Practice Sheet

MAT120

Separable Variables

(i) Solve the given differential equation by separation of variables:

$$1.\ \frac{dy}{dx} = \sin 5x, \quad 2.\ dx + e^{3x}dy = 0, \quad 3.\ x\frac{dy}{dx} = 4y, \quad 4.\ \frac{dy}{dx} + 2xy = 0, \quad 5.\ \frac{dy}{dx} = e^{3x+2y},$$

$$6.\ e^x y\frac{dy}{dx} = e^{-y} + e^{-2x-y}, \quad 7.\ y\ln x\frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2,$$

$$8.\ (e^y+1)^2 e^{-y}dx + (e^x+1)^3 e^{-x}dy = 0.$$

(ii) Solve the given initial-value problem:

$$1.\ \frac{dx}{dt} = 4(x^2+1), \quad x(\pi/4) = 1; \quad 2.\ \frac{dy}{dx} = \frac{y^2-1}{x^2-1}, \quad y(2) = 2$$

$$3.\ x^2\frac{dy}{dx} = y - xy, \quad y(-1) = -1; \quad 4.\ \frac{dy}{dt} + 2y = 1, \quad y(0) = \frac{5}{2}.$$

Linear Equations

(i) Find the general solution of the given differential equations:

$$1.\ \frac{dy}{dx} = 5y, \quad 2.\ \frac{dy}{dx} + 2y = 0, \quad 3.\ \frac{dy}{dx} + y = e^{3x}, \quad 4.\ (x^2-1)\frac{dy}{dx} + 2y = (x+1)^2,$$

$$5.\ x\frac{dy}{dx} - y = x^2\sin x, \quad 6.\ x\frac{dy}{dx} + 2y = 3, \quad 7.\ x\frac{dy}{dx} + 4y = x^3 - x,$$

$$8.\ (1+x)\frac{dy}{dx} - xy = x + x^2, \quad 9.\ x^2y' + x(x+2)y = e^x, \quad 10.\ ydx - 4(x+y^6)dy = 0,$$

$$11.\ (x+1)\frac{dy}{dx} + (x+2)y = 2xe^{-x}, \quad 12.\ x\frac{dy}{dx} + (3x+1)y = e^{-3x}, \quad 13.\ 3\frac{dy}{dx} + 12y = 4.$$

(ii) Solve the given initial-value problem:

$$1.\ xy' + y = e^x, \quad y(1) = 2; \quad 2.\ y\frac{dx}{dy} - x = 2y^2, \quad y(1) = 5;$$

$$3.\ (x+1)\frac{dy}{dx} + y = \ln x, \quad y(1) = 10; \quad 4.\ y' + (\tan x)y = \cos^2 x, \quad y(0) = -1.$$

Exact Equations

(i) Determine whether the given differential equation is exact. If it is exact, solve it.

1.$(2x-1)\,dx+(3y+7)\,dy=0,$ 2.$(2x+y)\,dx-(x+6y)\,dy=0,$

3. $2xy\,dx+(x^2-1)\,dy=0,$ 4.$(x^2-y^2)dx+(x^2-2xy)dy=0,$

5. $(e^{2y}-y\cos xy)\,dx+(2xe^{2y}-x\cos xy+2y)\,dy=0,$

6.$(\sin y-y\sin x)\,dx+(\cos x+x\cos y-y)\,dy=0,$

7. $\left(1+\ln x+\dfrac{y}{x}\right)dx=(1-\ln x)\,dy,$ 8. $x\dfrac{dy}{dx}=2xe^x-y+6x^2,$

9.$(x-y^3+y^2\sin x)\,dx=(3xy^2+2y\cos x)\,dy,$

10.$(y\ln x-e^{-xy})\,dx+\left(\dfrac{1}{y}+x\ln y\right)dy=0,$ 11. $\left(x^2y^3-\dfrac{1}{1+9x^2}\right)\dfrac{dx}{dy}+x^3y^2=0,$

12. $(\tan x-\sin x\sin y)\,dx+\cos x\cos y\,dy=0,$

13.$(4t^3y-15t^2-y)\,dt+(t^4+3y^2-t)\,dy=0.$

(ii) Solve the given initial-value problem:

1.$(x+y)^2\,dx+(2xy+x^2-1)\,dy=0,\quad y(1)=1;$

2.$(4y+2t-5)\,dt+(6y+4t-1)\,dy=0,\quad y(-1)=2;$

3. $\dfrac{dy}{dx}=\dfrac{xy^2-\cos x\sin x}{y(1-x^2)},\quad y(0)=2;$

4. $(y^2\cos x-3x^2y-2x)\,dx+(2y\sin x-x^3+\ln y)\,dy=0,\quad y(0)=e.$

BRAC UNIVERSITY

MAT 120 (Mathematics II)

Practice Sheet

Solutions by Substitutions

(i) Solve the given homogeneous equation by using an appropriate substitution:

1. $(x-y)dx + xdy = 0$, 2. $(x+y)\,dx + x\,dy = 0$, 3. $x\,dx + (y-2x)\,dy = 0$,
4. $y\,dx = 2(x+y)\,dy$, 5. $(y^2 + yx)\,dx - x^2\,dy = 0$, 6. $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$,
7. $-y\,dx + (x + \sqrt{xy})\,dy = 0$, 8. $(x^2 + y^2)\,dx + (x^2 - xy)\,dy = 0$.

(ii) Solve the given initial value problem:

1. $\dfrac{dy}{dx} = (-2x+y)^2 - 7, \quad y(0) = 0$;
2. $xy^2 \dfrac{dy}{dx} = y^3 - x^3, \quad y(1) = 2$;
3. $(x + ye^{y/x})\,dx - xe^{y/x}\,dy = 0, \quad y(1) = 0$.

Reduction of Order

The indicated function $y_1(x)$ is a solution of the given differential equation. Use reduction of order, to find a second solution $y_2(x)$.

1. $y'' - 4y' + 4y = 0; \quad y_1 = e^{2x}$, 2. $y'' + 2y' + y = 0; \quad y_1 = xe^{-x}$,
3. $y'' + 16y = 0; \quad y_1 = \cos 4x$, 4. $y'' + 9y = 0; \quad y_1 = \sin 3x$,
5. $9y'' - 12y' + 4y = 0; \quad y_1 = e^{2x/3}$, 6. $xy'' + y' = 0; \quad y_1 = \ln x$,
7. $(1 - 2x - x^2)y'' + 2(1+x)y' - 2y = 0; \quad y_1 = x + 1$,
8. $y'' - 25y = 0; \quad y_1 = e^{5x}$, 9. $x^2y'' + 2xy' - 6y = 0; \quad y_1 = x^2$,
10. $(1 - x^2)y'' + 2xy' = 0; \quad y_1 = 1$.

Homogeneous Linear Equations
With Constant Coefficients

(*i*) Find the general solution of the given second-order differential equations:

$1.\, 4y'' + y' = 0, \quad 2.\, y'' - y' - 6y = 0, \quad 3.\, y'' + 8y' + 16y = 0, \quad 4.\, y'' + 9y = 0$
$5.\, y'' - 4y' + 5y = 0, \quad 6.\ 3y'' + 2y' + y = 0, \quad 7.\, 2y'' - 3y' + 4y = 0.$

(*ii*) Find the general solution of the given higher-order differential equations:

$1.\, y''' - 4y'' - 5y' = 0, \quad 2.\, y''' - 5y'' + 3y' + 9y = 0, \quad 3.\, y^{(4)} + y''' + y'' = 0,$
$4.\, 16y^{(4)} + 24y'' + 9y = 0.$

(*iii*) Solve the given initial- value problems:

$1.\, y'' + 16y = 0, \ \ y(0) = 2, \ y'(0) = -2; \quad 2.\ \dfrac{d^2y}{d\theta^2} + y = 0, \ \ y(\pi/3) = 0, \ y'(\pi/3) = 2;$

$3.\, \dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} - 5y = 0, \ y(1) = 0, \ y'(1) = 2; \quad 4.\, y'' + y' + 2y = 0, \ y(0) = y'(0) = 0;$
$5.\, y''' + 12y'' + 36y' = 0, \ y(0) = 0, \ y'(0) = 1, \ y''(0) = -7.$

(*iv*) Solve the given boundary- value problems:

$1.\, y'' - 10y' + 25y = 0, \ y(0) = 1, \ y(1) = 0; \quad 2.\ y'' + y = 0, \ y'(0) = 0, \ y'(\pi/2) = 2.$

Undetermined Coefficients (Annihilator Approach)

Solve the given differential equation by undetermined coefficients.

$1.\ y'' - 9y = 54, \quad 2.\ y'' + 3y' = 4x - 5, \quad 3.\ y'' + 4y' + 4y = 2x + 6,$
$4.\ y'' - 2y' + y = x^3 + 4x, \quad 5.\ y'' + 6y' + 8y = 3e^{-2x} + 2x,$
$6.\ y'' + 25y = 6\sin x, \quad 7.\ y'' + 25y = 20\sin 5x, \quad 8.\ y'' - 2y' + 5y = e^x \sin x$

Variation of Parameters

Solve each differential equation by variation of parameters.

$1.\ y'' + y = \sec x, \quad 2.\ y'' + y = \tan x, \quad 3.\ y'' + y = \sin x,$

$4.\ y'' - 4y = \dfrac{e^{2x}}{x}, \quad 5.\ y'' + 3y' + 2y = \dfrac{1}{1+e^x}, \quad 6.\ y'' + 2y' + y = e^{-t}\ln t,$
$7.\ y'' + y = \cos^2 x, \quad 8.\ 3y'' - 6y' + 6y = e^x \sec x,$
$9.\ y'' + y = \sec^2 x, \quad 10.\ y'' - 9y = \dfrac{9x}{e^{3x}}.$