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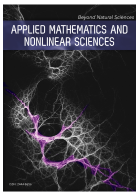

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Study on the training model of football movement trajectory drop point based on fractional differential equation

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Abstract

To study the landing point of the curved football track, the dynamic differential equations of the football were derived in this paper. The air resistance moment was taken into account, and the rotation axis was no longer confined to the vertical direction. We compare various soccer movement regularity of different initial angular velocity 0, in turn, using standard numerical methods to solve differential equations, the selection of the initial angular velocity of three typical 0s has been carried on the detailed numerical study, and the results show that: in the same velocity V play football, corresponding to different initial angular velocity 0, the movement of football is an obvious difference. Conclusion: For the same V = $5+28+11$ m/s, when no rotation Ω 0 = 0, the trajectory of the football is the usual trajectory of the projectile. When 0 = $2-2+16$ rad/s, the trajectory of the football is a typical banana ball trajectory; When 0 = 13+0+0 rad/s, the trajectory of the football shows the phenomenon of left-right fluttering, similar to the fallen leaf ball.

Keywords: football, fractional differential equation, magnus effect, digital to analog
AMS 2010 codes: 34A08

1 Introduction

In the video tracking system, to analyse the movement of the football, it is necessary to know the position of the football at every moment. In this way, it is necessary to identify and separate the target object football in each frame of the video sequence and determine its position, that is, the image segmentation based on the video sequence. At present, there are many methods of image segmentation, but it is difficult to find a general method. Some systems use grayscale information to segment video sequence images, others use colour information to segment video sequence images [1]. Grayscale segmentation often ignores the original colour information of

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the image, but for the moving target, colour is a relatively stable feature and can provide more information than gray. Therefore, this paper mainly discusses the algorithm of segmentation with colour information, and for the recognition and extraction of football trajectory, to find out the better algorithm [2].

In the current college physics textbooks, the oblique projectile motion is mostly discussed with the particle as the research object, and seldom involves the oblique projectile motion of a rotating object. In fact, the movement of the rotating projectile is a very common phenomenon, especially in ball games. When the ball is moving in the air, it is often accompanied by rotation, such as football, table tennis, basketball and tennis. And the rotation of the sphere itself must affect the trajectory of its motion, that is, in this case, the Magnus effect needs to be considered [3]. When a cylinder is around its own axis of rotation and the fluid is in the direction perpendicular to the axis flow, it will be a transverse force perpendicular to the flow direction. When the direction of the force is always never flowing in direction and the online speed cylinder is on the opposite side of the point to the same side, this phenomenon is called Magnus effect, and the literature written is for the Magnus effect. In fact, the Magnus effect is not limited to the rotating cylinder, the so-called ‘banana ball’ in the football game, the circular ball and the slice in the table tennis game all have the obvious effect. The literature has discussed the phenomenon of ‘banana ball’, but most of them are only limited to qualitative discussion, and lack quantitative research, and do not give strict mathematical results. In this paper, all kinds of curvilinear spheres are collectively called curveballs. Starting from the kinetic equation, the motion law and trajectory of the curveballs are studied [4].

In this paper, there are three key points to recognise moving objects. First of all, we must ensure the correctness of target identification, that is, weshould be able to accurately identify the required target, which is the premise of correctly analysing the state of football. If there is no correctness, that is, the discriminated football is not the actual football, and it will lead to the next movement prediction, state analysis and a series of tasks error. Second, it is necessary to ensure the rapidity of target recognition, also known as real-time, that is, to quickly identify the moving target in as short a time as possible, which requires the design of an appropriate recognition algorithm, which will reduce the complexity of the algorithm as far as possible, and improve the effectiveness [5]. The third is to maximise the accuracy of the target identification position, that is, to improve the accuracy of the identification, that is, to improve the accuracy of the final analysis results of the football movement state as far as possible. If the identified football position has a large error, it is reflected in the final analysis.

Various motion parameters have great influence, which reduces the significance of automatic video tracking by computers to a certain extent. Hao Chenghong, Yao-Qing Huang, Lu Sheng Yang, Jun-Sheng Duan, etc who were involved in the study of football movement made a further study of the dynamics equation of football movement. Considering the MagnusEffect and related parameters, the influence of the choreography is derived for the football curveball movement track curves, and the result has universal significance not only in football but it also provides the theoretical basis and calculation method for other related ball sports training (such as table tennis [6]). Liang Huiqiang, in ‘Preliminary Study on the Drift of a Curving Ball’, generalised the function of Magnus-Force concerning radius R, rotational speed n and relative velocity V by using the integral method, and obtained that the expression has universal significance for calculating the rotation motion of a homogeneous sphere in an ideal fluid with relative velocity V. Wu Junwei, Zhang Sheping, Ouyang Lin, Han Wei, et al. used mathematical methods to accurately describe the flight path of a football in the article ‘Theoretical Research on the Running Track of a Direct Free Kick in Football’. This paper quantifies the power, eccentricity, Angle and final landing point of the ball after passing the ‘human wall’ of the direct free kick, and calculates the trajectory parameters of the whole football movement [7]. The relevant factors used are very valuable for relevant research. Foreign scholars are very fond of and good at using theoretical physics to explain the phenomena in sports, especially to explain some seemingly unreasonable phenomena. On the analysis of all kinds of free kicks, foreign scholars have summed up several related research factors, which have universal significance for all trajectories.

The first factor is the vector of the flight speed. In foreign research, the ball sports research takes ‘spin first and then speed' as the mainstream, only football is the first to talk about speed, then considers rotation. It depends on the speed of the ball. The second element is the angular velocity vector. The model of a football rotating around the centre of a circle is established. The experiment proves that it is a general linear model with the ground as the reference frame. The other factors include the air condition inside the football, the roughness of the football surface and the comprehensive variables of the environment, which will be introduced in the text [8]. In Europe, for football players, lower limb strength research in the late 20th century has been embryonic. Since the start of the 21^{st} century, in foreign research, they have launched a series of studies, which have resulted in various innovation points: Based on the research literature, this paper further considers the air resistance moment, and the rotation axis is no longer limited to the vertical direction so that the physical model is closer to the actual situation. The standard numerical method is used to solve the dynamic differential equations, and the motion rules of three different rotating soccer balls are compared, and the three-dimensional trajectory of the curveball is quantitatively simulated by animation.

2 Research methods

2.1 Dynamic differential equations

The force that deflects a spinning ball in flight is called the Magnus force, and its magnitude and direction are given by the following equation

$$\vec{f} = \frac{8}{3}\pi p r^3 \vec{\omega} \times \vec{V} \tag{1}$$

In the formula, V is the velocity of the football relative to the air, ω bar is the instantaneous angular velocity of the football, r is the radius of the ball, and ρ is the density of the air. Suppose the moment of inertia about the central axis of the football is I, the initial rotational speed is, and the air resistance moment is proportional to the rotational angular velocity, then the rotational angular velocity of the football at time t is satisfied [9, 10].

$$I\frac{d\vec{\omega}}{dt} = \vec{M} = -\eta\vec{\omega} \tag{2}$$

Solve the system of Eq. (2)

$$\vec{f} = \frac{8}{3}\pi p r^3 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ V_x & V_y & V_z \end{vmatrix} \tag{3}$$

In addition to the gravity of Mg and Magnus force F, the ball is also subjected to the action of air resistance, which is proportional to the square of the speed and in the opposite direction of the speed

$$\vec{R} = -kV^2\vec{V} = -kV\vec{V} = -k\sqrt{x'^2 + y'^2 + z'^2} \tag{4}$$

Put formula (4) and gravity into Newton's second law, to obtain the football kinetic equations system (5):

$$mz'' = -k\sqrt{x'^2 + y'^2 + z'^2 y'} + \lambda\left(\omega_{0y}z' - \omega_{0z}y'\right)e^{-\mu} - mg \tag{5}$$

2.2 Numerical simulation of football motion rules

Formula (5) is a complex nonlinear system of differential equations with no analytical solution. In this paper, it will be solved by the standard numerical method of Mathematica9.0 software. In the calculation, m = 0.445 kg, r = 0.11 m, ρ = 1.29 kg/m^3, c = 0.5, μ = 0.1 s^{-1}, g = 9.8 m/s^2 according to the actual values; The width of the goal is 7.32 m and the height is 2.44 m; Assuming that the ball is shot from 30 m away from the goal and directly under the right goalpost, for the convenience of numerical calculation and beautiful drawing, the following coordinates are used: the coordinate below the left pillar is (0, 30.0), and the coordinate below the right pillar is (7.32, 30.0), then the initial position of the ball is r_0 = (7.23, 0, 0) m. Assuming that the ball is kicked at the speed V_0 = (5, 28, 11) m/s, and the angular velocity of the football is given, the numerical solution of the differential Eq. (5) can be determined, from which all the information of the football's motion law can be extracted. The following is a detailed study of the corresponding motions of the three representative ones. Figure 1 shows the schematic diagram of the three initial angular velocities

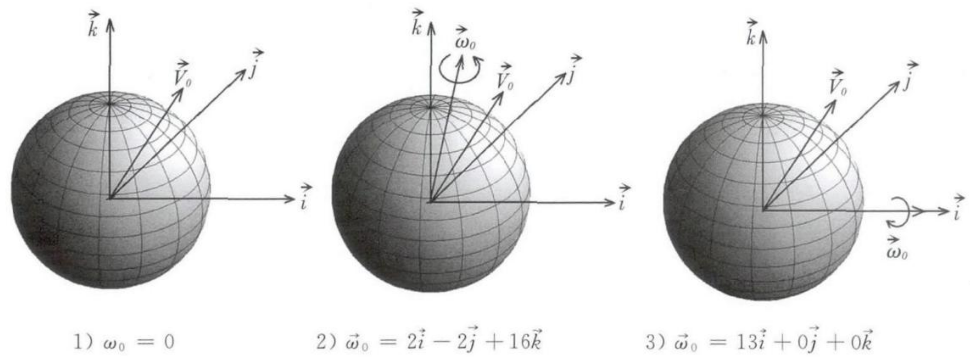

Fig. 1 Schematic diagram of three different initial angular velocities (the take-off velocity is also V_0).

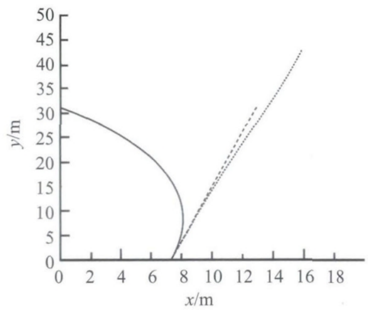

Fig. 2 Projection of the football track in the horizontal plane under three conditions: no rotation Ω_0 = 0; The solid line: 0 = 2-2+16 rad/s; the short line: 0 = 13+0+0 rad/s.

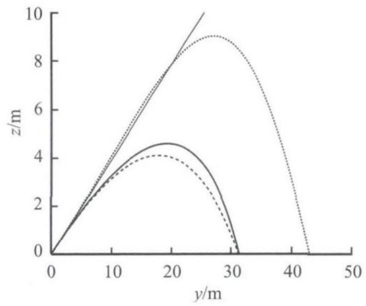

Fig. 3 Projection fine solid line of football trajectory in Yoz plane under three conditions: outgoing direction; Long line: no rotation Ω_0 = 0; Bold line: 0 = 2-2+16 rad/s Short line: 0 = 13+0+0 rad/s.

3 Research results

With r_0 = (7.23, 0, 0) and V PI 0 = (5, 28, 11) as the initial value conditions of the second-order nonlinear differential Eq. (5), Mathematica9.0 standard numerical method is used to solve the equations. The results are as follows:

1. When 0 = 0, the football reached the highest point in 0.83695 s, landed in 1.8113 s and reached a maximum height of 4.0974 m. The projection of the trajectory in the XOY plane and YOZ plane is shown in the long line in Figures 2 and 3, respectively. The projection of the trajectory in the XOY plane is a straight line. The trajectory of a football is a plane curve perpendicular to the ground.

2. When 0 = 2-2+16 rad/s, the football reached the highest point at 0.927011 s, landed at 1.98024 s, and reached a maximum height of 4.58502 m. The projection of the motion track in the XOY plane and the Yoz plane is shown in the thick solid lines in Figures 2 and 3, respectively. The projection of the trajectory in the xoy plane is a curve with a large curve, which is typical of the trajectory of a banana ball.

3. When 0 = 13+0+0 rad/s, the football reached the highest point in 1.68368 s, landed in 3.46416 s, and reached a maximum height of 9.03032 m. The projection of the motion track in the XOY plane and YOZ plane is shown in the short line in Figures 2 and 3, respectively. Figure 2 shows that the projection of the trajectory in the xoy plane presents the phenomenon of left-right fluttering; As can be seen from Figure 3, the position of the football in the pre-ascent stage is above the exit direction (above the thin solid line). If there is no effect of gravity and Magnus force, the exit direction is the movement trajectory of the football. Therefore, it is shown in Figure 3 that the Magnus force of the football in the first 8 m of the ascent is greater than the gravity. In order to simulate the movement law of football more vividly, the following cartoon is used to simulate the three-dimensional movement track of football. Divide the football flight time into 60 equal parts, and draw 60 pieces of the same size but gradually changing. Because the arc length of the flight within the same time interval is related to the linear velocity, the actual animation demonstration can realistically show the speed of the football flight at different moments. To avoid redundancy, this paper only gives the trajectory of the football near the peak and on the eve of entering the net (or the landing place). In addition, we also drew the actual size of the goal (to visually determine whether the ball went into the net), represented the ball with a small ball, and plotted the movement of the ball with a solid line.

4 Conclusion

In this paper, two improvements are made to the study of curved football based on literatures: (1) the air resistance moment is considered; (2) The axis of football rotation is no longer limited to the vertical direction. These two improvements make the simulated physical conditions closer to reality, and we can compare the football movement rules of different rotation directions. In this paper, three typical initial angular velocities 0 are selected to carry out specific numerical research. The numerical results show that when the football is kicked at the same initial velocity V = 5+28+11 m/s, there are obvious differences in the movement of the football corresponding to different initial angular velocities 0.

When there is no rotation 0, the trajectory of the football is the usual trajectory of the projectile. When 0 = 2-2+16 rad/s, the flight of the football is a typical trajectory of the banana ball, which only takes 1.98 s to run into the net in a wonderful arc, and reaches the highest point at 0.927 s (4.585 m off the ground). At this time, the football has not obviously turned to the left of the goal. However, in the second half of the following period, the ball turns sharply, goes left and down, and with about a second left, the goalkeeper has no time to react before the goal is broken! When 0 = 13+0+0 rad/s, in the pre-rise stage, the Magnus force of the football is larger than the force Mg so that the maximum height and range of the football can reach are far greater than the maximum height and range of the ball without rotation. The football is far away from the goal and flies out of the field, and the trajectory shows the phenomenon of fluttering from left to right, similar to the 'leaf ball'. This left-right flutter is also a good form of goal if the shooting spot is chosen. As shown in Figure **??**, when the shooting spot is moved 10 m to the left and 12 m to the back, it is a beautiful goal to kick the ball with V = (5.28, 11)m/s and 0 = 13+0+0 rad/s.

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