ASSIGNEMNT-1 PROBABILITY

Katherapaka Nikhil*

Question 35. Suppose that $(\mathbf{X_1}, \mathbf{X_2}, \mathbf{X_3})$ has $N_3(\mu, \mathbf{\Sigma})$ distrubution with $\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Given that $\phi(-0.5)$=0.3085, where $\phi(.)$ denotes the cumulative distribution function of a standard normal random variable, $P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)$ equals to

Solution:

Let $\mathbf{Y} = \mathbf{X_1} - \mathbf{2X_2} + \mathbf{2X_3}$ and $\mathbf{a}$=$\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$$\mu_Y = \mathbf{a}^T \mu \tag{1}$$

$$= \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$$= 0 \tag{3}$$

$$\sigma_Y^2 = \mathbf{a}^T \mathbf{\Sigma} \mathbf{a} \tag{4}$$

$$= \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \tag{5}$$

$$= 1 \tag{6}$$

$$\Pr\left(Y^2 < \frac{7}{2}\right) = \Pr\left(-\sqrt{\frac{7}{2}} < Y < \sqrt{\frac{7}{2}}\right) \tag{7}$$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{8}$$

$$\Phi\left(\sqrt{\tfrac{7}{2}}\right) = \int_{-\infty}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{9}$$

$$= \int_{-\infty}^{-\frac{1}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} + \int_{-\frac{1}{2}}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{10}$$

$$= \Phi(-0.5) + \int_{-\frac{1}{2}}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{11}$$

$$= 0.3085 + 0.66082 \tag{12}$$

$$= 0.96932 \tag{13}$$

$$\Phi\left(-\sqrt{\tfrac{7}{2}}\right) = \int_{-\infty}^{-\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{14}$$

$$= 0.03068 \tag{15}$$

$$\Pr\left(\left(X_1 - 2X_2 + 2X_3\right)^2 < \frac{7}{2}\right) = \Phi\left(\sqrt{\tfrac{7}{2}}\right) - \Phi\left(-\sqrt{\tfrac{7}{2}}\right) \tag{16}$$

$$= 0.96932 - 0.03068 \tag{17}$$

$$= 0.93864 \tag{18}$$

1 Simulation Steps

1) Generate Random Samples: Generate random samples for X_1, X_2, and X_3 from the given multivariate normal distribution with mean vector $\mu = [0, 0, 0]$ and covariance matrix Σ.
2) Calculate Y: Calculate $Y = X_1 - 2X_2 + 2X_3$.
3) Count Samples: Count the number of samples for which $Y^2 < \frac{7}{2}$.
4) Calculate Estimated Probability: Calculate the estimated probability by dividing the count by the total number of samples (1000000).