TORQUE ANALYSIS

$$\boxed{\tau = F \cdot d}$$

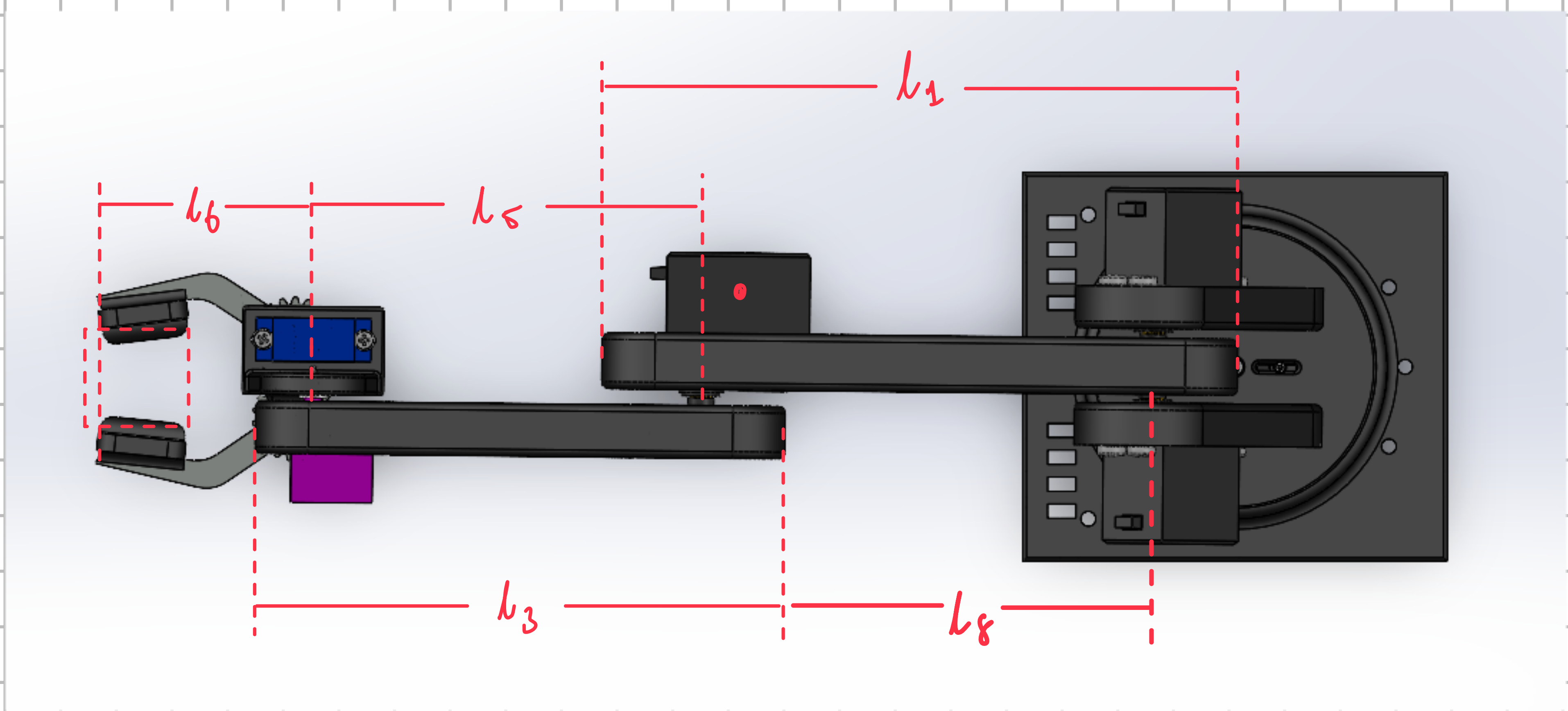
Torque analysis is done to determine the servo specifications required for smooth, safe and reliable operation of the Robotic Arm.

In order to do that, the analysis is carried out for the worst-case configuration which is when the arm is fully extended in a horizontal orientation while carrying maximum payload.

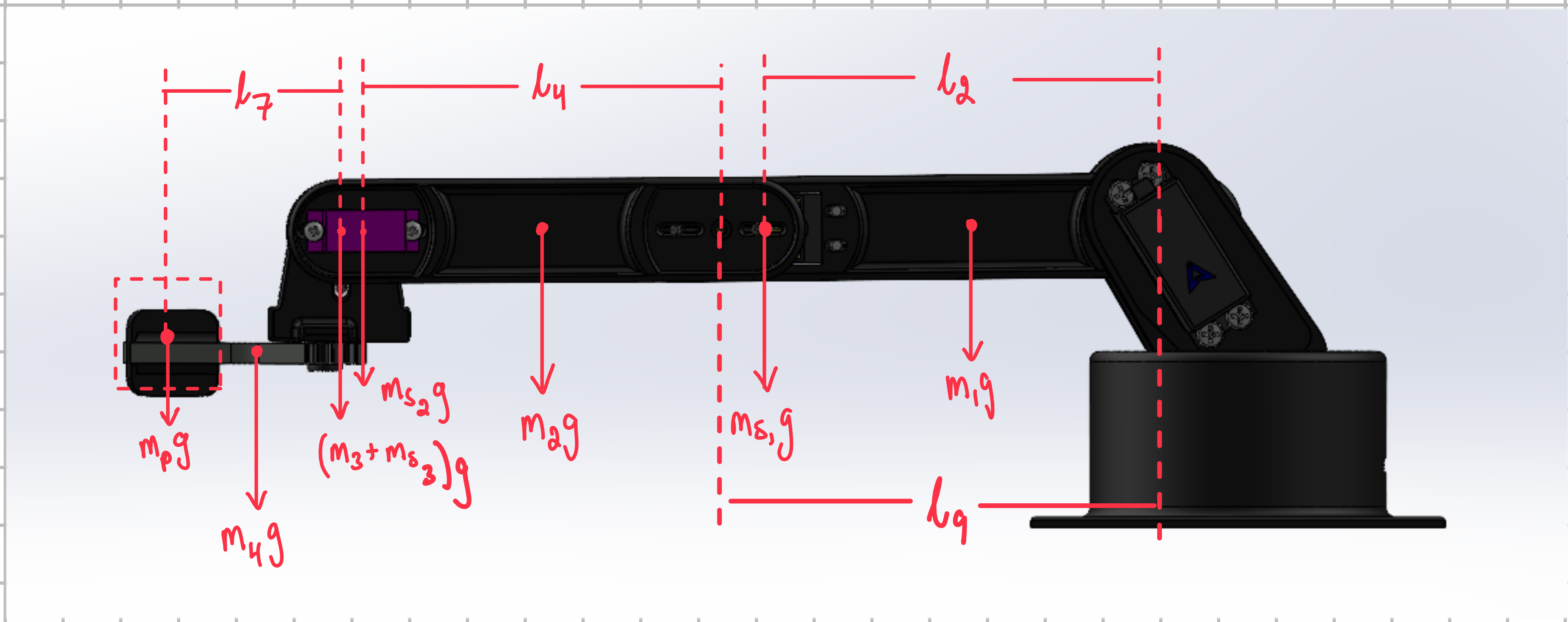
In this position, all links, the payload, and other masses act at the maximum distance relative to the shoulder and elbow joints.

This approach ensures that the servo motors at the shoulder and elbow joints are not overloaded during operation which prevents over-heating, excessive current draw and load on power supply, mechanical wear, and loss of position accuracy.

The image below shows the Robotic arm in its extended horizontal orientation:



Top View



Side View

l is the perpendicular distance.

Upper arm : $l_1 = 165\ mm = 0.165\ m$, $m_1 = 21.4\ g = 0.0214\ Kg$

Distance from shoulder to elbow servo : $l_2 = 130\ mm = 0.13\ m$

Mass of elbow servo : $m_{s_1} = 55\ g = 0.055\ Kg$

Forearm : $l_3 = 135\ mm = 0.135\ m$, $m_2 = 16.3\ g = 0.0163\ Kg$

Distance from elbow to wrist servo : $l_4 = 110\ mm = 0.11\ m$

Mass of wrist servo : $m_{s_2} = 11 g = 0.011\ Kg$

Distance from elbow to wrist and claw servo : $l_5 = 120\ mm = 0.12\ m$

Mass of wrist and claw servo respectively : $m_3 = 6.9 g = 0.0069\ Kg$,

$m_{s_3} = 11 g = 0.011\ Kg$

Claw : $l_6 = 60\ mm = 0.06\ m$, $m_4 = 7.3\ g = 0.0073\ Kg$

Distance from wrist to payload : $l_7 = 50\ mm = 0.05\ m$

This is an approximation as payload shapes may vary

Distance from shoulder to forearm link : $l_8 = 125\ mm = 0.125\ m$

Distance from shoulder to forearm servo shafts : $l_9 = 150\ mm = 0.15\ m$

Payload (Assumption) : $m_p = 250\ g = 0.25\ Kg$

Gravitational constant : $g = 9.81\ m/s^2$

Torque at elbow

$$\tau_{el} = \sum F \cdot d_{\perp}$$

$$\tau_{el} = g\left[\left(m_2 \cdot \frac{l_3}{2}\right) + \left(m_{s_2} \cdot l_4\right) + \left(\left(m_3 + m_{s_3}\right) \cdot l_5\right) + \left(m_4 \cdot \left(l_5 + \frac{l_6}{2}\right)\right) + \left(m_p \cdot \left(l_5 + l_7\right)\right)\right]$$

$$\Rightarrow \tau_{el} = 9.81\left[\left(0.0163 \cdot \frac{0.185}{2}\right) + \left(0.011 \cdot 0.11\right) + \left(\left(0.0069 + 0.011\right) \cdot 0.12\right) + \left(0.0073 \cdot \left(0.12 + \frac{0.06}{2}\right)\right) + \left(0.25\left(0.12 + 0.05\right)\right)\right]$$

$$\Rightarrow \tau_{el} = 9.81\left[\left(1.10025 \times 10^{-3}\right) + \left(1.21 \times 10^{-3}\right) + \left(2.148 \times 10^{-3}\right) + \left(1.095 \times 10^{-3}\right) + \left(0.0425\right)\right]$$

$$\Rightarrow \tau_{el} = 9.81\left(0.04805325\right) \Rightarrow \tau_{el} \approx 0.47\ N \cdot m$$

Converting to Kg·cm:

$$\Rightarrow \tau_{el} = \frac{0.46}{0.0981} \Rightarrow \boxed{\tau_{el} \approx 4.81\ Kg \cdot cm}$$ (250 g payload)

$$\Rightarrow \boxed{\tau_{el} \approx 9.4\ Kg \cdot cm}$$ (500 g payload)

This torque analysis at the elbow shows that with a 500g payload, the servo motor at the elbow requires approximately 9 Kg·cm of torque given worst-case orientation.

This requirement is within the rated capacity of the MG996r Servo motor, which provides about 10-12 Kg·cm of stall torque at 6V.

Torque at Shoulder

$$\tau_{sh} = \sum F \cdot d_{\perp}$$

$$\tau_{sh} = g\left[\left(m_1 \cdot \frac{l_1}{2}\right) + \left(m_{s_1} \cdot l_2\right) + \left(m_2 \cdot \left(l_8 + \frac{l_3}{2}\right)\right) + \left(m_{s_2} \cdot \left(l_4 + l_9\right)\right) + \right.$$
$$\left.\left(\left(m_3 + m_{s_3}\right) \cdot \left(l_5 + l_9\right)\right) + \left(m_4 \cdot \left(l_5 + \frac{l_6}{2} + l_9\right)\right) + \left(m_p \cdot \left(l_5 + l_7 + l_9\right)\right)\right]$$

$$\Rightarrow \tau_{sh} = 9.81\Big[\left(0.0214 \cdot \frac{0.165}{2}\right) + \left(0.055 \cdot 0.13\right) + \left(0.0163 \cdot \left(0.125 + \frac{0.135}{2}\right)\right) + \left(0.011 \cdot (0.11 + 0.15)\right) + \left((0.0069 + 0.011) \cdot (0.12 + 0.15)\right) + \left(0.0073 \cdot \left(0.11 + \frac{0.06}{2} + 0.15\right)\right) + \left(0.25 \cdot (0.11 + 0.06 + 0.15)\right)\Big]$$

$$\Rightarrow \tau_{sh} = 9.81\Big[(1.7655 \times 10^{-3}) + (7.15 \times 10^{-3}) + (3.13775 \times 10^{-3}) + (2.86 \times 10^{-3}) + (4.833 \times 10^{-3}) + (2.117 \times 10^{-3}) + 0.0775\Big]$$

$$\Rightarrow \tau_{sh} = 9.81(0.09936325) \Rightarrow \tau_{sh} \approx 0.975 \text{ N.m}$$

Converting to Kg.cm:

$$\Rightarrow \tau_{sh} = \frac{0.975}{0.0981} \Rightarrow \tau_{sh} \approx 9.94 \text{ Kg.cm} \quad (250\text{g payload})$$

$$\Rightarrow \tau_{sh} \approx 17.7 \text{ Kg.cm} \quad (500\text{g payload})$$

This torque analysis at the shoulder shows that with a 500 g payload, the servo motor at the shoulder requires approximately 17.7 Kg.cm of torque given worst-case orientation.

When compared to the rated torque specifications of the MG996r servo motor (≈ 10-12 Kg.cm @ 6V), these results show that a single servo is insuffecient.

However, the dual-servo configuration at the shoulder utilizes parrallel torque sharing, which provides a combined effective torque capacity of approximately ≈ 20 Kg.cm. This capacity is sufficient for payloads of up to 500 g.