BRAC University
Course Code: MAT 216

## Home work* Sheet # 1

**1.** Solve the following matrix equation for a, b, c and d .

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}.$$

**2.** Consider the matrices :

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix},$$

Compute the following (where possible)

**(a)** $D + E$ **(b)** $-7C$ , **(c)** $2B - C$, **(d)** $-3(D + 2E)$ , **(e)** $A - A$ , **(f)** $\operatorname{tr}(D - 3E)$.

**3.** Using the matrices in exercise **(2)** , compute the following (where possible) :

**(a)** $2A^T + C$, **(b)** $(2E^T - 3D^T)^T$, **(c)** $(D - E)^T$ , **(d)** $B^T + 5C^T$ , **(e)** $\frac{1}{2}C^T - \frac{1}{4}A$.

**4.** Using the matrices in exercise **(2)** , compute the following (where possible) .

**(a)** $AB$ , **(b)** $BA$ , **(c)** $(3E)D$ , **(d)** $(AB)C$ , **(e)** $A(BC)$ , **(f)** $(DA)^T$ ,

**(g)** $(C^T B) A^T$ , **(h)** $\operatorname{tr}(DD^T)$ , **(i)** $\operatorname{tr}(4E^T - D)$.

**5.** Using the matrices in exercise **(2)** , compute the following (where possible) :

**(a)** $(2D^T - E)A$ , **(b)** $(BA^T - 2C)^T$ .

**These problems are for the students only as home work. Search the reference books for more examples.*

BRAC University
Course Code: MAT 216

## Home work* Sheet # 2

**1.** Solve each of the following systems by Gaussain elimination or Gauss - Jordan elimination:

(i) $$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

(ii) $$\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$$

(iii) $$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}$$

**2.** Solve each of the following homogeneous system of linear equations by Gaussain elimination or Gauss - Jordan elimination:

(i) $$\begin{aligned} 2x_1 + 2x_2 - x_3 \quad + \quad x_5 &= 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\ x_1 + x_2 - 2x_3 \qquad - x_5 &= 0 \\ x_3 + x_4 + x_5 &= 0 \end{aligned}$$

(ii) $$\begin{aligned} 2x + 2y + 4z &= 0 \\ w \qquad - y - 3z &= 0 \\ 2w + 3x + y + z &= 0 \\ -2w + x + 3y - 2z &= 0 \end{aligned}$$

**3.** Determine the values of parameter $\lambda$ , such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution:

$$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + \lambda z &= 3 \\ x + \lambda y + 3z &= 2 \end{aligned} \quad .$$

**4.** Determine the values of parameters $\lambda$ & $\mu$, such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned} .$$

**5.** Determine the values of parameter (s) such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

(i) $$\begin{aligned} \lambda x + y + z &= 1 \\ x + \lambda y + z &= 1 \\ x + y + \lambda z &= 1 \end{aligned}$$

(ii ) $$\begin{aligned} x + y + kz &= 2 \\ 3x + 4y + 2z &= k \\ 2x + 3y - z &= 1 \end{aligned}$$

(iii) $$\begin{aligned} x \qquad - 3z &= -3 \\ 2x + \lambda y - z &= -2 \\ x + 2y + \lambda z &= 1 \end{aligned}$$

(iv) $$\begin{aligned} x + y + \lambda z &= 1 \\ x + \lambda y + z &= \lambda \\ \lambda x + y + z &= \lambda^2 \end{aligned}$$

**These problems are for the students only as home work. Search the reference books for more examples.*

6. Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$ ,

(a) Find all the minors of A
(b) Find all the cofactors of A,
(c) Find adj (A) ,
(d) Find $A^{-1}$ , Using $A^{-1} = \dfrac{1}{\det(A)}$ adj (A).

**7.** Find the inverse of the following matrices if it exists, using [**A: I]:**

(i) $\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (vii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

**8.** If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ , prove that $(AB)^{-1} = B^{-1}.\, A^{-1}$

**9.** Solve the following system of linear equations using **x = $\mathbf{A^{-1}}$ b**

(i) $\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 3 \end{aligned}$ (ii) $\begin{aligned} 5x_1 + 3x_2 + 2x_3 &= 4 \\ 3x_1 + 3x_2 + 2x_3 &= 2 \\ x_2 + x_3 &= 5 \end{aligned}$ (iii) $\begin{aligned} x + y + z &= 5 \\ x + y - 4z &= 10 \\ -4x + y + z &= 0 \end{aligned}$

**These problems are for the students only as home work. Search the reference books for more examples.*

BRAC University
Course Code: MAT 216

## Home work* Sheet # 3

**1.** Verify whether the following sets are subspace of $R^3 / R^4$ or not.

(i) S = {(x, 2y, 5) : x,y ϵ R }
(ii) S = {(x, x + y, 3z) : x, y, z ϵ R }
(iii) S = {(x, y, z) ϵ $R^3$: x - y + z = 0 }
(iv) S = {(x, y, z, t) ϵ $R^4$ : 3x - 2y - 2z - t = 0 }
(v) S = {(x, y, z) ϵ $R^3$: x + y + z = 0 }

**2.** Write the vectors (1, 0, 0) and (0, 0, 1) as linear combinations of vectors {(1, 0, -1), (0, 1, 0), (1, 0, 1)}

**3.** Determine whether or not,

(i) the vector (1, 2, 6) is a linear combination of (2, 1, 0), (1, -1, 2) & (0, 3, -4).
(ii) the vector (1, 1, 1) is a linear combination of (2, 1, 0), (1, -1, 2) & (0, 3, -4).
(iii) the vector (3, 9, -4, -2) is a linear combination of (1,-2, 0, 3), (2, 3, 0, -1) & (2, -1, 2, 1).
(iv) the vector (2, 3, -7, 3) is a linear combination of (2, 1, 0, 3), (3, -1, 5, 2) & (-1, 0, 2, 1).

**4.** Determine whether or not the following set of vectors span $R^3$,

(i) {(1, 1, 2), (1, -1, 2), (1, 0, 1)}
(ii) {(-1, 1, 0), (-1, 0, 1), (1, 1, 1)}
(iii) {(2, 1, 2), (0, 1, -1), (4, 3, 3)}

**5.** Determine whether each of the following sets are linearly independent or dependent:

(i) {(2 ,1 ,2) , (0 , 1 , - 1) , (4 , 3 , 3)} .
(ii) {(3 , 0 , 1 , -1) , (2 , -1, 0 , 1) , (1 , 1 , 1 , -2)} .
(iii) {(1 , - 4 , 2) , (3 , - 5 , 1) , (2 , 7 , 8) , (- 1 ,1 , 1)} .
(iv) {(0 , 1 , 0 , 1) , (1 , 2 , 3 , -1) , (8 , 4 , 3 , 2) , (0 , 3 , 2 , 0)} .
(v) {(1 , 3 , 2) , (1 , -7 , - 8) , (2 , 1 , - 1)} .
(vi) {(3 , 0 , 4 , 1) , (6 , 2 , -1 , 2) , (-1 , 3 , 5 , 1) , (- 3 , 7 , 8 , 3)}
(vii) {(4 , -4 , 8 , 0) , (2 , 2 , 4 , 0) , (6 , 0 , 0 , 2) , (6 , 3 , -3 , 0)} .

**6.** Determine whether each of the following sets form a basis for $R^3$ / $R^4$ :

(i) {(1 , 2 , 0) , (0 , 5 , 7) & (-1 , 1 , 3)}.
(ii) {(2 , 0 , 1) , (1 , 1 , 1)} .
(iii) {(1 , 1 , 1 , 1) , (0 , 1 , 1 , 1) , (0 , 0 , 1 , 1) , (0 , 0 , 0 , 1)} .

**These problems are for the students only as home work. Search the reference books for more examples.*

**7.** Find *a basis for the row space, a basis for the column space* and *the rank* of the following matrices:

**(i)** $A=\begin{bmatrix} 6 & 2 & 0 & 4 \\ -2 & -1 & 3 & 4 \\ -1 & -1 & 6 & 10 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

**(iv)** $A=\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **(v)** $A=\begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$.

**8.** Find *a basis for the Null space, the rank* and *the Nullity* of the following matrices:

(i) $A=\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

**(iv)** $A=\begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$

**9.** Find *a basis* and *dimension* of the subspace generated by the set of vectors
S = {(1 , 2 , 1) , (3 , 1 , 2) , (1 , -3 , 4)}.

**10.** Let **U** be the subspace of $R^3$ spanned (generated) by the set of vectors
S = {(1 , 2 , 1) , (0 , - 1 , 0) & (2 , 0 , 2)}. Find *a basis* and *dimension* of **U .**

**11.** Let **W** be the subspace of $R^4$ generated by the set of vectors
S = {(1 ,- 2 , 5 ,-3) , (2 , 3 , 1 , - 4) & (3 , 8 , - 3 , - 5)}. Find *a basis* and *dimension* of **W**.

**These problems are for the students only as home work. Search the reference books for more examples.*

BRAC University
Course Code: MAT 216

**Home work* Sheet # 4**

**1.** Determine whether each of the following Transformation is a linear transformation:
(i) $T(x,y,z)=(x-y,x-z)$ (ii) $T(x,y,z)=(3x-2y+z,x-3y-2z)$
(iii) $T(x,y,z)=(x+1,y+z)$. (iv) $T(x,y)=(x+y,1)$

**2.** Let T : $R^4 \to R^3$ be the linear transformation defined by
$T(x,y,z,t)=(x-y+z+t,x+2z-t,x+y+3z-3t)$.
Find a basis & dimension of the range space of (T) & the null space of (T).

**3.** Let T : $R^3 \to R^3$ be the linear transformation defined by
$T(x,y,z)=(x+2y-z,y+z,x+y-2z)$.
Find a basis & dimension of (i) Range(T) & (ii) Ker (T).

**4.** Let T : $R^3 \to R^3$ be the linear transformation defined by
$T(x,y,z)=(3x-y,y-z,3x-2y+z)$,
Find a basis & dimension of (i) Range(T) & (ii) Ker (T).

**5.** Let T : $R^3 \to R^3$ be the linear transformation defined by
$T(x,y,z)=(x+2y-3z,2x-y+4z,4x+3y-2z)$ ,
Find a basis & dimension of (i) Range (T) & (ii) Ker (T).

**6.** Find all eigenvalues and the corresponding eigenvectors of the following matrices:

**(i)** $A=\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

**7.** Find a matrix P that diagonalizes the following matrices , also find $P^{-1}AP$ :

**(i)** $A=\begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix}$

**Solve the following problems given in the book "Elementary linear algebra by Howard Anton and Chris Rorres, Application version, eigth edition."**

_ Ex 5.5: 3(a,b), 6(a,b), 7(a,b),8(a, b,c), 11(a,c),12(a,b)
_ Ex 5.6: 1, 2(a,b,c)
_ Ex 8.1: 13, 16
_ Ex 8.2: 3,4, 10, 11
_ Ex 7.1: Consider the matrix given in 4(a,c,d). Find the eigenvalues and their corresponding
Eigenvectors that form bases for eigenspace. If possible, diagonalize those matrices.

**These problems are for the students only as home work. Search the reference books for more examples.*