① Probability Homework Muni. M

1. 2 dice rolled at once. Find P( ) of sum of number being even and one of die shows 6.

P(A): Sum of even #
: 2, 4, 6, 8, 10, 12

1,1 1,3 1,5 3,1 3,3 3,5
5,1 5,3 5,5 2,4 2,6 2,2
4,2 4,4 4,6 6,2 6,4 6,6
18/36 = 1/2

P(B): one die 6.
6,1 6,2 6,3 6,4 6,5 6,6
(5) 5/36

P(AB) = 5

$P(A|B) = P(AB) / P(B) \Rightarrow \frac{5/36}{1/2} = \frac{10}{36} = \frac{5}{18}$?

2. Two Dice Rolled at once. P( ) of sum less than 7.

⇒ Total: P(S) = 36

⇒ P(<7): 1,5 1,4 1,3 1,2 1,1
2,4 2,3 2,2 2,1
3,3 3,2 3,1
4,2 4,1
5,1
} 15

⇒ $P(<7) = \frac{15}{36}$.

3. Toss fair coin 8 times. Given observed 1 head. P( ) of observe atleast 2 heads.

P(A): HHH HTH HTT HHT
TTT THT TTH THH
} 8

P(1H) = 7/8

P(2H) = 4/8

$P(1H|2H) = \frac{4/8}{7/8} = \frac{4}{7}$

4. Its Rainy 1/3 of days. Not Raing 2/3 of days muni

Raing, there is heavy traffic with $p=1/2$

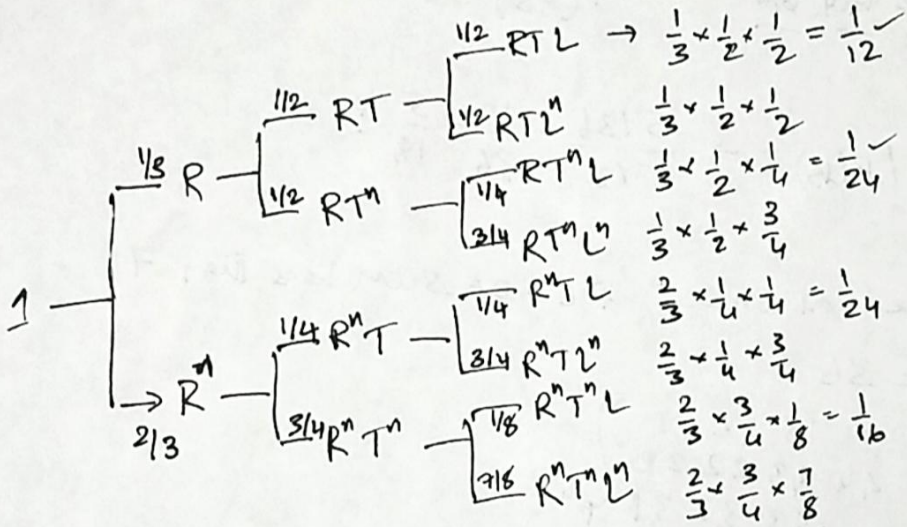
Not Raing, there is heavy traffic with $P() = \frac{1}{4}$

If R, arrive L for work with T $P() = 1/2$

If not R, and no heavy traffic P() of L = 1/8

In other situation (R & noT ; not R & T) $P() = \frac{1}{4} \times \frac{1}{4}$

Pick a random day. What is P() that its not raining & there is heavy T & I am not L.



(a) P() Not R & Heavy T & not Late.

$$P(R^n \cap T \cap L^n) \Rightarrow P(R^n)\, P(T|R^n)\, P(L^n|R^n \cap T) = \frac{2}{3} \times \frac{1}{4} \times \frac{3}{4} = \frac{1}{8}$$

(b) probability that I am late

$$P(L) = \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{4+2+2+3}{48} = \frac{11}{48}$$

(c) Given I arrived Late, what is prob that it rained

$$P(R \cap L) = \frac{1}{12} + \frac{1}{24} = \frac{2+1}{24} = \frac{3}{24} = \frac{1}{8}$$

$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{1/8}{11/48} = \frac{1}{8} \times \frac{48}{11} = \frac{6}{11}$$

(2)

5. A box → 3 coins. 2 Regular & One with two heads ((P(H) = 1)

You pick a coin at random and toss it

(a) What is P() that it lands heads up → 1/2, 2/3, 1, 1/3

R_c → Reg coin R_f → fake coin

$P(H/R_c) = 0.5$ $P(H/R_f) = 1$

$\Rightarrow P(H/R_c) \times P(R_c) + P(H/R_f) P(R_f)$

$\Rightarrow \frac{1}{2} \times \frac{2}{3} + 1 \cdot \frac{1}{3} = \frac{2}{3}$

(b) pick a coin at random and toss it & it gets H. What is P() that its a R_f.

$$P(R_f/H) = \frac{P(H/R_f)\, P(R_f)}{P(H)} = \frac{1 \times \frac{1}{3}}{2/3} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

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11. A is known to tell the truth 5/6. A states a white Ball was drawn from a bag of 8 B & 1 W. Find P() that W ball is drawn

$$P(W) = \frac{1}{9} \quad P(W') = 1 - \frac{1}{9} = \frac{8}{9}$$

$$P(T/W) = \frac{5}{6} \quad P(T/W') = 1 - \frac{5}{6} = \frac{1}{6}$$

$$P(W/T) = \frac{P(T/W) * P(W)}{P(T)} = \frac{\frac{5}{6} \times \frac{1}{9}}{\frac{13}{54}} = \frac{5}{13}$$

$$P(T) = P(T/W)\, P(W) + P(T/W')\, P(W')$$

$$= \frac{5}{6} \times \frac{1}{9} + \frac{1}{6} \times \frac{8}{9} = \frac{5}{54} + \frac{8}{54} = \frac{13}{54}$$

#7. A population has a mean of 50 & SD of 6.

a. what are μ and σ of sampling distribution of μ for $N=16$

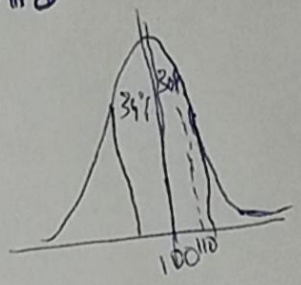
$$\mu = 50 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{6}{\sqrt{16}} = \frac{6}{4} = 1.5$$

b. what are μ & σ for $N=20$.

$$\mu = 50 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{6}{\sqrt{20}} = \frac{6}{2\sqrt{5}} = \frac{3}{\sqrt{5}} = \frac{3}{2.24} = 1.34$$

#8 Given a test that is N-dist with $\mu = 100$ & $\sigma = 12$.

a) prob that a single score drawn at random will be ≥ 110



$$z = \frac{110-100}{12} = \frac{10}{12} = \frac{5}{6} = 0.83 \Rightarrow 0.7967 = 0.80$$

$$1 - 0.8 = 0.2 = 20\%$$

b) prob that a sample of 25 will have $\mu > 105$

Since the sample is for $N=25$, effectively, σ is changed:

$\sigma = 12$ & for $N=25 \Rightarrow \sigma = \frac{12}{\sqrt{25}} = \frac{12}{5} = 2.4$

So: $z = \frac{105-100}{2.4} = \frac{5}{2.4} = 2.08 \Rightarrow 0.98124.$

Area to right of $2.08 \Rightarrow 1 - 0.9812 \Rightarrow 0.01876 \times 100 = \underline{1.87\%}$

(c) prob that $N=64$ for $\mu > 105$; repeating as above

$\sigma = 12$ & $N=64 \rightarrow \sigma = \frac{12}{\sqrt{64}} = \frac{12}{8} = \frac{3}{2} = 1.5$

$$z = \frac{105-100}{1.5} = \frac{5}{1.5} = 3.33 \Rightarrow 0.99957 \Rightarrow 1 - 0.99957 = 0.00043$$

$$\Rightarrow \underline{0.04\%}$$

(d) prob that mean of sample of 16 will be $\underline{\leq 95}$ & ~~105~~

$\sigma = 12$; $N=16 \Rightarrow \sigma = \frac{12}{\sqrt{16}} = \frac{12}{4} = 3$

$z = \frac{95-100}{3} = -\frac{5}{3} = 1.67 \Rightarrow 0.47746 \times 100 = 47.45 \rightarrow$ 47.8%

$z = \frac{105-100}{3} = \frac{5}{3} = 1.67 \Rightarrow 0.95254 \Rightarrow 1 - 0.9525 = 0.00476 \times 100 = 0.4$

Central Tendencies Assignment

1. Find mean of following

a) 9, 7, 11, 13, 2, 4, 5, 5 ⇒ $\frac{2+4+5+5+7+9+11+13}{8} = \frac{56}{8} = 7$

b) 2.2, 10.2, 14.9, 5.9, 4.4, 11.1, 10.5 ⇒ $\frac{2.2+4.9+5.9+10.2+10.5+11.1+14.7}{7}$

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(9) SAT $\mu = 1000$ would you be more likely ⇒ equally likely to get a $\mu = 1200$ if you sampled 10 students & sampled 30 students? python.

(10) Normally dist pop ⇒ with $\sigma = 2.8$

(a) Compute 95% CI based on sample of 9: 8, 9, 10, 13, 14, 16, 17, 20, 21

$$\mu = \frac{8+9+10+13+14+16+17+20+21}{9} = \frac{128}{9} = 14.2$$

95% CI ⇒ $\mu \pm 2\sigma$

$2\sigma = 2.8 \times 2 = 5.6$

$-2\sigma < \mu < 2\sigma \Rightarrow 8.6 < \mu < 19.8$

$14.2 - 5.6 = 8.6$ $\quad 14.2 + 5.6 = 19.8$

(b) 99% CI on same data as above.

$3\sigma = 2.8 \times 3 = 8.4$

CI → 99% = $\mu \pm 3\sigma$

$\mu + 3\sigma = 14.2 + 7.4 = 22.6$

$\mu - 3\sigma = 14.2 - 7.4 = 6.8$

$$\boxed{6.8 \leq \mu \leq 22.6}$$

(12) A Speaker Truth 4/5. A die is tossed. A reports its 6. (4) Mumi
What are chances that its actually 6.

$$P(6) = \frac{1}{6} \quad P(6') = \frac{5}{6}$$

$$P(T/6) = \frac{4}{5} \quad P(T/6') = \frac{1}{5}$$

$$P(6/T) = \frac{P(T/6) \times P(6)}{P(6)P(T/6) + P(6')P(T/6')} = \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}}$$

$$= \frac{\frac{4}{30}}{\frac{9}{30}} = \frac{4}{9}$$