

# CSE331: Automata and Computability

## Worksheet 3\_Solution (CFG)

**2. Give context-free grammars that generate the following languages.  $\Sigma = \{0,1\}$**

a)  $L = \{w \mid w \text{ contains at least three 1's}\}.$

b)  $L = \{w \mid w \text{ starts and ends with the same symbol}\}.$

$$S \rightarrow 0E0 \mid 1E1 \mid \epsilon$$

$$E \rightarrow 1E \mid 0E \mid \epsilon$$

c)  $L = \{w \mid \text{the length of } w \text{ is odd}\}$

$$S \rightarrow X \mid 0Z \mid 1Z$$

$$Z \rightarrow 0XZ \mid 1XZ \mid \epsilon$$

$$X \rightarrow 0 \mid 1$$

d)  $L = \{w \mid \text{the length of } w \text{ is odd and its middle is } 0\}$

$$S \rightarrow ZSZ \mid 0$$

$$Z \rightarrow 0 \mid 1$$

Or

$$S \rightarrow 0S0 \mid 0S1 \mid 1S1 \mid 1S0 \mid 0$$

e)  $L = \{w \mid w \text{ contains twice as many 1s as 0s}\}$

$$S \rightarrow SS \mid 11S0 \mid 10S1 \mid 01S1 \mid \epsilon$$

**1. Give a context-free grammar for each of the following languages.**

a)  $L(G) = \{0^n 1^m 0^m \mid n, m \geq 0\}$  over the terminals  $\{0,1\}$

b)  $L(G) = \{a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n=2m+3k\}$  over  $\Sigma = \{a,b,c\}$

$$S \rightarrow aaaSc \mid Z$$

$$Z \rightarrow aaZb \mid \epsilon$$

$$c) L(G) = \{ a^n b^m \mid 0 < n < m < 3n \}. \Sigma = \{a, b\}$$

$$S \rightarrow aSbb \mid Z$$

$$Z \rightarrow aZb \mid \varepsilon$$

$$d) L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j=k \}. \Sigma = \{a, b, c\}$$

$$e) L(G) = \{ a^i b^j c^k \mid j \neq i+k \}. \Sigma = \{a, b, c\}$$

$$S \rightarrow EcC \mid aAE \mid AU$$

$$A \rightarrow aA \mid \varepsilon$$

$$B \rightarrow bB \mid \varepsilon$$

$$C \rightarrow cC \mid \varepsilon$$

$$E \rightarrow aEc \mid F$$

$$F \rightarrow bFc \mid \varepsilon$$

$$U \rightarrow aUc \mid V$$

$$V \rightarrow bVc \mid bB$$

$$f) L(G) = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m > 0 \}.$$

**3. Consider the following context-free grammar  $\Sigma = \{0, 1\}$ .**

$$S \rightarrow A 1 B$$

$$A \rightarrow 0A \mid \varepsilon$$

$$B \rightarrow 0B \mid 1B \mid \varepsilon$$

Give leftmost and rightmost derivations and parse tree for the following strings

a) 0010101

b) 10100

c) 00011

**4. Which language generates the grammar G given by the productions.**

$$S \rightarrow aSa \mid aBa$$

$$B \rightarrow bB \mid b$$

*Ans: equal number of a's in beginning and end, at least 1 b in middle*

*Or*

$$L = \{ a^i b^j a^i, i, j > 0 \}$$

**5. Explain/Prove why the grammar below is ambiguous.**

$$S \rightarrow 0A \mid 1B$$

$$A \rightarrow 0AA \mid 1S \mid 1$$

$$B \rightarrow 1BB \mid 0S \mid 0$$

**6. Given the following ambiguous context free grammar**

$$S \rightarrow Ab \mid aaB$$

$$A \rightarrow a \mid Aa$$

$$B \rightarrow b$$

- Find leftmost and rightmost derivations for aaaaab, aabb, ab.
- Show the parse trees for the above strings in (a).
- Find an equivalent unambiguous context-free grammar.
- Give the unique leftmost derivation and parse tree for the above strings generated from the unambiguous grammar you designed in (c).

**6. a) Show that the following grammar is ambiguous.**

$$S \rightarrow aEbS$$

$$S \rightarrow aEbScS \mid \epsilon$$

$$E \rightarrow d$$

**b) Consider the grammar with start symbol D,  $\Sigma = \{c, a, b, ., 0, 1\}$**

$$D \rightarrow TL$$

$$T \rightarrow c \mid Tc$$

$$L \rightarrow L.V \mid V$$

$$V \rightarrow a \mid b \mid 0 \mid 1 \mid Va \mid Vb \mid V0 \mid V1$$

- i) Derive ccab.01 using leftmost derivation
- ii) Derive cabb0011.ab1 using rightmost derivation