

# ASSIGNMENT-1 PROBABILITY

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Question 35. Suppose that  $(X_1, X_2, X_3)$  has  $N_3(\mu, \Sigma)$  distribution with  $\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$   $\Sigma = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Given that  $\phi(-0.5)=0.3085$ , where  $\phi(\cdot)$  denotes the cumulative distribution function of a standard normal random variable,  $P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)$  equals to

**Solution:**

$$\mu_Y = \mathbf{a}^T \boldsymbol{\mu} = \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0, \quad (1)$$

$$\sigma_Y^2 = \mathbf{a}^T \boldsymbol{\Sigma} \mathbf{a} = \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = 1 \quad (2)$$

$$\Pr\left((Y^2 < \frac{7}{2})\right) = \Pr\left(\left(-\sqrt{\frac{7}{2}} < Y < \sqrt{\frac{7}{2}}\right)\right) \quad (3)$$

$$= \Phi\left(\sqrt{\frac{7}{2}}\right) - \Phi\left(-\sqrt{\frac{7}{2}}\right). \quad (4)$$

$$\Pr\left(\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)\right) = \Phi\left(\sqrt{\frac{7}{2}}\right) - \left(\Phi\left(-\sqrt{\frac{7}{2}}\right)\right) \quad (5)$$

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (6)$$

$$\Phi\left(\sqrt{\frac{7}{2}}\right) = \int_{-\infty}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (7)$$

$$= 0.96932 \quad (8)$$

$$\Phi\left(-\sqrt{\frac{7}{2}}\right) = \int_{-\infty}^{-\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (9)$$

$$= 0.03068 \quad (10)$$

$$= 0.96932 - 0.03068$$

$$(12)$$

$$= 0.93864 \quad (13)$$