



# 703308 VO High-Performance Computing

## MPI - Message Passing Interface

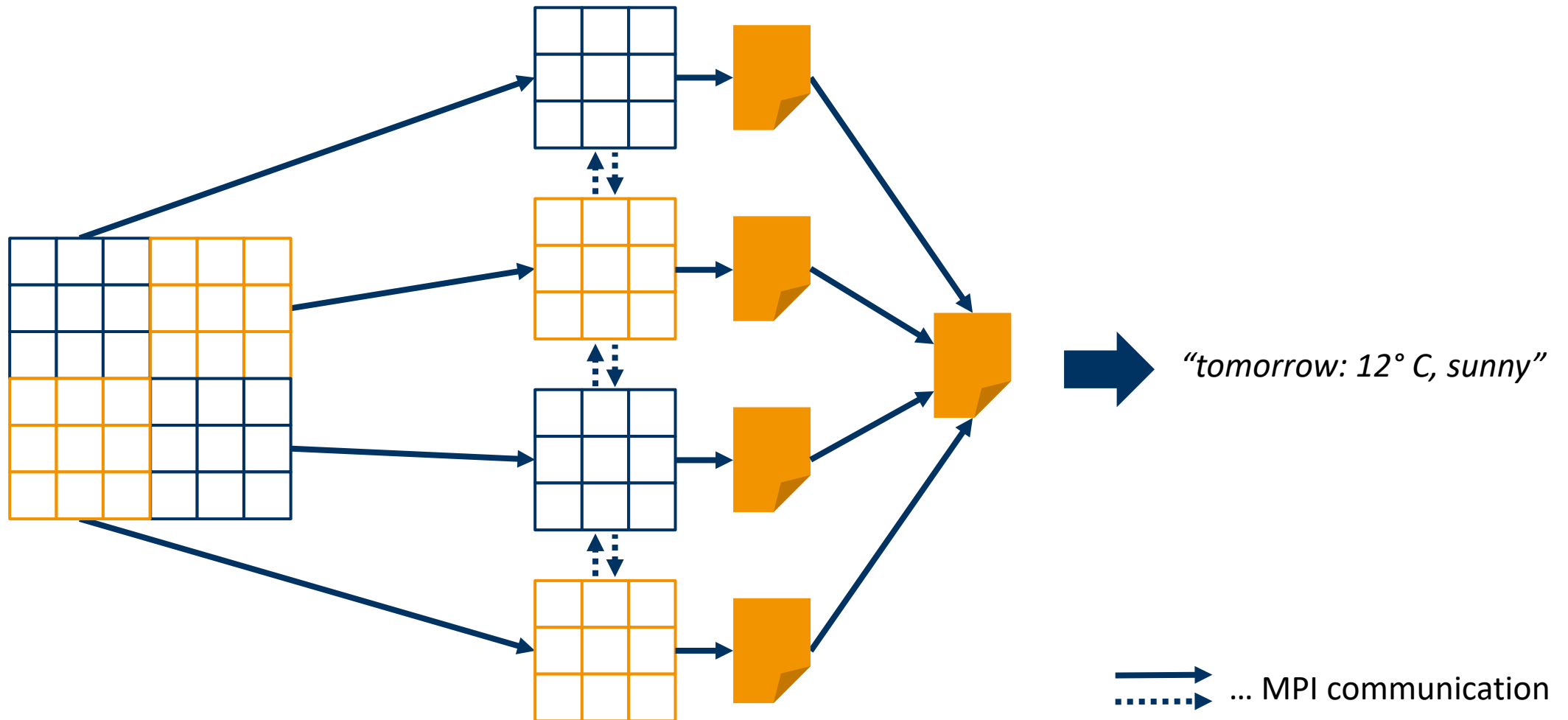
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# Overview

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- ▶ general concepts about MPI
  - ▶ characteristics
  - ▶ programming model
  - ▶ startup
- ▶ point-to-point communication
- ▶ collective communication
- ▶ practical example

# Motivation for using MPI: data distribution



# Message Passing Interface (MPI)

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- ▶ message passing library for distributed memory parallelism
- ▶ de-facto standard for C (C++) and Fortran
- ▶ maintained by the MPI Forum
  - ▶ initial release in 1994 (version 1.0)
  - ▶ updates in 1997 (2.0), 2012 (3.0), 2015 (3.1), 2021 (4.0), 202x (4.1), 202x (5.0)
  - ▶ specification updates slow, aim at stability and high TRL
    - ▶ “On June 30 2020, the MPI forum voted for version 3.3 of [these voting rules](#) (effective June 30th, 2020).”

# MPI implementations

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- ▶ OpenMPI
  - ▶ open source
  - ▶ merge of multiple previous MPI implementations
  - ▶ default on many systems
- ▶ MPICH
  - ▶ also open source
  - ▶ basis for many vendor implementations such as Intel, IBM, Cray, Microsoft, ...
    - ▶ default on many systems
- ▶ Do not confuse implementation versions with specification versions!
- ▶ Do not confuse implementation adherence with specification adherence!
- ▶ <https://www.mpi-forum.org/implementation-status/>

# Main characteristics

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- ▶ offers specific tools for
  - ▶ sending and receiving messages
  - ▶ waiting and synchronization
  - ▶ identification of individual processes
  - ▶ ...
- ▶ additional convenience tools for
  - ▶ partitioning and distributing data
  - ▶ organizing processes in structures
  - ▶ large-scale I/O operations
  - ▶ ...

## Main characteristics cont'd

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- ▶ a lot of user responsibility

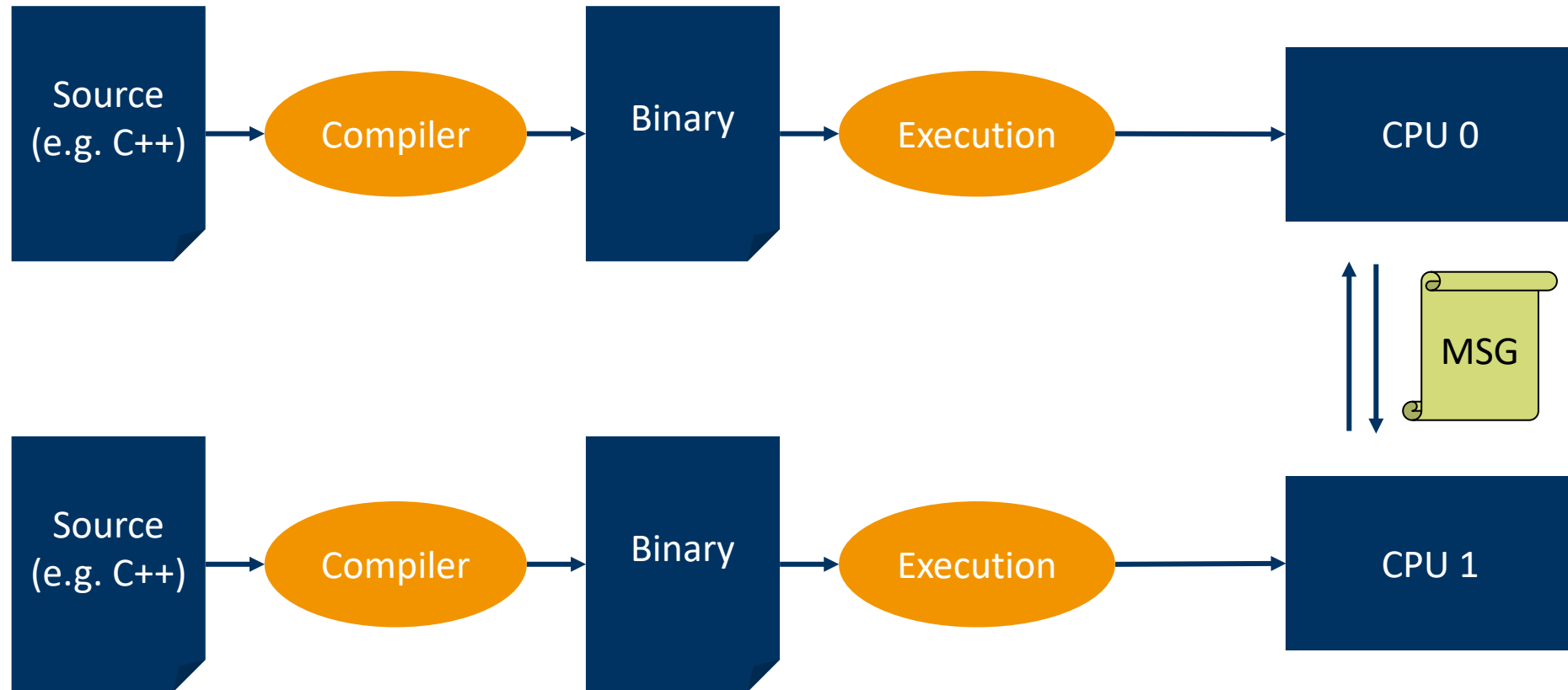
- ▶ explicit parallelism and communication
- ▶ program correctness
- ▶ performance optimization
  - ▶ (non-)blocking
  - ▶ (a)synchronous

- ▶ a lot of advantages

- ▶ available everywhere
- ▶ several implementations
- ▶ portable to many architectures
- ▶ very high performance

## Recap: MIMD: MPMD

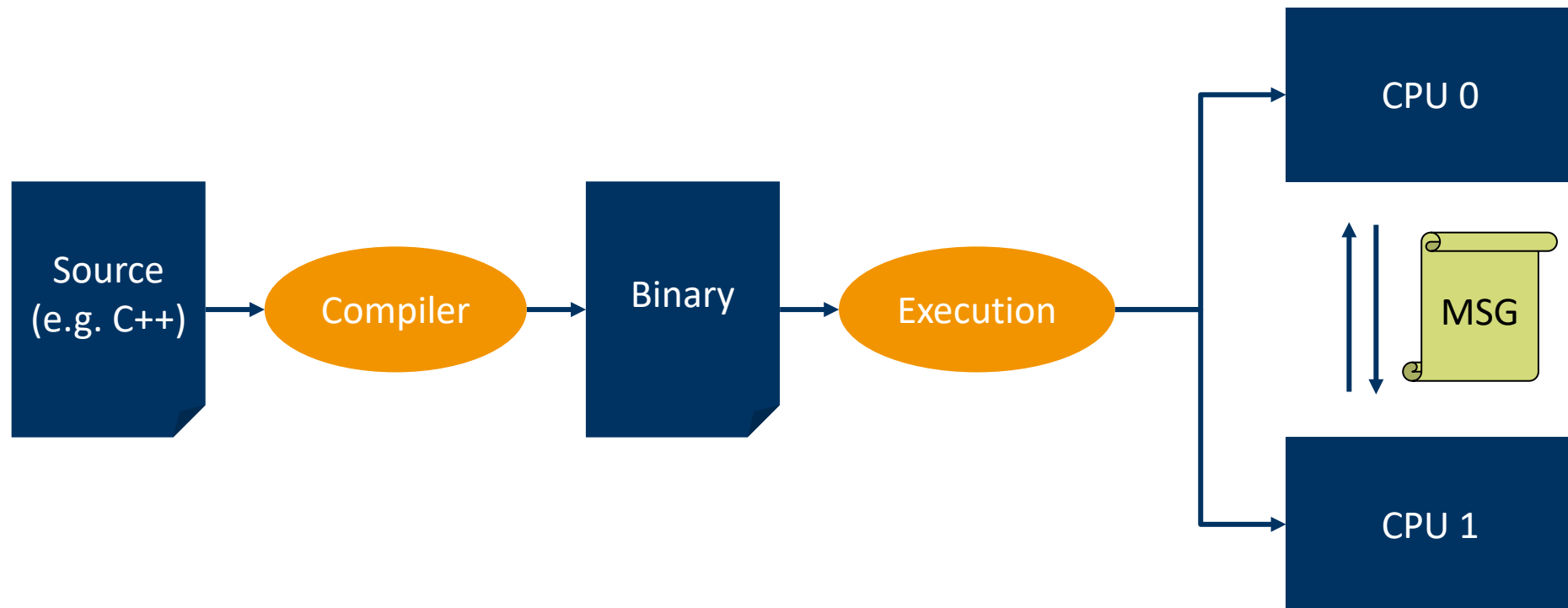
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## Recap: MIMD: SPMD

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## MPMD through SPMD

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- ▶ many MPI implementations support only SPMD
- ▶ SPMD can emulate MPMD

```
int main() {  
    // get id information  
    int cpuID = ...;  
    if(cpuID==0) {  
        ... // program A  
    } else {  
        ... // program B  
    }  
}
```

# Parallelism requires two mechanisms

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- ▶ a mechanism for spawning processes
  - ▶ multiple ways of achieving this
  - ▶ we won't look at this in too much detail
  - ▶ simply rely on `mpiexec` to do the work for you
- ▶ a mechanism for sending and receiving messages
  - ▶ many, many ways of exchanging messages
  - ▶ we will definitely look at this in a lot of detail
  - ▶ tons of functionality to choose from, as we'll see in a bit

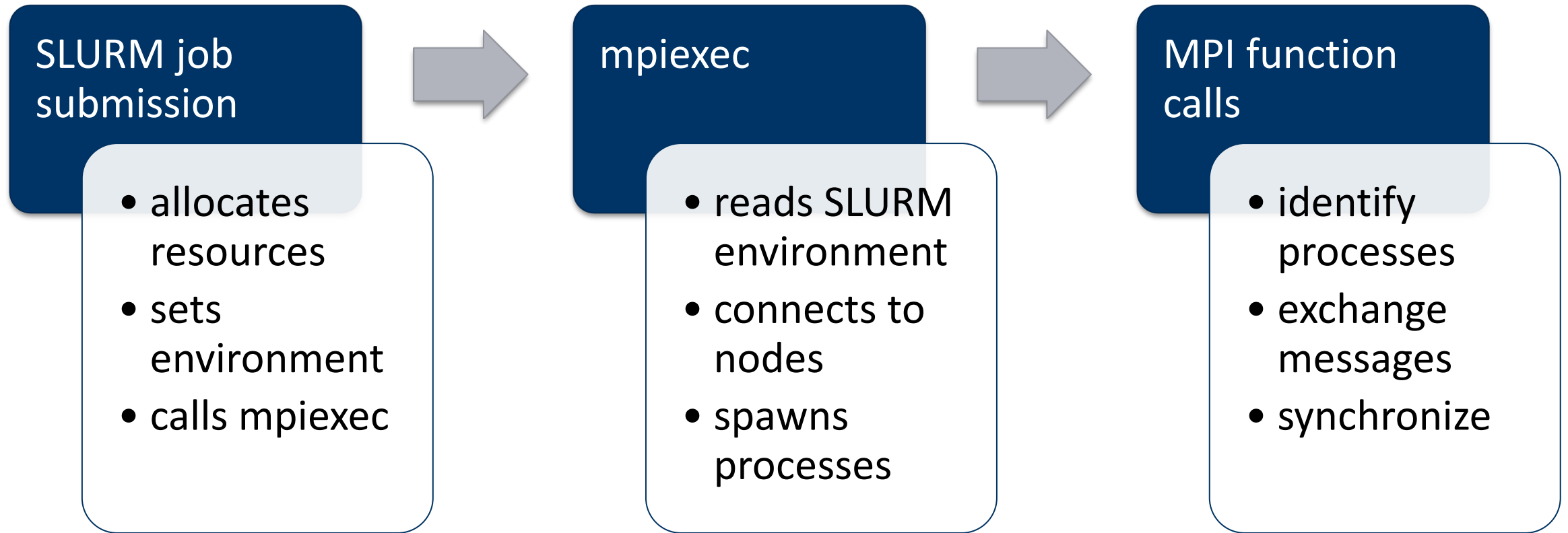
# Compiler and execution wrappers

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- ▶ `mpicc / mpic++` for compiling
  - ▶ OpenMPI: `--showme` prints compiler flags
  - ▶ passes all additional compiler flags to backend compiler (e.g. `mpicc -g`)
- ▶ `mpiexec` for running
  - ▶ formerly `mpirun`, but `mpiexec` is standardized
  - ▶ alternatively `srun` or whatever the platform documentation recommends

# Startup procedure of an MPI application

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# Hello world in MPI

---

```
#include <mpi.h>
#include <stdio.h>

int main(int argc, char** argv) {
    MPI_Init(&argc, &argv); // initialize the MPI environment
    int size;
    MPI_Comm_size(MPI_COMM_WORLD, &size); // get the number of ranks
    int rank;
    MPI_Comm_rank(MPI_COMM_WORLD, &rank); // get the rank of the caller
    printf("Hello world from rank %d of %d\n", rank, size);
    MPI_Finalize(); // cleanup
}
```

## Setup and teardown

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- ▶ `int MPI_Init( int* argc, char*** argv )`
  - ▶ must be called by every process before calling any other MPI function
  - ▶ initializes the MPI library
- ▶ `int MPI_Finalize( void )`
  - ▶ must be the last MPI function called by every process
  - ▶ user must ensure completion of all locally (!) pending communication
  - ▶ performs library cleanup

# Who am I talking to?

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- ▶ in MPI-speak, processes are known as “*ranks*”
  - ▶ numbered from 0 to N-1
  - ▶ own rank can be queried with

```
int MPI_Comm_rank( MPI_Comm comm, int* rank )
```
  - ▶ number of ranks can be queried with

```
int MPI_Comm_size( MPI_Comm comm, int* size)
```
- ▶ almost all MPI semantics are relative to a “*communicator*” or “*group*”
  - ▶ identifies a set of ranks
  - ▶ always available: MPI\_COMM\_WORLD (=everyone)
  - ▶ new communicators and groups that hold subsets of ranks can be created
  - ▶ when developing a library, always create your own communicator!
    - ▶ frequent MPI programming pattern





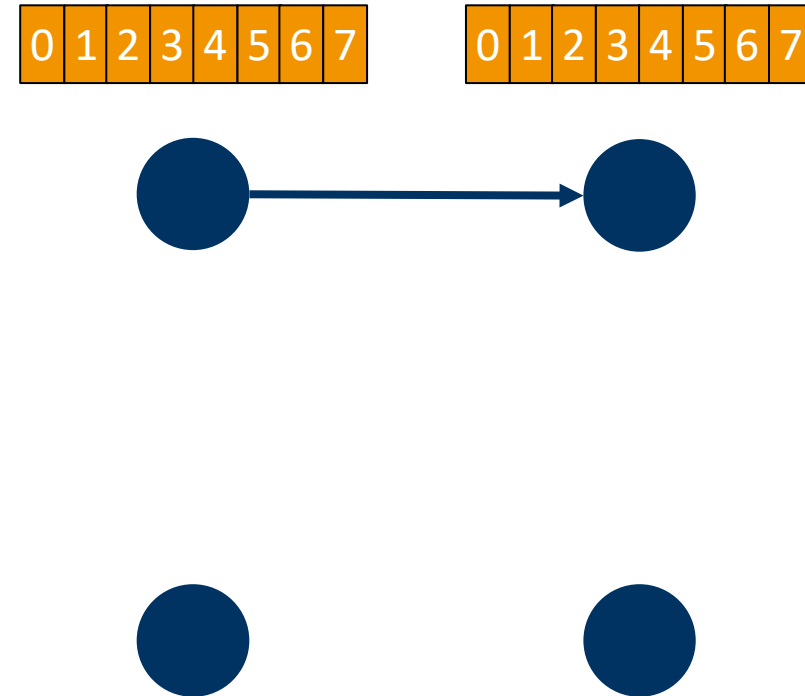
# Point-to-Point Communication



# Point-to-point communication

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- ▶ `MPI_Send(...)/MPI_Recv(...)`
  - ▶ single sender, single receiver  
(*“point-to-point”*)
- ▶ simplest form of communication available
  - ▶ not necessarily the best
- ▶ multiple different types
  - ▶ (a)synchronous
  - ▶ (non-)blocking



## MPI\_Send

---

- ▶ `int MPI_Send(const void* buf, int count, MPI_Datatype datatype, int dest, int tag, MPI_Comm comm)`
  - ▶ `buf`: source buffer to send data from
  - ▶ `count`: number of data elements to send
  - ▶ `datatype`: type of data to send
  - ▶ `dest`: destination rank
  - ▶ `tag`: user-defined message type or category
  - ▶ `comm`: communicator
- ▶ `MPI_Send(&number, 1, MPI_INT, 1, 42, MPI_COMM_WORLD);`

## MPI\_Recv

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- ▶ `int MPI_Recv(void* buf, int count, MPI_Datatype datatype, int source, int tag, MPI_Comm comm, MPI_Status* status)`
  - ▶ `buf`: destination buffer to save data to
  - ▶ `count`: number of data elements to receive
  - ▶ `datatype`: type of data to receive
  - ▶ `source`: source rank
  - ▶ `tag`: user-defined message type or category
  - ▶ `comm`: communicator
  - ▶ `status`: holds additional information (e.g. rank of sender or tag of message)
- ▶ `MPI_Recv(&number, 1, MPI_INT, 0, 42, MPI_COMM_WORLD, MPI_STATUS_IGNORE);`

## Basic send/receive example

---

```
int number;
if (rank == 0) {
    number = -1;
    MPI_Send(&number, 1, MPI_INT, 1, 42, MPI_COMM_WORLD);
} else if (rank == 1) {
    MPI_Recv(&number, 1, MPI_INT, 0, 42, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
    printf("Rank 1: Received %d from rank 0\n", number);
}
```

## Side note: MPI data type conversion

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- ▶ “representation conversion”
  - ▶ changes the binary representation of some data
  - ▶ allowed in MPI, required by supporting heterogeneous architectures
    - ▶ e.g. sender is ARM, receiver is x86, ...
    - ▶ e.g. big-endian vs. little-endian, ASCII vs. EBCDIC, ...
- ▶ “type conversion”
  - ▶ converts the actual data type, e.g. from float to integer
  - ▶ c.f. C++ numeric casts
  - ▶ not allowed in MPI

# Predefined MPI constants

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- ▶ datatypes

- ▶ MPI\_INT, MPI\_FLOAT, MPI\_DOUBLE, MPI\_BYTE, MPI\_CHAR, ...

- ▶ wildcards & misc

- ▶ MPI\_ANY\_SOURCE
- ▶ MPI\_ANY\_TAG
- ▶ MPI\_COMM\_WORLD
- ▶ MPI\_STATUS\_IGNORE
- ▶ ...

# (Non-)Blocking and (a)synchronous communication

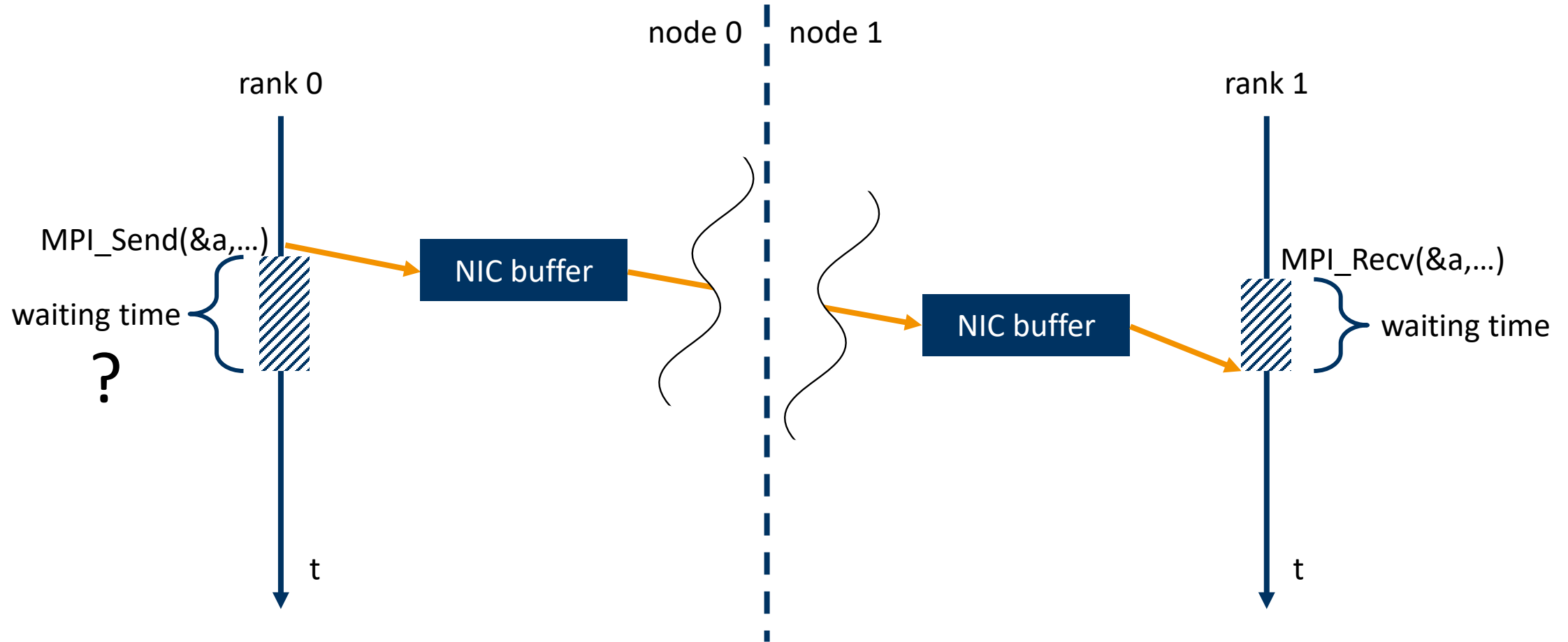
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- ▶ distinguish two important properties
  - ▶ When does the MPI function call return?
    - ▶ Can I overwrite the send buffer?
    - ▶ When is all the data in the receive buffer?
  - ▶ When does the corresponding message transfer happen?
    - ▶ Do I need to wait for the receiver to get the entire message?
    - ▶ Do I need to wait for the receiver to begin receiving?

```
if (rank == 0) {  
    MPI_Send(&number, ...);  
} else if (rank == 1) {  
    MPI_Recv(&number, ...);  
}
```



## (Non-)Blocking and (a)synchronous communication cont'd



# Blocking vs. non-blocking communication

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- ▶ blocking point-to-point:  
MPI\_Send() and MPI\_Recv()
  - ▶ allows to re-use send buffer after send call returns
  - ▶ allows to read receive buffer after receive call returns

```
if (rank == 0) {  
    MPI_Send(&number, ...);  
    // re-use number here  
} else if (rank == 1) {  
    MPI_Recv(&number, ...);  
    // use number here  
}
```

## Blocking vs. non-blocking communication cont'd

---

- ▶ non-blocking point-to-point:  
MPI\_Isend() and MPI\_Irecv()
  - ▶ send and receive return (almost) immediately
  - ▶ MPI\_Wait() call blocks until buffer can be read/re-used

```
MPI_Request request;
if (rank == 0) {
    MPI_Isend(&number, ..., &request);
    MPI_Wait(&request, MPI_STATUS_IGNORE);
    // re-use number here
} else if (rank == 1) {
    MPI_Irecv(&number, ..., &request);
    MPI_Wait(&request, MPI_STATUS_IGNORE);
    // re-use number here
}
```

## (A)Synchronous send modes

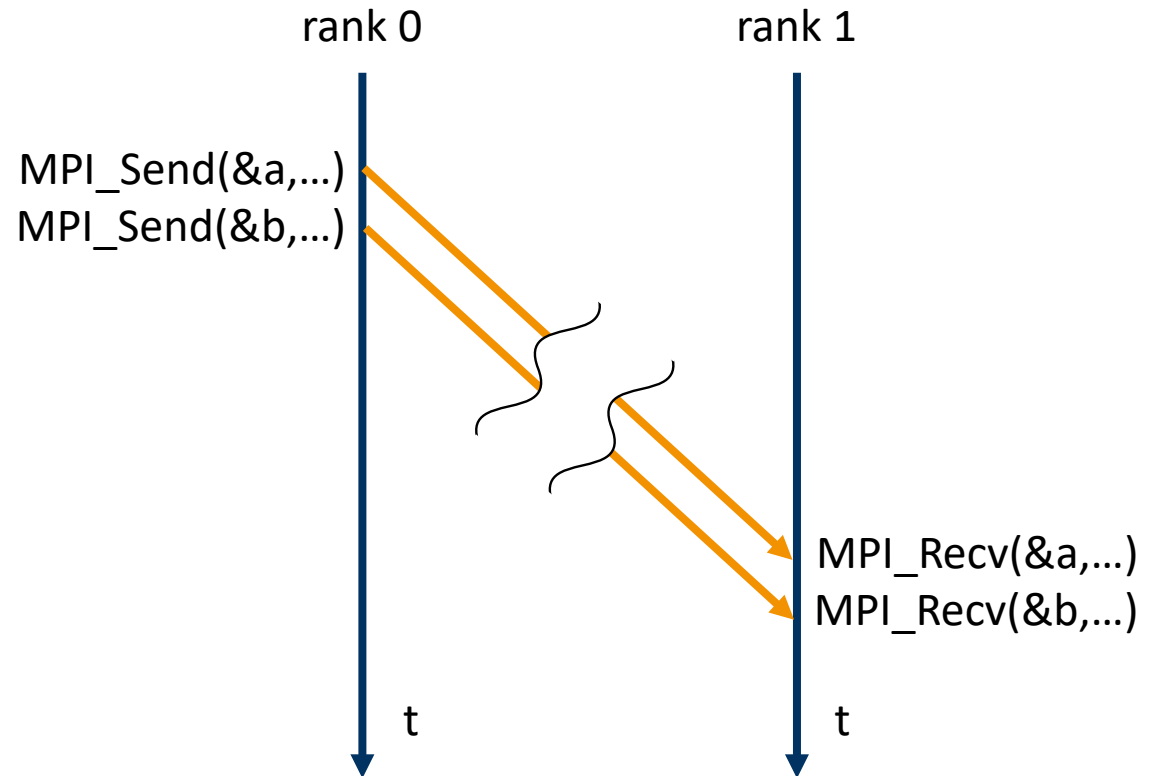
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- ▶ `MPI_Ssend( )` – synchronous mode
  - ▶ will wait for matching receive
- ▶ `MPI_Bsend( )` – buffered mode
  - ▶ will buffer, won't wait for a matching receive
- ▶ `MPI_Rsend( )` – ready mode
  - ▶ requires an already posted, matching receive (dangerous, developer responsibility!)
- ▶ `MPI_Send( )` – standard mode
  - ▶ may buffer (depends on message size)
  - ▶ may or may not wait for matching receive
- ▶ and there are also non-blocking variants for ALL of them...

# Message order preservation

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- ▶ messages do NOT overtake each other if
  - ▶ same communicator
  - ▶ same source rank
  - ▶ same destination rank
- ▶ regardless of blocking or synchronization mode
- ▶ mandated by MPI specification





# Collective Communication



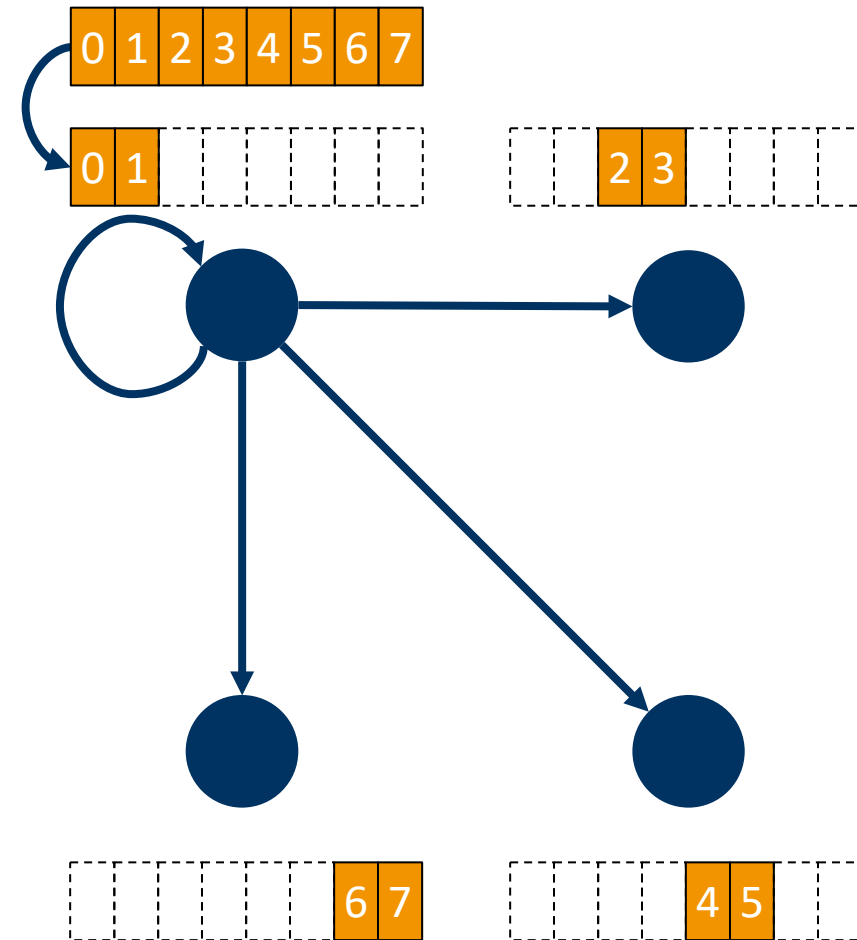
# Collective communication

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- ▶ convenience function for frequently-used programming patterns (e.g. distributing data)
  - ▶ can involve several ranks at the same time, not just 2
  - ▶ must be called by ALL ranks in the communicator
  - ▶ must be called in-order by all ranks (no interleaving of multiple collective communication calls!)
  - ▶ is locally finished when local operation has finished
  - ▶ is globally finished when all participating ranks are finished
  - ▶ available as blocking and non-blocking variants (but cannot be mixed!)

## MPI\_Scatter/MPI\_Scatterv

- ▶ sends chunks of data to multiple ranks, including root itself
- ▶ simple way of partitioning and distributing data
- ▶ MPI\_Scatterv() allows varying counts of elements to be distributed to each rank





# MPI\_Scatter

---

- ▶ `int MPI_Scatter(const void* sendbuf, int sendcount, MPI_Datatype sendtype, void* recvbuf, int recvcount, MPI_Datatype recvtype, int root, MPI_Comm comm)`
  - ▶ `sendbuf`: source buffer to send data from
  - ▶ `sendcount`: number of data elements to send to each rank
  - ▶ `sendtype`: type of data to send
  - ▶ `recvbuf`: destination buffer to save data to
  - ▶ `recvcount`: number of data elements to receive at each rank
  - ▶ `recvtype`: type of data to receive
  - ▶ `root`: rank of the sender
  - ▶ `comm`: communicator

## Scatter example

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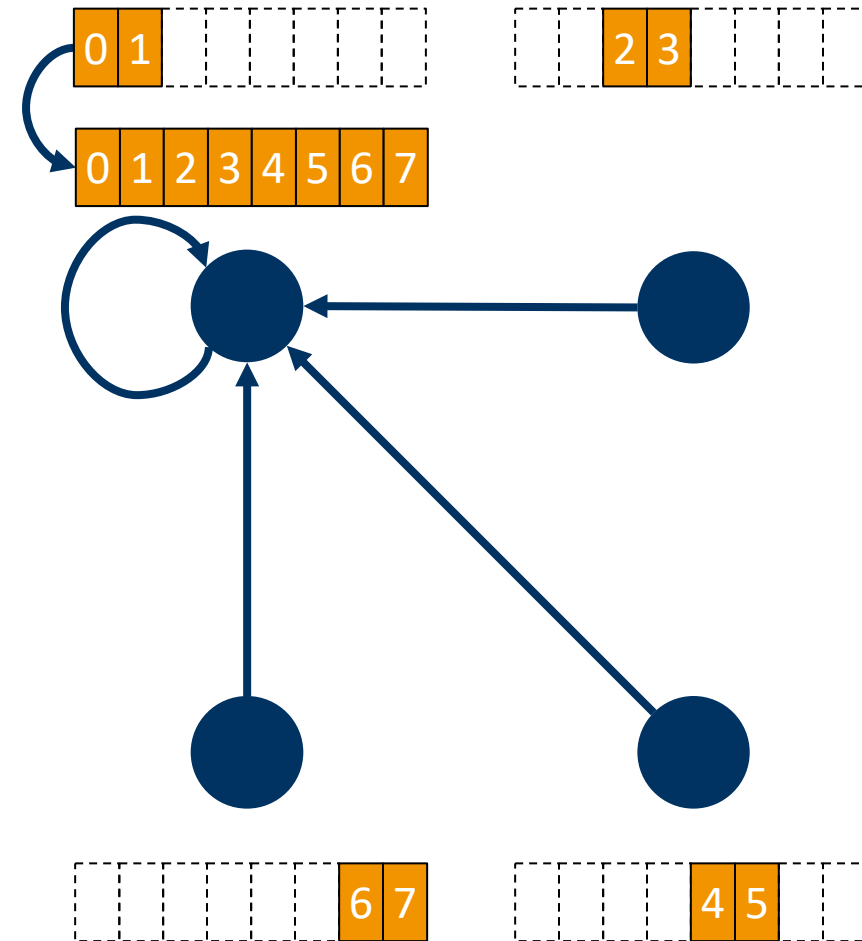
```
int globaldata[4];

if(rank==0) {
    for(int i = 0; i < 4; i++) {
        globaldata[i] = ...
    }
}

int localdata;
MPI_Scatter(globaldata, 1, MPI_INT, &localdata, 1, MPI_INT, 0,
            MPI_COMM_WORLD);
```

# MPI\_Gather/MPI\_Gatherv

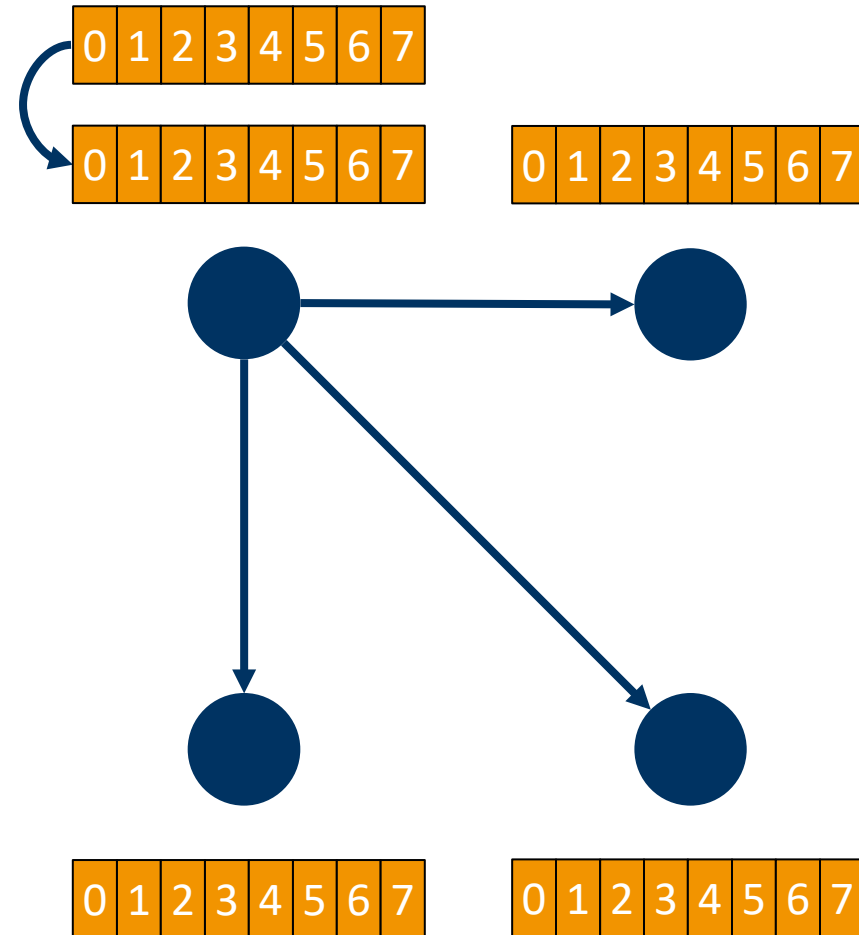
- ▶ sends chunks of data from multiple ranks, including root itself, to root
- ▶ simple way of collecting data
- ▶ MPI\_Gatherv() allows varying counts of elements to be collected from each rank



# MPI\_Bcast

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- ▶ broadcast operation
- ▶ sends copies of data to multiple ranks



## Q: emulating broadcast with point-to-point?

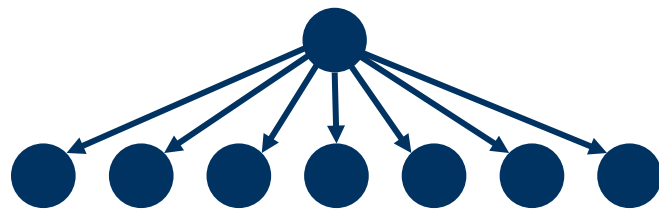
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```
MPI_Bcast(&buf, 1, MPI_INT, 0, MPI_COMM_WORLD);  
// ##### or instead: #####  
if (rank == 0) {  
    for (int i = 0; i < size; i++) {  
        if (i != rank) {  
            MPI_Send(&buf, 1, MPI_INT, i, 0, MPI_COMM_WORLD);  
        }  
    }  
} else {  
    MPI_Recv(&buf, 1, MPI_INT, 0, 0, MPI_COMM_WORLD, MPI_STATUS_IGNORE);  
}
```

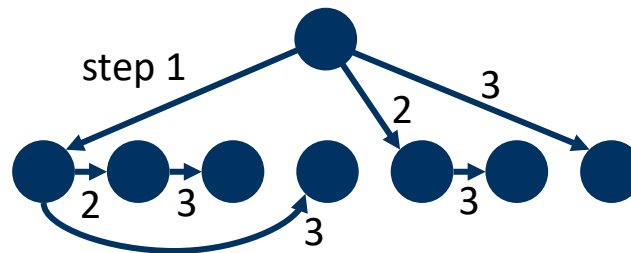
# Collective communication patterns

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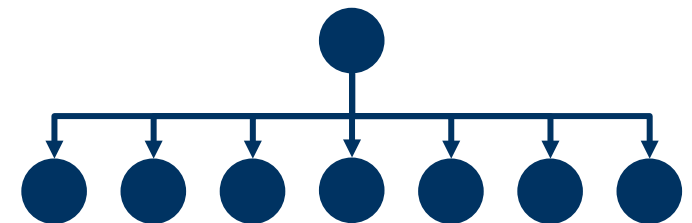
- ▶ chosen automatically at runtime by MPI implementation
- ▶ may depend on multiple parameters, including
  - ▶ type of operation (e.g. broadcast)
  - ▶ number and location of ranks
  - ▶ size and structure of data
  - ▶ hardware capabilities



sequential algorithm  
 $O(\text{num\_ranks})$



tree-based algorithm  
e.g.  $O(\log_2(\text{num\_ranks}))$



hardware operation  
 $O(1)$

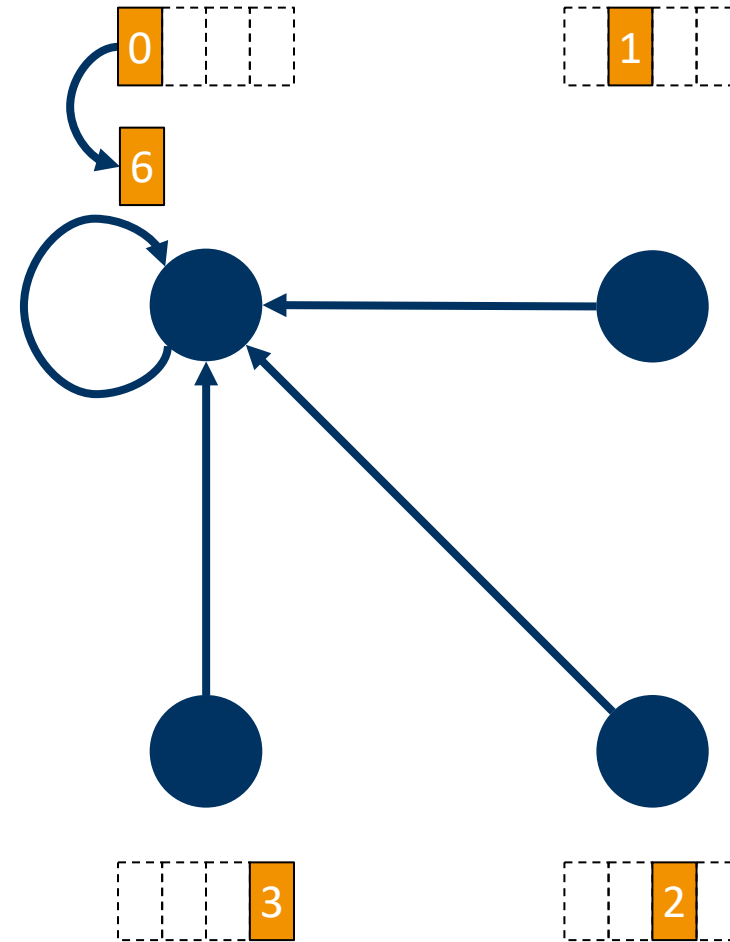
# Barrier

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- ▶ `int MPI_Barrier(MPI_Comm comm)`
  - ▶ `comm`: communicator
- ▶ causes all ranks to wait until everyone reached the barrier
  - ▶ normally not needed: explicit data communication usually also inherently synchronizes
  - ▶ often used for debugging and profiling
    - ▶ Don't forget to remove in production/release!

# MPI\_Reduce

- ▶ aggregate data from multiple ranks, including root itself, to root
  - ▶ e.g. MPI\_SUM
  - ▶ may use optimized collective communication patterns
- ▶ assumes associative reduction operation ◦ such that e.g.
$$(x_0 \circ x_1) \circ (x_2 \circ x_3) = ((x_0 \circ x_1) \circ x_2) \circ x_3$$
- ▶ be careful with floating point types!





## MPI\_Reduce cont'd

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- ▶ `int MPI_Reduce(const void* sendbuf, void* recvbuf, int count, MPI_Datatype datatype, MPI_Op op, int root, MPI_Comm comm)`
  - ▶ `sendbuf`: source buffer to reduce data from
  - ▶ `recvbuf`: destination buffer to reduce data into
  - ▶ `count`: number of data elements in source and destination buffers
  - ▶ `datatype`: type of data to reduce
  - ▶ `op`: reduction operation
  - ▶ `root`: rank of the destination of aggregated result
  - ▶ `comm`: communicator

# Available reduction operations

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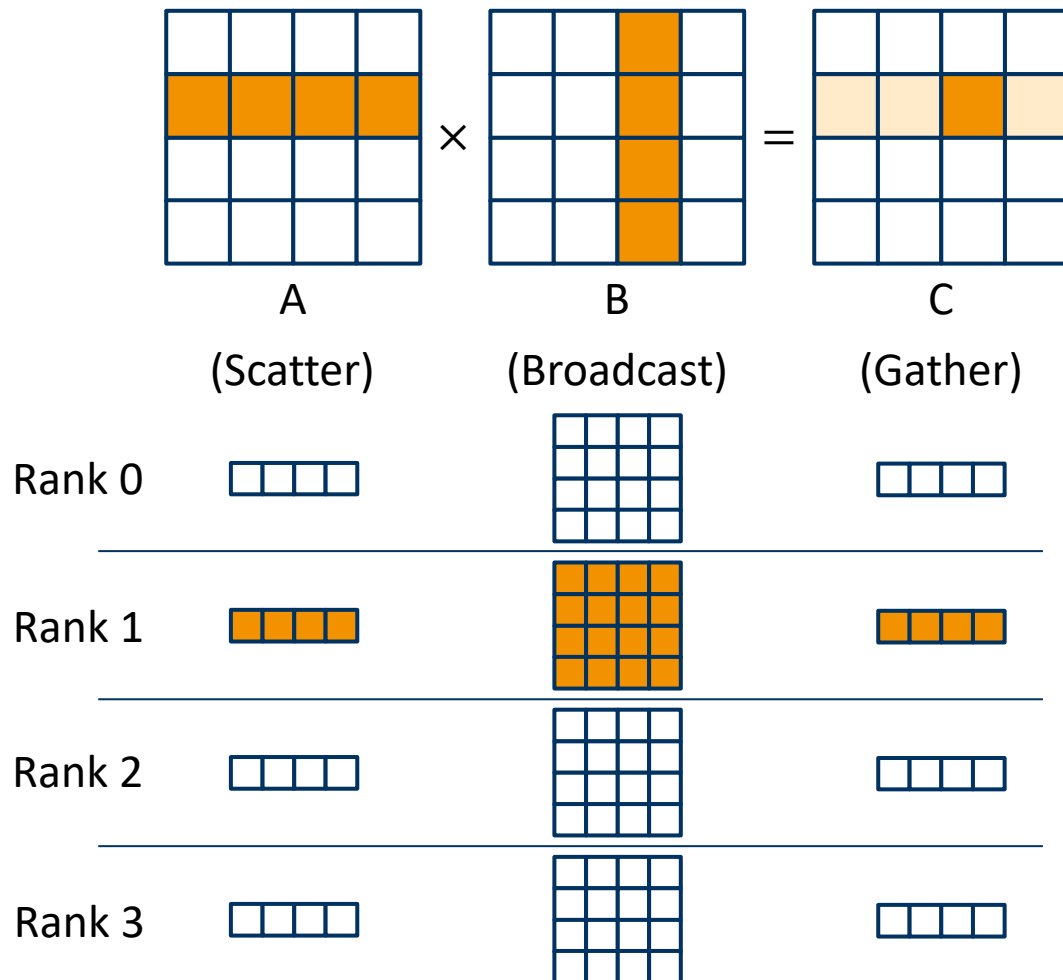
- ▶ several pre-defined
  - ▶ MPI\_MAX, MPI\_MIN
  - ▶ MPI\_SUM, MPI\_PROD
  - ▶ MPI\_LAND, MPI\_LOR, MPI\_LXOR
  - ▶ MPI\_BAND, MPI\_BOR, MPI\_BXOR
  - ▶ MPI\_MAXLOC, MPI\_MINLOC
- ▶ All pre-defined operations are associative and commutative
- ▶ also user-defined ops are possible
  - ▶ must be associative
  - ▶ may be commutative
  - ▶ requires a specific function signature
  - ▶ needs to be registered as an MPI handle

# Additional MPI functions

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- ▶ **MPI\_Wtime**
  - ▶ returns time in seconds since an arbitrary time in the past (Wall clock time)
- ▶ **MPI\_Sendrecv**
  - ▶ convenience wrapper for blocking send and receive
- ▶ **MPI\_Allreduce/MPI\_Allgather/...**
  - ▶ same as non-all versions, but result is available everywhere (performance impact!)
- ▶ **MPI\_Scan/MPI\_Exscan**
  - ▶ inclusive and exclusive prefix reductions
- ▶ **MPI\_Wait/MPI\_Test**
  - ▶ blocking/non-blocking check whether pending operation completed
- ▶ **MPI\_Probe/MPI\_Iprobe**
  - ▶ blocking/non-blocking check for new message without actually receiving it

## Example code: naïve matrix multiplication



```
#include <mpi.h>
#include <stdio.h>
#include <stdlib.h>
```

```
#define SIZE 4
```

```
int A[SIZE][SIZE];
int B[SIZE][SIZE];
int C[SIZE][SIZE];
```

```
void fill_matrix(int m[SIZE][SIZE]);
void print_matrix(int m[SIZE][SIZE]);
```

## Example code: naïve matrix multiplication cont'd

---

```
int myRank, numProcs;

MPI_Init(&argc, &argv);
MPI_Comm_rank(MPI_COMM_WORLD, &myRank);
MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
// if matrix size not divisible
if(SIZE % numProcs != 0) {
    MPI_Finalize();
    return EXIT_FAILURE;
}
// root generates input data
if(myRank == 0) {
    fill_matrix(A);
    fill_matrix(B);
}
```

```
// compute boundaries of local computation
int start = myRank*SIZE/numProcs;
int end = (myRank+1)*SIZE/numProcs;

// distribute rows of A to everyone
MPI_Scatter(A, SIZE*SIZE/numProcs, MPI_INT,
    A[start], SIZE*SIZE/numProcs, MPI_INT, 0,
    MPI_COMM_WORLD);

// send entire matrix B to everyone
MPI_Bcast(B, SIZE*SIZE, MPI_INT, 0,
    MPI_COMM_WORLD);
```

## Example code: naïve matrix multiplication cont'd

---

```
// local computation of every rank
for(int i = start; i < end; i++) {
    for(int j = 0; j < SIZE; j++) {
        C[i][j] = 0;
        for(int k = 0; k < SIZE; k++) {
            C[i][j] += A[i][k] * B[k][j];
        }
    }
}
```

```
// gather result rows back to root
MPI_Gather(C[start], SIZE*SIZE/numProcs,
          MPI_INT, C, SIZE*SIZE/numProcs, MPI_INT, 0,
          MPI_COMM_WORLD);

if(myRank == 0) { print_matrix(C); }

MPI_Finalize();

return EXIT_SUCCESS;
```

# Submitting to a cluster (SLURM & SGE)

---

```
#!/bin/bash

# submission partition
#SBATCH --partition std.q
# name of the job
#SBATCH --job-name my_test_job
# redirect output stream to this file
#SBATCH --output output.dat
# specify parallel environment
#SBATCH --ntasks 8
#SBATCH --ntasks-per-node 2

srun /path/to/application
# or
mpiexec -n $SLURM_NTASKS /path/to/application
```

```
#!/bin/bash

# submission queue
#$ -q std.q
# change to current directory
#$ -cwd
# name of the job
#$ -N my_test_job
# redirect output stream to this file
#$ -o output.dat
# join the output and error stream
#$ -j yes
# specify parallel environment
#$ -pe openmpi-2perhost 8

mpiexec -n 8 /path/to/application
```

# Tales from the proseminar: osu\_latency on LCC3 (WIP!)

|         | mpiexec                                     |                                  |                                  |                                 |  |   | mpirun                                      |                                  |                                  |                                 |  |   | srun  |                                  |                                  |                                 |  |   |
|---------|---|----------------------------------|----------------------------------|---------------------------------|--|---|---|----------------------------------|----------------------------------|---------------------------------|--|---|---|----------------------------------|----------------------------------|---------------------------------|--|---|
|         | intel-<br>mpi_2019<br>.10.317-<br>gcc-8.5.0 | openmpi<br>_3.1.6-<br>gcc-12.2.0 | openmpi<br>_4.1.4-<br>gcc-12.2.0 | openmpi<br>_4.1.4-<br>gcc-8.5.0 | openmpi<br>_4.1.4-<br>intel-<br>2021.7.1 | openmpi<br>_4.1.4-<br>oneapi-<br>2022.2.1 | intel-<br>mpi_2019<br>.10.317-<br>gcc-8.5.0 | openmpi<br>_3.1.6-<br>gcc-12.2.0 | openmpi<br>_4.1.4-<br>gcc-12.2.0 | openmpi<br>_4.1.4-<br>gcc-8.5.0 | openmpi<br>_4.1.4-<br>intel-<br>2021.7.1 | openmpi<br>_4.1.4-<br>oneapi-<br>2022.2.1 | intel-<br>mpi_2019<br>.10.317-<br>gcc-8.5.0 | openmpi<br>_3.1.6-<br>gcc-12.2.0 | openmpi<br>_4.1.4-<br>gcc-12.2.0 | openmpi<br>_4.1.4-<br>gcc-8.5.0 | openmpi<br>_4.1.4-<br>intel-<br>2021.7.1 | openmpi<br>_4.1.4-<br>oneapi-<br>2022.2.1 |
| 0       | 1,80  | 1,66                             | 95,88                            | 69,47                           | 28,93                                    | 85,42                                     | 1,81  | 1,66                             | 79,14                            | 56,00                           | 97,42                                    | 80,42                                     |   | 1,67                             | 71,90                            | 78,34                           | 66,89                                    | 92,06                                     |
| 1       | 1,76  | 1,63                             | 95,62                            | 70,13                           | 40,54                                    | 86,30                                     | 1,78  | 1,63                             | 28,03                            | 86,42                           | 82,59                                    | 85,00                                     |   | 1,63                             | 71,16                            | 76,03                           | 83,24                                    | 87,43                                     |
| 2       | 1,78  | 1,62                             | 94,80                            | 62,64                           | 75,30                                    | 86,50                                     | 1,77  | 1,64                             | 26,43                            | 86,48                           | 85,01                                    | 77,27                                     |   | 1,63                             | 71,98                            | 78,50                           | 82,65                                    | 58,13                                     |
| 4       | 1,74  | 1,61                             | 95,21                            | 40,60                           | 95,26                                    | 83,93                                     | 1,76  | 1,62                             | 31,36                            | 66,64                           | 93,13                                    | 28,45                                     |   | 1,62                             | 73,02                            | 78,09                           | 86,48                                    | 75,57                                     |
| 8       | 1,75  | 1,62                             | 91,95                            | 34,66                           | 95,93                                    | 64,33                                     | 1,77  | 1,62                             | 26,62                            | 25,96                           | 97,88                                    | 84,32                                     |   | 1,63                             | 69,35                            | 74,28                           | 86,32                                    | 92,21                                     |
| 16      | 1,75  | 1,62                             | 94,23                            | 51,73                           | 93,17                                    | 86,81                                     | 1,76  | 1,62                             | 27,07                            | 26,12                           | 96,36                                    | 84,41                                     |   | 1,63                             | 72,14                            | 69,28                           | 86,04                                    | 91,93                                     |
| 32      | 1,76  | 1,62                             | 93,55                            | 27,47                           | 94,14                                    | 87,76                                     | 1,78  | 1,63                             | 27,01                            | 44,32                           | 99,08                                    | 85,11                                     |   | 1,64                             | 72,14                            | 77,91                           | 84,58                                    | 89,96                                     |
| 64      | 1,90  | 1,76                             | 96,35                            | 60,40                           | 45,49                                    | 85,40                                     | 1,92  | 1,76                             | 27,84                            | 87,75                           | 99,55                                    | 85,52                                     |   | 1,78                             | 73,94                            | 78,47                           | 87,38                                    | 79,77                                     |
| 128     | 2,53  | 2,43                             | 96,78                            | 26,83                           | 35,30                                    | 88,23                                     | 2,53  | 2,43                             | 67,60                            | 91,09                           | 90,25                                    | 88,17                                     |   | 2,41                             | 75,87                            | 80,22                           | 88,34                                    | 95,52                                     |
| 256     | 2,68  | 2,58                             | 99,64                            | 59,55                           | 34,41                                    | 90,81                                     | 2,70  | 2,58                             | 88,90                            | 98,05                           | 101,17                                   | 91,23                                     |   | 2,56                             | 80,46                            | 87,47                           | 91,85                                    | 98,71                                     |
| 512     | 2,96  | 2,86                             | 107,64                           | 82,11                           | 110,03                                   | 100,77                                    | 2,97  | 2,86                             | 93,53                            | 103,54                          | 112,58                                   | 97,99                                     |   | 2,84                             | 83,85                            | 94,60                           | 97,54                                    | 104,10                                    |
| 1024    | 3,52  | 3,41                             | 120,97                           | 98,86                           | 106,42                                   | 112,24                                    | 3,52  | 3,41                             | 107,47                           | 118,41                          | 126,83                                   | 106,49                                    |   | 3,40                             | 97,34                            | 106,20                          | 112,88                                   | 114,55                                    |
| 2048    | 4,54  | 4,48                             | 113,75                           | 172,84                          | 108,50                                   | 146,21                                    | 4,55  | 4,48                             | 144,54                           | 126,49                          | 117,29                                   | 141,24                                    |   | 4,44                             | 192,35                           | 153,77                          | 147,66                                   | 137,15                                    |
| 4096    | 6,58  | 6,57                             | 251,72                           | 256,34                          | 247,65                                   | 244,15                                    | 6,58  | 6,58                             | 269,62                           | 230,88                          | 224,56                                   | 284,89                                    |   | 6,56                             | 282,53                           | 239,24                          | 275,62                                   | 274,82                                    |
| 8192    | 8,47  | 8,43                             | 250,83                           | 248,67                          | 250,95                                   | 249,15                                    | 8,47  | 8,45                             | 248,30                           | 248,97                          | 254,67                                   | 247,43                                    |   | 8,42                             | 261,81                           | 240,03                          | 251,19                                   | 250,45                                    |
| 16384   | 11,12                                       | 11,10                            | 328,66                           | 321,86                          | 334,00                                   | 323,64                                    | 11,14                                       | 11,10                            | 332,36                           | 335,69                          | 328,07                                   | 333,04                                    |   | 11,12                            | 313,01                           | 340,97                          | 330,24                                   | 333,83                                    |
| 32768   | 16,44                                       | 16,17                            | 492,69                           | 483,37                          | 494,25                                   | 490,36                                    | 16,27                                       | 16,16                            | 490,37                           | 497,96                          | 500,31                                   | 491,88                                    |   | 16,16                            | 484,16                           | 487,69                          | 501,96                                   | 492,64                                    |
| 65536   | 26,62                                       | 26,27                            | 362,85                           | 296,98                          | 260,67                                   | 376,14                                    | 26,47                                       | 26,21                            | 353,66                           | 331,07                          | 265,20                                   | 329,14                                    |   | 26,21                            | 390,03                           | 284,66                          | 379,37                                   | 318,44                                    |
| 131072  | 46,24                                       | 45,86                            | 488,97                           | 428,50                          | 398,00                                   | 460,38                                    | 46,22                                       | 45,86                            | 445,32                           | 451,03                          | 408,62                                   | 420,09                                    |   | 45,79                            | 496,38                           | 427,23                          | 487,01                                   | 408,33                                    |
| 262144  | 85,68                                       | 84,46                            | 555,13                           | 491,73                          | 487,80                                   | 545,97                                    | 85,67                                       | 83,83                            | 551,87                           | 518,01                          | 425,43                                   | 514,13                                    |   | 84,51                            | 563,26                           | 500,48                          | 551,89                                   | 481,68                                    |
| 524288  | 164,97                                      | 161,19                           | 770,21                           | 734,66                          | 662,11                                   | 740,81                                    | 165,59                                      | 161,27                           | 738,98                           | 728,89                          | 666,06                                   | 742,23                                    |   | 161,10                           | 762,28                           | 661,29                          | 848,54                                   | 703,71                                    |
| 1048576 | 324,72                                      | 315,54                           | 1095,23                          | 958,25                          | 996,85                                   | 1057,35                                   | 324,32                                      | 315,51                           | 1053,36                          | 1037,02                         | 955,35                                   | 944,12                                    |   | 315,53                           | 1170,47                          | 998,66                          | 1073,28                                  | 1008,58                                   |
| 2097152 | 625,85                                      | 624,34                           | 1874,31                          | 1453,91                         | 1289,10                                  | 1763,01                                   | 626,02                                      | 624,43                           | 1815,92                          | 1769,04                         | 1280,92                                  | 1492,41                                   |   | 624,48                           | 2025,38                          | 1390,87                         | 1603,72                                  | 1669,82                                   |
| 4194304 | 1243,62                                     | 1243,62                          | 3420,45                          | 2699,88                         | 2327,88                                  | 3177,13                                   | 1243,86                                     | 1243,49                          | 3290,81                          | 2947,66                         | 2329,35                                  | 2736,99                                   |   | 1243,16                          | 3706,90                          | 2748,61                         | 2925,97                                  | 3087,45                                   |



# Summary

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- ▶ general concepts about MPI
  - ▶ characteristics
  - ▶ program model
  - ▶ startup
- ▶ point-to-point communication
- ▶ collective communication
- ▶ practical example (matrix multiplication)