HomeWork * Sheet #2

1/

(i) $S = \{(x, 2y, 5) : x, y \in R\}$

$\Rightarrow$ $(0,0,0) \in R^3$, $(0,0,5) \notin S$

$\therefore$ S is not a sub spacce of R^3

(ii) $S = \{(x, x+y, 3z) : x, y, z \in R\}$

$\Rightarrow$ $(0,0,0) \in R^3$ $\therefore$ $(0,0,0) \in S$

$\therefore$ S is not empty

Let $\underline{u} = (x, x+y, 3z)$

$\underline{v} = (x', x'+y', 3z')$

and α, β are scalars

$$\alpha \underline{u} + \beta \underline{v} = \alpha (x, x+y, 3z) + \beta (x', x'+y', 3z')$$

$$= (\alpha x + \beta x', \alpha x + \beta x' + \alpha y + \beta y', 3\alpha z + 3\beta z')$$

$$= (\alpha x + \beta x', (\alpha x + \beta x') + (\alpha y + \beta y'), 3\alpha z + 3\beta z')$$

$\in S$

$\therefore S$ is a subspace of $\cancel{V}\ \mathbb{R}^3$

$S = \{ (x, y, z) \in \mathbb{R}^3 : x - y + z = 0 \}$

$(0,0,0) \in \mathbb{R}^3 \quad \therefore \quad (0,0,0) \in S$

$\therefore S$ is not empty

Let

$\underline{u} = (x, y, z)$ and $x - y + z = 0$

$\underline{v} = (x', y', z')$ and $x' - y' + z' = 0$

and α and β are scalars

$$\alpha \underline{u} + \beta \underline{v} = \alpha (x, y, z) + \beta (x', y', z').$$

$$= (\alpha x + \beta x', \ \alpha y + \beta y', \ \alpha z + \beta z')$$

$$\alpha x + \beta x' - \alpha y - \beta y' + \alpha z + \beta z' = 0$$

$$\alpha (x - y + z) + \beta (x' - y' + z') = 0$$

$$\alpha \cdot 0 + \beta \cdot 0 = 0$$

$0 = 0 \quad \therefore S$ is a subspace of $\cancel{V}\ \mathbb{R}^3$

(12)

$S = \{(x, y, z, t) \in R^4 : 3x - 2y - 2z - t = 0\}$

$(0,0,0,0) \in R^4 \therefore (0,0,0,0) \in S$

$\therefore S$ is not empty

Let

$\underline{u} = (x, y, z, t)$ and $3x - 2y - 2z - t = 0$

and $\underline{v} = (x', y', z', t)$ and $3x' - 2y' - 2z' - t' = 0$

and α and β are scalars

$\alpha \underline{u} + \beta \underline{v} = \alpha(x, y, z, t) + \beta(x', y', z', t)$

$= (\alpha x + \beta x', \alpha y + \beta y', \alpha z + \beta z', \alpha t + \beta t')$

$\therefore 3\alpha x + 3\beta x' - 2\alpha y - 2\beta y' - 2\alpha z - \beta z' - 2\alpha t - 2\beta t = 0$

$\alpha(3x - 2y - 2z - t) + \beta(3x' - 2y' - 2z' - t') = 0$

$\alpha . 0 + \beta . 0 = 0$

$0 = 0 \quad \therefore$ S is a subspace of of ~~R^3~~ R^4

$S = \{ (x, y, z) \in R^3 : x + y + z = 0 \}$

$(0, 0, 0) \in R^3 \therefore (0, 0, 0) \in S$

$\therefore S$ is not empty

let $\underline{u} = (x, y, z)$ and $x + y + z = 0$

$\underline{v} = (x', y', z')$ and $x' + y' + z' = 0$

and α and β are scalars.

$\alpha \underline{u} + \beta \underline{v} = \alpha(x, y, z) + \beta(x', y', z')$

$= ((\alpha x + \beta x'), (\alpha y + \beta y'), (\alpha z + \beta z'))$

$\alpha x + \beta x' + \alpha y + \beta y' + \alpha z + \beta z' = 0$

$\alpha(x + y + z) + \beta(x' + y' + z') = 0$

$\alpha \cdot 0 + \beta \cdot 0 = 0$

$0 = 0$

$\therefore S$ is a subspace of $\mathbb{R}^3$

2/

$\underline{w} = (1,0,0) \quad \underline{x} = (0,0,1)$

$(1,0,0) = k_1(1,0,-1) + k_2(0,1,0) + k_3(1,0,1)$

$= (k_1 + 0k_2 + k_3,\ 0k_1 + k_2 + 0k_3,\ -k_1 + 0k_2 + k_3)$

$$\left.\begin{aligned} k_1 + 0k_2 + k_3 &= 1 \\ 0k_1 + k_2 + 0k_3 &= 0 \\ -k_1 + 0k_2 + k_3 &= 0 \end{aligned}\right\} \qquad \left.\begin{aligned} k_1 + k_3 &= 1 \quad \text{—(i)} \\ k_2 &= 0 \quad \text{—(ii)} \\ -k_1 + k_3 &= 0 \quad \text{—(iii)} \end{aligned}\right\}$$

(i) + (iii)

$2k_3 = 1$

$\Rightarrow k_3 = 1/2$

$k_1 = k_3 = 1/2$

$(1,0,0) = \frac{1}{2}(1,0,-1) + 0\cdot(0,1,0) + \frac{1}{2}(1,0,1)$

$(0,0,1) = k_1(1,0,-1) + k_2(0,1,0) + k_3(1,0,1)$

$= (k_1 + k_3), (k_2), (-k_1 + k_3)$

$\therefore$

$k_1 + k_3 = 0$ —(i)

$k_2 = 0$

$-k_1 + k_3 = 1$ —(ii)

(i) + (ii)

$2k_3 = 1$

$k_3 = 1/2$

$k_1 = -1/2$

$k_2 = 0$

$(0,0,1) = -\frac{1}{2}(1,0,-1) + 0(0,1,0) + \frac{1}{2}(1,0,1)$

5) $\underline{w} = (1, 2, 6)$
$\underline{v_1} = (2, 1, 0)$
$\underline{v_2} = (1, -1, 2)$
$\underline{v_3} = (0, 3, -4)$

$w = k_1 v_1 + k_2 v_2 + k_3 v_3$

$(1, 2, 6) = k_1(2, 1, 0) + k_2(1, -1, 2) + k_3(0, 3, -4)$

$= (2k_1 + k_2, k_1 - k_2 + 3k_3, 2k_2 - 4k_3)$

$\therefore$ $2k_1 + k_2 = \cancel{0}\ 1$
$k_1 - k_2 + 3k_3 = \cancel{0}\ 2$
$2k_2 - 4k_3 = \cancel{0}\ 6$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & \cancel{0}1 \\ 1 & -1 & 3 & \cancel{0}2 \\ 0 & 2 & -4 & \cancel{0}6 \end{array}\right]$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & -4 & 6 \end{bmatrix} \xleftarrow[R_2' = R_1]{R_1' = R_2}$$

$$\begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 3 & -6 & -3 \\ 0 & 2 & -4 & 6 \end{bmatrix} \xleftarrow{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & 3 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{2} R_3]{R_2' = \frac{1}{3} R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

Since there is no solution the vector $(1, 2, 6)$ is not linear combination of $(2, 1, 0)$, $(1, -1, 2)$ and $(0, 3, -4)$

(11) $\underline{w} = (1, 1, 1)$

$\underline{v_1} = (2, 1, 0)$
$\underline{v_2} = (1, -1, 2)$
$\underline{v_3} = (0, 3, -4)$

$\underline{w} = k_1 v_1 + k_2 v_2 + k_3 v_3$

$(1, 1, 1) = k_1 (2, 1, 0) + k_2 (1, -1, 2) + k_3 (0, 3, -4)$

$(1, 1, 1) = (2k_1 + k_2), (k_1 - k_2 + 3k_3), (2k_2 - 4k_3)$

$\therefore$ $2k_1 + k_2 = 1$

$k_1 - k_2 + 3k_3 = 1$

$2k_2 - 4k_3 = 1$

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & -1 & 3 & 1 \\ 0 & 2 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & -4 & 1 \end{bmatrix} \xleftarrow[R_2' = R_1]{R_1' = R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 3 & -6 & -1 \\ 0 & 2 & -4 & 1 \end{bmatrix} \xleftarrow{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 1 & -2 & -1/3 \\ 0 & 2 & -4 & 1 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{3}R_2} \qquad \frac{2}{3}+1 \qquad \frac{2+3}{3}$$

$$= \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 1 & -2 & -1/3 \\ 0 & 0 & 0 & 5/3 \end{bmatrix}$$

no solution

Since there is no solution $(1,1,1)$ can't be linear combination of $(2,1,0)$ $(1,-1,2)$

(iii)

the vectors,

$\underline{w} = (3,9,-4,-2)$

$\underline{v}_1 = (1,-2,0,3)$

$\underline{v}_2 = (2,3,0,-1)$

$\underline{v}_3 = (2,-1,2,1)$

$\underline{w} = k_1\underline{v}_1 + k_2\underline{v}_2 + k_3\underline{v}_3 = (k_1,-2k_1,0,3k_1) + (2k_2,3k_2,0,-k_2) + (2k_3,-k_3,2k_3,k_3)$

$(3,9,-4,-2) = k_1(1,-2,0,3) + k_2(2,3,0,-1) + k_3(2,-1,2,1)$

or $(3,9,-4,(-2)) = ((k_1+2k_2+2k_3),(-2k_1+3k_2-k_3),(2k_3),(3k_1-k_2+k_3))$

$k_1+2k_2+2k_3 = 3$

$-2k_1+3k_2-k_3 = 9$

$2k_3 = -4$

$3k_1-k_2+k_3 = -2$

-6
$+1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ -2 & 3 & -1 & 9 \\ 0 & 0 & 2 & -4 \\ 3 & -1 & 1 & -2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 7 & 3 & 15 \\ 0 & 0 & 2 & -4 \\ 0 & -7 & -5 & -11 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = 2R_1+R_2 \\ R_4' = -3R_1+R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 7 & 5 & 11 \\ 0 & 0 & 2 & -4 \end{array}\right] \longleftarrow$$

$R_2' = 1/7 R_2$

~~$R_3' = -\frac{1}{7} R_4$~~

$R_4' = R_3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & \cancel{-14/7}\ -2 & \cancel{-94}\ 4 \\ 0 & 0 & 2 & -4 \end{array}\right] \longleftarrow R_3' = -7R_2 + R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & \cancel{116}\ -2 & \cancel{94}\ 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \longleftarrow \cancel{R_3' = -7R_3/6}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & \cancel{16}\ -1 & 94\ 2 \end{array}\right]$$

$R_3' = \frac{1}{2} R_4$

$R_4 = R_3$

$= -2\times3 + 2\times2\times3$

$= -6 + 9 + 3$

$=$

$=$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3/7 & 15/7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

linear combination

$(3,9,-4,-2) = 1(1,-2,0,3)$
$+3(2,3,0,-1)$
$-2(2,-1,2,1)$

$k_3 = -2 \qquad k_2 + 3/7 k_3 = 15/7 \qquad k_1 = 1$

$k_2 + 3/7(-2) = 15/7$

$k_2 = 3$

(iv)

$\underline{w} = (2,3,-7,3)$

$v_1 = (2,1,0,3)$

$v_2 = (3,-1,5,2)$

$v_3 = (-1,0,2,1)$

$$\underline{w} = k_1 \underline{v_1} + k_2 \underline{v_2} + k_3 \underline{v_3}$$

$$(2,3,-7,3) = k_1(2,1,0,3] + k_2(3,-1,5,2) + k_3(-1,0,2,1)$$

$$(2,3,-7,3) = (2k_1+3k_2-k_3),(k_1-k_2), (5k_2+2k_3),(3k_1+2k_2+k_3)$$

$2k_1+3k-k_3 = \cancel{0}\ 2$

$k_1-k_2 = \cancel{0}\ 3$

$5k_2+2k_3 = \cancel{0}\ -7$

$3k_1+2k_2+k_3 = \cancel{0}\ 3$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow[R_2' = R_1]{R_1' = R_2} \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 3 & -1 & 2 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 5 & -1 & -4 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix} \xleftarrow[R_4' = -3R_1 + R_4]{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{5} R_2}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 2 & -2 \end{bmatrix} \xleftarrow[R_4' = -5R_2 + R_4]{R_3' = -5R_2 + R_3}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xleftarrow[R_4' = \frac{1}{2} R_2]{R_3' = \frac{1}{3} R_3}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_4' = -R_3 + R_4}$$

$k_3 = -1$

$k_2 = -1/5 - 4/5 = \frac{-1-4}{5} = -1$

$k_3 = 2$

linear combination

$(2,3,-7,3) = -1(2,1,0,3)$
$-1(3,-1,5,2)$
$+2(-1,0,2,1)$

4

① $S = \{(1,1,2)\ (1,-1,2)\ (1,0,1)\}$

Let

$\underline{b} = (b_1, b_2, b_3) \in \mathbb{R}^3$

$k_1 v_1 + k_2 v_2 + k_3 v_3 = \underline{b}$

$k_1(1,1,2) + k_2(1,-1,2) + k_3(1,0,1) = (b_1, b_2, b_3)$

Or $(k_1 + k_2 + k_3), (k_1 - k_2), (k_1 \times 2 + 2k_2 + k_3) = (b_1, b_2, b_3)$

$\therefore\ k_1 + k_2 + k_3 = b_1$

$k_1 - k_2 = b_2$

$2k_1 + 2k_2 + k_3 = b_3$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$\det(A) = 1(-1-0) - 1(1-0) + 1(2+2)$

$= -1 - 1 + 4$

$= -2 + 4 = 2$

$\det(A) \neq 0$

$\therefore$ A is invertible

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = A^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

~~So span is R^3~~ So S spans $\mathbb{R}^3$

b) $\{(-1,1,0),(-1,0,1)(1,1,1)\}$

Let,

$\underline{b} = (b_1, b_2, b_3)$

$k_1 \underline{v_1} + k_2 \underline{v_2} + k_3 \underline{v_3} = \underline{b}$

$k_1(-1,1,0) + k_2(-1,0,1) + k_3(1,1,1) = \underline{b}$

$(-k_1 - k_2 + k_3), (k_1 + k_3), (k_2 + k_3) = (b_1, b_2, b_3)$

$-k_1 - k_2 + k_3 = b_1$

$k_1 + k_3 = b_2$

$k_2 + k_3 = b_3$

$$A = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(A) = -1(0-1) + 1(1-0) + 1(1-0)$$

$$= 1 + 1 + 1$$

$$= 3$$

$\therefore \det(A) \neq 0$

$\therefore$ A is invertible

$\therefore$ so span is R^3.

(iii) $S = \{ (2,1,2), (0,1,-1), (4,3,3) \}$

Let

$\underline{b} = (b_1, b_2, b_3)$

$k_1 v_1 + k_2 v_2 + k_3 v_3 = \underline{b}$

$k_1(2,1,2) + k_2(0,1,-1) + k_3(4,3,3) = (b_1, b_2, b_3)$

or $(2k_1 + 4k_3), (k_1 + k_2 + 3k_3), (2k_1 - k_2 + 3k_3) = (b_1, b_2, b_3)$

$2k_1 + 4k_3 = b_1$

$k_1 + k_2 + 3k_3 = b_2$

$2k_1 - k_2 + 3k_3 = b_3$

$$\begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & 3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\det(A) = 2(3+3) - 0 + 4(-1-2)$$
$$= \cancel{6}\,6\ \cancel{+} - 12$$
$$= \cancel{6}\ 0$$

$\det(A) = 0$

$\therefore$ so S ~~span~~ ~~is~~ does not span $\mathbb{R}^3$

5/

① $\{(2,1,2), (0,1,-1), (4,3,3)\}$

$\underline{0} = (0,0,0,\cancel{0}) \in R^3$

$k_1v_1 + k_2v_2 + k_3v_3 = (0,0,0)$

$k_1(2,1,2) + k_2(0,1,-1) + k_3(4,3,3) = (0,0,0)$

$(2k_1 + 4k_3), (k_1 + k_2 + 3k_3), (2k_1 - k_2 + 3k_3) = (0,0,0)$

$$= \begin{bmatrix} 2 & 0 & 4 & 0 \\ 1 & 1 & 3 & 0 \\ 2 & -1 & 3 & 0 \end{bmatrix}$$

$$\det(A) = 2(3+3) + 4(-1-2)$$
$$= 12 - 12$$
$$= 0$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 0 & 4 & 0 \\ 2 & -1 & 3 & 0 \end{bmatrix} \xleftarrow{R_1 \leftrightarrows R_2}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -3 & -3 & 0 \end{bmatrix} \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = -2R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \xleftarrow[R_3' = -\frac{1}{3} R_3]{R_2' = \frac{1}{-2} R_2}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$= \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_1' = -R_2 + R_1}$$

$3-2=1$

$x_3 = K \quad x_2 = -K \quad x_1 = -2K$

So we have non trivial solution

The the set of vectors are linearly dependent.

(11) $\{(3,0,1,-1),(2,-1,0,1)(1,1,1,-2)\}$

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$k_1 v_1 + k_2 v_2 + k_3 v_3 + k_4 v_4 = (0,0,0,0)$

$k_1(3,0,1,-1) + k_2(2,-1,0,1) + k_3(1,1,1,-2) = (0,0,0,0)$

$(3k_1 + 2k_2 + k_3), (-k_2 + k_3), (k_1 + k_3), (-k_1 + k_2 - 2k_3) = (0,0,0,0)$

$$\begin{aligned} 3k_1 + 2k_2 + k_3 &= 0 \\ -k_2 + k_3 &= 0 \\ k_1 + k_3 &= 0 \\ -k_1 + k_2 - 2k_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ -1 & 1 & -2 & 0 \end{bmatrix} \xleftarrow{R_1 \rightleftarrows R_3}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \xleftarrow[R_4' = R_1 + R_4]{R_3' = -3R_1 + R_3}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \xleftarrow[R_3' = \frac{1}{2} R_3]{R_2' = -1 R_2}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_2 + R_4]{R_3' = -R_2 + R_3}$$

$\therefore$ this is a non trivial solution

$x_3 = K$

$x_2 = K$

$x_1 = -K$

So the set of the vectors are linearly dependant

$\{(1,-4,2),(3,-5,1),(2,7,8),(-1,1,1)\}$

$\underline{0} = (0,0,0) \in \mathbb{R}^3$

$\underline{0} = k_1\underline{v}_1 + k_2\underline{v}_2 + k_3\underline{v}_3 + k_4\underline{v}_4$

$(0,0,0) = k_1(1,-4,2) + k_2(3,-5,1) + k_3(2,7,8) + k_4(-1,1,1)$

$(0,0,0) = (k_1 + 3k_2 + 2k_3 - k_4), (-4k_1 - 5k_2 + 7k_3 + k_4),$

$(2k_1 + k_2 + 8k_3 + k_4)$

$\therefore k_1 + 3k_2 + 2k_3 - k_4 = 0$

$-4k_1 - 5k_2 + 7k_3 + k_4 = 0$

$2k_1 + k_2 + 8k_3 + k_4 = 0$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 0 \\ -4 & -5 & 7 & 1 & 0 \\ 2 & 1 & 8 & 1 & 0 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 7 & 15 & -3 & 0 \\ 0 & -5 & 4 & 3 & 0 \end{bmatrix} \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = 4R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & -5 & 4 & 3 & 0 \end{bmatrix} \longleftarrow R_2' = \tfrac{1}{7} R_2$$

$$\frac{5\times 15}{7} + 4$$

$$\frac{5\times 15 + 28}{7}$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & 0 & 103/7 & 6/7 & 0 \end{bmatrix} \longleftarrow R_3' = -5R_2 + R_3$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & -3/7 & 0 \\ 0 & 0 & 103 & 6 & 0 \end{bmatrix} \longleftarrow R_3' = 7R_3$$

$$= \begin{bmatrix} 1 & 3 & 2 & -1 & 0 \\ 0 & 1 & 15/7 & 3/7 & 0 \\ 0 & 0 & 1 & 6/103 & 0 \end{bmatrix} \longleftarrow R_3' = \frac{1}{103} R_3$$

$K_1 + 3K_2 + 2K_3 - K_4 = 0$

$\Rightarrow K_1 = -3K_2 - 2K_3 + K_4.$

$K_2 = -\frac{15}{7} K_3 + \frac{3}{7} K_4$

$K_3 = -\frac{6}{103} K_4$ [non trivial]

So dependent

(7)

$\{(0,1,0,1), (1,2,3,-1), (8,4,3,2), (0,3,2,0)\}$.

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$(0,0,0,0) = k_1(0,1,0,1) + k_2(1,2,3,-1) + k_3(8,4,3,2)$
$\quad + k_4(0,3,2,0)\}$

$(0,0,0,0) = (k_2 + 8k_3), (k_1 + 2k_2 + 4k_3 + 3k_4),$
$\quad (3k_2 + 3k_3 + 2k_4)\ (k_1 - k_2 + 2k_3)$

$$\begin{bmatrix} 0 & 1 & 8 & 0 & 0 \\ 1 & 2 & 4 & 3 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 1 & -1 & 2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 1 & -1 & 2 & 0 & 0 \end{bmatrix} \longleftarrow R_1 \rightleftarrows R_2$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 0 & -3 & -2 & -3 & 0 \end{bmatrix} \longleftarrow R_4' = -R_1 + R_4$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & -21 & 2 & 0 \\ 0 & 0 & 22 & -3 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_3' = -3R_2 + R_3 \\ R_4' = 3R_2 + R_4 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 2/-21 & 0 \\ 0 & 0 & 22 & -3 & 0 \end{bmatrix} \xleftarrow{R_3' = \frac{1}{-21} R_3}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 3 & 0 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 2/-21 & 0 \\ 0 & 0 & 0 & -19/21 & 0 \end{bmatrix} \xleftarrow{R_4' = -22R_3 + R_4}$$

$\therefore \frac{-19}{21} K_4 = 0$

$K_3 + \frac{2}{-21} K_4 = 0$

$K_3 = 0$ $\quad K_2 + 8K_3 = 0$

$K_2 = 0$

$K_1 = 0$

as trivial solution

so the set of the vectors

are linearly independent

$\{(1,3,2),(1,-7,-8),(2,1,-1)\}$

$\underline{0} = (0,0,0) \in \mathbb{R}^3$

$0 = k_1(1,3,2) + k_2(1,-7,-8) + k_3(2,1,-1)$

$(0,0,0) = (k_1 + k_2 + 2k_3), (3k_1 - 7k_2 + k_3), (2k_1 - 8k_2 - k_3)$

$\therefore k_1 + k_2 + 2k_3 = 0$

$3k_1 - 7k_2 + k_3 = 0$

$2k_1 - 8k_2 - k_3 = 0$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & -7 & 1 & 0 \\ 2 & -8 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & -10 & -5 & 0 \end{bmatrix} \xleftarrow[R_3' = -2R_1 + R_3]{R_2' = -3R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -10/-10 & -5/-10 & 0 \\ 0 & -10 & -5 & 0 \end{bmatrix} \xleftarrow{R_2' = -\frac{1}{10} R_2}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & -10 & -5 & 0 \end{bmatrix} \longleftarrow$$

$$= \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = 10R_2 + R_3$$

$\therefore$ Non trivial

$k_3 = \alpha$

$k_2 + k_3/2 = 0$

$k_2 = -\alpha/2$

$k_1 = \alpha/2 - 2\alpha$

$= \alpha(-\frac{1}{2} - 2)$

$= \alpha(-\frac{3/2}{})$

$= -3/2\,\alpha$

$\therefore$ So the all these vectors are ~~not~~ linearly dependant

(vi)

$\{(3,0,4,1), (6,2,-1,2), (-1,3,5,1), (-3,7,8,3)\}$

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$(0,0,0,0) = k_1(3,0,4,1) + k_2(6,2,-1,2) + k_3(-1,3,5,1)$
$+ k_4(-3,7,8,3)$

$(0,0,0,0) = (3k_1 + 6k_2 - k_3 - 3k_4), (2k_2 + 3k_3 + 7k_4),$
$(4k_1 - k_2 + 5k_3 + 8k_4), (k_1 + 2k_2 + k_3 + 3k_4)$

$k_1 + 6k_2 - k_3 - 3k_4 = 0$

$2k_2 + 3k_3 + 7k_4 = 0$

$4k_1 - k_2 + 5k_3 + 8k_4 = 0$

$k_1 + 2k_2 + k_3 + 3k_4 = 0$

$$\begin{bmatrix} 3 & 6 & -1 & -3 & 0 \\ 0 & 2 & 3 & 7 & 0 \\ 4 & -1 & 5 & 8 & 0 \\ 1 & 2 & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 3 & 7 & 0 \\ 4 & -1 & 5 & 8 & 0 \\ 3 & 6 & -1 & -3 & 0 \end{bmatrix} \xleftarrow{R_1 \rightleftharpoons R_4}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 3 & 7 & 0 \\ 0 & -9 & 1 & -4 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \xleftarrow[R_4' = -3R_4 + R_4]{R_3' = -4R_1 + R_3}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & -9 & 1 & -4 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{2}R_2}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 29/2 & 55/2 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \longleftarrow R_3' = 9R_2 + R_3$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & \cancel{29/2}\ 1 & \cancel{55/2}\ 55/29 & 0 \\ 0 & 0 & -4 & -12 & 0 \end{bmatrix} \longleftarrow R_3' = R_3 / \tfrac{29}{2}$$

$$\frac{220}{29} - 12$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 1 & 55/29 & 0 \\ 0 & 0 & 0 & -128/29 & 0 \end{bmatrix} \longleftarrow R_4' = 4R_3 + R_4$$

$$\frac{220 - 348}{29}$$

$$\frac{128}{29}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 3/2 & 7/2 & 0 \\ 0 & 0 & 1 & 55/29 & 0 \\ 0 & 0 & 0 & \cancel{-128/29}\ 1 & 0 \end{bmatrix} \longleftarrow R_4' = -\frac{29}{128} R_4$$

$\therefore k_4 = 0$

$k_3 = 0$ $\quad\therefore$ Trivial solution

$k_2 = 0$ $\quad\therefore$ So all the vectors

$k_1 = 0$ $\quad$ are linearly independent

$\{(4,-4,8,0),(2,2,4,0),(6,0,0,2),(6,3,-3,0)\}$

$(0,0,0,0) = k_1(4,-4,8,0) + k_2(2,2,4,0) + k_3(6,0,0,2) + k_4(6,3,-3,0)$

$(0,0,0,0) = (4k_1 + 2k_2 + 6k_3 + 6k_4), (-4k_1 + 2k_2 + 3k_4),$

$(8k_1 + 4k_2 - 3k_4), (2k_3)$

$\therefore\ 4k_1 + 2k_2 + 6k_3 + 6k_4 = 0$

$-4k_1 + 2k_2 + 3k_4 = 0$

$8k_1 + 4k_2 - 3k_4 = 0$

$2k_3 = 0$

$$\begin{bmatrix} 4 & 2 & 6 & 6 & 0 \\ -4 & 2 & 0 & 3 & 0 \\ 8 & 4 & 0 & -3 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ -4 & 2 & 0 & 3 & 0 \\ 8 & 4 & 0 & -3 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \xleftarrow{R_1' = \frac{1}{4}R_1}$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 4 & 6 & 9 & 0 \\ 0 & 0 & -6\times2 & -15 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \longleftarrow \begin{matrix} R_2' = 4R_1 + R_2 \\ R_3' = -8R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 1 & 6/4 & 9/4 & 0 \\ 0 & 0 & -12 & -15 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \longleftarrow$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 1 & 6/4 & 9/4 & 0 \\ 0 & 0 & 1 & 15/12 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \xleftarrow{R_3' = -\frac{1}{12} R_3}$$

$$= \begin{bmatrix} 1 & 1/2 & 6/4 & 6/4 & 0 \\ 0 & 1 & 6/4 & 9/4 & 0 \\ 0 & 0 & 1 & 15/12 & 0 \\ 0 & 0 & 0 & -15/6 & 0 \end{bmatrix}$$

$k_4 = 0$

$k_3 = 0$ Trivial solution

$k_2 = 0$

$k_1 = 0$

so the elements are linearly independent.

b//

① $S = \{ (1,2,0), (0,5,7) \ \& \ (-1,1,3) \}$

$\underline{0} = (0,0,0) \in \mathbb{R}^3$

$(0,0,0) = k_1(1,2,0) + k_2(0,5,7) + k_3(-1,1,3)$

$(0,0,0) = (k_1 - k_3), \ (2k_1 + 5k_2 + k_3), \ (7k_2 + 3k_3)$

$k_1 - k_3 = 0$

$2k_1 + 5k_2 + k_3 = 0$

$7k_2 + 3k_3 = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 5 & 1 \\ 0 & 7 & 3 \end{bmatrix} \qquad \det(A) = 1(15-7) - 1(14-5)$$

$$= 8 - 9$$

$$= -1$$

$$\neq 0$$

∴ A is independent and A is invertible

again,

$\underline{b} = (b_1, b_2, b_3) \in \mathbb{R}^3$

$(b_1, b_2, b_3) = k_1(1,2,0) + k_2(0,5,7) + k_3(-1,1,3)$

$= (k_1 - k_3) \ (2k_1 + 5k_2 + k_3) \ (7k_2 + 3k_3$

$\therefore$

$K_1 - K_3 = b_1$

$2K_1 + 5K_2 + K_3 = b_2$

$7K_2 + 3K_3 = b_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 5 & 1 \\ 0 & 7 & 3 \end{bmatrix}$$

$\det(A) = -1$

$\det(A) \neq -1$

$\therefore$ $\therefore$ S spans $\mathbb{R}^3$

$\therefore$ The set S. ~~is~~ forms a basis for $\mathbb{R}^3$

(11) $S = \{(2,0,1), (1,1,1)\}$

$\underline{0} = (0,0,0)$

$\therefore (0,0,0) = K_1(2,0,1) + K_2(1,1,1)$

$(0,0,0) = (2K_1 + K_2), (K_2), (K_1 + K_2)$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

Not getting any solution

(11)

$S = \{(1,1,1,1), (0,1,1,1), (0,0,1,1), (0,0,0,1)\}$

$\underline{0} = (0,0,0,0) \in \mathbb{R}^4$

$(0,0,0,0) = k_1(1,1,1,1) + k_2(0,1,1,1) + k_3(0,0,1,1)$
$+ k_4(0,0,0,1)$

$(0,0,0,0) = \cancel{k} (k_1), (k_1+k_2), (k_1+k_2+k_3),$
$(k_1+k_2+k_3+k_4)$

$k_1 = 0$

$k_1 + k_2 = 0$

$k_1 + k_2 + k_3 = 0$

$k_1 + k_2 + k_3 + k_4 = 0$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$\det(A) = 1 \cdot 1 \cdot 1 \cdot 1$
$= 1$

$\therefore$ A is ~~not invertible~~ invertible

so S is independent

$\underline{b} = (b_1, b_2, b_3, b_4) \in \mathbb{R}^4$

$(b_1, b_2, b_3, b_4) = k_1(1,1,1,1) + k_2(0,1,1,1) + k_3(0,0,1,1)$
$+ k_4(0,0,0,1)$

$(b_1, b_2, b_3, b_4) = (k_1), (k_1 + k_2), (k_1 + k_2 + k_3), (k_1 + k_2 + k_3 + k_4)$

$k_1 = b_1$

$k_1 + k_2 = b_2$

$k_1 + k_2 + k_3 = b_3$

$k_1 + k_2 + k_3 + k_4 = b_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\det(A) = 1 \cdot 1 \cdot 1 \cdot 1$
$= 1$

$\det(A) \neq 0$

$\therefore$ A ~~is~~ span, ~~of~~ $\mathbb{R}^4$

$\therefore$ The set S form a basis for $\mathbb{R}^4$.

7/

(1)

$$A = \begin{bmatrix} 6 & 2 & 0 & 4 \\ -2 & -1 & 3 & 4 \\ -1 & -1 & 6 & 10 \end{bmatrix}$$

~~-2~~ $-12 \; 3$

$-20 \; 4$

-16

~~-2~~

$$= \begin{bmatrix} 1 & 1 & -6 & -10 \\ -2 & -1 & 3 & 4 \\ 6 & 2 & 0 & 4 \end{bmatrix} \longleftarrow (\ ) R_1 \rightleftarrows R_3 (-1)$$

$$= \begin{bmatrix} 1 & 1 & -6 & -10 \\ 0 & 1 & -9 & -16 \\ 0 & -4 & 36 & 64 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = 2R_1 + R_2 \\ R_3' = -6R_1 + R_3 \end{array}$$

$$= \begin{bmatrix} \boxed{1 \quad 1 \quad -6 \quad -10} \\ 0 \quad \boxed{1 \quad -9 \quad -16} \\ 0 \quad 0 \quad 0 \quad 0 \end{bmatrix} \longleftarrow R_3' = 4R_2 + R_3$$

$\boxed{x_3 = \text{free}}$ $\boxed{x_2 = -16 + 9x_3}$

$x_1 = -x_2 + 6x_3 - 10$

$= 16 - 9x_3 + 6x_3 - 10$

$\boxed{x_1 = 6 - 3x_3}$

Basis for Row space

$R_1 = \{[1, 1, -6, -10]\}$

$R_2 = \{[0, 1, -9, -16]\}$

Basis for (A) column space

$c_1 = \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

Basis for R column space

$c_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad c_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

~~asis for null space~~

Rank = 2