Homework Sheet #2

1

(i) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3 \quad - 7 \quad + 4 \quad = 10$

$$(A|B) = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] \xleftarrow[R_3' = -3R_1 + R_3]{R_2' = R_1 + R_2}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -5 & -1 & -7 \end{array}\right] \xleftarrow[R_3' = \frac{1}{2} R_3]{R_2' = (-1) R_2}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -26 & -52 \end{array}\right] \xleftarrow{R_3' = 5(R_2) + R_3}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right] \xleftarrow{R_3' = -\frac{1}{26} R_3}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & \cancel{0} & -9+5\times 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \leftarrow R_2^1 = 5R_3 + R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \leftarrow R_2' = -2R_3 + R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \leftarrow R_1' = -R_2 + R_1$$

$\therefore$ $x = 3$, $y = 1$, $z = 2$

$\therefore$ $x_1 = 3$, $x_2 = 1$, $x_3 = 2$

Ans.

(11)

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

$$(A|B) = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \leftarrow R_1' = \tfrac{1}{2}R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right] \leftarrow \begin{array}{l} R_2' = 2R_1 + R_2 \\ R_3' = -8R_1 + R_3 \end{array}$$

$$= \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -1 & -4/7 & -1/7 \end{array}\right] \leftarrow \begin{array}{l} R_2' = \tfrac{1}{7}R_2 \\ R_3' = \tfrac{1}{7}R_3 \end{array}$$

$$= \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Free variables $= 3 - 2 = \cancel{2}\,1$

$\therefore \; x_1 + x_2 + x_3 = 0$ — (i)

$x_2 + \frac{4}{7}x_3 = \frac{1}{7}$ — (ii)

$0 \cdot x_3 = 0$

let $x_3 = r$

(ii) —

$x_2 = \frac{1}{7} - \frac{4}{7}r$

(i) $x_1 = -\frac{1}{7} + \frac{4}{7}r \cancel{+r}$

$= -\frac{1}{7} - \frac{3}{7}r$

Ans.

(iii) $x - y + 2z - w = -1$

$2x + y - 2z - 2w = -2$

$-x + 2y - 4z + w = 1$

$3x \qquad -3w = -3$

$$A|B = \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -4 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = -2R_1 + R_2 \\ R_3' = R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -4 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = R_3 \\ R_3' = R_2 \end{matrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -3(R_2) + R_4]{R_3' = -3(R_2) + R_3}$$

free variable = 4 - 3 = 1

$x_1 - x_2 + 2x_3 - x_4 = -1$ — (i)

$x_2 - 2x_3 = 0$ — (ii)

$x_3 = 2$ — (iii)

$x_4 = t$ (let)

(ii)

$x_2 = 2x_3$

$= 2 \times 2 = 4$

(i)

$x_1 = x_2 - 2x_3 + x_4 - 1$

$= 4 - 2 \times 2 + t - 1$

$= 4 - 4 + t - 1$

$= t - 1$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3' = R_3 + R_2$$

free variables = 4 - 3 = 1

$x_1 - x_2 + 2x_3 - x_4 = -1$ — (i)

$x_2 - 2x_3 = 0$ — (ii)

$2x_3 = 0$ — (iii)

$x_4 = t$ (let)

(iii) $x_3 = 0$

(ii) $x_2 = 0$

(i) $x_1 = t - 1$

2

(i) $2x_1 + 2x_2 - x_3 + 0 + x_5 = 0$

$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$

$x_1 + x_2 - 2x_3 + 0 - x_5 = 0$

$0 + 0 + x_3 + x_4 + x_5 = 0$

$$\begin{bmatrix} 2 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_1' = R_3 \\ R_3' = R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & -3 & 3 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = R_1 + R_2 \\ R_3' = -2R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & -3 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = R_4 \\ R_4' = R_2 \end{array}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \longleftarrow \begin{array}{l} R_3' = R_3 \times \frac{1}{3} \\ R_4' = R_3 \times (-1/3) \end{array}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = (-1) R_2 + R_3$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = -1/2 \times R_3$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow R_4' = -R_3 + R_4$$

$\therefore\ x_1 + x_2 - 2x_3 + 0 - x_5 = 0$ — (i) free variables

$x_3 + x_4 + x_5 = 0$ — (ii) $= 5 - 3$

$x_4 = 0$ —— $= 2$

$x_5 = t$ (let)

$x_2 = p$ (let)

(ii) ——

$x_3 = -0 - t$

$= -t$

(i) ——

$x_1 = -p + 2(-t) + t$

$= -p - 2t + t$

$= -p - t$

(ii) $0 + 2x + 2y + 4z = 0$

$w + 0 - y - 3z = 0$

$2w + 3x + y + z = 0$

$-2w + x + 3y - 2z = 0$

$$\begin{bmatrix} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix} \xleftarrow[R_2' = R_1]{R_1' = R_2}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 2 & -5 & 0 \end{bmatrix} \xleftarrow[R_4' = R_1 + R_4]{R_3' = -2(R_1) + R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 2 & -5 & 0 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{2} R_2}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -7 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_2 + R_4]{R_3' = -3 R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & -7 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow{\substack{R_3' = R_4 \\ R_4' = R_3}}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_3 + R_4]{R_3' = -\frac{1}{9} R_3}$$

$x_1 + 0 - x_3 - 3x_4 = 0$ —— (i)

$x_2 + x_3 + 2x_4 = 0$ —— (ii)

Free $4 - 3 = 1$

$x_4 = 0$

$x_3 = t$ (let)

(ii) ——

$x_2 = -t - 2 \cdot 0$

$= -t$

(i) ——

$x_1 = t + 3 \times 0$

$= t$

Ans.

(2)

$x + y - z = 1$

$2x + 3y + \lambda z = 3$

$x + \lambda y + 3z = 2$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & \lambda & 3 \\ 1 & \lambda & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2+\lambda & 1 \\ 0 & \lambda-1 & 4 & 1 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -2(R_1) + R_2}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2+\lambda & 1 \\ 0 & 0 & -(\lambda+3)(\lambda-2) & -(\lambda-2) \end{bmatrix} \xleftarrow{R_3' = -(\lambda-1)R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2+\lambda & 1 \\ 0 & 0 & (\lambda+3)(\lambda-2) & (\lambda-2) \end{bmatrix} \xleftarrow{R_3' = (-1)R_3}$$

$-\lambda + 1 + 1$

$-\lambda + 2$

$-(\lambda - 2)$

$-(\lambda-1)(2+\lambda)$

$(-\lambda+1)(2+\lambda)$

$-2\lambda - \lambda^2 + 2 + \lambda$

$-\lambda^2 - 2\lambda + \lambda + 2$

$-\lambda^2 - \lambda + 2 + 4$

$-\lambda^2 - \lambda + 6$

$-(\lambda^2 + \lambda - 6)$

$-(\lambda^2 + 3\lambda - 2\lambda - 6)$

$-\{\lambda(\lambda+3) - 2(\lambda+3)\}$

$-(\lambda+3)(\lambda-2)$

(i) For no solution

$(\lambda+3)(\lambda-2)=0$ and $\lambda-2\neq 0$

$\lambda\neq 2$

$\lambda\neq -3$

(ii) for unique solution

$(\lambda+3)(\lambda-2)\neq 0$ $\lambda-2\neq 0$

$\lambda\neq 2$

$\lambda\neq -3$ $\lambda\neq 2$

(iii) for more then one solution

$(\lambda+3)(\lambda-2)=0$ — (i) $\lambda-2=0$ — (ii)

$\lambda=-3$ $\lambda=2$

$\lambda=2$

$\lambda\neq -3$

$\lambda=2$ [as it will make both (i) and (ii) valid as $0=0$]

(4)

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

$\underline{0}$ $\ne 0$

$-2 + \lambda - 1$
$= \lambda - 3$
$-4 + \mu - 6$
$\mu - 10$

(i) For no solution

$\lambda - 3 = 0 \quad \mu - 10 \ne 0$

$\lambda = 3 \quad \mu \ne 10$

(ii) For more than one solution

$\lambda - 3 = 0 \quad \mu - 10 = 0$

$\lambda = 3 \quad \mu = 10$

(iii) For ~~more~~ unique solution

$\lambda - 3 \ne 0 \quad \mu - 10 \ne 0$

$\lambda \ne 3 \quad \mu \ne 10$

$\mu = 10$

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(i) $\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

$$\begin{bmatrix} \lambda & 1 & 1 & 1 \\ 1 & \lambda & 1 & 1 \\ 1 & 1 & \lambda & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & \lambda & 1 \\ 1 & \lambda & 1 & 1 \\ \lambda & 1 & 1 & 1 \end{bmatrix} \xleftarrow[R_3' = R_1]{R_1' = R_3}$$

$$= \begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & 1-\lambda & 0 \\ 0 & 1-\lambda & 1-\lambda^2 & 1-\lambda \end{bmatrix} \xleftarrow[R_3' = -\lambda R_1 + R_3]{R_2' = -R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & 1-\lambda & 0 \\ 0 & 0 & \underset{(\lambda+2)\{-(\lambda-1)\}}{1+\lambda} & 1-(\lambda-1) \end{bmatrix} \xleftarrow[R_3' = \cancel{\frac{1}{1-\lambda} R_3}\ R_2 + R_3]{\cancel{R_2' = \frac{1}{\lambda-1} R_2}}$$

$$= \begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & \lambda+2 & 1 \end{bmatrix}$$

(i) For no solution

$\lambda + 2 = 0$ $\quad (\lambda - 1) = 0$ $\quad \lambda - 1 \neq 0$ $\quad \lambda \neq 1$

$\lambda = -2$ $\quad \lambda = 1$

$\therefore \boxed{\lambda = -2}$

(ii) For more than one solution

$\cancel{\lambda + 2 = 0}$ $\quad \lambda = 1$ $\quad (\lambda + 2)(\lambda - 1) = 0$ $\quad \lambda - 1 = 0$

$\cancel{\lambda = -2}$ $\quad \lambda = -2$ $\quad \lambda = 1$

$\lambda = 1$ $\quad \boxed{\lambda = 1}$

(iii) For Unique Solution

$\lambda \neq -2$, $\lambda \neq 1$.

(ii) $x + y + kz = 2$

$3x + 4y + 2z = k$

$2x + 3y - z = 1$

$$\begin{bmatrix} 1 & 1 & k & 2 \\ 3 & 4 & 2 & k \\ 2 & 3 & -1 & 1 \end{bmatrix}$$

=

$$= \begin{bmatrix} 1 & 1 & k & 2 \\ 0 & 1 & -3k+2 & k-6 \\ 0 & 1 & -2k-1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & k & 2 \\ 0 & 1 & -3k+2 & k-6 \\ 0 & 0 & k-3 & -k+3 \end{bmatrix}$$

(i) For no solution : $\boxed{\text{no value.}}$

$\cancel{k-3=0}$ $\qquad$ $\cancel{-k+3 \neq 0}$

$\cancel{k=3}$ $\qquad$ $\cancel{-k \neq -3}$

$\cancel{k \neq 3}$

(ii) For more than one solution

$k-3=0$ $\qquad$ $-k+3=0$

$k=3$ $\qquad$ $k=3$

(iii) For Unique solution

$k-3 \neq 0$ $\qquad$ $-k+3 \neq 0$ $\qquad$ $\therefore k \neq 3$

(117)

$x + 0 - 3z = -3$

$2x + \lambda y - z = -2$

$x + 2y + \lambda z = 1$

$$\begin{bmatrix} 1 & 0 & -3 & -3 \\ 2 & \lambda & -1 & -2 \\ 1 & 2 & \lambda & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -3 & -3 \\ 0 & \lambda & 5 & 4 \\ 0 & 2 & \lambda+3 & 4 \end{bmatrix} \leftarrow \begin{matrix} R_2' = -2R_1 + R_2 \\ R_3' = -R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & -3 & -3 \\ 0 & 1 & 5/\lambda & 4/\lambda \\ 0 & 2 & \lambda+3 & 4 \end{bmatrix} \leftarrow R_2' = R_2/\lambda$$

$$= \begin{bmatrix} 1 & 0 & -3 & -3 \\ 0 & 1 & 5/\lambda & 4/\lambda \\ 0 & 0 & \frac{(\lambda-2)(\lambda+5)}{\lambda} & \frac{4(\lambda-2)}{\lambda} \end{bmatrix}$$

$-10/\lambda + \lambda + 3$

$\frac{-10 + \lambda^2 + 3\lambda}{\lambda}$

$\frac{\lambda^2 + 3\lambda - 10}{\lambda}$

$\frac{\lambda^2 + 5\lambda - 2\lambda - 10}{\lambda}$

$\lambda(\lambda+5) - 2(\lambda+5)$

$\frac{(\lambda-2)(\lambda+5)}{\lambda}$

$-8/\lambda + 4$

$\frac{-8 + 4\lambda}{\lambda}$

① For no solution

$$\frac{(\lambda-2)(\lambda+5)}{\lambda} = 0 \qquad \frac{4\lambda - 8}{\cancel{4\lambda}} \neq 0$$

$\lambda = 2$, $\boxed{\lambda = -5}$, $\lambda = 0$

$\cancel{\lambda \neq 0}$, $\lambda \neq 2$

(ii) More than One solution

$$\frac{(\lambda-2)(\lambda+5)}{\lambda} = 0 \qquad \frac{4\lambda-8}{\lambda} = 0$$

$\lambda = 2, \lambda = -5$ $\qquad$ $\boxed{\lambda = 2}$

$\cancel{\lambda = 0}$ $\qquad$ $\cancel{\lambda = 0}$

invalid $\qquad$ invalid

(iii) For Unique solution

$$\frac{(\lambda-2)(\lambda+5)}{\lambda} \neq 0 \qquad \frac{4\lambda-8}{\lambda} \neq 0$$

$\lambda \neq 2 \quad \lambda \neq -5$ $\qquad$ $\lambda \neq 2$

$\cancel{\lambda \neq 0}$ $\qquad$ $\cancel{\lambda \neq 0}$

(iv)

$$x + y + \lambda z = 1$$
$$x + \lambda y + z = \lambda$$
$$\lambda x + y + z = \lambda^2$$

$$\begin{bmatrix} 1 & 1 & \lambda & 1 \\ 1 & \lambda & 1 & \lambda \\ \lambda & 1 & 1 & \lambda^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & 1-\lambda & -1+\lambda \\ 0 & \cancel{0}\; 1-\lambda & 1-\lambda^2 & \cancel{1-\lambda}\; -\lambda+\lambda^2 \end{bmatrix}$$

$R_2' = -R_1 + R_2$

$R_3' = -\lambda R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & -(\lambda-1) & \lambda-1 \\ 0 & 1-\lambda & 1-\lambda^2 & -\lambda(1-\lambda) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & 1-\lambda & \lambda-1 \\ 0 & 0 & (\lambda+2)(1-\lambda) & (\lambda+1)(\lambda-1) \end{bmatrix}$$

$R_2' = \frac{1}{\lambda-1} R_2$

$\longleftarrow$

$R_3' = \frac{1}{1-\lambda} R_3$

$$\begin{bmatrix} 1 & 1 & \lambda & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & \lambda+2 & -\lambda-1 \end{bmatrix}$$

$R_3' = -R_1 + R_3$

$\longleftarrow$

$1-\lambda^2-\lambda+1$

$-\lambda^2-\lambda+2$

$-\lambda^2-2\lambda+\lambda+2$

$(\lambda+2)(-\lambda+1)$

$\lambda^2-1-\lambda+\lambda^2$

$\cancel{(\lambda-1)(\lambda^2}$

λ^2-1

$(\lambda+1)(\lambda$

(i) For no solution

$\lambda+2=0$

$\lambda=-2$

$-\lambda-1 \neq 0$

$\lambda \neq -1$

(ii) For more than one solution

$\lambda+2=0$

$\lambda=-2$

$-\lambda-1=0$

$\lambda=-1$

(iii) For Unique solution

$\lambda+2 \neq 0$

$\lambda \neq -2$

$-\lambda-1 \neq 0$

$\lambda \neq -1$

6/

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

(a) $M_{11} = \cancel{7\times 4} \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix} = 28 + 1 = 29$

$M_{12} = \begin{vmatrix} 6 & -1 \\ -3 & 4 \end{vmatrix} = 24 - 3 = 21$

$M_{13} = \begin{vmatrix} 6 & 7 \\ -3 & 1 \end{vmatrix} = 6 + 21 = 27$

$M_{21} = \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix} = -8 - 3 = -11$

$M_{22} = \begin{vmatrix} 1 & 3 \\ -3 & 4 \end{vmatrix} = 4 + 9 = 13$

$M_{23} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} = 1 - 6 = -5$

$M_{31} = \begin{vmatrix} -2 & 3 \\ 7 & -1 \end{vmatrix} = 2 - 21 = -19$

$M_{32} = \begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix} = -1 - 18 = -19$

$M_{33} = \begin{vmatrix} 1 & -2 \\ 6 & 7 \end{vmatrix} = 7 + 12 = 19$

(b) Cofactor, $C_{ij} = (-1)^{i+j} \cdot M_{ij}$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} \quad A_{co} = \begin{bmatrix} 29 & -21 & 27 \\ 11 & 13 & 5 \\ -19 & 19 & 19 \end{bmatrix}$$

(c)

$$Adj(A) = C_o^T = \begin{bmatrix} 29 & 11 & -19 \\ -21 & 13 & 19 \\ 27 & 5 & 19 \end{bmatrix}$$

$$\det(A) = 1\,(28+1) + 2(24-3) + 3\,(6+21)$$

$$= 29 + 2\times 21 + 3\times(26+1)$$

$$= \cancel{23544}\ 152 \neq 0$$

(d)

$$\therefore A^{-1} = \frac{1}{\det(A)}\, adj(A)$$

$$= \frac{1}{152}\begin{bmatrix} 29 & 11 & -19 \\ -21 & 13 & 19 \\ 27 & 5 & 19 \end{bmatrix}$$

⑦

(i) $A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

$\det(A) = 2(-3-0) - 5(-3-0) + 5(-4+2)$

$= -6 + 15 - 10$

$= -1 \neq 0$

$\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

$A : I = \left[\begin{array}{ccc|ccc} 2 & 5 & 5 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right]$

$R_1' = R_2$

$R_2' = R_1$

$= \left[\begin{array}{ccc|ccc} -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 1 & 0 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right]$

$R_1' = (-1) R_1$

$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 2 & 5 & 5 & 1 & 0 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right]$

$R_2' = (-2) R_1 + R_2$

$R_3' = (-2) R_1 + R_3$

$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 3 & 5 & 1 & 2 & 0 \\ 0 & 2 & 3 & 0 & 2 & 1 \end{array}\right]$

$R_2' = \frac{1}{3} R_2$

$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 3/3 & 5/3 & 1/3 & 2/3 & 0 \\ 0 & 2 & 3 & 0 & 2 & 1 \end{array}\right]$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 5/3 & 1/3 & 2/3 & 0 \\ 0 & 2 & 3 & 0 & 2 & 1 \end{array}\right] \longleftarrow$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 5/3 & 1/3 & 2/3 & 0 \\ 0 & 0 & -1/3 & -2/3 & 2/3 & 1 \end{array}\right] \longleftarrow R_3' = (-2)R_2 + R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 5/3 & 1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] \longleftarrow R_3' = -1/3\, R_3 \qquad \frac{-2/3}{-1/3} \quad 2/3 \times 3/1$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -3 & 4 & 5 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] \longleftarrow R_2' = -5/3\, R_3 + R_2$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & -5 \\ 0 & 1 & 0 & -3 & 4 & 5 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] \longleftarrow R_1' = -R_2 + R_1$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$$

(11) $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

$$\det(A) = 2(2-0) + 3(0-0) + 5(0) = 2 \times 2 = 4$$

$= A : I$

$$= \left[\begin{array}{ccc|ccc} 2 & -3 & 5 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \longleftarrow R_1' = \tfrac{1}{2} R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right] \longleftarrow R_3' = \tfrac{1}{2} R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & -3/2 & 0 & 1/2 & 0 & -5/4 \\ 0 & 1 & 0 & 0 & 1 & 3/2 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right] \longleftarrow \begin{array}{l} R_1' = -5/2 R_3 + R_1 \\ R_2' = 3 R_3 + R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 3/2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 3/2 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1/2 & 3/2 & 1 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

(iii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$$M_{11} = \begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} = 5$$

$$M_{12} = \begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix} = 10$$

$$M_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} = -4 -$$

$$\left[\begin{array}{ccc|ccc} -1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 0 & 0 & 1 \end{array}\right] \leftarrow R_1' = -R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 5 & -6 & 2 & 1 & 0 \\ 0 & 6 & -7 & 4 & 0 & 1 \end{array}\right] \xleftarrow[R_3' = -4R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 1 & -6/5 & 2/5 & 1/5 & 0 \\ 0 & 6 & -7 & 4 & 0 & 1 \end{array}\right] \longleftarrow R_2' = \tfrac{1}{5} R_2$$

$-36/5 + 7$

$\frac{36-35}{5}$

$-12/5 + 4$

$\frac{-12+20}{5}$

$8/5$

$-6/5$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 1 & -6/5 & 2/5 & 1/5 & 0 \\ 0 & 0 & 1/5 & 8/5 & -6/5 & 1 \end{array}\right] \longleftarrow R_3' = -6R_2 + R_3$$

$-1-24$

$\frac{6 \times 5}{5} + 2/5$

$\frac{48+12}{5}$ $\quad 50/5 \quad 10$

$-6/5 \quad -3/5 + 1/5$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 1 & -6/5 & 2/5 & 1/5 & 0 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{array}\right] \quad R_3' = 5R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -25 & 18 & -15 \\ 0 & 1 & 0 & 10 & -7 & 6 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = \tfrac{6}{5} R_3 + R_2 \\ R_1' = -3R_3 + R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 4 & -3 \\ 0 & 1 & 0 & 10 & -7 & 6 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{array}\right] \longleftarrow R_1' = 2R_2 + R_1$$

0

1 0

(iv) $$\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 3 & 4 & -1 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 2 & 5 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 3 & 4 & -1 & 1 & 0 & 0 \\ 2 & 5 & -4 & 0 & 0 & 1 \end{array}\right] \xleftarrow{R_1 \rightleftharpoons R_2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 4 & -10 & 1 & -3 & 0 \\ 0 & 5 & -10 & 0 & -2 & 1 \end{array}\right] \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = -3R_1 + R_2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -10/4 & 1/4 & -3/4 & 0 \\ 0 & 5 & -10 & 0 & -2 & 1 \end{array}\right] \xleftarrow{R_2' = 1/4\, R_2}$$

$\frac{50}{4} - 10$

$\frac{50-40}{4}$ 10

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -10/4 & 1/4 & -3/4 & 0 \\ 0 & 0 & 10/4 & -5/4 & 7/4 & 1 \end{array}\right] \xleftarrow{R_3' = -3R_2 + R_3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -10/4 & 1/4 & -3/4 & 0 \\ 0 & 0 & 1 & -1/2 & 7/10 & 4/10 \end{array}\right] \xleftarrow{R_3' = 4/10\, R_3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -11/10 & -12/10 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1/2 & 7/10 & 9/10 \end{array}\right]$$

(v)

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 8 & 9 & 0 & 0 & 1 & 0 \\ 1 & 3 & 2 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 7 & 8 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = -2R_1 + R_2 \\ R_3' = -R_1 + R_3 \\ R_4' = -R_1 + R_4 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 7 & 8 & -1 & 0 & 1 & 0 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = -R_2 \\ R_3 \rightleftarrows R_4 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 6 & 0 & 1 & -7 \end{array}\right] \longleftarrow R_4' = -7R_3 + R_4$$

$$= \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 0 & -5 & 0 & -1 & 7 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -7 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 & 6 & 0 & 1 & -7 \end{array}\right] \quad \begin{array}{l} R_3' = -R_4 + R_3 \\ \longleftarrow \\ R_1' = -R_4 + R_1 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -7 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 & 6 & 0 & 1 & -7 \end{array}\right] \quad \begin{array}{l} R_1' = -R_3 + R_1 \\ \longleftarrow \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -4 & 3 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -7 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 & 6 & 0 & 1 & -7 \end{array}\right] \quad \begin{array}{l} R_1' = -3R_2 + R_1 \\ \longleftarrow \end{array}$$

(vi)

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 \end{array}\right] \xleftarrow{R_1 \rightleftharpoons R_3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & -1 & 1 & 1 & 0 & -1 \end{array}\right] \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right] \xleftarrow[R_3' = -R_3]{R_2' = -R_2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & -1 & -1 & 1 & -1 \end{array}\right] \xleftarrow{R_3' = -R_2 + R_3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right] \xleftarrow{R_3' = -R_3}$$

$$= \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$$

(vii)

$$\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 0 & 0 & 1 \end{array}\right] \longleftarrow R_1' = -R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 5 & -6 & 2 & 1 & 0 \\ 0 & 6 & -7 & 4 & 0 & 1 \end{array}\right] \longleftarrow \begin{matrix} R_2' = -2R_1 + R_2 \\ R_3' = -4R_1 + R_3 \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 5/5 & -6/5 & 2/5 & 1/5 & 0 \\ 0 & 6 & -7 & 4 & 0 & 1 \end{array}\right] \longleftarrow R_2' = 1/5\, R_2$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 1 & -6/5 & 2/5 & 1/5 & 0 \\ 0 & 0 & 1/5 & 8/5 & -6/5 & 1 \end{array}\right] \quad \longleftarrow \quad R_3' = -6R_2 + R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 1 & -6/5 & 2/5 & 1/5 & 0 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{array}\right] \quad \longleftarrow \quad R_3' = 5R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -25 & 18 & 15 \\ 0 & 1 & 0 & 10 & -7 & 6 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 9 & -3 \\ 0 & 1 & 0 & 10 & -7 & 6 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{array}\right]$$

viii

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 2 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & -4 & -1 & -3 & 1 & 0 & 0 \\ 0 & 3 & -5 & -1 & -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \xleftarrow[R_4 = -R_1 + R_4]{R_2' = -3R_1 + R_2,\; R_3' = -2R_1 + R_3}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 3 & -5 & -1 & -2 & 0 & 1 & 0 \\ 0 & 3 & -4 & -1 & -3 & 1 & 0 & 0 \end{array}\right] \xleftarrow{R_2 \rightleftharpoons R_4}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -2 & -1 & 1 & 0 & 1 & -3 \\ 0 & 0 & -1 & -1 & 0 & 1 & 0 & -3 \end{array}\right] \xleftarrow[R_4' = -3R_2 + R_4]{R_3' = -3R_2 + R_3}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & 1 & 0 & -3 \\ 0 & 0 & -2 & -1 & 1 & 0 & 1 & -3 \end{array}\right] \longleftarrow R_3 \rightleftharpoons R_4$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 & 3 \\ 0 & 0 & -2 & -1 & 1 & 0 & 1 & -3 \end{array}\right] \longleftarrow R_3' = -R_3$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 & -2 & 1 & 3 \end{array}\right] \longleftarrow R_4' = 2R_3 + R_4$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 0 & 0 & 2 & -1 & -3 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & \cancel{0}1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & 1 & 3 \end{array}\right] \longleftarrow \begin{array}{l} R_3' = -R_4 + R_2 \\ R_1' = -R_4 + R_1 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 2 & 0 & 1 & -3 \\ 0 & 1 & 0 & 0 & -2 & \cancel{0}1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & \cancel{0}1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & 1 & 3 \end{array}\right] \longleftarrow \begin{array}{l} R_1' = -2R_3 + R_1 \\ R_2' = R_2 + R_3 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -2 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & 1 & 3 \end{array}\right] \longleftarrow R_1' = R_2 + R_1$$

(8) $(AB)^{-1} = B^{-1} \cdot A^{-1}$

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+3+1 & 5+1+2 & 3+2+ \\ 1+6+3 & 5+2+6 & 3+4+ \\ 2+12+9 & 5+4+18 & 3+ \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 & 6 \\ 10 & 13 & 10 \\ 23 & 27 & 20 \end{bmatrix}$$

$$AB^{-1} = \begin{bmatrix} -5/4 & 1/4 & 1/4 \\ 5/4 & -9/4 & 3/4 \\ \cancel{-29/16}\ -1/4 & 11/4 & -5/4 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} -3/4 & 1/4 & 7/4 \\ -1/4 & -1/4 & 5/4 \\ 5/4 & 1/4 & -13/4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -5/2 & 1/2 \\ -3 & 4 & -1 \\ 1 & -3/2 & 1/2 \end{bmatrix}$$

$$B^{-1} \cdot A^{-1} = \begin{bmatrix} -5/4 & 1/4 & 1/4 \\ 5/4 & -9/4 & 3/4 \\ -1/4 & 11/4 & -5/4 \end{bmatrix}$$

rough

(AB^{-1}) = Calculating by calculator

B^{-1} =
A^{-1} = } calculating by calculator

(9) $x = A^{-1}b$

(1)

$x_1 + 3x_2 + x_3 = 4$

$2x_1 + 2x_2 + x_3 = -1$

$2x_1 + 3x_2 + x_3 = 3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 2 & 2 & 1 & -1 \\ 2 & 3 & 1 & 3 \end{array}\right] \qquad A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$M_{11} = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2-3 = -1$ $\quad M_{12} = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 2-2 = 0$ $\quad M_{13} = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} = 6-4 = 2$

$M_{21} = \begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} = 3-3 = 0$ $\quad M_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$ $\quad M_{23} = \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 3-6 = -3$

$M_{31} = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = 3-2 = 1$ $\quad M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$ $\quad M_{33} = \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = 2-6 = -4$

$$C = \begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 3 \\ 1 & 1 & -4 \end{bmatrix} \qquad adj(A) = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$\det(A) = 1$

$$\therefore A^{-1} = \frac{1}{1}\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4 + 0 + 3 \\ 0 + 1 + 3 \\ 8 - 3 - 12 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

Ans.