

BRAC University
MAT-215
Practice Sheet # 6

1. Find the Laplace transformation of each of the following function:

$$(i) 3e^{-2t} \quad (ii) 4t^3 - e^{-t} \quad (iii) 7\sin 2t - 3\cos 2t \quad (iv) (t^2 + 1)^2 \quad (v) (4e^{2t} - 2)^3.$$

2. Evaluate each of the following:

$$(i) \mathcal{L}\{t^3 e^{-3t}\} \quad (ii) \mathcal{L}\{5e^{3t} \sin 4t\} \quad (iii) \mathcal{L}\{(t+2)^2 e^t\} \quad (iv) \mathcal{L}\{e^{-t}(3\sinh 2t - 5\cosh 2t)\} \\ (v) \mathcal{L}\{e^{-4t} \cosh 2t\} \quad (vi) \mathcal{L}\{e^{2t}(3\sin 4t - 4\cos 4t)\}.$$

3. Determine each of the following:

$$(i) \mathcal{L}^{-1}\left\{\frac{12}{4-3s}\right\} \quad (ii) \mathcal{L}^{-1}\left\{\frac{2s-5}{s^2-9}\right\} \quad (iii) \mathcal{L}^{-1}\left\{\frac{23s-15}{s^2+8}\right\} \quad (iv) \mathcal{L}^{-1}\left\{\frac{1}{s^{3/2}}\right\} \\ (v) \mathcal{L}^{-1}\left\{\frac{s+1}{s^{4/3}}\right\} \quad (vi) \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} \quad (vii) \mathcal{L}^{-1}\left\{\frac{1}{\sqrt{2s+3}}\right\}.$$

4. Evaluate each of the following using partial fraction:

$$(i) \mathcal{L}^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\} \quad (ii) \mathcal{L}^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\} \quad (iii) \mathcal{L}^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\} \\ (iv) \mathcal{L}^{-1}\left\{\frac{5s^2-15s-11}{(s+1)(s-2)^3}\right\} \quad (v) \mathcal{L}^{-1}\left\{\frac{3s+1}{(s^2+1)(s-1)}\right\} \quad (vi) \mathcal{L}^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\} \\ (vii) \mathcal{L}^{-1}\left\{\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}.$$

5. Solve the given differential equation:

$$(i) Y'' - 3Y' + 2Y = 4e^{2t}, \quad Y(0) = -3, \quad Y'(0) = 5 \\ (ii) Y'' + 9Y = \cos 2t, \quad Y(0) = 1, \quad Y(\pi/2) = -1 \\ (iii) Y'' + 2Y' + 5Y = e^{-t} \sin t, \quad Y(0) = 0, \quad Y'(0) = 1 \\ (iv) Y''' - 3Y'' + 3Y' - Y = e^t t^2, \quad Y(0) = 0, \quad Y'(0) = 0, \quad Y''(0) = -2.$$