1. BELGIUM'S BEST BEER

The Assignment (June 1991)

A small Belgian brewery, "Belgium's Best Beer" (BBB) has the option of producing two different kinds of beer, hereafter referred to as *Strong* and *Light* respectively. The relevant data are:

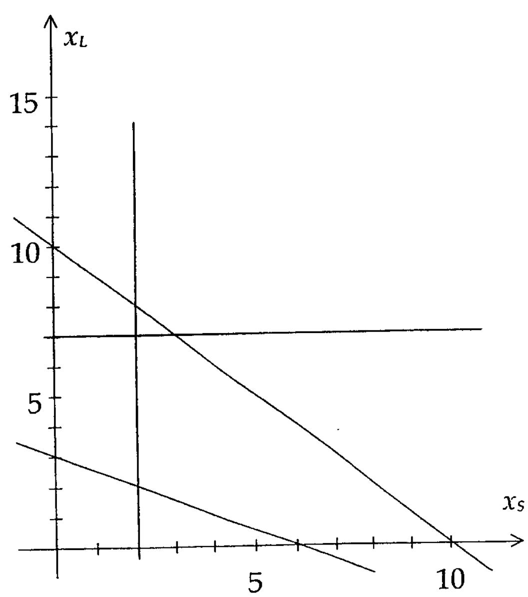
At least 2,000 litres of *Strong* must be produced per day.
At most 7,000 litres of *Light* can be produced per day.
The total production cannot exceed 10,000 litres per day.
The production of 1,000 litres of *Strong* requires 6 man-hours.
The production of 1,000 litres of *Light* requires 3 man-hours.
Total number of man-hours available per day: 48.

Let c_S be the profit made per litre *Strong* (c_S is measured in Belgian Francs (BEF) per litre), and let c_L be the corresponding number for *Light*. Furthermore, let x_S and x_L be the amounts (measured in 1,000 litres per day) produced of *Strong* and *Light* respectively.

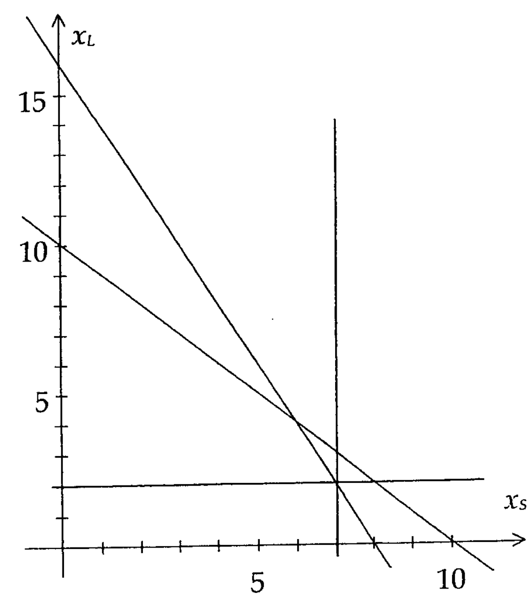
Questions Q1-Q3: The basic model

A *production plan* is a combination of x_S and x_L and will be denoted by (x_S, x_L). For c_S = 6 and c_L = 4, the manager of BBB wishes to find a production plan which maximises profit and satisfies all the requirements made above.

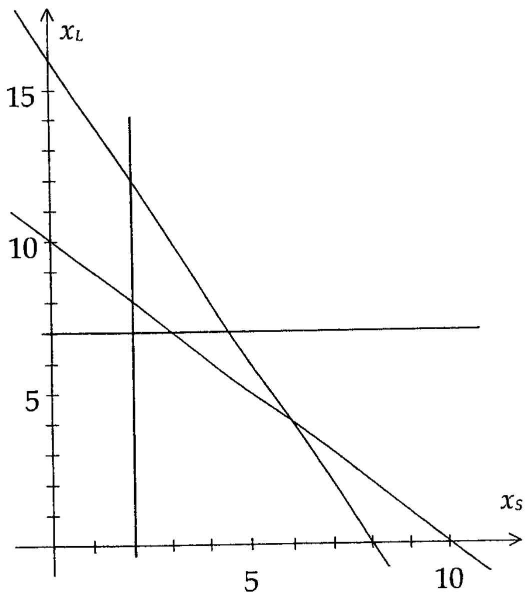
Q1: Which one of the four figures 1a, 1b, 1c, 1d is an adequate graphical representation of the constraints to be observed by the manager?



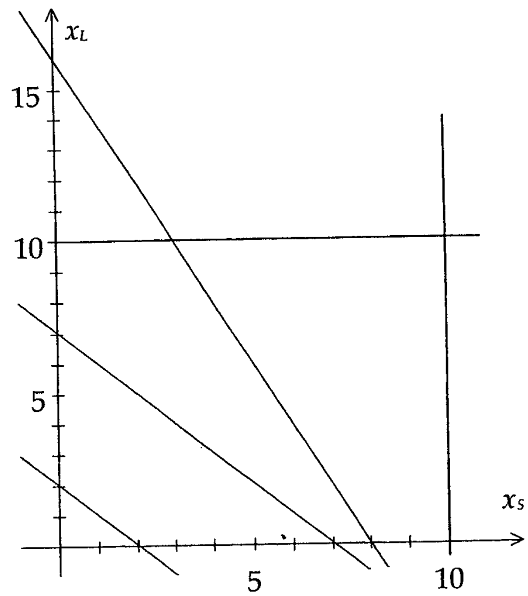
a: Figure 1a



b: Figure 1b



c: Figure 1c



d: Figure 1d

Q2: Which one of the following production plans is infeasible?

a: $(x_S, x_L) = (3, 7)$ b: $(x_S, x_L) = (9, 1)$ c: $(x_S, x_L) = (7, 2)$

Q3: Solve the problem graphically. Which one of the following production plans represents an optimal solution?

a: $(x_S, x_L) = (6, 4)$ b: $(x_S, x_L) = (3, 7)$ c: $(x_S, x_L) = (8, 0)$

Questions Q4-Q5: Variation of c_S and stability of solution

The present value of c_S is 6. Suppose we allow c_S to take other values whereas everything else is kept as before.

Q4: It appears from Q3 that there exists an optimal solution in which both x_S and x_L are integers. If the problem is solved by some standard linear programming (LP) technique for other values of c_S, will we then always obtain such an integer-valued optimal solution?

a: Yes b: No

Q5: Suppose that c_S is increased. For c_S sufficiently large, an optimal production plan is obviously to produce *Strong* only. What is the minimum value of c_S in this case?

a: 7 b: 8 c: 9 d: 10

Questions Q6-Q8: Capacity extension

The present capacity (or upper bound on the total production) is 10,000 litres per day. The next three questions deal with deliberations with respect to an extension of that capacity. $c_S = 6$ as before.

Let y (measured in 1,000 litres per day) be the amount by which the capacity per day is extended and let k (measured in BEF per 1,000 litres) be the cost incurred by extending the capacity by 1,000 litres per day. y is not restricted to be an integer, that is, the capacity per day can in principle be extended by <u>any</u> amount.

Q6: If a production plan maximising total profit is still sought for, how should this feature be incorporated in the model?

a: $\max 6{,}000\, x_S + 4{,}000\, x_L - ky,$ $\quad x_S + x_L \leq 10 + y$

b: $\max 6{,}000\, x_S + 4{,}000\, x_L - 1{,}000\, ky,$ $\quad x_S + x_L \leq 10 + y$

c: $\max 6{,}000\, x_S + 4{,}000\, x_L - 1{,}000\, ky,$ $\quad x_S + x_L \leq y$

Q7: Is it profitable for BBB to extend the capacity from 10,000 litres per day to 11,500 litres per day if $k = 800$?

a: Yes b: No c: Indifferent

Q8: Is it profitable for BBB to extend the capacity by 10,000 litres if $k = 500$?

a: Yes b: No c: Indifferent

Questions Q9-Q10: The mayor's worries

The mayor of the town in which BBB is located is worried about the high consumption of *Strong* and wants to impose a restriction on BBB in order to increase the proportion of *Light* in the daily production plan.

Q9: For each litre *Strong* produced, the mayor requires that at least 4 litres of *Light* should be produced. The manager of BBB objects vigorously, because BBB then has to operate under conflicting constraints. Is the manager right?

a: Yes b: No

Q10: The mayor and the manager reconsider the situation and agree on a less rigorous requirement: For each litre *Strong* produced per day <u>exceeding</u> 2,000 litres per day, BBB must produce at least 3 litres *Light*. Three different extensions of the model are proposed below. Which one reflects correctly the agreement made?

a: $3x_L - x_S \geq 2$ b: $3x_S + x_L \leq 2$ c: $3x_S - x_L \leq 6$

Questions Q11-Q12: Work faster, earn more?

A spokesman representing the BBB-workers suggests a change of the production process enabling the workers to work faster. In return, a reasonable raise of the salary should be considered.

Q11: How should the original model be modified if the workers agree

- to work $33\frac{1}{3}$ % faster, <u>and</u>, in return,
- to obtain an increase of their salary by w BEF per man-hour?

Work faster (new constraint)	Earn more (new objective function)
a: $6x_S + 3x_L \leq 72$	$\max\ (6{,}000 - 4w)\, x_S + (4{,}000 - 2w)\, x_L$
b: $6x_S + 3x_L \leq 64$	$\max\ 6{,}000x_S + 4{,}000x_L - (4.5x_S + 2.25x_L)w$
c: $2x_S + x_L \leq 48$	$\max\ 6x_S + 4x_L - (6x_S + 3x_L)w$

Q12: Is it profitable for BBB to accept the spokesman's offer of faster work if $w = 200$?

a: Yes b: No c: Indifferent