

EGMO : Chapter 3

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§1 Menelaus's and Ceva's Theorem

Theorem 1.1 (Menelaus's Theorem)

Let X, Y, Z be points on lines BC, CA, AB in triangle $\triangle ABC$, distinct from its vertices. Then X, Y, Z are collinear if and only if

$$\frac{BX}{XC} \cdot \frac{CY}{YA} \cdot \frac{AZ}{ZB} = -1$$

where the lengths are directed^a.

^aGiven collinear points X, Y, Z , we say that $\frac{XY}{YZ}$ is positive if Y lies between X and Z

Theorem 1.2 (Ceva's Theorem)

Let X, Y, Z be points on lines BC, CA, AB in triangle $\triangle ABC$, distinct from its vertices. Then X, Y, Z are concurrent if and only if

$$\frac{BX}{XC} \cdot \frac{CY}{YA} \cdot \frac{AZ}{ZB} = +1$$

where the lengths are directed.

§2 Homothety

Definition 2.1. A **homothety** \mathcal{H} is a transformation defined by a center O and a real number k . It sends a point P to another point $\mathcal{H}(P)$, multiplying the distance from O by k . The number k is called the scale factor.

