HomeWork * Sheet #2

1/

(i) $S = \{(x, 2y, 5) : x, y \in R\}$

$\Rightarrow$ $(0,0,0) \in R^3$, $(0,0,5) \notin S$

$\therefore$ S is not a sub spacce of R^3

(ii) $S = \{(x, x+y, 3z) : x, y, z \in \mathbb{R}\}$

$\Rightarrow$ $(0,0,0) \in R^3$ $\therefore$ $(0,0,0) \in S$

$\therefore$ S is not empty

Let $\underline{u} = (x, x+y, 3z)$

$\underline{v} = (x', x'+y', 3z')$

and α, β are scalars

$$\alpha \underline{u} + \beta \underline{v} = \alpha (x, x+y, 3z) + \beta (x', x'+y', 3z')$$

$$= (\alpha x + \beta x', \alpha x + \beta x' + \alpha y + \beta y', 3\alpha z + 3\beta z')$$

$$= (\alpha x + \beta x', (\alpha x + \beta x') + (\alpha y + \beta y'), 3\alpha z + 3\beta z')$$

$\in S$

$\therefore S$ is a subspace of $\cancel{V}\ \mathbb{R}^3$

(iii) $S = \{(x, y, z) \in \mathbb{R}^3 : x - y + z = 0\}$

$(0, 0, 0) \in \mathbb{R}^3 \quad \therefore (0, 0, 0) \in \cancel{\mathbb{R}}\ S$

$\therefore S$ is not empty

Let

$\underline{u} = (x, y, z)$ and $x - y + z = 0$

$\underline{v} = (x', y', z')$ and $x' - y' + z' = 0$

and α and β are scalars

$\alpha \underline{u} + \beta \underline{v} = \alpha(x, y, z) + \beta(x', y', z').$

$= (\alpha x + \beta x', \ \alpha y + \beta y', \ \alpha z + \beta z')$

$\alpha x + \beta x' - \alpha y - \beta y' + \alpha z + \beta z' = 0$

$\alpha(x - y + z) + \beta(x' - y' + z') = 0$

$\alpha \cdot 0 + \beta \cdot 0 = 0$

$0 = 0 \quad \therefore S$ is a subspace of $\cancel{V}\ \mathbb{R}^3$

(12)

$S = \{(x, y, z, t) \in R^4 : 3x - 2y - 2z - t = 0\}$

$(0,0,0,0) \in R^4 \quad \therefore (0,0,0,0) \in S$

$\therefore S$ is not empty

Let

$\underline{u} = (x, y, z, t)$ and $3x - 2y - 2z - t = 0$

and $\underline{v} = (x', y', z', t)$ and $3x' - 2y' - 2z' - t' = 0$

and α and β are scalars

$\therefore \alpha\underline{u} + \beta\underline{v} = \alpha(x, y, z, t) + \beta(x', y', z', t)$

$= (\alpha x + \beta x', \alpha y + \beta y', \alpha z + \beta z', \alpha t + \beta t')$

$\therefore 3\alpha x + 3\beta x' - 2\alpha y - 2\beta y' - 2\alpha z - \beta z'$
$- 2\alpha t - 2\beta t = 0$

$\alpha(3x - 2y - 2z - t) + \beta(3x' - 2y' - 2z' - t')$
$= 0$

$\alpha . 0 + \beta . 0 = 0$

$0 = 0 \quad \therefore S$ is a subspace of ~~of~~ R^4

(v) $S = \{ (x, y, z) \in \mathbb{R}^3 : x + y + z = 0 \}$

$(0, 0, 0) \in \mathbb{R}^3$ $\because$ $(0, 0, 0) \in S$

$\therefore S$ is not empty

let $\underline{u} = (x, y, z)$ and $x + y + z = 0$

$\underline{v} = (x', y', z')$ and $x' + y' + z' = 0$

and α and β are scalars.

$\alpha \underline{u} + \beta \underline{v} = \alpha (x, y, z) + \beta (x', y', z')$

$= ((\alpha x + \beta x'), (\alpha y + \beta y'), (\alpha z + \beta$

$\alpha x + \beta x' + \alpha y + \beta y' + \alpha z + \beta z' = 0$

$\alpha (x + y + z) + \beta (x' + y' + z') = 0$

$\alpha \cdot 0 + \beta \cdot 0 = 0$

$0 = 0$

$\therefore S$ is a subspace of $\mathbb{R}^3$

2/

$\underline{w} = (1,0,0) \quad \underline{x} = (0,0,1)$

$(1,0,0) = k_1(1,0,-1) + k_2(0,1,0) + k_3(1,0,1)$

$= (k_1 + 0k_2 + k_3,\ 0k_1 + k_2 + 0k_3,\ -k_1 + 0k_2 + k_3)$

$$\left.\begin{aligned} k_1 + 0k_2 + k_3 &= 1 \\ 0k_1 + k_2 + 0k_3 &= 0 \\ -k_1 + 0k_2 + k_3 &= 0 \end{aligned}\right\} \quad \left.\begin{aligned} k_1 + k_3 &= 1 \quad \text{—(i)} \\ k_2 &= 0 \quad \text{—(ii)} \\ -k_1 + k_3 &= 0 \quad \text{(iii)} \end{aligned}\right\}$$

(i) + (iii)

$2k_3 = 1$

$\Rightarrow k_3 = 1/2$

$k_1 = k_3 = 1/2$

$(1,0,0) = \frac{1}{2}(1,0,-1) + 0 \cdot (0,1,0) + \frac{1}{2}(1,0,1)$

$(0,0,1) = k_1(1,0,-1) + k_2(0,1,0) + k_3(1,0,1)$

$= (k_1 + k_3), (k_2), (-k_1 + k_3)$

$k_1 + k_3 = 0$ —(i)

$k_2 = 0$

$-k_1 + k_3 = 1$ —(ii)

(i) + (ii)

$2k_3 = 1$

$k_3 = 1/2$

$k_1 = -1/2$

$k_2 = 0$

$(0,0,1) = -\frac{1}{2}(1,0,-1) + 0(0,1,0) + \frac{1}{2}(1,0,1)$

3/ (i) $\underline{w} = (1,2,6)$

$\underline{v_1} = (2,1,0)$

$\underline{v_2} = (1,-1,2)$

$\underline{v_3} = (0,3,-4)$

$\underline{w} = k_1 v_1 + k_2 v_2 + k_3 v_3$

$\therefore (1,2,6) = k_1(2,1,0) + k_2(1,-1,2) + k_3(0,3,-4)$

$= (2k_1 + k_2,\ k_1 - k_2 + 3k_3,\ 2k_2 - 4k_3)$

$\therefore 2k_1 + k_2 = \cancel{0}\ 1$

$k_1 - k_2 + 3k_3 = \cancel{0}\ 2$

$2k_2 - 4k_3 = \cancel{0}\ 6$

$$\begin{bmatrix} 2 & 1 & 0 & \cancel{0}1 \\ 1 & -1 & 3 & \cancel{0}2 \\ 0 & 2 & -4 & \cancel{0}6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & -4 & 6 \end{bmatrix} \xleftarrow[R_2' = R_1]{R_1' = R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 3 & -6 & -3 \\ 0 & 2 & -4 & 6 \end{bmatrix} \xleftarrow{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & 3 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{2} R_3]{R_2' = \frac{1}{3} R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

Since there is no solution the vector $(1,2,6)$ is not linear combination of $(2,1,0)$, $(1,-1,2)$ and $(0,3,-4)$

(11) $\underline{w} = (1,1,1)$

$\underline{v_1} = (2,1,0)$

$\underline{v_2} = (1,-1,2)$

$\underline{v_3} = (0,3,-4)$

$\underline{w} = k_1 v_1 + k_2 v_2 + k_3 v_3$

$(1,1,1) = k_1(2,1,0) + k_2(1,-1,2) + k_3(0,3,-4)$

$(1,1,1) = (2k_1 + k_2), (k_1 - k_2 + 3k_3), (2k_2 - 4k_3)$

$\therefore 2k_1 + k_2 = 1$

$k_1 - k_2 + 3k_3 = 1$

$2k_2 - 4k_3 = 1$

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & -1 & 3 & 1 \\ 0 & 2 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & -4 & 1 \end{bmatrix} \xleftarrow[R_2' = R_1]{R_1' = R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 3 & -6 & -1 \\ 0 & 2 & -4 & 1 \end{bmatrix} \xleftarrow{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 1 & -2 & -1/3 \\ 0 & 2 & -4 & 1 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{3}R_2} \qquad 2/3 + 1 \quad \frac{2+3}{3}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 1 & -2 & -1/3 \\ 0 & 0 & 0 & 5/3 \end{bmatrix}$$

no solution

since there is no solution (1,1,1) can't be linear combination of (2,1,0) (1,-1,2)

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 7 & 5 & 11 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

$R_2' = \frac{1}{7} R_2$

~~$R_3' = -\frac{1}{7} R_4$~~

$R_4' = R_3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

$R_3' = -7R_2 + R_3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & \cancel{+16}\ -2 & \cancel{94}\ 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & \cancel{+6}\ -1 & \cancel{94}\ 2 \end{array}\right]$$

$R_3' = \frac{1}{2} R_4$

$R_4 = R_3$

$= -2\times3+2\times2+3$

$= -6+4+3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

linear combination

$(3,9,-4,-2) = 1(1,-2,0,3)$
$+3(2,3,0,-1)$
$-2(2,-1,2,1)$

$k_3 = -2 \qquad k_2 + \frac{3}{7}k_3 = \frac{15}{7} \qquad k_1 = 1$

$k_2 + \frac{3}{7}(-2) = \frac{15}{7}$

$k_2 = 3$

(iv)

$\underline{w} = (2, 3, -7, 3)$

$v_1 = (2, 1, 0, 3)$

$v_2 = (3, -1, 5, 2)$

$v_3 = (-1, 0, 2, 1)$

$\underline{w} = k_1 \underline{v_1} + k_2 \underline{v_2} + k_3 \underline{v_3}$

$(2, 3, -7, 3) = k_1 (2, 1, 0, 3] + k_2 (3, -1, 5, 2)$
$+ k_3 (-1, 0, 2, 1)$

$(2, 3, -7, 3) = (2k_1 + 3k_2 - k_3), (k_1 - k_2),$
$(5k_2 + 2k_3), (3k_1 + 2k_2 + k_3)$

$2k_1 + 3k - k_3 = \cancel{0}\ 2$

$k_1 - k_2 = \cancel{0}\ 3$

$5k_2 + 2k_3 = \cancel{0}\ -7$

$3k_1 + 2k_2 + k_3 = \cancel{0}\ 3$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow[R_2' = R_1]{R_1' = R_2} \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 3 & -1 & 2 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 5 & -1 & -4 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix} \xleftarrow[R_4' = -3R_1 + R_4]{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{5} R_2}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 2 & -2 \end{bmatrix} \xleftarrow[R_4' = -5R_2 + R_4]{R_3' = -5R_2 + R_3}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xleftarrow[R_4' = \frac{1}{2} R_2]{R_3' = \frac{1}{3} R_3}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_4' = R_3 + R_4}$$

$K_3 = -1$

$K_2 = -1/5 - 4/5 = \frac{-1-4}{5} = -1$

$K_3 = 2$

linear combination

$(2,3,-7,3) = -1(2,1,0,3)$
$-1(3,-1,5,2)$
$+2(-1,0,2,1$

4

① $S = \{(1,1,2)\,(1,-1,2)\,(1,0,1)\}$

Let

$\underline{b} = (b_1, b_2, b_3) \in \mathbb{R}^3$

$k_1 v_1 + k_2 v_2 + k_3 v_3 = \underline{b}$

$k_1(1,1,2) + k_2(1,-1,2) + k_3(1,0,1) = (b_1, b_2, b_3)$

Or $(k_1 + k_2 + k_3), (k_1 - k_2), (k_1 \times 2 + 2k_2 + k_3) = (b_1, b_2, b_3)$

$\therefore\ k_1 + k_2 + k_3 = b_1$

$k_1 - k_2 = b_2$

$2k_1 + 2k_2 + k_3 = b_3$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$\det(A) = 1(-1-0) - 1(1-0) + 1(2+2)$

$= -1 - 1 + 4$

$= -2 + 4 = 2$

$\therefore \det(A) \neq 0$

$\therefore A$ is invertible

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = A^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

~~So Span is $\mathbb{R}^3$~~

So S spans $\mathbb{R}^3$

(ii) $\{(-1,1,0),(-1,0,1)(1,1,1)\}$

Let,

$\underline{b} = (b_1, b_2, b_3)$

$k_1 \underline{v_1} + k_2 \underline{v_2} + k_3 \underline{v_3} = \underline{b}$

$k_1(-1,1,0) + k_2(-1,0,1) + k_3(1,1,1) = \underline{b}$

$(-k_1 - k_2 + k_3), (k_1 + k_3), (k_2 + k_3) = (b_1, b_2, b_3)$

$-k_1 - k_2 + k_3 = b_1$

$k_1 + k_3 = b_2$

$k_2 + k_3 = b_3$

$$A = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(A) = -1(0-1) + 1(1-0) + 1(1-0)$$

$$= 1 + 1 + 1$$

$$= 3$$

$\therefore \det(A) \neq 0$

$\therefore$ A is invertible

$\therefore$ so span is $\mathbb{R}^3$.

(iii) $S = \{(2,1,2), (0,1,-1), (4,3,3)\}$

Let

$\underline{b} = (b_1, b_2, b_3)$

$k_1 v_1 + k_2 v_2 + k_3 v_3 = \underline{b}$

$k_1(2,1,2) + k_2(0,1,-1) + k_3(4,3,3) = (b_1, b_2, b_3)$

or $(2k_1 + 4k_3), (k_1 + k_2 + 3k_3), (2k_1 - k_2 + 3k_3) = (b_1, b_2, b_3)$

$2k_1 + 4k_3 = b_1$

$k_1 + k_2 + 3k_3 = b_2$

$2k_1 - k_2 + 3k_3 = b_3$

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & 3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\det(A) = 2(3+3) - 0 + 4(-1-2)$$

$$= 6 \cancel{+} - 12$$

$$= \cancel{6}\ 0$$

$\det(A) = 0$

$\therefore$ so S ~~span~~ does ~~is~~ not span $\mathbb{R}^3$

5/

① $\{(2,1,2), (0,1,-1), (4,3,3)\}$

$\underline{0} = (0,0,0,\cancel{0}) \in \mathbb{R}^3$

$k_1v_1 + k_2v_2 + k_3v_3 = (0,0,0)$

$k_1(2,1,2) + k_2(0,1,-1) + k_3(4,3,3) = (0,0,0)$

$(2k_1 + 4k_3), (k_1 + k_2 + 3k_3), (2k_1 - k_2 + 3k_3) = (0,0,0)$

$$A = \begin{bmatrix} 2 & 0 & 4 & 0 \\ 1 & 1 & 3 & 0 \\ 2 & -1 & 3 & 0 \end{bmatrix}$$

$$\det(A) = 2(3+3) + 4(-1-2)$$

$$= 12 - 12$$

$$= 0$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 0 & 4 & 0 \\ 2 & -1 & 3 & 0 \end{bmatrix} \xleftarrow{R_1 \leftrightarrows R_2}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -3 & -3 & 0 \end{bmatrix} \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \xleftarrow[R_3' = -\frac{1}{3} R_3]{R_2' = \frac{1}{-2} R_2}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$= \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_1' = -R_2 + R_1}$$

$3-2=1$

$x_3 = k \quad x_2 = -k \quad x_1 = -2k$

so we have non trivial solution

the the set of vectors are

linearly dependant.

(11) $\{ (3,0,1,-1), (2,-1,0,1) (1,1,1,-2) \}$

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$k_1 v_1 + k_2 v_2 + k_3 v_3 + k_4 v_4 = (0,0,0,0)$

$k_1(3,0,1,-1) + k_2(2,-1,0,1) + k_3(1,1,1,-2) = (0,0,0,0)$

$(3k_1 + 2k_2 + k_3), (-k_2 + k_3), (k_1 + k_3), (-k_1 + k_2 - 2k_3) = (0,0,0,0)$

$3k_1 + 2k_2 + k_3 = 0$

$-k_2 + k_3 = 0$

$k_1 + k_3 = 0$

$-k_1 + k_2 - 2k_3 = 0$

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ -1 & 1 & -2 & 0 \end{bmatrix} \xleftarrow{R_1 \rightleftarrows R_3}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \xleftarrow[R_4' = R_1 + R_4]{R_3' = -3R_1 + R_3}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{2} R_3]{R_2' = -1 R_2}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_2 + R_4]{R_3' = -R_2 + R_3}$$

$\therefore$ this is a non trivial solution

$x_3 = k$

$x_2 = k$

$x_1 = -k$

So the set of the vectors are

linearly dependant

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{2} R_3]{R_2' = -1 R_2}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_2 + R_4]{R_3' = -R_2 + R_3}$$

$\therefore$ this is a non trivial solution

$x_3 = k$

$x_2 = k$

$x_1 = -k$

So the set of the vectors are linearly dependant

(111)

$\{(1,-4,2),(3,-5,1),(2,7,8),(-1,1,1)\}$

$\underline{0} = (0,0,0) \in R^3$

$\underline{0} = k_1\underline{v_1} + k_2\underline{v_2} + k_3\underline{v_3} + k_4\underline{v_4}$

$(0,0,0) = k_1(1,-4,2) + k_2(3,-5,1) + k_3(2,7,8), + k_4(-1,1$

$(0,0,0) = (k_1 + 3k_2 + 2k_3 - k_4), (-4k_1 - 5k_2 + 7k_3 + k_4$

$(2k_1 + k_2 + 8k_3 + k_4)$

$\therefore k_1 + 3k_2 + 2k_3 - k_4 = 0$

$-4k_1 - 5k_2 + 7k_3 + k_4 = 0$

$2k_1 + k_2 + 8k_3 + k_4 = 0$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 0 \\ -4 & -5 & 7 & 1 & 0 \\ 2 & 1 & 8 & 1 & 0 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 7 & 15 & -3 & 0 \\ 0 & -5 & 4 & 3 & 0 \end{bmatrix} \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = 4R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & -5 & 4 & 3 & 0 \end{bmatrix} \longleftarrow R_2' = \frac{1}{7} R_2$$

$$\frac{5 \times 15}{7} + 4$$

$$\frac{5 \times 15 + 28}{7}$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & 0 & 103/7 & 6/7 & 0 \end{bmatrix} \longleftarrow R_3' = -5R_2 + R_3$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & 0 & 103 & 6 & 0 \end{bmatrix} \longleftarrow R_3' = 7R_3$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & 0 & 1 & 6/103 & 0 \end{bmatrix} \longleftarrow R_3' = \frac{1}{103} R_3$$

$$K_1 + 3K_2 + 2K_3 - K_4 = 0$$

$$\Rightarrow K_1 = -3K_2 - 2K_3 + K_4.$$

$$K_2 = -\frac{15}{7} K_3 + \frac{3}{7} K_4$$

$$K_3 = -\frac{6}{103} K_4$$

[non trivial]

So dependent

(iv)

$\{(0,1,0,1), (1,2,3,-1), (8,4,3,2), (0,3,2,0)\}$.

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$(0,0,0,0) = k_1(0,1,0,1) + k_2(1,2,3,-1) + k_3(8,4,3,2)$
$+ k_4(0,3,2,0)\}$

$(0,0,0,0) = (k_2 + 8k_3), (k_1 + 2k_2 + 4k_3 + 3k_4),$
$(3k_2 + 3k_3 + 2k_4)(k_1 - k_2 + 2k_3)$

$$\begin{bmatrix} 0 & 1 & 8 & 0 & 0 \\ 1 & 2 & 4 & 3 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 1 & -1 & 2 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 1 & -1 & 2 & 0 & 0 \end{bmatrix} \xleftarrow{R_1 \rightleftharpoons R_2}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 0 & -3 & -2 & -3 & 0 \end{bmatrix} \xleftarrow{R_4' = -R_1 + R_4}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & -21 & 2 & 0 \\ 0 & 0 & 22 & -3 & 0 \end{bmatrix} \xleftarrow[R_4' = 3R_2 + R_4]{R_3' = -3R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 2/-21 & 0 \\ 0 & 0 & 22 & -3 & 0 \end{bmatrix} \leftarrow R_3' = \frac{1}{-21} R_3$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 2/-21 & 0 \\ 0 & 0 & 0 & -19/21 & 0 \end{bmatrix} \leftarrow R_4' = -22R_3 + R_4$$

$\therefore \frac{-19}{21} K_4 = 0$

$K_3 + \frac{2}{-21} K_4 = 0$

$K_3 = 0$ $\quad\quad K_2 + 8K_3 = 0$

$K_2 = 0$

$K_1 = 0$

as trivial solution

so the set of the vectors

are linearly independent

(v)

$\{(1,3,2),(1,-7,-8),(2,1,-1)\}$.

$\underline{0} = (0,0,0) \in R^3$

$\underline{0} = k_1(1,3,2) + k_2(1,-7,-8) + k_3(2,1,-1)$

$(0,0,0) = (k_1+k_2+2k_3),(3k_1-7k_2+k_3),(2k_1-8$

$-k$

$\therefore k_1+k_2+2k_3=0$
$3k_1-7k_2+k_3=0$
$2k_1-8k_2-k_3=0$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & -7 & 1 & 0 \\ 2 & -8 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & -10 & -5 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = -3R_1+R_2 \\ R_3' = -2R_1+R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -10/-10 & -5/10 & 0 \\ 0 & -10 & -5 & 0 \end{bmatrix} \longleftarrow R_2' = -\frac{1}{10}R_2$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & -10 & -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 2 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = 10R_2 + R_3}$$

$\therefore$ Non trivial

$k_3 = \alpha$

$k_2 + k_3/2 = 0$

$k_2 = -\alpha/2$

$k_1 = \alpha/2 - 2\alpha$

$= \alpha(-\frac{1}{2} - 2)$

$= \alpha(-\frac{3/2}{})$

$= -3/2\,\alpha$

So the ~~se~~ all these vectors are ~~not~~ linearly dependant

(vi)

$\{(3,0,4,1), (6,2,-1,2), (-1,3,5,1), (-3,7,8,3)\}$

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$(0,0,0,0) = k_1(3,0,4,1) + k_2(6,2,-1,2) + k_3(-1,3,5,1) + k_4(-3,7,8,3)$

$(0,0,0,0) = (3k_1 + 6k_2 - k_3 - 3k_4), (2k_2 + 3k_3 + 7k_4), (4k_1 - k_2 + 5k_3 + 8k_4), (k_1 + 2k_2 + k_3 + 3k_4)$

$$3k_1 + 6k_2 - k_3 - 3k_4 = 0$$

$$2k_2 + 3k_3 + 7k_4 = 0$$

$$4k_1 - k_2 + 5k_3 + 8k_4 = 0$$

$$k_1 + 2k_2 + k_3 + 3k_4 = 0$$

$$\begin{bmatrix} 3 & 6 & -1 & -3 & 0 \\ 0 & 2 & 3 & 7 & 0 \\ 4 & -1 & 5 & 8 & 0 \\ 1 & 2 & 1 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 3 & 7 & 0 \\ 4 & -1 & 5 & 8 & 0 \\ 3 & 6 & -1 & -3 & 0 \end{bmatrix} \longleftarrow R_1 \rightleftharpoons R_4$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 3 & 7 & 0 \\ 0 & -9 & 1 & -4 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \xleftarrow[R_4' = -3R_1 + R_4]{R_3' = -4R_1 + R_3}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & -9 & 1 & -4 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{2}R_2}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 29/2 & 55/2 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \xleftarrow{R_3' = 9R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 1 & 55/29 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \xleftarrow{R_3' = R_3 / 29/2}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 1 & 55/29 & 0 \\ 0 & 0 & 0 & -128/29 & 0 \end{bmatrix} \xleftarrow{R_4' = 4R_3 + R_4}$$

$$\frac{220}{29} - 12$$

$$\frac{220 - 348}{29}$$

$$\frac{128}{29}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 1 & 55/29 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow{R_4' = -\frac{29}{128} R_4}$$

$\therefore k_4 = 0$

$k_3 = 0$ $\quad\therefore$ Trivial solution

$k_2 = 0$ $\quad\therefore$ so all the vectors

$k_1 = 0$ $\quad$ are linearly independent

(vii) $\{(4,-4,8,0),(2,2,4,0),(6,0,0,2),(6,3,-3,0)\}$

$\underline{0}=(0,0,0,0)=k_1(4,-4,8,0)+k_2(2,2,4,0)+k_3(6,0,0,2)+k_4(6,3,-3$

$(0,0,0,0)=(4k_1+2k_2+6k_3+6k_4),(-4k_1+2k_2+3k_4)$

$(8k_1+4k_2-3k_4),(2k_3)$

$\therefore\ 4k_1+2k_2+6k_3+6k_4=0$

$-4k_1+2k_2+3k_4=0$

$8k_1+4k_2-3k_4=0$

$2k_3=0$

$$\begin{bmatrix} 4 & 2 & 6 & 6 & 0 \\ -4 & 2 & 0 & 3 & 0 \\ 8 & 4 & 0 & -3 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ -4 & 2 & 0 & 3 & 0 \\ 8 & 4 & 0 & -3 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \xleftarrow{R_1' = \frac{1}{4}R_1}$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 4 & 6 & 9 & 0 \\ 0 & 0 & -6\times2 & -15 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = 4R_1 + R_2 \\ R_3' = -8R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 1 & 6/4 & 9/4 & 0 \\ 0 & 0 & -12 & -15 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \longleftarrow$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 1 & 6/4 & 9/4 & 0 \\ 0 & 0 & 1 & 15/12 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = -\frac{1}{12} R_3$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 1 & 6/4 & 9/4 & 0 \\ 0 & 0 & 1 & 15/12 & 0 \\ 0 & 0 & 0 & -15/6 & 0 \end{bmatrix}$$

$k_4 = 0$

$k_3 = 0$ Trivial solution

$k_2 = 0$

$k_1 = 0$

So the elements are linearly independent.

6

① $S = \{ (1,2,0), (0,5,7) \ \& \ (-1,1,3) \}$

$\underline{0} = (0,0,0) \in \mathbb{R}^3$

$(0,0,0) = k_1(1,2,0) + k_2(0,5,7) + k_3(-1,1,3)$

$(0,0,0) = (k_1 - k_3), (2k_1 + 5k_2 + k_3), (7k_2 + 3k_3)$

$k_1 - k_3 = 0$

$2k_1 + 5k_2 + k_3 = 0$

$7k_2 + 3k_3 = 0$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 5 & 1 \\ 0 & 7 & 3 \end{bmatrix}$$

$\det(A) = 1(15-7) - 1(14-5)$

$= 8 - 9$

$= -1$

$\neq 0$

$\therefore$ A is independent and A is invertible

again,

Let $\underline{b} = (b_1, b_2, b_3) \in \mathbb{R}^3$

$\therefore (b_1, b_2, b_3) = k_1(1,2,0) + k_2(0,5,7) + k_3(-1,1,3)$

$= (k_1 - k_3)\ (2k_1 + 5k_2 + k_3)\ (7k_2 + 3k_3$

$\therefore$

$k_1 - k_3 = b_1$

$2k_1 + 5k_2 + k_3 = b_2$

$7k_2 + 3k_3 = b_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 5 & 1 \\ 0 & 7 & 3 \end{bmatrix}$$

$\det(A) = -1$

$\det(A) \neq -1$

$\therefore$ $\therefore$ S pans $\mathbb{R}^3$

$\therefore$ The set S. ~~is~~ forms a basis for $\mathbb{R}^3$

(11) $S = \{(2,0,1), (1,1,1)\}$

$\underline{0} = (0,0,0)$

$\therefore (0,0,0) = k_1(2,0,1) + k_2(1,1,1)$

$(0,0,0) = (2k_1 + k_2), (k_2), (k_1 + k_2)$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

Not getting any solution

(iii)

$S = \{(1,1,1,1), (0,1,1,1), (0,0,1,1), (0,0,0,1)\}$

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$(0,0,0,0) = k_1(1,1,1,1) + k_2(0,1,1,1) + k_3(0,0,1,1)$
$+ k_4(0,0,0,1)$

$(0,0,0,0) = (k_1), (k_1+k_2), (k_1+k_2+k_3), (k_1+k_2+k_3+k_4)$

$k_1 = 0$

$k_1 + k_2 = 0$

$k_1 + k_2 + k_3 = 0$

$k_1 + k_2 + k_3 + k_4 = 0$

$$A = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$\det(A) = 1 \cdot 1 \cdot 1 \cdot 1$
$= 1$

$\therefore$ A is ~~not invertible~~ invertible

so S is independent

$\underline{b} = (b_1, b_2, b_3, b_4) \in \mathbb{R}^4$

$(b_1, b_2, b_3, b_4) = k_1(1,1,1,1) + k_2(0,1,1,1) + k_3(0,0,1,1)$
$+ k_4(0,0,0,1)$

~~Kop~~

$(b_1, b_2, b_3, b_4) = (k_1), (k_1+k_2), (k_1+k_2+k_3), (k_1+k_2+k_3$
$+k_4)$

$k_1 = b_1$

$k_1 + k_2 = b_2$

$k_1 + k_2 + k_3 = b_3$

$k_1 + k_2 + k_3 + k_4 = b_4$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\det(A) = 1 \cdot 1 \cdot 1 \cdot 1$
$= 1$

$\det(A) \neq 0$

$\therefore$ A ~~is~~ span, ~~of~~ $\mathbb{R}^4$

$\therefore$ The set S form a basis for $\mathbb{R}^4$.

I/

1

$$A = \begin{bmatrix} 6 & 2 & 0 & 4 \\ -2 & -1 & 3 & 4 \\ -1 & -1 & 6 & 10 \end{bmatrix}$$

−1213
−2024
−16

$$= \begin{bmatrix} 1 & 1 & -6 & -10 \\ -2 & -1 & 3 & 4 \\ 6 & 2 & 0 & 4 \end{bmatrix} \longleftarrow (\ )R_1 \rightleftarrows R_3(-1)$$

$$= \begin{bmatrix} 1 & 1 & -6 & -10 \\ 0 & 1 & -9 & -16 \\ 0 & -4 & 36 & 64 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = 2R_1 + R_2 \\ R_3' = -6R_1 + R_3 \end{array}$$

$$= \begin{bmatrix} \boxed{1 \quad 1 \quad -6 \quad -10} \\ 0 \quad \boxed{1 \quad -9 \quad -16} \\ 0 \quad 0 \quad 0 \quad 0 \end{bmatrix} \longleftarrow R_3' = 4R_2 + R_3$$

$\boxed{x_3 = \text{free}}$ $\boxed{x_2 = -16 + 9x_3}$

$x_1 = -x_2 + 6x_3 - 10$

$= 16 - 9x_3 + 6x_3 - 10$

$\boxed{x_1 = 6 - 3x_3}$

Basis for Row space

$R_1 = \{[1, 1, -4, -10]\}$

$R_2 = \{[0, 1, -7, -16]\}$

Basis for (A) column space

$c_1 = \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

Basis for R column space

$c_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad c_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

~~Basis for null space~~

Rank = 2

(11)

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 1 & 5 \\ 0 & 7 & 2 & 3 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = -3R_1 + R_2 \\ R_3' = 2R_1 + R_3 \end{array}$$

$$= \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & \cancel{-2}\,1 & 1/-2 & 5/-2 \\ 0 & 7 & 2 & 3 \end{bmatrix} \longleftarrow R_2' = \frac{1}{-2} R_2$$

$$= \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & \frac{1}{-2} & 5/-2 \\ 0 & 0 & 11/2 & 41/2 \end{bmatrix} \longleftarrow R_3' = -7R_2 + R_3$$

$$= \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & -1/2 & -5/2 \\ 0 & 0 & 1 & 41/11 \end{bmatrix} \longleftarrow R_3' = \frac{1}{11/2} R_3$$

Basis for row space

$r_1 = [1\ 2\ 0\ -1]$

$r_2 = [0\ 1\ -1/2\ -5/2]$

$r_3 = [0\ \ 0\ \ 1\ \ 41/11]$

Basis for column space of A

$c_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \quad c_2 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} \quad c_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

rank = 3

(iii)

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1/2 & 3/2 & 4/2 \\ 0 & 3 & 4 & 1 \\ 0 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix} \leftarrow R_1' = \frac{1}{2} R_1$$

$$= \begin{bmatrix} 1 & -1/2 & 3/2 & 4/2 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 6 & 8 & 2 \end{bmatrix} \leftarrow \begin{array}{l} R_3' = -2R_1 + R_3 \\ R_4' = -2R_1 + R_4 \end{array}$$

$$= \begin{bmatrix} 1 & -1/2 & 3/2 & 4/2 \\ 0 & 1 & 4/3 & 1/3 \\ 0 & 4 & 4 & 1 \\ 0 & 6 & 8 & 2 \end{bmatrix} \leftarrow R_2' = \frac{1}{3} R_2$$

$$= \begin{bmatrix} 1 & -1/2 & 3/2 & 4/2 \\ 0 & 1 & 4/3 & 1/3 \\ 0 & 0 & -4/3 & -1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow \begin{array}{l} R_3' = -4R_2 + R_3 \\ R_4' = -6R_2 + R_4 \end{array}$$

$$= \begin{bmatrix} 1 & -1/2 & 3/2 & 9/2 \\ 0 & 1 & 4/3 & 1/3 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for row space of A

$r_1 = [1 \ \ -1/2 \ \ 3/2 \ \ 9/2]$

$r_2 = [0 \ \ 1 \ \ 4/3 \ \ 1/3]$

$r_3 = [0 \ \ 0 \ \ 1 \ \ 1/4]$

Basis for column space of A

$c_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 2 \end{bmatrix}$ $c_2 = \begin{bmatrix} -1 \\ 3 \\ 3 \\ 5 \end{bmatrix},$ $c_3 = \begin{bmatrix} 3 \\ 9 \\ 7 \\ 4 \end{bmatrix}$

rank = 3

$$= \begin{bmatrix} 1 & -1/2 & 3/2 & 9/2 \\ 0 & 1 & 4/3 & 1/3 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for row space of A

$r_1 = [1 \ \ -1/2 \ \ 3/2 \ \ 9/2]$

$r_2 = [0 \ \ 1 \ \ 4/3 \ \ 1/3]$

$r_3 = [0 \ \ 0 \ \ 1 \ \ 1/4]$

Basis for column space of A

$$c_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 2 \end{bmatrix} \quad c_2 = \begin{bmatrix} -1 \\ 3 \\ 3 \\ 5 \end{bmatrix} \quad c_3 = \begin{bmatrix} 3 \\ 4 \\ 7 \\ 11 \end{bmatrix}$$

rank = 3

(v)

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix} \qquad R_1 \rightleftarrows R_4$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & -1 & 3 & 1 \\ 0 & -2 & 6 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix} \qquad \begin{aligned} R_2' &= -1R_1+R_2 \\ R_3' &= -3R_1+R_3 \end{aligned}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & -2 & 6 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix} \qquad R_2' = -1R_2$$

$$R_3' = 2R_1+R_3$$

$$R_4' = -R_2+R_3$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for row space

$r_1 = [1 \quad 1 \quad -2 \quad 0]$

$r_2 = [0 \quad 1 \quad -3 \quad -1]$

Basis for column space

$$c_1 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix} \qquad c_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Rank = 2

(v) $A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & -3 & -6 & -3 & 3 \\ 0 & -1 & -2 & -1 & 1 \end{bmatrix} \quad \xleftarrow[R_4' = -3R_1 + R_4]{R_2' = -R_1 + R_2,\ R_3' = -2R_1 + R_3}$$

$$= \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \xleftarrow[R_4' = R_2 + R_4]{R_3' = 3R_2 + R_3}$$

Basis for row space

$r_1 = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \end{bmatrix}$

$r_2 = \begin{bmatrix} 0 & 1 & 2 & 1 & -1 \end{bmatrix}$

Basis for column space

$c_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad c_2 = \begin{bmatrix} 3 \\ 4 \\ 3 \\ 8 \end{bmatrix}$

Rank $= 2$

8

①

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 0 & -4 & -5 & 3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix} \longleftarrow R_1' = (-1)R_1$$

$$= \begin{bmatrix} 1 & -2 & 0 & -4 & -5 & 3 \\ 0 & -1 & 2 & 12 & 16 & -5 \\ 0 & -1 & 2 & 12 & 16 & -5 \\ 0 & -1 & 2 & 12 & 16 & -5 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = -3R_1 + R_2 \\ R_3' = -2R_1 + R_2 \\ R_4' = -4R_1 + R_4 \end{matrix}$$

$$R_2' = -1R_2$$

$$= \begin{bmatrix} 1 & -2 & 0 & -4 & -5 & 3 \\ 0 & 1 & -2 & -12 & -16 & +5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_3' = R_2 + R_3 \\ R_4' = R_3 + R_4 \end{matrix}$$

$x_1 \cancel{-2x_2 + 4x_4} +$

$A \cdot x = 0$

$\cancel{x = A^{-1}}\ 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 & -4 & -5 & 3 \\ 0 & 1 & -2 & -12 & -16 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 2x_2 - 4x_4 - 5x_5 + 3x_6 = 0$

$x_2 - 2x_3 - 12x_4 - 16x_5 + 5x_6 = 0$

$x_3 = p$

$x_4 = a$

$x_5 = r$

$x_6 = s$

$x_1 = 2x_2 + 4x_4 + 5x_5 - 3x_6$

$= 2(2P + 12a + 16r - 5s) + 4a + 5r - 3s$

$= 4P + 24a + 32r - 10s + 4a + 5r - 3s = 4P + 28a + 37r - 13s$

$x_2 = 2x_3 + 12x_4 + 16x_5 - 5x_6 = 0$

$= 2P + 12a + 16r - 5s$

$\therefore x_1 = 4p + 28q + 37r - 13s$

$x_2 = 2p + 12q + 16r - 5s$

$x_3 = p + 0 + 0 + 0$

$x_4 = 0 + q + 0 + 0$

$x_5 = 0 + 0 + r + 0$

$x_6 = 0 + 0 + 0 + s$

$\therefore$ basis for null space

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = p\begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + q\begin{bmatrix} 28 \\ 12 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} 37 \\ 16 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -13 \\ -5 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

<u>nullity</u> $= 4$

Rank $= 2$

(11)

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -19 \\ 0 & 1 & -19 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = -5R_1 + R_2 \\ R_3' = -7R_1 + R_3 \end{array}$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} A = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - x_2 + 3x_3 = 0$

$x_2 - 19x_3 = 0 \qquad x_2 = 19r$

$x_3 = r$

$\therefore \; x_1 = x_2 - 3r = 19r - 3r = 16r$

basis for null space

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = r \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \qquad \text{Rank} = 2 \qquad \text{nullity} = 1$$

(iii)

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 7 & 7 & 4 \end{bmatrix} \leftarrow \begin{matrix} R_2' = -2R_1 + R_2 \\ R_3' = R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & 4/7 \\ 0 & 7 & 7 & 4 \end{bmatrix} \leftarrow R_2' = \frac{1}{-7} R_2$$

$$= \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow R_3' = -7R_2 + R_3$$

$\therefore$ $A \cdot x = \underline{0}$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 4x_2 + 5x_3 + 2x_4 = 0$

$x_2 + x_3 + 4/7\, x_4 = 0$

$4/7 - 2$

$\frac{4-14}{7} -1$

$x_3 = r$

$x_4 = s$

$x_1 = -4x_2 - 5x_3 - 2x_4 \quad = -4(-r - 4/7 s) - 5r -$

$= +4r + \cancel{16}/7 s - 5r - 2$

$x_2 = -x_3 - 4/7\, x_4 = -r - 4/7 s$

$x_1 = -\cancel{6}r + \cancel{10}^{2}/7 s$

$x_2 = -r - 4/7 s$

$x_3 = r + 0$

$x_4 = 0 + s$

Basis for null space

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} \cancel{0}\,\cancel{10}^{2}/7 \\ -4/7 \\ 0 \\ 1 \end{bmatrix}$$

nullity $= 2$

Rank $= 2$

(iv)

$$A = \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & 6 & 0 & -3 \end{bmatrix} \xleftarrow[R_5' = 2R_1 + R_5]{R_3' = -2R_1 + R_3,\ R_4' = -3R_1 + R_4}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & 6 & 0 & -3 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{3} R_2}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_5' = -3R_2 + R_5]{R_3' = -3R_2 + R_3,\ R_4' = -3R_2 + R_4}$$

(iv)

$$A = \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & 6 & 0 & -3 \end{bmatrix} \xleftarrow[R_5' = 2R_1 + R_5]{\substack{R_3' = -2R_1 + R_3 \\ R_4' = -3R_1 + R_4}}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & 3 & 6 & 0 & -3 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{3} R_2}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_5' = -3R_2 + R_5]{\substack{R_3' = -3R_2 + R_3 \\ R_4' = -3R_2 + R_4}}$$

$$
= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -5/12 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = \frac{1}{-12} R_3
$$

$$
= \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -5/12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -5/12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$x_1 - 3x_2 + 2x_3 + 2x_4 + x_5 = 0$

$x_2 + 2x_3 - x_5 = 0$

$x_3 - 5/12\, x_5 = 0 \quad x_3 = 5/12\, t$

$\cancel{x_3 = r}$

$x_4 = s$

$x_5 = t$

$$
\boxed{\begin{aligned} x_2 &= -2r + t \\ x_1 &= 3(-2r + t) - 2r - 2s - \\ &= -6r + 3t - 2r - 2s - t \\ &= -8r + 2t - 2s \end{aligned}}
$$

$x_1 = -\cancel{4}r + 2s + 2t$

$x_2 = -2r + 0 + t$

$x_2 = -2\ 5/12\,t + t$

$= -5/6\,t + t$

$= \frac{(-5+6)t}{6}$ $\quad 6, 12$

$= 1/6\,t$

$x_1 = 3(1/6\,t) - 2(5/12\,t) - 2s - t$

$= 1/2\,t - 5/6\,t - 2s - t$

$= \frac{6t - 10t - 12t}{12} - 2s$

$= \frac{-16t}{12} - 2s$

$= -\frac{4t}{3} - 2s$

$\therefore x_1 = -\frac{4}{3}t - 2s = -2s - 4/3\,t$

$x_2 = \qquad = 0 + 1/6\,t$

$x_3 = \qquad = 0 + 5/12\,t$

$x_4 = \qquad = s + 0$

$x_5 = \qquad = 0 + t$

Basis for null space

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -4/3 \\ 1/6 \\ 5/12 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ nullity $= 2$

~~and~~ Rank $= 3$

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$$S = \{(1,2,1),(3,1,2),(1,-3,4)\}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & -1 \\ 0 & -5 & 3 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = -3R_1 + R_2 \\ R_3' = -R_1 + R_2 \end{array}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1/5 \\ 0 & -5 & 3 \end{bmatrix} \longleftarrow R_2' = -1/5 R_1$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1/5 \\ 0 & 0 & 4 \end{bmatrix} \longleftarrow \quad R_3' = 5R_2 + R_3$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1/5 \\ 0 & 0 & 1 \end{bmatrix} \quad R_3' = \frac{1}{4} R_3$$

Basis for row space

$r_1 = [1 \quad 2 \quad 1]$

$r_2 = [0 \quad 1 \quad 1/5]$

$r_3 = [0 \quad 0 \quad 1]$

dimention = ~~2~~ 3 $\longleftarrow$ check it (verify it)

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$S = \{(1,2,1), (0,-1,0), (2,0,2)\}$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -4 & 0 \end{bmatrix} \xleftarrow[R_2' = -R_2]{R_3' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = 4R_2 + R_3}$$

Basis for row space [sub space]

$r1 = [1\ 2\ 1]$

$r2 = [0\ 1\ 0]$ dim = 2

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$S = \{ (1, -2, 5, -3)$