BRAC University
Course: MAT-216(Mathematics III)
Practice sheet (Calculus) # 4

1. Evaluated the iterated integrals:

(a) $\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{x^2} \frac{1}{x}\cos\frac{y}{x}\,dy\,dx$ (b) $\int_0^1\int_0^1 \frac{x}{(xy+1)^2}dy\,dx$ (c) $\int_1^2\int_0^{y^2} e^{\frac{x}{y^2}}\,dx\,dy$ (d) $\int_0^2\int_0^{\sqrt{4-x^2}} e^{\sqrt{x^2+y^2}}\,dy\,dx$.

2. (a) Find the area of the region inside the circle r = $4\sin\theta$ and outside the circle r = 2.

(b) $\iint_R \frac{1}{x^2+y^2+1}dA$, where R is the sector in the first quadrant bounded by

y = 0 , y = x and $x^2 + y^2 = 4$.

3. Use polar coordinates to evaluate the double integral $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \left(x^2+y^2\right)^{1/2}dydx$.

4. (a) Find the volume of the solid that is bounded by the cylinder $y = x^2$ and by the planes y + z = 4 and z =0.

(b) Find the volume of the surface enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

5. Evaluated the iterated integral by converting to polar coordinates:

(a) $\int_0^1\int_0^{\sqrt{1-x^2}} \left(x^2+y^2\right)dydx$ (b) $\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\;dydx$

(c) $\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{dydx}{\left(1+x^2+y^2\right)^{\frac{3}{2}}}$ $(a>0)$.

6. (a) Evaluate $\int_{y=0}^{4}\int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \frac{2x-y}{2}\,dxdy$ by applying transformation T :

where $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and integrating over an appropriate region in *uv*–plane.

(b) Evaluate $\iint_R \frac{x-y}{x+y}\,dA$, where R is the region enclosed by the lines

$x - y = 0$, $x - y = 1$, $x + y = 1$ & $x + y = 3$, using the transformation .

Double Integral

Exercise- 14.1- 1-16.
Exercise- 14.2- 1-26.
Exercise- 14.3- 1-12, 23-34.

Surface Area from Double Integral

Exercise- 14.4- 1-9.

Triple Integral

Exercise- 14.5- 1-12, 15-18.

Change of variables

Exercise- 14.7- 1-12, 21-24, 35-37.

Book: Elementary Calculus- Howard Anton (10^{th} Edition), Soft Copy