Maximum Flow

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1 Notes

Flow: A flow is a function $f : V \times V \to \mathbb{R}, u, v \in V$ statisfies: capacity constraint and flow conservation

Capacity constraint: For all $u, v \in V$, we require: $0 \leq f(u, v) \leq c(u, v)$

Flow conservation: For all $u \in V - \{s, t\} : \sum_{v \in V} f(v, u) = \sum_{v \in V} f(u, v)$

Residual network: Same as the original but with edges going back and forth telling how much we can add/"subtract" to/from each edge

Augmenting path: Path from source to sink having a least minimum capacity 1

Cancellation: When you move flow in opposite direction of the paths

Cut: A partition of V into two sets S and T where $s \in S$ and $t \in T$

Net flow: Total flow over a cut: $f(S, T) = \sum_{u \in S} \sum_{v \in T} f(u, v) - \sum_{u \in S} \sum_{v \in T} f(v, u)$

Capacity of cut: $c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v)$

Minimum cut: Cut of minimum capacity over all cuts of the network

Ford-Fulkerson: Search for augmenting path: $O(V + E') = O(E)$ (ie. depth-first search.

If capacities are integral, there can be as many as $|f^*|$ iterations and hence: $O(E|f^*|)$

Edmunds-Karp algorithm Same as Ford-Fulkerson but always choosing the shortest augmenting path (each path unitweight) / breadth-first search. Runningtime: $O(VE^2)$

Max-Flow min-cut 1 => 2. Contradiction, 2 => 3 Get the right cut, 3 => 1 Simple lemma 26.?

$c_f(p)$ $c_f(p) = \min\{c_f(u, v) : (u, v) \text{ is on } p\}$

2 Agenda for exam

1. Definition
2. Algorithm
3. Cut, $|f| \leq c(S,T)$
4. Proof of Max Flow-Min Cut
5. Time complexity (Lemma 1, Lemma 2)