Ex. 3

Advanced algorithms 2013

MCMF recap

proof of min cut max flow

29.2-4

- max $f_{sv1} + f_{sv2}$

Such that

$f_{sv1} \leq 16, f_{sv2} \leq 13, f_{sv3} \leq 0, f_{sv4} \leq 0, f_{st} \leq 0, f_{v1v2} \leq 0, f_{v1v3} \leq 12, f_{v1v4} \leq 0, f_{v1t} \leq 0, f_{v1s} \leq 0, f_{v2v1} \leq 4, f_{v2v3} \leq 0, f_{v2v4} \leq 14, f_{v2t} \leq 0, f_{v2s} \leq 0, f_{v3v1} \leq 0, f_{v3v2} \leq 9, f_{v3v4} \leq 0, f_{v3s} \leq 0, f_{v3t} \leq 20, f_{v4v1} \leq 0, f_{v4v2} \leq 0, f_{v4v3} \leq 7, f_{v4s} \leq 0, f_{v4t} \leq 4, f_{tv1} \leq 0, f_{tv2} \leq 0, f_{tv3} \leq 0, f_{tv4} \leq 0, f_{ts} \leq 0$

$f_{sv1} + f_{tv1} + f_{v2v1} + f_{v3v1} + f_{v4v1} = f_{v1v2} + f_{v1v3} + f_{v1v4} + f_{v1s} + f_{v1t}$
$f_{sv2} + f_{tv2} + f_{v1v2} + f_{v3v2} + f_{v4v2} = f_{v2v1} + f_{v2v3} + f_{v2v4} + f_{v2s} + f_{v2t}$
$f_{sv3} + f_{tv3} + f_{v1v3} + f_{v2v3} + f_{v4v3} = f_{v3v1} + f_{v3v2} + f_{v3v4} + f_{v3s} + f_{v3t}$
$f_{sv4} + f_{tv4} + f_{v1v4} + f_{v2v4} + f_{v3v4} = f_{v4v1} + f_{v4v2} + f_{v4v3} + f_{v4s} + f_{v4t}$

$f_{sv1} \geq 0, f_{sv2} \geq 0, f_{sv3} \geq 0, f_{sv4} \geq 0, f_{st} \geq 0, f_{v1v2} \geq 0, f_{v1v3} \geq 0, f_{v1v4} \geq 0, f_{v1t} \geq 0, f_{v1s} \geq 0, f_{v2v1} \geq 0, f_{v2v3} \geq 0, f_{v2v4} \geq 0, f_{v2t} \geq 0, f_{v2s} \geq 0, f_{v3v1} \geq 0, f_{v3v2} \geq 0, f_{v3v4} \geq 0, f_{v3s} \geq 0, f_{v3t} \geq 0, f_{v4v1} \geq 0, f_{v4v2} \geq 0, f_{v4v3} \geq 0, f_{v4s} \geq 0, f_{v4t} \geq 0, f_{tv1} \geq 0, f_{tv2} \geq 0, f_{tv3} \geq 0, f_{tv4} \geq 0, f_{ts} \geq 0$

29.2-5

Let $G = \langle V, E \rangle$ denote the flow network with s and t denoting the source and sink respectively and c_{ij} denoting the capacity of arc $(v_i, v_j) \in E$. Let x_{ij} denote the flow on arc (v_i, v_j) of the network. The linear programming formulation of the max flow problem is:

$$
\begin{aligned}
& \max \sum_{(s,v)\in E} x_{(s,v)} - \sum_{(v,s)\in E} x_{(v,s)} \\
s.t. \quad & \sum_{(v_i, v_k \in E)} x_{ik} - \sum_{(v_k, v_r)\in E} x_{kr} = 0 \; \forall v_k \in V - \{s, t\} \\
& x_{ij} \leq c_{ij}, \; \forall (v_i, v_j) \in E \\
& x_{ij} \geq 0, \; \forall (v_i, v_j) \in E
\end{aligned}
$$

First session

Available at:

https://docs.google.com/file/d/0By5fc2BvnLW9R19SMTZQazR6RkU/edit?usp=sharing

29.1-2, 29.1-3,29.1-4,29.1-5,29.1-6

29.1-7

- Suppose we increase $x1$ to ∞ and we keep x2 fixed. Thus we have $\infty >> x2$
- The constraints are:

1. $-(2\ \infty) + x2 \leq -1$ TRUE
2. $-\infty - 2x2 \leq -2$ TRUE
3. $\infty \geq 0$ TRUE

29.1-8

- Suppose we have the following linear program, not in standard form:

$$\min \sum_{j=1}^{n} c_j x_j$$

such that

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1..m$$

$$x_j \ unbounded$$

For some values of b_i and a_{ij}

29.1-8(cont)

- Given the transformation to standard form we will have:
  - 2 * n variables since none of the initial ones had the nonnegativity constraint
  - 2 * m constraints since we have to transform all equality constraint into two pairs of ‘opposite’ inequality constraints
  - Thus the upper bound is 2 * (n + m)

29.1-9

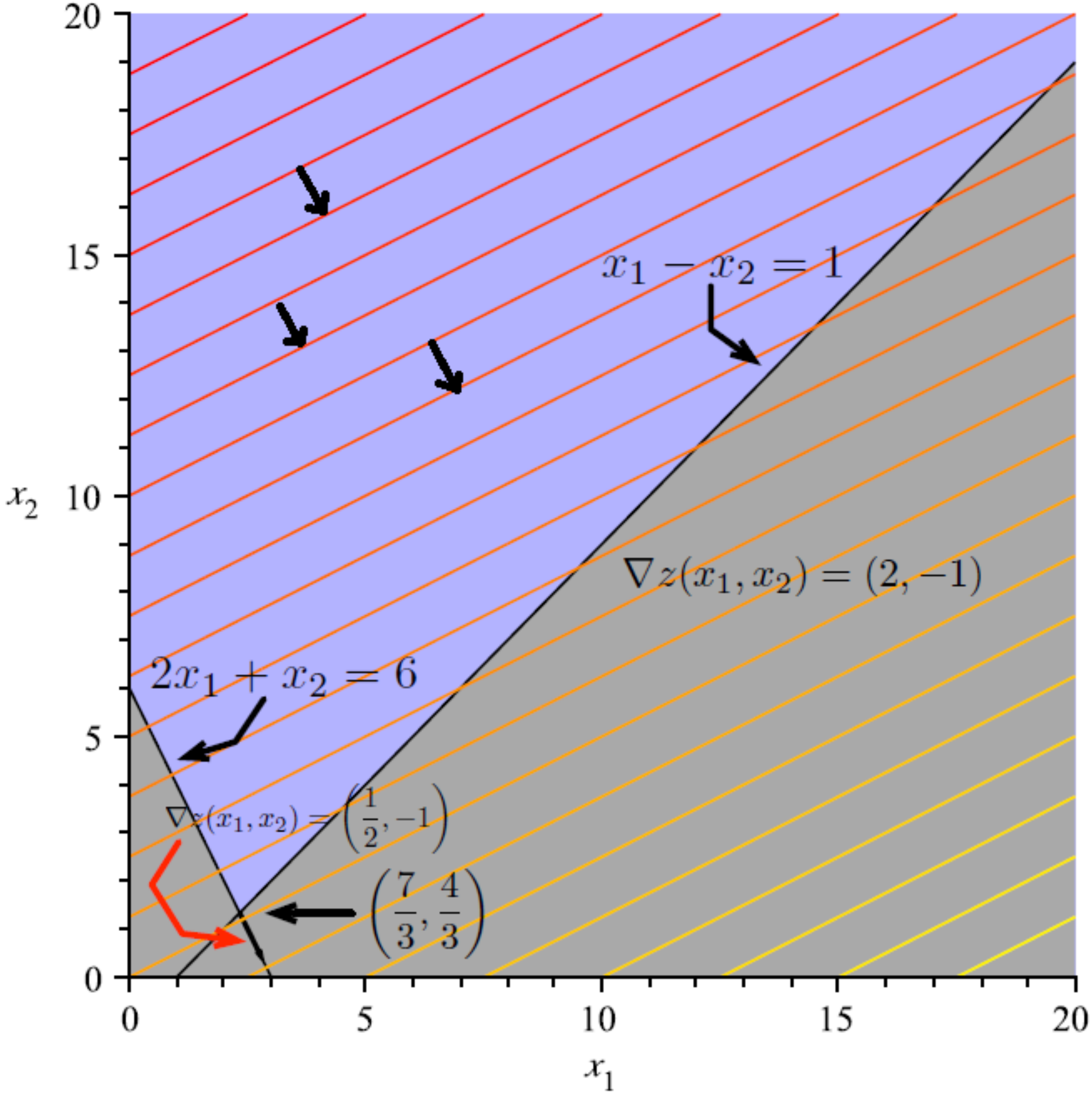
$$\max \frac{1}{2}x_1 - x_2$$

such that

$$x_1 - x_2 \leq 1$$
$$2x_1 + x_2 \geq 6$$
$$x_1, x_2 \geq 0$$

The direction of increase of the objective function is away from the direction in which the feasible region is unbounded. As a result, the point in the feasible region with the largest objective value is the intersection of the lines formed by the constraints, thus the point (7/3,4/3)

29.1-9



$x_1 - x_2 = 1$
$\nabla z(x_1, x_2) = (2, -1)$
$2x_1 + x_2 = 6$
$\nabla z(x_1, x_2) = \left(\frac{1}{2}, -1\right)$
$\left(\frac{7}{3}, \frac{4}{3}\right)$
x_1
x_2