Department of Mathematics and Natural Sciences

Class Test/Quiz 2 (**Solution**)

Semester: Fall 2016

Course Title: Mathematics I: Differential Calculus and Coordinate Geometry

Course No.: MAT110

Section: 06

Student Name	:	-----------------------------	Student ID	:	----------------
Time	:	20 min	Date	:	----------------
Total marks	:	25	Marks Obtained	:	----------------

1. Show that [15+10]

$$f(x) = \begin{cases} x^2 + 1, & x > 1 \\ 2\sqrt{x}, & x \le 1 \end{cases}$$

is continuous but not differentiable at $x = 1$. Sketch the graph of f and justify your answer.

[Hint: For continuity compute

$$f(1), \lim_{x\to 1^-} f(x)\text{, and } \lim_{x\to 1^+} f(x)$$

and for differentiability compute

$$f'_-(1) = \lim_{\Delta x\to 0^-} \frac{f(1+\Delta x) - f(1)}{\Delta x}, \qquad f'_+(1) = \lim_{\Delta x\to 0^+} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

]

Solution

Continuity

$f(1) = 2\sqrt{1} = 2$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2\sqrt{x} = 2\sqrt{1} = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2 + 1) = 2$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1) = 2$$

The function f is continuous at $x = 1$.

Differentiability

$$f'_-(1) = \lim_{\Delta x \to 0^-} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{2\sqrt{1+\Delta x} - 2}{\Delta x}$$

$$= 2\times \lim_{\Delta x \to 0^-} \frac{\left(\sqrt{1+\Delta x} - 1\right)\left(\sqrt{1+\Delta x} + 1\right)}{\Delta x\left(\sqrt{1+\Delta x} + 1\right)}$$

$$= 2\times \lim_{\Delta x \to 0^-} \frac{(1+\Delta x - 1)}{\Delta x\left(\sqrt{1+\Delta x} + 1\right)}$$

$$= 2\times \lim_{\Delta x \to 0^-} \frac{1}{\left(\sqrt{1+\Delta x} + 1\right)} = 2\times \frac{1}{2} = 1$$

$$f'_+(1) = \lim_{\Delta x \to 0^+} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{(1+\Delta x)^2 + 1 - 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0^+} \frac{1 + 2\Delta x + (\Delta x)^2 - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0^+} (2 + \Delta x) = 2$$

Therefore

$$\boxed{f'_-(1) \neq f'_+(1)}$$

The function f is not differentiable at $x = 1$.

Sketch