WORLDQUANT UNIVERSITY

Compiled Content Module 7

MScFE 640

Portfolio Theory and Asset Pricing

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Module 7: Transaction Costs, Incentives, Trading and Market Frictions

Module 7 begins by introducing measures of market efficiency and the costs of trading. Then, market orders, limit orders, iceberg orders, and algorithmic trading are discussed along with the major types of markets. The module ends by showing how to calculate the cost of trading using VWAP and implementation shortfall methods.

Unit 1: Measures of Market Efficiency

Introduction

One important reason that financial markets[1] exist is to ensure that capital – i.e. money – gets from where it is available to where it is needed. If financial markets functioned perfectly, it would be possible for an entrepreneur in Alaska in need of capital to obtain a loan or equity capital from a Mozambican investor in a very short space of time and at very low cost.

In reality, financial markets are not that efficient. Many countries impose limits on the amount of capital citizens can transfer offshore; differences in taxation rules between countries complicate matters; people use different currencies and speak different languages; legal documents need to be drawn up; there are various entities charging fees on international (and national) capital transfers, such as lawyers, brokers, governments, etc. As a result of these and other factors, capital does not flow 100% freely around the globe.

This module will investigate what market efficiency is and how it is measured. It will also cover how different types of markets work and discuss the most common types of orders.

Functions of exchanges

Securities exchanges (or simply "exchanges") serve a number of important functions including (Madhavan, et al., 2018):

1 Providing **liquidity**:

- namely, the ability to trade desired quantities of securities timeously.

2 Providing **transparency**:

- providing current information on bids, offers, and quantities;
- recording and providing historical trade information.

3 Providing **surety of completion**:

- Ensuring the securities and money are exchanged without hassle.

[1] A brief note on terminology: the term "market" is a broad definition used to describe any type of commercial activity. It can refer to a regulated venue where securities are traded, but not exclusively. The term "securities exchange" or simply "exchange" would be more specific terminology. The terms "market" and "exchange" will be used interchangeably in these notes. It will be possible to interpret their specific meaning from the context.

Liquidity

A liquid market will have the following characteristics (Madhavan, et al., 2018):

1. Relatively low bid-ask[2] spreads. If bid-ask spreads are high, investors cannot profitably trade on information except when it is of great value. The **bid-ask** spread is the difference between the lowest offer and the highest bid.
2. Be deep. A **deep** market is one which has many shares available for trade at any given time. The deeper the market, the less of a price impact large trades will have on the market price.
3. Be resilient. A **resilient** market is one where discrepancies between market price and the intrinsic value of securities is corrected quickly.

The following factors will contribute to making a market more liquid:

- The market has many participants. The greater the number of participants, the more likely an investor is to find a counterparty to their desired trade.
- There is a diversity of opinion, investment needs, and information among market participants. Consider the following situation: a market may be deep with many participants, but if all those participants wish to sell, the market will not be liquid because there will be no one taking the opposite side of the trade – i.e. buying – and therefore no transactions will occur.
- The market is convenient to access.
- The market is run fairly. This includes:
    - accurate accounting and auditing at public companies;
    - securities laws and enforcement that prevent market manipulation and trading on inside information.

[2] Bid-ask spreads will be covered in more detail later in the reading.

Transparency

Markets serve to provide useful information to market participants in a convenient manner. Since the advent of the internet, access to the market and market information has become much easier and more convenient. Via a single website, investors can obtain a plethora of useful information on public companies from around the world, use tools such as filters to find companies that have certain characteristics, and submit trade orders directly to the market.

Surety of completion

Markets are places where two counterparties, often strangers, meet to trade. By implementing rules and policies, the market can ensure that each party has peace of mind with regards to delivery and payment in a transaction. The buyer can rest assured that the security will be delivered in the time specified, and the seller can be assured that they will be paid in the time specified. By providing this surety, the market encourages trading and contributes to a healthy financial system.

Costs of trading

The cost of trading is an extremely important consideration in investment management. In some cases, an investment decision may appear profitable before considering trading costs but be unprofitable after these are considered.

Trading costs can be thought of as having two major components: explicit costs and implicit costs (Madhavan, et al., 2018). Explicit costs are easy to identify: they involve the payment of cash. These include fees and taxes. Implicit costs include the following:

- the bid-ask spread;
- price impact;
- missed trade opportunity costs; and
- delay costs.

Bid-ask spread

The **bid-ask spread** is essentially the price an investor pays for gaining the ability to trade a security conveniently. For example: imagine an importer of a rare spice. The importer sources the spice in a foreign territory and incurs the cost and risk of bringing that spice to their domestic

territory. The importer sets a price (called the base price in this example) that covers their costs and then adds a profit to compensate them for their time, risk and enterprise. The profit in this scenario is equivalent to the bid-ask spread in securities markets. A consumer of the spice could acquire the spice from the foreign territory themselves and pay the base price, but in most cases they are willing to pay the importer's profit for the convenience that they offer.

Similarly, in a securities market, if a dealer purchases a security for $62 and sells it for $64, their bid-ask spread is $2. The extra $2 that investors pay is the cost of the convenience of being able to quickly trade a security.

Price impact

The **price impact** of a trade is the impact that a trade has on the market price of a security. Example A will illustrate this.

Example A: Price impact

At a particular point in time, the following order book[3] for ABC exists:

Bids to buy		Offers to sell	
Quantity	Price	Quantity	Price
4 000	$61	5 000	$63
5 500	$60	6 200	$64
3 000	$58	4 500	$66
4 200	$57	3 800	$67

Let us define the market price as the price at which the most recently exchanged security was exchanged. A trader is instructed to purchase 10 000 shares of ABC at market. The first 5 000 are purchased for $63 and the next 5 000 are purchased for $64. If the market price before the trader's arrival was $62, then the market price (after the trader's transactions) is $64. The price impact of the trader's trades was $2.

[3] A limit order book is a list of bids and offers for a particular security.

Missed trade opportunity costs

Missed trade opportunity costs arise when the market price moves against the investor's desired trade. Example B will illustrate this.

Example B: Missed trade opportunity costs

At a particular point in time the following order book for ABC exists:

Bids to buy		Offers to sell	
Quantity	Price	Quantity	Price
4 000	$61	5 000	$63
5 500	$60	6 200	$64
3 000	$58	4 500	$66
4 200	$57	3 800	$67

An investor wishes to purchase 5 000 shares of ABC when the market price is $63. They place a limit order[4] for 5 000 shares with a limit price of $62 and that expires in one day. During the day, other trades are executed at $64 and the market price moves to $64. The trader's order is not filled. The **missed trade opportunity cost** for the trader is $1 because they miss out on the $1 gain they would have earned if they had purchased ABC for $63 at the start of the day.

Delay costs

Delay costs arise when a trade of a desired size is not executed immediately due to a market being insufficiently deep or liquid. Example C will illustrate this.

Example C: Delay costs

At a particular point in time the following order book for ABC exists:

Bids to buy		Offers to sell	
Quantity	Price	Quantity	Price
4 000	$61	5 000	$63
5 500	$60	6 200	$64
3 000	$58	4 500	$66
4 200	$57	3 800	$67

[4] Limit orders and market orders will be explained later in the module.

An investor wishes to purchase 25 000 shares of ABC. The opening price on the day is \$63. They place a market order[5] for 25 000. The investor immediately purchases 19 500 (5 000 + 6 200 + 4 500 + 3 800) shares. At the close of trading, 5 500 shares of the trade are unfilled. The next day, a new offer to sell arrives on the market. This offer is to sell 6 000 shares for \$66. 5 500 of these shares are bought for the investor at \$66. Because there was insufficient depth to fill the order on the first day, the delay cost is \$3 (\$66-\$63) per share.

The **implementation shortfall method** is a formal method for calculating the explicit and implicit costs of trading. This will be covered later in the module.

Summary

In these notes we have studied the functions of markets, measures of the efficiency of markets, and the costs of trading.

[5] Limit orders and market orders will be explained later in the module.

Unit 2: Market Orders Versus Limit Orders

Introduction

In order-driven markets, brokers receive trade instructions – i.e. **orders** – from investors that they execute for a fee. Brokers need to know what the investor's priorities are. Is the investor more concerned with trading quickly or trading at a certain price? Are there any special conditions that need to be adhered to? These notes will discuss common types of orders.

Types of orders

There are two major types of orders:

1. market orders; and
2. limit orders.

The types of orders will be demonstrated with the following hypothetical limit order book for ABC. The **limit order book** is a list of currently active limit orders for an asset.

Bids to buy		Offers to sell	
Quantity	Price	Quantity	Price
4 000	$61	5 000	$63
5 500	$60	6 200	$64
3 000	$58	4 500	$66
4 200	$57	3 800	$67

Table 1: Limit order book

Market orders

A **market order** is an instruction to execute a trade at the best available price. For **a buy order** this will be the lowest offered price; for a **sell order** this will be the highest offered price. The following buy instruction is a market order:

INSTRUCTION: Purchase 6 000 shares of ABC

To fill this order, the broker will purchase 5 000 shares for $63 and the remaining 1 000 shares for $64.

The following sell instruction is a market order:

INSTRUCTION: Sell 5 500 shares of ABC

To fill this order, the broker will sell 4 000 shares for $61 and the remaining 1 500 shares for $60.

With a market order the investor is uncertain what price the order will be executed at. This is known as **price uncertainty**. The advantage of a market order, however, is that it will be executed quickly (known as **execution certainty**).

A market order will typically be used when the trade decision is time sensitive. This means that the investor has information the value of which will expire within a certain time period. For example: if an investor discovers that a rival is intending to make an offer to purchase ABC in two days' time, the investor must acquire ABC shares within two days because after the offer is made, the price of ABC shares will rise and the opportunity for profit will be lost.

Limit orders

A **limit order** is an order to execute a trade at a certain price or better. A limit order is always accompanied by a validity period. The order expires when the validity period passes.

A buy order must be executed at the lowest available price, which must not exceed the limit price. A sell order must be executed at the highest available price, which must not be below the limit price. A market order is executed at whatever price is available on the market at the time it is submitted.

The following buy instruction is a limit order:

INSTRUCTION: Purchase 6 000 shares of ABC at $64 or better. This instruction is valid for 24 hours.

To fill this order, the broker will purchase 5 000 shares for $63 and the remaining 1 000 shares for $64. If the instruction had instead given a limit price of $62, the order would not be filled. Instead the limit order book would update as follows:

Bids to buy		Offers to sell	
Quantity	Price	Quantity	Price
6 000	$62	5 000	$63
4 000	$61	6 200	$64
5 500	$60	4 500	$66
3 000	$58	3 800	$67
4 200	$57		

The following sell instruction is a limit order:

INSTRUCTION: Sell 6 000 shares of ABC at $59 or better. This instruction is valid for 24 hours.

This instruction would be filled as follows (see Table 1): 4 000 shares would be sold for $61 and 2 000 shares would be sold for $60

Compared to a market order, a limit order has *price certainty* but may not execute immediately (or at all).

A limit order will typically be used when the trade decision is price sensitive. The investor is more concerned about trading at a certain price than trading within a certain time frame. For example: if an investor determines after thorough analysis that the intrinsic value of ABC is $60 per share, and that they feel this assessment will be valid until the next set of financial results are released in 3 months' time, they can submit a limit order to buy at $57 or better, expiring in a month. If they had submitted a market order, they would purchase ABC for $63 (more than intrinsic value) and they would be overpaying.

Iceberg orders

The limit order book is visible to all market participants. Therefore, any investor can see, at a given time, what the demand is to buy or sell a security.

Imagine an investor has a large order, for example: they wish to purchase 20 000 shares of ABC. Note that the order quantity exceeds what is currently being offered (19 500 shares). If they submitted this order as a limit order at a certain price, the large increase in demand for ABC would likely result in an increase in market price. This is known as the **price impact** (also known as **market impact**) of an order.

Investors who wish to avoid the price impact of their orders can submit **iceberg orders**. This is a limit order where only a fraction of the total order is made visible at any given time. Returning to our example, the full 20 000 order may be broken up into 10 orders of 2 000 each, with each being submitted after the prior order fills.

Algorithmic trading

Algorithmic trading is often used to assist with issues such as unfavorable price impact. A trading algorithm is a computer system that operates automatically and according to certain rules. The appeal of algorithmic trading is that it can process new information and react much quicker than a human trader can.

Algorithms can also be used to trade more efficiently. For example: to prevent price impact, an algorithm can perform a variation of the iceberg order which involves monitoring the ebb and flow of trade on a market. The algorithm breaks up large orders and submits smaller portions in order to match this ebb and flow and minimize the market impact. This is known as a **logical participation strategy**. Besides for participating in markets based on trade volumes, algorithms can also break up an order and submit it in smaller portions at regular time intervals. This is known as a **time-weighted average price strategy** because the goal is to match or beat the equal-weighted average price. Algorithms can also monitor multiple markets and direct incoming orders to the most liquid markets, a process known as **smart routing**.

Summary

These notes have presented the major types of orders (market and limit) and variations of these (iceberg orders and algorithmic trading).

Unit 3: Types of Markets

Introduction

Sometime in the first half of 1886[6], a wandering English prospector stumbled upon the rocky outcrop of what would become the richest deposit of gold ever discovered (Meredith, 2007). The discovery was made in a rural, impoverished republic known as the Transvaal, in present day Gauteng Province in the Republic of South Africa. Within a short space of time, the town of Johannesburg would come into existence as a result of the influx of people attracted by the discovery. At the corner of two streets, Commissioner and Simmonds, stood a single-story brick building that housed the Johannesburg Exchange and Chambers Company (Limited, 2013). In this simple venue, ownership interests in the future wealth or ruin of thousands of companies associated with the gold reef were created and exchanged. Trading was in such demand that it would frequently spill out into the street after the exchange had closed for the day. The company continues to exist today as the Johannesburg Stock Exchange (JSE).

The JSE (and other exchanges throughout the world) have evolved into efficient, computerized, regulated venues where many different types of financial instruments, including equity, bonds, and derivatives, are traded. Although modern exchanges are more complicated, the reason they exist remains the same: to provide a venue to deal in financial interests. This set of notes will explore the structure of modern exchanges.

Types of exchanges

There are three main ways in which trading is organized:

1. **Quote-driven** (or **dealer**) markets. Investors trade with dealers.
2. **Order-driven markets**. Investors trade with each other.
3. **Brokered markets**. Investors rely on a broker to assist them or represent them in trading.

[6] Note that the details of the discovery of gold in the Transvaal are disputed. There were earlier discoveries of lesser deposits prior to the 1886 discovery.

Quote-driven (dealer) markets

A grocery store maintains an inventory of groceries, including milk, eggs, etc. It sells this inventory to shoppers and purchases replacement inventory from its suppliers. **Quote-driven** (or **dealer**) markets work the same way, except that instead of groceries they have an inventory of securities. They sell securities to buyers and purchase securities from sellers. The difference between their buying and selling prices (known as the bid-ask spread) is a factor determining their profit (along with operating costs and the like). A dealer is sometimes known as a **market maker** due to the fact that they make or create a market for particular securities.

Major dealer markets are the markets for currencies and bonds. A multitude of financial institutions across the world maintain inventories of currencies. They post bid and ask prices, which are available for view by market participants. A dealer's bid price is what they are willing to pay for a currency, and their ask price is what they are willing to sell the currency for. These bid-ask prices will only be valid for a certain amount of time, and specify a quantity associated with each price, so they are in fact limit orders.

If we imagine that company ABC is traded on a dealer market, then the order book would be comprised of dealers' limit orders:

Bids to buy		Offers to sell	
Quantity	Price	Quantity	Price
4 000	\$61	5 000	\$63
5 500	\$60	6 200	\$64
3 000	\$58	4 500	\$66
4 200	\$57	3 800	\$67

The highest bid price is known as the **inside bid** or **market bid**. The lowest offer to sell is known as the **inside ask** or **market ask**. The **midquote** is halfway between the inside bid and inside ask. So, for ABC, the midquote is:

$$\frac{\$63 + \$61}{2} = \$62$$

In a dealer market, factors besides price are also used to decide whom to trade with. For example: an investor may decide to trade with a certain dealer, even if they are not offering the inside bid or ask price, because they are reliable in terms of delivery or payment.

Order-driven markets

In **order-driven** markets, investors trade with each other – i.e. not exclusively with dealers – on a market that provides various support services. Dealers may also participate in an order-driven market, but they do so alongside regular members of the public. Limit orders submitted by traders are publicly visible and are executed when there is a match.

In order-driven markets, it is more likely (relative to dealer markets) that a certain security might have very low liquidity, since there might not be any dealer who "makes a market" in that security – i.e. holds the security in inventory and trades the security.

A sub-class of order-driven markets is an electronic crossing network.

Electronic crossing networks

Electronic crossing networks are networks for batching and executing large orders at set times. They execute the orders at prices taken from other markets. The following example will illustrate the functioning of an electronic crossing network.

Example: An electronic crossing network

The average daily trading volume in ABC shares on the New York Stock Exchange (NYSE) is 10 000. An electronic crossing network receives the following orders from institutional investors:

- Investor A: purchase 100 000 ABC shares
- Investor B: sell 60 000 ABC shares
- Investor C: sell 35 000 ABC shares

Every day at noon, the electronic crossing network batches and **crosses** – i.e. executes – the orders on its books based on the midquote taken from the NYSE. At noon on a certain day, the midquote of ABC is $62. Investor A purchases 95 000 ABC shares for this price, and investor B and C both sell theirs.

Electronic crossing networks have the following characteristics:

- They serve investors with large orders. In the example, the smallest order (Investor C's) was three times the daily average trading volume. Investors who use electronic crossing networks wish to avoid the impact that their large trades will have on the market price.

- Orders are not publicly displayed. Therefore, they have no impact on price. In the example, the fact that demand for ABC shares (100 000) is greater than supply (95 000) is not known to any of the participants (investors A - C) and does not cause the price to increase.

Brokered markets

In **brokered markets**, investors are represented by brokers whose job it is to find counterparties to trades and assist in the execution. The broker charges a fee or commission for this service. The broker's services include:

- finding a counterparty to the trade;
- negotiating price;
- providing advice; and
- assisting with administration.

Brokered markets typically tend to be markets in which counterparties are difficult to find – i.e. the market is illiquid. Brokered markets also typically tend to serve the markets for assets that are heterogeneous. Heterogeneous assets are diverse in their characteristics, and each asset needs to be analyzed individually. For example, the market for public-company shares is more homogenous that the real estate market. Two shares of ABC are identical, but two pieces of real estate are rarely so.

Summary

These notes have presented and described the major types of markets (dealer, order-driven, and brokered markets).

Unit 4: Measures of Transaction Costs, Including the Implementation Shortfall Method

Introduction

Trading costs have a significant impact on long-term investment returns. They decrease returns (all else being equal) by:

- costing money (the fees charged by brokers);
- causing any unrealized capital gains to be realized and necessitating the payment of capital gains taxes.

The significant impact of frequent trading is illustrated in the following table, which compares two identical investors, Ms. Patient and Ms. Restless, who both earn a gross return of 14% per year for 10 years on an initial investment of R 100 000. The only difference between them is that Ms. Restless trades twice as frequently as Ms. Patient: Ms. Patient turns over her portfolio once every 12 months, whereas Ms. Restless does so once every 6 months.

Net Investment Value (net of fees and taxes)	
Ms. Patient (turnover = 100%)	Ms. Restless (turnover = 200%)
R 272 520	R 249 451

Ms. Restless' investment gains are 13.37% ($\frac{R272\,520-R249\,451}{R272\,520-R100\,000}$) lower than Ms. Patient's. See Appendix 1 for a full description of these calculations.

Put differently: Ms. Restless would have to earn a gross return 1.22% higher than Ms. Patient to earn a net return (net of fees and taxes) equal to Ms. Patient.

The analysis above considers only the explicit costs of trading. However, as you will likely remember from the first set of notes, trading incurs both explicit and implicit costs. In these notes, we will learn a formal calculation methodology for explicit and implicit costs (the implementation shortfall method), as well as a simpler but less informative method: the volume-weighted average price method. We will begin with the latter.

Volume-weighted average price

A simple way for a trader to assess the quality of their trading is to compare an execution price to the volume-weighted average price (VWAP). A good-quality, purchase trade will have an execution price lower than the VWAP. A good-quality, sale trade will have an execution price higher than the VWAP. The following example will illustrate this.

Example: Volume-weighted average price

On a particular day, 19 500 shares of ABC were traded. The table below lists the volume of shares traded at each execution price. The weight column is the portion of the total volume at each execution price.

Transaction prices		
Quantity	Price	Weight
5 000	$63.00	25.64%
6 200	$64.00	31.79%
4 500	$66.00	23.08%
3 800	$67.00	19.49%

The VWAP is $64.79. Any trader who has managed to purchase ABC shares for less than $64.79, or sell shares for more the $64.79, would be considered (based on the VWAP method) to have executed good quality trades. Note that the trader's trade contributes to the calculation of the VWAP.

The VWAP method has two limitations:

- If the trader's trades are a large portion of the volume traded, their own trades will have a large impact on the VWAP and limit its usefulness as a benchmark.
- The VWAP can be predicted and gamed. For example: if the trading day is close to over, a trader can delay their trades if they think their trades will look poor relative to the VWAP. This may result in delay costs, which are not measured by VWAP.

Implementation shortfall method

The implementation shortfall method allows the quantification of both explicit and implicit costs. The implementation shortfall can be quantified by summing four costs:

1. Explicit costs (fees and taxes)

2. Realized profit/loss
   - The difference between the execution price and the opening price on the day an order is executed.
   - Only the portion of the trade *executed* on the same day the order is placed is considered.
3. Missed trade opportunity cost
   - The difference between the closing price on the day the order is cancelled or expires and the decision price. When calculating missed trade opportunity cost, the decision price is usually taken to be the price when the original decision to trade is made.
   - Only the portion of the trade that is *not executed* is considered.
4. Delay cost
   - The cost due to an order not being completely filled on the day it is placed.
   - Measured as the difference between the closing price on the day an order is placed and the closing price on the day prior to when the order is placed.
   - Only the portion of the trade executed on the second day is considered.

The following example will illustrate the calculation of the implementation shortfall for a trade.

Example: The implementation shortfall method (Madhavan, et al., 2018).

The calculation of the implementation shortfall for a trade will be based on the following facts:

- On Monday, the closing price of ABC is $10.
- Before the market opens on Tuesday, an investor places a limit order to purchase 1 000 shares of ABC with a limit price of $9.98 and expiring at the end of the day. No part of the order is filled, and the closing price of ABC is $10.05.
- On Wednesday, the investor places another limit order to purchase 1 000 shares with a limit price of $10.07 and expiring at the end of the day. There are no offers to sell at $10.07 until an offer arrives at 12 pm. 700 shares are purchased for $10.07 and fees of $14 are paid. The closing price of ABC is $10.08.
- No further attempt is made to purchase ABC shares.

To calculate the implementation shortfall, we calculate the difference between the changes in value of the paper portfolio and the actual portfolio. The **paper portfolio** is a theoretical portfolio that incurs no costs (either explicit or implicit) and where trades are executed immediately at the decision price.

In this example, the gain on the paper portfolio is:

$$\Delta\ Paper\ Portfolio\ Value = V_{P_1} - V_{P_0}$$

$$\Delta\ Paper\ Portfolio\ Value = (1000 \times \$10.08) - (1000 \times \$10)$$

$$\Delta\ Paper\ Portfolio\ Value = \$80$$

The gain on the actual portfolio is:

$$\Delta\ Actual\ Portfolio\ Value = V_{P_1} - V_{P_0} - explicit\ costs$$

$$\Delta\ Actual\ Portfolio\ Value = (700 \times \$10.08) - (700 \times \$10.07) - \$14$$

$$\Delta\ Actual\ Portfolio\ Value =$$-\$7

$$Implementation\ shortfall = \Delta\ Paper\ Portfolio - \Delta\ Actual\ Portfolio$$
$$Implementation\ shortfall = \$80 - (-\$7) = \$87$$

We can now break up the total implementation shortfall into its four components.

1. Explicit costs
   The explicit costs are the fees amounting to \$14.
2. Realized profit/loss
   The price at which a portion of the trade is executed is \$10.07. The opening price is \$10.05. 700 shares are traded; therefore, the realized profit/loss is:

$$Realized\ profit/loss = (\$10.07 - 10.05) \times 700 = \$14$$

3. Missed trade opportunity costs
   The closing price on the day the order expires (Wednesday) is \$10.08. The decision price is \$10. 300 shares are not traded. Therefore, the missed trade opportunity cost is:

$$Missed\ trade\ opportunity\ cost = (\$10.08 - \$10) \times 300 = \$24$$

4 Delay cost

The investor wishes to purchase 1 000 shares on Tuesday but is unable to purchase any. On Wednesday, they succeed in purchasing 700. The closing price on Tuesday is \$10.05, and the closing price on Monday is \$10.00. Therefore, the delay cost (due to being unable to purchase shares on Tuesday as desired) is:

$$Delay\ cost = (\$10.05 - \$10.00) \times 700 = \$35$$

The sum of the four components will be the implementation shortfall:

$$Implementation\ shortfall = \$14 + \$14 + \$24 + \$35 = \$87$$

The following figure provides a graphical illustration of the implementation shortfall.

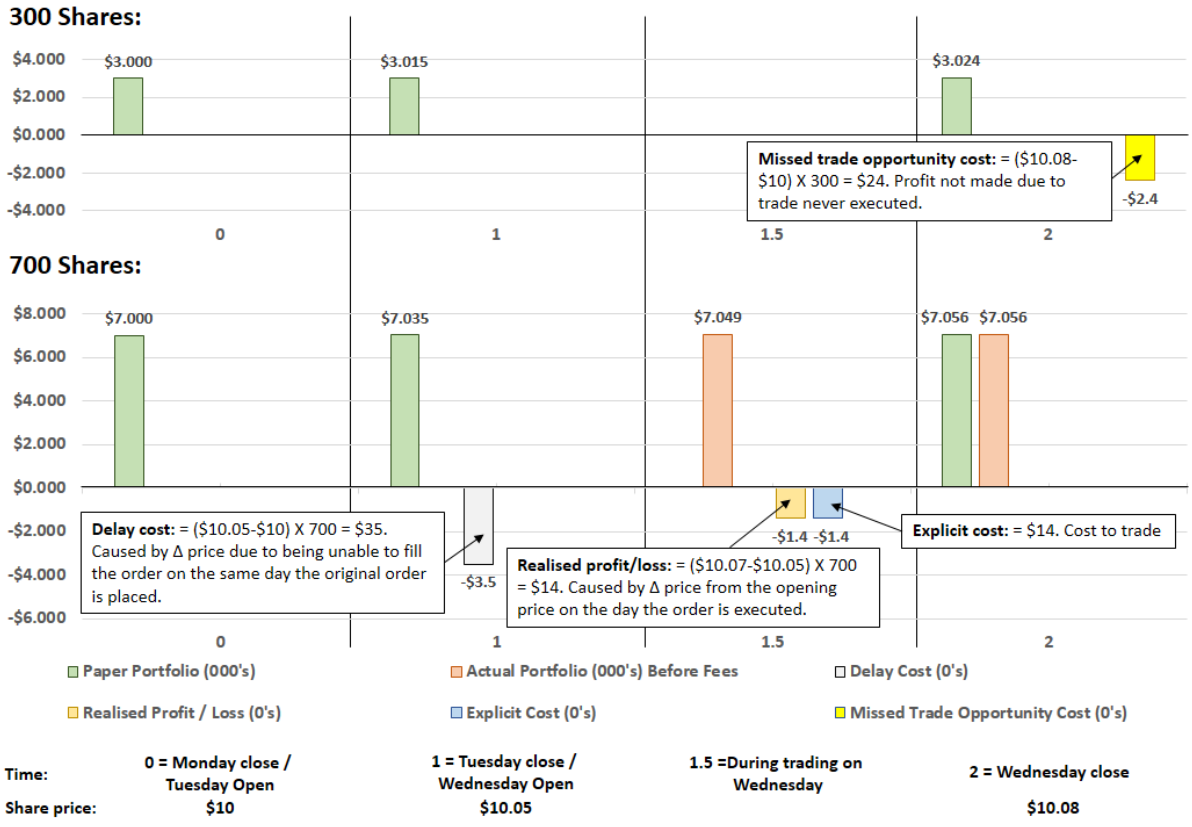

Figure 1: A graphical illustration of the implementation shortfall

Summary

These notes have described how to calculate the costs of trading, including the VWAP and implementation shortfall method.

Appendix 1: The Cost of Trading

In this analysis we will examine the investment outcome for two, nearly identical investors, Ms. Restless and Ms. Patient. The only difference between the investors is that Ms. Restless trades twice as frequently as Ms. Patient. Ms. Restless has a portfolio turnover of 200% per year – i.e. will reallocate their entire portfolio every six months – and Ms. Patient has a portfolio turnover of 100% – i.e. will reallocate their entire portfolio every 12 months. The following table summarizes calculation information for both investors:

	Ms. Patient	Ms. Restless
Initial investment	R 100 000	R 100 000
Periodicity (payments per year)	1	2
Total payments (trading costs and taxes)	10	20
Return (gross, annual)	14.00%	14.00%
Return (gross, periodic)	14.00%	6.77%
Trading cost (%, periodic)	1.00%	1.00%
Return (net of trading costs, periodic)	12.86%	5.70%
Tax rate on capital gains	18%	18%

Please note:

- trading costs are 0.5% per transaction (1% to sell their existing holdings and purchase new holdings);
- trading costs are considered a tax-deductible cost.

Example: Calculating Ms. Patient's future wealth

Trading costs are a tax-deductible expense, so only the return net of these expenses is subject to capital gains tax. The return net of trading expenses is $(1 + 14\%)(1 - 1\%) = 12.86\%$.

The future value net of capital gains tax and trading costs is calculated as follows:

$$Future\ value = R100\ 000 * [1 + r(1 - t_{CG})]^n$$

$$Future\ value = R100\ 000 * [1 + 12.86\%(1 - 18\%)]^{10}$$

$$Future\ value = R272\ 520.$$

The following table summarizes the investors' wealth at the end of ten years:

Net Investment Value (fees and taxes paid)	
Ms. Patient (turnover = 100%)	Ms. Restless (turnover = 200%)
R 272 520	R 249 451

A common tax rule in developed markets is to tax capital gains from short-term investment periods at a higher rate than from long-term investment periods. If we assume that, due to her more frequent trading, Ms. Restless pays a higher rate of tax on capital gains (28% versus Ms. Patient's 18%), then their future wealth is the following:

Net Investment Value (fees and taxes paid)	
Ms. Patient (turnover = 100%)	Ms. Restless (turnover = 200%)
R 272 520	R 223 632

Bibliography

The Johannesburg Stock Exchange Limited. (2013). *JSE Overview*. [Online] Available at: https://www.jse.co.za/about/history-company-overview. [Accessed 06 December 2018].

Madhavan, A., Treynor, J., and Wagner, W. (2018). Execution of Portfolio Decisions. *Trading and Rebalancing, Performance Evaluation, and Global Investment Performance Standards*, *CFA Program Curriculum, Level III,* 6.

Meredith, M. (2007). *Diamonds, Gold and War: The British, the Boers and the Making of South Africa*. New York: PublicAffairs.