BRAC University
Course Code: MAT 216
Home Work SHEET # 2

1. Determine the values of parameter λ , such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

$$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + \lambda z &= 3 \\ x + \lambda y + 3z &= 2 \end{aligned} .$$

2. Determine the values of parameters λ & μ, such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10. \\ x + 2y + \lambda z &= \mu \end{aligned}$$

3. Determine the values of parameter (s) such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

(i) $$\begin{aligned} \lambda x + y + z &= 1 \\ x + \lambda y + z &= 1 \\ x + y + \lambda z &= 1 \end{aligned}$$

(ii) $$\begin{aligned} x + y + kz &= 2 \\ 3x + 4y + 2z &= k \\ 2x + 3y - z &= 1 \end{aligned}$$

(iii) $$\begin{aligned} x \qquad\quad - 3z &= -3 \\ 2x + \lambda y - z &= -2 \\ x + 2y + \lambda z &= 1 \end{aligned}$$

(iv) $$\begin{aligned} x + y + \lambda z &= 1 \\ x + \lambda y + z &= \lambda \\ \lambda x + y + z &= \lambda^2 \end{aligned}$$

4. Solve each of the following systems by Gaussain elimination or Gauss - Jordan elimination:

(i) $$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

(ii) $$\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$$

(iii) $$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad\quad - 3w &= -3 \end{aligned}$$

**These problems are for the students only as home work. Search the reference books for more examples.*

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5. Solve each of the following homogeneous system of linear equations by Gaussain elimination or Gauss - Jordan elimination :

(i)
$$\begin{aligned} 2x_1+2x_2-x_3 \quad + \quad x_5 &= 0 \\ -x_1-x_2+2x_3-3x_4+x_5 &= 0 \\ x_1+x_2-2x_3 \qquad -x_5 &= 0 \\ x_3+x_4+x_5 &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} 2x+2y+4z &= 0 \\ w \qquad -y-3z &= 0 \\ 2w+3x+y+z &= 0 \\ -2w+x+3y-2z &= 0 \end{aligned}$$

6. Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$,

(a). Find all the minors of A
(b) Find all the cofactors, (c) Find adj (A) ,
(d) Find A^{-1} , Using $A^{-1} = \dfrac{1}{\det(A)}$ adj (A).

7. Find the inverse of the following matrices if it exists, using [**A: I]:**

(i) $\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (vii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

8. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, prove that $(AB)^{-1} = B^{-1}.\,A^{-1}$

9. Solve, using **x = A^{-1} b**

(i)
$$\begin{aligned} x_1+3x_2+x_3 &= 4 \\ 2x_1+2x_2+x_3 &= -1 \\ 2x_1+3x_2+x_3 &= 3 \end{aligned}$$

(ii)
$$\begin{aligned} 5x_1+3x_2+2x_3 &= 4 \\ 3x_1+3x_2+2x_3 &= 2 \\ x_2+x_3 &= 5 \end{aligned}$$

(iii)
$$\begin{aligned} x+y+z &= 5 \\ x+y-4z &= 10 \\ -4x+y+z &= 0 \end{aligned}$$

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