ASSIGNEMNT-1 PROBABILITY

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Question 18
A card is selected from a deck of 52 cards. The probability of its being a red face card is
Solution: :

$$p_X(k) = \begin{cases} \frac{12}{52} & k = \{1,2,3...12\} \\ \frac{40}{52} & k = \{13,14,15....52\} \\ 0 & otherwise \end{cases} \tag{1}$$

$$p_Y(k) = \begin{cases} \frac{26}{52} & k = \{1,2,3,....26\} \\ \frac{26}{52} & k = \{27,28,29....52\} \\ 0 & otherwise \end{cases} \tag{2}$$

Parameter	Value	Description
X	0	Not a face card
	1	A face card
Y	0	A black card
	1	A red card

TABLE 0: Table with random varaible outcomes

$$Z = XY \tag{3}$$

$$p_Z(k) = p\left(XY = k\right) \tag{4}$$

$$= \Pr\left(X = \frac{k}{Y}\right) \tag{5}$$

$$= E\left(p_X\left(\frac{k}{Y}\right)\right) \tag{6}$$

$$= \sum_{i=1}^{52} p_X\left(\frac{k}{i}\right) p_Y\left(i\right) \tag{7}$$

$$= \frac{1}{2}\sum_{i=1}^{52} p_X\left(\frac{k}{i}\right) \tag{8}$$

$$Z = 1 \quad X = 1, Y = 1 \tag{9}$$

$$\implies p_Z(1) = \frac{1}{2}\{p_X\left(1\right) + 0 + 0...\} \tag{10}$$

$$= \frac{1}{2}\left(\frac{12}{52}\right) \tag{11}$$

$$= \frac{3}{26} \tag{12}$$

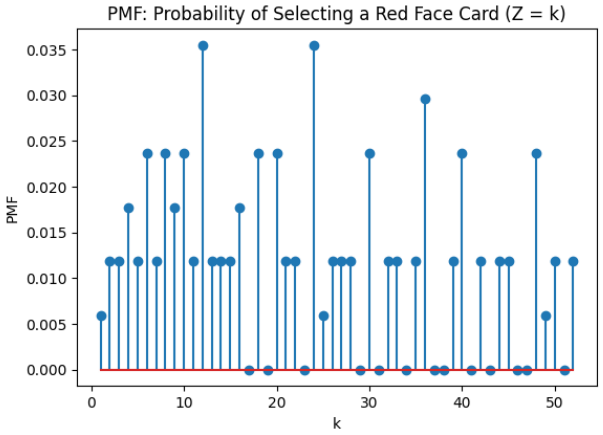

Fig. 0: Plot of Probability Mass Function