Problem Solving Strategies

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1 Invariants

Proposition 1.0.1

Let a_n be sequence of integers recursively defined as,

$$a_n = \sum_{j=1}^{k} N_j a_{n-j}$$

where $N_j \in \mathbb{Z}$. If N is an integer such that, $\sum_{j=1}^{k} N_j \equiv 1 \pmod{N}$ then the sequence

$$X_n = \sum_{i=1}^{m} k_m a_{n-m+1}$$

where,

$$k_m = \sum_{r=0}^{m-1} N_{k-r}$$

is invariant modulo N, that is, $X_n \equiv X_{n-1} \pmod{N}$ for all n.