Double Integral

① $\int_0^1 \int_0^2 (x+3)\, dy\, dx$

$= \int_0^1 [xy + 3y]_0^2\, dx$

$= \int_0^1 [(2x+6) - (0+0)]\, dx$

$= \int_0^1 [2x+6]\, dx$

$= \left[\frac{2x^2}{2} + 6x\right]_0^1$

$= [x^2 + 6x]_0^1$

$= (1+6) - (0+0)$

$= 7$

Ans

② $\int_1^3 \int_{-1}^1 (2x - 4y)\, dy\, dx$

$= \int_1^3 \left[2xy - 4\frac{y^2}{2}\right]_{-1}^1 dx$

$= \int_1^3 ((2x-2) - (2x-2))\, dx$

$= \int_1^3 2x - 2 - 2x + 2\, dx$

$= \int_1^3 4x\, dx$

$= 4\left[\frac{x^2}{2}\right]_1^3$

$= 4 \times \left(\frac{9}{2} - \frac{1}{2}\right)$

$= 4 \times \frac{9-1}{2}$

$= 2 \times 8 = 16$

③ $$\int_2^4\int_0^1 x^2 y \, dx\, dy$$

$$= \int_2^4 \left[y\frac{x^3}{3} \right]_0^1 dy$$

$$= \int_2^4 y\left(\frac{1}{3} - 0\right) dy$$

$$= \int_2^4 y \, \tfrac{1}{3} \, dy$$

$$= \tfrac{1}{3} \left[\frac{y^2}{2} \right]_2^4$$

$$= \tfrac{1}{3} \times \left(\frac{16}{2} - \tfrac{4}{2} \right)$$

$$= \tfrac{1}{3} \times \frac{16-4}{2}$$

$$= \tfrac{6}{3}$$

$$= 2$$

④ $$\int_{-2}^{0}\int_{-1}^{2} (x^2+y^2)\, dx\, dy$$

$$= \int_{-2}^{0} \left[\tfrac{x^3}{3} + xy^2 \right]_{-1}^{2} dy$$

$$= \int_{-2}^{0} \left(\left(\tfrac{8}{3} + 2y^2\right) - \left(-\tfrac{1}{3} - y^2\right) \right) dy$$

$$= \int_{-2}^{0} \left(\tfrac{8}{3} + 2y^2 + \tfrac{1}{3} + y^2 \right) dy$$

$$= \int_{-2}^{0} \left(\tfrac{9}{3} + 3y^2 \right) dy$$

$$= \left[\tfrac{9}{3}y + 3\tfrac{y^3}{3} \right]_{-2}^{0}$$

$$= \left(3y + y^3 \right)_{-2}^{0}$$

$$= (0+0) - (-6-8)$$

$$= -(-14$$

$$= 14$$

⑤ $$\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\,dy\,dx$$

$$= \int_0^{\ln 3}\int_0^{\ln 2} e^{x}\cdot e^{y}\,dy\,dx$$

$$= \int_0^{\ln 3} e^{x}\left[e^{y}\right]_0^{\ln 2}\,dx$$

$$= \int_0^{\ln 3} e^{x}\left(e^{\ln 2}-e^{0}\right)dx$$

$$= \int_0^{\ln 3} e^{x}(2-1)\,dx$$

$$= \int_0^{\ln 3} e^{x}\,dx$$

$$= \left[e^{x}\right]_0^{\ln 3}$$

$$= e^{\ln 3}-e^{0}$$

$$= 3-1$$

$$= 2$$

⑥ ** $$\int_0^{2}\int_0^{1} y\,\sin x\,dy\,dx$$

$$= \int_0^{2} \sin x\left[\frac{y^2}{2}\right]_0^{1}dx \quad \frac{1}{2}$$

$$= \int_0^{2} \frac{\sin x}{2}\,dx$$

$$= -\frac{1}{2}\left[\cos x\right]_0^{2}$$

$$= -\frac{1}{2}\left[\cos 2-\cos 0\right]$$

$$= -\frac{1}{2}\left(\cos 2-1\right)$$

~~.956~~

.4782

(7) $\int_{-1}^{0}\int_{2}^{5} dx\,dy$

$= \int_{-1}^{0} [x]_{2}^{5}\, dy$

$= \int_{-1}^{0} (5-2)\, dy$

$= 3\int_{-1}^{0} dy$

$= 3\,[y]_{-1}^{0}$

$= 3\,(0+1)$

$= 3$

(8) $\int_{4}^{6}\int_{-3}^{7} dy\,dx$

$= \int_{4}^{6} [y]_{-3}^{7}\, dx$

$= \int_{4}^{6} (7+3)\, dx$

$= 10\int_{4}^{6} dx$

$= 10\,[x]_{4}^{6}$

$= 10\,(6-4)$

$= 10 \times 2 = 20$

(9) $$\int_0^1\int_0^1 \frac{x}{(xy+1)^2}\,dy\,dx$$

let

$xy+1=z$

$\frac{d}{dy}(xy+1)=\frac{dz}{dy}$

$x=\frac{dz}{dy}$

$dz = xdy$

limit

$y=0$ $\quad y=1$

$z=1$ $\quad z=x+1$

$$=\int_0^1\int_1^{x+1}\frac{1}{z^2}\,dz\,dx$$

$$=-\int_0^1\left[\frac{1}{z}\right]_1^{x+1}dx$$

$$=-\int_0^1\left(\frac{1}{x+1}-1\right)dx$$

$$=-\int_0^1\frac{1-x-1}{x+1}\,dx$$

$$=\int_0^1\frac{x}{x+1}\,dx$$

$$=-\int_0^1\frac{1}{x+1}-1\,dx$$

$$=-\Big[\ln x+1\Big]_0^1+\Big[x\Big]_0^1$$

$$=-(\ln 2-\ln 1)+(1-0)$$

$$=-\ln 2+\ln 1+1$$

$$=1-\ln 2$$

(10)

$$\int_{\pi/2}^{\pi}\int_{1}^{2} x\cos xy \, dy\, dx$$

$$= \int_{\pi/2}^{\pi} \frac{x}{x}\Big[\sin xy\Big]_1^2 \, dx$$

$$= \int_{\pi/2}^{\pi} (\sin 2x - \sin x)\, dx$$

$$= -\left[\frac{\cos 2x}{2}\right]_{\pi/2}^{\pi} + \Big[\cos x\Big]_{\pi/2}^{\pi}$$

$$= -\frac{1}{2}(\cos 2\pi - \cos\pi) + (\cos\pi - \cos\pi/2)$$

$$= -2$$

(11)

$$\int_0^{\ln 2}\int_0^1 xy\, e^{y^2 x}\, dy\, dx$$

Let

$$\frac{d(y^2x)}{dy} = \frac{dz}{dy}$$

$$2yx = \frac{dz}{dy}$$

$$dz = 2xy\, dy$$

$$= \frac{1}{2}\int_0^{\ln 2}\int_0^1 2xy\, e^{y^2x}\, dy\, dx$$

$$= \frac{1}{2}\int_0^{\ln 2}\int_0^{x} e^{z}\, dz\, dx$$

limit

$y = 0$ $y = 1$

$z = 0$ $z = x$

$$= \frac{1}{2}\int_0^{\ln 2}\Big[e^{z}\Big]_0^{x} dx$$

$$= \frac{1}{2} \int_0^{\ln 2} (e^x - e^0)\, dx$$

$$= \frac{1}{2} \int_0^{\ln 2} e^x - 1\, dx$$

$$= \frac{1}{2} \left[e^x - x \right]_0^{\ln 2}$$

$$= \frac{1}{2} \left(e^{\ln 2} - \ln 2 - e^0 \right) + (1 - 0) \quad = \frac{1}{2} (2 - \ln 2 - 1)$$

$$= \cancel{\frac{1}{2}(2 - 0 + 1)} \;\; \frac{1}{2} (1 - \ln 2)$$

$$= \cancel{\frac{1}{2}} \;\; \frac{1 - \ln 2}{2}$$

(12) $$\int_3^4 \int_1^2 \frac{1}{(x+y)^2}\, dy\, dx$$

let

$$\frac{d(x+y)}{dy} = \frac{dz}{dy}$$

$$0 + 1 = \frac{dz}{dy}$$

$$dz = dy$$

limit

$y = 1 \quad y = 2$

$z = 1 + x \quad z = 2 + x$

$$= \int_3^4 \int_{1+x}^{2+x} \frac{1}{z^2}\, dz\, dx$$

$$= -\int_3^4 \cancel{\int_{1+x}^{2+x}} \left[\frac{1}{z} \right]_{1+x}^{2+x} dx$$

$$= -\int_3^4 \left(\frac{1}{2+x} - \frac{1}{1+x} \right) dx$$

$$= -\int_3^4 \frac{1+x-2-x}{(1+x)(2+x)} dx$$

$$= +\int_3^4 \frac{1}{(1+x)(2+x)} dx$$

$$= -\int_3^4 \frac{1}{2+x} - \frac{1}{1+x} dx$$

$$= -\left[\ln 2+x\right]_3^4 + \left[\ln 1+x\right]_3^4$$

$$= -(\ln 6 - \ln 5) + (\ln 5 - \ln 4)$$

$$= -\ln 6 + \ln 5 + \ln 5 - \ln 4$$

$$= -\ln 6 + 2\ln 5 - \ln 4$$

$$\frac{A}{1+x} + \frac{B}{2+x}$$

$$1 = A(2+x) + B(1+x)$$

$$1 = 0 - B$$

$$B = -1$$

$$1 = A(2-1) + 0$$

$$A = 1$$

$$\frac{1}{1+x} - \frac{1}{2+x}$$

$$= \frac{2+x-1-x}{(1+x)(2+x)}$$

$$= \frac{1}{(1+x)(2+x)}$$

(13)

$$\iint_R 4xy^3 \, dA \quad R = \{(x,y): -1 \le x \le 1, -2 \le y \le 2\}$$

$$\int_{-1}^{1}\int_{-2}^{2} 4xy^3 \, dy \, \cancel{dx}$$

$$= \int_{-1}^{1} x \left[4 \times \frac{y^4}{4}\right]_{-2}^{2} dx$$

$$= \int_{-1}^{1} x(2^4 - 2^4) \, dx$$

$$= \int_{-1}^{1} 0 \, dx$$

$$= 0$$

(14)

$$\iint_R \frac{xy}{\sqrt{x^2+y^2+1}} \, dA$$

$$R = \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$$

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}} \, dx \, dy$$

let

$$\frac{d(x^2+y^2+1)}{dx} = \frac{dz}{dx}$$

$$2x = \frac{dz}{dx}$$

$$dz = 2x \, dx$$

$x = 0 \quad x = 1$

$z = 1+y^2 \quad z = 2+y^2$

$$= \frac{1}{2}\int_0^1\int_{1+y^2}^{2+y^2} \frac{xy}{\sqrt{z}} \, dz \, dy$$

$$= \int_0^1 y \int_{1+y^2}^{2+y^2} \frac{1}{2\sqrt{z}} \, dz \, dy$$

$$= \int_0^1 y \left[\sqrt{z}\right]_{1+y^2}^{2+y^2} dy$$

$$= \int_0^1 y\left(\sqrt{2+y^2} - \sqrt{1+y^2}\right) dy$$

$$= \int_0^1 y\sqrt{2+y^2} \, dy - \int_0^1 y\sqrt{1+y^2} \, dy$$

(13)

$$\iint_R 4xy^3 \, dA \quad R = \{(x,y) : -1 \le x \le 1,\ -2 \le y \le 2\}$$

$$\int_{-1}^{1}\int_{-2}^{2} 4xy^3 \, dy\, dx$$

$$= \int_{-1}^{1} x \left[4 \times \frac{y^4}{4}\right]_{-2}^{2} dx$$

$$= \int_{-1}^{1} x\left(2^4 - 2^4\right) dx$$

$$= \int_{-1}^{1} 0 \, dx$$

$$= 0$$

(14)

$$\iint_R \frac{xy}{\sqrt{x^2+y^2+1}} \, dA$$

$$R = \{(x,y) : 0 \le x \le 1,\ 0 \le y \le 1\}$$

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}} \, dx\, dy$$

let

$$= \frac{1}{2}\int_0^1\int_{1+y^2}^{2+y^2} \frac{xy}{\sqrt{z}} \, dz\, dy \qquad \frac{d(x^2+y^2+1)}{dx} = \frac{dz}{dx}$$

$$2x = \frac{dz}{dx}$$

$$= \int_0^1 y \int_{1+y^2}^{2+y^2} \frac{1}{2\sqrt{z}} \, dz\, dy \qquad dz = 2x\,dx$$

$x = 0 \quad x = 1$

$z = 1+y^2 \quad z = 2+y^2$

$$= \int_0^1 y \left[\sqrt{z}\right]_{1+y^2}^{2+y^2} dy$$

$$= \int_0^1 y\left(\sqrt{2+y^2} - \sqrt{1+y^2}\right) dy$$

$$= \int_0^1 y\sqrt{2+y^2}\, dy - \int_0^1 y\sqrt{1+y^2}\, dy$$

$$= \int_0^1 y\sqrt{y^2+2}\,dy - \int_0^1 y\sqrt{y^2+1}\,dy$$

$$= \frac{1}{2}\int_2^3 \sqrt{z}\,dz - \frac{1}{2}\int_1^2 \sqrt{s}\,ds$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}\left[z^{3/2}\right]_2^3 - \frac{1}{2} \times \frac{2}{3}\left[\cancel{z}^{3/2}\right]_1^2$$

$$= \frac{1}{3}\left(3^{3/2} - 2^{3/2}\right) - \frac{1}{3}\left[2^{3/2} - 1\right]$$

$$= \frac{1}{3}\left(3^{3/2} - 2^{3/2} - 2^{3/2} + 1\right)$$

let

$y^2+2 = z$

$\frac{d(y^2+2)}{dy} = \frac{dz}{dy}$

$2y = \frac{dz}{dy}$

$y=0 \quad y=1$

$z=2 \quad z=3$

$y^2+1 = s$

$2y = \frac{ds}{dy}$

$y=0 \quad y=1$

$s=1 \quad s=2$

(15)

$$\iint_R x\sqrt{1-x^2}\,dA \quad R = \{(x,y) : 0 \le x \le 1;\ 2 \le y \le 3\}$$

$$\int_2^3 \int_0^1 x\sqrt{1-x^2}\,dx\,dy$$

$$= -\frac{1}{2}\int_2^3 \int_1^0 \sqrt{z}\,dz\,dy$$

let

$1-x^2 = z$

$\frac{d(1-x^2)}{dx} = \frac{dz}{dx}$

$-2x = \frac{dz}{dx}$

limit

$x=0 \quad x=1$

$z=1 \quad z=0$

$$= -\frac{1}{2} \times \frac{2}{3} \int_2^3 \left[z^{3/2} \right]_1^0 dy$$

$$= \frac{1}{3} \int_2^3 dy$$

$$= \frac{1}{3} \left[y \right]_2^3$$

$$= \frac{1}{3} (3-2)$$

$$= 1/3$$

(16) $\iint_R (x \sin y - y \sin x) \, dA \quad R = \{ (x,y) ; \, 0 \le x \le \pi/2 , \, 0 \le y \le \pi/3 \}$

$$= \int_0^{\pi/3} \int_0^{\pi/2} x \sin y - y \sin x \, dx \, dy$$

$$= \int_0^{\pi/3} \cancel{\int_0^{\pi}} \sin y \left[\frac{x^2}{2} \right]_0^{\pi/2} + y \left[\cos x \right]_0^{\pi/2} dy$$

$$= \int_0^{\pi/3} \sin y \left(\frac{\pi^2/4}{2} - 0 \right) + y \left(\cos \pi/2 - \cos 0 \right) dy$$

$$= \int_0^{\pi/3} \frac{\pi^2}{8} \sin y - y \, dy$$

$$= -\frac{\pi^2}{8} \left[\cos y \right]_0^{\pi/3} - \left[\frac{y^2}{2} \right]_0^{\pi/3}$$

$$= -\frac{\pi^2}{8}\left(\cos \pi/3 - \cos 0\right) - \left(\frac{\pi^2}{18} - 0\right)$$

$$= -\pi^2/8 \left(\frac{1}{2} - 1\right) - \left(\pi^2/18\right)$$

$$= -\pi^2/8 \times \frac{1}{2} + \pi^2/8 - \pi^2/18$$

$$= -\frac{\pi^2}{16} + \pi^2/8 - \pi^2/18$$

$$= .07$$

14.2

(1) $$\int_0^1 \int_{x^2}^{x} x y^2 \, dy \, dx$$

$$= \int_0^1 \frac{x}{3}\left[y^3\right]_{x^2}^{x} dx$$

$$= \frac{1}{3}\int_0^1 x\left(x^3 - x^6\right) dx$$

$$= 1/3 \int_0^1 x^4 - x^7 \, dx$$

$$= \frac{1}{3}\left(\left[\frac{x^5}{5}\right]_0^1 - \left[\frac{x^8}{8}\right]_0^1\right)$$

$$= 1/3\left(\frac{1}{5}(1-0) - \frac{1}{8}(1-0)\right)$$

$$= 1/3\left(1/5 - 1/8\right)$$

$$= 1/3\left(\frac{8-5}{40}\right)$$

$$= 1/3 \times 3/40 = 1/40$$

(2) $$\int_1^{3/2}\int_y^{3-y} y\,dx\,dy$$

$$= \int_1^{3/2} y\,[x]_y^{3-y}\,dy$$

$$= \int_1^{3/2} y\,(3-y-y)\,dy$$

$$= \int_1^{3/2} y(3-2y)\,dy$$

$$= \int_1^{3/2} 3y - 2y^2\,dy$$

$$= \frac{3}{2}[y^2]_1^{3/2} - \frac{2}{3}[y^3]_1^{3/2}$$

$$= 3/2\left((3/2)^2 - (1)^{3/2}\right) - 2/3\left((3/2)^3 - (1)^3\right)$$

$$= 7/24$$

(3) $$\int_0^3 \int_0^{\sqrt{9-y^2}} y \, dx \, dy$$

$$= \int_0^3 y \left[x \right]_0^{\sqrt{9-y^2}} dy$$

$$= \int_0^3 y \left(\sqrt{9-y^2} \right) dy$$

$$= -\frac{1}{2} \int_9^0 \sqrt{z} \, dz$$

$$= -\frac{1}{2} \times \frac{2}{3} \left[z^{3/2} \right]_9^0$$

$$= -\frac{1}{3} \left(-9^{3/2} \right)$$

$$= 9$$

let

$$9 - y^2 = z$$

$$\frac{d(9-y^2)}{dy} = \frac{dz}{dy}$$

$$-2y = \frac{dz}{dy}$$

if $y = 0$	$y = 3$
then $z = 9$	$z = 0$

(4) $$\int_{1/4}^1 \int_{x^2}^{x} \frac{\sqrt{x}}{\sqrt{y}} \, dy \, dx$$

$$= \int_{1/4}^1 \sqrt{x} \int_{x^2}^{x} y^{-1/2} \, dy \, dx$$

$$= \int_{1/4}^1 \sqrt{x} \, 2 \left[y^{1/2} \right]_{x^2}^{x} dx \quad = 2 \int_{1/4}^1 \sqrt{x} \left(\sqrt{x} - x \right) dx$$

$$= 2\left[\int_{1/4}^{1} x\,dx - \int_{1/4}^{1} x^{3/2}\,dx\right]$$

$3/2 + 1 \quad \frac{3+}{2}$

$$= 2\left[\left[\frac{x^2}{2}\right]_{1/4}^{1} - \frac{2}{5}\left[x^{5/2}\right]_{1/4}^{1}\right]$$

$$= 2\left[\left(\frac{1}{2} - \frac{1}{32}\right) - \frac{2}{5}\left(1 - \frac{1}{32}\right)\right]$$

$$= 13/80$$

(5)

$$\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_{0}^{x^3} \sin\frac{y}{x}\,dy\,dx$$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\frac{1}{x}} x \left[\cos y/x\right]_0^{x^3} dx$$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\frac{1}{x}} x\left(\cos x^3/x - \cos 0\right) dx$$

let
$x^2 = z$
$2x\,dx = dz$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\frac{1}{x}} x\left(\cos x^2 - 1\right) dx$$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \frac{x\cos x^2}{x}\,dx + \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\frac{1}{x}} x\,dx = -\frac{1}{2}\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cos z\,dz + \frac{1}{2}\left[x^2\right]_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= -\frac{1}{2}\left[\sin z\right]_{\sqrt{\pi}}^{\sqrt{2\pi}} + \frac{1}{2}\left[2\pi - \pi\right]$$

$$= -1/2\left(\sin 2\pi - \sin\pi\right) + 1/2\,\pi$$

$$= 2\left[\int_{1/4}^{1} x\,dx - \int_{1/4}^{1} x^{3/2}\,dx\right]$$

$3/2 + 1 \quad \frac{3+}{2}$

$$= 2\left[\left[\frac{x^2}{2}\right]_{1/4}^{1} - \frac{2}{5}\left[x^{5/2}\right]_{1/4}^{1}\right]$$

$$= 2\left[\left(\frac{1}{2} - \frac{1}{32}\right) - \frac{2}{5}\left(1 - \frac{1}{32}\right)\right]$$

$$= 13/80$$

(5)

$$\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_{0}^{x^3} \sin\frac{y}{x}\,dy\,dx$$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\tfrac{1}{x}}x\left[\cos y/x\right]_0^{x^3} dx$$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\tfrac{1}{x}}x\left(\cos x^3/x - \cos 0\right) dx$$

Let
$x^2 = z$
$2x\,dx = dz$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\tfrac{1}{x}}x\left(\cos x^2 - 1\right) dx$$

$$= -\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \frac{x\cos x^2}{x}\,dx + \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cancel{\tfrac{1}{x}}\,x\,dx = \cancel{\tfrac{1}{x}} - \frac{1}{2}\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \cos z\,dz + \frac{1}{2}\left[z^2\right]_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= -\frac{1}{2}\left[\sin z\right]_{\sqrt{\pi}}^{\sqrt{2\pi}} + \frac{1}{2}\left[2\pi - \pi\right]$$

$$= -1/2\left(\sin 2\pi - \sin\pi\right) + 1/2\,\pi$$

$$= -\tfrac{1}{2}(0 - 0) + \tfrac{1}{2}\pi$$

$$= \pi/2$$

6 $$\int_{-1}^{1}\int_{-x^2}^{x^2} (x^2 - y)\, dy\, dx$$

$$= \int_{-1}^{1} \left[x^2 y - \frac{y^2}{2}\right]_{-x^2}^{x^2} dx$$

$$= \int_{-1}^{1} \left(\left(x^2 x^2 - \frac{x^4}{2}\right) - \left(-x^2 \cdot x^2 - \frac{x^4}{2}\right)\right) dx$$

$$= \int_{-1}^{1} \left(x^4 - \frac{x^4}{2} + x^4 + \frac{x^4}{2}\right) dx$$

$$= \int_{-1}^{1} 2x^4 \cdot dx = 2\int_{-1}^{1} x^4 \cdot dx = \frac{2}{5}\left[x^5\right]_{-1}^{1}$$

$$= 2/5\,(1+1) = 4/5 = 4/5$$

(7) $$\int_0^1\int_0^x y\sqrt{x^2-y^2}\,dy\,dx$$

$x^2-y^2=z$

$-2y\,dy=dz$

$y=0 \quad z=x^2$

$y=x \quad z=0.$

$$=-\frac{1}{2}\int_0^1\int_0^x -2y\sqrt{x^2-y^2}\,dy\,dx$$

$$=-\frac{1}{2}\int_0^1\int_0^x\sqrt{z}\,dz\,dx$$

$$=-\frac{1}{2}\int_0^1\frac{2}{3}\left[z^{3/2}\right]_{\cancel{0}\,x^2}^{\cancel{x}\,0}dx$$

$$=-\frac{1}{2}\times\frac{2}{3}\int_0^1 \cancel{(x^{3/2}-x^0)\,dx} \qquad -\frac{1}{3}\int_0^1 -x^3\,dx$$

$$=-\frac{1}{3}\int_0^1 x^{3/2}-1\,dx \qquad =\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$$

$$=-\frac{1}{3}\times\frac{2}{5}\left[x^{5/2}\right]_0^1+\frac{1}{3}\left[\cancel{x}\right]_0^1$$

$3\,\underline{|\,15\,|\,3}$

$\;\;5\,|\,1\,|\,1$

$$=-\frac{2}{15}\times 1+\frac{1}{3}$$

$$=\frac{-2+5}{15}=\frac{3}{15}=\frac{1}{5}$$

(8) $\int_1^2 \int_0^{y^2} e^{x/y^2}\,dx\,dy$

$= \int_1^2 y^2 \left[e^{x/y^2} \right]_0^{y^2} dy$

$= \int_1^2 y^2 \left(e^{y^2/y^2} - e^0 \right) dy$

$= \int_1^2 y^2 (e-1)\, dy$

$= (e-1) \int_1^2 y^2\, dy$

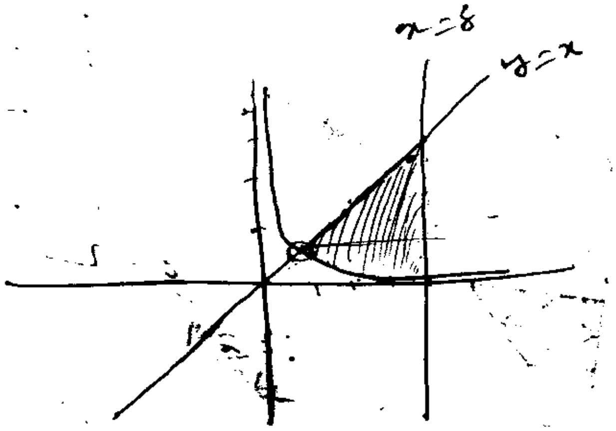
$= \frac{e-1}{3} (2^3 - 1^3)$

$= \frac{e-1}{3} \times 7$

$= \frac{7(e-1)}{3}$

(15)

$\iint_R x^2\, dA$ R is the region bounded by $y = 16/x$, $y = x$, $x=8$



$y = x$ $\quad x = \frac{16}{x}$

$y = \frac{16}{x}$ $\quad x^2 = 4$

$x = y$ $\quad x = \pm 4$

$y = x$ $\quad y = 16/x$

$x = y$

$x = \pm 4$

$x = 4$

Type-2

$\cancel{\int_{4}^{8}\int_{y}^{8} x^2 \, dx\, dy}$

Type-1

$$\int_4^8 \int_{16/x}^{x} x^2 \, dy\, dx$$

$$= \int_4^8 x^2 \Big[y \Big]_{16/x}^{x} dx$$

$$= \int_4^8 x^2 (x - 16/x)\, dx$$

$$= \int_4^8 x^2 \left(\frac{x^2 - 16}{x} \right) dx$$

$$= \int_4^8 x(x^2-16)\, dx = \int_4^8 x^3 - \int_4^8 16x$$

$$= \frac{1}{4}\left[x^4\right]_4^8 - 8\left[x^2\right]_4^8$$

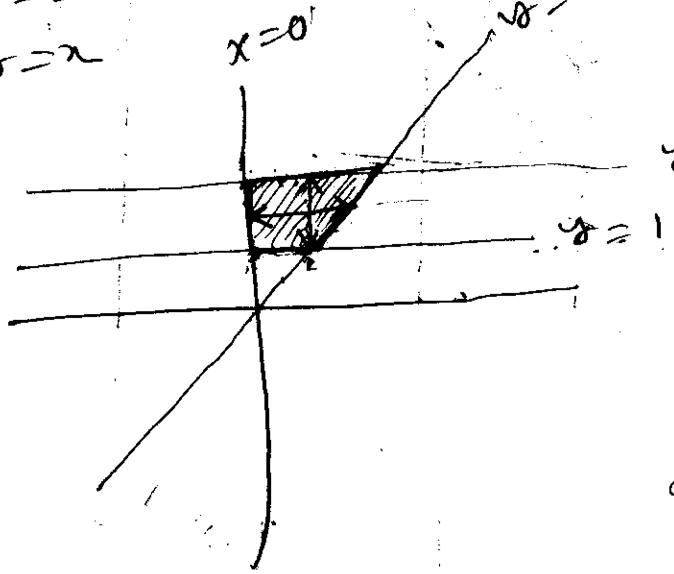
$$= \frac{1}{4}(8^4 - 4^4) - 8(8^2 - 4^2)$$

$$= 576$$

(6) $\iint_R xy^2\,dA$ R is the region bounded by

$y=1$
$y=x$
$x=1$

$y=1$, $y=2$
$x=0$ $y=x$



$y=1$
$y=x$

ask mam

Type I

$$\cancel{\int_1^2\int_0^1 xy^2\,dx\,dy + \int_1^2\int_1^2 xy^2\,dx\,dy}\ dy$$

$$\int_0^1\int_1^2 xy^2\,dy\,dx + \int_1^2\int_x^2 xy^2\,dy\,dx$$ ← ask mam

$$= \int_0^1 x\left[\frac{y^3}{3}\right]_1^2 dx + \int_1^2 x\left[\frac{y^3}{3}\right]_x^2 dx$$

$$= \frac{1}{3}\int_0^1 x(8-1)\,dx + \frac{1}{3}\int_1^2 x(8-x^3)\,dx$$

$$= \frac{1}{3}\int_0^1 8x - x \, dx + \frac{1}{3}\int_1^2 8x - x^4 \, dx$$

$$= \frac{1}{3}\left[\frac{8x^2}{2} - \frac{x^2}{2}\right]_0^1 + \frac{1}{3}\left[\frac{8x^2}{2} - \frac{x^5}{5}\right]_1^2$$

$$= \frac{1}{3}\left(\frac{7}{2} - 0\right) + \frac{1}{3}\left(\cancel{\frac{8}{2} - \frac{1}{8}}\right)\left(\frac{8 \times 4}{2} - \frac{32}{5} - \frac{8}{2} + \frac{1}{5}\right)$$

$$= \frac{1}{3} \cancel{\times \frac{7}{2} + \frac{1}{3} \times \frac{40 - 2}{10}} = \frac{7}{6} + \frac{1}{3} \times \frac{29}{5}$$

$$= \cancel{\frac{7}{6} + \frac{38}{30} = \frac{73}{30}} = \frac{31}{10}$$

Type = 2

$$\int_1^2 \int_0^y x y^2 \, dx \, dy$$

$$= \int_1^2 y^2 \left[\frac{x^2}{2}\right]_0^y dy$$

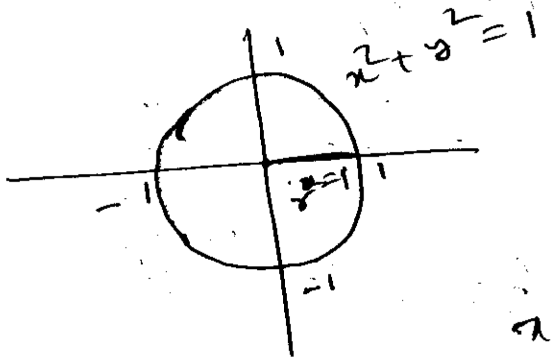
$$= \int_1^2 y^2 \left(\frac{y^2}{2}\right) dy$$

$$= \frac{1}{2}\int_1^2 y^4 \, dy$$

$$= \frac{1}{10}\left[y^5\right]_1^2 = \frac{31}{10}$$

(17)

$\iint_R x(3x-2y)\, dA$ R is enclosed by the circle $x^2+y^2=1$



$x = \sqrt{1-y^2}$

$y = \sqrt{1-x^2}$

$x(3x-2y)$

Typ 2

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3x - 2y \; dx\, dy$$

$$= \int_{-1}^{1}\left[\left[3\frac{x^2}{2}\right]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} - \left[2xy\right]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\right] dy$$

$$= \int_{-1}^{1}\left[3\left(\frac{1-y^2}{2} - \frac{1-y^2}{2}\right) - 2x\left(\sqrt{1-y^2}+\sqrt{1-y^2}\right)\right] dy$$

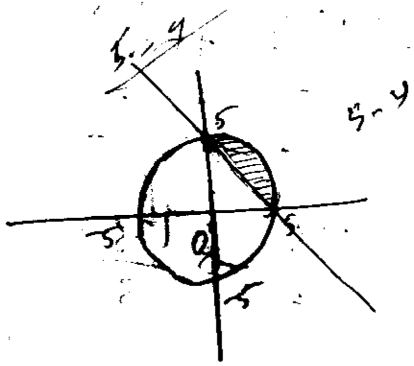
$$= \int_{-1}^{1}\left[3\left(\frac{1-y^2-1+y^2}{2}\right) - 2x\,2\sqrt{1-y^2}\right] dy$$

$$= -4\int_{-1}^{1} x\sqrt{1-y^2}\, dy = -4\sqrt{1-y^2}\left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$= -4\sqrt{1-y^2}\left(\frac{1}{2}-\frac{1}{2}\right) = 0$$

(18) $\iint_R y\,dA$ R is the region in the first quadrant enclosed between the circle $x^2+y^2=25$ and the line $x+y=5$

$x = \sqrt{5-y^2}$



$x+y=5.$ $x = 5-y$

$y = 5-x$

$x^2+y^2=25$

$x^2+(5-x)^2=25$

$x^2 = 25-(25-10x+x^2)$

$x^2 = 25-25+10x-x^2$

$2x^2 = 10x$

$2x^2-10x=0$

$x^2-5x=0$

$x(x-5)=0$

$x=0$

$x=5$

Type-2

$$\int_0^5 \int_{5-y}^{\sqrt{25-y^2}} y\,dx\,dy$$

$$= \int_0^5 y\,\Big[x\Big]_{5-y}^{\sqrt{25-y^2}} dy$$

$$= \int_0^5 y\left(\sqrt{25-y^2}-5+y\right) dy$$

$$= \int_0^5 y\sqrt{25-y^2} - 5y + y^2\,dy$$

$$= \int_0^5 y\sqrt{25-y^2}\,dy - 5\int_0^5 y\,dy + \int_0^5 y^2\,dy$$

$$= -\frac{1}{2}\int_{25}^{0}\sqrt{z}\,dz - 5\left[\frac{y^2}{2}\right]_0^5 + \frac{1}{3}\left[y^3\right]_0^5$$

$$= -\frac{1}{2}\times\frac{2}{3}\left[z^{3/2}\right]_{25}^{0} - 5\times\frac{25}{2} + \frac{1}{3}\times 125$$

$$\cancel{= \frac{1}{3}\left[(25-y^2)^{3/2}\right]_0^5 - \frac{125}{2} + \frac{125}{3}}$$

$$\cancel{= -\frac{1}{3}(25-25) - \frac{125}{2} + \frac{125}{3}}$$

$$\cancel{= \frac{-125\times 3 + 125\times 2}{6}} = -\frac{125}{6}$$

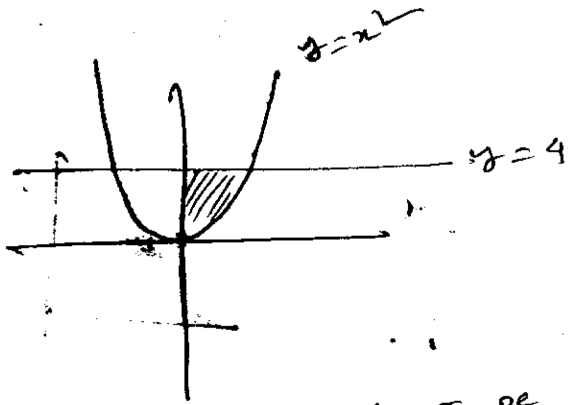
$$= -\frac{1}{3}\left[z^{\frac{3}{2}}\right]_{25}^{0} - \frac{125}{2} + \frac{125}{3}$$

$$= -\frac{1}{3}\left[-25^{\frac{3}{2}}\right] - \frac{125}{2} + \frac{125}{3}$$

$$= \frac{125}{3} - \frac{125}{2} + \frac{125}{3}$$

$$= \frac{250 - 375 + 250}{6} = \frac{500-375}{6} = \frac{125}{6}$$

(10) $\iint_R x(1+y^2)^{-1/2}\,dA$: R is the region in the first quadrant enclosed by $y=x^2$, $y=4$ and $x=0$



Type-1

$$\int_0^2 \int_{x^2}^4 x(1+y^2)^{-1/2}\,dy\,dx$$

$$= \int_0^2 x \int_{x^2}^4 \frac{1}{1+2y^2+y^4}$$

Type 2

$$\int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-1/2}\,dx\,dy$$

$$= \frac{1}{2}\int_0^4 y(1+y^2)^{-1/2}\,dy$$

$$= \frac{1}{2}\int_0^4 \frac{y}{\sqrt{(1+y^2)}}\,dy$$

$$= \frac{1}{2}\int_0^4 \frac{2y}{2\sqrt{1+y^2}}\,dy$$

$$= \frac{1}{2}\left[\sqrt{1+y^2}\right]_0^4$$

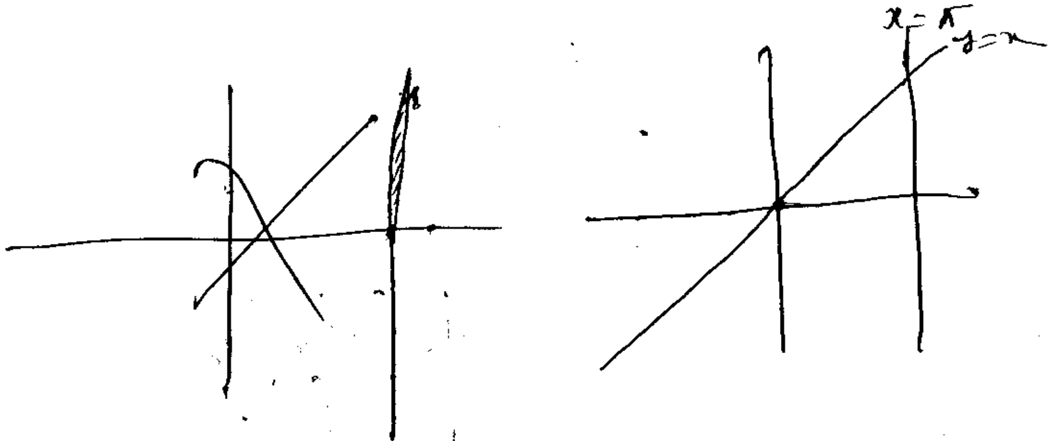
$$= 1/2\left[\sqrt{1+16}-1\right]$$

$$= \frac{1}{2}\left(\sqrt{17}-1\right)$$

(28)

$\iint_R x \cos y \, dA$: R is the triangular region

bounded by lines $y = x$, $y = 0$, $x = \pi$



Type 2

$$\int_0^\pi \int_y^\pi x \cos y \, dx \, dy$$

$$= \frac{1}{2}\int_0^\pi \cos y \left[x^2\right]_y^\pi dy$$

$$= \frac{1}{2} \int_0^\pi \cos y \left(\pi^2 - y^2\right) dy$$

$$= \frac{1}{2}\pi^2 \int_0^\pi \cos y - \frac{1}{2} \int_0^\pi y^2 \cos y \, dy$$

$$= \pi^2/2 \left[\sin y\right]_0^\pi - \frac{1}{2} \left\{ y^2 \int_0^\pi \cos y - \int_0^\pi \frac{d}{dy}(y^2) \int_0^\pi \cos y \, dy \, dy \right.$$

$$= -\frac{1}{2}\left(y^2 \left[\sin y\right]_0^\pi - 2 \int y \left[\sin y\right]_0^\pi dy \right)$$

$= -\cancel{\tfrac{1}{2}(y^2 \sin y -}$

$= \frac{1}{2}\left[\pi^2 \int_0^{\pi} \cos y - \int_0^{\pi} y^2 \cos y \, dy\right]$

$= \frac{1}{2}\left[\pi^2 \cancel{\int} [\sin y]_0^{\pi} - \int_0^{\pi} y^2 \cos y \, dy\right]$

$= -\frac{1}{2}\int_0^{\pi} y^2 \cos y \, dy$

$\int y^2 \cos y \, dy$

$= y^2 \int \cos y - \int \frac{d\,y^2}{dy} \int \cos y \, dy \, dy$

$= y^2 \sin y - 2\int y \sin y \, dy$

$= y^2 \sin y - 2\left[y \cos y - \int \frac{d}{dy}(y) \int \sin y \, dy \, dy\right]$

$= y^2 \sin y - 2\left[-y \cos y - \int \cos y \, dy\right]$

$= y^2 \sin y - 2\left[-y \cos y - \sin y\right]$

$= y^2 \sin y + 2y \cos y + 2 \sin y$

$\int_0^{\pi} y^2 \cos y = (\pi^2 \sin \pi + 2\pi \cos \pi + 2 \sin \pi) - (0 + 0 + 0)$

$= 2\pi$

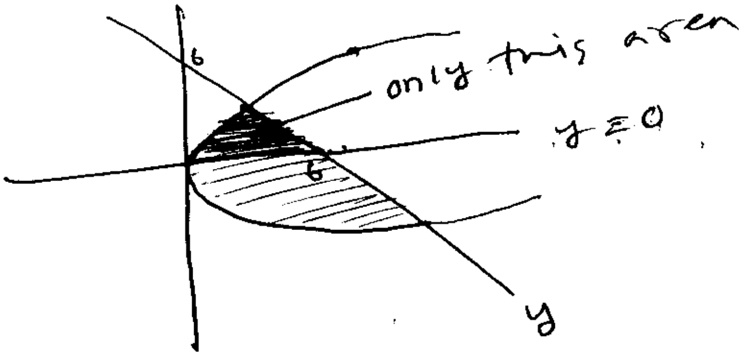
Ans will be pi (π)

$\therefore$ π Ans

(21)

$\iint_R xy\,dA$; R is the region enclosed by

$y = \sqrt{x}$, $y = 6 - x$, $y = 0$



$$\int_0^{\overset{2}{\cancel{\sqrt{x}}}} \int_{y^2}^{6-y} x\,y\,dx\,dy$$

$x = 6 - y$
$x = y^2$
$y^2 = 6 - y$
$y^2 + y - 6 = 0$
$y^2 + 3y - 2y - 6 = 0$
$y(y+3) - 2(y+3)$
$y = 2 \quad y = -3$

$$= \frac{1}{2}\int_0^2 y\left[x^2\right]_{y^2}^{6-y} dy$$

$$= \tfrac{1}{2}\int_0^2 y\left(6^2 - 12y + y^2 - y^4\right) dy$$

$$= \tfrac{1}{2}\int_0^2 y\left(36 - 12y + y^2 - y^4\right) dy$$

$$= \tfrac{1}{2}\int_0^2 36y - 12y^2 + y^3 - y^5 \; dy$$

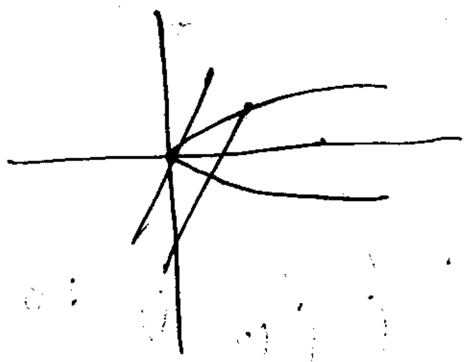
$$= \frac{1}{2}\left[\frac{36}{2}\left[y^2\right]_0^2 - \frac{12}{3}\left[y^3\right]_0^2 + \frac{1}{4}\left[y^4\right]_0^2 - \frac{1}{6}\left[y^6\right]_0^2\right]$$

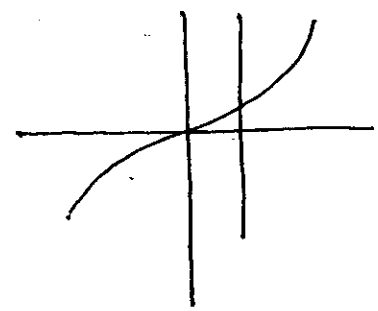
$= \frac{1}{2}\left(18 \times 4 - 4\times 8 + 4 - 32/3\right)$

$= 56/3$

(22) $\iint_R x\,dA$; R is the region enclosed by ~~$y=\sqrt{x}$, $y=6-x$~~

~~and $y=0$~~ $\quad y = \sin^{-1}x$, $x = \frac{1}{\sqrt{2}}$ $\quad y=0$





$$\int_0^{\pi/4}\int_{\sin y}^{1/\sqrt{2}} x\,dx\,dy$$

$$= \frac{1}{2}\int_0^{\pi/4}\left[x^2\right]_{\sin y}^{1/\sqrt{2}} dy = \frac{1}{2}\int_0^{\pi/4}\left(\frac{1}{2} - \sin^2 y\right) dy$$

$$= \frac{1}{2}\int_0^{\pi/4}\frac{1}{2} - \frac{1}{2}\int_0^{\pi/4}\sin^2 y\,dy$$

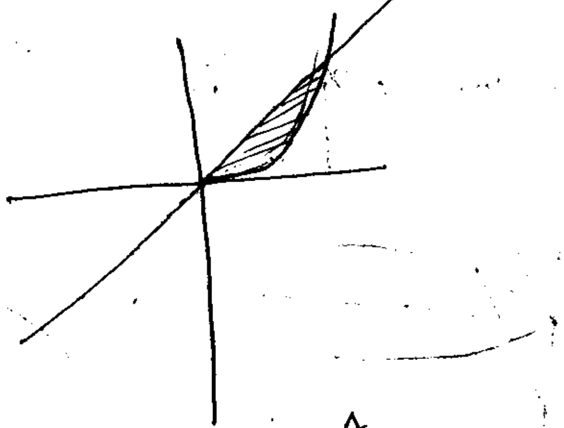
$$= \frac{1}{4}\left[x\right]_0^{\pi/4} - \frac{1}{2}\times\frac{1}{2}\int_0^{\pi/4}(1-\cos 2y)\,dy$$

$$= \frac{1}{4}(\pi/4) - \frac{1}{4}\left[y - \frac{\sin 2y}{2}\right]_0^{\pi/4} = \frac{\pi}{16} - \frac{1}{4}\left(\pi/4 - \frac{\sin \pi/2}{2}\right)$$

$$= \frac{\pi}{16} - \frac{\pi}{16} + 1/8 = 1/8$$

(23)

$\iint_R (x-1)\,dA$; R is the region in the first quadrant enclosed between $y=x$ and $y=x^3$;



$x^3 = x$

$x^3 - x = 0$

$x(x^2-1) = 0$

$x = 0$

$x^2 - 1 = 0$

$x = \pm 1$

$\cancel{\int\int_{\sqrt[3]{y}}^{x} (x-1)\,dx\,dy}$

$$\int_0^1 \int_{x^3}^{x} (x-1)\,dy\,dx$$

$$= \int_0^1 (x-1)\,(x-x^3)\,dx$$

$$= \int_0^1 x(x-x^3) - 1\,(x-x^3)\,dx$$

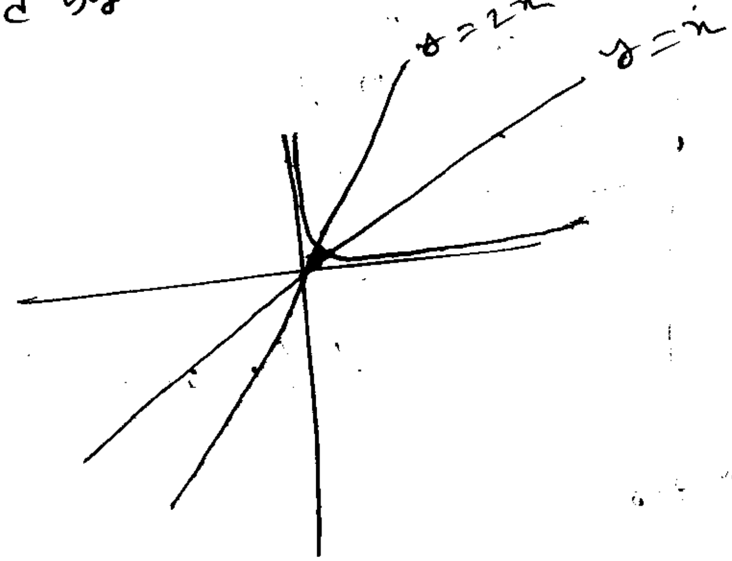
$$= \int_0^1 x^2 - x^4 - x + x^3\,dx = -7/60$$

**

(24)

$\iint_R x^2 \, dA$; R is the region in the first quadrant

enclosed by $xy = 1$ $y = x$ $y = 2x$



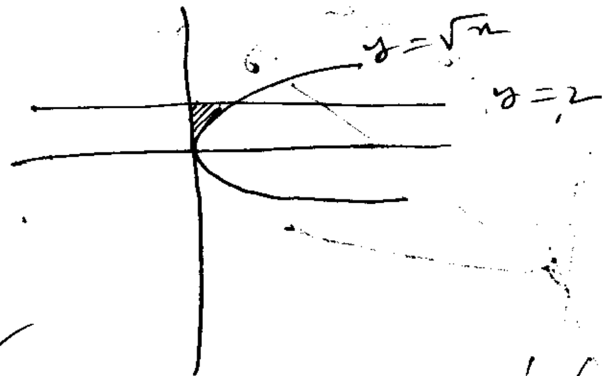
$y = 2x$

$y = 1/x$

$\iint_{2x}^{1/x}$

(23) $\iint_R \sin(y^3)\, dA$; R is the region enclosed by

$\cancel{y = \sin^{-1} x}$ $\quad y = \sqrt{x}$, $y = 2$ $\quad x = 0$



$\frac{1}{3}\left(1 - \cos\frac{8}{3}\right)$

$$\int_0^2 \int_0^{y^2} \sin y^3 \, dx \, dy$$

$= \cancel{\int_0^2 y^2 \, dy}$ $\quad \int_0^2 y^2 \sin y^3 \, dy$

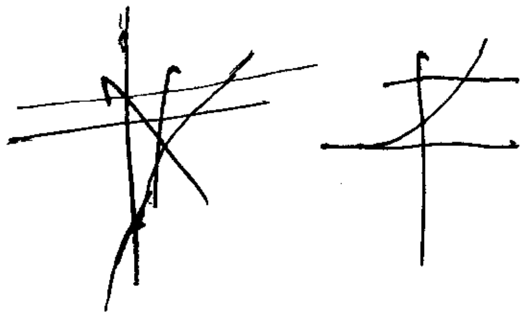
$= \cancel{\frac{1}{3}(8-0)}$ $= 8/3$ $\quad = -\frac{1}{3}\left[\cos y^3\right]_0^2$

$= -1/3\left[\cos 8 - 1\right]$

$= \frac{1}{3}(1 - \cos 8)$

Ans

(26) Evaluate $\iint_R x\,dA$, where R is the region bonded by $x = \ln y$, $x = 0$ and $y = e$



$\ln y = p$

$\frac{d}{dy}(\ln y) = \frac{dp}{dy}$

$\frac{1}{y} = dp$

$dy = y\,dp$

$x = \ln y$

$e^x = y$

$e^0 = y$

$y = 1$

$$\int_1^e \int_0^{\ln y} x\,dx\,dy$$

$$\frac{1}{2}\int_0^e (\ln y)^2\,dy$$

$$\frac{1}{2}\int_0^e \ln y^2\,dy$$

$$\frac{1}{2}\left(\ln y^2 \int dy - \int \frac{d}{dy}\ln(y^2) \int dy\right)$$

$$\frac{1}{2}\left(y\ln y^2 - \int \frac{1}{y^2}\,2y\,y\right)$$

$$\frac{1}{2}\left(y\ln y^2 - 2\int dy\right)$$

$$\frac{1}{2}\left(y\ln y^2 - 2y\right)\Big|_1^e$$

$$\frac{1}{2}\left(e\ln e^2 - 2e\right.$$

$$\frac{1}{2}(2e - 2e) - (1\ln 1 - 1) \quad = \frac{1}{2}\cancel{(2e - 2 + 1)}$$

$$= \cancel{\frac{1}{2}(e+1)} \quad , 1$$

$\underline{e - e^2/2}$