Power of a Point

Munir Uz Zaman

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§1 Power of a Point

Theorem 1.1 (Power of a Point)

Consider a circle ω and an arbitrary point P. The **power of** P with respect to circle ω is defined by

$$\text{Pow}_\omega(P) = OP^2 - r^2$$

where O is the center of ω and r is the radius.

- $\text{Pow}_\omega(P)$ is negative, zero or positive according to whether P is inside, on or outside the circle ω, respectively.
- If a line ℓ through point P intersects ω at points X and Y then

$$PX \times PY = |\text{Pow}_\omega(P)|$$

- If the line PA is tangent to circle ω at point A then

$$PA^2 = \text{Pow}_\omega(P)$$

Theorem 1.2 (Converse of the Power of a Point)

Let A, B, X, Y be four distinct points in the plane and let $P = \overline{AB} \cap \overline{XY}$ be the intersection point of lines $\overline{AB}$ and $\overline{XY}$. If $PA \times PB = PX \times PY$ and if P lies on both segments AB and XY, or in neither segment then the points A, B, X, Y are concyclic.