Non-linear optics

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February 26, 2014

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1 Introduction

During the better part of the course we have studied linear phenomena, that is the interaction between matter and light described by the wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \mathbf{P}}{\partial t^2}, \tag{1}$$

where the polarization $\mathbf{P}$ is linear wrt. the electric field of the light $\mathbf{E}$, as described by $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$, where χ is electric suscpetibility of the medium. The course has touched upon non-linear phenomena, where the polarization can be expanded as $\mathbf{P} = \epsilon_0(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}^2 + \dots)$.

In this project we describe and discuss two such effects, namely beam self-focusing using the Kerr effect, and wave mixing.

2 The Kerr effect

The Kerr effect is found in media, where the index of refraction is dependent on the electric field of the propagating light. For real life laser beams, where the (time avaraged) amplitude of electric field depends on the position in the beam, this leads to a index of refraction varying over the spacial position within the beam. Under certain and experimentally obtainable conditions, this leads to self-focusing of the beam within a passive medium.

2.1 Mathematical formulation

As for second harmonic generation, which has been described in the course, our model is a two-state atom and a monochromatic field with semi-classical light/matter interaction. As in [1], the frequency of monochromatic laser field is determined by the energies of the two states of the atom, $\omega = \frac{E_2 - E_1}{\hbar}$, leading to the expression

$$E = \mathcal{E}(r, \omega) e^{-i\omega t}, \tag{2}$$

for the electric field in a isotropic material with radial symmetry; here $\mathcal{E}$ is the complex field amplitude. The polarization can then be described similarily as $\mathcal{P}(r, \omega) e^{-i\omega t}$, where strength of the polarization $\mathcal{P}$ has both a linear and a non-linear contribution.

In this formulation, the wave equation becomes

$$\nabla^2 \mathcal{E} + \frac{\omega^2}{c^2} \mathcal{E} = \frac{1}{\epsilon_0 c^2} \mathcal{P} \tag{3}$$

3 Wave mixing

4 Bibliography

[1] P.W. Milonni and J.H. Eberly. *Laser Physics*. Wiley, 2010. ISBN: 978-0-470-38771-9.