

 Python
NumPy

Mathematical Computing with Python (NumPy)

Quick Recap : List

A list is a collection of values. You can individually add, remove, or update these values. A single list can contain multiple data types.

List

```
distance=[10,15,17,26]  
time=[.30,.47,.55,1.20]
```

Collection of values

Multiple types (heterogeneous)

Add, remove, update

Limitations of lists

Though you can change individual values in a list, you cannot apply a mathematical operation over the entire list.

```
distance=[10,15,17,26]
time=[.30,.47,.55,1.20]
```

```
speed=distance/time
```

← Mathematical operation over the entire “distance” and “time” lists

```
-----
TypeError                                Traceback (most recent call last)
<ipython-input-37-b779bad68500> in <module>()
----> 1 speed=distance/time
```

```
TypeError: unsupported operand type(s) for /: 'list' and 'list'
```

← Error

Why Numpy

Numerical Python (NumPy) supports multidimensional arrays over which you can easily apply mathematical operations.

```
distance=[10,15,17,26]  
time=[.30,.47,.55,1.20]
```

```
import numpy as np
```

 ← Import NumPy

```
np_distance = np.array(distance)  
np_time=np.array(time)  
speed=np_distance/np_time
```

 } Create "distance" and "time" NumPy arrays
← Mathematical function applied over the entire "distance" and "time" arrays

```
speed
```

```
array([ 33.33333333,  31.91489362,  30.90909091,  21.66666667])
```

 ← Output

NumPy Overview

NumPy is the foundational package for mathematical computing in Python.

It has the following properties:

Supports fast and efficient
multidimensional arrays (ndarray)



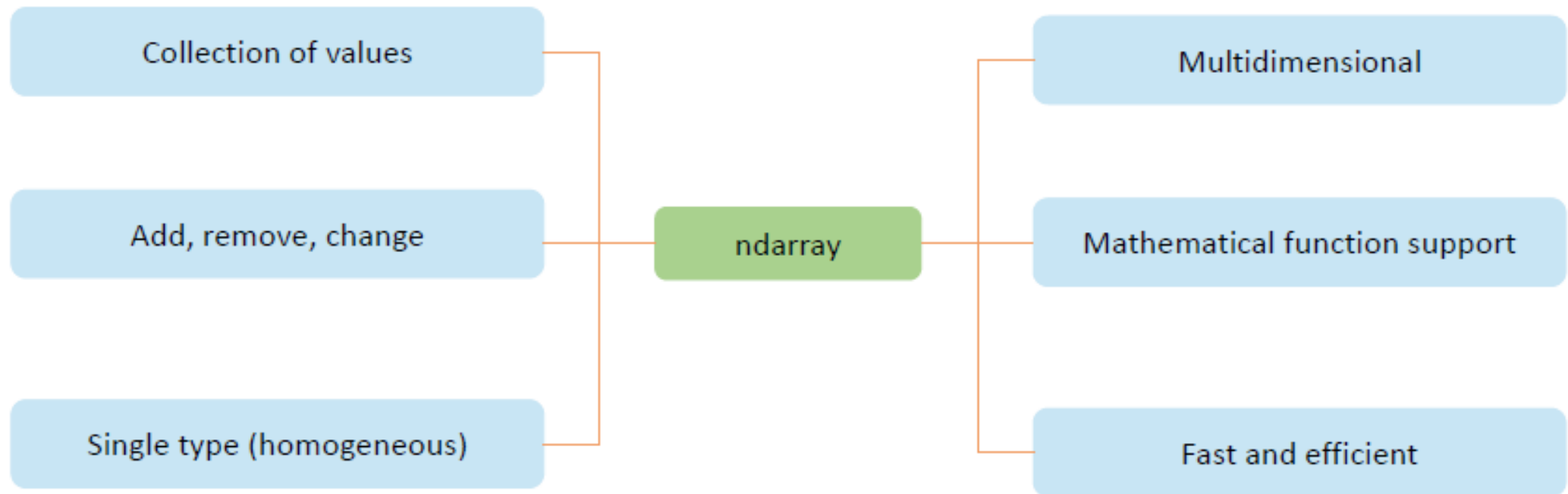
Performs linear algebraic
operations, Fourier transforms,
and random number generation



Executes element-wise
computations and mathematical
calculations

Properties of ndarray

An array in NumPy has the following properties:



Knowledge Check – Sequence it Right !

The code here is buggy. You have to correct its sequence to debug it.

1

```
distance=[10,15,17,26]  
time=[.30,.47,.55,1.20]
```

2

```
np_distance = np.array(distance)  
np_time=np.array(time)
```

3

```
import numpy as np
```

4

```
speed=np_distance/np_time  
  
speed  
  
array([ 33.33333333,  31.91489362,  30.90909091,  21.66666667])
```

Knowledge Check – Sequence it Right !

The code here is buggy. You have to correct its sequence to debug it.

1

```
distance=[10,15,17,26]  
time=[.30,.47,.55,1.20]
```

2

```
import numpy as np
```

3

```
np_distance = np.array(distance)  
np_time=np.array(time)
```

4

```
speed=np_distance/np_time  
  
speed  
array([ 33.33333333,  31.91489362,  30.90909091,  21.66666667])
```


Types of Arrays

Arrays can be one-dimensional, two dimensional, three-dimensional, or multi-dimensional.

One-Dimensional Array

Printed as rows

`array([5, 7, 9])` ← 1 axis
rank 1
Length = 3

5	7	9
---	---	---

0 1 2
x axis

Two-Dimensional Array

Printed as matrices (2x3)

`array([[0, 1, 2],
[5, 6, 7]])` ← 2 axes
rank 2
Length = 3

0 (0,0)	1 (0,1)	2 (0,2)
5 (1,0)	6 (1,1)	7 (1,2)

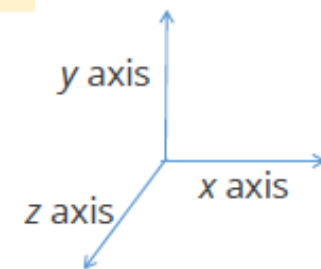
y axis

x axis

Three-Dimensional Array

Printed as list of matrices (3x3x3)

`array([[[0, 1, 2],
[3, 4, 5],
[6, 7, 8]],
[[9, 10, 11],
[12, 13, 14],
[15, 16, 17]],
[[18, 19, 20],
[21, 22, 23],
[24, 25, 26]]])` ← 3 axes
rank 3
Length = 3



Class and Attributes of ndarray.ndim

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

This refers to the number of axes (dimensions) of the array. It is also called the rank of the array.

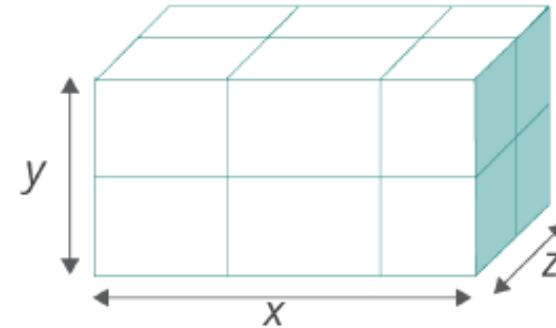
ndarray.shape

ndarray.size

ndarray.dtype



Two axes or 2D array



Three axes or 3D array

Concept

Example

Class and Attributes of ndarray.ndim

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

The array "np_city" is one-dimensional, while the array "np_city_with_state" is two-dimensional.

ndarray.shape

```
In [108]: np_city = np.array(['NYC', 'LA', 'Miami', 'Houston'])
```

```
In [109]: np_city.ndim
```

```
Out[109]: 1
```

ndarray.size

```
In [110]: np_city_with_state = np.array(['NYC', 'LA', 'Miami', 'Houston'], ['NY', 'CA', 'FL', 'TX'])
```

```
In [111]: np_city_with_state.ndim
```

```
Out[111]: 2
```

ndarray.dtype

Concept

Example

Class and Attributes of ndarray-.shape

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

ndarray.shape

ndarray.size

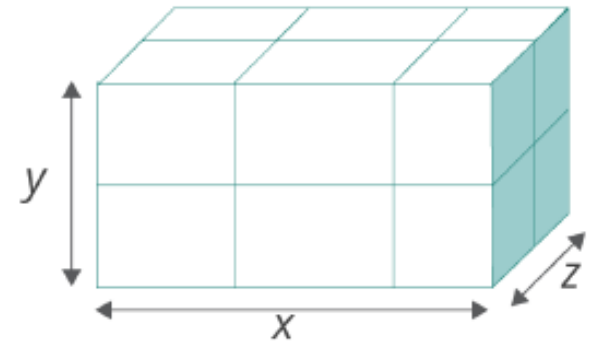
ndarray.dtype

This consists of a tuple of integers showing the size of the array in each dimension. The length of the "shape tuple" is the rank or ndim.



2 rows, 3 columns

Shape: (2, 3)



2 rows, 3 columns, 2 ranks

Shape: (2, 3, 2)

Concept

Example

Class and Attributes of ndarray-.shape

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

ndarray.shape

ndarray.size

ndarray.dtype

The shape tuple of both the arrays indicate their size along each dimension.

```
In [108]: np_city = np.array(['NYC', 'LA', 'Miami', 'Houston'])
```

```
In [110]: np_city_with_state = np.array([[ 'NYC', 'LA', 'Miami', 'Houston'], [ 'NY', 'CA', 'FL', 'TX']])
```

```
In [112]: np_city.shape
```

```
Out[112]: (4L,)
```

```
In [113]: np_city_with_state.shape
```

```
Out[113]: (2L, 4L)
```

Concept

Example

Class and Attributes of ndarray-.size

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

ndarray.shape

ndarray.size

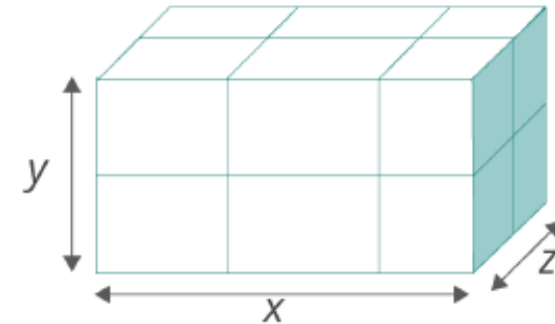
ndarray.dtype

It gives the total number of elements in the array. It is equal to the product of the elements of the shape tuple.



Array contains 6 elements

Array a = (2, 3)
Size = 6



Array contains 12 elements

Array b = (2, 3, 2)
Size = 12

Concept

Example

Class and Attributes of ndarray.size

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

ndarray.shape

ndarray.size

ndarray.dtype

Look at the examples to see how the shape tuples of the arrays are used to calculate their size.

```
In [112]: np_city.shape
```

```
Out[112]: (4L,)
```

```
In [113]: np_city_with_state.shape
```

```
Out[113]: (2L, 4L)
```

```
In [114]: np_city.size
```

```
Out[114]: 4
```

```
In [115]: np_city_with_state.size
```

```
Out[115]: 8
```

Concept

Example

Class and Attributes of ndarray.dtype

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

ndarray.shape

ndarray.size

ndarray.dtype

It's an object that describes the type of the elements in the array. It can be created or specified using Python.

Array contains integers

Array a = [3, 7, 4]
 [2, 1, 0]

Array contains floats

Array b = [1.3, 5.2, 6.7]
 [0.2, 8.1, 9.4]

 [2.6, 4.2, 3.9]
 [7.8, 3.4, 0.8]

Concept

Example

Class and Attributes of ndarray.dtype

Numpy's array class is "ndarray," also referred to as "numpy.ndarray." The attributes of ndarray are:

ndarray.ndim

ndarray.shape

ndarray.size

ndarray.dtype

Both the arrays are of "string" data type (dtype) and the longest string is of length 7, which is "Houston."

```
In [116]: np_city
```

```
Out[116]: array(['NYC', 'LA', 'Miami', 'Houston'],  
               dtype='<S7')
```

```
In [117]: np_city_with_state
```

```
Out[117]: array([[ 'NYC', 'LA', 'Miami', 'Houston'],  
               [ 'NY', 'CA', 'FL', 'TX']],  
               dtype='<S7')
```

```
In [118]: np_city_with_state.dtype
```

```
Out[118]: dtype('<S7')
```

Concept

Example

Basic Operations

Using the following operands, you can easily apply various mathematical, logical, and comparison operations on an array.

Mathematical Operations

Addition	+
Subtraction	-
Multiplication	*
Division	/
Exponentiation	**

Logical Operations

And	&
Or	
Not	~

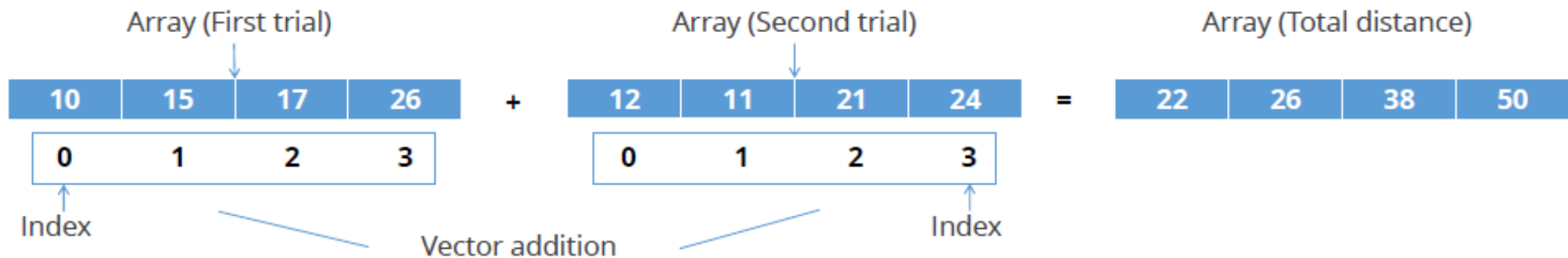
Comparison Operations

Greater	>
Greater or equal	>=
Less	<
Less or equal	<=
Equal	==
Not equal	!=

Basic Operation – Example

NumPy uses the indices of the elements in each array to carry out basic operations. In this case, where we are looking at a dataset of four cyclists during two trials, vector addition of the arrays gives the required output.

```
In [99]: first_trial_cyclist = [10, 15, 17, 26] ← First trial
In [100]: second_trial_cyclist = [12, 11, 21, 24] ← Second trial
In [101]: np_first_trial_cyclist = np.array(first_trial_cyclist)
In [102]: np_second_trial_cyclist = np.array(second_trial_cyclist)
In [103]: np_first_trial_cyclist + np_second_trial_cyclist ← Total distance
Out[103]: array([22, 26, 38, 50])
```



Accessing Array Elements: Indexing

You can access an entire row of an array by referencing its axis index.

1st set data 2nd set data

```
In [117]: cyclist_trials = np.array([[10, 15, 17, 26], [12, 11, 21, 24]]) ← Create 2D array using cyclist trial data shown earlier
```

```
In [118]: first_trial = cyclist_trials[0] ← First trial data
```

```
In [119]: first_trial
```

```
Out[119]: array([10, 15, 17, 26])
```

```
In [120]: second_trial = cyclist_trials[1] ← Second trial data
```

```
In [121]: second_trial
```

```
Out[121]: array([12, 11, 21, 24])
```

2D array containing cyclists' data

10	15	17	26	← First trial (axis 0)
12	11	21	24	← Second trial (axis 1)

Accessing Array Elements: Indexing (contd.)

You can refer the indices of the elements in an array to access them. You can also select a particular index of more than one axis at a time.

```
In [122]: first_cyclist_firstTrial = cyclist_trials[0][0] ← First cyclist: first trial data
```

```
In [123]: first_cyclist_firstTrial
```

```
Out[123]: 10
```

```
In [124]: first_cyclist_all_trials = cyclist_trials[:,0] ← First cyclist: all trial data  
(Use ":" to select all the rows of an array)
```

```
In [125]: first_cyclist_all_trials
```

```
Out[125]: array([10, 12])
```

Cyclist 1, first trial data →

(0, 0)	(0, 1)	(0, 2)	(0, 3)
10	15	17	26
12	11	21	24
(1, 0)	(1, 1)	(1, 2)	(1, 3)

Cyclist 1, all trials data {

(0, 0)	(0, 1)	(0, 2)	(0, 3)
10	15	17	26
12	11	21	24
(1, 0)	(1, 1)	(1, 2)	(1, 3)

Accessing Array Elements: Slicing

Use the slicing method to access a range of values within an array.

Shape of the array

```
In [152]: cyclist_trials.shape
Out[152]: (2L, 4L)
```

Shape of the array

```
In [153]: two_cyclist_trial_data=cyclist_trials[:,1:3]
In [154]: two_cyclist_trial_data
Out[154]: array([[15, 17],
                 [11, 21]])
```

Slicing the array data `[:, 1:3]`
where 1 is inclusive but 3 is not

Shape of the array

10	15	17	26
12	11	21	24

2 rows

4 columns

Slicing the array

10	15	17	26
12	11	21	24

Use ':' to select all rows

0 1 2 3

Starting index (1) Ending index (2)

Accessing Array Elements: Iteration

Use the iteration method to go through each data element present in the dataset.

```
In [117]: cyclist_trials = np.array([[10,15,17,26],[12,11,21,24]])
```

```
In [153]: two_cyclist_trial_data=cyclist_trials[:,1:3]
```

```
In [154]: two_cyclist_trial_data
```

```
Out[154]: array([[15, 17],  
                [11, 21]])
```

```
In [159]: for iterate_cyclist_trials_data in cyclist_trials:  
           print (iterate_cyclist_trials_data)
```

```
[10 15 17 26]  
[12 11 21 24]
```

Iterate with "for loop"
through entire dataset

```
In [160]: for iterate_two_cyclist_trial_data in two_cyclist_trial_data:  
           print (iterate_two_cyclist_trial_data)
```

```
[15 17]  
[11 21]
```

Iterate with "for loop" through
the "two cyclist" datasets

Indexing with Boolean Arrays

Boolean arrays are useful when you need to select a dataset according to set criteria. Here, the original dataset contains test scores of two students. You can use a Boolean array to choose only the scores that are above a given value.

Test scores

Student 1	83	71	57	63
Student 2	54	68	81	45

Test 1 Test 2 Test 3 Test 4

Test score > 60

83	71	57	63
54	68	81	45

```
In [234]: test_scores = np.array([[83, 71, 57, 63], [54, 68, 81, 45]])
```

```
In [235]: passing_score = test_scores > 60
```

Setting the passing score

```
In [236]: passing_score
```

```
Out[236]: array([[ True,  True, False,  True],
                 [False,  True,  True, False]], dtype=bool)
```

Shows data elements which fit the criteria (Boolean array)

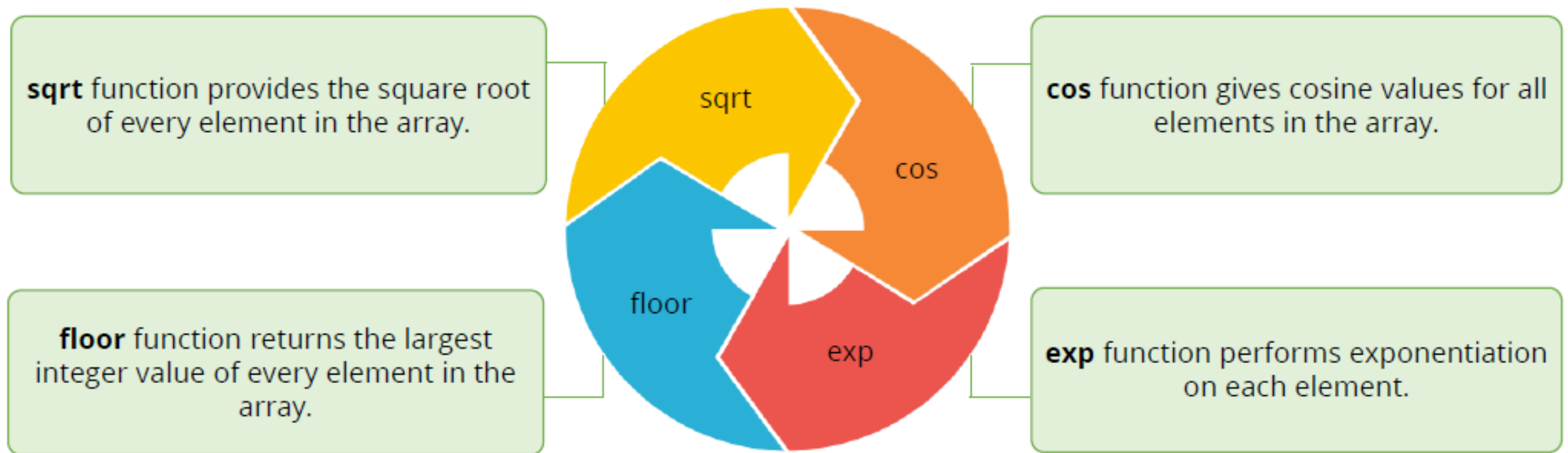
```
In [237]: test_scores[passing_score]
```

Send "passing score" as an argument to "test scores" object

```
Out[237]: array([83, 71, 63, 68, 81])
```


Universal Functions (ufunc)

NumPy provides useful mathematical functions called Universal Functions. These functions operate element-wise on an array, producing another array as output. Some of these functions are listed here:



ufunc-Examples

Let's look at some common ufunc examples:

In [186]: `np_sqrt = np.sqrt([2,4,9,16])` ← Numbers for which square root will be calculated

In [187]: `np_sqrt`

Out[187]: `array([1.41421356, 2. , 3. , 4.])` ← Square root values

In [188]: `from numpy import pi` ← Import pi*
`np.cos(0)`

Out[188]: `1.0`

In [189]: `np.sin(pi/2)` ← Trigonometric functions

Out[189]: `1.0`

In [190]: `np.cos(pi)` ← Trigonometric functions

Out[190]: `-1.0`

In [191]: `np.floor([1.5,1.6,2.7,3.3,1.1,-0.3,-1.4])` ← Return the floor of the input element wise

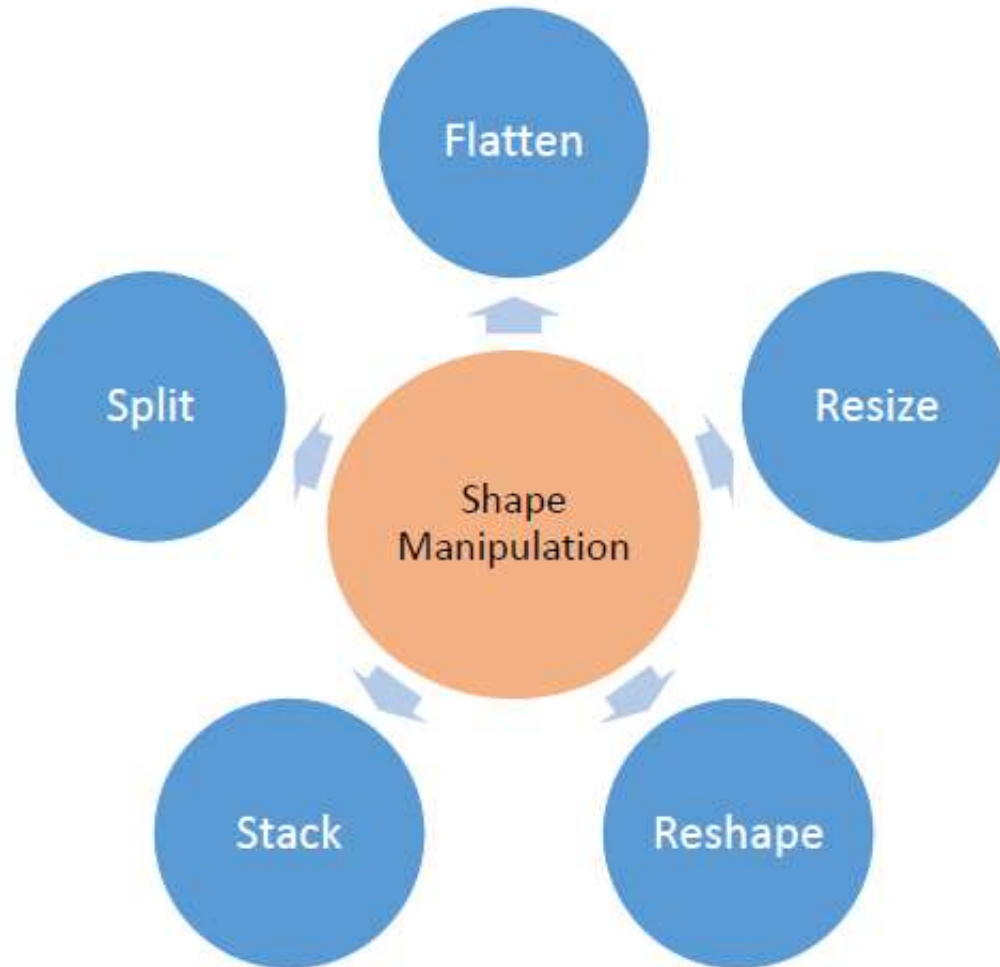
Out[191]: `array([1., 1., 2., 3., 1., -1., -2.])`

In [192]: `np.exp([0,1,5])` ← Exponential functions for complex mathematical calculations

Out[192]: `array([1. , 2.71828183, 148.4131591])`

Shape Manipulation

You can use certain functions to manipulate the shape of an array to do the following:



Shape Manipulation - Example

You can use certain functions to manipulate the shape of an array to do the following:

```
In [383]: new_cyclist_trials = np.array([[10,15,17,26,13,19],[12,11,21,24,14,23]])
```

```
In [384]: new_cyclist_trials.ravel()
```

← Flattens the dataset

```
Out[384]: array([10, 15, 17, 26, 13, 19, 12, 11, 21, 24, 14, 23])
```

```
In [385]: new_cyclist_trials.reshape(3,4)
```

← Changes or reshapes the dataset to 3 rows and 4 columns

```
Out[385]: array([[10, 15, 17, 26],  
                [13, 19, 12, 11],  
                [21, 24, 14, 23]])
```

```
In [386]: new_cyclist_trials.resize(2,6)
```

← Resizes again to 2 rows and 6 columns

```
In [387]: new_cyclist_trials
```

```
Out[387]: array([[10, 15, 17, 26, 13, 19],  
                [12, 11, 21, 24, 14, 23]])
```

```
In [388]: np.hsplit(new_cyclist_trials,2)
```

← Splits the array into two

```
Out[388]: [array([[10, 15, 17],  
                [12, 11, 21]]), array([[26, 13, 19],  
                [24, 14, 23]])]
```

```
In [389]: new_cyclist_1 = np.array([10,15,17,26,13,19])
```

```
In [390]: new_cyclist_2 = np.array([12,11,21,24,14,23])
```

```
In [391]: np.hstack((new_cyclist_1,new_cyclist_2))
```

← Stacks the arrays together

```
Out[391]: array([10, 15, 17, 26, 13, 19, 12, 11, 21, 24, 14, 23])
```

NumPy uses broadcasting to carry out arithmetic operations between arrays of different shapes. In this method, NumPy automatically broadcasts the smaller array over the larger array.

```
In [9]: import numpy as np

In [10]: #Create two arrays of the same shape
array_a = np.array([2, 3, 5, 8])
array_b = np.array([.3, .3, .3, .3])

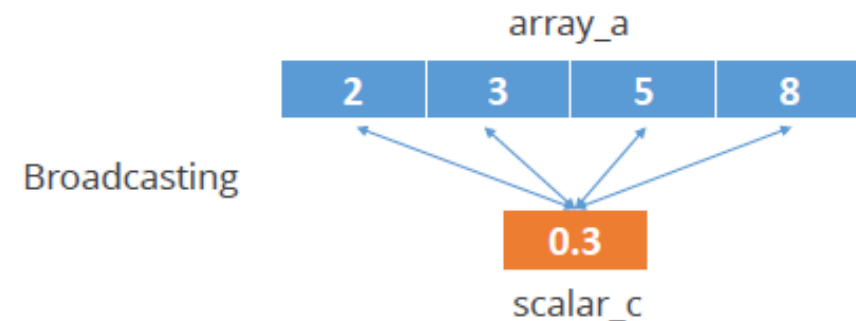
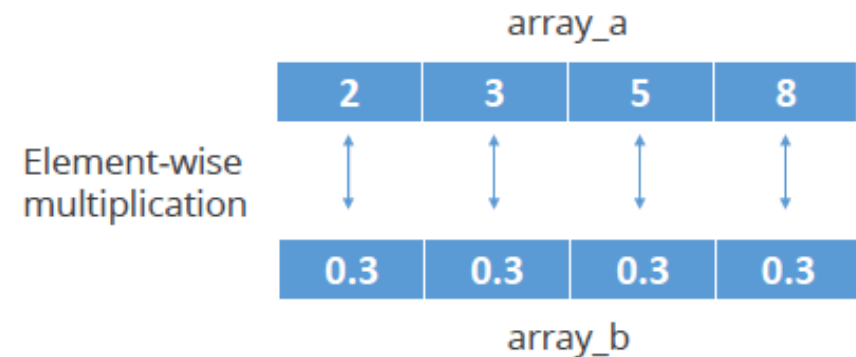
In [11]: #Multiply arrays
array_a * array_b

Out[11]: array([ 0.6,  0.9,  1.5,  2.4])

In [12]: #Create a variable with a scalar value
scalar_c = .3

In [13]: #Multiply 1D array with a scalar value
array_a * scalar_c

Out[13]: array([ 0.6,  0.9,  1.5,  2.4])
```



Broadcasting - Constraints

Though broadcasting can help carry out mathematical operations between different-shaped arrays, they are subject to certain constraints as listed below:

```
In [9]: import numpy as np
```

```
In [10]: #Create two arrays of the same shape
array_a = np.array([2, 3, 5, 8])
array_b = np.array([.3, .3, .3, .3])
```

```
In [11]: #Multiply arrays
array_a * array_b
```

```
Out[11]: array([ 0.6,  0.9,  1.5,  2.4])
```

```
In [14]: #Create array of a different shape
array_d = np.array([4, 3])
```

```
In [15]: array_a * array_d
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-15-43adcf6f7a54> in <module>()
----> 1 array_a * array_d

ValueError: operands could not be broadcast together with shapes (4,) (2,)
```

- When NumPy operates on two arrays, it compares their shapes element-wise. It finds these shapes compatible only if:
 - Their dimensions are the same or
 - One of them has a dimension of size 1.
- If these conditions are not met, a "ValueError" is thrown, indicating that the arrays have incompatible shapes.

Broadcasting - Example

Let's look at an example to see how broadcasting works to calculate the number of working hours of a worker per day in a certain week.

```
In [246]: np_week_one = np.array([105, 135, 195, 120, 165])  
          np_week_two = np.array([123, 156, 230, 200, 147])
```

← Week one earnings
← Week two earnings

```
In [247]: total_earning = np_week_one + np_week_two
```

```
In [248]: total_earning
```

```
Out[248]: array([228, 291, 425, 320, 312])
```

← Total earning for 2 weeks

```
In [249]: np_week_one_hrs = np_week_one / 15
```

← Calculate week one hours

```
In [250]: np_week_one_hrs
```

← Hourly wage

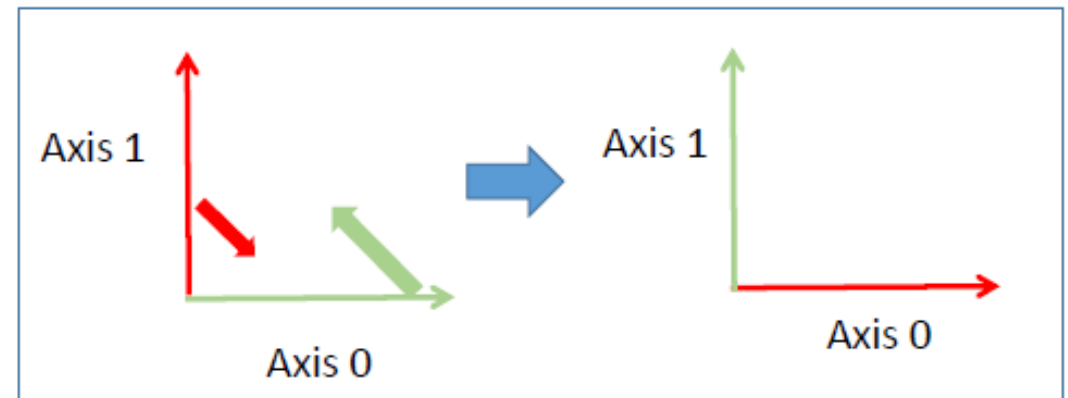
```
Out[250]: array([ 7,  9, 13,  8, 11])
```

← Number of working hours per day in week one

Linear Algebra - Transpose

NumPy can carry out linear algebraic functions as well. The “transpose()” function can help you interchange rows as columns, and vice-versa.

transpose()			
83	71	57	63
54	68	81	45



```
In [397]: test_scores = np.array([[83, 71, 57, 63], [54, 68, 81, 45]])
```

```
In [398]: test_scores.transpose()
```

```
Out[398]: array([[83, 54],  
                [71, 68],  
                [57, 81],  
                [63, 45]])
```


Linear Algebra – Inverse and Trace Functions

Using NumPy, you can also find the inverse of an array and add its diagonal data elements.

np.linalg.inv()

```
In [411]: inverse_array = np.array([[10,20],[15,25]])
```

```
In [412]: np.linalg.inv(inverse_array)
```

```
Out[412]: array([[ -0.5,  0.4],  
                [ 0.3, -0.2]])
```

← Inverse of the given array

* Can be applied only on a square matrix

np.trace()

```
In [420]: trace_array = np.array([[10,20],[22,31]])
```

```
In [421]: np.trace(trace_array)
```

```
Out[421]: 41
```

← Sum of diagonal elements "10" and "31"



Q2.

The `np.trace()` method gives the sum of _____.

- a. the entire array
- b. the diagonal elements from left to right
- c. the diagonal elements from right to left
- d. consecutive rows of an array

Q3.

Which statement will slice the highlighted data?

11	14	21	32	53	64
----	----	----	----	----	----

- a. `[3 : 5]`
- b. `[3 : 6]`
- c. `[2 : 5]`
- d. `[2 : 4]`



Assignment

Problem

Instructions

Evaluate the dataset of the Summer Olympics, London 2012 to:

- Find and print the name of the country that won maximum gold medals,
- Find and print the countries who won more than 20 gold medals,
- Print the medal tally,
- Print each country name with the corresponding number of gold medals, and
- Print each country name with the total number of medals won.

```
olympic_country=['GBR','China','RUS','US','KOR','JPN','GER']  
olympic_country_Gold=[29,38,24,46,13,7,11]  
olympic_country_Silver=[17,28,25,28,8,14,11]  
olympic_country_Bronze=[19,22,32,29,7,17,14]
```