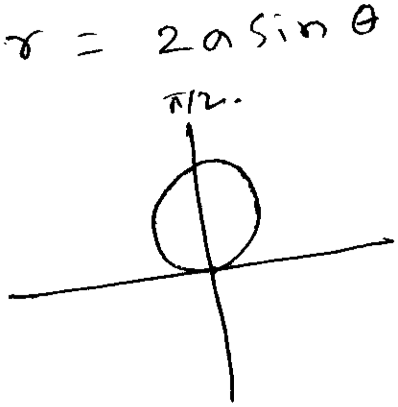
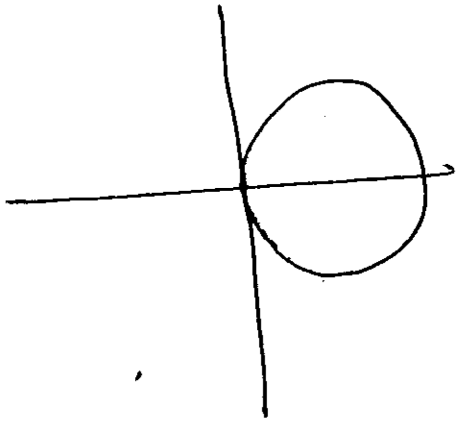
1.4.3.

$r = 2a\sin\theta$

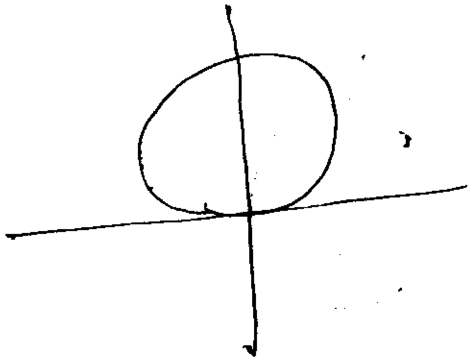


θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
r	0	$1a$	$\sqrt{3}a$	$2a$	$\sqrt{3}a$	1	0

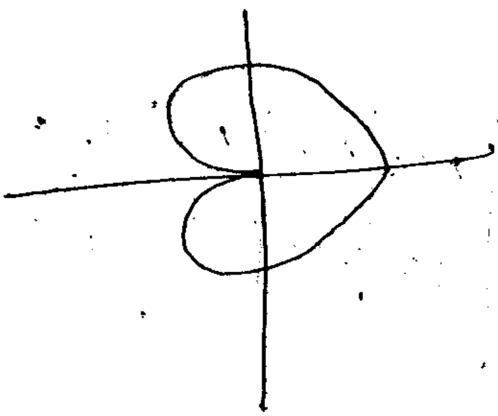
$r = 2a\cos\theta$



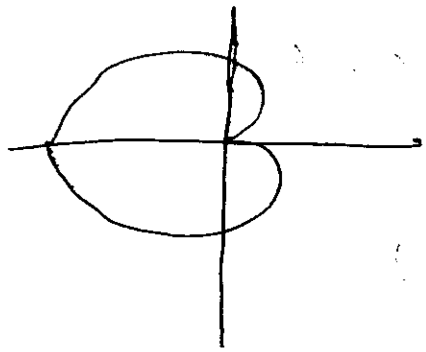
$r = 2a\sin\theta$



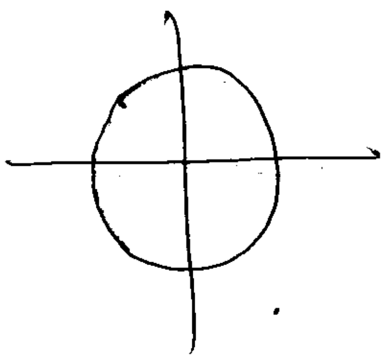
$r = a(1+\cos\theta)$



$r = a(1-\cos\theta)$

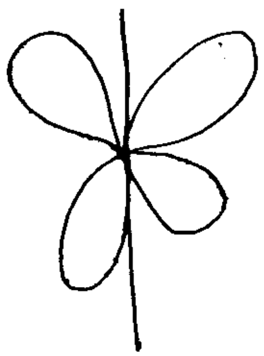


$r = a$

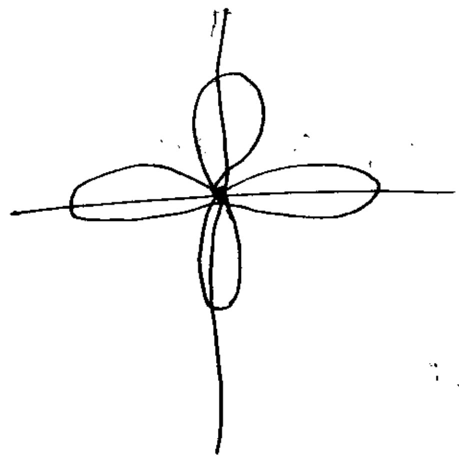


Rose

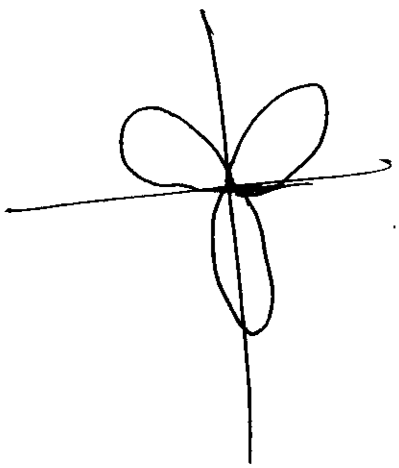
$r = a\sin 2\theta$



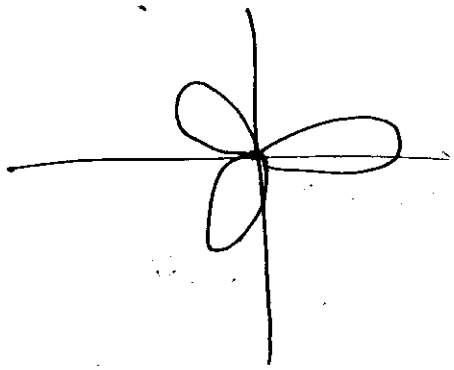
$r = a\cos 2\theta$



$r = a\sin 3\theta$



$r = a\cos 3\theta$



①

$$\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta \, dr\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \cos\theta \sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} u^2 \, d\theta$$

$$= \frac{1}{6}\left[u^3\right]_0^{\pi/2}$$

$$= \frac{1}{6}\left[\sin\theta\right]_0^{\pi/2} = \frac{1}{6}(\sin 90 - \sin 0)$$

$$= \frac{1}{6}$$

②

$$\int_0^{\pi}\int_0^{1+\cos\theta} r \, dr\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} (1+\cos\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 1 + 2\cos\theta + \cancel{\cos^2\theta \, d\theta} \quad \frac{1}{2}(1+\cos 2\theta$$

$$= \frac{1}{2}\left[\theta - 2\sin\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$= \frac{1}{2}\left(\pi - 2\sin\pi + \frac{1}{2}\pi - \frac{1}{4}\sin 2\pi\right)$$

$$= \frac{1}{2}\left(\pi + \frac{1}{2}\pi\right)$$

$$= \frac{1}{2} \times \frac{3}{2}\pi$$

$$= \frac{3\pi}{4}$$

③ $$\int_0^{\pi/2}\int_0^{a\sin\theta} r^2\,dr\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2}\left[r^3\right]_0^{a\sin\theta} dr\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2} a^3\sin^3\theta\,dr\,d\theta$$

$$= \frac{1}{3}a^3\int_0^{\pi/2}\sin\theta\sin^2\theta\,d\theta$$

$$= \frac{1}{3}a^3\int_0^{\pi/2}\sin\theta(1-\cos^2\theta)\,d\theta$$

$$= \frac{1}{3}a^3\int_0^{\pi/2}\sin\theta - \sin\theta\cos^2\theta\,d\theta$$

$$= \frac{1}{3} a^3 \int_0^{\pi/2} \sin\theta + \frac{1}{3} a^3 \int_0^{\pi/2} u^2 \, du$$

$$= -\frac{1}{3} a^3 \left(\cos \pi/2 - \cos 0\right) + \frac{1}{3} a^3 \frac{1}{3} \left[u^3\right]_0^{\pi/2}$$

$$= \frac{1}{3} a^3 + \frac{1}{9} a^3 \left((\cos \pi/2)^3 - \cos^3 0\right)$$

$$= \frac{1}{3} a^3 + \frac{1}{9} a^3 (0 - 1)$$

$$= \frac{1}{3} a^3 - \frac{1}{9} a^3$$

$$= \frac{2}{9} a^3$$

(4) $$\int_0^{\pi/6} \int_0^{\cos 3\theta} r \, dr \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/6} \cos^2 3\theta \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/6} \left(4\cos^3\theta - 3\cos\theta\right)^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi/6} 4\cos^3\theta \, d\theta - \frac{1}{2} \int_0^{\pi/6} 3\cos\theta \, d\theta$$

$$= 2\int_0^{\pi/6} \cos\theta \cos^2\theta \, d\theta - 3/2 \left[\sin \pi/6 - \sin 0\right]$$

$$= 2\int_0^{\pi/6} \cos\theta(1-\sin^2\theta) \, d\theta - \frac{3}{4}$$

$$= 2\int_0^{\pi/6} \cos\theta - \cos\theta \sin^2\theta \, d\theta - 3/4$$

$$= 2\int_0^{\pi/6} \cos\theta \, d\theta - 2\int_0^{\pi/6} \cos\theta \sin^2\theta \, d\theta - 3/4$$

$$= 2 \times \frac{1}{2} - 2\int_0^{\pi/6} u^2 \, du - 3/4$$

$$= 1 - \frac{2}{3}\left[\sin^3\theta\right]_0^{\pi/6} - 3/4$$

$$= 1 - 2/3 \times \frac{1}{8} - 3/4$$

$$= 1 - \frac{1}{12} - 3/4$$

$$= 1/6$$

(4) $\int_0^{\pi/6}\int_0^{\cos 3\theta} r\,dr\,d\theta$

Not right

$= \frac{1}{2}\int_0^{\pi/6} \cos^2 3\theta \, d\theta$

$= \frac{1}{2}\int_0^{\pi/6} \frac{1}{2}(1+\cos 6\theta)\, d\theta$

$= \frac{1}{4}\int_0^{\pi/6} 1 + \cos 6\theta \, d\theta$

$= \frac{1}{4}\left[1 + \frac{\sin 6\theta}{6}\right]_0^{\pi/6}$

$= \frac{1}{4}\left(\pi/6 + \frac{\sin 6\frac{\pi}{6}}{6} - 1 - \frac{\sin 0}{6}\right)$ $\quad \frac{\pi}{6} - 1$ $\quad \frac{\pi - 6}{24}$

$= 1/4\left(\pi/6 + \frac{0}{6} - 1 - \frac{0}{6}\right)$

$= \frac{1}{4} \times \left(\frac{\pi}{6} - 3/3\right)$

$= \frac{1}{4} \times \frac{\pi - 18}{6}$ $\quad = \frac{1}{4}\left(\frac{\pi - 18}{24}\right)$

(5)

$$\int_0^{\pi}\int_0^{1-\sin\theta} r^2 \cos\theta \, dr \, d\theta$$

$$= \frac{1}{3}\int_0^{\pi} \cos\theta \left[r^3\right]_0^{1-\sin\theta} d\theta$$

$$= \frac{1}{3}\int_0^{\pi} \cos\theta \, (1-\sin\theta)^3 \, d\theta$$

$$= \frac{1}{3}\int_0^{\pi} \cos\theta \left(1 - 3\sin\theta + 3\sin^2\theta - \sin^3\theta\right) d\theta$$

$$= \frac{1}{3}\int_0^{\pi} \cos\theta - 3\sin\theta\cos\theta + 3\cos\theta\sin^2\theta - \cos\theta\sin^3\theta \, d\theta$$

$$= \frac{1}{3}\left[\sin\theta - \frac{3}{2}\cos^2\theta + \frac{3}{3}\sin^3\theta - \frac{1}{4}\sin^4\theta\right]_0^{\pi}$$

$$= \frac{1}{3} \times \left(-\frac{3}{2} + \frac{3}{2}\right) = 0$$

⑥ $$\int_0^{\pi/2}\int_0^{\cos\theta} r^3\,dr\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/2}\cos^4\theta\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/2}\cos^2\theta\cos^2\theta\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/2}\cancel{\cos^2\theta}\left(\frac{1}{2}\times\frac{1}{2}(1+\cos2\theta)(1+\cos2\theta)\,d\theta\right.$$

$$=\frac{1}{16}\int_0^{\pi/2}(1+\cos2\theta)^2\,d\theta$$

$$=\frac{1}{16}\int_0^{\pi/2}1+2\cos2\theta+\cos^2 2\theta\,d\theta$$

$$=\frac{1}{16}\int_0^{\pi/2}1+2\cos2\theta+\frac{1}{2}+\frac{1}{2}\cos4\theta\,d\theta$$

$$=\frac{1}{16}\left[\theta+\frac{2}{2}\sin2\theta+\frac{1}{2}\theta+\frac{1}{8}\sin4\theta\right]_0^{\pi/2}$$

$$=\frac{1}{16}\left[\pi/2+\sin\pi+\frac{1}{2}\,\pi/2+\frac{1}{8}\sin2\pi\right]$$

$$=\frac{1}{16}\left[\frac{\pi}{2}+\frac{\pi}{4}\right]=\frac{1}{16}\times\frac{3\pi}{4}=3\pi/64$$

(6) $\displaystyle\int_0^{\pi/2}\int_0^{\cos\theta} r^3\,dr\,d\theta$

$$=\frac{1}{4}\int_0^{\pi/2}\cos^4\theta\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/2}\cos^2\theta\cos^2\theta\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/2}\cancel{\cos^2\theta}\left(\frac{1}{2}\times\frac{1}{2}(1+\cos 2\theta)(1+\cos 2\theta)\right)d\theta$$

$$=\frac{1}{16}\int_0^{\pi/2}(1+\cos 2\theta)^2\,d\theta$$

$$=\frac{1}{16}\int_0^{\pi/2}1+2\cos 2\theta+\cos^2 2\theta\,d\theta$$

$$=\frac{1}{16}\int_0^{\pi/2}1+2\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\,d\theta$$

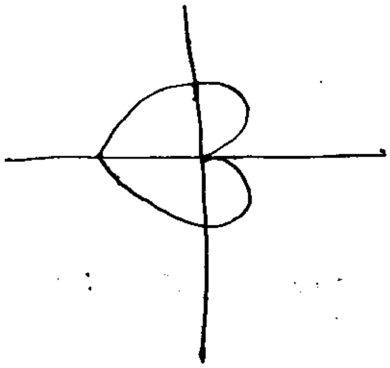
$$=\frac{1}{16}\left[\theta+\frac{2}{2}\sin 2\theta+\frac{1}{2}\theta+\frac{1}{8}\sin 4\theta\right]_0^{\pi/2}$$

$$=\frac{1}{16}\left[\pi/2+\sin\pi+\frac{1}{2}\,\pi/2+\frac{1}{8}\sin 2\pi\right]$$

$$=\frac{1}{16}\left[\frac{\pi}{2}+\frac{\pi}{4}\right]=\frac{1}{16}\times\frac{3\pi}{4}=3\pi/64$$

(7) The region enclosed by the cardioid

$r = 1 - \cos\theta$



$$\int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (1-\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} 1 - 2\cos\theta + \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} 1 - 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\,d\theta$$

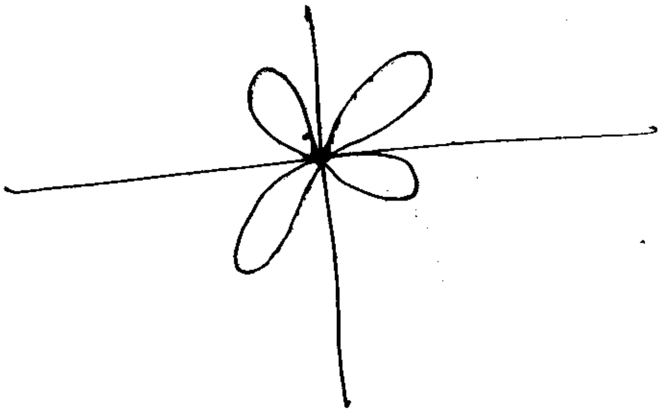
$$= \frac{1}{2}\left[\theta - 2\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}\left[2\pi - 0 + \frac{1}{2}\times 2\pi + 0\right]$$

$$= \frac{1}{2}\pi = \pi/2$$

(8) The region enclosed by the rose

$r = \sin 2\theta$



$$\int_0^{2\pi}\int_0^{\sin 2\theta} r\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \sin^2 2\theta \, d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{2\pi}$$

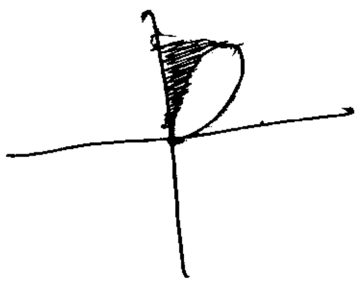
$$= \frac{1}{4}\left[2\pi - \frac{\sin 8\pi}{4} - \overset{0}{\cancel{2\pi}} + \frac{\sin 0}{4}\right]$$

$$= \frac{1}{4}\left[2\pi - \overset{0}{\cancel{8\pi}}\right] = \pi/2$$

Ans.

(9)

The region ~~enclose~~ in the first quadrant bounded by $r=1$ and $r=\sin 2\theta$, with $\pi/4 \le \theta \le \pi/2$



$$\int_{\pi/4}^{\pi/2}\int_{\sin 2\theta}^{1} r\,dr\,d\theta$$

$$= \frac{1}{2}\int_{\pi/4}^{\pi/2} 1-\sin^2 2\theta \, d\theta$$

$$= \frac{1}{2}\int_{\pi/4}^{\pi/2} 1-\frac{1}{2}(1-\cos 4\theta)\, d\theta$$

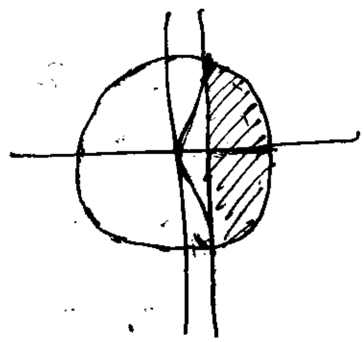
$$= \frac{1}{2}\int_{\pi/4}^{\pi/2} 1-\frac{1}{2}+\frac{1}{2}\cos 4\theta \, d\theta = \frac{1}{2}\left[\theta-\frac{1}{2}\theta+\frac{1}{4}\sin 4\theta\right]_{\pi/4}^{\pi/2}$$

$\pi/16$

(10)

The region inside the circle $x^2+y^2=4$ and to the right of the line $x=1$

$r^2=4 \quad r=\pm 2$



$y^2=4-1$

$y^2=3$

$y=\pm\sqrt{3}$

$(1,\sqrt{3})\ (1,-\sqrt{3})$

$$2\int_0^{\pi/3}\int_{\sec\theta}^{2} r\,dr\,d\theta$$

$x=1$

$r\cos\theta=1$

$r=\frac{1}{\cos\theta}$

$=\sec\theta$

$\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$=\pi/3$

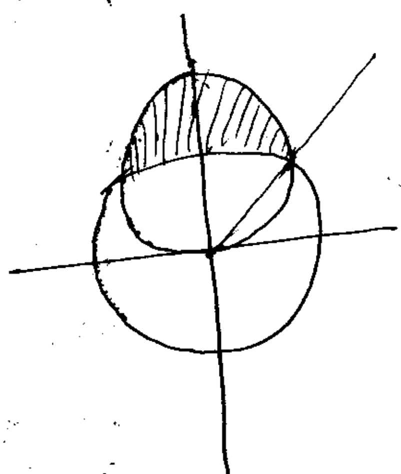
$$=\frac{2}{2}\int_0^{\pi/3} 4-\sec^2\theta\,d\theta$$

$$=\int_0^{\pi/3} 4-\sec^2\theta\,d\theta$$

$$=\Big[4\theta-\tan\theta\Big]_0^{\pi/3}$$

$$=\frac{4\pi}{3}-\sqrt{3}-0 = \frac{4\pi-3\sqrt{3}}{3}$$ (Ans.

(11) The region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$



$r = 4\sin\theta$

$r = 2$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}(1/2)$

$= 30° = \pi/6$

$$2\int_{\pi/6}^{\pi/2}\int_{2}^{4\sin\theta} r\,dr\,d\theta$$

$$= \frac{2}{2}\int_{\pi/6}^{\pi/2} 16\sin^2\theta - 4\,d\theta$$

$$= 4\int_{\pi/6}^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos 2\theta - 1\,d\theta$$

$$= 2\left[\theta - \frac{1}{2}\sin 2\theta - 2\theta\right]_{\pi/6}^{\pi/2}$$

$$= 2\left(\pi/2 - \frac{1}{2}\times 0 - \pi - \pi/6 + \sqrt{3}/2 \times 1/2 + \pi/3\right)$$

$$= 2\left(\frac{\pi}{2} - \pi - \frac{\pi}{6} + \frac{\sqrt{3}}{4} + \pi/3\right) = 2\left(\frac{\sqrt{3}}{4} - \pi/3\right)$$

$$= 2 \times \frac{3\sqrt{3} - 4\pi}{12}$$

$$= \frac{6\sqrt{3}}{12} - \frac{8\pi}{12}$$

$$= \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$$

$$= 4 \int_{\pi/6}^{\pi/2} 4\sin^2\theta - 1 \, d\theta$$

$$= 4 \int_{\pi/6}^{\pi/2} \frac{4}{2}(1 - \cos 2\theta) - 1 \, d\theta$$

$$= 4 \int_{\pi/6}^{\pi/2} 2 - 2\cos 2\theta - 1 \, d\theta$$

$$= 4 \int_{\pi/6}^{\pi/2} 1 - 2\cos 2\theta \, d\theta$$

$$= 4 \left[\theta - \sin 2\theta\right]_{\pi/6}^{\pi/2}$$

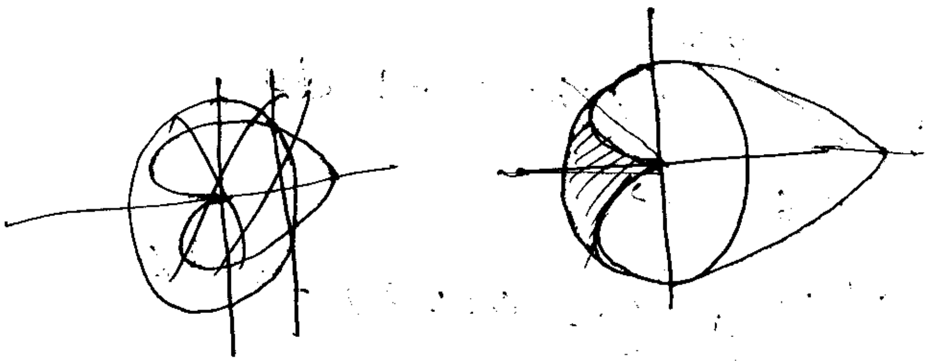
$$2 \,\underline{|\, 2, 6, 2}$$
$$1, 1, 3$$

$$= 4\left(\pi/2 - 0 - \pi/6 + \sqrt{3}/2\right)$$

$$= \cancel{4}^{2} \times \frac{3\pi - \pi + 3\sqrt{3}}{\cancel{6}\,3} = \frac{4\pi}{3} + 2\sqrt{3}$$

Ans.

(12) The region inside the circle $r=1$ and outside the Cardioid $r=1+\cos\theta$



$$2\int_{\pi/2}^{\pi}\int_{1}^{1+\cos\theta} r\,dr\,d\theta$$

$1+\cos\theta=1$

$\cos\theta=0$

$\theta=\cos^{-1}(0)$

$$=\frac{2}{2}\int_{\pi/2}^{\pi}(1+\cos\theta)^2-1\,d\theta$$

$$=\int_{\pi/2}^{\pi} 1+2\cos\theta+\cos^2\theta-1\,d\theta$$

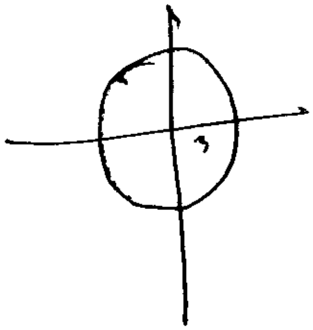
$$=\int_{\pi/2}^{\pi} 2\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta\,d\theta$$

$$=\left[2\sin\theta+\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\right]_{\pi/2}^{\pi}$$

$$=\frac{1}{2}\pi-\left(2+\frac{\pi}{4}\right)=\frac{1}{2}\pi-\left(\frac{8+\pi}{4}\right)=\frac{\pi}{2}-\frac{8+\pi}{4}=\frac{2\pi-8-\pi}{4}=\frac{\pi-8}{4}$$

(23)

$\iint_R \sin(x^2+y^2)\, dA$ where R is the region enclosed by the circle $x^2+y^2=9$



$r^2=9$

$r=\pm 3$

$$2\int_0^{2\pi}\int_0^3 r\sin r^2\, dr\, d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\int_{-3}^{3}\sin p\, dp\, d\theta$$

$$=-\frac{1}{2}\int_0^{2\pi}\left[\cos r^2\right]_0^3 d\theta$$

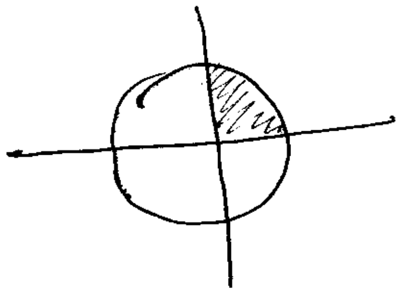
$$=-\frac{2}{2}\int_0^{2\pi}(\cos 9-\cos 0)\, d\theta$$

$$=-\int_0^{2\pi}\cos 9-1\, d\theta$$

$$=-(\cos 9-1)2\pi = 2\pi(1-\cos 9)$$

(24)

$\iint_R \sqrt{9-x^2-y^2}\, dA$ where R is the region in the first quadrant within the circle $x^2+y^2=9$



$$\int_0^{\pi/2}\int_0^3 \sqrt{9-r^2}\; r\,dr\,d\theta$$

$$= -\frac{1}{2}\int_0^{\pi/2}\int \sqrt{p}\; dp\,d\theta$$

$$= -\frac{1}{2}\cdot\frac{2}{3}\int_0^{\pi/2} \left[(9-p^2)^{3/2}\right]_0^3 d\theta$$

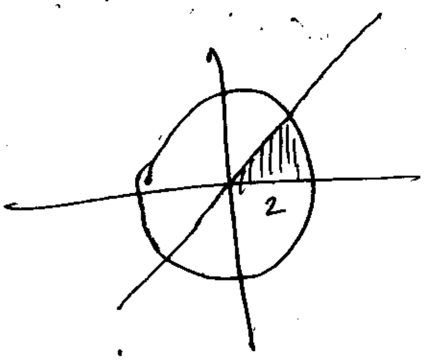
$$= -\frac{1}{3}\int_0^{\pi/2} (0-27)\, d\theta$$

$$= 9\int_0^{\pi/2} d\theta = 9\left(\frac{\pi}{2}-0\right) = \frac{9\pi}{2}$$

(25) $\iint \frac{1}{1+x^2+y^2} \, dA$, where R is the sector in the first quadrant bounded by $y=0$, $y=x$, $x^2+y^2=4$

$r^2 = 4$

$r = \pm 2$



$x^2+y^2=4$

$y = x$

$x^2+x^2=4$

$2x^2=4$

$x^2=2$

$r^2\cos^2\theta = 2$

$x = \pm\sqrt{2}$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = \cos^{-1}(1/\sqrt{2})$

$$\int_0^{\pi/4}\int_0^{2} \frac{1}{1+r^2} \, r \, dr \, d\theta$$

$$\frac{1}{2}\int_0^{\pi/4}\int_0^{2} \frac{2r}{1+r^2} \, dr \, d\theta$$

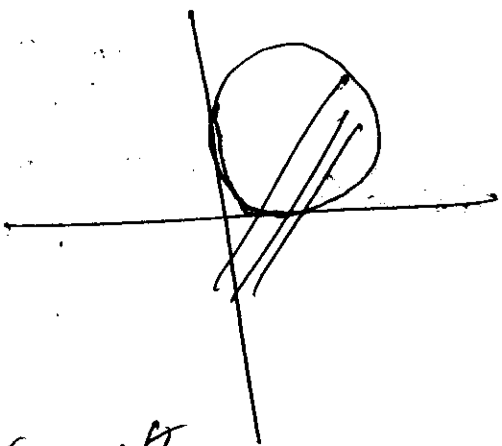
$$= \frac{1}{2}\int_0^{\pi/4} \left[\ln 1+r^2\right]_0^2 \, d\theta$$

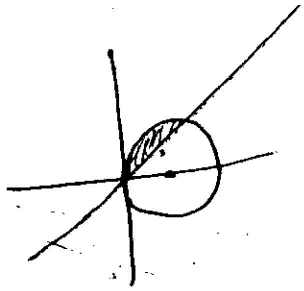
$$= \frac{1}{2}\int_0^{\pi/4} (\ln 5 - \ln 1) \, d\theta$$

$$= \frac{\ln 5}{2} \, \pi/4 = \frac{\pi \ln 5}{8}$$

(26)

$\iint_R 2y\,dA$ where R is the region in the first quadrant bounded above by the circle $(x-1)^2 + y^2 = 1$ and below by the line $y = x$





$$\int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} 2r\sin\theta\, r\,dr\,d\theta$$

$x^2 - 2x + 1 + y^2 = 1$

$x^2 + y^2 - 2x = 0$

$r^2 - 2r\cos\theta = 0$

$r(r - 2\cos\theta) = 0$

$r = 0$

$r = 2\cos\theta$

$x^2 + x^2 - 2x = 0$

$2x^2 - 2x = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$

$x = 1.$

$(1,1)$

$\tan^{-1}(1) = \pi/4$

$$\therefore \int_{\pi/4}^{\pi/2} \int_{0}^{2\cos\theta} 2r^2 \sin\theta \, dr \, d\theta$$

$$\frac{2}{3} \int_{\pi/4}^{\pi/2} 8\cos^3\theta \sin\theta \, d\theta$$

$$= \frac{\cancel{2}}{3} \times \frac{\cancel{8}^{2}}{\cancel{4}} \left[\cos^4\theta \right]_{\pi/4}^{\pi/2}$$

$$= \frac{4}{3} \left(0 - \frac{1}{4} \right)$$

$$= -\frac{1}{3}$$

~~(28)~~ (27)

$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)\, dy\, dx$$

$$= \int_0^{\pi/2} \int_0^1 r^2 r\, dr\, d\theta$$

$$= \frac{1}{4} \int_0^{\pi/2} d\theta$$

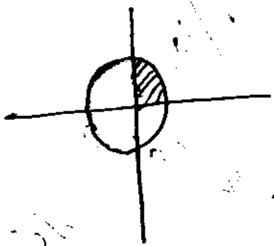
$$= \frac{1}{4} \times \pi/2 = \pi/8$$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$

$r^2 = 1$

$r = \pm 1$



(28) $$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\, dx\, dy$$

$x^2 + y^2 = 4$

$r^2 = 4$

$r = \pm 2$

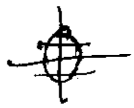
$y = 2$

$y = -2$

$r \sin\theta = 2$

$\sin\theta = 1$

$\theta = \pi/2$



$$= \int_0^{2\pi} \int_0^2 e^{-r^2}\, r\, dr\, d\theta$$

$$= -\frac{1}{2} \int_0^{2\pi} \left[e^{-r^2}\right]_0^2 d\theta = -\frac{1}{2} \int_0^{2\pi} e^{-4} - 1\, d\theta$$

$$= \frac{1}{2}(1 - e^{-4})\, 2\pi$$

$$= (1 - e^{-4})\, \pi$$

(29) $$\int_0^2 \int_0^{\sqrt{2x-x^2}} \cancel{\cos x^2} \sqrt{x^2+y^2}\; dy\, dx$$

$$= \int_0^{\pi/2} \int_0^{2\cos\theta} \sqrt{r^2}\; r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{2\cos\theta} r^2\, dr\, d\theta$$

$$= \frac{1}{3} \int_0^{\pi/2} 8\cos^3\theta \; d\theta$$

$$= 8/3 \int_0^{\pi/2} \cos^2\theta \cos\theta\, d\theta$$

$$= 8/3 \int_0^{\pi/2} (1-\sin^2\theta)\cos\theta\, d\theta$$

$$= 8/3 \int_0^{\pi/2} \cos\theta - \cos\theta\sin^2\theta\, d\theta$$

$$= 8/3 \left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2}$$

$$= 8/3\left[1 - 1/3\right] = \frac{8}{3} \times \frac{2}{3}$$

$$= \frac{16}{9}$$

$y^2 = 2x - x^2$

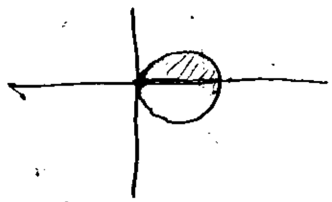
$x^2 + y^2 - 2x = 0$

$x^2 - 2x + 1 + y^2 = 1$

$(x-1)^2 + y^2 = 1$

$x = 2 \quad r\cos\theta = 2$

r



$r^2\cos^2\theta + r^2\sin^2\theta - 2r\cos\theta = 0$

$r^2 - 2r\cos\theta = 0$

$r(r - 2\cos\theta) = 0$

$r = 0$

$r = 2\cos\theta$

$x = 0$

$r\cos\theta = 0$

$\cos\theta = 0$

$x = 2$

$r\cos\theta = 2$

$\cos\theta = \frac{2}{2\cos\theta}$

$=$

(30)

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\, dx\, dy$$

$$\int_0^{\pi/2}\int_0^1 \cos r^2 \cdot r\, dr\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \left[\sin r^2\right]_0^1 d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \sin 1\, d\theta$$

$$= \frac{\sin 1}{2}\,(\pi/2)$$

$$= \frac{\pi \sin 1}{4}$$

$x^2 = 1 - y^2$

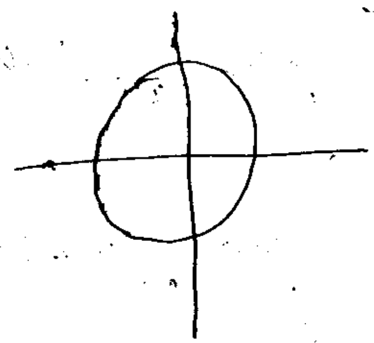
$x^2 + y^2 = 1$

$r^2 = 1$

$r = \pm 1$

$r = 1$

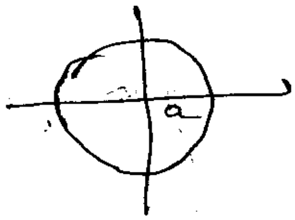
$r \sin\theta = 1$



(31) $$\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{1}{(1+x^2+y^2)^{3/2}}\, dy\, dx$$

$y^2 = a^2 - x^2$

$x^2 + y^2 = a^2$



$$\int_0^{\pi/2} \int_0^{a} \frac{r}{(1+r^2)^{3/2}}\, dr\, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \int \frac{1}{p^{3/2}}\, dp\, d\theta$$

$$= \cancel{\tfrac{1}{2}} - \int_0^{\pi/2} \left[p^{-1/2}\right]_0^a \cancel{dp}\, d\theta$$

$$= - \int_0^{\pi/2} \left[(1+r^2)^{-1/2}\right]_0^a d\theta$$

$$= - \int_0^{\pi/2} (1+a^2)^{-1/2} - 1 \, d\theta$$

$$= - \int_0^{\pi/2} \frac{1}{\sqrt{1+a^2}} - 1 \, d\theta$$

$$= 1 - \frac{1}{\sqrt{1+a^2}} \, \pi/2$$

$$= - \int_0^{\pi/2} \frac{1-1-a^2}{\sqrt{1+a^2}}$$

$$= \int_0^{\pi/2} \frac{a^2}{\sqrt{1+a^2}}\, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{\sqrt{1+a^2}}\, d\theta$$

$$= \left[\theta - \tan^{-1} a\right]_0^{\pi}$$

$$= \pi/2$$

(32)

$$\int_0^1 \int_y^{\sqrt{y}} \sqrt{x^2+y^2}\, dx\, dy$$

$$= \int_0^{\pi/4} \int_0^{\tan\theta\sec\theta} r\, r\, dr\, d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \tan^3\theta \sec^3\theta\, d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \tan^2\theta \sec^2\theta \tan\theta\sec\theta\, d\theta$$

$$= 1/3 \int_0^{\pi/4} (\sec^2\theta - 1)\sec^2\theta \tan\theta\sec\theta\, d\theta$$

$$= 1/3 \int_0^{\pi/4} (p-1)\, p\, d\theta$$

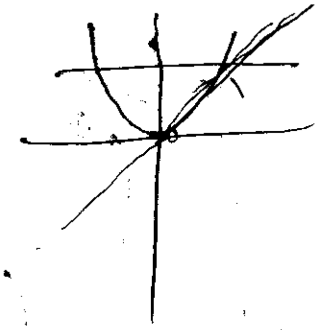
$$= \frac{1}{3}\int_0^{\pi/4} p^2 - p\, d\theta$$

$$= \frac{1}{3}\left[\frac{\sec^6\theta}{3} - \frac{\sec^4\theta}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{3}\cancel{\left(-\frac{\sqrt{}}{66} + 1/6\right)} \quad 2(\sqrt{2}+1)/45$$

$x = \sqrt{y}$

$x^2 = y$



$x^2 - y = 0$

$r^2\cos^2\theta - r\sin\theta = 0$

$r(r\cos^2\theta - \sin\theta = 0)$

$r = 0$

$r\cos^2\theta = \sin\theta$

$r = \frac{\sin\theta}{\cos^2\theta}$

$r = \tan\theta\sec\theta$

$\sqrt{y} = y$

$y = y^2$

$y^2 - y = 0$

$y(y-1) = 0$

$y = 0$

$y = 1$

$r\sin\theta = 1$

$\sin\theta = \frac{1}{\tan\theta\sec\theta}$

$=$

(33)

(*)

$$\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dx\,dy$$

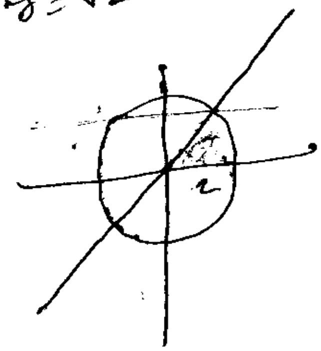
$$= \int_0^{\pi/4}\int_0^{2} \frac{r}{\sqrt{1+r^2}}\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/} p^{-1/2}$$

$x^2 = 4 - y^2$

$x^2 + y^2 = 4$

$y = \sqrt{2}$



$r^2 = 4$

$r = 2$

$1 + r^2 = p$

$2r = \frac{dp}{dr}$

(3) $\displaystyle\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x \, dy \, dx$

$\displaystyle = 2\int_{\pi/2}^{\pi}\int_{0}^{4} 3r^2\cos\theta \, dr \, d\theta$

$\displaystyle = \frac{\cancel{6}}{2}\int_{\pi/2}^{\pi}\cos\theta \int 64 \, d\theta$

$\displaystyle = \frac{2\cancel{6}\times 64}{\cancel{3}}\Big[\sin\theta\Big]_{\pi/2}^{\pi}$

$= -128$

Ans

$x^2+y^2=4^2$

$r^2=4^2$

$r=4$