

CSE331: Automata and Computability

Worksheet 4 (PDA)

1. Design pushdown automata for the following languages.

a) $L = \{w \mid w \text{ contains at least three 1's}\}.$

b) $L = \{w \mid w \text{ starts and ends with the same symbol}\}.$

d) $L = \{w \mid \text{the length of } w \text{ is odd and its middle is } 0\}.$ Show that your PDA works for the following string “10011” using instantaneous descriptions.

e) $L = \{w \mid w \text{ contains twice as many 1s as 0s}\}$

f) $L(G) = \{0^n 1^m 0^m \mid n, m \geq 0\}$ over the terminals $\{0, 1\}$

g) $L(G) = \{a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n = 2m + 3k\}$ over $\Sigma = \{a, b, c\}$

h) $L(G) = \{a^n b^m \mid 0 < n < m < 3n\}.$ $\Sigma = \{a, b\}.$ Give the instantaneous descriptions for the following string “aabbabb”

i) $L(G) = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j=k\}.$ $\Sigma = \{a, b, c\}$

j) $L(G) = \{a^i b^j c^k \mid j \neq i+k\}.$ $\Sigma = \{a, b, c\}$

~~k) $L(G) = \{a^n b^m c^m d^{2n} \mid n \geq 0, m > 0\}, \Sigma = \{a, b, c, d\}$~~

l) $L = \{a^{2m} b^{n/3} c^n a^m \mid m \geq 1, n \geq 1\}$ $\Sigma = \{a, b, c\}$

2. Convert the following context free grammars to its equivalent pushdown automata.

a) $S \rightarrow A 1 B$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 0B \mid 1B \mid \epsilon$

b) $S \rightarrow aSa \mid aBa$

$B \rightarrow bB \mid b$

c) $S \rightarrow EcC \mid aAE \mid AU$

$A \rightarrow aA \mid \epsilon$

$B \rightarrow bB \mid \epsilon$

$C \rightarrow cC \mid \epsilon$

$E \rightarrow aEc \mid F$

$F \rightarrow bFc \mid \epsilon$

$$\begin{aligned}U &\rightarrow aUc \mid V \\V &\rightarrow bVc \mid bB\end{aligned}$$

$$\begin{aligned}\text{d) } S &\rightarrow aSbb \mid Z \\Z &\rightarrow aZb \mid \epsilon\end{aligned}$$

$$\begin{aligned}\text{e) } S &\rightarrow aaaSc \mid Z \\Z &\rightarrow aaZb \mid \epsilon\end{aligned}$$