

# Nonlinear optics

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## 1 Introduction

During the better part of the course we have studied linear phenomena, that is the interaction between matter and light described by the wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}, \quad (1)$$

where the polarization  $\mathbf{P}$  is linear wrt. the electric field of the light  $\mathbf{E}$ , as described by  $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$ , where  $\chi$  is electric susceptibility of the medium. The course has touched upon nonlinear phenomena, where the polarization can be expanded as  $\mathbf{P} = \epsilon_0 (\chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{E}^2 + \dots)$ .

In this project we describe and discuss two such effects, namely beam self-focusing using the Kerr effect, and wave mixing.

## 2 The Kerr effect

The Kerr effect is found in media, where the index of refraction is dependent on the electric field of the propagating light. For real life laser beams, where the (time averaged) amplitude of electric field depends on the position in the beam, this leads to a index of refraction varying over the spacial position within the beam. Under certain and experimentally obtainable conditions, this leads to self-focusing of the beam within a passive medium.

### 2.1 Mathematical details at a glance

The mathematical formulation of the Kerr effect is derived in Milonni and Eberly [2, sec. 10.5]. For a monochromatic laser field  $E = \mathcal{E}e^{-i\omega t}$  with frequency  $\omega$  in an isotropic and centrosymmetric medium (ie. a medium which quantum states have definite parity), an alternative wave equation for the complex laser field and polarization amplitudes is found. The most important point here is, that the polarization has a linear and a nonlinear contribution  $\mathcal{P} = \mathcal{P}^L + \mathcal{P}^{\text{NL}}$ :

$$\nabla^2 \mathcal{E} + \frac{\omega^2}{c^2} \mathcal{E} = -\frac{\omega^2}{\epsilon_0 c^2} \mathcal{P} = -\frac{\omega^2}{\epsilon_0 c^2} (\epsilon_0 \chi_1 \mathcal{E} + \epsilon_0 \chi_3 |\mathcal{E}|^2 \mathcal{E}). \quad (2)$$

It should be noted, that the susceptibilities here are not simple scalars, but are certain values of the nonlinear susceptibility tensor. For a monochromatic field,  $\chi_1$  and  $\chi_3$  are constant as  $\omega$  is constant. Defining

$$-\chi \frac{\omega^2}{c^2} \mathcal{E} \equiv -\chi_1 \frac{\omega^2}{c^2} \mathcal{E} - \chi_3 \frac{\omega^2}{c^2} |\mathcal{E}|^2 \mathcal{E} \quad (3)$$

and using  $n^2 = 1 + \chi$  leads to

$$n = \sqrt{1 + \chi_1 + \chi_3 |\mathcal{E}|^2} = n_0 \sqrt{1 + \frac{\chi_3}{n_0^2} |\mathcal{E}|^2} \quad (4)$$

$$\simeq n_0 + \frac{\chi_3}{2n_0} |\mathcal{E}|^2 \equiv n_0 + \frac{1}{2} |\mathcal{E}|^2 n_2, \quad (5)$$

where  $n_0 = \sqrt{1 + \chi_1}$  is the linear refractive index in the medium. Clearly, the effective refractive index felt by the propagating wave, depends on the form of the electric field.

### 2.2 Qualitative interpretation

As shown i figure 1, a gaussian beam, which is a beam exhibiting cylindrical symmetry and where the intensity distribution is or nearly is gaussian, feels a larger refractive index closer to the beam axis. Eqn. (5) shows, that this nonlinear correction is proportional to the norm square of the field amplitude, ie. the beam intensity.

The beam size of an gaussian beam is not constant, but varies along the beam axis  $z$ , with minimum beam cross section found at the beam waist.

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Figure 1: Gaussisk beamprofil og effektivt brydningsindeks.

Using a geometric interpretation of the light, rays moving away from the waist have increasing distance from the axis, ie. radial distance  $r$ , leading to a broadening of the beam and a lower intensity. As such, the rays move from a region of high to lower refraction index, leading to a reflection of the ray back towards the beam axis.

This self-focusing competes with diffractive spreading and below a certain critical intensity, self-focusing is not observed. This limit is normally given in terms of total beam power, for example:

$$P_{\text{crit}}^{\text{gauss}} \sim \frac{c\epsilon_0 \lambda^2}{8\pi n_2} \quad \text{Gaussian beam [2],} \quad (6)$$

$$P_{\text{crit}}^{\text{cyl}} \approx \frac{ca^2 \lambda^2}{64 n_2} \quad \text{Cylindrical beam of uniform intensity [1],} \quad (7)$$

where the angular diffractive divergence of the cylindrical beam is  $a\lambda/n_0 2r$ . Note here, that the critical power decreases linearly in  $n_2$  but increases quadratically in wavelength.

### **3 Wave mixing**

## 4 Bibliography

- [1] R. Y. Chiao, E. Garmire, and C. H. Townes. “Self-Trapping of Optical Beams”. In: *Phys. Rev. Lett.* 14 (25 June 1965), pp. 1056–1056. doi: 10.1103/PhysRevLett.14.1056.
- [2] P.W. Milonni and J.H. Eberly. *Laser Physics*. Wiley, 2010. isbn: 978-0-470-38771-9.