ASSIGNEMNT-1 PROBABILITY

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Question 35. Suppose that (X_1, X_2, X_3) has $N_3(\mu, \Sigma)$ distrubution with $\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Given that $\phi(-0.5)$=0.3085, where $\phi(.)$ denotes the cumulative distribution function of a standard normal random variable, $P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)$ equals to

Solution:

Let $Y = X_1 - 2X_2 + 2X_3$ and a=$\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$$\mu_Y = \mathbf{a}^T \boldsymbol{\mu} \tag{1}$$

$$= \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$$= 0 \tag{3}$$

$$\sigma_Y^2 = \mathbf{a}^T \boldsymbol{\Sigma} \mathbf{a} \tag{4}$$

$$= \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \tag{5}$$

$$= 1 \tag{6}$$

$$\Pr\left((Y^2 < \frac{7}{2})\right) = \Pr\left(\left(-\sqrt{\frac{7}{2}} < Y < \sqrt{\frac{7}{2}}\right)\right) \tag{7}$$

$$= \Phi\left(\sqrt{\frac{7}{2}}\right) - \Phi\left(-\sqrt{\frac{7}{2}}\right). \tag{8}$$

$$\Pr\left(\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)\right) = \Phi\left(\sqrt{\frac{7}{2}}\right) - \left(\Phi\left(-\sqrt{\frac{7}{2}}\right)\right) \tag{9}$$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{10}$$

$$\Phi\left(\sqrt{\frac{7}{2}}\right) = \int_{-\infty}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{11}$$

$$= \int_{-\infty}^{-\frac{1}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} + \int_{-\frac{1}{2}}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{12}$$

$$= \Phi(-0.5) + \int_{-\frac{1}{2}}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{13}$$

$$= 0.3085 + 0.66082 \tag{14}$$

$$= 0.96932 \tag{15}$$

$$\Phi\left(-\sqrt{\frac{7}{2}}\right) = \int_{-\infty}^{-\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{16}$$

$$= 0.03068 \tag{17}$$

$$\Pr\left(\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)\right) = \Phi\left(\sqrt{\frac{7}{2}}\right) - \left(\Phi\left(-\sqrt{\frac{7}{2}}\right)\right) \tag{18}$$

$$= 0.96932 - 0.03068 \tag{19}$$

$$= 0.93864 \tag{20}$$