BRAC UNIVERSITY

Inspiring Excellence

MNS Department
Final Examination
Spring Semester, 2017
Course Title: Linear Algebra and Fourier Analysis
Course ID: MAT 216

Total Marks: 50
Times: 3 hours

Date: April 13, 2017

Instructions:

- **DO NOT** make any rough work on the question paper. Do it on the last page of your answer script.
- You **MUST** write your faculty initials at the top of the front page of your answer script.
- You **must return the question paper** along with your answer script.
- You must use your own calculator if needed.

SECTION A: Compulsory

Answer all questions

1. (i) "The $n \times n$ matrix A does have n distinct eigenvalues and one of these is zero, so matrix A is nonsingular", true or false? [1]

   (ii) If the rank of $n \times n$ matrix A is m, then what will be the nullity of this matrix? [1]

   (iii) When does a linear transformation have inverse transformation ? [1]

   (iv) Write the geometrical significance of $\iiint_G dV$. [1]

   (v) Sketch the odd extension of the function $f(x) = \sin x$, $0 < x < \pi$ and find its period. [1]

SECTION B

Answer any **One** from the following:

2. (i) When do you say that a matrix A is diagonalizable? [2]

   (ii) Find the eigenvalues of the matrix: [3]

$$A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{pmatrix}.$$

   (iii) Find the matrix P, if there is any, which will diagonalize the matrix A. [4]

3. (i) Define the kernel of a linear transformation. What is the rank of a linear transformation? [2]

(ii) Find the rank of the following matrix: [3]

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{pmatrix}.$$

(iii) Find out the condition on p, q, r so that the following system of non-homogeneous linear equations has a solution: [4]

$$\begin{aligned} x + 2y - 3z &= p \\ 3x - y + 2z &= q \\ 2x - 10y + 16z &= 2r. \end{aligned}$$

SECTION C

Answer any **Three** from the following:

4. (i) Write the transformation formulas from three dimensional Cartesian coordinates to spherical coordinates. [2]

(ii) Evaluate: $\iint_R (3x - 2y)dA$, R is the region enclosed by the circle $x^2 + y^2 = 1$. [3]

(iii) Use triple integral to find out the volume of the solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$. [4]

5. (i) Evaluate: $\oint_C (x^2 - y)dx + xdy$, where C is the circle $x^2 + y^2 = 4$. [2]

(ii) Determine whether the given vector field is a conservative: [3]

$$\mathbf{F}(x, y) = e^{-y}\cos x\,\hat{i} - e^{-y}\sin x\,\hat{j}.$$

If it is, then find the potential function for it.

(iii) Use the transformation $u = x - y$ and $v = x + y$ to find: [4]

$$\iint_R \frac{e^{x-y}}{x+y}\,dA$$

over the rectangular region R enclosed by the lines $y = x$, $y = 5 + x$, $y = 2 - x$, and $y = 4 - x$.

6. (i) Convert the following integration into cylindrical coordinates: [2]

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2 + y^2 + z^2}\,dz\,dy\,dx.$$

(ii) Convert the following integration into spherical coordinates: [3]

$$\int_{0}^{a} \int_{0}^{\sqrt{a^2-x^2}} \int_{0}^{a^2-x^2-y^2} x^2 dz\,dy\,dx.$$

(iii) Find the volume of the solid enclosed between the paraboloids: $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$. [4]