BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences
Final Examination
Semester: Summer 2016
Course Title: Linear Algebra and Fourier Analysis
Course No.: MAT216

Time: 3 hours
Total Marks: 50

Date: August 11, 2016

SECTION A

Answer all of the following:

1. (a) "An $n \times n$ matrix A does have n distinct eigenvalues, so matrix A is diagonalizable", Is the statement true or false? If any of its eigenvalue is zero then is it invertible? [1]

   (b) Suppose that a system of linear equations can be put into the form $AX = B$ where $\det A = 0$. Then the system has: [1]
   i) no solution ii) infinitely many solutions iii) an unique solution.
   Find the possible answer(s).

   (c) Do not evaluate. Tell what does the following integral represent. [1]

   $$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\, dx; \quad a > 0.$$

   (d) Write the geometrical significance of $\iiint\limits_{G} dV$. [1]

   (e) If the odd extension of the function $f(x) = x$, $0 < x < 2$ is periodic, then find its period. [1]

SECTION B

Answer any **TWO**.

2. (a) Write the characteristic equation of a square matrix A. [1]

   (b) What do you mean by a *diagonalizable* matrix A? [2]

   (c) Find the matrix P, if it exists, that diagonalizes the following matrix: [6]

   $$A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}.$$

3. (a) Write the standard matrix of a linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by:

$$T(x, y) = (-x, y)$$

that maps each vector into its symmetric image about the y-axis. [2]

(b) Find the rank of the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{pmatrix}.$$

[6]

(c) Consider the basis $B = \{u_1, u_2, u_3\}$ where $u_1 = (1, -1, 2) \in \mathbb{R}^3$, $u_2 = (2, 1, -3) \in \mathbb{R}^3$, and $u_3 = (1, 0, -2) \in \mathbb{R}^3$ and let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that:

$$\begin{aligned} T(u_1) &= (-3, -1) \\ T(u_2) &= (9, 0) \\ T(u_3) &= (2, -2). \end{aligned}$$

Find $T(5, -2, 7)$.

[1]

4. (a) Define a symmetric matrix. [2]

(b) State the elementary row operations. [6]

(c) Find $\lambda \in \mathbb{R}$ for which the vector $v = (1, \lambda, 5) \in \mathbb{R}^3$ is a linear combination of the vectors $v_1 = (1, -3, 2)$ and $v_2 = (2, -1, 1)$.

SECTION C

Answer any **TWO**. [2]

5. (a) Use double integral to find the area of the region R enclosed between the parabola $y = x^2/2$ and the line $y = 2x$.

$\left[\textbf{Hint:} \iint_R dA = \int_{\square}^{\square} \int_{\square}^{\square} dy dx.\right]$

[3]

(b) Evaluate:

$$\int_0^2 \int_{y/2}^1 e^{x^2} dx\, dy.$$

$\left[\textbf{Hint:} \int_0^2 \int_{y/2}^1 e^{x^2} dx\, dy = \int_{\square}^{\square} \int_{\square}^{\square} e^{x^2} dy\, dx.\right]$

[4]

(c) Use triple integral to find out the volume of the solid in the first octant bounded by the coordinate planes and the plane defined by $3x + 6y + 4z = 12$.

[2]

6. (a) Evaluate:

$$\iint_R (x^2 + y^2) dA,$$

where R is the region bounded by the unit circle centered at the origin.

$\left[\textbf{Hint:} \text{ Polar coordinates may be useful.}\right]$