ASSIGNEMNT-1 PROBABILITY

Katherapaka Nikhil*

Question 35. Suppose that (X_1, X_2, X_3) has $N_3(\mu, \Sigma)$ distrubution with $\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Given that $\phi(-0.5)$=0.3085, where $\phi(.)$ denotes the cumulative distribution function of a standard normal random variable, $P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)$ equals to

Solution:

$$\mu_Y = \mathbf{a}^T \boldsymbol{\mu} = \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0, \tag{1}$$

$$\sigma_Y^2 = \mathbf{a}^T \mathbf{\Sigma} \mathbf{a} = \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = 1 \tag{2}$$

$$P(Y^2 < \frac{7}{2}) = P\left(-\sqrt{\frac{7}{2}} < Y < \sqrt{\frac{7}{2}}\right) \tag{3}$$

$$= \Phi\left(\sqrt{\frac{7}{2}}\right) - \Phi\left(-\sqrt{\frac{7}{2}}\right). \tag{4}$$

$$P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right) = \Phi\left(\sqrt{\frac{7}{2}}\right) - \left(\Phi\left(-\sqrt{\frac{7}{2}}\right)\right) \tag{5}$$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dt \tag{6}$$

$$\Phi\left(\sqrt{\frac{7}{2}}\right) = \int_{-\infty}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dt \tag{7}$$

$$= 0.96932 \tag{8}$$

$$\Phi\left(-\sqrt{\frac{7}{2}}\right) = \int_{-\infty}^{\sqrt{\frac{7}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dt \tag{9}$$

$$= 0.03068 \tag{10}$$

$$P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right) = \Phi\left(\sqrt{\frac{7}{2}}\right) - \left(\Phi\left(-\sqrt{\frac{7}{2}}\right)\right) \tag{11}$$

$$= 0.96932 - 0.03068 \tag{12}$$

$$= 0.93864 \tag{13}$$