WORLDQUANT UNIVERSITY

Compiled Content Module 6

MScFE 640

Portfolio Theory and Asset Pricing

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Module 6: Indexation

This module begins by introducing various methods to calculate security market index values and the issues related to each method. The module continues by discussing index replication methods, and ends by explaining various types of passively managed funds such as index mutual funds, exchange-traded funds, and separate or pooled accounts. Fixed income indexes are introduced as well.

Unit 1: Index Calculation Methods (part 1)

Introduction

A **security market index** is a method of calculating a single number which represents the value, at a particular point in time, of a large number of securities. Hereafter we will simply call a security market index an **index**. It is comprised of a group of securities known as the **constituent securities** (henceforth simply referred to as the **constituents**). An index value only has meaning relative to its own past values. For example, indexes are normally set to equal a nice round number at initiation, such as 100. If the index value is 105 one period later, then the return of the index for the period is 5%. Since an index is created to represent a certain group of securities (which we will call a **target market**) as accurately as possible, an interested party can conclude that an investment in that market would have returned 5%.

A brief history of indexes

1884: The first index for measuring the performance of a security market, the Dow Jones Average, is created.

1896: The Dow Jones Industrial Average, which exists to this day, is created.

1971: Wells Fargo creates the first pension fund which simply replicates an index.

1975: Vanguard Group creates the first broad-market index fund for retail investors.

The world's first index – the Dow Jones Average – was created by the publishers Charles H. Dow and Edward D. Jones in 1884. This index has ceased to exist but the Dow Jones Industrial Average, which they created 12 years later, exists to this day and is a widely recognized measure of the U.S. equity market.

An index can be one of two types: either a price return index or a total return index. A **price return index** measures the performance of the constituents' prices *only*. However, the majority of assets have an income return component in addition to a capital return[1] component. A **total return index** measures both the performance of the constituents' prices and the reinvestment of all income received since initiation. At initiation a price return index and a total return index will have the same value. However, in a short space of time the total return index value will be greater than the price return index value, due to the receipt of income. At the time of this writing, the Standard and

[1] i.e. return due to a change in price

Poor's 500 (S&P 500) price return index value and total return index value were in the region of 2800 and 5700 respectively.

Functions of indexes

Since their creation, the functions of indexes have grown from simply measuring the performance of markets. In present financial markets, they can also serve the following functions (Kaplan & Kelly, 2015):

1. Serve as gauges of market sentiment.
2. Determine systematic risk:

   In the Capital Asset Pricing Model, an asset's return on equity is a function of the risk-free rate and the asset's beta relative to the market return. Indexes are often used to determine market return.
3. Represent asset classes:

   When undertaking financial planning and constructing an optimal portfolio for an investor, a wealth manager will need to predict the return and volatility of various asset classes. The historical return and volatility of indexes representing those asset classes form the basis of these predictions.
4. Serve as a benchmark for actively managed funds. This will be explained in more detail later in the reading.

Multi-period index returns

The price return of an index in a single period is calculated as follows:

$$Price\ return = \frac{V_{I_1} - V_{I_0}}{V_{I_0}},$$

where:

$V_{I_1} = the\ value\ of\ the\ index\ at\ time\ 1\ (end\ of\ the\ period)$

$V_{I_0} = the\ value\ of\ the\ index\ at\ time\ 0\ (beginnning\ of\ the\ period).$

The total return of an index in a single period is calculated as follows:

$$Total\ return = \frac{V_{I_1} - V_{I_0} + Inc_1}{V_{I_0}},$$

where:

$Inc_1 = total\ income(dividends\ and\ interest\ received\ at\ time\ 1)$.

Calculating an index's value based on a series of single-period returns is done by geometrically linking the single-period returns. Geometric linking is performed as follows: for a series of periodic returns denoted r_1 to r_x, the value at t_x of an initial value, v_0, is calculated as follows:

$$v_x = v_0 \times (1 + r_1) \times (1 + r_2) \times \ldots \times (1 + r_x).$$

For example, given the price returns listed in Table 1, the index value at the end of period 3 is calculated as follows. (Note that the calculation below assumes that the index had been set to equal 100 at initiation):

Period	Price return
1	5.2%
2	4.1%
3	-2.6%

Table 1: Price return index

$$V_{I_3} = 100 \times (1 + 5.2\%) \times (1 + 4.1\%) \times \big(1 + (-2.6\%)\big) = 106.6659.$$

If we were calculating a total return index's values, we would add income return to price return to calculate total return as shown in Table 2:

Period	Price return	Income return	Total return
1	5.2%	1.5%	6.7%
2	4.1%	1.5%	5.6%
3	-2.6%	1.5%	-1.1%

Table 2: Total return index

And we would geometrically link the total returns as before:

$$V_{I_3} = 100(1 + 6.7\%)(1 + 5.6\%)\big(1 + (-1.1\%)\big) = 111.4358.$$

111.4358 is the value of the index including the accumulated income.

Issues related to index construction

Now that we have an introductory understanding of indexes, it is time to discuss the issues that an index creator, such as Morgan Stanley Capital International (MSCI), needs to deal with in more detail. These can be broken down into five issues (Kaplan & Kelly, 2015):

1. Which target market should the index represent?
2. Which securities should be selected from that market?
3. How much weight should be allocated to each security in the index?
4. When should the index be rebalanced?
5. When should the security selection and weighting decision be re-examined?

The question relating to constituents' weighting (Question 3) is technical and important enough to justify its own chapter. The other four questions are discussed in the following sections.

Selecting a target market

An index can be constructed to measure any market of interest. A market may be distinguished by any characteristic desired. The more common characteristics that are used to define markets are:

- Asset class

  For example: equity, fixed income, real estate, commodities, etc.

- Market segment

  For example: within the equity asset class we may categorize equities based on size. The term used to describe a company's size is **market capitalization** (simply abbreviated to "market-cap"), which is equal to the number of outstanding shares multiplied by the market price per share. Equities are frequently distinguished into **large-capitalization** (simply abbreviated to "large cap"), **mid-capitalization** ("mid cap"), and **small-capitalization** ("small cap") based on the relative size of the calculated value.

- Geographic region

  For example: an exchange such as the Johannesburg Stock Exchange, a country, a continent, or the world.

It is important to note that an index can be constructed to measure any market desirable. Due to the fact that index providers earn fees from portfolios that track an index they own, they compete to create the most accurate (and therefore most widely recognized) indexes. Some of the other most well-known index providers include MSCI and Vanguard.

Selecting securities from the target market

Once a target market has been selected, the next decision is to decide how to select securities from the **investment universe** – i.e. all the securities in the target market – to adequately represent the target market. One possibility is to include the entire investment universe. For example: if the target market is publicly-listed South African equity, all shares listed on the Johannesburg Stock Exchange will be included. Another possibility is to include a sample of securities from the investment universe. For example, the S&P 500 index measures the performance of large-cap U.S. equities (the target market) by including 500 companies out of those with a primary listing on any eligible U.S. exchange (the sample). There were eleven eligible U.S. exchanges in September 2018. A committee chooses the 500 constituents based on a number of criteria (Anon., 2018).

Rebalancing

Rebalancing refers to returning constituent securities' weights to their appropriate weighting for the index type. Only equal-weighted and fundamental-weighted indexes need to be rebalanced regularly. Market-capitalization-weighted indexes mostly rebalance themselves; only special events such as mergers, acquisitions, liquidations and other corporate actions create the need for rebalancing by the index manager (Kaplan & Kelly, 2015). (This will be better understood after reading the section on constituent weighting.) For those indexes that require it, rebalancing is normally done quarterly, as is the rebalancing for fundamentally-weighted indexes since their fundamentals – e.g. revenue, dividends – are usually available every quarter.

Note that there is a trade-off between the accuracy of the index and the associated costs: a more frequently rebalanced index will be more accurate but will result in higher trading costs for the funds that are indexed[2] to them. Indexes whose constituents are chosen by a committee (such as the S&P 500) will have lower turnover than those whose constituents are chosen by an algorithm (such as the Russell 1000, which competes with the S&P 500 to be the most widely recognized U.S.

[2] The word "index" is a noun and refers to a security market index as defined earlier in this reading. The word "indexing" is a verb and refers to the act of replicating an index. Indexing is performed by passively managed portfolios which do not attempt to beat a benchmark index (which is the case in active investment management) but simply to match it. This is discussed in more detail later in the module.

large-cap index). This is because a committee will avoid changing constituents too frequently, as they realize that this will incur trading costs for indexed funds.

Reconstitution

Reconstitution is the process of changing the constituent securities in an index. It may be based on clear rules – e.g. Russell 2000 index – or be decided by a committee – e.g. S&P 500.

Security weighting

There are four potential weighting methodologies that an index can use. It can be either:

1. price-weighted;
2. equal-weighted;
3. market-cap-weighted, including float-adjusted market-cap weighted; or
4. fundamental-weighted.

The weighting methodology chosen will have a significant impact on index values. Each has advantages and disadvantages. Each will be discussed and demonstrated in the following sections. The information contained in Table 3 will be used in the calculation of all index values.

Security	$Price_0$ ps[3]	Return	$Price_1$ ps	Dividend ps	Shares outstanding	Float adjustment factor[4]	Market float
A	$60.00	9.50%	$65.70	$1.05	3300	1.00	3300
B	$30.00	-11.40%	$26.58	$0.14	11000	0.70	7700
C	$15.00	-34.20%	$9.87	$0.00	5500	0.90	4950
D	$12.00	38.00%	$16.56	$0.07	8800	0.25	2200
E	$4.80	47.50%	$7.08	$0.00	7700	0.80	6160

Table 3: Constituent security information

In each index, a divisor is normally chosen at inception to allow the index to be a nice round number initially, such as 100 or 1 000. The index manager adjusts the value of the divisor as

[3] "ps" stands for per share

[4] The terms "float adjustment factor" and "market float" are explained later in these notes.

necessary to avoid changes to the index value that are unrelated to changes in the prices of the constituent securities.

Price-weighted index

In a price-weighted index, the index is comprised of *1 share* of each of the constituent securities. The weight of a constituent security in the index is its price divided by the sum of the prices of all constituent securities. The index value is calculated by dividing the sum of the prices of all constituent securities by the number of constituent securities.

Example: Calculating price-weighted index values: price return and total return

We will consider a price-weighted index consisting of five securities (A-E) from Table 3. The **divisor** is the number of constituent securities in the index under consideration. In this instance the divisor is 5.

Security	Shares in index	$Price_0$ ps	$Value_0$ $(Shares \times Price_0)$	$Weight_0$
A	1	\$60.00	\$60.00	49.26%
B	1	\$30.00	\$30.00	24.63%
C	1	\$15.00	\$15.00	12.32%
D	1	\$12.00	\$12.00	9.85%
E	1	\$4.80	\$4.80	3.94%
Total			**\$121.80**	
Price return index value			**24.36**	
Divisor			5	

To calculate the initial index value (which is the index value at t_0), we divide the total value of the securities (at t_0) by the divisor. Here, that looks like the following:

$$Index\ value = \frac{TOTAL}{DIVISOR} = \frac{121.80}{5} = 24.36.$$

Thus the initial index value is calculated at 24.36.

If we recalculate the index value one time period later (at t_1), we simply plug the new values into our equation.

Security	Shares in index	$Price_1$ ps	$Value_1$ $(Shares \times Price_1)$	$Weight_1$
A	1	\$65.70	\$65.70	52.23%
B	1	\$26.58	\$26.58	21.13%
C	1	\$9.87	\$9.87	7.85%
D	1	\$16.56	\$16.56	13.16%
E	1	\$7.08	\$7.08	5.63%
Total			**\$125.79**	
Price return index value			**25.16**	
Divisor			5	

The new index value calculation (at t_1) would then be:

$$Index\ value = \frac{TOTAL}{DIVISOR} = \frac{125.79}{5} = 25.16.$$

The index value (at t_1) is thus calculated as 25.16.

With these index values, we can now also calculate the price return over this period, keeping in mind that:

$$Price\ Return = \frac{V_{I_1} - V_{I_0}}{V_{I_0}}.$$

The price return here is 3.28%.

If we were calculating total return we would need to include dividends paid as well.

Security	Shares in index	$Price_1$ ps	Dividend ps	$Value_1$ $[Shares \times (Price_1 + Dividend)]$
A	1	\$65.70	\$1.05	\$66.75
B	1	\$26.58	\$0.14	\$26.72
C	1	\$9.87	\$0.00	\$9.87
D	1	\$16.56	\$0.07	\$16.63
E	1	\$7.08	\$0.00	\$7.08
Total				**\$127.05**
Price return index value				**25.41**
Divisor				5

Remember that we can calculate the total return as follows:

$$Total\ return = \frac{V_{I_1} - V_{I_0} + Inc_1}{V_{I_0}}.$$

The index value is thus now calculated as 25.41. The total return is 4.31%.

The advantages of a price-weighted index relative to an equal-weighted or market-cap weighted index are that it is an easy index to create and manage.

The disadvantages are that the index is more strongly influenced by the highest priced constituent securities. This is not ideal because a company's absolute share price is an arbitrary figure; a company can simply change its share price through stock splits, stock dividends, or reverse splits (Gastineau, et al., 2018).

Well known examples of price-weighted indexes include (Gastineau, et al., 2018):

- *The Dow Jones Industrial Average*. This is the oldest and most widely followed US equity index. The constituent securities are 30 blue chip[5] US companies chosen by the editors of the Wall Street Journal.
- *The Nikkei Stock Average*. Its constituent securities are 225 Japanese blue chip companies. It is considered to be representative of the performance of Japanese equities. To minimize bias towards high share price companies due to the wide variation in price in constituent securities, high share price constituents are adjusted in order to reduce their impact on the index.

[5] The term "blue chip" is market jargon for a company that is considered to be high-quality and a reliable investment by the majority of market participants.

Index Calculation Methods (part 2)

Equal-weighted index

As the name implies, an equal-weighted index weights each constituent equally. The weight of each constituent is calculated with the following equation:

$$w_i = \frac{1}{N},$$

where:

$w_i = weight\ of\ constituent\ security\ i\ in\ the\ equal - weighted\ index$

$N = number\ of\ securities\ in\ the\ index.$

The steps for constructing an equal-weighted index are as follows:

- Calculate w_i.
- Allocate an equal amount in each constituent security according to w_i multiplied by the initial index value.

Example: Calculating equal-weighted index values: price return and total return

An initial index value of 1 000 is calculated as indicated below. There are 5 constituent securities so $w_i = \frac{1}{5} = 20\%$. The index creator wishes the index to have an initial round number, such as 1 000. So each constituent security must contribute 20% to this value. For example, the number of shares of Security A to include in the index is calculated as follows:

$$\frac{2000}{\$60} = 33.33.$$

Security	$Weight_0$	Shares in index	$Price_0$ ps	$Value_0$ $(Shares \times Price_0)$
A	20.00%	33.33	$60.00	2 000
B	20.00%	66.67	$30.00	2 000
C	20.00%	133.33	$15.00	2 000
D	20.00%	166.67	$12.00	2 000
E	20.00%	416.67	$4.80	2 000
Total				10 000
Price return index value				1 000.00
Divisor				10

One time period later (at t_1) the new index value is:

Security	Shares in Index	$Price_1$ ps	$Value_1$ $(Shares \times Price_1)$	$Weight_1$
A	33.33	\$65.70	2 190	20%
B	66.67	\$26.58	1 772	16%
C	133.33	\$9.87	1 316	12%
D	166.67	\$16.56	2 760	25%
E	416.67	\$7.08	2 950	27%
Total			10 988	
Price return index value			1 098.80	
Divisor			10	

The index value is now 1 098.80. The price return is 9.88%, calculated as follows:

$$Price\ return = \frac{1098.80}{1000} - 1 = 9.88\%.$$

If we were calculating the total return, we would need to include dividends paid as follows:

Security	$Weight_0$	Shares in index	$Price_1$ ps	Dividend ps	$Value_1$ $[Shares \times (Price_1 + Dividend)]$
A	20.00%	33.33	\$65.70	\$1.05	2 225
B	20.00%	66.67	\$26.58	\$0.14	1 781
C	20.00%	133.33	\$9.87	\$0.00	1 316
D	20.00%	166.67	\$16.56	\$0.07	2 772
E	20.00%	416.67	\$7.08	\$0.00	2 950
Total					11 044
Price return index value					1 104.40
Divisor					10

The index value is now 1 104.40. The total return is 10.44%, calculated as follows:

$$Total\ return = \frac{1104.40}{1000} - 1 = 10.44\%.$$

The advantages of an equal-weighted index relative to a price-weighted or market-cap weighted index are that all constituent securities impact the index equally, irrespective of their share price or market value.

The disadvantages are that:

- After one period has passed the index will no longer be equally weighted and will need to be rebalanced to become equally-weighted again. Therefore, an indexed portfolio will need to trade frequently to maintain constituent securities in the same weighting as the index. This will lead to higher costs.

- An equal-weighted index cannot be exactly replicated by an indexed portfolio, due to fractional constituents. It is impossible to purchase anything other than a whole share. In the example considered, we saw that the equal-weighted index contained 33.33 shares of constituent A at t_0. An indexed portfolio would have to purchase 33 or 34 shares of constituent A, meaning that the fund's results would differ from the index.

- Since an equal-weighted index will include a greater weight of small cap constituents (relative to a market-capitalization-weighted index), there may not be sufficient liquidity in these small capitalization constituent securities for portfolios to effectively track the index.

A well-known example of an equal-weighted index is the Value Line Arithmetic Composite Index, which consists of approximately 1 700 U.S. equities covered in the *Value Line Investment Survey* (Gastineau, et al., 2018).

Market-cap-weighted index

In a market-cap-weighted index, the weight of each constituent security is calculated by dividing the security's market-cap by the total market-cap of all securities in the index. The **market-cap** of a company is the number of shares outstanding multiplied by the market price per share.

Example: Calculating market-cap-weighted index values: price return and total return

An initial index value of 100 is calculated as indicated below:

Security	Shares outstanding	$Price_0$ ps	$Market\ Cap_0$ (000's)	$Weight_0$
A	3 300	\$60.00	\$198.0	26.29%
B	11 000	\$30.00	\$330.0	43.82%
C	5 500	\$15.00	\$82.5	10.96%
D	8 800	\$12.00	\$105.6	14.02%
E	7 700	\$4.80	\$37.0	4.91%
Total			753.1	
Price return index value			100.00	
Divisor			7.531	

For example, the weight of Security A in the index at t_0 is calculated as follows:

$$Weight_{\ A} = \frac{Market\ Cap\ of\ Company\ A}{Total\ Market\ Cap\ of\ All\ Constituents} = \frac{\$198\ 000}{\$753\ 100} = 0.2629.$$

The index value at t_0 is calculated by dividing the total market cap by the divisor:

$$Market\ cap\ weighted\ index\ value = \frac{Total\ Market\ Cap\ of\ All\ Constituents}{Divisor} = \frac{753.1}{7.531}$$

$$= 100.$$

One period later, the market-cap-weighted price index value is:

Security	Shares outstanding	$Price_1$ ps	$Market\ Cap_1$ (000's)	$Weight_1$
A	3 300	\$65.70	\$216.81	28.39%
B	11 000	\$26.58	\$292.38	38.28%
C	5 500	\$9.87	\$54.29	7.11%
D	8 800	\$16.56	\$145.73	19.08%
E	7 700	\$7.08	\$54.52	7.14%
Total			763.719	
Price return index value			101.41	
Divisor			7.531	

The weight of Security A in the index at t_1 is:

$$Weight_{A} = \frac{\$216\,810}{\$763\,719} = 0.2839.$$

The index value at t_1 is calculated by dividing the total market cap by the divisor:

$$Market\ cap\ weighted\ index\ value = \frac{Total\ Market\ Cap\ of\ all\ Constituents}{Divisor} = \frac{763.719}{7.531} = 101.41.$$

The market-capitalization-weighted index price return is 1.41%.

$$Price\ return = \frac{101.41}{100} - 1 = 1.41\%.$$

If we were calculating total return we would need to include dividends paid as follows:

Security	Shares outstanding	$Price_1$ ps	Dividend ps	$Market\ Cap_1 + Total\ Dividends$ (000's)	$Weight_1$
A	3 300	\$65.70	\$1.05	\$220.28	28.39%
B	11 000	\$26.58	\$0.14	\$293.92	38.28%
C	5 500	\$9.87	\$0.00	\$54.29	7.11%
D	8 800	\$16.56	\$0.07	\$146.34	19.08%
E	7 700	\$7.08	\$0.00	\$54.52	7.14%
Total				769.34	
Price return index value				102.16	
Divisor				7.531	

The market-cap-weighted index total return is 2.16%.

$$Total\ return = \frac{102.16}{100} - 1 = 2.16\%.$$

Float-adjusted market-cap-weighted index

A variation of a market-cap-weighted index is a float-adjusted market-cap-weighted index. Instead of using *total* shares outstanding, a **float-adjusted market-cap-weighted index** uses market float. **Market float** refers to shares that are available to the investing public. For example, if a company has 10 million shares outstanding and, of these, 3 million are held by the company's founder – who has never sold any – and 3 million are held by the company for distribution to employees as part of an incentive package, then the market float would be 4 million shares.

Let us further consider another example. Company B has 11 000 total shares outstanding. Only 7 700 of these are available to the investing public, therefore the float-adjusted market-cap of Company B at t_0 is: $7\ 700 \times \$30 = \$231\ 000$.

The float adjustment factor is calculated as follows:

$$Float\ adjustment\ factor = \frac{Market\ float}{Shares\ outstanding}.$$

So, the float adjustment factor for Company B is $7\ 700/11\ 000 = 0.7$.

It is at the discretion of index providers to determine what the market float for a security is. For example: a U.S. index provider who is creating an index to measure the performance of the Brazilian stock market in U.S. dollar terms might consider the market float to only include shares available to non-Brazilian investors.

Example: Calculating float-adjusted market-cap-weighted index values: price return and total return

An initial index value of 100 is calculated as indicated below:

Security	Market float	$Price_0$ ps	$Float-adjusted\ Market\ Cap_0$ (000's)	$Weight_0$
A	3 300	\$60.00	\$198.00	35.41%
B	7 700	\$30.00	\$231.00	41.31%
C	4 950	\$15.00	\$74.25	13.28%
D	2 200	\$12.00	\$26.40	4.72%
E	6 160	\$4.80	\$29.57	5.29%
Total			559.22	
Price return index value			100.00	
Divisor			5.5922	

For example, the weight of Security A in the index at t_0 is calculated as follows:

$$Weight_A = \frac{Float-adjusted\ Market\ Cap\ of\ Company\ A}{Total\ Float-adjusted\ Market\ Cap\ of\ All\ Constituents} = \frac{\$198\ 000}{\$559\ 220} = 0.3541.$$

The index value at t_0 is calculated by dividing the total float-adjusted market cap by the divisor:

$$Float-adjusted\ Market\ cap\ weighted\ index\ value = \frac{Total\ Float-adjusted\ Market\ Cap\ of\ All\ Constituents}{Divisor} = \frac{559.22}{5.5922} = 100.$$

One period later, the float-adjusted market-cap-weighted price index value is:

Security	Market float	$Price_1$ ps	$Float-adjusted\ Market\ Cap_1$ (000's)	$Weight_1$
A	3 300	\$65.70	\$216.81	39.39%
B	7 700	\$26.58	\$204.67	37.19%
C	4 950	\$9.87	\$48.86	8.88%
D	2 200	\$16.56	\$36.43	6.62%
E	6 160	\$7.08	\$43.61	7.92%
Total			550.38	
Price return index value			98.42	
Divisor			5.5922	

The weight of Security A in the index at t_1 is:

$$Weight_A = \frac{\$216\,810}{\$550\,380} = 0.3939.$$

The index value at t_1 is calculated by dividing the total market cap by the divisor:

$$Float-adjusted\ Market\ cap\ weighted\ index\ value = \frac{Total\ Float-adjusted\ Market\ Cap\ of\ All\ Constituents}{Divisor} = \frac{550.38}{5.5922} = 98.42.$$

The float-adjusted market-cap-weighted index price return is -1.58%.

$$Price\ return = \frac{98.42}{100} - 1 = -1.58\%.$$

If we were calculating total return we would need to include dividends paid as follows:

Security	Market float	$Price_1$ ps	Dividend ps	$Float-adjusted\ Market\ Cap_1 + Total\ Dividends$ (000's)	$Weight_1$
A	3 300	\$65.70	\$1.05	\$220.28	39.39%
B	7 700	\$26.58	\$0.14	\$205.74	37.19%
C	4 950	\$9.87	\$0.00	\$48.86	8.88%
D	2 200	\$16.56	\$0.07	\$36.59	6.62%
E	6 160	\$7.08	\$0.00	\$43.61	7.92%
Total				555.0743	
Price return index value				99.26	
Divisor				5.5922	

The float-adjusted market-cap-weighted index total return is -0.74%.

$$Total\ return = \frac{99.26}{100} - 1 = -0.74\%.$$

The advantages of a market-cap-weighted index relative to a price-weighted or equal-weighted index are that:

- Constituent securities are held in proportion to their value in the target market.
- It self-corrects for stock splits, reverse stock splits, and dividends because such actions are directly reflected in the number of shares outstanding and price per share for the company affected (Anon., 2018).

The disadvantages are that:

- Securities whose prices increase will have a greater weight in the index. Those whose prices decrease will have a lower weight in the index. Securities that have increased in price may then be overvalued, whereas securities that have decreased in price may then be undervalued.
- It is biased towards companies with the largest market capitalizations. In effect, this means that the index will be biased towards large, and probably mature, companies and companies that are overvalued.

Well-known market-capitalization or float-adjusted market-capitalization-weighted indexes include:

- S&P 500. This is a float-adjusted index consisting of 500 of the largest (as measured by market capitalization) U.S. companies. The constituents are chosen by a committee of S&P employees.
- FTSE 100. This is a float-adjusted index consisting of 100 of the largest (as measured by market capitalization) companies traded on the London Stock Exchange.
- DAX 30. This is a float-adjusted index consisting of 30 blue chip German companies. It is a widely recognized measure of the performance of German equities.

It is worthwhile to note that the majority of well-recognized indexes are float-adjusted market-cap indexes.

Fundamental-weighted index

In a fundamental-weighted index, the weight of each security is determined based on some fundamental factor that is independent of its market price. Examples of fundamental factors include: book value, cash flow, revenues, earnings, dividends, and number of employees (Kaplan & Kelly, 2015). Fundamental-weighted indexes may use a single factor to weight securities or a composite of factors.

Summary

The four index returns are compared in Table 4. You'll notice that these values are very different from one another, despite being calculated based on exactly the same securities.

	Price return	Total return
Price-weighted index	3.28%	4.31%
Equal-weighted index	9.88%	10.44%
Market-cap weighted index	1.41%	2.16%
Float-adjusted market-cap weighted index	-1.58%	-0.74%

Table 4: Summary of index returns

An investor would have earned:

- The price-weighted index return if they had bought an equal number of each constituent security.
- The equal-weighted index return if they had spent an equal amount of money purchasing each constituent security.
- The market-cap-weighted index return if they had bought all the outstanding shares of each constituent security.
- The float-adjusted market-cap-weighted index return if they had bought all the market float of each constituent security.

Unit 2: Index Replication Methods

Introduction

Active investment management refers to an investment process where an asset manager attempts to provide better returns (after fees) than a relevant benchmark by gathering information and making decisions. A benchmark in this context will typically be an index that represents the investment manager's **investment universe** – i.e. the set of investments from which the manager selects his investments. The investment manager's return relative to this benchmark is known as his **active return**.

$$Active\ return = Portfolio\ return - Benchmark\ return.$$

If the relative return (after fees and costs) is positive, then the investment manager has added value. Due to the fact that relative return is often not positive, more and more funds have become placed under passive management.

Passive investment management refers to an investment approach where an investor does not attempt to beat a benchmark but only to match it. Passive investment management is achieved by **indexing,** which refers to purchasing the same securities in the same proportions as the benchmark. This is done in order for the passively managed fund to achieve a return as close as possible to the benchmark. In the following sections we will discuss three methods that passive investment managers may employ in order to replicate an index's returns: full replication, stratified sampling, and optimization. The effectiveness of each replication method will be measured in terms of its f. **Tracking risk** is the standard deviation of the differences between a portfolio's returns and its benchmark's returns (Anon., 2018). Tracking risk is also known as **active risk** or **tracking error**. A perfectly indexed portfolio will have a tracking risk of 0. What this would mean is that indexed portfolio returns are exactly equal to index returns. Due to management fees, trading costs, and cash drag – all of which cost money and therefore reduce returns – a perfectly indexed portfolio will still have lower returns than its reference index.

Full replication

Implementing the **full replication indexing method** requires that the manager purchase the same securities as the reference index and in the same weights. If implemented correctly, this method should result in the lowest tracking risk.

This method is most appropriate when:

- the number of constituent securities is less than 1 000;
- constituent securities are liquid.

When full replication is not appropriate, alternative methods of indexing are stratified sampling and optimization. Since both of these methods do not require that the index be mimicked exactly, they are both likely to have a higher tracking risk than a portfolio indexed with the full replication method.

Stratified sampling

In **stratified sampling**, all the constituent securities are classified (or "stratified") based on distinguishing characteristics. What this means is that similar securities are grouped together. A sample of securities are then selected from each group for inclusion in the indexed portfolio. As a result, the total number of securities in the indexed portfolio is less than the number of constituent securities in the index. The reason for classifying the securities is to ensure that a representative sample is selected. The larger the number of distinct groups created, the more representative the sample will be (the extreme would be for the number of groups to equal the number of constituent securities, in which case the portfolio manager would be implementing the full replication method).

Optimization

Optimization is a more mathematical approach to indexing. It involves the calculation of the factor exposures of the index. Factor exposures are variables that the index has a reasonably consistent and predictable relationship with. The securities of the indexed portfolio are then selected to ensure that the factor exposures of the index and the portfolio match one another.

Summary

In this set of notes, we have studied the methods that a passive manager can use to replicate an index, and the effectiveness of each of these techniques.

Unit 3: Structures for Passively Managed Funds and Replication of Equity Versus Bond Indexes

Introduction

Now that we have a solid understanding of how indexes are created and how indexing is achieved, we will discuss the types of investment vehicles available to various types of investors. The three most important categories of indexed portfolios are (Gastineau, et al., 2018):

1. index mutual funds;
2. exchange-traded funds (ETFs);
3. separate accounts or pooled accounts.

Index mutual fund

In an index mutual fund:

- an asset manager takes in money from investors;
- the pooled money (the mutual fund) is invested in the same securities and in the same weights as the reference index.

When investors wish to trade shares in the mutual fund, they trade with the asset manager. The trade price is based on the net-asset-value of all the constituent securities. Time period "windows" in which to trade are available on a regular basis. This arrangement is illustrated in Figure 1 below.

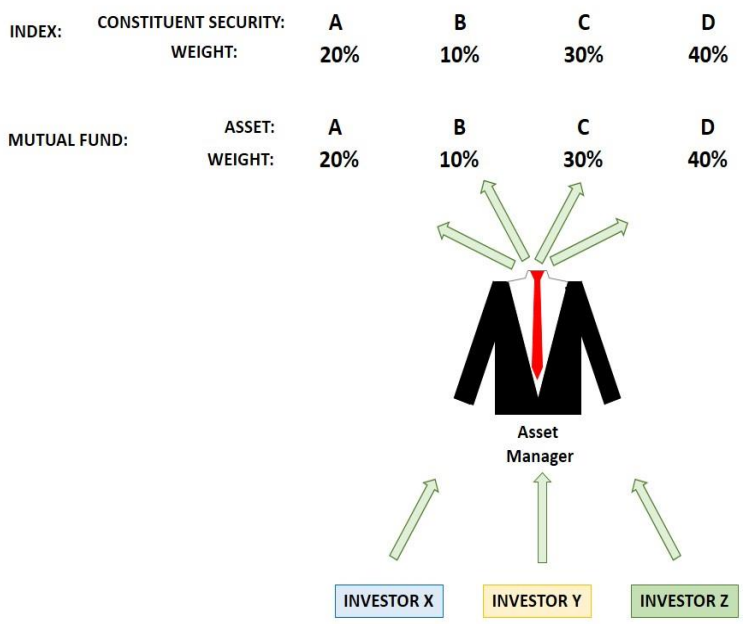

Figure 1: The structure of an index mutual fund

Exchange-traded fund

In an ETF:

- a fund is created, shares of which are listed on a securities exchange;
- the fund is invested in the same securities and in the same weights as the reference index;
- the shares of the ETF are traded among investors on the exchange.

This arrangement is illustrated in Figure 2.

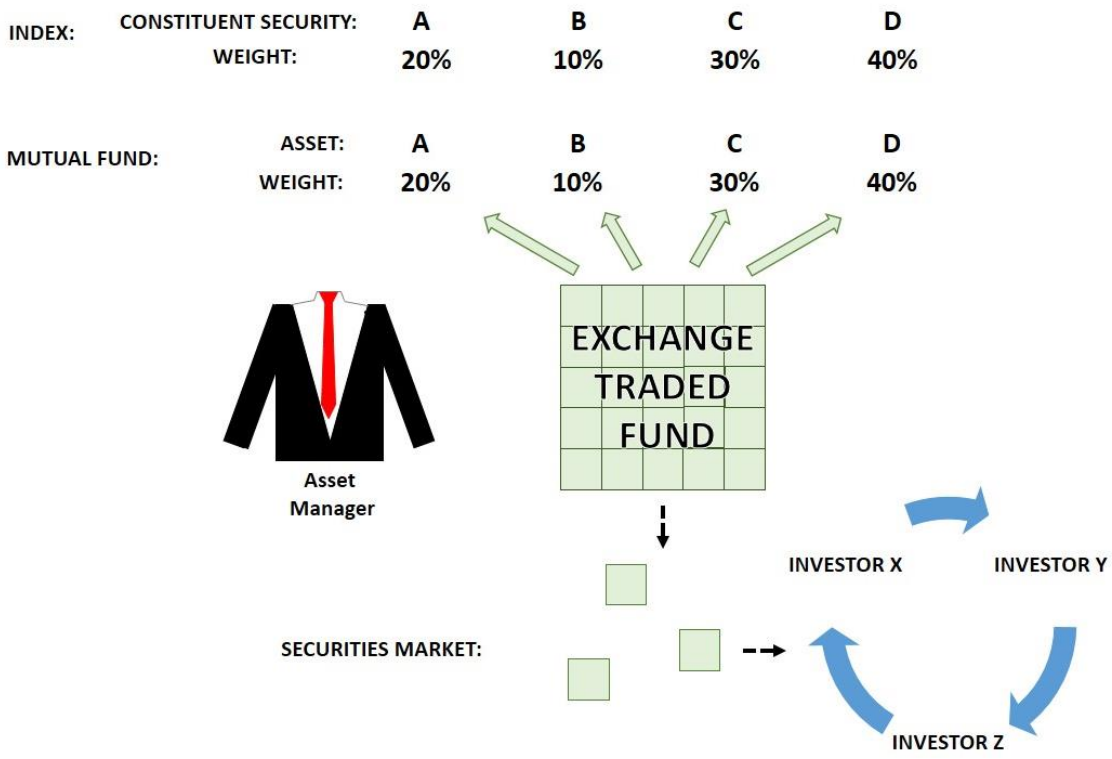

Figure 2: The structure of an ETF

In an ETF, an asset manager is responsible for managing the ETF and ensuring that it continues to match the reference index. An authorized dealer, known as an **authorized participant**, is allowed to undertake arbitrage when the market price of an ETF strays from the net asset value of its constituents. In this way, the market price of the ETF is kept approximately equal to its net asset value.

The fact that ETF shares are traded on an exchange makes them more liquid than an index mutual fund where trade is only possible at certain times.

Separate accounts or pooled accounts

Indexed separate accounts or pooled accounts are very similar in structure to an index mutual fund. The difference is that there are far fewer and much larger (based on money invested) investors. An investor may be a single, very large entity (such as a sovereign wealth fund or a large pension fund) or consist of a few slightly smaller entities (such as a few extremely wealthy individuals) that have been pooled in order to achieve economies of scale. Indexed separate accounts or pooled accounts have lower costs than index mutual funds, partly due to less arduous record-keeping requirements (Gastineau, et al., 2018). ETFs have no shareholder accounting at

the fund level, and therefore they have the lowest expenses of the three in this regard (Gastineau, et al., 2018).

Fixed income indexes

Creating and replicating a fixed income index presents unique challenges due to the characteristics of the fixed income market and fixed income securities. These challenges are due to:

1. the number of securities in the fixed income market;
2. the availability of pricing information; and
3. liquidity.

Number of securities in the fixed income market

One reason that a fixed income index is more difficult to create and replicate is due to the fact that there are far more unique securities in the fixed income market than compared to the equity market.

One factor contributing to the larger number of securities is the fact that fixed income securities can be issued by a wide variety of organizations, including governments, government agencies, and corporations. In contrast, equity securities can only be issued by corporations.

Secondly, a single entity can issue multiple fixed income securities, whereas a corporation can only issue a few types of equity (a couple of classes of common stock and preferred stock). The multiple fixed income securities that a single entity can issue vary by maturity date, embedded options – e.g. puttable bond, callable bond, convertible bond – and coupon rate. The fact that fixed income securities can mature and cease to exist increases the turnover of fixed income indexes relative to equity indexes.

Thirdly, an entity can issue bonds in more than one currency. Governments, for example, can issue bonds in their domestic currency and in foreign currencies.

Availability of pricing information

Fixed income markets are predominantly dealer markets. This means that specific dealers trade in specific, fixed income securities. Consequently, obtaining the current price of a specific security may be more challenging than obtaining the price of an equity security, which are predominantly traded on exchange markets. In exchange markets, market participants trade with each other (not

necessarily only with dealers) and bid, offer, and closing prices are posted to a publicly available board.

Liquidity

Fixed income markets are generally less liquid than equity markets. As a result, the prices for fixed income securities may be **stale** (meaning that it has been a long time since a new market price was established) or possibly even not available.

Summary

In this set of notes, we have studied the different types of fund structures that can provide passive exposure. We have also examined the difference between replicating equity indexes compared to fixed income indexes.

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