Probability Homework — Murat M.

1. 2 dice rolled at once. Find P( ) of sum of numbers being even and one of die shows 6.

P(A): Sum of even #
: 2, 4, 6, 8, 10, 12

1,1 1,3 1,5 3,1 3,3 3,5
5,1 5,3 5,5 2,4 2,6 2,2
4,2 4,4 4,6 6,2 6,4 6,6
18/36 = 1/2

P(B): one die 6.
6,1 6,2 6,3 6,4 6,5 6,6
(5) 5/36

P(AB) = 5

$P(A|B) = P(AB) / P(B) \Rightarrow \frac{5/36}{1/2} = \frac{10}{36} = \frac{5}{18}$?

2. Two Dice Rolled at Once. P( ) of sum less than 7.

→ Total: P(B) = 36

⇒ P(<7): 1,5 1,4 1,3 1,2 1,1
2,4 2,3 2,2 2,1
3,3 3,2 3,1
4,2 4,1
5,1
} 15

⇒ $P(<7) = \frac{15}{36}$.

3. Toss fair coin 3 times. Given observed 1 head. P( ) of observe atleast 2 heads.

P(A): HHH HTH HTT HHT
TTT THT TTH THH
} 8

P(1H) = 7/8

P(2H) = 4/8

$P(1H|2H) = \frac{4/8}{7/8} = \frac{4}{7}$

4. Its Rainy 1/3 of days. Not Rainy 2/3 of days muni

Rainy, there is heavy traffic with $p=1/2$

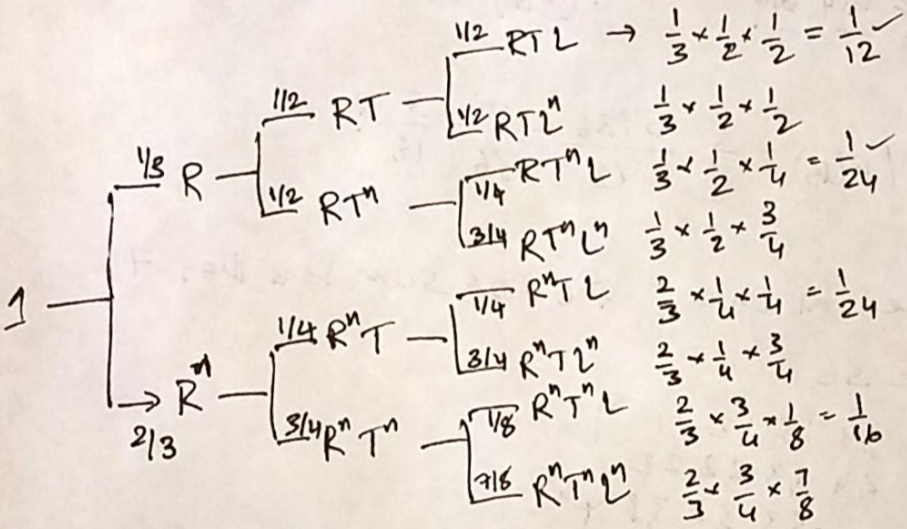
Not Rainy, there is heavy traffic with $P() = \frac{1}{4}$

If R, arrive L for work with T $P() = 1/2$

If not R, and no heavy traffic $P()$ of $L = 1/8$

In other situations (R & noT ; not R & T) $P() = \frac{1}{4} \times \frac{1}{4}$

Pick a random day. What is $P()$ that its not raining & there is heavy T & I am not L.



(a) $P()$ Not R & Heavy T & not Late.

$$P(R^n \cap T \cap L^n) \Rightarrow P(R^n)\, P(T|R^n)\, P(L^n|R^n \cap T) = \frac{2}{3} \times \frac{1}{4} \times \frac{3}{4} = \frac{1}{8}$$

(b) probability that I am late

$$P(L) = \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{4+2+2+3}{48} = \frac{11}{48}$$

(c) Given I arrived Late, what is prob that it rained

$$P(R \cap L) = \frac{1}{12} + \frac{1}{24} = \frac{2+1}{24} = \frac{3}{24} = \frac{1}{8}$$

$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{1/8}{11/48} = \frac{1}{8} \times \frac{48}{11} = \frac{6}{11}$$

~~Maths~~
Muni

5. A box → 3 coins. 2 Regular & one with two heads ((P(H) = 1)
You pick a coin at random and toss it

(a) What is ~~probab~~ P( ) that it lands heads up → $\Rightarrow P(H|R_c) \times P(R_c) + P(H|R_f)\, P(R_f)$

R_c → Reg coin R_f → fake coin

$P(H/R_c) = 0.5$ $P(H|R_f) = 1$

$\Rightarrow \frac{1}{2} \times \frac{2}{3} + 1 \cdot \frac{1}{3} = \frac{2}{3}$

(b) pick a coin at random and toss it & it gets H. What is P( ) that its a R_f.

$$P(R_f|H) = \frac{P(H|R_f)\, P(R_f)}{P(H)} = \frac{1 \times \frac{1}{3}}{2/3} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

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11. 'A' is known to tell the truth 5/6. A states a white Ball was drawn from a bag of 8 B & 1 W. Find P( ) that W ball is drawn

$P(W) = \frac{1}{9}$ $P(W') = 1 - \frac{1}{9} = \frac{8}{9}$

$P(T/W) = \frac{5}{6}$ $P(T/W') = 1 - \frac{5}{6} = \frac{1}{6}$

$$P(W/T) = \frac{P(T/W) \times P(W)}{P(T)} = \frac{\frac{5}{6} \times \frac{1}{9}}{\frac{13}{54}} = \frac{5}{13}$$

$P(T) = P(T/W)\, P(W) + P(T/W')\, P(W')$

$= \frac{5}{6} \times \frac{1}{9} + \frac{1}{6} \times \frac{8}{9} = \frac{5}{54} + \frac{8}{54} = \frac{13}{54}$

(12) A Speaker Truth 4/5. A die is tossed. A reports its 6. What are chances that its actually 6.

Mumi

$$P(6) = \frac{1}{6} \qquad P(6') = \frac{5}{6}$$

$$P(T/6) = \frac{4}{5} \qquad P(T/6') = \frac{1}{5}$$

$$P(6/T) = \frac{P(T/6) \times P(6)}{P(6)P(T/6) + P(6')P(T/6')} = \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}}$$

$$= \frac{\frac{4}{30}}{\frac{9}{30}} = \frac{4}{9}$$