

## MAT 120

### Beta and Gamma Functions

1. Evaluate in terms of gamma function:

$$\begin{aligned} & (i) \int_0^4 x^{3/2} (4-x)^{5/2} dx, \quad (ii) \int_0^b y^5 \sqrt{b^2 - y^2} dy, \quad (iii) \int_0^\infty e^{-t^2} dt, \\ & (iv) \int_0^\infty x^5 e^{-4x} dx, \quad (v) \int_0^\infty e^{-y^2} y^5 dy, \quad (vi) \int_0^\infty e^{-x^2} dx, \quad (vii) \int_0^\infty x^6 e^{-3x} dx, \\ & (viii) \int_0^\infty e^{-x^2} x^9 dx, \quad (ix) \int_0^\infty \sqrt{x} e^{-x^2} dx, \quad (x) \int_0^1 \frac{x^3}{\sqrt{1-x^3}} dx, \\ & (xi) \int_0^1 \frac{1}{\sqrt{x \ln(1/x)}} dx, \quad (xii) \int_0^1 \left(1 - \frac{1}{x}\right)^{1/3} dx. \end{aligned}$$

2. Evaluate in terms of beta function:

$$\begin{aligned} & (i) \int_0^1 \frac{x^2}{\sqrt{1-x}} dx, \quad (ii) \int_0^1 x^7 (1-x)^3 dx, \quad (iii) \int_0^1 \frac{1}{\sqrt{1-x^3}} dx, \\ & (iv) \int_0^1 (1-x)^{1/2} x^3 dx, \quad (v) \int_0^1 x^{5/2} (1-x)^{3/2} dx, \\ & (vi) \int_0^a y^7 \sqrt{a^4 - y^4} dy, \quad (vii) \int_0^4 y^3 \sqrt{64 - y^3} dy, \\ & (viii) \int_0^1 x^2 (1-x^3)^{3/2} dx, \quad (ix) \int_0^\infty \frac{1}{1+x^4} dx. \end{aligned}$$

3. Evaluate the following integrals:

$$\begin{aligned} & (i) \int_0^\pi \sin^5 \theta \cos^4 \theta d\theta, \quad (ii) \int_0^\pi \sin^6 \theta \cos^7 \theta d\theta, \\ & (iii) \int_0^{\pi/6} \sin^2 6x \cos^4 3x dx, \quad (iv) \int_0^{\pi/4} \sin^2 4\theta \cos^3 2\theta d\theta, \\ & (v) \int_0^{\pi/2} \sin^4 \theta \cos^2 \theta d\theta, \quad (vi) \int_0^{\pi/8} \sin^2 8x \cos^4 4x dx. \end{aligned}$$

## **Formula**

1.  $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx, \quad \text{where } n > 0$
2.  $\beta(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx, \quad \text{where } m > 0, n > 0.$
3.  $\int_0^{\pi/2} \sin^p x \cos^q x dx = \frac{\Gamma(\frac{p+1}{2}) \Gamma(\frac{q+1}{2})}{2 \Gamma(\frac{p+q+2}{2})}$
4.  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
5.  $\Gamma(n) = (n-1)!$
6.  $\Gamma(n+1) = n\Gamma(n) = n!$
7.  $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
8.  $\Gamma(1) = 1$
9.  $\Gamma(\frac{p}{2}) = (\frac{p}{2}-1)(\frac{p}{2}-2)(\frac{p}{2}-3)\dots\dots\dots\frac{1}{2} \cdot \Gamma(\frac{1}{2})$
10.  $\Gamma(n) = (n-1)(n-2)\dots\dots\dots 3.2.1$