

Step 4 – Get the second derivative when slope is zero

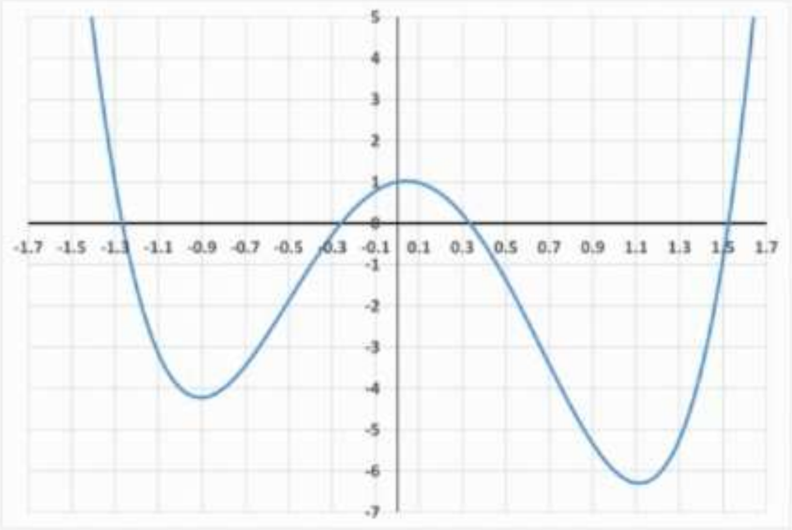
Step 5 – Apply the rules for maxima and minima

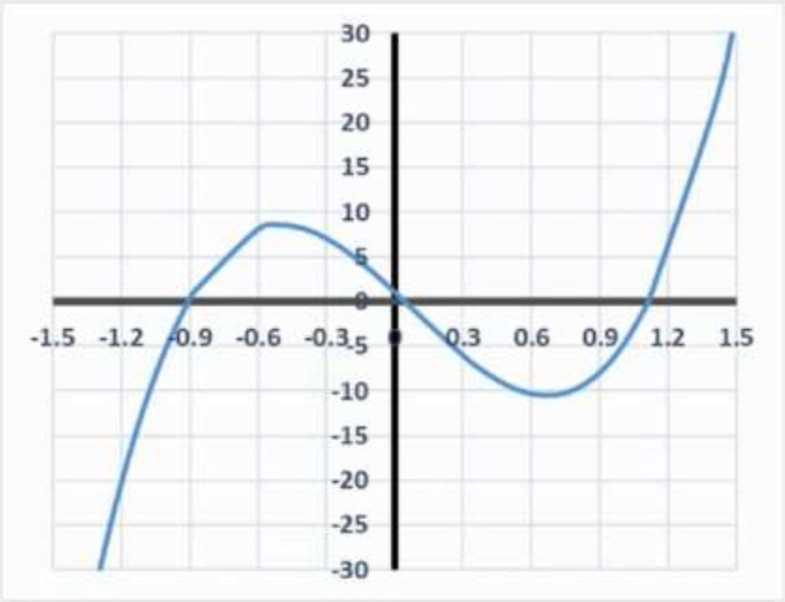
Derivative for Maxima and Minima

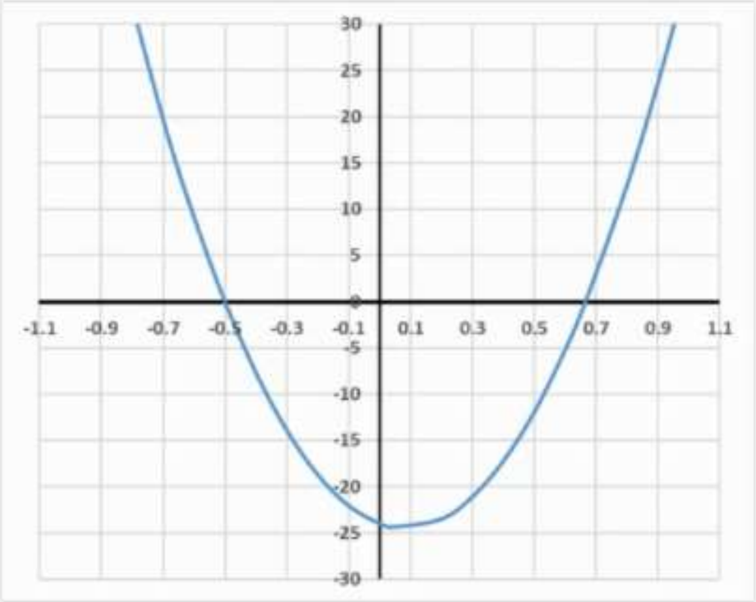
$$y = 6x^4 - 2x^3 - 12x^2 + x + 1$$

$$\frac{dy}{dx} = 24x^3 - 6x^2 - 24x + 1$$

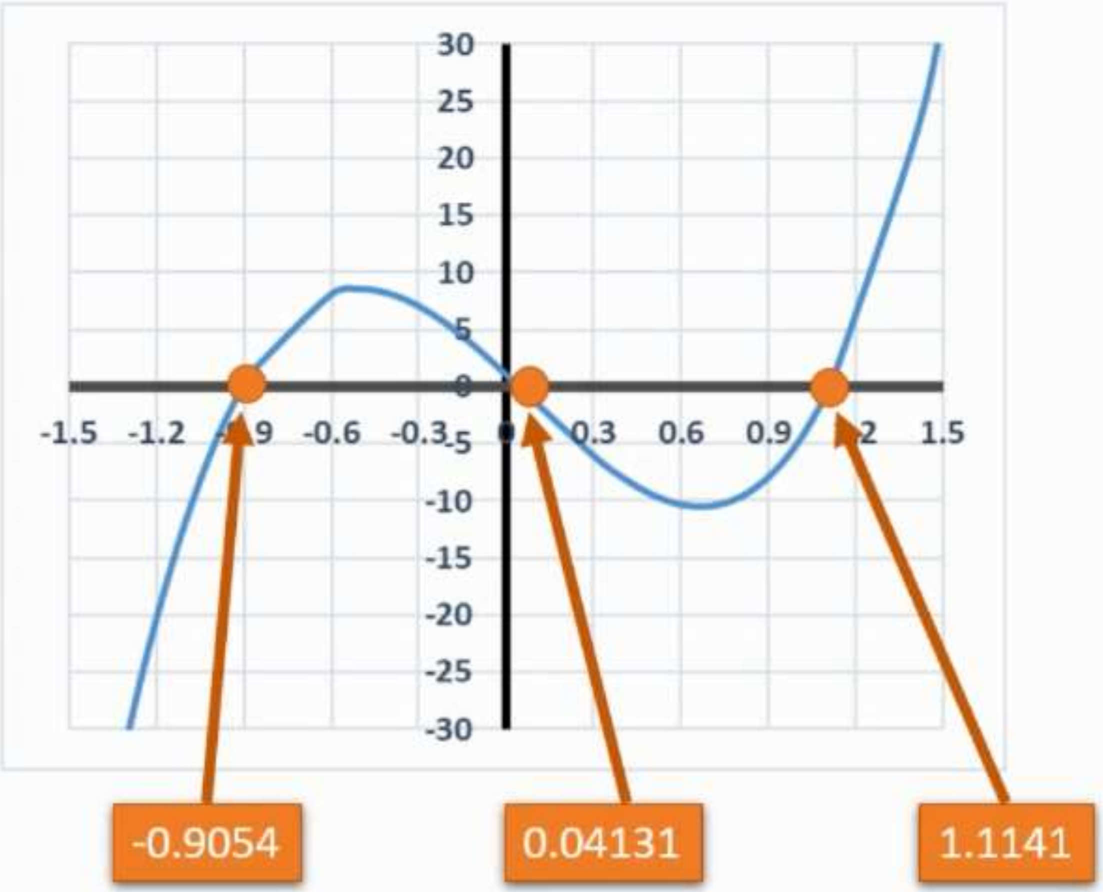
$$\frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$







First Derivative for zero slope

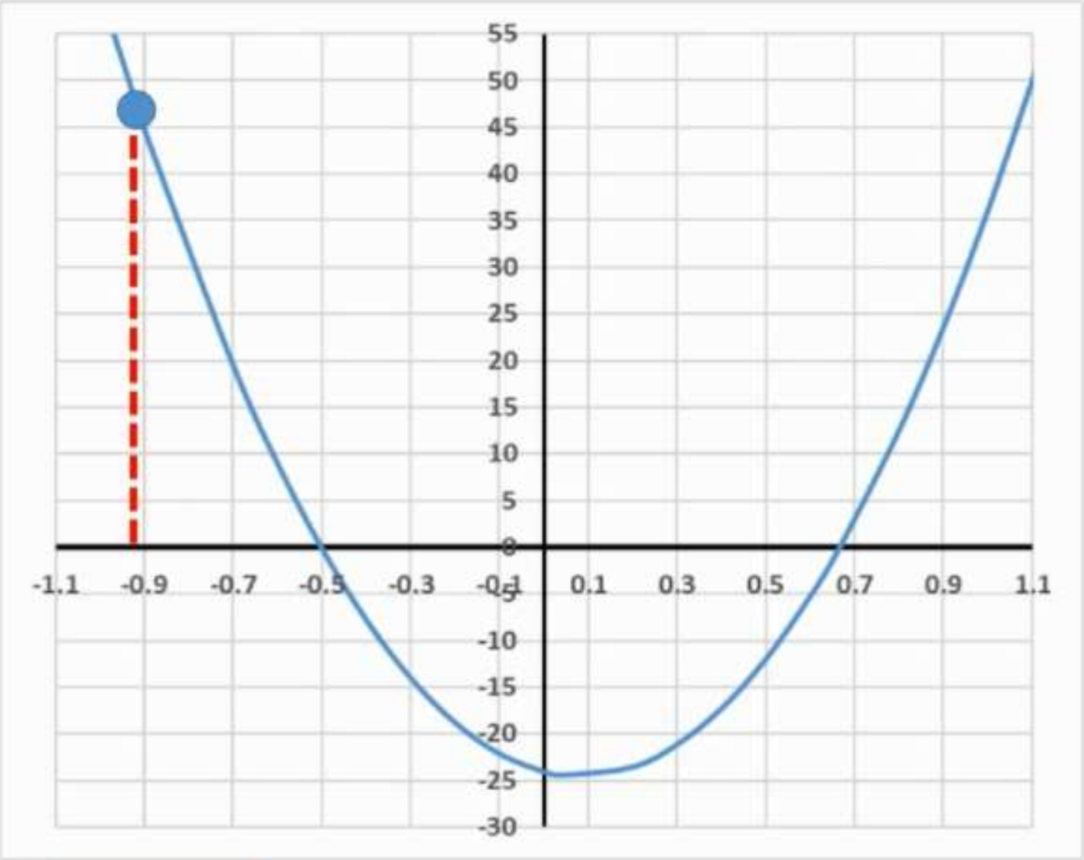


$$\frac{dy}{dx} = 24x^3 - 6x^2 - 24x + 1$$

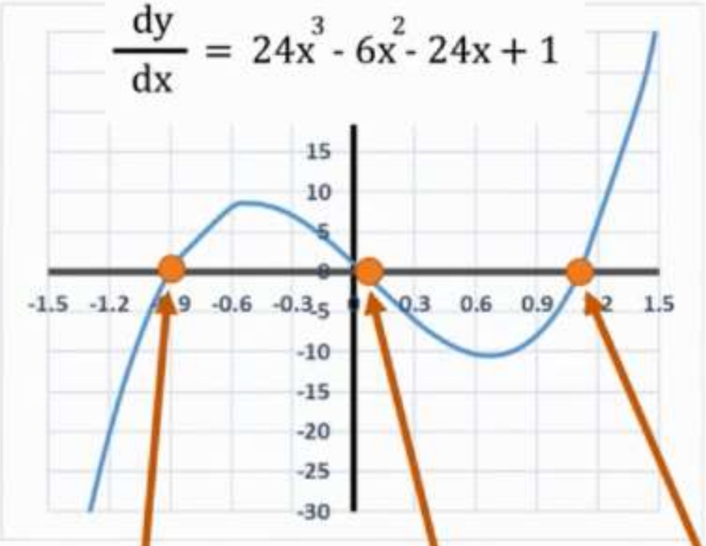
Second Derivative

$$\frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$

$$f(-0.9054) = 72x^2 - 12x - 24 = +45.88$$



-0.9054

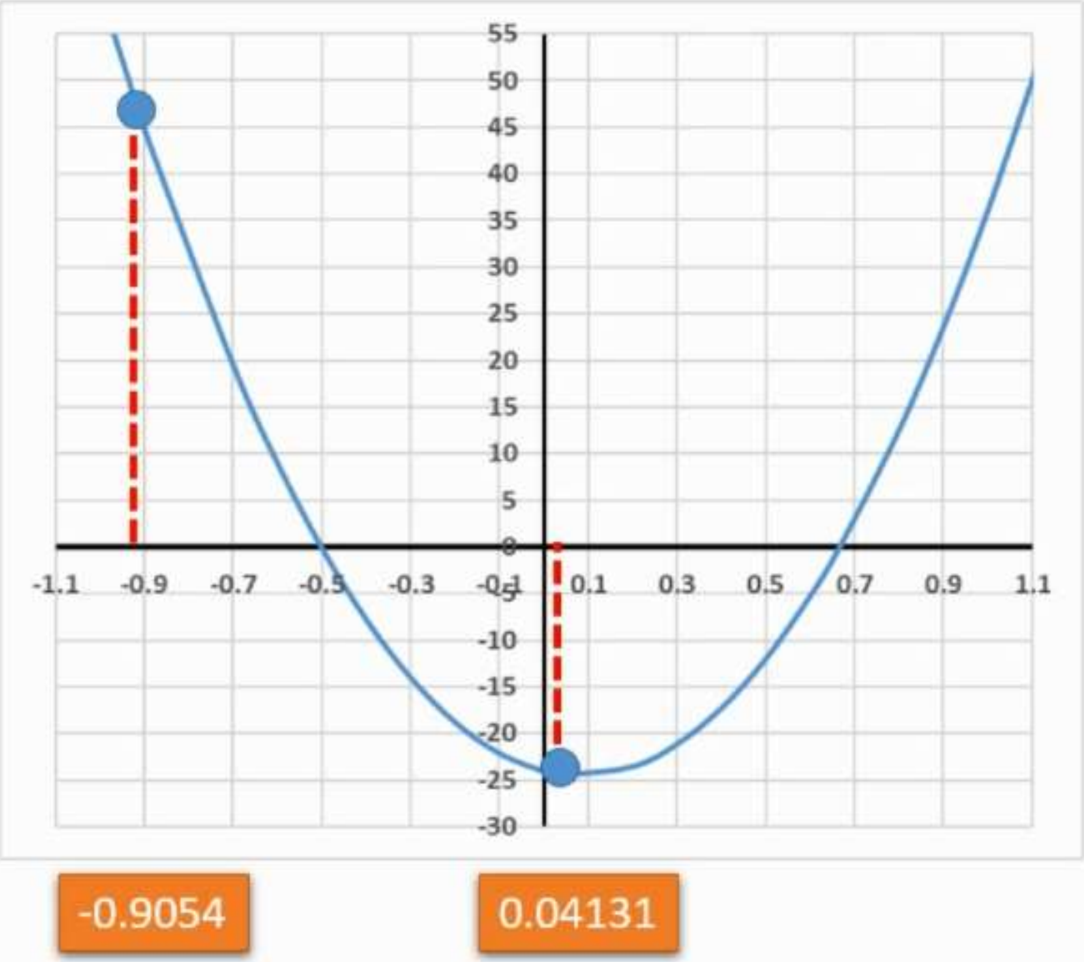


-0.9054

0.04131

1.1141

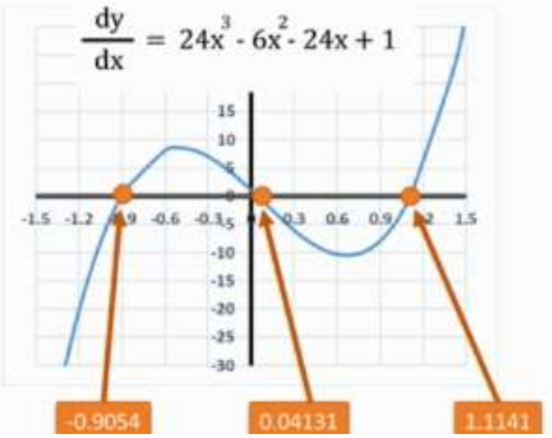
Second Derivative



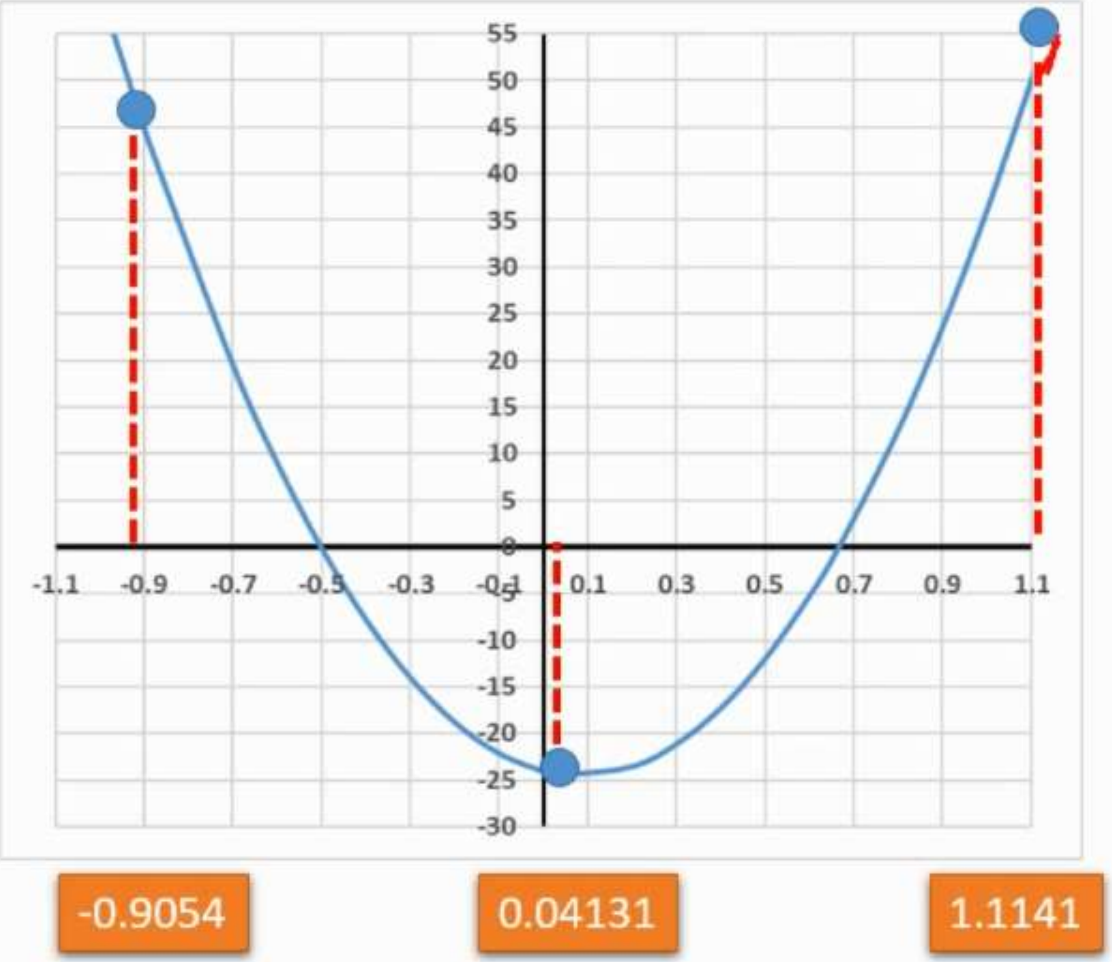
$$\frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$

$$f(-0.9054) = 72x^2 - 12x - 24 = +45.88$$

$$f(\underline{0.0413}) = 72x^2 - 12x - 24 = -24$$



Second Derivative



$$\frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$

$$f(-0.9054) = 72x^2 - 12x - 24 = +45.88$$

$$f(0.0413) = 72x^2 - 12x - 24 = -24$$

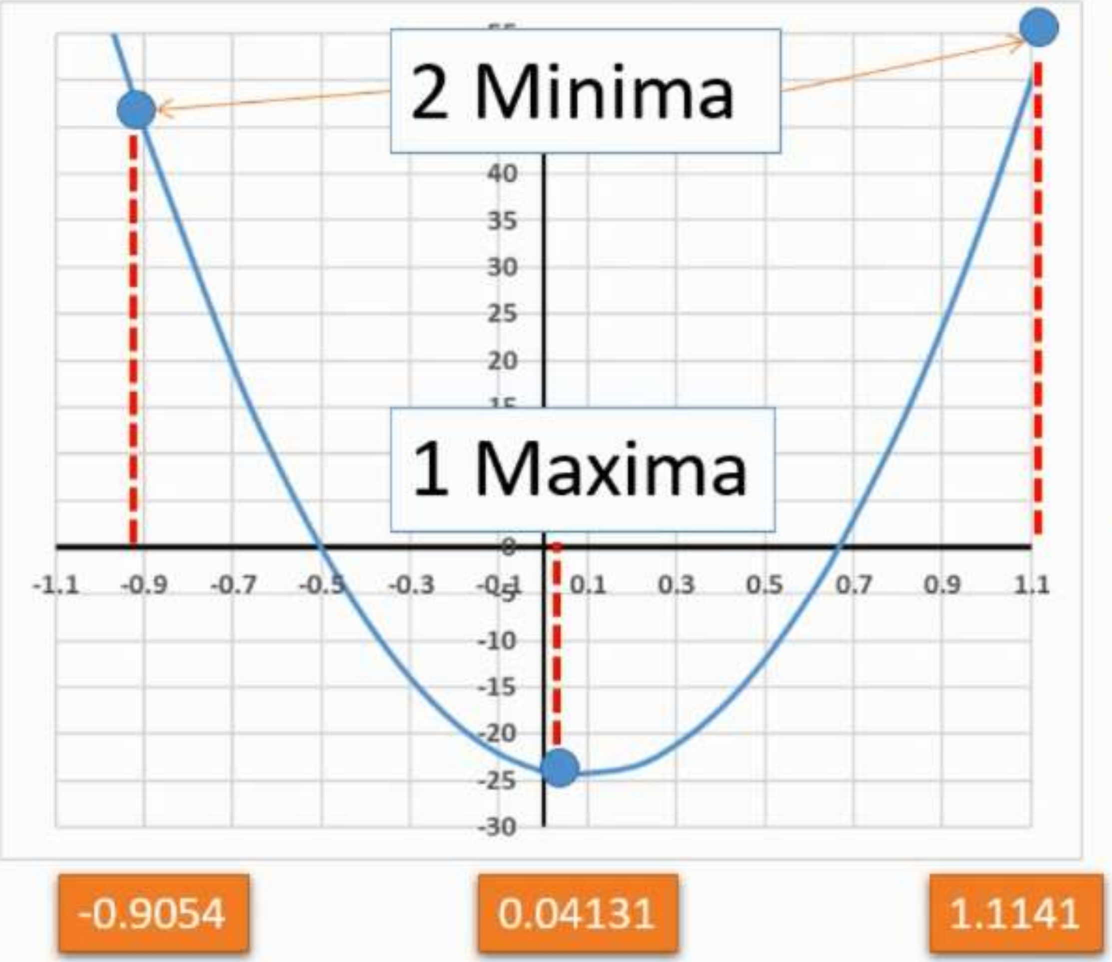
$$f(1.1141) = 72x^2 - 12x - 24 = +52$$

Rules for Maxima and Minima

Second Derivative < 0 → Local Maxima

Second Derivative > 0 → Local Minima

Get Maxima and Minima

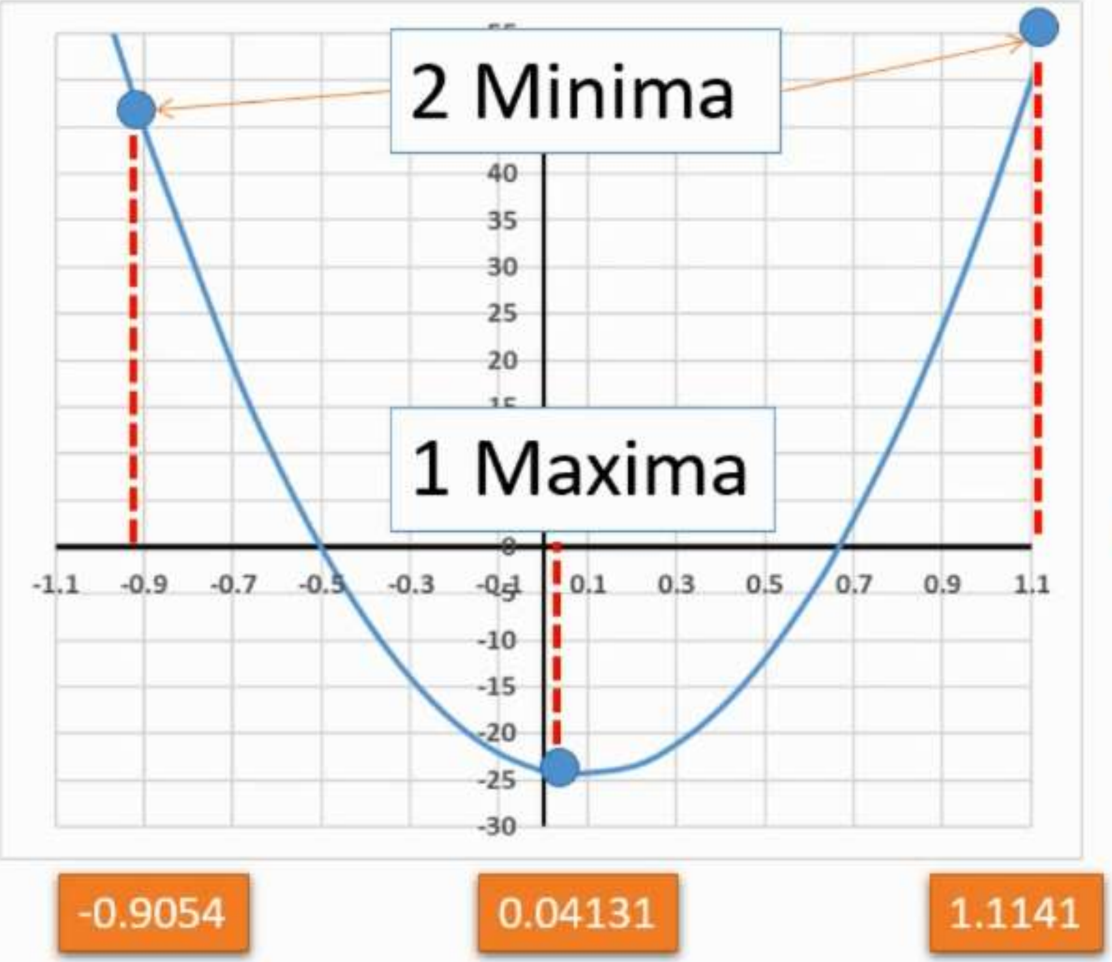


$$f(-0.9054) = 72x^2 - 12x - 24 = +45.88$$

$$f(-0.0413) = 72x^2 - 12x - 24 = -24$$

$$f(1.1141) = 72x^2 - 12x - 24 = +52$$

Get Maxima and Minima

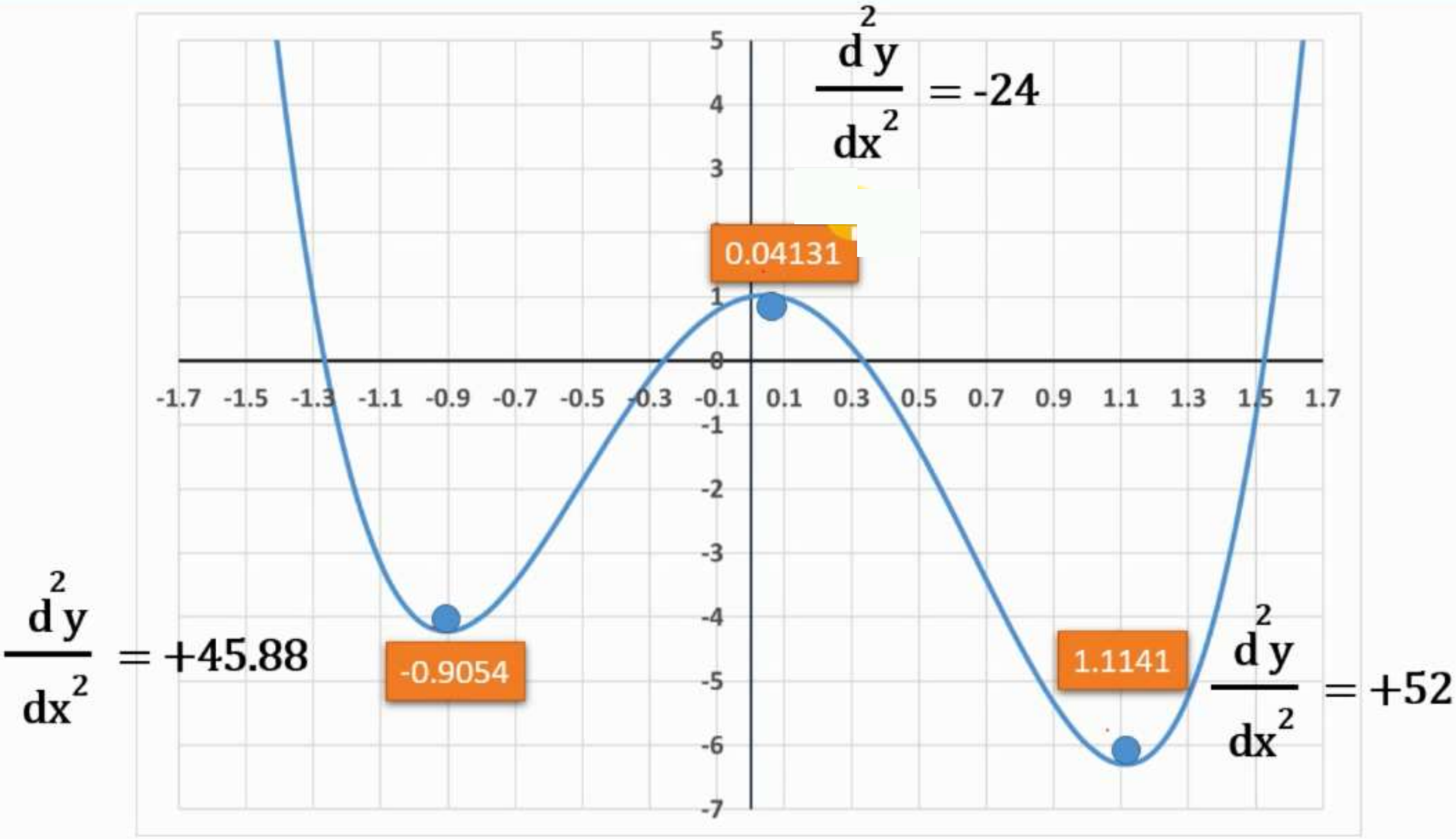


$$f(-0.9054) = 72x^2 - 12x - 24 = +45.88$$

$$f(-0.0413) = 72x^2 - 12x - 24 = -24$$

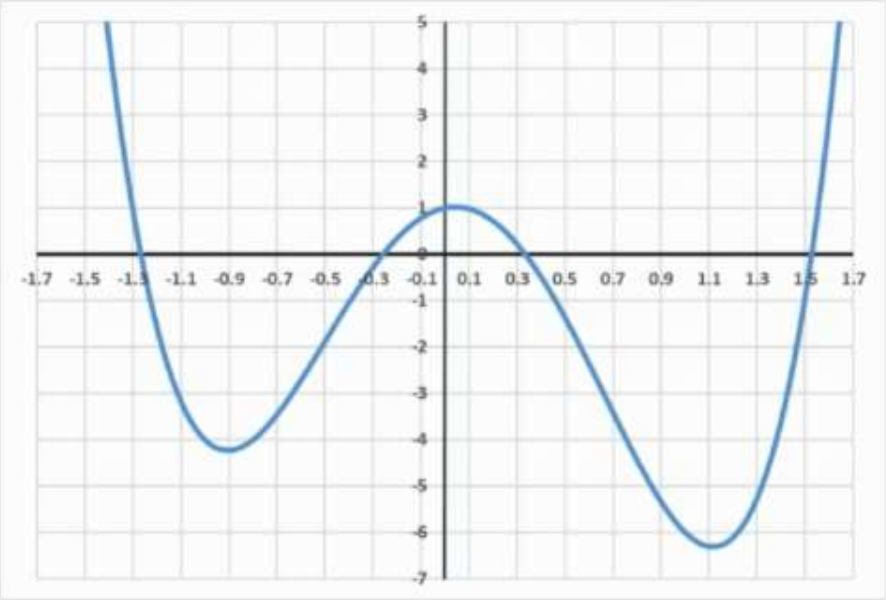
$$f(1.1141) = 72x^2 - 12x - 24 = +52$$

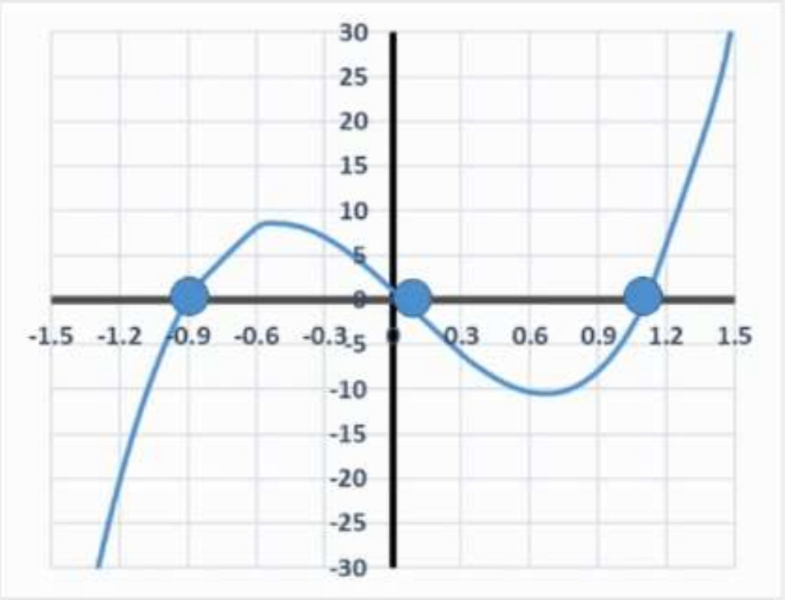
Actual Minima and Maxima

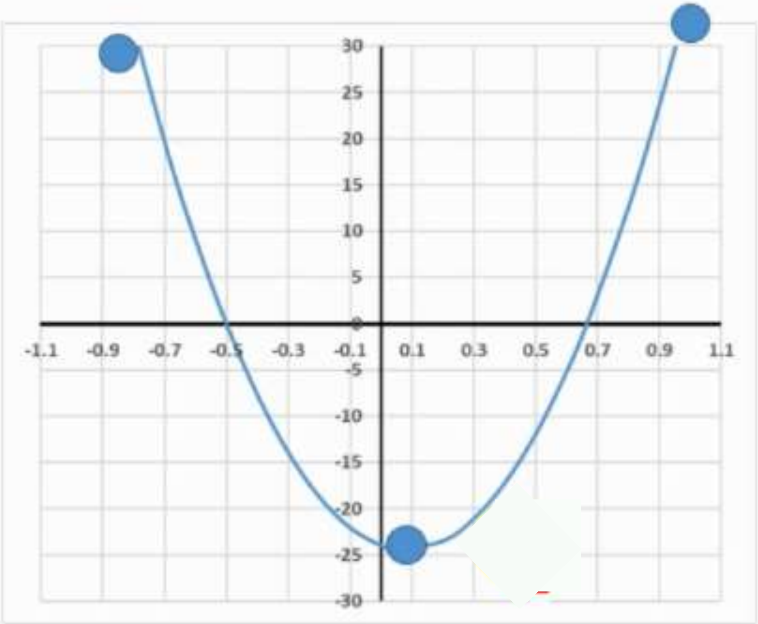


Derivative for Maxima and Minima

$$y = 6x^4 - 2x^3 - 12x^2 + x + 1 \qquad \frac{dy}{dx} = 24x^3 - 6x^2 - 24x + 1 \qquad \frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$







-0.9054 0.04131 1.1141

-0.9054 0.04131 1.1141

Derivative for Maxima and Minima

$$y = 6x^4 - 2x^3 - 12x^2 + x + 1$$

$$\frac{dy}{dx} = 24x^3 - 6x^2 - 24x + 1$$

$$\frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$

-0.9054	0.04131	1.1141

-0.9054	0.04131	1.1141

Quiz

Question 1:

Y = 1/x

What will be the answer of the following limit?

$$\lim_{x \to 2} \frac{1}{x} = ?$$

- 1
- 2
- Undefined
- 0.5

Question 2:

First Order Derivative of a horizontal line is zero. True or False?

- True
- False

Question 3:

$$y = 2x^3 + x^2 + 4x + 9$$

Using the chain rule of derivative, what will be the first order derivative of the y, in the following equation?

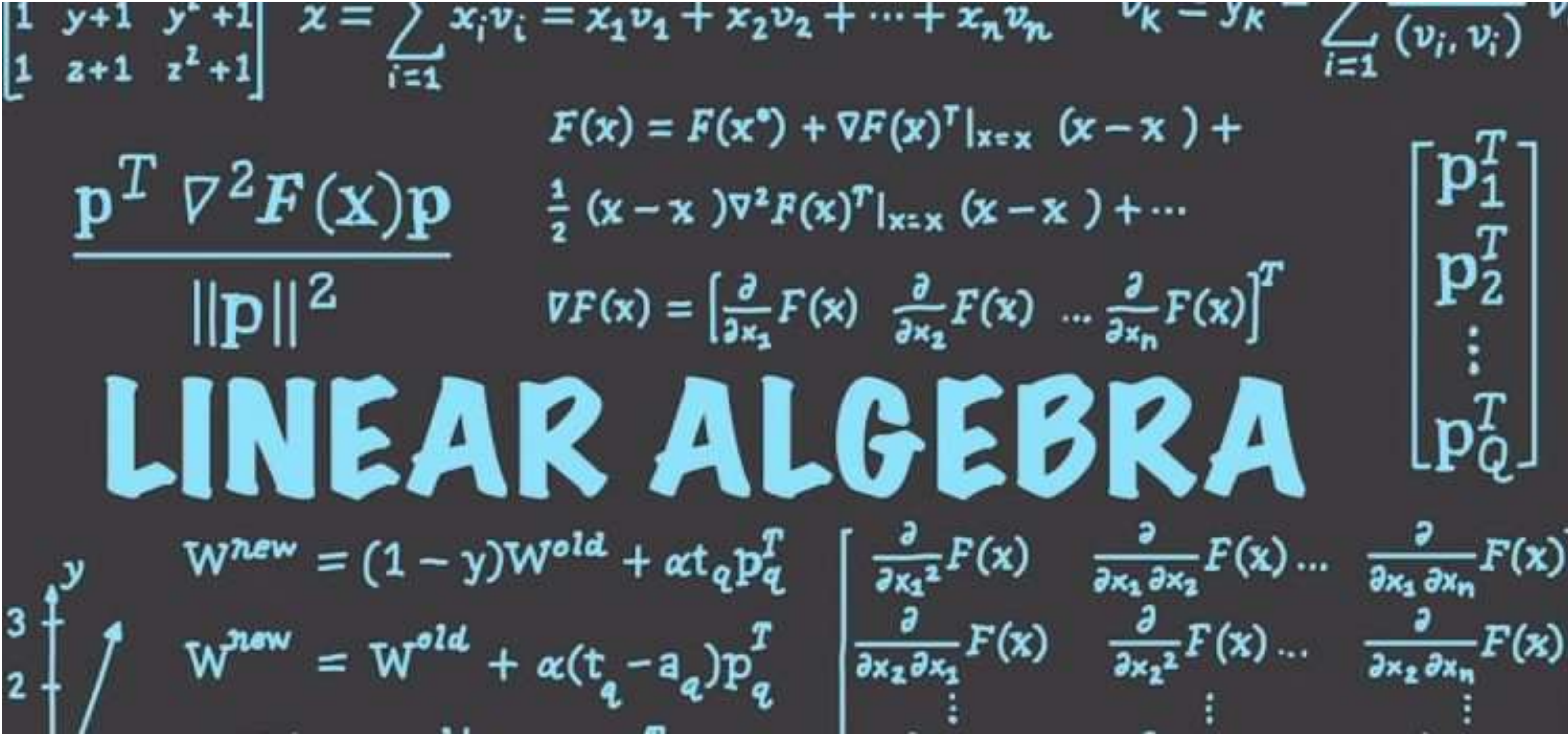
$$\frac{dy}{dx} = ?$$

- ○ `3x^2 + 2x + 4`
- ○ `6x^2 + 2x + 4`
- ○ `2x^2 + x + 4`
- ○ None of the above

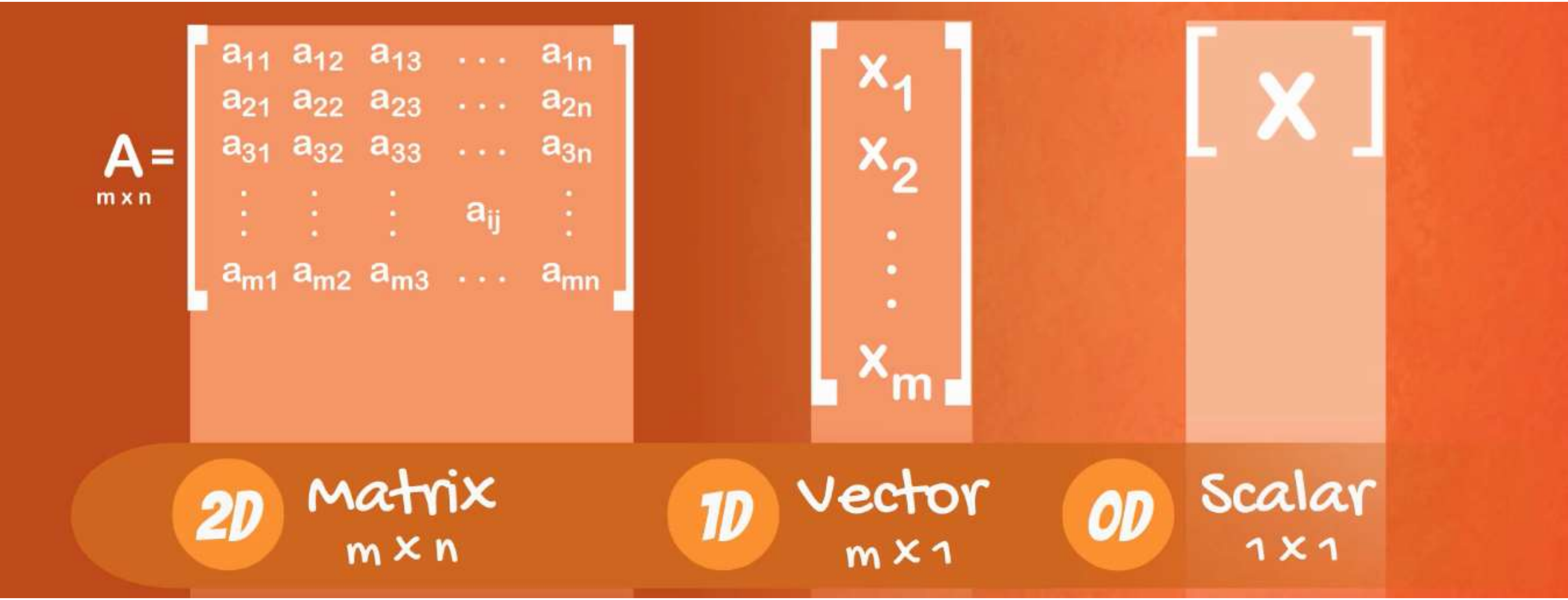
Question 4:

If the double derivative of the function is less than zero, we have got the minima of the function. True or False?

- **True**
- **False**

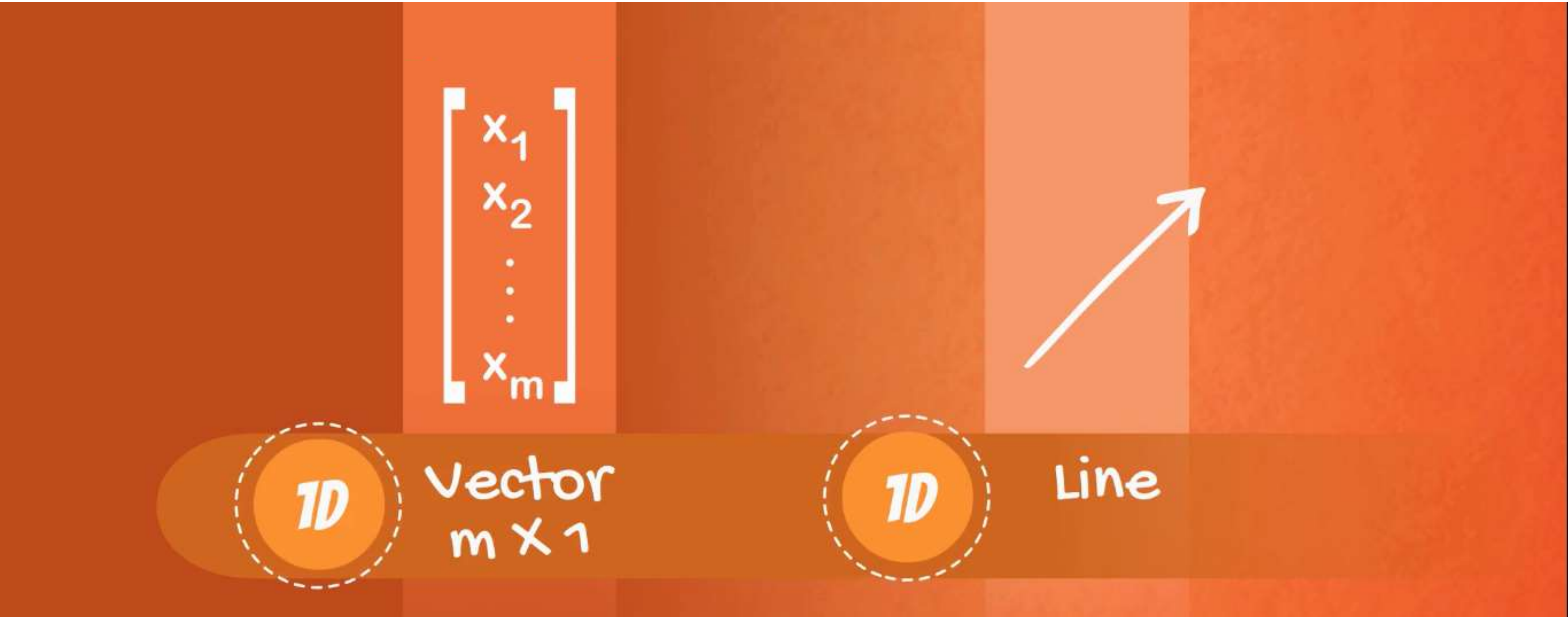

LINEAR ALGEBRA

Matrix Vs. Vector Vs. Scalar

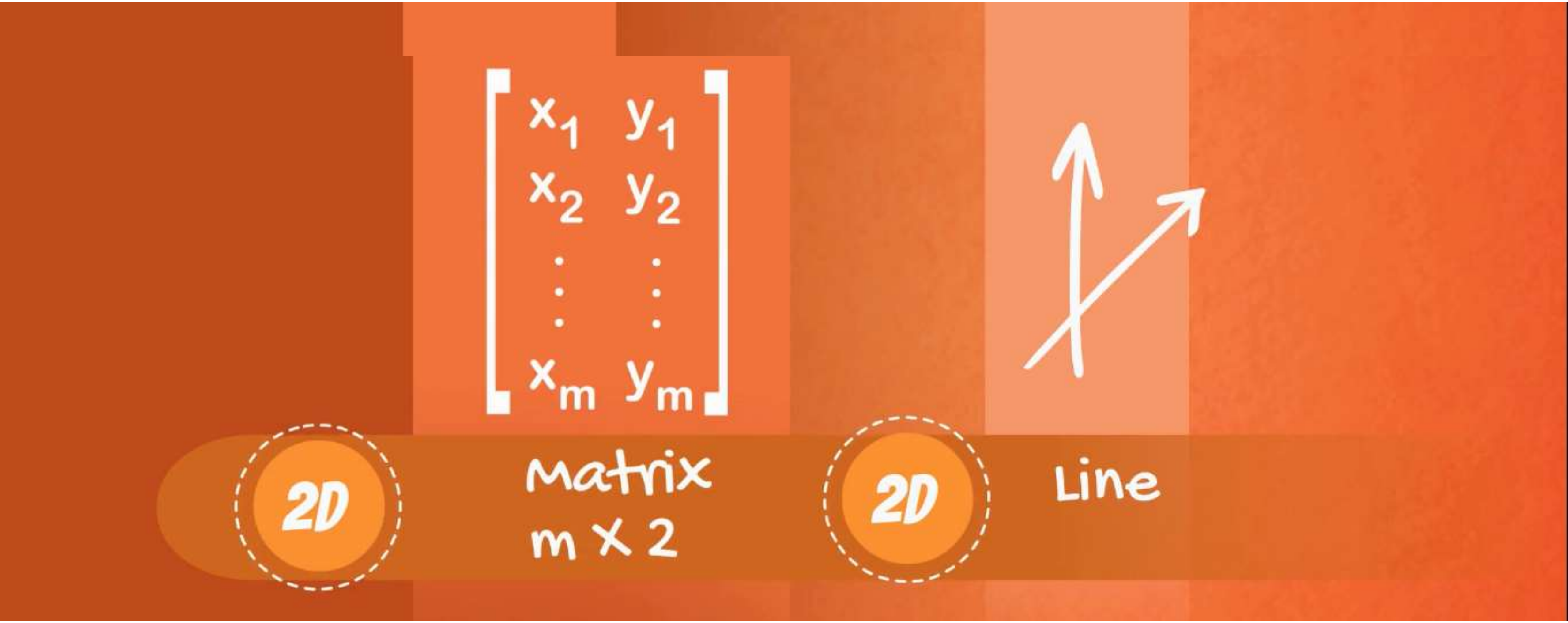


Scalar

Vector



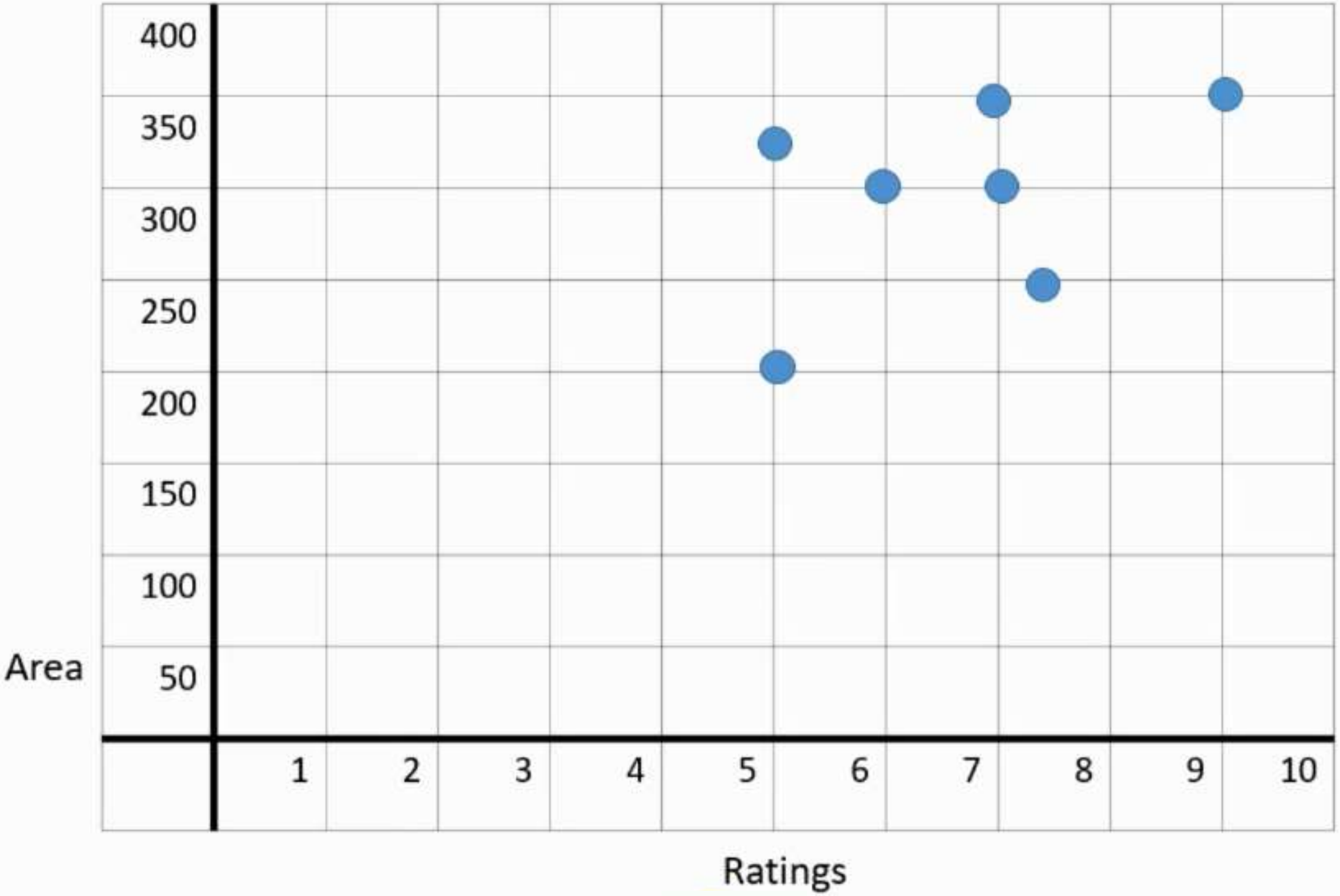
Matrix



$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_m & y_m \end{bmatrix}$
2D
Matrix
m x 2
2D
Line

Vectors in Machine Learning

Rating	Area sq. mtr
5	200
7	300
5	325
8	250
6	300
7	350
7.5	250
9	350



Vectors in Machine Learning

Rating	Area sq. mtr
5	200
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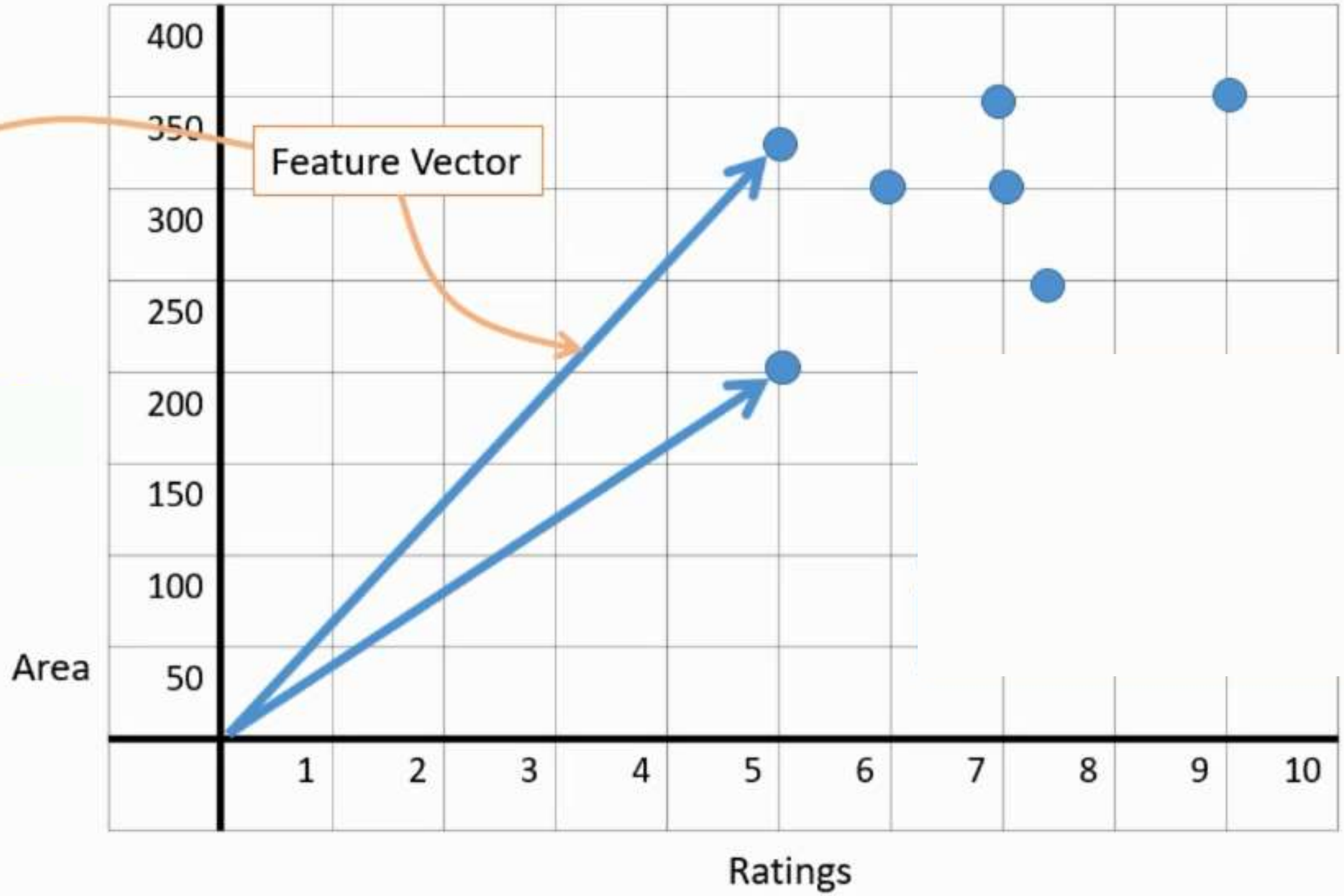
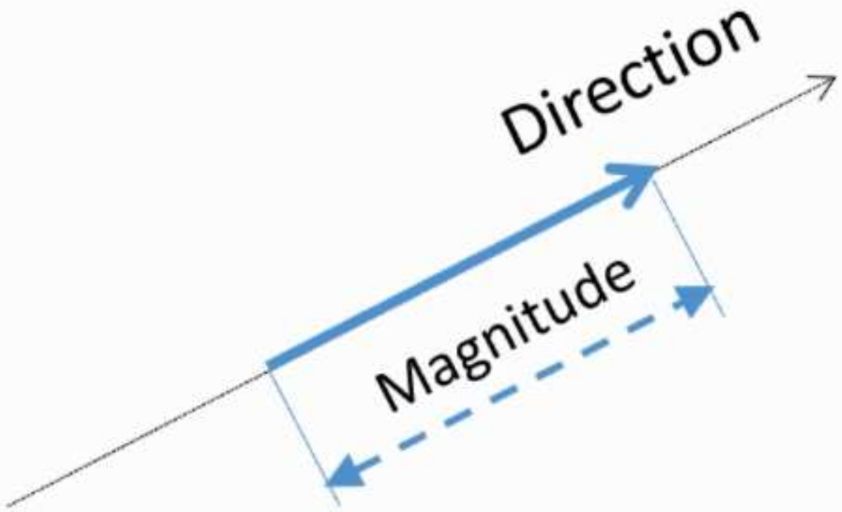


What is a vector?

$\overrightarrow{V}$



What is a vector?

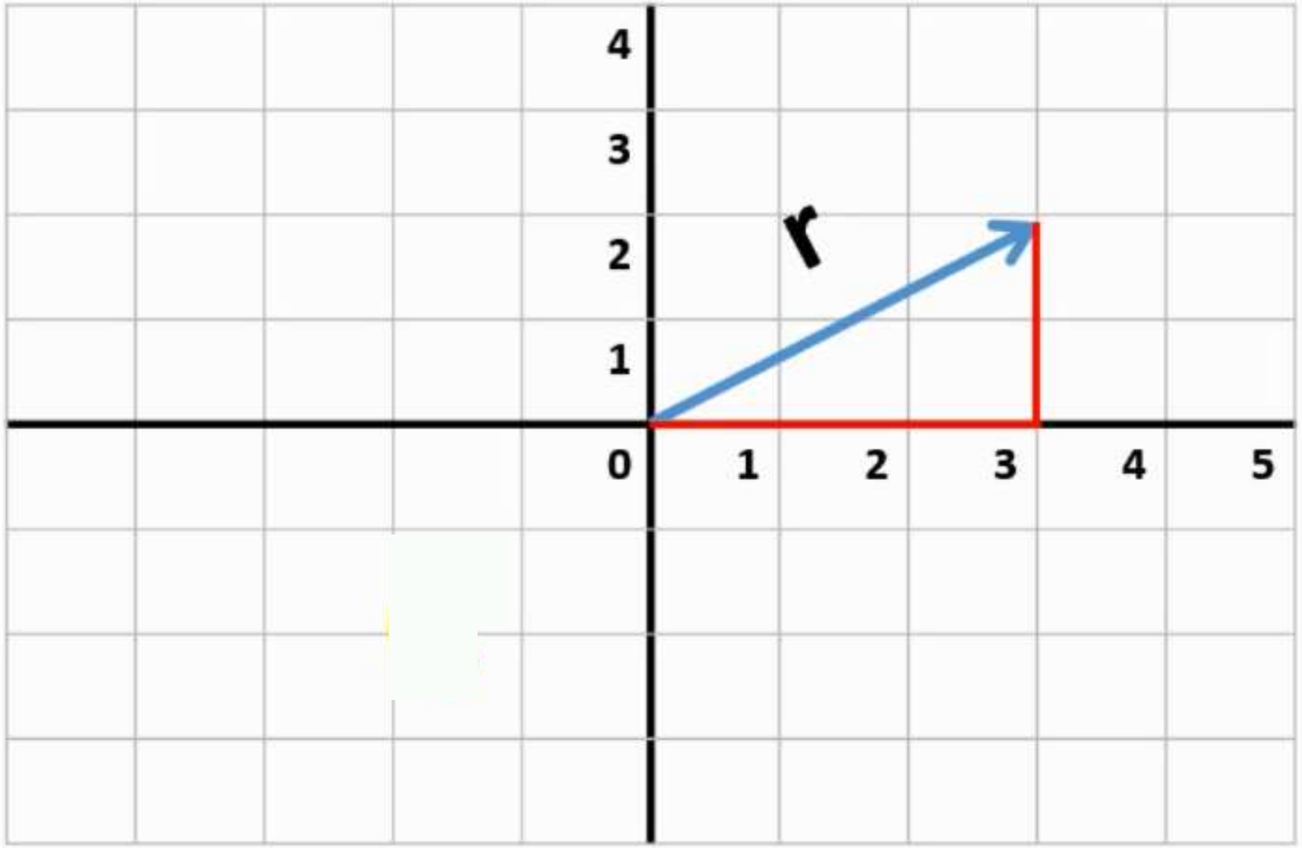
Cartesian:

$$\vec{V} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$r^2 = a^2 + b^2$$

$$= 9 + 4$$

$$= \sqrt{13}$$



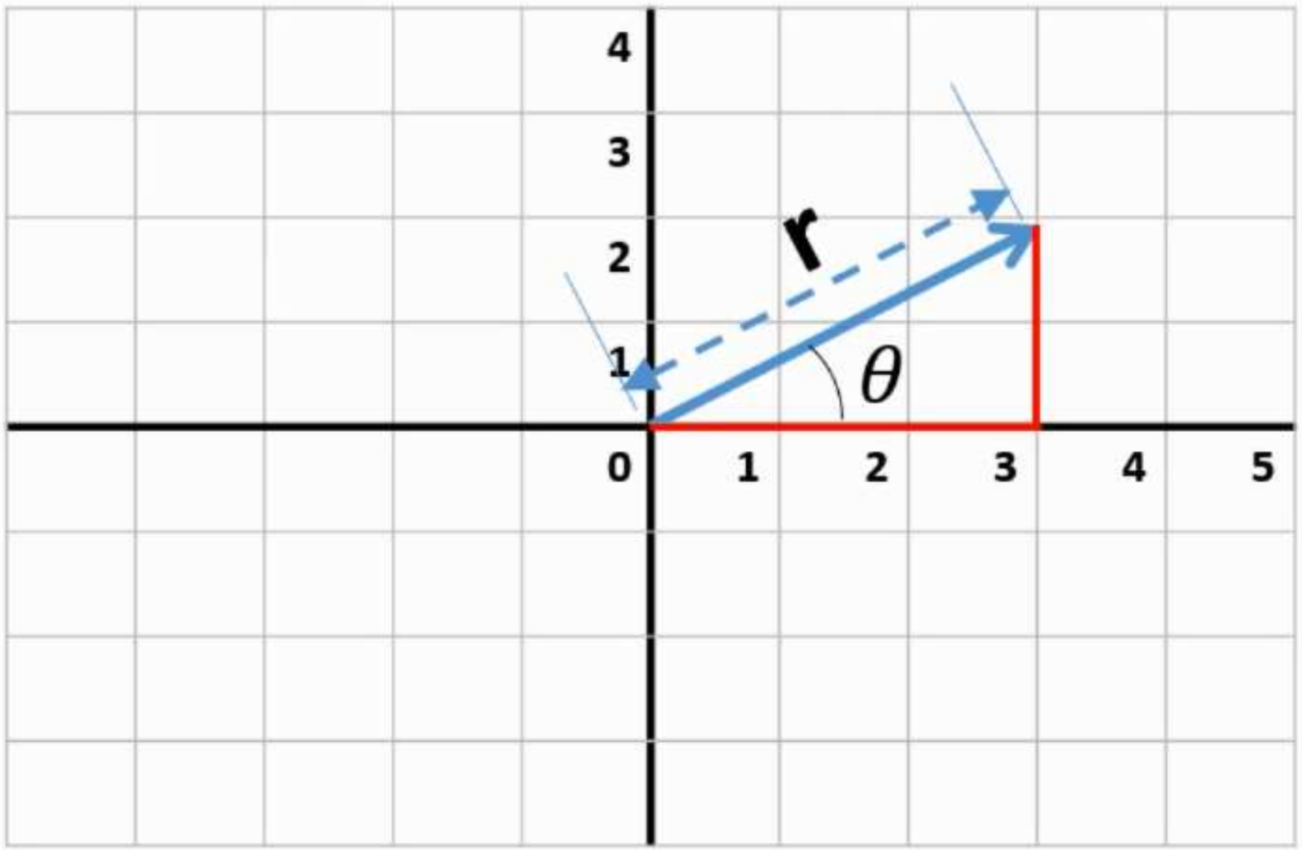
What is a vector?

Cartesian:

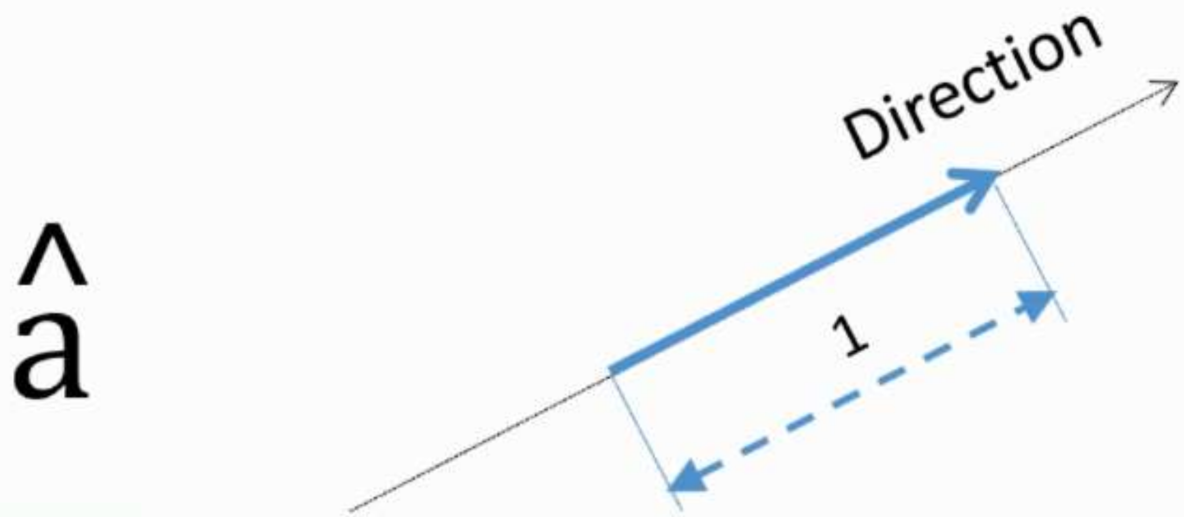
$$\vec{V} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Polar:

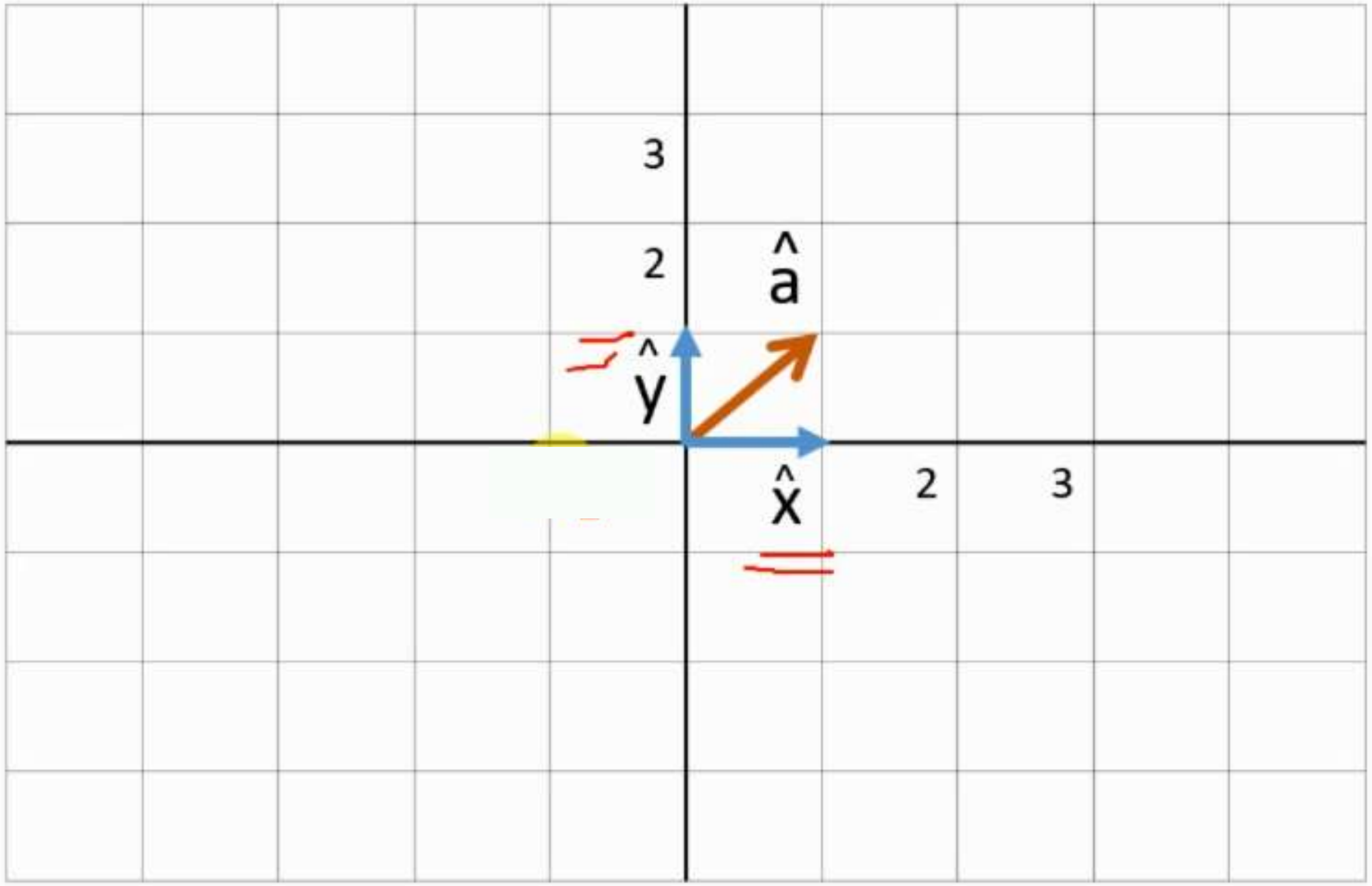
$$\vec{V} = (r, \theta)$$



Unit Vector



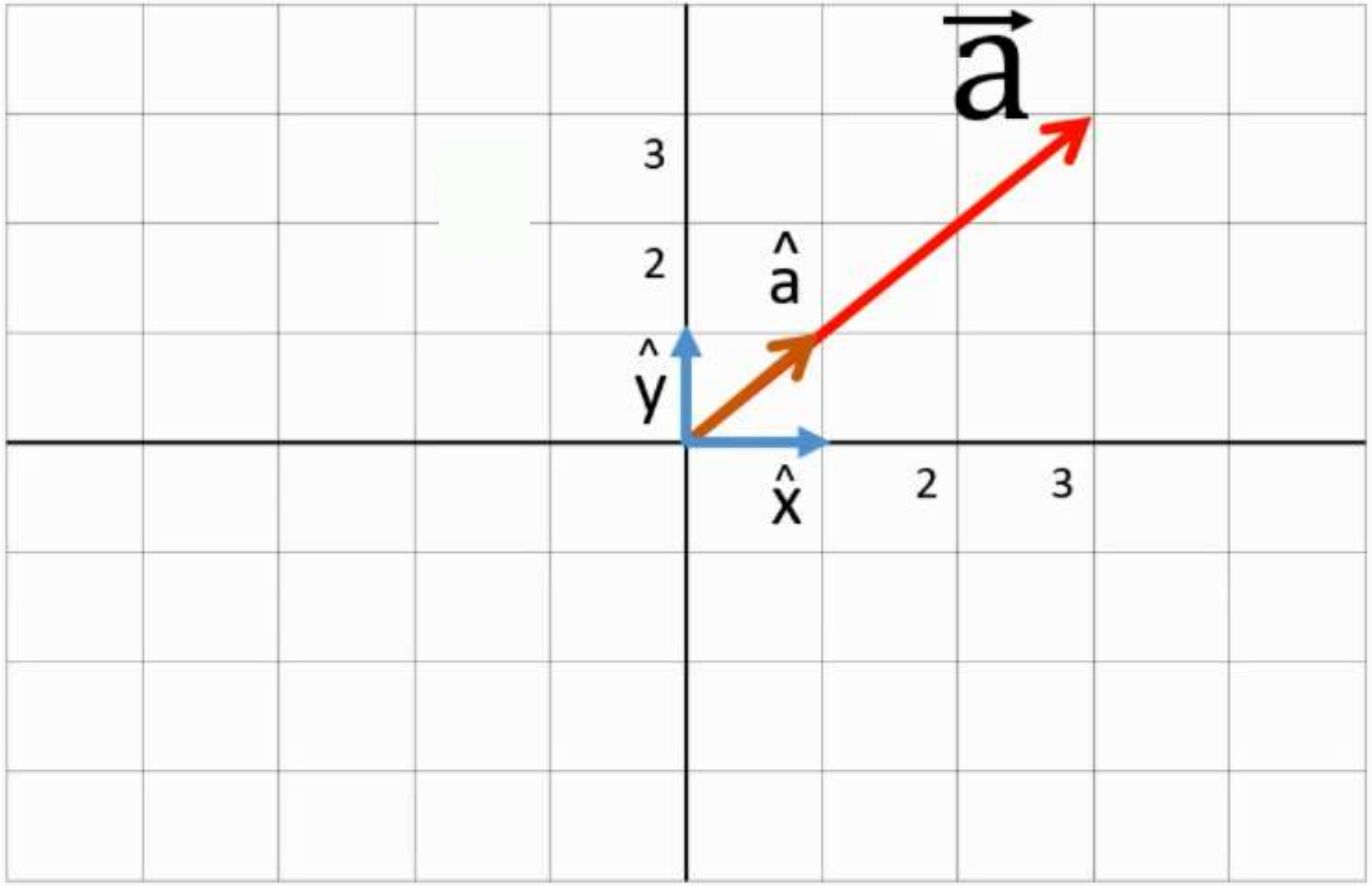
Unit Vector



Unit Vector

$$\vec{a} = 3 * \hat{a}$$

$$\vec{a} = 3*\hat{x} + 3*\hat{y}$$

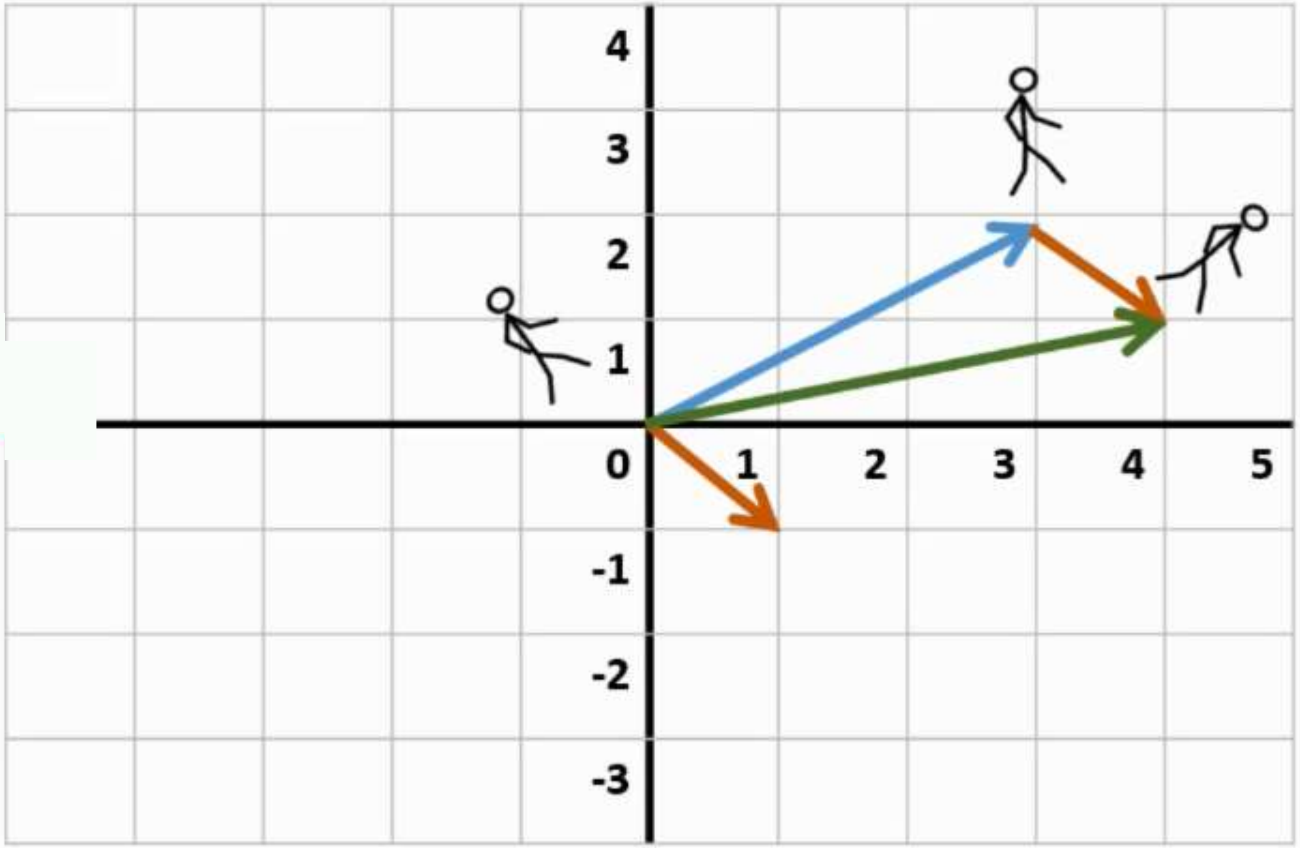


Vector Addition

$$\vec{V}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\vec{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{V}_1 + \vec{V}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

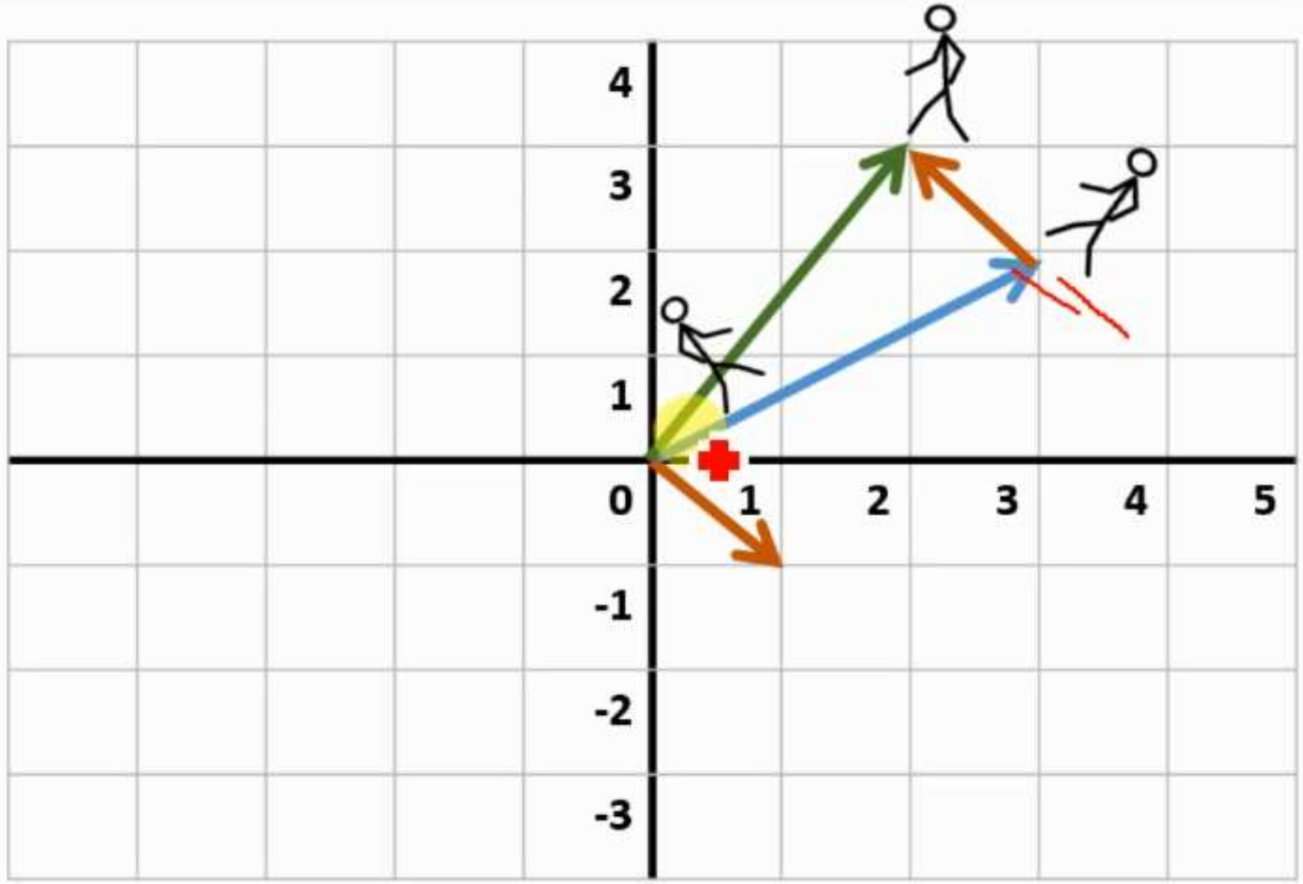


Vector Subtraction

$$\vec{V}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\vec{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{V}_1 - \vec{V}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



Why should we learn Matrices?

What is a Matrix?

$$X = \underbrace{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}}_{\text{Columns}} \left.\vphantom{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}\right\} \text{Rows}$$

Why should we learn Matrices?

Fixed Acidity	Volatile Acidity	Citric Acid	Residual Sugar	Chlorides	Free Sulfur Dioxide	Total Sulfur Dioxide	Density	pH	Sulphates	Alcohol	Quality
7.4	0.7	0	1.9	0.076	11	34	0.9978	3.51	0.56	9.4	5
7.8	0.88	0	2.6	0.098	25	67	0.9968	3.2	0.68	9.8	5
7.8	0.76	0.04	2.3	0.092	15	54	0.997	3.26	0.65	9.8	5
11.2	0.28	0.56	1.9	0.075	17	60	0.998	3.16	0.58	9.8	6
7.4	0.7	0	1.9	0.076	11	34	0.9978	3.51	0.56	9.4	5
7.4	0.66	0	1.8	0.075	13	40	0.9978	3.51	0.56	9.4	5
7.9	0.6	0.06	1.6	0.069	15	59	0.9964	3.3	0.46	9.4	6
7.3	0.65	0	1.2	0.065	15	21	0.9946	3.39	0.47	10	7
7.8	0.58	0.02	2	0.073	9	18	0.9968	3.36	0.57	9.5	7

Why should we learn Matrices?

7.4	0.7	0	1.9	0.076	11	34	0.9978	3.51	0.56	9.4	5
7.8	0.88	0	2.6	0.098	25	67	0.9968	3.2	0.68	9.8	5
7.8	0.76	0.04	2.3	0.092	15	54	0.997	3.26	0.65	9.8	5
11.2	0.28	0.56	1.9	0.075	17	60	0.998	3.16	0.58	9.8	6
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Why should we learn Matrices?

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7.8	0.58	0.02	2	0.073	9	18	0.9968	3.36	0.57	9.5	7

Why should we learn Matrices?

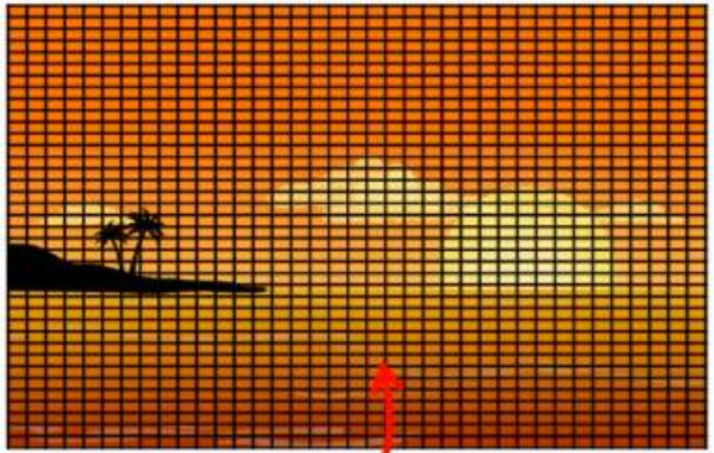
$$\begin{bmatrix}
7.4 & 0.7 & 0 & 1.9 & 0.076 & 11 & 34 & 0.9978 & 3.51 & 0.56 & 9.4 & 5 \\
7.8 & 0.88 & 0 & 2.6 & 0.098 & 25 & 67 & 0.9968 & 3.2 & 0.68 & 9.8 & 5 \\
7.8 & 0.76 & 0.04 & 2.3 & 0.092 & 15 & 54 & 0.997 & 3.26 & 0.65 & 9.8 & 5 \\
11.2 & 0.28 & 0.56 & 1.9 & 0.075 & 17 & 60 & 0.998 & 3.16 & 0.58 & 9.8 & 6 \\
7.4 & 0.7 & 0 & 1.9 & 0.076 & 11 & 34 & 0.9978 & 3.51 & 0.56 & 9.4 & 5 \\
7.4 & 0.66 & 0 & 1.8 & 0.075 & 13 & 40 & 0.9978 & 3.51 & 0.56 & 9.4 & 5 \\
7.9 & 0.6 & 0.06 & 1.6 & 0.069 & 15 & 59 & 0.9964 & 3.3 & 0.46 & 9.4 & 6 \\
7.3 & 0.65 & 0 & 1.2 & 0.065 & 15 & 21 & 0.9946 & 3.39 & 0.47 & 10 & 7 \\
7.8 & 0.58 & 0.02 & 2 & 0.073 & 9 & 18 & 0.9968 & 3.36 & 0.57 & 9.5 & 7
\end{bmatrix}$$

Why should we learn Matrices?

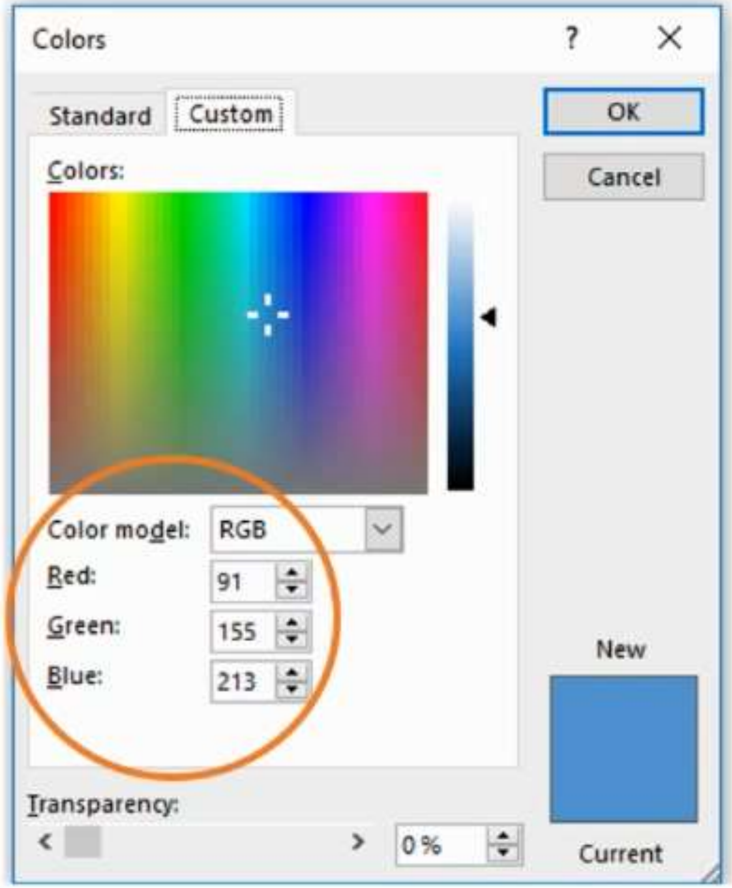
$$
\begin{bmatrix}
7.4 & 0.7 & 0 & 1.9 & 0.076 & 11 & 34 & 0.9978 & 3.51 & 0.56 & 9.4 & 5 \\
7.8 & 0.88 & 0 & 2.6 & 0.098 & 25 & 67 & 0.9968 & 3.2 & 0.68 & 9.8 & 5 \\
7.8 & 0.76 & 0.04 & 2.3 & 0.092 & 15 & 54 & 0.997 & 3.26 & 0.65 & 9.8 & 5 \\
11.2 & 0.28 & 0.56 & 1.9 & 0.075 & 17 & 60 & 0.998 & 3.16 & 0.58 & 9.8 & 6 \\
7.4 & 0.7 & 0 & 1.9 & 0.076 & 11 & 34 & 0.9978 & 3.51 & 0.56 & 9.4 & 5 \\
7.4 & 0.66 & 0 & 1.8 & 0.075 & 13 & 40 & 0.9978 & 3.51 & 0.56 & 9.4 & 5 \\
7.9 & 0.6 & 0.06 & 1.6 & 0.069 & 15 & 59 & 0.9964 & 3.3 & 0.46 & 9.4 & 6 \\
7.3 & 0.65 & 0 & 1.2 & 0.065 & 15 & 21 & 0.9946 & 3.39 & 0.47 & 10 & 7 \\
7.8 & 0.58 & 0.02 & 2 & 0.073 & 9 & 18 & 0.9968 & 3.36 & 0.57 & 9.5 & 7
\end{bmatrix}
$$

Why should we learn Matrices?

Matrix of Pixels



$$\begin{array}{c} R \\ G \\ B \end{array} \begin{bmatrix} 230 \\ 169 \\ 43 \end{bmatrix}$$



Matrix Addition

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 8 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$X + Y = \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Matrix Addition

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 8 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$X + Y = \begin{bmatrix} 2+1 & 3+8 & 4+(-1) \\ 1+5 & 6+(-2) & 7+(-3) \end{bmatrix} = \begin{bmatrix} 3 & 11 & 3 \\ 6 & 4 & 4 \end{bmatrix}$$

Matrix Subtraction

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 8 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Matrix Subtraction

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 8 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 2 - 1 & 3 - 8 & 4 - (-1) \\ 1 - 5 & 6 - (-2) & 7 - (-3) \end{bmatrix} = \begin{bmatrix} 1 & -5 & 5 \\ -4 & 8 & 10 \end{bmatrix}$$

Matrix Multiplication

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

Scalar Multiplication

$$2 * X = 2 * \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

Matrix Multiplication

$$X \,.\, A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

Matrix Multiplication – Scalar

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$2 * X = 2 * \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

Matrix Multiplication – Scalar

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$2 * X = 2 * \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 6 & 8 \\ 2 & 12 & 14 \end{bmatrix}$$

Let's see an example

	Average Price
Sports Shoes	$ 40
Formal	$ 30
Sandals	$ 20

	2016	2017	2018
Sports Shoes	2	3	3
Formal	3	4	3
Sandals	6	8	9

	2016	2017	2018
Sports Shoes	2 * 40	3 * 40	3 * 40
Formal	3 * 30	4 * 30	3 * 30
Sandals	6 * 20	8 * 20	9 * 20

	2016	2017	2018
Sports Shoes	80	120	120
Formal	90	120	90
Sandals	120	160	180
Total	**290**	**400**	**390**

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$2 \times \boxed{\underline{3} \qquad\qquad \underline{3}} \times 2$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X \,.\, A =$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X \,.\, A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X\,.\,A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X \,.\, A = \begin{bmatrix} 2 & 3 & 4 \\ \color{red}{1} & \color{red}{6} & \color{red}{7} \end{bmatrix} \begin{bmatrix} 2 & \color{red}{3} \\ 1 & \color{red}{6} \\ 2 & \color{red}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} 15 & 36 \\ 22 & 60 \end{bmatrix}$$

$$(1*3) + (6*6) + (7*3) = 60$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix} \qquad X \, . \, A = \begin{bmatrix} 15 & 36 \\ 22 & 60 \end{bmatrix}$$

$$2 \; X \; \boxed{3 \qquad\qquad\qquad 3} \; X \; 2 \qquad\qquad\qquad 2 \; X \; 2$$

Matrix Division

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 7 & -3 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$\frac{A}{X} =$$

Matrix Division

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 7 & -3 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$\frac{A}{X} = \cancel{\frac{\begin{bmatrix} 3 & 4 & -1 \\ 7 & -3 & 2 \end{bmatrix}}{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}}}$$

Matrix Division

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 7 & -3 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$\frac{A}{X} = A \,.\, X^{-1}$$

Inverse of a Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$1/A = \text{Inverse of } A = A^{-1}$$

$$A^{-1} =$$

Inverse of a Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$1/A = \text{Inverse of } A = A^{-1}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Determinant of a Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Determinant} = ad - bc$$

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 3 & 8 \end{bmatrix}$$

$$A * I =$$

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 3 & 8 \end{bmatrix}$$

$$A * I = A$$

Transposing Vectors/Matrix

- The values are not changing or transforming only their position is.
- Transposing the same vector (object) twice yields the initial vector (Object).
- A 3X1 matrix transposed is a 1X3 matrix.

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \Longrightarrow X^T = \begin{bmatrix} \quad \end{bmatrix}$$

Transposing Vectors/Matrix

- The values are not changing or transforming only their position is.
- Transposing the same vector (object) twice yields the initial vector (Object).
- A 3X1 matrix transposed is a 1X3 matrix.

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \longrightarrow X^T = \begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$$