ASSIGNEMNT-4 PROBABILITY

Katherapaka Nikhil EE22BTECH11028*

Question 12.13.3.65
If A and B are independent, then
$\Pr(exactlyoneofA, Boccurs)$
$=\Pr(B)\Pr(A')+\Pr(A)\Pr(B')$
Solution: Let E be the event for getting exactly one of A,B occurs.
If A and B are independent events

$$\Pr(AB) = \Pr(A)\Pr(B) \tag{1}$$

$$\begin{aligned}
\Pr(B) &= \Pr(B(A + A')) && (2)\\
&= \Pr(BA + BA') && (3)\\
&= \Pr(BA) + \Pr(BA') + \Pr((BA)(BA')) && (4)\\
&= \Pr(BA) + \Pr(BA') + \Pr((BB)(AA')) && (5)\\
&= \Pr(BA) + \Pr(BA') && (6)\\
\implies \Pr(BA') &= \Pr(B) - \Pr(BA) && (7)\\
&= \Pr(B) - \Pr(A)\Pr(B) && (8)\\
&= \Pr(B)(1 - \Pr(A)) && (9)\\
&= \Pr(B)\Pr(A') && (10)
\end{aligned}$$

$$\Pr(A'B) = \Pr(A')\Pr(B) \tag{11}$$

$$\Pr(AB') = \Pr(A)\Pr(B') \tag{12}$$

$$\begin{aligned}
\Pr(E) &= \Pr(A'B + AB') && (13)\\
&= \Pr(A'B) + \Pr(AB') - \Pr(A'BAB') && (14)\\
&= \Pr(A')\Pr(B) + \Pr(A)\Pr(B') - 0 \quad (AA' = 0) && (15)\\
&= \Pr(A')\Pr(B) + \Pr(A)\Pr(B') && (16)
\end{aligned}$$

$\therefore$ The statement is true