Compiled Content
Module 1

MScFE 640

Portfolio Theory and Asset Pricing

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Module 1: Single Period Mean-variance Portfolio Theory and Asset Pricing

Module 1 begins by introducing mean return and the volatility of a multi-asset portfolio. The module continues by discussing the Sharpe ratio and the efficient frontier, before proceeding to discuss common emotional biases and price versus intrinsic value. At the end of the module, the single stage Gordon Growth Model and residual income valuation are explained.

Unit 1: Mean Return and Volatility of a Multi-asset Portfolio

These notes will demonstrate how to calculate the mean return and volatility of a portfolio of assets. They will define risk, and explain the importance of correlation between individual asset returns, and the impact this has on overall portfolio volatility

Harry Markowitz and Modern Portfolio Theory

Diversification of a portfolio has always been an intuitively appealing idea but it wasn't until 1952 that Harry Markowitz's classic article on portfolio selection provided the underlying theory of this concept and its application to investments (Markowitz, 1952). This article provided the foundation for what is now known as modern portfolio theory (Conroy and Byrne, 2015). The fundamental goal of portfolio theory is to optimally allocate a portfolio between different assets. In this context, the optimal portfolio is the one which has the highest return for the amount of risk taken. A less optimal portfolio will have the same return but higher risk, or the same amount of risk but lower return. But what is risk?

In the world of finance, **risk** is measured as the volatility of returns. The **volatility of returns** is the standard deviation of returns, although the term volatility is typically used instead of standard deviation. Standard deviation is calculated as follows:

$$Standard\ deviation\ (\sigma) = \sqrt{\frac{\sum |x - \bar{x}|^2}{n}}.$$

Volatility is denoted with the lower-case Greek alphabet letter *Sigma*, σ.

Example: Calculating asset volatility

Year	Return (x)	$(x-\bar{x})^2$
1	3.2%	0.0028%
2	7.8%	0.2632%
3	5.1%	0.0590%
4	-2.2%	0.2372%
5	-1.0%	0.1347%
6	2.6%	0.0000%
7	4.1%	0.0204%
8	9.2%	0.4264%
9	-4.9%	0.5730%
10	2.8%	0.0002%
∑ =	2.7%	1.7169%
n =	10	

$$\sigma = \sqrt{\frac{0.01717}{10}} = 0.0414 = 4.14\%$$

(Note: Microsoft Excel will calculate volatility of 4.36%. This is because it uses $(n-1)$ in the denominator instead of n which has been used in this calculation. Excel's formula calculates the *sample* volatility, while the formula used in the example calculates *population* volatility.)

An important insight of modern portfolio theory is that when there is more than one asset in a portfolio the correlation between the assets has an important impact on portfolio risk. If the correlation between two assets is low or negative, when one asset's performance is bad the other asset's performance will be better. By combining assets with low or negative correlation, the volatility of a portfolio can be reduced without impacting expected return of the portfolio.

Asset portfolios

Mean return

The **mean return** of a portfolio in a period is the sum of the individual asset returns, weighted based on beginning-of-period value. A **period** is defined as the time between the inflow or outflow of any cash, including withdrawals or contributions and asset distributions such as dividends or interest.

Example: Calculating mean portfolio return

The table below presents information on a portfolio consisting of three assets. The period ends with the receipt of the dividend of $25 from asset DEF.

Asset	Beginning of period value	Change in capital value	Dividend	End of period value	Return
ABC	$800	-$50		$750	-6.25%
DEF	$1 100	$75	$25	$1 200	9.09%
GHI	$600	$30		$630	5.00%
∑	$2 500			$2 580	

The return of the portfolio in the period is:

$$= \left({}^{800}/_{2500}\right)(-6.25\%) + \left({}^{1100}/_{2500}\right)(9.09\%) + \left({}^{600}/_{2500}\right)(5\%)$$

$$= 3.2\%$$

Arithmetic versus geometric mean return

The difference between arithmetic mean and geometric mean is important to understand when dealing with asset returns. Arithmetic mean is calculated with the following familiar equation:

$$\bar{x}_{Arithmetic} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}.$$

Geometric mean is calculated with the following, possibly less familiar, equation:

$$\bar{x}_{Geometric} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n}.$$

The geometric mean should be used when the observations are not independent. This is the case in asset returns. If your portfolio declines by 50% in the first period, you have significantly less capital to invest in the second period. The geometric mean of periodic returns will take account of this, while the arithmetic mean will not. The following example will demonstrate this.

Example: Demonstrating the difference between arithmetic and geometric mean return

In the previous example, the mean return of the portfolio was calculated to be 3.2%. Let's call this period 1, and add two more periods:

PERIOD 2:					
Asset	Beginning of period value	Change in capital value	Dividend	End of period value	Return
ABC	\$750	\$100	\$20	\$870	16.00%
DEF	\$1 200	-\$600		\$600	-50.00%
GHI	\$630	-\$25		\$605	-3.97%
∑	\$2 580			\$2 075	

Return of portfolio in period 2 = -19.57%

PERIOD 3:					
Asset	Beginning of period value	Change in capital value	Dividend	End of period value	Return
ABC	\$870	\$50		\$920	5.75%
DEF	\$600	-\$25		\$575	-4.17%
GHI	\$605	\$30	\$15	\$650	7.44%
∑	\$2 075			\$2 145	

Return of portfolio in period 3 = 3.37%

The arithmetic mean of the three periods' returns = -4.33% and the geometric mean = -4.98%. The geometric mean is calculated as follows:

$$= \sqrt[3]{(1+3.2\%)(1+(-19.57\%))(1+3.37\%)} - 1$$

$$= -4.977\%$$

Which rate of return is correct? Let us calculate the future value of the period 1 beginning value (\$2 500) using both rates:

Using arithmetic mean return:

$$Future\ value = (\$2500)\big(1+(-4.33\%)\big)^3$$

$$Future\ value = \$2188.88$$

Using geometric mean return:

$$Future\ value = (\$2500)\big(1 + (-4.98\%)\big)^3$$

$$Future\ value = \$2145.00$$

The geometric mean return calculates the correct ending value of the portfolio.

Note: if you perform the above calculations on a calculator you will calculate a future value of \$2189.11 and \$2144.79 for the arithmetic and geometric mean respectively. The figures of \$2188.88 and \$2145.00 were calculated using Excel, therefore there were no rounding errors.

Portfolio volatility

As mentioned previously, the correlation between two assets in a portfolio consisting of only these two assets (called a **two-asset portfolio**) has an important impact on portfolio volatility. The volatility of a two-asset portfolio is calculated with the following equation:

$$\sigma_p = \sqrt{\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\sigma_1\omega_2\sigma_2\rho_{1,2}},$$

where:

σ_p = volatility of the portfolio

ω_1 = weight of asset 1 in the portfolio

σ_1 = volatility of asset 1

ω_2 = weight of asset 2 in the portfolio

σ_2 = volatility of asset 2

$\rho_{1,2}$ = correlation between asset 1 and asset 2 returns.

Example: Calculating the volatility of a two-asset portfolio

A portfolio manager intends to construct a portfolio consisting of two assets with the following characteristics:

Asset	Weight in portfolio	Volatility	Correlation with MNO
JKL	30%	20%	0.4
MNO	70%	12%	1

The resulting two-asset portfolio will have a volatility of:

$$\sigma_p = \sqrt{(0.3)^2(0.2)^2 + (0.7)^2(0.12)^2 + (2)(0.3)(0.2)(0.7)(0.12)(0.4)}$$

$$\sigma_p = 12.11\%$$

The volatility of a multi-asset portfolio can be calculated using an extension of the formula for a two-asset portfolio. For a portfolio consisting of n assets the portfolio volatility will be given by:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i,j=1,i\neq j}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{i,j}} \, .$$

Unit 2: Sharpe Ratio and the Efficient Frontier

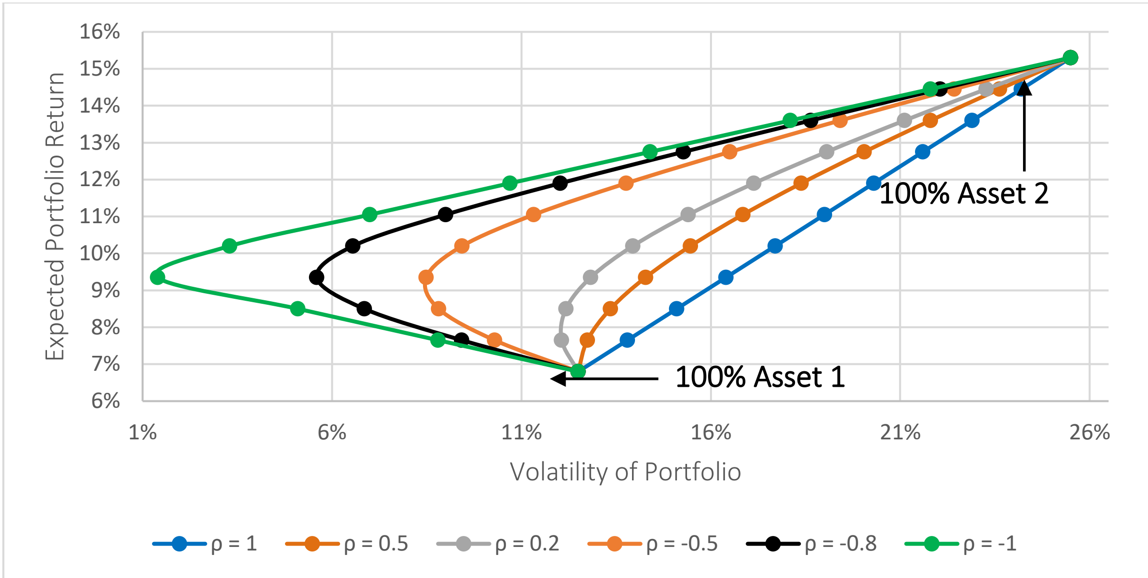
In the prior set of notes, we learnt that the mean return of a portfolio is the weighted-average of the returns of the assets making up the portfolio. We learnt that the volatility of a portfolio is dependent on the weights of the assets, their volatilities, and, importantly, the correlation between asset returns. We learnt that when the correlation between asset returns in a two-asset portfolio is less than 1, the volatility of the portfolio will always be less than the weighted-average of the individual asset volatilities. Thus, by combining assets in a portfolio whose returns are less than perfectly positively correlated, we can create more efficient portfolios – i.e. portfolios that achieve the same return but with lower risk compared to other portfolios. The following example will illustrate this.

Example: Mean return and volatility of a two-asset portfolio

There are two assets available for investment. Asset 1 has an expected return of 6.8% and an expected volatility of 12%. Asset 2 has an expected return of 15.3% and an expected volatility of 25%. If we assume correlations between the two assets of 1, 0.5, 0.2, -0.5, -0.8, and -1 we get the following expected portfolio return and volatility for various combinations of asset weights:

Weight of asset 1	Portfolio expected return	Portfolio expected volatility (ρ = 1)	Portfolio expected volatility (ρ = 0.5)	Portfolio expected volatility (ρ = 0.2)	Portfolio expected volatility (ρ = -0.5)	Portfolio expected volatility (ρ = -0.8)	Portfolio expected volatility (ρ = -1)
100%	6.8%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%
90%	7.7%	13.3%	12.2%	11.6%	9.8%	8.9%	8.3%
80%	8.5%	14.6%	12.9%	11.7%	8.3%	6.4%	4.6%
70%	9.4%	15.9%	13.8%	12.3%	8.0%	5.1%	0.9%
60%	10.2%	17.2%	15.0%	13.4%	8.9%	6.1%	2.8%
50%	11.1%	18.5%	16.3%	14.9%	10.8%	8.5%	6.5%
40%	11.9%	19.8%	17.9%	16.6%	13.3%	11.5%	10.2%
30%	12.8%	21.1%	19.6%	18.6%	16.0%	14.8%	13.9%
20%	13.6%	22.4%	21.3%	20.6%	18.9%	18.1%	17.6%
10%	14.5%	23.7%	23.1%	22.8%	21.9%	21.6%	21.3%
0%	15.3%	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%

The table graphed appears as follows:

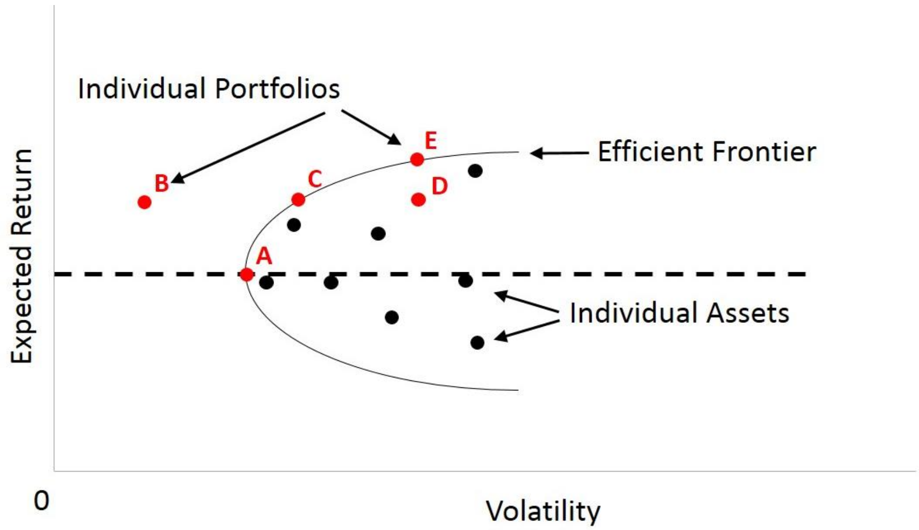


Modern portfolio theory and the benefits of diversification are clearly illustrated in the graph above. When the two assets are perfectly positively correlated (ρ=1),

increasing the weight of asset 2 in the portfolio increases expected portfolio return but also increases portfolio volatility. However, when ρ<1, adding asset 2 to the portfolio can increase expected portfolio return while reducing portfolio volatility. This is more pronounced the less positively correlated the assets are. In these notes we will continue to explore diversification.

Risky assets and the efficient frontier

When there are multiple assets available for investors to invest in, the number of possible portfolios (combinations of the assets) is large. All of the available portfolios, taken together, is known as the **opportunity set of investments**. The various assets are correlated with each other to various degrees, ranging between -1 (perfectly negatively correlated) and 1 (perfectly positively correlated). Consider the figure below:



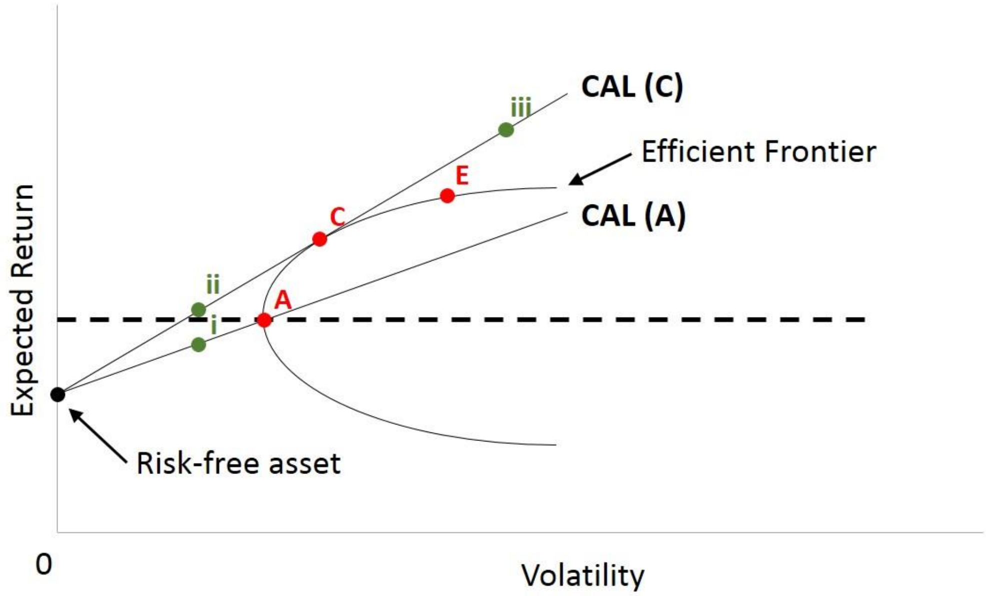
The investment opportunity set is all points on the curve and to the right of the curve. Portfolio A is known as the **global minimum variance portfolio**. This is the portfolio on the efficient frontier which has the lowest volatility. The portion of the curve that lies above and to the right of the global minimum variance portfolio is known as the **efficient frontier**. A portfolio on the efficient frontier has the highest return for a given level of volatility. The portfolio on the efficient frontier is said to **dominate** – i.e. be superior to – all other portfolios that have the same level of volatility.

Considering the portfolios B, C, and D, a risk-averse investor will choose portfolio B because it has the same return as C and D but lower volatility. However, portfolio B is not attainable because it is not within the investment opportunity set. If we must earn B, C,b and D's return then the most desirable portfolio is C as it has lower volatility than D. All portfolios to the right of portfolio C are feasible but have higher risk, making them inefficient.

Risk-free asset and the capital allocation line

Up until this point in the notes we have only considered risky assets – i.e. assets with an uncertain payoff in the future. However, in finance there is a class of securities known as **risk-free assets**. The risk-free asset is typically considered to be the bonds issued by first-world stable democracies. They have very low return compared to other available assets, but they will almost certainly be repaid. This is why they are considered to be the closest practical example of an asset that cannot lose value, and therefore is considered risk-free.

By definition, the risk-free asset must have a volatility of zero. A **capital allocation line** is a line drawn between the risk-free asset and a portfolio of risky assets on the efficient frontier. The figure below illustrates two capital allocation lines: one between the risk-free asset and portfolio A [CAL (A)] and another between the risk-free asset and portfolio C [CAL (C)].



The points along the capital allocation line graph the expected return and risk of some combination of the risk-free asset and a portfolio of risky assets. If we consider portfolio i on CAL (A) and portfolio ii on CAL (C): portfolio ii is superior to i as it has a higher return for the same amount of volatility. In fact, the capital allocation line that is tangent to the efficient frontier graphs the most efficient portfolios consisting of both the risk-free and risky assets. Portfolio iii can be achieved by leveraging portfolio C at the risk-free rate – i.e. borrowing money at the risk-free rate and purchasing more of portfolio C. As you can see, portfolio iii is superior to a portfolio on the efficient frontier.

The capital allocation line helps investors to choose how much to invest in a risk-free asset and a portfolio of risky assets. If the volatility of the global minimum variance portfolio is still too high for an investor's purposes, he can combine a portfolio of risky assets with the risk-free asset to lower the portfolio's volatility. The most efficient portfolio to choose would be along the capital allocation line that is tangent to the efficient frontier.

Sharpe ratio

The Sharpe ratio is a measure for comparing the performance of portfolios. The Sharpe ratio for a particular portfolio is calculated with the following equation:

$$Sharpe\ ratio = \frac{E(R_p) - R_f}{\sigma_p},$$

where:

$E(R_p)$ = expected portfolio return; or portfolio return in a past period
R_f = the rate of return on the risk-free asset
σ_p = the volatility of portfolio returns.

The most attractive portfolio amongst a choice of portfolios will be the one with the highest Sharpe ratio, as it will have the highest return for the risk taken.

The Sharpe ratio uses the total risk of the portfolio, whereas only systematic risk is priced. This is a limitation. To understand systematic risk, we need to introduce the Capital Asset Pricing Model (CAPM). The **Capital Asset Pricing Model** is a tool for predicting the expected return of an asset. The expected return is calculated as follows:

$$E(R_i) = R_f + \beta_i\left(R_m - R_f\right),$$

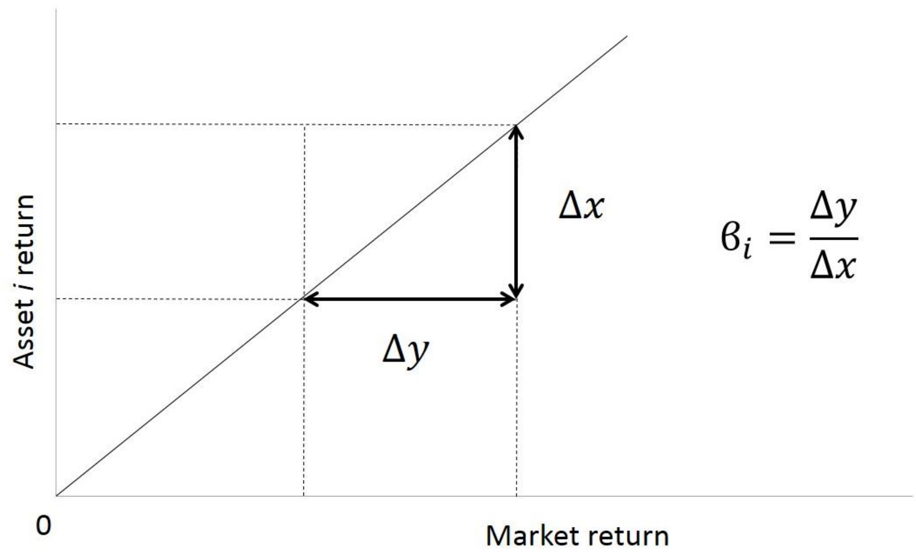
where:

$E(R_i)$ = the expected return for asset i
R_f = the rate of return on the risk-free asset
$\left(R_m - R_f\right)$ = the return of the market in excess of the risk-free rate
β_i = the beta of asset i with respect to $\left(R_m - R_f\right)$.

(R_m - R_f) is a term for the **market risk premium**. The **market** is a theoretical portfolio of all investable equity. In practice, a representative equity index is normally used as a proxy for the market portfolio. By investing in the market, you take on **systematic risk**. This is risk that is inherent in market equity and cannot be diversified away. Systematic risk can be contrasted with **non-systematic risk**, which is risk that is unique to a particular asset based on its unique characteristics. Non-systematic risk can be diversified away by combining the asset with other assets with which it has a low positive or negative correlation. The sum of systematic risk and non-systematic risk is **total risk**. The total risk of a portfolio (σ_p) is used in the Sharpe ratio.

It is impossible to be more diversified than owning the entire market. The entire market's return is R_m in the CAPM. Therefore, the risk associated with owning the market (systematic risk) cannot be diversified away because you're already as diversified as it is possible to be. $R_m - R_f$ is the return in excess of the risk-free rate earned by being willing to take on systematic risk. However, every asset has idiosyncratic, or non-systematic risk. Beta in the CAPM calculates a particular asset's equity risk premium, $E(R_i)$ by adjusting the market risk premium, $R_m - R_f$, for this fact.

Asset i's beta (β_i) is a measure of the sensitivity of asset i's return to the market return. If asset i's expected risk premium were plotted on the y-axis, and the market risk premium on the x-axis, the beta of asset i would be the slope of the line:



Beta is normally calculated using historical returns. The betas for many familiar public companies are provided by financial data providers such as Bloomberg.

Unit 3: Market Price Vs. Intrinsic Value and Common Emotional Biases

In these notes we will review the difference between market price and intrinsic value, and investigate some of the common behavioral biases (which include cognitive errors and emotional biases) that are among the factors which influence market price.

Asset pricing: market price versus intrinsic value

As an investor in securities, it is important to be able to distinguish between the market price and the intrinsic value of a security. Paying market price may well result in a poor investment outcome.

According to the CFA Institute, **market price** is the amount at which an investment can be exchanged in an arm's length transaction between willing parties in which the parties each act knowledgably and prudently. On the other hand, **intrinsic value** is considered by most investors to be the present value of a security's future cash flows.

Determining intrinsic value is subject to a number of uncertainties, including the amount and timing of future cash flows, and the appropriate discount rate to use to discount these cash flows. The market price of an asset will be determined by a number of factors, including market participants' individual assessments of intrinsic value, market sentiment (driven by emotions), investors' liquidity needs, the trading of index funds, etc.

Behavioral biases

People are not perfectly rational. Investors suffer from both cognitive errors and emotional biases, to a greater or lesser degree. **Cognitive errors** stem from basic statistical, information-processing or memory errors (Pompian, 2018). **Emotional biases** stem from both impulse and intuition. Asset pricing theories from the traditional finance school of thought assume that investors are perfectly rational and are not subject to any biases. This field is an area of ongoing debate amongst researchers.

Cognitive errors

Some of the more prominent cognitive errors present amongst investors include the following (Pompian, 2018):

- **Conservatism bias** is a tendency for investors to maintain their initial views or forecasts and inadequately respond to new information. This generally leads to an underreaction to new information.

- **Confirmation bias** is a tendency for investors to look for and notice information that confirms their views and undervalue or ignore contradictory information. This generally leads to inadequate diversification of portfolios and consequently exposure to significant risk.

- **Representative bias** is the tendency to classify new information based on past experiences and classifications. Investors use a heuristic to process new information quickly, in order to avoid the time cost and mental effort required to thoroughly and rationally analyze new information.

- **Illusion of control bias** is a tendency for investors to believe they can control or influence outcomes when they cannot. For example, the researcher Ellen Langer found that people permitted to choose their own numbers in a hypothetical lottery game were willing to pay a higher price for a ticket compared to people who were assigned random numbers (Langer, 1983). The illusion of control bias tends to lead people to be unreasonably certain of their predictions of unpredictable outcomes.

- **Hindsight bias** is a tendency to see past events as predictable. Investors believe that, had they been present during past events, they would have been able to predict the future and act accordingly. This leads to overconfidence in their current predictions.

- **Anchoring or adjustment bias** is similar to conservatism bias, except that investors fail to update their views in relation to an initially formed "anchor", while in conservatism bias they fail to update a view. The "anchor" is formed when investors are required to estimate an unknown number. New information causes them to adjust their estimation in relation to this "anchor". This bias stems from the fact that people are generally better at estimating relative comparisons than absolute figures.

- **Mental accounting bias** is a tendency to treat money differently based on its source or intended use. For example, people may spend money less strictly on leisure activities than they would on groceries. In investing, mental accounting may lead investors to treat assets in their trading account differently to their investing account. In a trading account, an investor will own speculative positions that carry high risk; while in the investing account he will own only conservative investments held for the long term.

- **Framing bias** is a tendency to interpret information based on how it is presented or framed. An investor needs to be wary of this bias as the issuers of equity or debt will be inclined to present company news in the most positive light possible.
- **Availability bias** is a tendency for investors to attach more weight to information that is most available to them. This availability may stem from a number of sources, including how recently the information was received, the investor's personal familiarity with the subject matter, etc.

Emotional biases

Some of the more prominent emotional biases present amongst investors include the following (Pompian, 2018):

- **Loss aversion bias** is exhibited by an investor who feels significantly more pain from a loss of a certain amount, compared to pleasure from a gain of the same amount. This bias leads people to strongly prefer avoiding a loss than achieving a gain of the same amount. In investing, this bias often leads people to hold on to investment positions that have declined in value with little prospect of improving, in order to avoid realizing the loss.
- **Overconfidence bias** is self-explanatory: it is the tendency for an investor to be overly confident about their decisions, skill, level of knowledge, and forecasts. This generally leads to an underestimation of investment risks and inappropriately concentrated portfolios.
- **Self-control bias** is a lack of self-discipline. In decision making, short-term satisfaction always trumps long-term goals. This generally leads to insufficient saving.
- **Status quo bias** is a tendency for investors to do nothing, or to procrastinate about making decisions. If there is no urgent problem that needs a solution, nothing is done, and the status quo is maintained. For an investor, this bias often results in portfolios being infrequently rebalanced – this leads them to drift from the investor's optimal asset allocation. This also leads to investors being inactive in searching for investment opportunities.

- **Endowment bias** is a tendency for people to value something more because they own it. It frequently applies to inherited assets, or assets with some other sentimental value for the owner. It can also apply to purchased assets – i.e. assets that have not been inherited.

- **Regret aversion bias** is a tendency to avoid regret. In investing, regret can stem from making the wrong decision – either selling an investment that is subsequently successful or buying an investment that subsequently turns out to be poor. This generally leads to fewer decisions being taken than necessary.

Unit 4: Single Stage Gordon Growth Model and Residual Income Valuation

In Unit 1 and Unit 2 of this module we have studied mean-variance optimization in portfolio construction (Modern Portfolio Theory). We now turn to the important topic of determining the intrinsic value of those building blocks of a portfolio: assets. In Unit 3 we studied common emotional biases that impact asset prices. In these notes, we will study various ways in which an asset's intrinsic value can be rationally determined, and the challenges associated with this task. We will focus on single-stage models. However, in reality the intrinsic value of many assets is more accurately calculated with a multiple-stage model. The difference between these will be explained. We will focus on valuing equity.

Determining intrinsic value requires that the analyst combine accurate forecasts with an appropriate valuation model (Pinto et al., 2016a). This requires a thorough understanding of the business being valued.

Broadly, a business can be valued on a going-concern or liquidation basis. When valued on a going-concern basis, it is assumed that the business will continue to operate in the future. In this case, its present value is based on expectations of the future. When valued on a liquidation basis, it is assumed that the business will cease to operate, and its assets will be sold off. The cash that remains after paying all expenses and repaying debt will be distributed to shareholders on a pro rata basis. Later in the reading we will learn about the state of a business that may call for its liquidation.

A business may also have a unique value to a purchaser based on the synergy that can be created by combining the business with the purchaser's existing business. This value is known as **investment value** and is unique to each potential purchaser.

Present value models

A **present value model** derives the value of equity as the present or discounted value of its expected future cash flows (Pinto et al., 2016b). The definition of cash flows depends on various factors. There are essentially four choices available to an analyst: dividends, free cash flow to the firm (FCFF), free cash flow to equity (FCFE), and residual income.

Dividends

A **dividend** is a payment from a business to its shareholders. The timing and size of the dividend is decided by the directors and management of the company. More mature and stable companies normally set a predictable formula for the payment of dividends. The reason for this is to give certainty and confidence to its investors. A decrease in the expected dividend can be met with a harsh reaction from the market (if it is publicly traded), therefore businesses generally try to avoid this. If an investor only owns (or is seeking to purchase) a small proportion of a business's shares, then using dividends as the definition of cash flow is appropriate because the investor has very little control over the dividend policy.

Free cash flow to the firm and free cash flow to equity

Free cash flow to the firm (FCFF) is defined as the cash available for distribution to providers of capital (debt and equity) without impairing the operation of the business. Free cash flow to the firm is the cash that remains after all expenses have been paid, and necessary investments in fixed assets and working capital have been made.

Free cash flow to equity (FCFE) is FCFF net of all payments due to debt holders. Since payment of interest and principal to debt holders is contractually required, FCFE is the appropriate cash flow to use when valuing the equity of a business. The management and directors of a business essentially have two choices regarding what to do with their FCFE: pay it out to shareholders as a dividend or retain it in the business and reinvest it. Free cash flow to the firm and free cash flow to equity should be used as the definition of cash flow when the investor assumes they will be able to control or significantly influence the business. If they have control, they will be able to decide how the business's profits are treated.

Residual income

The residual income of a business is based on the idea of equity having a cost. If a business borrows money, the cost of this debt is easy to determine: it is the interest rate charged by the lender. The cost of equity is a similar concept: if an investor provides equity capital to a business, he is not contractually entitled to any interest payments; he is only entitled to the business's profits (whether this be in the form of dividends paid out or profits retained and reinvested). However, by providing equity capital the investor gives up the opportunity to consume that money – i.e. spend it on things that he likes – or invest it in other attractive opportunities (called

opportunity costs). The capital asset pricing model (studied previously) is one way of determining the cost of equity.

The residual income is what remains of a company's profit after deducting the cost of equity.

Example: Calculating residual income

Earnings before interest and taxes	$320 000
Less: interest expense	$70 000
Pretax income	$250 000
Less: income tax expense	$75 000
Net income	$175 000

Equity charge = equity capital x cost of equity capital
Equity charge = $ 1 000 000 x 10%
Equity charge = $ 100 000

∴ Residual income = Net income – equity charge
∴ Residual income = $ 175 000 - $100 000
∴ Residual income = $75 000

According to the residual income model, a business that is not earning its cost of equity – i.e. residual income is negative – is a poor investment. The result of this will be a decline in book value of the business over time.

Estimating the required return on equity

When using present value models, two forecasts are critically important and subject to a lot of uncertainty:

1. The cost of equity, denoted r_e or R_e. This is also called the **equity discount rate**.

2 The growth rate of cash flows, denoted g. In a single-stage model, g is considered to be constant in all future periods. In multiple-stage models, g changes in the multiple stages modeled, reflecting the business cycle.

Methods for estimating the required return on equity include the Capital Asset Pricing Model (CAPM), Fama-French Model (FFM), Pastor-Stambaugh Model, and build-up methods (Pinto et al., 2016b). Refer to the second set of notes for this module (Sharpe Ratio and the Efficient Frontier) for a description of the CAPM.

Fama-French Model (FFM)

To recap, the cost of equity for a particular asset, i, was calculated with the CAPM as follows:

$$E(R_i) = R_f + \beta_i(R_m - R_f).$$

In this case, a single risk premium called the **market risk premium** $(R_m - R_f)$, adjusted for asset i specific beta (β_i), was added to the risk-free rate.

If we consider systematic risk to be a factor influencing a particular asset's returns, the FFM simply builds on the CAPM by adding factors that are assumed to influence an asset's returns. Specifically, the FFM uses the following factors:

- RMRF, standing for the market return in excess of the risk-free return $(R_m - R_f)$. This is the market risk premium from the CAPM.
- SMB, standing for Small Minus Big. SMB is the average return of three small-cap portfolios minus the average return of three large-cap portfolios. SMB is a market capitalization factor.
- HML, standing for High Minus Low. This is the average return of two low price-to-book portfolios minus the average return of two high price-to-book portfolios.

The FFM estimate of an asset's cost of equity is as follows:

$$E(R_i) = R_f + \beta_i^{mkt} RMRF + \beta_i^{size} SMB + \beta_i^{value} HML.$$

Data for the three factors is normally obtained from historical data (Pinto et al., 2016b).

Models that use more than a single factor to estimate R_e are known as Multi Factor Models, of which the Arbitrage Pricing Theory is included.

Pastor-Stambaugh Model

The Pastor-Stambaugh Model builds on the Fama-French Model simply by adding an additional factor: liquidity. Therefore, the full Pastor-Stambaugh Model is as follows:

$$E(R_i) = R_f + \beta_i^{mkt} RMRF + \beta_i^{size} SMB + \beta_i^{value} HML + \beta_i^{liq} LIQ.$$

An average liquidity equity should have a liquidity beta (β_i^{liq}) of zero. If an equity has below-average liquidity, β_i^{liq} will be greater than 0 to reflect the increased return required to compensate the investor for his inability to sell the equity quickly if desired. If the equity has above-average liquidity, the opposite is true.

Build-up methods

Build-up methods essentially give the analyst the freedom to add or subtract as many factors from the risk-free rate as they see fit. Many financial institutions have developed their own proprietary models for calculating r_e. Accurate models will require an in-depth understanding of the asset being valued. In build-up methods, beta adjustments are not applied to factors.

Single-period asset pricing

Gordon Growth Model

The single-stage Gordon Growth Model (GGM) calculates the present value of a share as follows:

$$V_0 = \frac{D_0(1+g)}{r-g},$$

where:

V_0 = the present value (at time 0) of the share

D_0 = the most recently paid dividend per share

g = the periodic growth rate of dividends

r = the periodic equity discount rate.

Example: The Gordon Growth Model for valuing equity

Foley Industries has just paid a dividend of \$5 per share. Foley Industries is a stable and mature company with operations throughout the United States. The target sustainable real GDP growth rate has been set by the US Federal Reserve at 3%. The bank has also set an inflation target of 2%. As Foley is a mature company, its growth rate in earnings is expected to be approximately equal to the nominal economic growth rate in the US. The discount rate for Foley's equity, calculated using the CAPM, is equal to 8%. What is the intrinsic value per share of Foley calculated using the GGM?

Solution:

$$V_0 = \frac{\$5(1+0.05)}{0.08-0.05} = \$175$$

Residual income valuation

The main shortcoming of the GGM is the heavy reliance on an accurate prediction of r and g. Changing these values slightly can result in large fluctuations in the calculation of V_0. The residual income model alleviates these shortcomings by deriving the majority of the present value from the current book value per share.

If a company has negative residual income, it is not earning enough to cover its cost of equity capital. In this case, shares are expected to sell at a discount to book value. If residual income is positive, shares are expected to sell at a premium to book value.

The accuracy of the residual income model depends on the clean surplus relation holding. The **clean surplus relationship** sets out the relationship among earnings, dividends, and book value (excluding ownership transactions) as follows (Pinto et al., 2016c):

$$B_t = B_{t-1} + E_t - D_t.$$

In words: the book value per share in the subsequent period (B_t) is equal to the book value per share in the prior period (B_{t-1}) plus subsequent period earnings (E_t) less dividends paid (D_t), assuming no ownership transactions.

The residual income model is given as follows:

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{(ROE_t - r)B_{t-1}}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T},$$

where:

B_0 = beginning book value per share

ROE_t = return on equity in time period t

r = the periodic equity discount rate

B_{t-1} = book value per share at the beginning of time period t

P_T = the price per share in the terminal period (time T)

B_T = the book value per share in the terminal period (time T).

The return on equity (ROE) for a business is calculated as follows:

$$ROE = \frac{Net\ income}{Book\ value\ of\ equity\ (beginning\ of\ period)}.$$

Net income is the profit due to the shareholders. The book value of equity at the beginning of the period is the equity that was invested to generate that profit. Therefore, the ROE is an accurate measure of the health of a business. A business should earn a ROE higher than its equity discount rate, otherwise it is not earning an adequate return for shareholders and they should consider liquidating it.

Example: Residual income valuation

Jackson Company has just released its financial statements for the year ending 31 December 2017. The book value per share is $11.50. Jackson is a mature company with limited growth prospects. As a result, its return on equity is close to its equity discount rate. The return on equity has been 10% in the recent past and is expected to persist in the future. The discount rate for Jackson's equity (estimated using the CAPM) is 8.2%. Jackson company has adopted a policy of paying out 25% of net income as dividends. An analyst decides to estimate the intrinsic value of Jackson Company using a residual income model. He assumes an 8-year time horizon, since he believes that forecasts beyond this time horizon will be inaccurate. As Jackson Company is a stable company with few growth prospects, he assumes that the market price per share will be 1.5 times the book value upon sale.

Solution:

The analyst forecasts the book value, dividends, and residual income as follows:

Time	Book value (Beginning of period)	Net income	Dividends	Residual income	Book value (End of period)
1	\$11.50	\$1.15	\$0.29	\$0.20700	\$12.36
2	\$12.36	\$1.24	\$0.31	\$0.22253	\$13.29
3	\$13.29	\$1.33	\$0.33	\$0.23921	\$14.29
4	\$14.29	\$1.43	\$0.36	\$0.25716	\$15.36
5	\$15.36	\$1.54	\$0.38	\$0.27644	\$16.51
6	\$16.51	\$1.65	\$0.41	\$0.29718	\$17.75
7	\$17.75	\$1.77	\$0.44	\$0.31946	\$19.08
8	\$19.08	\$1.91	\$0.48	\$0.34342	\$20.51

The calculations for time period 1 are as follows:

$Net\ income$ =book value (beginning of period) x ROE

$Net\ income =$ \$11.50 x 10% = \$1.15

$Dividends =$ net income x dividend payout ratio

$Dividends =$ \$1.15 x 25% = \$0.29

$Residual\ income =$ net income – equity charge

$Residual\ income =$ \$1.15 – (\$11.50 x 8.2%) = \$0.21

$Book\ value\ (end\ of\ period) =$ book value (beginning of period) + net income – dividends

$Book\ value\ (end\ of\ period) =$ \$11.50 +\$1.15 - \$0.29 = \$12.36

Discounting the residual income at 8.2% per year and summing the results yields the following:

Time	Present value (Residual income)
1	\$0.19131
2	\$0.19007
3	\$0.18885
4	\$0.18762
5	\$0.18641
6	\$0.18520
7	\$0.18401
8	\$0.18281
∑	\$1.50

For example, the present value of residual income for time period 6 is as follows:

$$= \frac{\$0.29718}{(1.082)^6} = \$0.1852.$$

Therefore, the present value per share of Jackson Company is:

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{(ROE_t - r)B_{t-1}}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$$

$$V_0 = \$11.50 + \$1.50 + \frac{(1.5\; x\; \$20.51) - \$20.51}{(1.082)^8}$$

$$V_0 = \$18.46$$

The main shortcoming of the residual income model is its reliance on numbers determined by accounting definitions. Book value and earnings are accounting numbers and may not necessarily reflect the business's true economic situation.

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Collaborative Review Task

In this module, you are required to complete a collaborative review task, which is designed to test your ability to apply and analyze the knowledge you have learned during the week.

Brief

Mclean Technology is a software engineering company. The Board of Directors has established a policy of paying out 30% of net income as dividends. In the financial year recently ended, Mclean earned a return on equity (ROE) of 18%. Beginning shareholders' equity was $10 500 000. A dividend was paid to shareholders. Mclean has 100 000 shares outstanding.

Jason Rissik is investigating Mclean with the intention of purchasing shares, if they are reasonably priced. He decides that he will purchase shares in Mclean if his estimate of intrinsic value is 20% or higher than the current market price. The current market price is $107.02. Rissik decides to use the Gordon Growth Model with dividends as the cash flow input. To estimate g he decides to use the formula:

$$g = b \times ROE,$$

where:

$b =$ earnings retention ratio

$ROE =$ return on equity

The earnings retention ratio (b) is that portion of net income not paid out as dividends. He decides to assume that Mclean's most recent year's ROE is the company's average ROE.

To calculate the equity discount (r) he decides to use the Capital Asset Pricing Model (CAPM). The inputs into the CAPM are as follows:

Real yield on 10-year treasury bonds	3.2%
Historical nominal return on the S&P500	15.3%
Beta of Mclean with respect to the market risk premium	1.21
Federal Reserve inflation rate target	2.8%

Rissik estimates that the price of Mclean in one year's time will be $118. He expects to receive a dividend of $6.38 per share at that time as well.

Rissik decides to use the Sharpe ratio to assess the attractiveness of Mclean compared to similar companies. Mclean has an estimated volatility of 8.2%, which is similar to peer companies. The Sharpe ratios for peer companies are as follows:

Emslie	0.81
Nel	1.02
Van Zyl	1.33

A correlation matrix for the four peer companies appears below:

	Mclean	Emslie	Nel	Van Zyl
Mclean	1	0.86	0.02	-0.45
Emslie		1	-0.21	0.54
Nel			1	0.18
Van Zyl				1

Questions:

1. Calculate the most recent dividend per share for Mclean.
2. Calculate the intrinsic value of Mclean using the Gordon Growth Model with dividends as the cash flow input.
3. Based on his criteria for investment, will Rissik decide to purchase Mclean shares?
4. Based on his assumption about the share price of Mclean in one year's time, what is Rissik's expected total return for his Mclean investment?
5. Based on the Sharpe ratio, is Mclean the most attractive investment?
6. Considering the correlation of Mclean with its peer companies, the combination with which company (in a two-asset portfolio) will result in a portfolio with the lowest volatility?