

Practice sheet 4

Theme:

Date: / /
☐ Sat ☐ Sun ☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri

$$1c) \int_{(0,1)}^{(5,5)} (3u+y)du + (2y-u)dy$$

Equation of straight line: $\frac{y-0}{5-0} = \frac{x-0}{1-0}$

$$\Rightarrow \frac{y}{5} = \frac{x-1}{-4}$$

$$\Rightarrow 4u = 2y - 2$$

$$\Rightarrow y = 2u + 1$$

$$\therefore dy = 2du$$

$$\int_0^2 [(3u + 2u + 1)du + (4u + 2 - u)2du]$$

$$= \int_0^2 \left[\frac{3u^2}{2} + u^2 + 1 + \frac{2u(3u) + 4u^2}{2} \right] du$$

$$= \left[\frac{3}{2}u^2 + \frac{1}{3}u^3 + u + \frac{1}{2}(3u^2 + 4u^2) \right]_0^2$$

$$= 32 \text{ [Ans]}$$

c) For (0,1) to (0,5)

$$u=0$$

$$du=0$$

$$\int_1^5 0 + (2y-0)dy$$

$$= \left[\frac{2y^2}{2} \right]_1^5 = (25-1)$$

$$= 24$$

Theme:

Date: / /
Sat Sun Mon Tue Wed Thu FriAlong the line $(0, 5)$ to $(2, 5)$

$$y = 5, \quad dy = 0$$

$$\int_0^2 (3u+5) du + (10-u) = 0$$

$$= \int_0^2 (3u+3) du$$

$$= \frac{3}{2} [u^2]_0^2 + 3[u]_0^2$$

$$= 6 + 10 = 16$$

\therefore Required value $\rightarrow 29 + 16 = 45$ [Ans]

3. (c)

$$\int_C (x - iy) dz$$

Equation of the line, $\frac{y-1}{1-2} = \frac{y-1}{1-2}$

$$\Rightarrow \frac{y-1}{-1} = \frac{y-1}{-2}$$

$$\Rightarrow 2y - 2 = y - 1$$

$$\Rightarrow y = 2y - 6$$

$$\therefore dy = 2 du$$

$$\int_1^2 (u - i(2u-6)) (1+xi) du$$

$$\Rightarrow (1+xi) \int_1^2 (u - 49iu + 84u - 36i) du$$

$$\Rightarrow (1+xi) \left[\frac{u^2}{2} - \frac{49u^2 i}{2} + \frac{84u^2}{2} - \frac{36u^2 i}{2} \right]_1^2$$

$$\Rightarrow (1+xi) \left(\frac{2^2}{2} - \frac{49 \cdot 2^2 i}{2} + \frac{84 \cdot 2^2}{2} - \frac{36 \cdot 2^2 i}{2} \right)$$

$$\Rightarrow$$

$$\Rightarrow 57\frac{8}{3} - 8i \quad [Ans]$$

6 (a) $z_1 = i = 0 + 1.i \quad (0, 1)$

$$z_2 = 2-i = 2 + i(-1) \quad (2, -1)$$

Along $(0, 1)$ to $(2, -1)$.

$$\frac{u-v}{0-2} = \frac{u-1}{1+i}$$

$$\Rightarrow 2u = -2(y-i)$$

$$\Rightarrow u = -y+i$$

$$\Rightarrow y = 1-u$$

$$\therefore dy = -du$$

$$\therefore du = -dy$$

$$\int_0^1 \int_{-1}^1 3y(y+1) + i(f) \{ (i-1)dy$$

$$\Rightarrow \int_0^1 (-3y^2 + 3y + iy') (i-1) dy$$

$$\Rightarrow (i-1) \left[-y^3 + \frac{3y^2}{2} + \frac{iy^3}{3} \right]_0^1$$

$$\Rightarrow (i-1) (2 - \frac{2i}{3})$$

$$\Rightarrow \frac{8i}{3} - \frac{4}{3} \quad [Ans]$$

$$\begin{aligned} z &= x+iy \\ dz &= dx+idy \\ &= -dy+idy \\ &= (i-1)dy \end{aligned}$$

$$f(b) \quad u=0 \quad 2+2=0 \quad +=01$$

$$\begin{array}{r} u=2 \\ 2 \rightarrow 2+2 \\ 2+2=7 \\ 7+2=2 \end{array}$$

$$u = 24.2$$

$$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$$

$$\Delta n = 2d +$$

$$y = 1 + 4 + 7 + \dots$$

$$dy_t = 1 - 2t$$

$$\frac{1}{2} \Delta + \frac{1}{2} \Delta = \Delta$$

$$\int^2 [3(2+2)(1+1+1) + i(1+1+1)^2] 22 + i(1-2) \int dt$$

$$\Rightarrow \int_1^2 3(2t-2+2t^2-2t^3+2t^4-2t^5+2t^6-2t^7+2t^8-2t^9+2t^{10}-2t^{11}+2t^{12}-2t^{13}+2t^{14}-2t^{15}+2t^{16}-2t^{17}+2t^{18}-2t^{19}+2t^{20}-2t^{21}+2t^{22}-2t^{23}+2t^{24}-2t^{25}+2t^{26}-2t^{27}+2t^{28}-2t^{29}+2t^{30}-2t^{31}+2t^{32}-2t^{33}+2t^{34}-2t^{35}+2t^{36}-2t^{37}+2t^{38}-2t^{39}+2t^{40}-2t^{41}+2t^{42}-2t^{43}+2t^{44}-2t^{45}+2t^{46}-2t^{47}+2t^{48}-2t^{49}+2t^{50}-2t^{51}+2t^{52}-2t^{53}+2t^{54}-2t^{55}+2t^{56}-2t^{57}+2t^{58}-2t^{59}+2t^{60}-2t^{61}+2t^{62}-2t^{63}+2t^{64}-2t^{65}+2t^{66}-2t^{67}+2t^{68}-2t^{69}+2t^{70}-2t^{71}+2t^{72}-2t^{73}+2t^{74}-2t^{75}+2t^{76}-2t^{77}+2t^{78}-2t^{79}+2t^{80}-2t^{81}+2t^{82}-2t^{83}+2t^{84}-2t^{85}+2t^{86}-2t^{87}+2t^{88}-2t^{89}+2t^{90}-2t^{91}+2t^{92}-2t^{93}+2t^{94}-2t^{95}+2t^{96}-2t^{97}+2t^{98}-2t^{99}+2t^{100}-2t^{101}+2t^{102}-2t^{103}+2t^{104}-2t^{105}+2t^{106}-2t^{107}+2t^{108}-2t^{109}+2t^{110}-2t^{111}+2t^{112}-2t^{113}+2t^{114}-2t^{115}+2t^{116}-2t^{117}+2t^{118}-2t^{119}+2t^{120}-2t^{121}+2t^{122}-2t^{123}+2t^{124}-2t^{125}+2t^{126}-2t^{127}+2t^{128}-2t^{129}+2t^{130}-2t^{131}+2t^{132}-2t^{133}+2t^{134}-2t^{135}+2t^{136}-2t^{137}+2t^{138}-2t^{139}+2t^{140}-2t^{141}+2t^{142}-2t^{143}+2t^{144}-2t^{145}+2t^{146}-2t^{147}+2t^{148}-2t^{149}+2t^{150}-2t^{151}+2t^{152}-2t^{153}+2t^{154}-2t^{155}+2t^{156}-2t^{157}+2t^{158}-2t^{159}+2t^{160}-2t^{161}+2t^{162}-2t^{163}+2t^{164}-2t^{165}+2t^{166}-2t^{167}+2t^{168}-2t^{169}+2t^{170}-2t^{171}+2t^{172}-2t^{173}+2t^{174}-2t^{175}+2t^{176}-2t^{177}+2t^{178}-2t^{179}+2t^{180}-2t^{181}+2t^{182}-2t^{183}+2t^{184}-2t^{185}+2t^{186}-2t^{187}+2t^{188}-2t^{189}+2t^{190}-2t^{191}+2t^{192}-2t^{193}+2t^{194}-2t^{195}+2t^{196}-2t^{197}+2t^{198}-2t^{199}+2t^{200}-2t^{201}+2t^{202}-2t^{203}+2t^{204}-2t^{205}+2t^{206}-2t^{207}+2t^{208}-2t^{209}+2t^{210}-2t^{211}+2t^{212}-2t^{213}+2t^{214}-2t^{215}+2t^{216}-2t^{217}+2t^{218}-2t^{219}+2t^{220}-2t^{221}+2t^{222}-2t^{223}+2t^{224}-2t^{225}+2t^{226}-2t^{227}+2t^{228}-2t^{229}+2t^{230}-2t^{231}+2t^{232}-2t^{233}+2t^{234}-2t^{235}+2t^{236}-2t^{237}+2t^{238}-2t^{239}+2t^{240}-2t^{241}+2t^{242}-2t^{243}+2t^{244}-2t^{245}+2t^{246}-2t^{247}+2t^{248}-2t^{249}+2t^{250}-2t^{251}+2t^{252}-2t^{253}+2t^{254}-2t^{255}+2t^{256}-2t^{257}+2t^{258}-2t^{259}+2t^{260}-2t^{261}+2t^{262}-2t^{263}+2t^{264}-2t^{265}+2t^{266}-2t^{267}+2t^{268}-2t^{269}+2t^{270}-2t^{271}+2t^{272}-2t^{273}+2t^{274}-2t^{275}+2t^{276}-2t^{277}+2t^{278}-2t^{279}+2t^{280}-2t^{281}+2t^{282}-2t^{283}+2t^{284}-2t^{285}+2t^{286}-2t^{287}+2t^{288}-2t^{289}+2t^{290}-2t^{291}+2t^{292}-2t^{293}+2t^{294}-2t^{295}+2t^{296}-2t^{297}+2t^{298}-2t^{299}+2t^{300}-2t^{301}+2t^{302}-2t^{303}+2t^{304}-2t^{305}+2t^{306}-2t^{307}+2t^{308}-2t^{309}+2t^{310}-2t^{311}+2t^{312}-2t^{313}+2t^{314}-2t^{315}+2t^{316}-2t^{317}+2t^{318}-2t^{319}+2t^{320}-2t^{321}+2t^{322}-2t^{323}+2t^{324}-2t^{325}+2t^{326}-2t^{327}+2t^{328}-2t^{329}+2t^{330}-2t^{331}+2t^{332}-2t^{333}+2t^{334}-2t^{335}+2t^{336}-2t^{337}+2t^{338}-2t^{339}+2t^{340}-2t^{341}+2t^{342}-2t^{343}+2t^{344}-2t^{345}+2t^{346}-2t^{347}+2t^{348}-2t^{349}+2t^{350}-2t^{351}+2t^{352}-2t^{353}+2t^{354}-2t^{355}+2t^{356}-2t^{357}+2t^{358}-2t^{359}+2t^{360}-2t^{361}+2t^{362}-2t^{363}+2t^{364}-2t^{365}+2t^{366}-2t^{367}+2t^{368}-2t^{369}+2t^{370}-2t^{371}+2t^{372}-2t^{373}+2t^{374}-2t^{375}+2t^{376}-2t^{377}+2t^{378}-2t^{379}+2t^{380}-2t^{381}+2t^{382}-2t^{383}+2t^{384}-2t^{385}+2t^{386}-2t^{387}+2t^{388}-2t^{389}+2t^{390}-2t^{391}+2t^{392}-2t^{393}+2t^{394}-2t^{395}+2t^{396}-2t^{397}+2t^{398}-2t^{399}+2t^{400}-2t^{401}+2t^{402}-2t^{403}+2t^{404}-2t^{405}+2t^{406}-2t^{407}+2t^{408}-2t^{409}+2t^{410}-2t^{411}+2t^{412}-2t^{413}+2t^{414}-2t^{415}+2t^{416}-2t^{417}+2t^{418}-2t^{419}+2t^{420}-2t^{421}+2t^{422}-2t^{423}+2t^{424}-2t^{425}+2t^{426}-2t^{427}+2t^{428}-2t^{429}+2t^{430}-2t^{431}+2t^{432}-2t^{433}+2t^{434}-2t^{435}+2t^{436}-2t^{437}+2t^{438}-2t^{439}+2t^{440}-2t^{441}+2t^{442}-2t^{443}+2t^{444}-2t^{445}+2t^{446}-2t^{447}+2t^{448}-2t^{449}+2t^{450}-2t^{451}+2t^{452}-2t^{453}+2t^{454}-2t^{455}+2t^{456}-2t^{457}+2t^{458}-2t^{459}+2t^{460}-2t^{461}+2t^{462}-2t^{463}+2t^{464}-2t^{465}+2t^{466}-2t^{467}+2t^{468}-2t^{469}+2t^{470}-2t^{471}+2t^{472}-2t^{473}+2t^{474}-2t^{475}+2t^{476}-2t^{477}+2t^{478}-2t^{479}+2t^{480}-2t^{481}+2t^{482}-2t^{483}+2t^{484}-2t^{485}+2t^{486}-2t^{487}+2t^{488}-2t^{489}+2t^{490}-2t^{491}+2t^{492}-2t^{493}+2t^{494}-2t^{495}+2t^{496}-2t^{497}+2t^{498}-2t^{499}+2t^{500}-2t^{501}+2t^{502}-2t^{503}+2t^{504}-2t^{505}+2t^{506}-2t^{507}+2t^{508}-2t^{509}+2t^{510}-2t^{511}+2t^{512}-2t^{513}+2t^{514}-2t^{515}+2t^{516}-2t^{517}+2t^{518}-2t^{519}+2t^{520}-2t^{521}+2t^{522}-2t^{523}+2t^{524}-2t^{525}+2t^{526}-2t^{527}+2t^{528}-2t^{529}+2t^{530}-2t^{531}+2t^{532}-2t^{533}+2t^{534}-2t^{535}+2t^{536}-2t^{537}+2t^{538}-2t^{539}+2t^{540}-2t^{541}+2t^{542}-2t^{543}+2t^{544}-2t^{545}+2t^{546}-2t^{547}+2t^{548}-2t^{549}+2t^{550}-2t^{551}+2t^{552}-2t^{553}+2t^{554}-2t^{555}+2t^{556}-2t^{557}+2t^{558}-2t^{559}+$$

$$dz = \frac{dz}{dz} dz$$

$$\Rightarrow \int_1^2 (6t - 6t^2 - 6t^3 + 6t^4 + t + 4)(2i - 2t) dt$$

$$\Rightarrow \int_1^2 (-8x^3 + 11x^2 - 6x + 9 + 2i + i - 8t^3; t + 11t^2 - 6it + 9$$

$$+2 + i\sqrt{4} + 16 + 4 - 22 + 3 + 12 + -2 + 3 - 9 + i - 2 + i]$$

$$1 - \frac{m_0}{m_1}$$

$$8. (v) |z-1|=7$$

$$\Rightarrow z-1=7e^{i\theta}$$

$$\Rightarrow z=7e^{i\theta}+1$$

$$dz=7ie^{i\theta}d\theta$$

$$\int_0^{2\pi} \frac{7ie^{i\theta}d\theta}{7e^{i\theta}+1-2}$$

$$= \int_0^{2\pi} \frac{7ie^{i\theta}d\theta}{7e^{i\theta}-1}$$

$$= \int_0^{2\pi} \frac{dv}{v}$$

$$= [\ln v]_0^{2\pi}$$

$$= \ln 8 - \ln 8$$

$$= 0 \quad [\text{Ans}]$$

$$\oint_C (5z^4 - z^3 + 2) dz \text{ around the circle } |z|=1$$

$$|z|=1$$

$$z=e^{i\theta}$$

$$dz=ie^{i\theta}d\theta$$

$$\int_0^{2\pi} (5e^{4i\theta} - e^{3i\theta} + 2)ie^{i\theta}d\theta$$

$$= i \int_0^{2\pi} [5e^{5i\theta} - e^{4i\theta} + 2e^{i\theta}]d\theta$$

$$\text{let } 7e^{i\theta}-1=4$$

$$dv=7ie^{i\theta}d\theta$$

$$\text{if } \theta=0, \quad v=7-1=6$$

$$\theta=2\pi, \quad v=7-1=6$$

Theme:

Date:

☐ Sat ☐ Sun ☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri

$$\begin{aligned}
 &= i \left[\frac{e^{5i\theta}}{5i} - \frac{e^{7i\theta}}{4i} + \frac{2e^{i\theta}}{1} \right]_0^{2\pi} \\
 &= \left[\frac{e^{5i\theta}}{5} - \frac{e^{4i\theta}}{4} + 2e^{i\theta} \right]_0^{2\pi} \\
 &= \left[\cos 5\theta + i \sin 5\theta \right]_0^{2\pi} - \frac{1}{4} \left[\cos 4\theta + i \sin 4\theta \right]_0^{2\pi} + 2 \left[\cos \theta + i \sin \theta \right]_0^{2\pi}
 \end{aligned}$$

$$= \cancel{(0+0)}$$

$$= (1+0-1+0) - \frac{1}{4} (1+0-1+0) + 2(1+0-1+0)$$

$$= 0 - 0 + 0$$

$$= 0$$

[Ans]