Department of Mathematics and Natural Sciences

Quiz 1 (**Solution**)

Semester: Summer 2016

Course Title: Mathematics I: Differential Calculus and Coordinate Geometry

Course No.: MAT110

Section: 02

Student Name	:		Student ID	:	
Time	:	25 min	Date	:	May 22, 2016
Total marks	:	25	Marks Obtained	:	

Answer the following:

1. Find $g(3), g(0)$, and $g(1)$. [03]

$$g(x) = \begin{cases} \sqrt{x+1}, & x > 1 \\ 3 & x < 1 \end{cases}$$

Solution

$g(3) = \sqrt{3+1} = \sqrt{4} = 2$

$g(0) = 3$

$g(1)$ is undefined.

2. Find the natural domain and determine the range of each function. [08]

i) $f(x) = \dfrac{x}{|x|}$ ii) $F(x) = \sqrt{4 - x^2}$

Solution

i)

$D_f = (-\infty, 0) \cup (0, +\infty)$

$R_f = \{-1, 1\}$

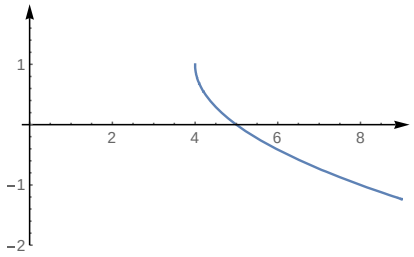
ii)

$D_f = [-2, 2]$

$R_f = [0, 2]$

3. Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = \sqrt{x}$. [05]

$$y = 1 - \sqrt{x - 4}$$



4. Find formulas for $f \circ g$, and state the domain of composition. [05]

$$f(x) = \frac{1+x}{1-x}, \qquad g(x) = \frac{1}{x}$$

Solution

$D_f = (-\infty, 1) \cup (1, +\infty)$ and $D_g = (-\infty, 0) \cup (0, +\infty)$

$$(f \circ g)(x) = f\big(g(x)\big) = \frac{1+g(x)}{1-g(x)} = \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{x+1}{x-1}$$

$D_{f\circ g} = (-\infty, 0) \cup (0,1) \cup (1, +\infty)$

5. Find g and h such that $f = g \circ h$. [02]

$$f(x) = (x^3+1)^2$$

Solution

Define,

$$g(x) = x^2 \text{ and } h(x) = x^3 + 1$$

$$(g \circ h)(x) = g\big(h(x)\big) = \big(h(x)\big)^2 = (x^3+1)^2 = f(x).$$

6. Classify the function as even, odd, or neither. [02]

$$f(x) = \frac{x^5 - x}{1+x^2}$$

Solution

$$\begin{aligned} f(-x) &= \frac{(-x)^5 - (-x)}{1+(-x)^2} \\ &= \frac{-x^5 + x}{1+x^2} \\ &= -\frac{x^5 - x}{1+x^2} \\ &= -f(x) \end{aligned}$$

Since

$$f(-x) = -f(x)$$

Therefore, f is an odd function. The graph of f is symmetric about the origin of the coordinate system.