Fourier Integrals

1. Find the Fourier transform of $f(x) = \begin{cases} 1/2 & , \ |x| < a \\ 0 & , \ |x| > a \end{cases}$

2. Find the Fourier sine transform and the Fourier cosine transform of $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

3. (a) Determine the Fourier Cosine transform of $f(x) = e^{-mx}$, $\ m > 0$

   (b) Use the result in (a) show that $\displaystyle\int_0^\infty \frac{Cos \ pv}{v^2 + \beta^2} \, dv = \frac{\pi}{2\beta} e^{-p\beta} \quad (p > 0 \, , \ \beta > 0)$.

4. (a) Determine the Fourier Sine transform of $f(x) = e^{-x}$, $\ x > 0$

   (b) Use the result in (a) to show that $\displaystyle\int_0^\infty \frac{x \sin mx}{x^2 + 1} \, dx = \frac{\pi}{2} e^{-m\beta} \quad (m > 0)$