Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad (1)$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -6 \\ 2 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$	CA
$\|A - B\|$	4	length of AB
$\|B - C\|$	6.32	length of BC
$\|C - A\|$	2.83	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 0 \\ -4 \end{pmatrix}$	AB
c_1	4	
$\mathbf{n_2}$	$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$	BC
c_2	4	
$\mathbf{n_3}$	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	CA
c_3	0	
Area	4	Area of Triangle
$\angle A$	135°	
$\angle B$	18.43°	Angles
$\angle C$	26.57°	

TABLE 1: Vectors.

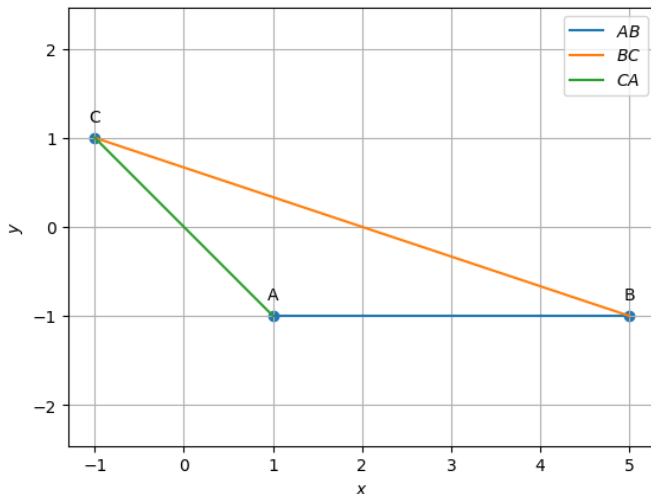Fig. 1: triangle plotted using python

2 Median

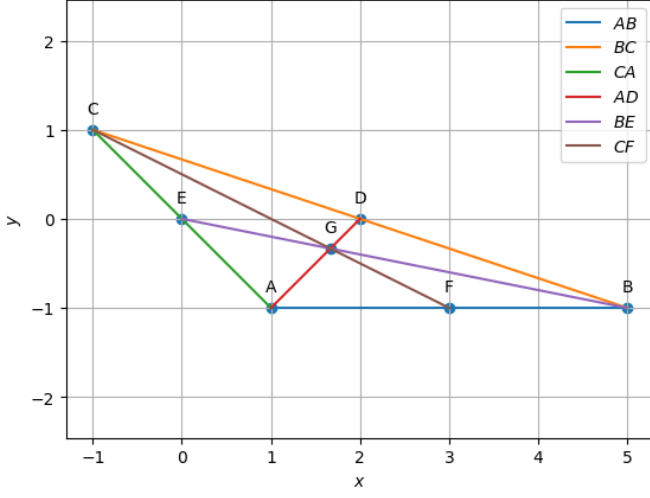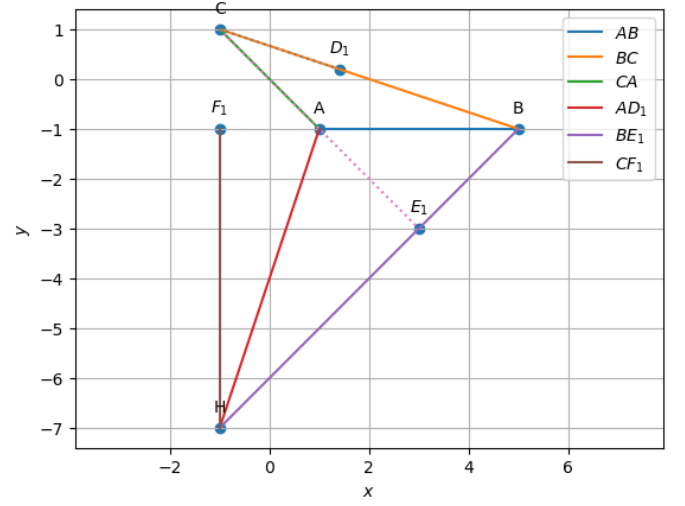| parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
c_4	2	
$\mathbf{m_5}$	$\begin{pmatrix} -5 \\ 1 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} 1 \\ 5 \end{pmatrix}$	
c_5	0	
$\mathbf{m_6}$	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} -2 \\ -4 \end{pmatrix}$	
c_6	-2	
$\mathbf{G}$	$\begin{pmatrix} 1.67 \\ -0.33 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$ $\frac{CG}{GF}$ $\frac{AG}{GD}$	2	Division ratio by $\mathbf{G}$
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

Fig. 2: medians plotted using python

Fig. 3: altitudes plotted using python

3 Altitude

4 Perpendicular Bisector

TABLE 3: Altitude.

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix} 1.4 \\ 0.2 \end{pmatrix}$	Foot of altitude from $\mathbf{A}$
$\mathbf{E_1}$	$\begin{pmatrix} 3 \\ -3 \end{pmatrix}$	Foot of altitude from $\mathbf{B}$
$\mathbf{F_1}$	$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$	Foot of altitude from $\mathbf{C}$
$\mathbf{m_7}$	$\begin{pmatrix} 0.4 \\ 1.2 \end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix} 1.2 \\ -0.4 \end{pmatrix}$	
c_7	1.6	
$\mathbf{m_8}$	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$	
c_8	-12	
$\mathbf{m_9}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	
c_9	2	
$\mathbf{H}$	$\begin{pmatrix} -1 \\ -7 \end{pmatrix}$	Orthocentre

TABLE 4: Perpendicular Bisector.

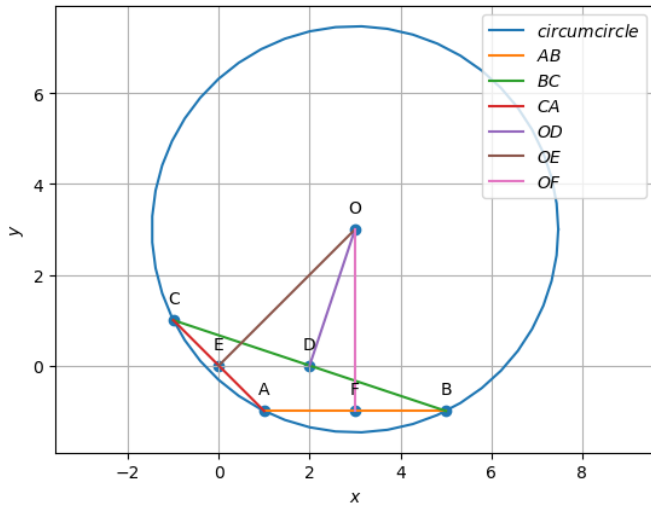| parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix} -6 \\ 2 \end{pmatrix}$	
c_{10}	-12	
$\mathbf{m_{11}}$	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$	
c_{11}	0	
$\mathbf{m_{12}}$	$\begin{pmatrix} 0 \\ 4 \end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix} -4 \\ 0 \end{pmatrix}$	
c_{12}	-12	
$\mathbf{O}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	Circumcentre
$\|\mathbf{O-A}\|$		$OA = OB = OC = R$
$\|\mathbf{O-B}\|$		
$\|\mathbf{O-C}\|$	4.48	
R		
$\angle BOC$	90°	$\angle BOC = 2\angle BAC$
$\angle BAC$	135°	
$\angle AOC$	36.8°	$\angle AOC = 2\angle ABC$
$\angle ABC$	18.4°	
$\angle AOB$	306.9°	$\angle AOB = 2\angle BCA$
$\angle BCA$	26.6°	

Fig. 4: perpendicular bisectors plotted using python

| parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix} -0.29 \\ -0.7 \end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix} -0.7 \\ 0.29 \end{pmatrix}$	
c_{13}	-1	
$\mathbf{m_{14}}$	$\begin{pmatrix} -1.95 \\ 0.32 \end{pmatrix}$	BI
$\mathbf{n_{14}}$	$\begin{pmatrix} -0.32 \\ -1.95 \end{pmatrix}$	
c_{14}	0.37	
$\mathbf{m_{15}}$	$\begin{pmatrix} -1.66 \\ 1.02 \end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix} -1.02 \\ -1.66 \end{pmatrix}$	
c_{15}	-0.63	
$\mathbf{I}$	$\begin{pmatrix} 1.26 \\ -0.39 \end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix} 1.44 \\ 0.19 \end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix} 0.82 \\ -0.82 \end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix} 1.25 \\ -1 \end{pmatrix}$	Point of contact with AB
$\|\mathbf{I-D_3}\|$		$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I-E_3}\|$		
$\|\mathbf{I-F_3}\|$	0.61	
r		
$\angle BAI$	67.5°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	9.21°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	13.28°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.
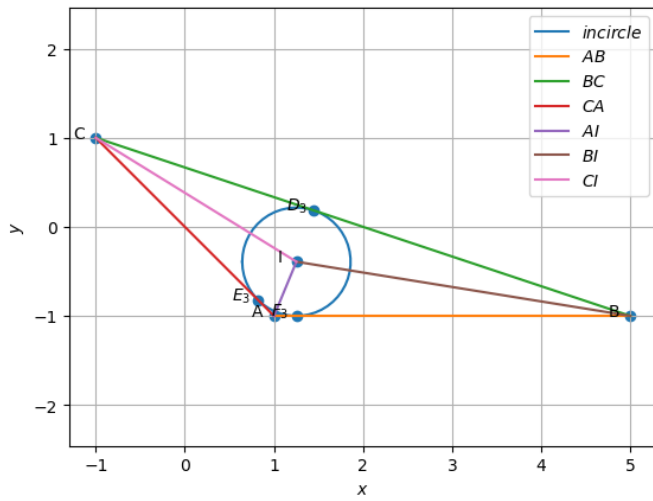
Fig. 5: Angle bisectors plotted using python