HomeWork Sheet # 4

(1)

(i) $T(x, y, z) = (x-y, x-z)$

$\Rightarrow \underline{u} = \cancel{x-y, x-z}\ (x, y, z)$

$\underline{v} = (x', y', z')$

$$T(\underline{u} + \underline{v}) = T(x+x', y+y', z+z')$$

$$= \left(\left(x+x' - (y+y')\right), \left(x+x' - (z+z')\right) \right)$$

$$= (x+x'-y-y', x+x'-z-z')$$

$$= \left((x-y)+(x'-y'), (x-z)+(x'-z')\right)$$

$$= \left((x-y), (x-z)\right) + \left((x'-y'), (x'-z')\right)$$

$$= T(x, y, z) + T(x', y', z')$$

$$= T(\underline{u}) + T(\underline{v})$$

$$T(c\underline{u}) = T(cx, cy, cz)$$

$$= (cx - cy,\ cx - cz)$$

$$= (c(x-y),\ c(x-z))$$

$$= c((x-y),\ (x-z))$$

$$= c\,T(x, y, z)$$

$$= c\,T(\underline{u})$$

$\therefore$ This transformation is a linear transformation

(11) $T(x, y, z) = (3x - 2y + z,\ x - 3y - 2z)$

Let $\underline{u} = (x, y, z)$

$\underline{v} = (x', y', z')$

$\therefore T(\underline{u} + \underline{v}) = T(x + x', y + y', z + z')$

$$= (3x + 3x' - 2y - 2y' + z + z',\ x + x' - 3y - 3y' - 2z - 2z')$$

$$= (3x - 2y + z + 3x' - 2y' + z',\ x - 3y - 2z + x' - 3y' - 2z')$$

$$= ((3x - 2y + z), (x - 3y - 2z)) + ((3x' - 2y' + z'), (x' - 3y' - 2z'))$$

$$= T(x, y, z) + T(x', y', z')$$

$$= T(\underline{u}) + T(\underline{v})$$

$$T(c\underline{u}) = T(cx, cy, cz)$$

$$= (3cx - 2cy + cz, cx - 3cy - 2cz)$$

$$= c(3x - 2y + z, x - 3y - z)$$

$$= c\,T(x, y, z)$$

$$= c\,T(\underline{u})$$

So this transformation is a linear transformation

(iii)

$$T(x, y, z) = (x+1, y+z)$$

Let

$$\underline{u} = (x, y, z)$$

$$\underline{v} = (x', y', z')$$

$$\therefore T(\underline{u}+\underline{v}) = (x+x', y+y', z+z')$$

$$= (x+x'+1, y+y'+z+z')$$

$$=$$

iv) $T(x,y) = (x+y, 1)$

$u = (x, y)$, $v(x', y')$

$T(u+v) = T(x+x', y+y') = (x+x'+y+y', 1)$

$= ((x+y)+(x'+y'), 1) = ((x+y), 1) + ((x'+y'), 1)$

$= T(u) + T(v)$

$T(cu) = T(cx, cy) = (cx+cy, 1) = (c(x+y), 1)$

(2)

$T(x,y,z,t) = (x - y + z + t,\ x + 2z - t,\ x + y + 3z - 3t)$

standard basis for $\mathbb{R}^4$

$= \{(1,0,0,0),\ (0,1,0,0),\ (0,0,1,0),\ (0,0,0,1)\}$

$T(1,0,0,0) = (1,1,1)$
$T(0,1,0,0) = (-1,0,1)$
$T(0,0,1,0) = (1,2,3)$
$T(0,0,0,1) = (1,-1,-3)$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & -2 & -4 \end{bmatrix} \xleftarrow[\substack{R_3' = -R_1 + R_3 \\ R_4' = -R_1 + R_4}]{R_2' = R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xleftarrow{R_4' = -\frac{1}{2} R_4}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_2 + R_4]{R_3' = -R_2 + R_3}$$

Basis of $R(T) = \{ (1,1,1), (0,1,2) \}$

dimention of $R(T) = 2 = rank(T)$

$T(x,y,z,t) = (0,0,0)$

$\Rightarrow (x-y+z+t,\ x+2z-t,\ x+y+3z-3t) = (0,0,0)$

$\therefore x-y+z+t=0$

$x+0+2z-t=0$

$x+y+3z-3t=0$

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & 2 & -1 & 0 \\ 1 & 1 & 3 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 2 & 2 & -4 & 0 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -2R_2 + R_3}$$

$x - y + z + t = 0$

$y + z - 2t = 0$

Let

$z = p$

$t = a$

$\therefore$ ~~x~~ $y = -p + 2a$

$x = -p + 2a - p - a$

$= -2p + a$

$x = -2p + a$

$y = -p + 2a$

$z = p + 0$

$t = 0 + a$

$$\therefore \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix} p + \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} a$$

$\therefore$ ~~nullity(T) = 2~~

$\therefore$ Basis of Kernel $(T) = \{(-2,-1,1,0), (1,2,0,1)\}$

$\therefore$ dim of $Ker(T) = 2 = \text{nullity}(T)$

$(x, y, z) = (x+2y-z,\ y+z,\ x+y-2z)$

ndard basis for $\mathbb{R}^3 = \{(1,0,0), (0,1,0), (0,0,1)\}$

$(1,0,0) = (1,0,1)$

$(0,1,0) = (2,1,1)$

$(0,0,1) = (-1,1,-2)$

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \xleftarrow[R_3' = R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

is of $R(T) = \{(1,0,1), (0,1,-1)\}$

nsion of $R(T) = 2 = \text{rank}(T)$

$$T(x, y, z) = (0, 0, 0)$$

$$\therefore (x + 2y - z,\; y + z,\; x + y - 2z) = (0, 0, 0)$$

$$x + 2y - z = 0$$

$$y + z = 0$$

$$x + y - 2z = 0$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & \overset{-1}{\cancel{0}} & 0 \end{bmatrix} \longleftarrow R_3' = -R_1 + R_3$$

$$= \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = R_2 + R_3$$

let $z = p$ $\quad \cancel{x_2 + x_3}$ $\quad y + z = 0$

$$y = -p$$

$$x + 2y - z = 0$$

$$x = 2p + p = 3p$$

$\therefore x = 2p$

$y = -p$

$z = p$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} p$$

$\therefore$ Basis of kernel $(T) = \{(3, -1, 1)\}$

$\therefore$ dim of $\ker(T) = 1 = \text{nullity}(T)$.

$T(x, y, z) = (3x - y, y - z, 3x - 2y + z)$

Standard basis for $\mathbb{R}^3 = \{(1,0,0), (0,1,0), (0,0,1)\}$

$(1,0,0) = (3, 0, 3)$

$(0,1,0) = (-1, 1, -2)$

$(0,0,1) = (0, -1, 1)$

$$\begin{bmatrix} 3 & 0 & 3 \\ -1 & 1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \\ 0 & -1 & 1 \end{bmatrix} \longleftarrow R_1' = \tfrac{1}{3} R_1$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \longleftarrow R_2' = R_1 + R_2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = R_2 + R_3$$

~~Let~~

~~$z = p$~~

~~$y =$~~

$\therefore$ basis for $R(T) = \{ (1,0,1), (0,1,-1) \}$

dimention of $R(T) = 2 = \text{Rank}(T)$

$T(x, y, z) = (0, 0, 0)$

$(3x - y, y - z, 3x - 2y + z) = (0, 0, 0)$

$3x - y = 0$

$y - z = 0$

$3x - 2y + z = 0$

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 3 & -2 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 3 & -2 & 1 & 0 \end{bmatrix} \xleftarrow{R_1' = 1/3 R_1}$$

$$\begin{bmatrix} 1 & -1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \xleftarrow{R_3' = -3R_1 + R_3}$$

$$\begin{bmatrix} 1 & -1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = R_2 + R_3}$$

Let $z = p$

$y = p$

$x = 1/3\, p$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1 \\ 1 \end{bmatrix} p$$

$$T(x,y,z) = (0,0,0)$$

$$(2x-y,\ y-z,\ 3x-2y+z) = (0,0,0)$$

$$\begin{matrix} 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 3 & -2 & 1 & 0 \end{matrix}$$

$\therefore$ Basis of kernel $(T) = \{(1/3, 1, 1)\}$

dim of Ker(T) $= 1 =$ nullity(T)

5/ $T(x,y,z) = (x+2y-3z,\ 2x-y+4z,\ 4x+3y-2z)$

$\therefore$ standart basis of $\mathbb{R}^3 = ((1,0,0),(0,1,0),(0,0,1)\}$

$T(1,0,0) = (1,2,4)$

$T\cdot(0,1,0) = (2,-1,3)$

$T(0,0,1) = (-3,4,-2)$

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ -3 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & 10 & 10 \end{bmatrix} \xleftarrow[R_3' = 3R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{10} R_3]{R_2' = -\frac{1}{5} R_2}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$\therefore$ Basis for $R(T) = \{(1,2,4), (0,1,1)\}$

$\therefore$ dim of $R(T) = 2 = \text{Rank}(T)$

$T(x, y, z) = (0, 0, 0)$

$(x + 2y - 3z, \; 2x - y + 4z, \; 4x + 3y - 2z) = (0, 0, 0)$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & -1 & 4 & 0 \\ 4 & 3 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -5 & +10 & 0 \\ 0 & -5 & -10 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = -2R_1 + R_2 \\ R_3' = -4R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = -1/5 \, R_2 \\ R_3' = -1/5 \, R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = -R_2 + R_3$$

let

$z = p$

$y = -2p$

$x = -4p + 3p = -p$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} p$$

$\therefore$ Basis for kernel $(T) = \{(-1, -2, 1)\}$

dim of $\ker(T) = 1 =$ nullity (T)

b

(i) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$

$\lambda I_3 - A$

$$= \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda-1 & -2 & 1 \\ 0 & \lambda+2 & 0 \\ 0 & 5 & \lambda-2 \end{bmatrix}$$

$$\det(\lambda I_3 - A) = 0$$

$$\begin{vmatrix} \lambda-1 & -2 & 1 \\ 0 & \lambda+2 & 0 \\ 0 & 5 & \lambda-2 \end{vmatrix} = 0$$

$$(\lambda-1)\{(\lambda-2)(\lambda+2)-0\}+2(0-0)+1(0) = 0$$

$$\Rightarrow (\lambda-1)(\lambda^2-4) = 0$$

$$\Rightarrow \lambda^3 - 4\lambda - \lambda^2 + 4 = 0$$

$$\Rightarrow \lambda^3 - \lambda^2 - 4\lambda + 4 = 0$$

$$\Rightarrow \lambda^2(\lambda-1) - 4(\lambda-1) = 0$$

$$\Rightarrow (\lambda^2-4)(\lambda-1) = 0$$

$$\Rightarrow (\lambda+2)(\lambda-2)(\lambda-1) = 0$$

$\therefore \lambda = -2$

$\lambda = 2$

$\lambda = 1$

Now

for $\lambda = -2$

$$\begin{bmatrix} -3 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 5 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_2 \rightleftharpoons R_3}$$

$$= \begin{bmatrix} 1 & 2/3 & -1/3 \\ 0 & 1 & -4/5 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_2' = \frac{1}{5} R_2]{R_1' = -\frac{1}{3} R_1}$$

~~Let~~ $\cancel{x} + 2/3\, y \cancel{+ z\, 1/3} = 0$

$\cancel{z = p}$

$\cancel{x_2 = 4/5}$

$$\begin{bmatrix} 1 & 2/3 & 1/3 \\ 0 & 1 & -4/5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 2/3\, y - 1/3\, z = 0$

$y - 4/5\, z = 0$

let $z = p$

$$3 \,\underline{|\, 15, 3}$$
$$\quad 5, 1$$

$y = 4/5\, p$

$x = -2/3 \times 4/5\, p + 1/3\, p$

$= -\frac{8}{15} p + 1/3\, p$

$= \frac{(-8+5)\,p}{15}$

$= -\cancel{1/3\, p}\ \cancel{15} \quad -3/15\, p = -1/5\, p$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\cancel{1}/5 \\ 4/5 \\ 1 \end{bmatrix} p$$

for $\lambda = 2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 0 \\ 0 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{5} R_3]{R_2' = \frac{1}{4} R_2}$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - 2y + z = 0$

$y = 0$

let

$z = \delta$

$x = -\delta$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \delta$$

for $\lambda = 1$

$$\begin{bmatrix} 0 & -2 & 1 \\ 0 & 3 & 0 \\ 0 & 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 0 \\ 0 & -2 & 1 \\ 0 & 5 & -1 \end{bmatrix} \xleftarrow{R_1 \rightleftharpoons R_2}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 5 & -1 \end{bmatrix} \xleftarrow{R_1' = \frac{1}{3} R_1}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \xleftarrow{R_2' = 2R_1 -}$$

$$\begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 1 & 0 \\ 0 & 5 & -1 \end{bmatrix} \xleftarrow[R_2' = \frac{1}{3} R_2]{R_1' = -\frac{1}{2} R_1}$$

$$= \begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1/2 \\ 0 & 0 & 3/2 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -R_1 + R_2}$$

$$= \begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xleftarrow[R_3' = 2/3 R_3]{R_2' = 2 R_2}$$

$$\begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$\begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{y}a = 1/2 \cancel{z} \qquad y = 0$

$z = 0$

Let

$x = R$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} R$$

$$P = \begin{bmatrix} -\overset{1/5}{\cancel{15/15}} & -1 & 1 \\ 4/5 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = -R_2 + R_3$$

$$\begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \quad y = 1/2\, z \qquad y = 0$

$z = 0$

Let

$x = R$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} R$$

$$\therefore P = \begin{bmatrix} -\overset{1/5}{\cancel{16/15}} & -1 & 1 \\ 4/5 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\det(P) = -\tfrac{1}{5}(0) + 1(0) + 1\,\tfrac{4}{5}$$

$$= \tfrac{4}{5}$$

$$\neq 0$$

$\therefore P^{-1}$ exists

(11)

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

$\therefore \lambda I - A$

$$= \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda-2 & 2 & -1 \\ -2 & \lambda+8 & 2 \\ -1 & -2 & \lambda-2 \end{bmatrix}$$

$$(\lambda-2)\left((\lambda+8)(\lambda-2)+4\right)-2\left(-2(\lambda-2)+2\right)-1\left(4+\lambda+8\right)=0$$

$$(\lambda-2)(\lambda^2-2\lambda+8\lambda-16+4)-2(-2\lambda+4+2)-(4+\lambda+8)=0$$

$$\Rightarrow(\lambda-2)(\lambda^2+6\lambda-12)-(-4\lambda+12)-(\lambda+12)=0$$

$$\Rightarrow \lambda^3+6\lambda^2-12\lambda-2\lambda^2-12\lambda+24+4\lambda-12-\lambda-12=0$$

$$\cancel{+2\lambda^2+12\lambda}$$

$$\lambda^3+6\lambda^2-2\lambda^2-12\lambda-12\lambda+4\lambda-\lambda+24-12-12=0$$

$$\Rightarrow \lambda^3+4\lambda^2-21\lambda=0$$

$$\Rightarrow \lambda(\lambda^2+4\lambda-21)=0$$

$$\lambda=0$$

$$\lambda^2+4\lambda-21=0$$

$$\lambda^2+7\lambda-3\lambda-21=0$$

$$\lambda(\lambda+7)-3(\lambda+7)=0$$

$$(\lambda-3)(\lambda+7)=0$$

$$\therefore \lambda=0,3,-7$$

for $\lambda = -7$

$$\begin{bmatrix} -9 & 2 & -1 \\ -2 & 1 & 2 \\ -1 & -2 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 9 \\ -2 & 1 & 2 \\ -9 & 2 & -1 \end{bmatrix} R_1 \rightleftharpoons -R_3$$

$$= \begin{bmatrix} 1 & 2 & 9 \\ 0 & 5 & 20 \\ 0 & 20 & 80 \end{bmatrix} \xleftarrow[R_3' = 9R_1 + R_2]{R_2' = 2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 2 & 9 \\ 0 & 1 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xleftarrow[R_3 = 1/20 R_3]{R_2 = 1/5 R_2}$$

$$= \begin{bmatrix} 1 & 2 & 9 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$x + 2y + 9z = 0$

$y + 4z = 0$

Let

$z = p$

$\therefore$ $y = -4p$

$x = 8p - 9p = -p$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ 1 \end{bmatrix} p$$

for $\lambda = 0$

$$\begin{bmatrix} -2 & 2 & -1 \\ -2 & 8 & 2 \\ -1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ -2 & 8 & 2 \\ -2 & 2 & -1 \end{bmatrix} R_1 \rightleftarrows R_3$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 0 & 12 & 6 \\ 0 & 6 & 3 \end{bmatrix} \xleftarrow[R_3' = 2R_1 + R_3]{R_2' = 2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \xleftarrow[R_3' = 1/3\, R_3]{R_2' = 1/6\, R_2}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1/2 \\ 0 & 2 & 1 \end{bmatrix} \xleftarrow{R_2' = 1/2\, R_2}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -2R_2 + R_3}$$

$x + 2y + 2z = 0$

$y + 1/2\, z = 0$

Let

$z = a$

$y = -1/2\, a$

$x = -2(-1/2\, a) - 2a$

$= 1/2\, a - 2a$

$= \dfrac{\cancel{a - 4a}}{2} \quad a - 2a = -a$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix} a$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 2 & -1 \\ -2 & 11 & 2 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 15 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_3' = R_1 + R_2]{R_2' = 2R_1 + R_2}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $x + 2y - z = 0$

$y = 0$

Let

$z = R$

$\therefore x = R$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} R$$

(iii)

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

$\therefore \lambda I - A$

$$= \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda - 3 & -2 & -4 \\ -2 & \lambda & -2 \\ -4 & -2 & \lambda - 3 \end{bmatrix}$$

$\det(\lambda I - A) = 0$

$\lambda - 3\left(\lambda(\lambda-3) - 4\right) + 2\left(-2(\lambda-3) - 8\right) - 4(4 + 4\lambda) = 0$

$\Rightarrow (\lambda-3)(\lambda^2 - 3\lambda - 4) + 2(-2\lambda + 6 - 8) - 16 - 16\lambda = 0$

$\Rightarrow \lambda^3 - 3\lambda^2 - 4\lambda - 3\lambda^2 + 9\lambda + 12 - 4\lambda - 4 - 16 - 16\lambda = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 - 15\lambda - 8 = 0$

$\Rightarrow \lambda^3 + \lambda^2 - 7\lambda^2 - 7\lambda - 8\lambda - 8 = 0$

$\Rightarrow \lambda^2(\lambda+1) - 7\lambda(\lambda+1) - 8(\lambda+1) = 0$

$\cancel{\lambda^3 + \lambda^2} -$

$(\lambda+1)(\lambda^2 - 7\lambda - 8) = 0$

$\lambda = -1 \qquad \lambda^2 - 8\lambda + \lambda - 8 = 0$

$\lambda(\lambda - 8) + 1(\lambda - 8) = 0$

$(\lambda+1)(\lambda - 8) = 0$

$\lambda = -1 \quad \lambda = +8$

for $\lambda = -1$

$$\begin{bmatrix} -4 & -2 & -4 \\ -2 & -1 & -2 \\ -4 & -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} \xleftarrow[\substack{R_2 = -R_2 \\ R_3 = -\frac{1}{2}R_3}]{R_1' = -\frac{1}{2}R_1}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 1 \\ 2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} \xleftarrow{R_1' = \frac{1}{2}R_1}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$x + \frac{1}{2}y + z = 0$

let

$y = p$

$z = a$

$\therefore \quad x = -\frac{1}{2}p - a$

$y = p + 0$

$z = 0 + a$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} p + a \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda = 8$

$$\begin{bmatrix} 5 & -2 & -4 \\ -2 & 8 & -2 \\ -4 & -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 5 & -2 & -4 \\ -4 & -2 & 5 \end{bmatrix} \xleftarrow{R_1 \rightleftharpoons -\frac{1}{2}R_2}$$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 0 & 18 & -9 \\ 0 & -18 & 9 \end{bmatrix} \quad \begin{matrix} R_2' = -5R_1 + R_2 \\ \longleftarrow \\ R_3' = 4R_1 + R_2 \end{matrix}$$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 0 & 1 & -1/2 \\ 0 & -18 & 9 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{18} R_2}$$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3' = 18 R_2 + R_3$$

$x - 4y + z = 0$

$y - \frac{1}{2} z = 0$

let $z = R$

$y = \frac{1}{2} R$

$x = 2R - R = R$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix} R$$

(7)

(1) $A \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

$\lambda I - A$

$= \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

$= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

$= \begin{bmatrix} \lambda+14 & -12 \\ 20 & \lambda-17 \end{bmatrix}$

$\det(\lambda I - A) = 0$

$(\lambda+14)(\lambda-17) + 12\times 20 = 0$

$\Rightarrow \lambda^2 - 17\lambda + 14\lambda - 14\times 17 + 12\times 20 = 0$

$\Rightarrow \lambda^2 - 3\lambda + 2 = 0$

$\Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$\Rightarrow \lambda(\lambda-2) - 1(\lambda-2) = 0$

$\Rightarrow (\lambda-1)(\lambda-2) = 0$

$\lambda = 1 \quad \lambda = 2$

for $\lambda = 1$

$$\begin{bmatrix} 15 & -12 \\ 20 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -12/15 \\ 20 & -16 \end{bmatrix} \xleftarrow{R_1' = 1/15 R_1}$$

$$\begin{bmatrix} 1 & -4/5 \\ 0 & 0 \end{bmatrix} \xleftarrow{R_2' = -20R_1 + R_2}$$

$\therefore x - 4/5\, y = 0$

let $y = R$

$x = 4/5\, R$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4/5 \\ 1 \end{bmatrix} R$$

for $\lambda = 2$

$$\begin{bmatrix} 16 & -12 \\ 20 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -12/16 \\ 20 & -15 \end{bmatrix} \xleftarrow{R_1' = 1/16 R_1}$$

$$= \begin{bmatrix} 1 & -12/16 \\ 0 & 0 \end{bmatrix} \xleftarrow{R_2' = -20R_1 + R_2}$$

$\therefore \; x - 12/16 \, y = 0$

let

$y = t$

$\therefore \; x = 3/4 \, t$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3/4 \\ 1 \end{bmatrix} t$$

$$\therefore P = \begin{bmatrix} 4/5 & 3/4 \\ 1 & 1 \end{bmatrix}$$

$$\det(P) = 4/5 - 3/4 = \frac{1}{20}$$

$$P_{co} = \begin{bmatrix} 1 & -3/4 \\ -1 & 4/5 \end{bmatrix}$$

$$P^{-1} = \frac{1}{\frac{1}{20}} \begin{bmatrix} 1 & -3/4 \\ -1 & 4/5 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -15 \\ -20 & 16 \end{bmatrix}$$

$\therefore P^{-1}AP$

$$= \begin{bmatrix} 20 & -15 \\ -20 & 16 \end{bmatrix} \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix} \begin{bmatrix} 4/5 & 3/4 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -280+300 & 240-255 \\ 280-320 & -240+272 \end{bmatrix} \begin{bmatrix} 4/5 & 3/4 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -15 \\ -40 & 32 \end{bmatrix} \begin{bmatrix} 4/5 & 3/4 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16-15 & 15-15 \\ -32+32 & -30+32 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

So the matrix A is diagonalizeable.

the matrix P diagonalizes A.

(11)

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$\lambda I - A$

$$= \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda+1 & 2 & 2 \\ -1 & \lambda-2 & -1 \\ 1 & 1 & \lambda \end{bmatrix}$$

$\det(\lambda I - A)$

$(\lambda+1)\{\lambda(\lambda-2)+1\} - 2(-\lambda+1) + 2(-1-\lambda+2) = 0$

$\Rightarrow (\lambda+1)(\lambda^2-2\lambda+1) + 2\lambda - 2 + 2 - 2\lambda = 0$

$\Rightarrow (\lambda+1)(\lambda^2-2\lambda+1) - \cancel{4\lambda} = 0$

$\Rightarrow \lambda^3 - 2\lambda^2 + \lambda + \lambda^2 - 2\lambda + 1 - \cancel{4\lambda} = 0$

$\Rightarrow \lambda^3 - \lambda^2 - \cancel{5}\lambda + 1 = 0$

$\lambda^3 - \lambda^2 - \lambda + 1 = 0$

$\lambda^2(\lambda - 1) - 1(\lambda - 1) = 0$

$(\lambda - 1)(\lambda^2 - 1) = 0$

$(\lambda - 1)(\lambda + 1)(\lambda - 1) = 0$

$\lambda = 1 \quad \lambda = -1$

for $\lambda = 1$

$$\begin{bmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \xleftarrow[R_2' = -R_2]{R_1' = \frac{1}{2}R_1}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -R_1 + R_2}$$

$x + y + z = 0$

Let $y = R$

$z = t$

$x = -R - t$

$y = R + 0$

$z = 0 + t$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} R + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} t$$

for $\lambda = -1$

$$\begin{bmatrix} 0 & 2 & 2 \\ -1 & -3 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ -1 & -3 & -1 \\ 0 & 2 & 2 \end{bmatrix} \leftarrow R_1 \rightleftharpoons R_3$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & -2 \\ 0 & 2 & 2 \end{bmatrix} \leftarrow R_2' = R_1 + R_2$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \leftarrow \begin{matrix} R_2' = -\frac{1}{2} R_2 \\ R_3' = \frac{1}{2} R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow R_3' = -R_2 + R_3$$

$x + y - z = 0$

$y + z = 0$

let $z = a$

$y = -a$

$x = a + a$

$= 2a$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\therefore \; P = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = 0+1 = 1 \qquad M_{12} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1 \qquad M_{13} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$M_{21} = \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = -1-2 = -3 \qquad M_{22} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1 \qquad M_{23} = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = -1$$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 \qquad M_{32} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1-2 = -1 \qquad M_{33} = \begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} = 1$$

$$C_0 = \begin{vmatrix} 1 & -1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \longrightarrow C^T = \begin{vmatrix} 1 & 3 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\det(A) = 2$$

$$\therefore \; P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 3 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 3/2 & 1/2 \\ -1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$\therefore P^{-1} A P$

$$= \begin{bmatrix} 1/2 & 3/2 & 1/2 \\ -1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 & 2 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

Reaction Rate:

1. Rate expression
2. order
3. pre-condition

$$A + B \rightarrow P$$

Rate $\rightarrow \dfrac{-d[A]}{dt}$ $\quad A$ = Reactant

$[A]$ = concentration of reactant

Rate expression: $\dfrac{-d[A]}{dt} = \dfrac{-d[B]}{dt} = \dfrac{d[P]}{dt}$

Rate of consumption for $A \Rightarrow \dfrac{-d[A]}{dt}$

for $B \Rightarrow \dfrac{-d[B]}{dt}$

Rate of formation of $P \Rightarrow \dfrac{d[P]}{dt}$

$$A + 2B \rightarrow 3P$$

$A \rightarrow \dfrac{-d[A]}{dt}$ $\quad 2\times10^{-3}\ mL^{-1}s^{-1}$

$B \rightarrow \dfrac{-\cancel{2}d[\cancel{2}B]}{dt}$

$P \rightarrow \dfrac{\cancel{3}d[P]}{dt}$

$$\frac{-d[A]}{dt} = \frac{-\cancel{1}\,d[B]}{2\,dt} = \frac{\cancel{1}\,d[P]}{3\,dt}$$

(মোলকে fraction হিসেবে বসায় দিতে হবে)

$$\therefore -\frac{d[B]}{dt} = 4\times10^{-3}\ ml\ L^{-1}s^{-1}$$

$$\frac{d[P]}{dt} = 6\times10^{-3}\ ml\ L^{-1}s^{-1}$$

Rate equation / Law: $\quad A \longrightarrow P$

It is the relationship between rate & reactants' concentration.

Rate $\propto$ reactants' concentration

$$\frac{-d[A]}{dt} \propto [A] \Rightarrow \frac{-d[A]}{dt} = \underset{\text{rate constant}}{k}\,[A]$$

K - equilibrium constant

* reaction rate always depend on reactants' concentration.

except - <u>auto-catalytic reaction</u> (যেই reaction এ product catalyst হিসেবে কাজ করে)

$$KMnO_4 \rightarrow MnO_2$$

$A + 2B \rightarrow 3P$ write the rate eqn w.r. to A, B, P

$$-\frac{d[A]}{dt} = -\frac{1}{2}\frac{d[B]}{dt} = k[A][2B]$$

* In the rate eqn, power of the reactants' concentration is known as order. It is the experimental value.

$A + B \rightarrow P \rightarrow -\frac{d[A]}{dt} = k[A]^{\alpha}$ $[\alpha = 0, \frac{1}{2}, 1, 2, 3 \dots]$

$$-\frac{d[A]}{dt} = k[A]^m[B]^n$$

(order w.r.t. A = m, w.r.t. B = n, overall = m+n)

* for the elementary reaction: order = molecularity

$A + 2B \rightarrow 3P$

$-\frac{d[A]}{dt} = k[A]^1[B]^2$ (elementary)

order w.r.t. A = 1

" B = 2

overall = 1+2 = 3

(auto catalyst) $\rightarrow \frac{d[P]}{dt} = 3k[A]^m[B]^n$

$A + 2B \rightarrow 3P$

$$-\frac{dA}{dt} = k[A]^m[B]^n$$

$$-\frac{dB}{dt} = 2k[A]^m[B]^n$$

$$-\frac{d(A)}{dt} = -\frac{1}{2}\frac{d(B)}{dt} = \frac{1}{3}\frac{dP}{dt}$$

pre-condition to occur a reaction;

collision 2: to overcome the activation energy

proper orientation