Nonlinear optics

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February 26, 2014

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1 Introduction

During the better part of the course we have studied linear phenomena, that is the interaction between matter and light described by the wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \mathbf{P}}{\partial t^2}, \tag{1}$$

where the polarization $\mathbf{P}$ is linear wrt. the electric field of the light $\mathbf{E}$, as described by $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$, where χ is electric suscpetibility of the medium. The course has touched upon nonlinear phenomena, where the polarization can be expanded as $\mathbf{P} = \epsilon_0(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}^2 + \dots)$.

In this project we describe and discuss two such effects, namely beam self-focusing using the Kerr effect, and wave mixing.

2 The Kerr effect

The Kerr effect is found in media, where the index of refraction is dependent on the electric field of the propagating light. For real life laser beams, where the (time avaraged) amplitude of electric field depends on the position in the beam, this leads to a index of refraction varying over the spacial position within the beam. Under certain and experimentally obtainable conditions, this leads to self-focusing of the beam within a passive medium.

2.1 Mathematical formulation

As for second harmonic generation, which has been described in the course, our model is a two-state atom and a monochromatic field with semi-classical light/matter interaction. The frequency of monochromatic laser field is determined by the energies of the two states of the atom, $\omega = \frac{E_2 - E_1}{\hbar}$. In the following we follow Milonni and Eberly [1, sec. 10.3], which gives the expression

$$E = \mathcal{E}(r, \omega) e^{-i\omega t}, \tag{2}$$

for the electric field in a centrosymmetric (ie. the quantum states have definite parity) and isotropic material; here $\mathcal{E}$ is the complex field amplitude. The polarization can then be described similarly as $\mathcal{P}(r, \omega) e^{-i\omega t}$, where strength of the polarization $\mathcal{P}$ has both a linear and a nonlinear contribution.

In this formulation, the wave equation becomes

$$\nabla^2 \mathcal{E} + \frac{\omega^2}{c^2} \mathcal{E} = -\frac{\omega^2}{\epsilon_0 c^2} \mathcal{P}, \tag{3}$$

where the lowest-order nonlinear polarization is

$$\mathcal{P} = \mathcal{P}^{\mathrm{L}} + \mathcal{P}^{\mathrm{NL}} = \epsilon_0 \chi_1 \mathcal{E} + \epsilon_0 \chi_3 |\mathcal{E}|^2 \mathcal{E}. \tag{4}$$

It should be noted, that the succeptibilities here are not simple scalars, but are certain values of the nonlinear succeptibility tensor, which is discussed in Milonni and Eberly [1, sec. 10.2]. In our model using a monochromatic field, χ_1 and χ_3 are constant as ω is constant.

When the field intensity is not large enough to require higher than third order terms in the polarization, the polarization term of the wave equation (3) becomes

$$-\chi_1 \frac{\omega^2}{c^2} \mathcal{E} - \chi_3 \frac{\omega^2}{c^2} |\mathcal{E}|^2 \mathcal{E} \equiv -\chi \frac{\omega^2}{c^2} \mathcal{E}. \tag{5}$$

Permittivity, succeptibility, and refractive index i related as $n^2 = \sqrt{\epsilon}^2 = 1 + \chi$ and we find that

$$n = \sqrt{1 + \chi_1 + \chi_3 |\mathcal{E}|^2} = n_0 \sqrt{1 + \frac{\chi_3}{n_0^2} |\mathcal{E}|^2} \simeq n_0 + \frac{\chi_3}{2n_0} |\mathcal{E}|^2 \tag{6}$$

$$\equiv n_0 + \tfrac{1}{2} |\mathcal{E}|^2 n_2, \tag{7}$$

where $n_0 = 1 + \chi$ is the linear refractive index in the medium. Defining $E = \frac{1}{2}(\mathcal{E}e^{-i\omega t} + \text{c.c.})$ we arrive at the Kerr nonlinearity $n = n_0 + n_2 E^2$, which is an approximation, where we have discarded terms of order higher than two (only even orders appear in the full expression). We are assured by Milonni and Eberly [1], that this is indeed an excellent approximation.

3 Wave mixing

4 Bibliography

[1] P.W. Milonni and J.H. Eberly. *Laser Physics*. Wiley, 2010. ISBN: 978-0-470-38771-9.