Project Report for ECE 351

Lab 7: Fourier Series Approximation of a Square Wave

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Contents

1 Introduction

The purpose of this lab is to use Fourier Series to approximate periodic time-domain signals.

2 Methodology

2.1 Part 1

The first prelab task was to solve the Fourier series of Figure 1 using these equations.

$$x(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_k cos(w_0 t) + b_k sin(k w_0 t)$$

$$a_k = \frac{2}{T}\int_0^T x(t)cos(kw_0t)dt$$

$$b_k = \frac{2}{T}\int_0^T x(t)sin(kw_0t)dt$$

$$w_0 = \frac{2\pi}{T}$$

Realizing since this is an odd function a_k and a_0 is zero and only b_k needs solved.

$$a_k = 0$$

$$b_k = \frac{2}{k\pi}[1 - cos(k\pi)]$$

$$x(t) = \sum_{n=1}^{\infty} b_k sin(kw_0t)$$

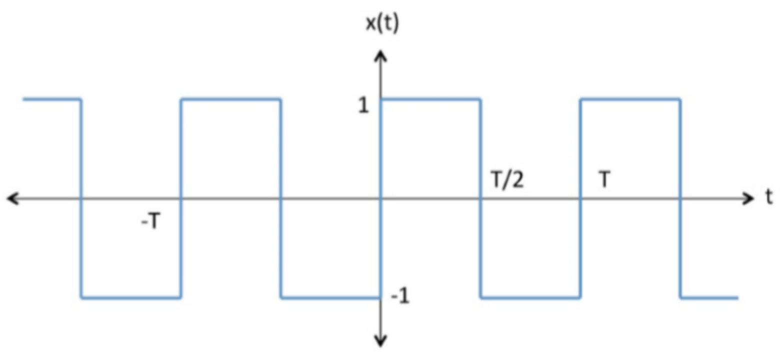

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Figure 1: Block Diagram

The first task was to solve for a_k and b_k explicitly and obtain values. Figure 2 shows the code for doing so.

```

steps = 0.01
t = np.arange(0, 20 + steps, steps)
T = 8 #whatver value
w = (2*np.pi)/T
ak = 0  #this is zeroe because it is odd function
# Part 1 Tasks
#plotting the step response found by hand

# steps = .001 # Define step size
# t = np . arange (0 , 2 + steps , steps )
#k=2
def bkfunc(k): #found from

    y=(2/(k*np.pi))*(1-np.cos(k*np.pi))
    return y


#trying to do for loop butits nooooooooot workrringng
    # for i in range (k):

    #     z= print('This is bk term ' bkfunc(k))
    # return z


#Printing the values to the third term of bk and 2nd term of ak
print("first term ak(0): ", ak)
print("second term ak(1): ", ak)
print("first term bk(1): ", bkfunc(1))
print("second term bk(2): ", bkfunc(2))
print("third term bk(3): ", bkfunc(3))


# ##plotting the fouriers task 2
#    for i in np.arange(1, n + 1):
#         y = y + (2/(i * np.pi)) * (1 - np.cos(i * np.pi)) * (np.sin(i * w * t))
#     return y

```

Figure 2: Block Diagram

I made a function to print out the values of b_k at k= 1,2 and 3. This was straightforward and the challenging part was whether or not b_k was found correctly in the pre-lab. The confirmation of the values can be found under the result section.

I then moved to task 2 and it was to plot the Fourier of Figure 1 at different k values. The code is in Figure 3 below. This was tricky because I ran into a few problems doing so.

```
63  def fourierfunc(k,t):
64      #k=n or number of iterations
65      y=0
66      for i in range(1,k+1):
67          # y+=bkfunc(i)*np.sin((2*np.pi*i*t)/T)
68          # y+=(2/(i*np.pi))*(1-np.cos(i*np.pi))*np.sin((i*w*t)
69           y +=  (2/(i * np.pi)) * (1 - np.cos(i * np.pi)) * (np.sin(i * w * t))
70
71      return y
72
73   # y=0
74   # for i in range(1,1):
75   #     # y+=bkfunc(i)*np.sin((2*np.pi*i*t)/T)
76   #     y+=(2/(i*np.pi))*(1-np.cos(i*np.pi))*np.sin((2*np.pi*i*t)/8)
77  fouri1=fourierfunc(1,t)
78  fouri3=fourierfunc(3, t)
79  fouri5=fourierfunc(10, t)
80
81  fouri50=fourierfunc(50, t)
82  fouri150=fourierfunc(150, t)
83  fouri1500=fourierfunc(1500, t)
84
85  plt.figure(figsize=(10,7))
86  plt.subplot(3,1,1)
87  plt.plot(t,fouri1)
88  plt.grid()
89  plt.ylabel('y output at k=1')
90  plt.title('first 3 graphs k=1,3,15')
91
92  plt.subplot(3,1,2)
93  plt.plot(t,fouri3)
94  plt.grid()
95  plt.ylabel('y output at k=3')
96
97  plt.subplot(3,1,3)
98  plt.plot(t,fouri5)
99  plt.grid()
100 plt.ylabel('y output at k=15')
101
102 ##########second plots
```

Figure 3: Block Diagram

As can be seen I ultimately left everything in variables for w and T. I then wrote the for loop to encapsulate both the b_k term with the Fourier and obtain that expression. I then called the function and passed it the time x-axis and the number of iterations I wanted it to add. The issue came with line 66 and having $range(1, k+1)$ instead of $range(1, k)$ because this produces and error and your not starting at them same place. Aside from that it worked fine. The plots for different k values are found in the result section.

3 Results

3.1 Part 1

```
first term ak(0):  0
second term ak(1):  0
first term bk(1):  1.2732395447351628
second term bk(2):  0.0
third term bk(3):  0.4244131815783876
```

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Figure 4: Verifying the b_k and a_k terms

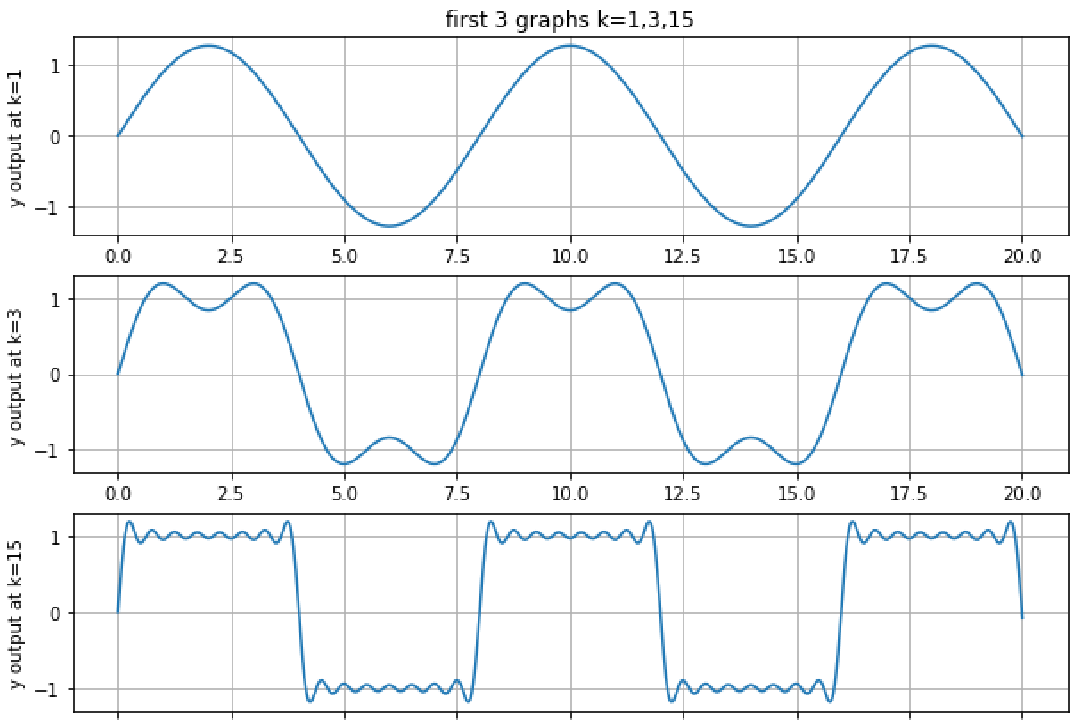

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Figure 5: Results of $k = 1, 3, 15$

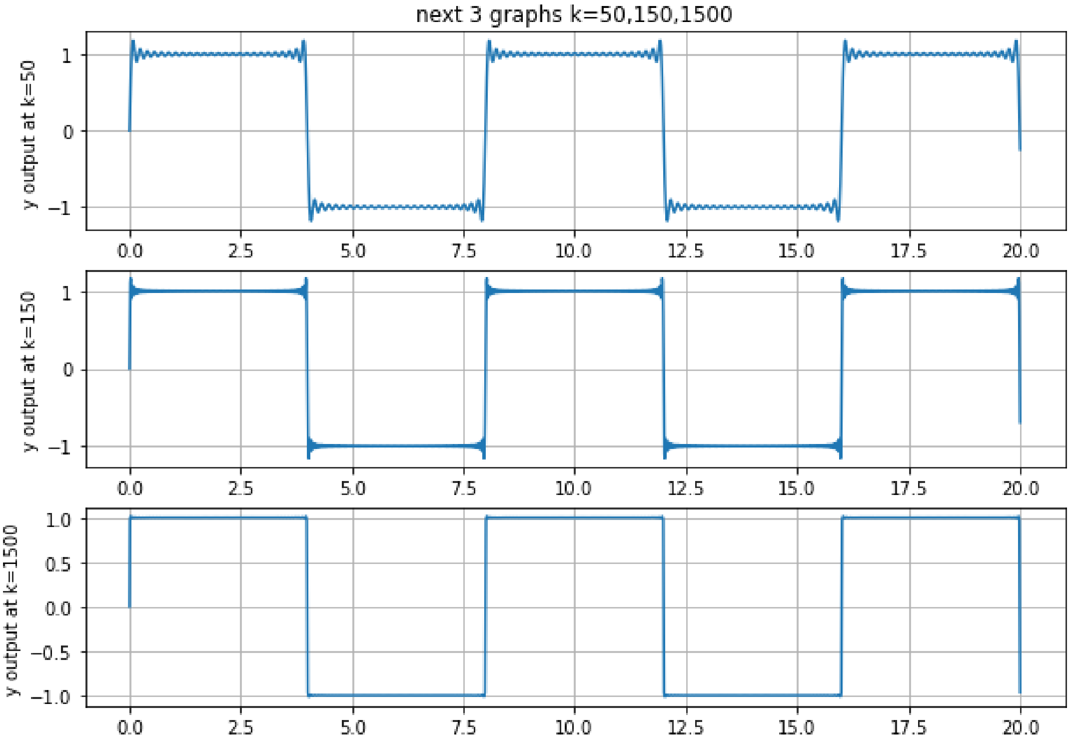

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Figure 6: Results of $k = 50, 15, 1500$

4 Questions

1. Is x(t) an even or an odd function? Explain why.

It is an odd function because it looks like a sine and sine is an odd function.

2. Based on your results from Task 1, what do you expect the values of a2, a3, . . . , an to be? Why?
0 because its on going forever and it's always going to be an odd function.

3. How does the approximation of the square wave change as the value of N increases? In what way does the Fourier series struggle to approximate the square wave?
The more iterations the close it looks like the square wave. The value evaluated for example inputting an x term or number of iterations into the sine and cosine outputs a y value that the more the iterations are the closer they are hence the flatter the top of the square wave it is.

4. What is occurring mathematically in the Fourier series summation as the value of N increases?.
The frequency increases meaning less variation between the rising and decreasing of the wave or a flatter top meaning a good perfect square approximation.

5. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.
It was straightforward I think.