BRAC University
Course: MAT-216(Mathematics III)
Homework Sheet (Calculus) # 4

1. Evaluated the iterated integrals:

(a) $\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{x^2} \frac{1}{x}\cos\frac{y}{x}\,dy\,dx$ (b) $\int_0^1\int_0^1 \frac{x}{(xy+1)^2}dy\,dx$ (c) $\int_1^2\int_0^{y^2} e^{\frac{x}{y^2}}dx\,dy$ (d) $\int_0^2\int_0^{\sqrt{4-x^2}} e^{\sqrt{x^2+y^2}}dy\,dx$.

2. (a) Find the area of the region inside the circle r = 4sinθ and outside the circle r = 2.

(b) $\iint_R \frac{1}{x^2+y^2+1}dA$, where R is the sector in the first quadrant bounded by
y = 0 , y = x and $x^2 + y^2 = 4$.

3. Use polar coordinates to evaluate the double integral $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \left(x^2+y^2\right)^{1/2}dydx$.

4. (a) Find the volume of the solid that is bounded by the cylinder $y = x^2$ and by the planes y + z = 4 and z =0.

(b) Find the volume of the surface enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

5. Evaluated the iterated integral by converting to polar coordinates:

(a) $\int_0^1\int_0^{\sqrt{1-x^2}}\left(x^2+y^2\right)dydx$ (b) $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}\;dydx$

(c) $\int_0^a\int_0^{\sqrt{a^2-x^2}}\frac{dydx}{\left(1+x^2+y^2\right)^{\frac{3}{2}}}$ $(a>0)$.

6. (a) Evaluate $\int_{y=0}^{4}\int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1}\frac{2x-y}{2}\,dxdy$ by applying transformation T :
where $u=\frac{2x-y}{2}$, $v=\frac{y}{2}$ and integrating over an appropriate region in uv–plane.

(b) Evaluate $\iint_R \frac{x-y}{x+y}\,dA$, where R is the region enclosed by the lines
$x-y=0$, $x-y=1$, $x+y=1$ & $x+y=3$, using the transformation .

Double Integral

Exercise- 14.1- 1-16.

EXERCISE SET 14.1

C CAS

1–12 Evaluate the iterated integrals.

1. $\int_0^1 \int_0^2 (x+3)\,dy\,dx$

2. $\int_1^3 \int_{-1}^1 (2x-4y)\,dy\,dx$

3. $\int_2^4 \int_0^1 x^2 y\,dx\,dy$

4. $\int_{-2}^0 \int_{-1}^2 (x^2+y^2)\,dx\,dy$

5. $\int_0^{\ln 3} \int_0^{\ln 2} e^{x+y}\,dy\,dx$

6. $\int_0^2 \int_0^1 y \sin x\,dy\,dx$

7. $\int_{-1}^0 \int_2^5 dx\,dy$

8. $\int_4^6 \int_{-3}^7 dy\,dx$

9. $\int_0^1 \int_0^1 \frac{x}{(xy+1)^2}\,dy\,dx$

10. $\int_{\pi/2}^{\pi} \int_1^2 x \cos xy\,dy\,dx$

11. $\int_0^{\ln 2} \int_0^1 xye^{y^2x}\,dy\,dx$

12. $\int_3^4 \int_1^2 \frac{1}{(x+y)^2}\,dy\,dx$

13–16 Evaluate the double integral over the rectangular region R.

13. $\iint_R 4xy^3\,dA$; $R = \{(x,y) : -1 \le x \le 1, -2 \le y \le 2\}$

14. $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\,dA$;
$R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$

15. $\iint_R x\sqrt{1-x^2}\,dA$; $R = \{(x,y) : 0 \le x \le 1, 2 \le y \le 3\}$

16. $\iint_R (x \sin y - y \sin x)\,dA$;
$R = \{(x,y) : 0 \le x \le \pi/2, 0 \le y \le \pi/3\}$

Exercise- 14.2- 1-26.

EXERCISE SET 14.2 Graphing Utility CAS

1–8 Evaluate the iterated integral.

1. $\int_0^1 \int_{x^2}^{x} xy^2\, dy\, dx$

2. $\int_1^{3/2} \int_y^{3-y} y\, dx\, dy$

3. $\int_0^3 \int_0^{\sqrt{9-y^2}} y\, dx\, dy$

4. $\int_{1/4}^1 \int_{x^2}^{x} \sqrt{\frac{x}{y}}\, dy\, dx$

5. $\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^3} \sin \frac{y}{x}\, dy\, dx$

6. $\int_{-1}^1 \int_{-x^2}^{x^2} (x^2 - y)\, dy\, dx$

7. $\int_0^1 \int_0^x y\sqrt{x^2 - y^2}\, dy\, dx$

8. $\int_1^2 \int_0^{y^2} e^{x/y^2}\, dx\, dy$

FOCUS ON CONCEPTS

9. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

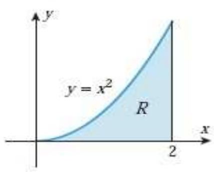
(a) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dy\, dx$

(b) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dx\, dy$

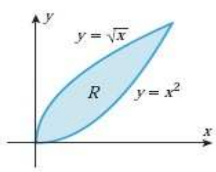
10. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

(a) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dy\, dx$

(b) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dx\, dy$



▲ Figure Ex-9



▲ Figure Ex-10

11. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

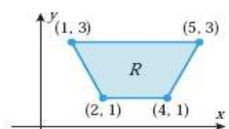
(a) $\iint_R f(x, y)\, dA = \int_1^2 \int_\square^\square f(x, y)\, dy\, dx + \int_2^4 \int_\square^\square f(x, y)\, dy\, dx + \int_4^5 \int_\square^\square f(x, y)\, dy\, dx$

(b) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dx\, dy$

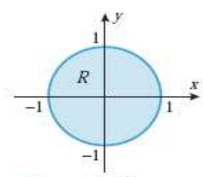
12. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

(a) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dy\, dx$

(b) $\iint_R f(x, y)\, dA = \int_\square^\square \int_\square^\square f(x, y)\, dx\, dy$



▲ Figure Ex-11



▲ Figure Ex-12

13. Evaluate $\iint_R xy\, dA$, where R is the region in

(a) Exercise 9 (b) Exercise 11.

14. Evaluate $\iint_R (x + y)\, dA$, where R is the region in

(a) Exercise 10 (b) Exercise 12.

15–18 Evaluate the double integral in two ways using iterated integrals: (a) viewing R as a type I region, and (b) viewing R as a type II region.

15. $\iint_R x^2\, dA$; R is the region bounded by $y = 16/x$, $y = x$, and $x = 8$.

16. $\iint_R xy^2\, dA$; R is the region enclosed by $y = 1$, $y = 2$, $x = 0$, and $y = x$.

17. $\iint_R (3x - 2y)\, dA$; R is the region enclosed by the circle $x^2 + y^2 = 1$.

18. $\iint_R y\, dA$; R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$.

19–24 Evaluate the double integral.

19. $\iint_R x(1 + y^2)^{-1/2}\, dA$; R is the region in the first quadrant enclosed by $y = x^2$, $y = 4$, and $x = 0$.

20. $\iint_R x \cos y\, dA$; R is the triangular region bounded by the lines $y = x$, $y = 0$, and $x = \pi$.

21. $\iint_R xy\, dA$; R is the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and $y = 0$.

20. $\iint_R x \cos y\, dA$; R is the triangular region bounded by the lines $y = x$, $y = 0$, and $x = \pi$.

21. $\iint_R xy\, dA$; R is the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and $y = 0$.

22. $\iint_R x\, dA$; R is the region enclosed by $y = \sin^{-1} x$, $x = 1/\sqrt{2}$, and $y = 0$.

23. $\iint_R (x - 1)\, dA$; R is the region in the first quadrant enclosed between $y = x$ and $y = x^3$.

24. $\iint_R x^2\, dA$; R is the region in the first quadrant enclosed by $xy = 1$, $y = x$, and $y = 2x$.

25. Evaluate $\iint_R \sin(y^3)\, dA$, where R is the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$. [*Hint:* Choose the order of integration carefully.]

26. Evaluate $\iint_R x\, dA$, where R is the region bounded by $x = \ln y$, $x = 0$, and $y = e$.

Exercise- 14.3- 1-12, 23-34.

1. $\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta\,dr\,d\theta$

2. $\int_0^{\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta$

3. $\int_0^{\pi/2}\int_0^{a\sin\theta} r^2\,dr\,d\theta$

4. $\int_0^{\pi/6}\int_0^{\cos 3\theta} r\,dr\,d\theta$

5. $\int_0^{\pi}\int_0^{1-\sin\theta} r^2\cos\theta\,dr\,d\theta$

6. $\int_0^{\pi/2}\int_0^{\cos\theta} r^3\,dr\,d\theta$

7–10 Use a double integral in polar coordinates to find the area of the region described.

7. The region enclosed by the cardioid $r = 1 - \cos\theta$.
8. The region enclosed by the rose $r = \sin 2\theta$.
9. The region in the first quadrant bounded by $r = 1$ and $r = \sin 2\theta$, with $\pi/4 \le \theta \le \pi/2$.
10. The region inside the circle $x^2 + y^2 = 4$ and to the right of the line $x = 1$.

FOCUS ON CONCEPTS

11–12 Let R be the region described. Sketch the region R and fill in the missing limits of integration.

$$\iint_R f(r,\theta)\,dA = \int_{\square}^{\square}\int_{\square}^{\square} f(r,\theta)r\,dr\,d\theta$$

11. The region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$.
12. The region inside the circle $r = 1$ and outside the cardioid $r = 1 + \cos\theta$.

13–16 Express the volume of the solid described as a double integral in polar coordinates.

23. $\iint_R \sin(x^2 + y^2)\,dA$, where R is the region enclosed by the circle $x^2 + y^2 = 9$.

24. $\iint_R \sqrt{9 - x^2 - y^2}\,dA$, where R is the region in the first quadrant within the circle $x^2 + y^2 = 9$.

25. $\iint_R \frac{1}{1 + x^2 + y^2}\,dA$, where R is the sector in the first quadrant bounded by $y = 0$, $y = x$, and $x^2 + y^2 = 4$.

26. $\iint_R 2y\,dA$, where R is the region in the first quadrant bounded above by the circle $(x-1)^2 + y^2 = 1$ and below by the line $y = x$.

27–34 Evaluate the iterated integral by converting to polar coordinates.

27. $\int_0^1\int_0^{\sqrt{1-x^2}} (x^2 + y^2)\,dy\,dx$

28. $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\,dx\,dy$

29. $\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\,dy\,dx$

30. $\int_0^1\int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2)\,dx\,dy$

31. $\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{dy\,dx}{(1 + x^2 + y^2)^{3/2}} \quad (a > 0)$

32. $\int_0^1\int_y^{\sqrt{y}} \sqrt{x^2 + y^2}\,dx\,dy$

33. $\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1 + x^2 + y^2}}\,dx\,dy$

34. $\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x\,dy\,dx$

Surface Area from Double Integral

Exercise- 14.4- 1-9.

EXERCISE SET 14.4 Graphing Utility CAS

1–4 Express the area of the given surface as an iterated double integral, and then find the surface area.

1. The portion of the cylinder $y^2 + z^2 = 9$ that is above the rectangle $R = \{(x, y) : 0 \le x \le 2, -3 \le y \le 3\}$.
2. The portion of the plane $2x + 2y + z = 8$ in the first octant.
3. The portion of the cone $z^2 = 4x^2 + 4y^2$ that is above the region in the first quadrant bounded by the line $y = x$ and the parabola $y = x^2$.
4. The portion of the surface $z = 2x + y^2$ that is above the triangular region with vertices $(0, 0)$, $(0, 1)$, and $(1, 1)$.

5–10 Express the area of the given surface as an iterated double integral in polar coordinates, and then find the surface area.

5. The portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 2x$.
6. The portion of the paraboloid $z = 1 - x^2 - y^2$ that is above the xy-plane.
7. The portion of the surface $z = xy$ that is above the sector in the first quadrant bounded by the lines $y = x/\sqrt{3}$, $y = 0$, and the circle $x^2 + y^2 = 9$.
8. The portion of the paraboloid $2z = x^2 + y^2$ that is inside the cylinder $x^2 + y^2 = 8$.
9. The portion of the sphere $x^2 + y^2 + z^2 = 16$ between the planes $z = 1$ and $z = 2$.

Triple Integral

Exercise- 14.5- 1-12, 15-18.

EXERCISE SET 14.5 C CAS

1–8 Evaluate the iterated integral.

1. $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}(x^2+y^2+z^2)\,dx\,dy\,dz$

2. $\int_{1/3}^{1/2}\int_{0}^{\pi}\int_{0}^{1} zx\sin xy\,dz\,dy\,dx$

3. $\int_{0}^{2}\int_{-1}^{y^2}\int_{-1}^{z} yz\,dx\,dz\,dy$

4. $\int_{0}^{\pi/4}\int_{0}^{1}\int_{0}^{x^2} x\cos y\,dz\,dx\,dy$

5. $\int_{0}^{3}\int_{0}^{\sqrt{9-z^2}}\int_{0}^{x} xy\,dy\,dx\,dz$

6. $\int_{1}^{3}\int_{x}^{x^2}\int_{0}^{\ln z} xe^y\,dy\,dz\,dx$

7. $\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dz\,dy\,dx$

8. $\int_{1}^{2}\int_{z}^{2}\int_{0}^{\sqrt{3}y}\frac{y}{x^2+y^2}\,dx\,dy\,dz$

9–12 Evaluate the triple integral.

9. $\iiint_G xy\sin yz\,dV$, where G is the rectangular box defined by the inequalities $0 \le x \le \pi$, $0 \le y \le 1$, $0 \le z \le \pi/6$.

10. $\iiint_G y\,dV$, where G is the solid enclosed by the plane $z = y$, the xy-plane, and the parabolic cylinder $y = 1 - x^2$.

11. $\iiint_G xyz\,dV$, where G is the solid in the first octant that is bounded by the parabolic cylinder $z = 2 - x^2$ and the planes $z = 0$, $y = x$, and $y = 0$.

12. $\iiint_G \cos(z/y)\,dV$, where G is the solid defined by the inequalities $\pi/6 \le y \le \pi/2$, $y \le x \le \pi/2$, $0 \le z \le xy$.

15–18 Use a triple integral to find the volume of the solid.

15. The solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$.

16. The solid bounded by the surface $z = \sqrt{y}$ and the planes $x + y = 1$, $x = 0$, and $z = 0$.

17. The solid bounded by the surface $y = x^2$ and the planes $y + z = 4$ and $z = 0$.

18. The wedge in the first octant that is cut from the solid cylinder $y^2 + z^2 \le 1$ by the planes $y = x$ and $x = 0$.

Change of variables

Exercise- 14.7- 1-12, 21-24, 35-37.

EXERCISE SET 14.7

1–4 Find the Jacobian $\partial(x, y)/\partial(u, v)$.

1. $x = u + 4v$, $y = 3u - 5v$
2. $x = u + 2v^2$, $y = 2u^2 - v$
3. $x = \sin u + \cos v$, $y = -\cos u + \sin v$
4. $x = \dfrac{2u}{u^2+v^2}$, $y = -\dfrac{2v}{u^2+v^2}$

5–8 Solve for x and y in terms of u and v, and then find the Jacobian $\partial(x, y)/\partial(u, v)$.

5. $u = 2x - 5y$, $v = x + 2y$
6. $u = e^x$, $v = ye^{-x}$
7. $u = x^2 - y^2$, $v = x^2 + y^2$ $(x > 0, y > 0)$
8. $u = xy$, $v = xy^3$ $(x > 0, y > 0)$

9–12 Find the Jacobian $\partial(x, y, z)/\partial(u, v, w)$.

9. $x = 3u + v$, $y = u - 2w$, $z = v + w$
10. $x = u - uv$, $y = uv - uvw$, $z = uvw$
11. $u = xy$, $v = y$, $w = x + z$
12. $u = x + y - z$, $v = x + y - z$, $w = x - y + z$

21. Use the transformation $u = x - 2y$, $v = 2x + y$ to find
$$\iint_R \frac{x-2y}{2x+y}\,dA$$
(cont.)

where R is the rectangular region enclosed by the lines $x - 2y = 1$, $x - 2y = 4$, $2x + y = 1$, $2x + y = 3$.

22. Use the transformation $u = x + y$, $v = x - y$ to find
$$\iint_R (x-y)e^{x^2-y^2}\,dA$$
over the rectangular region R enclosed by the lines $x + y = 0$, $x + y = 1$, $x - y = 1$, $x - y = 4$.

23. Use the transformation $u = \frac{1}{2}(x + y)$, $v = \frac{1}{2}(x - y)$ to find
$$\iint_R \sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y)\,dA$$
over the triangular region R with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$.

24. Use the transformation $u = y/x$, $v = xy$ to find
$$\iint_R xy^3\,dA$$
over the region R in the first quadrant enclosed by $y = x$, $y = 3x$, $xy = 1$, $xy = 4$.

35–38 Evaluate the integral by making an appropriate change of variables.

35. $\iint\limits_R \frac{y-4x}{y+4x}\,dA$, where R is the region enclosed by the lines $y=4x$, $y=4x+2$, $y=2-4x$, $y=5-4x$.

36. $\iint\limits_R (x^2-y^2)\,dA$, where R is the rectangular region enclosed by the lines $y=-x$, $y=1-x$, $y=x$, $y=x+2$.

37. $\iint\limits_R \frac{\sin(x-y)}{\cos(x+y)}\,dA$, where R is the triangular region enclosed by the lines $y=0$, $y=x$, $x+y=\pi/4$.

Book: Elementary Calculus- Howard Anton (10th Edition), Soft Copy