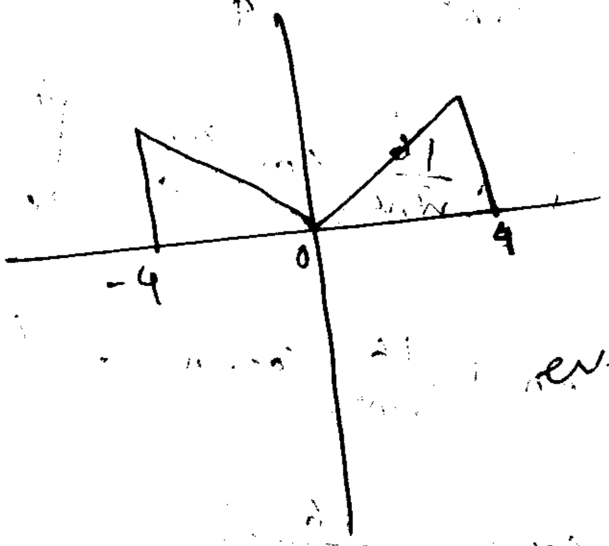
Fourier Analysis

1) a)

$$f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases}$$

period = 8



$2L = 8$

$L = 4$

even function

$b_n = 0$

$$a_n = \frac{2}{L} \int_0^4 x \cos \frac{n\pi x}{4} \, dx$$

$$= \frac{2}{4} \int_0^4 x \cos \frac{n\pi x}{4} \, dx$$

$$= \frac{1}{2} \int_0^4 x \cos \frac{n\pi x}{4} \, dx$$

$$\int x\cos\frac{n\pi x}{4}\,dx = x\int \cos\frac{n\pi x}{4} - \int\frac{d}{dx}(x)\int\cos\frac{n\pi x}{4}\,dx\,dx$$

$$= x\,\frac{4}{n\pi}\sin\frac{n\pi x}{4} - \frac{4}{n\pi}\int\sin\frac{n\pi x}{4}\,dx$$

$$= \frac{4x}{n\pi}\sin\frac{n\pi x}{4} + \frac{16}{n^2\pi^2}\cos\frac{n\pi x}{4}$$

$$\therefore a_n = \frac{1}{2}\left[\frac{4x}{n\pi}\sin\frac{n\pi x}{4} + \frac{16}{n^2\pi^2}\cos\frac{n\pi x}{4}\right]_0^4$$

$$= \frac{1}{2}\left(\left(\frac{16}{n\pi}\sin n\pi + \frac{16}{n^2\pi^2}\cos n\pi\right) - \left(0+\frac{16}{n^2\pi^2}\right)\right)$$

$$= \frac{1}{2}\left(0 + \frac{16}{n^2\pi^2}\cos n\pi - \frac{16}{n^2\pi^2}\right)$$

$$= \frac{1}{2}\times\frac{16}{n^2\pi^2}\left(\cos n\pi - 1\right)$$

$$= \frac{8}{n^2\pi^2}\left(\cos n\pi - 1\right)$$

$$a_0 = \frac{1}{L}\int_{-4}^{4} f(x)\,dx$$

$$= \frac{1}{4}\left\{-\int_{-4}^{0} x\,dx + \int_0^4 x\,dx\right\}$$

$$= \frac{1}{4}\left\{-\left[x^2/2\right]_{-4}^{0} + \left[\frac{x^2}{2}\right]_0^4\right\}$$

$$= \frac{1}{4}\left\{ -(0-8) + (16/2 - 0) \right\}$$

$$= \frac{1}{4}(8+8)$$

$$= 16/4$$

$$= 4$$

$$\therefore f(x) = a_0/2 + \sum_{n=1}^{\infty} \left\{ a_n \cos\frac{n\pi x}{4} + b_n \sin\frac{n\pi x}{4} \right\}$$

$$= 4/2 + \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2}(\cos n\pi - 1)\cos\frac{n\pi x}{4}$$

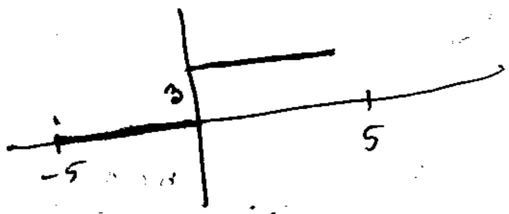
$$= 2 + \frac{8}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos n\pi - 1}{n^2}\cos\frac{n\pi x}{4}$$

(b)

$$f(x) = \begin{cases} 0, & -5 < x < 0 \\ 3, & 0 < x < 5 \end{cases} \quad \text{period} = 10$$

$2L = 10$

$L = 5$



neighter ~~Nor~~ odd nor even

$$a_0 = \frac{1}{L}\int_{-5}^{5} f(x)\,dx$$

$$= \frac{1}{5}\left\{ \int_{-5}^{0} 0\,dx + \int_{0}^{5} 3\,dx \right\}$$

$$= \frac{1}{5}\left(0 + 3\,[x]_0^5\right)$$

$$= 3/5\,(5-0)$$

$$= 3$$

$$a_n = \frac{1}{5}\int_{-5}^{5} f(x)\cos\frac{n\pi x}{5}\,dx$$

$$= \frac{1}{5}\left\{\int_{-5}^{0} 0\cos\frac{n\pi x}{5}\,dx + \int_0^5 3\cos\frac{n\pi x}{5}\,dx\right\}$$

$$= \frac{3}{5}\int_0^5 \cos\frac{n\pi x}{5}\,dx$$

$$= 3/5 \times \frac{5}{n\pi}\left[\sin\frac{n\pi x}{5}\right]_0^5$$

$$= \frac{3}{n\pi}\left(\sin n\pi - \sin 0\right)$$

$$= 0$$

$$b_n = \frac{1}{5}\int_{-5}^{5} f(x)\sin\frac{n\pi x}{5}\,dx$$

$$= \frac{1}{5}\left\{\int_{-5}^{0} 0\sin\frac{n\pi x}{9}\,dx + 3\int_0^5 \sin\frac{n\pi x}{5}\,dx\right\}$$

$$= -3/5 \times \frac{5}{n\pi}\left[\cos\frac{n\pi x}{5}\right]_0^5$$

$$= -\frac{3}{n\pi}(\cos n\pi - 1)$$

$$= \frac{3}{n\pi}(1 - \cos n\pi)$$

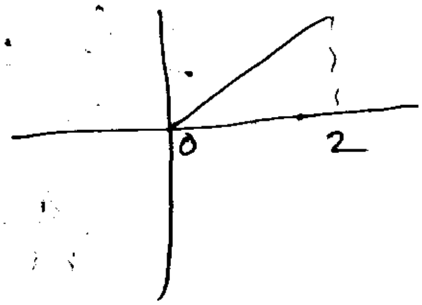
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{ a_n \cos\frac{n\pi x}{5} + b_n \sin\frac{n\pi x}{5} \right\}$$

$$= \frac{3}{2} + \frac{3}{\pi}\sum_{n=1}^{\infty} \frac{1-\cos n\pi}{n} \sin\frac{n\pi x}{5}$$

(2) $f(x) = x, \; 0 < x < 2$ in a half range

(i) sine series

(ii) cosine series



(i) As sine series

$L = 2$

$a_n = 0$

$$a_0 = \frac{2}{2}\int_0^2 x\,dx$$

$$= \frac{2}{2}\left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{2}{2}\left(\frac{4}{2} - 0\right)$$

$$= \frac{2}{2} \times 2 = 1 \times 2 = 2$$

$$b_n = \frac{2}{2}\int_0^{2} x \sin\frac{n\pi x}{2}\,dx$$

$$= \int_0^{2} x \sin\frac{n\pi x}{2}\,dx$$

$$\int x\sin\frac{n\pi x}{2}\,dx = x\int \sin\frac{n\pi x}{2} - \int \frac{d}{dx}(x)\int \sin\frac{n\pi x}{2}\,dx\,dx$$

$$= -x\,\frac{2}{n\pi}\cos\frac{n\pi x}{2} + \frac{2}{n\pi}\int\cos\frac{n\pi x}{2}\,dx$$

$$= -\frac{2x}{n\pi}\cos\frac{n\pi x}{2} + \frac{4}{n^2\pi^2}\sin\frac{n\pi x}{2}$$

$$\int_0^2 x\sin\frac{n\pi x}{2} = \left(-\frac{4}{n\pi}\cos n\pi + \frac{4}{n^2\pi^2}\sin n\pi\right)$$

$$-\left(-0+0\right)$$

$$= -\frac{4}{n\pi}\cos n\pi$$

$$b_n = -\frac{4}{n\pi}\cos n\pi$$

$$f(x) = \frac{2}{2} - \sum_{n=1}^{\infty}\frac{4}{n\pi}\cos n\pi \sin\frac{n\pi x}{2}$$

(11). cosine series

$b_n = 0$

$$a_0 = 2/2 \int_0^2 x \, dx$$

$$= \left[x^2/2 \right]_0^2$$

$$= 4/2 - 0$$

$$= 2$$

$$a_n = \frac{2}{2} \int_0^2 x \cos \frac{\cancel{n}\pi x}{\cancel{2}} \, dx$$

$$= \int_0^2 x \cos \frac{n\pi x}{2} \, dx$$

$$\int x \cos \frac{n\pi x}{2} \, dx$$

$$= x \int \cos \frac{n\pi x}{2} - \int \frac{d}{dx}(x) \int \cos \frac{n\pi x}{2} \, dx \, dx$$

$$= \frac{2x}{n\pi} \sin \frac{n\pi x}{2} - \frac{2}{n\pi} \int \sin \frac{n\pi x}{2} \, dx$$

$$= \frac{2x}{n\pi} \sin \frac{n\pi x}{2} + \frac{4}{n^2\pi^2} \cos \frac{n\pi x}{2}$$

$$\int_0^2 x \cos\frac{n\pi x}{2}\, dx$$

$$= \left(\frac{4}{n\pi}\times 0 + \frac{4}{n^2\pi^2}\cos n\pi\right)$$

$$- \left(0 + \frac{4}{n^2\pi^2}\right)$$

$$= \frac{4}{n^2\pi^2}(\cos n\pi - 1)$$

$$a_n = \frac{4}{n^2\pi^2}(\cos n\pi - 1)$$

$$f(x) = 2/2 + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}(\cos n\pi - 1)\cos\frac{n\pi x}{2}$$

$$\int_0^2 x \cos \frac{n\pi x}{2} \, dx$$

$$= \left(\frac{4}{n\pi} \times 0 + \frac{4}{n^2\pi^2} \cos n\pi \right)$$

$$- \left(0 + \frac{4}{n^2\pi^2} \right)$$

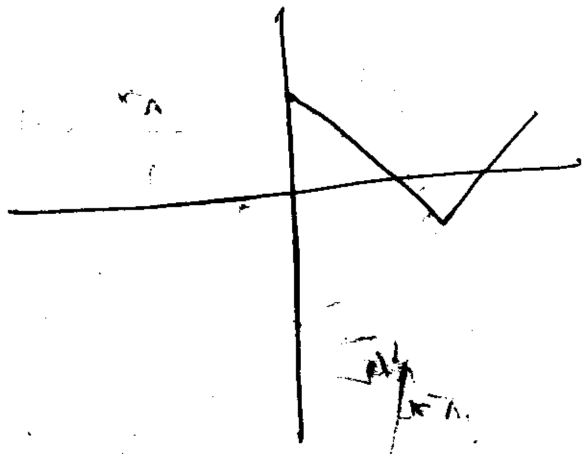
$$= \frac{4}{n^2\pi^2} (\cos n\pi - 1)$$

$$a_n = \frac{4}{n^2\pi^2} (\cos n\pi - 1)$$

$$f(x) = 2/2 + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} (\cos n\pi - 1) \cos \frac{n\pi x}{2}$$

(3)

$$f(x) = \begin{cases} 1/4 - x & ; 0 < x < 1/2 \\ x - 3/4 ; & 1/2 < x < 1 \end{cases} \quad ; L = 1$$



sine series

$a_n = 0$

$a_0 = 2/1 \left\{ \int_0^1 f(x)\, dx \right\}$

$= 2 \left[\int_0^{1/2} 1/4 - x\, dx + \int_{1/2}^1 x - 3/4\, dx \right]$

$= 2 \left[\frac{1}{4} [x]_0^{1/2} - \left[\frac{x^2}{2}\right]_0^{1/2} + \left[x^2/2\right]_{1/2}^1 - 3/4 [x]_{1/2}^1 \right]$

$= 2 \left[\frac{1}{4} \left(\frac{1}{2} - 0\right) - \left(\frac{1}{8} - 0\right) + \left(\frac{1}{2} - \frac{1}{8}\right) - \frac{3}{4}(1 - 1/2) \right]$

$= 2 \left[1/4 \times \frac{1}{2} - 1/8 + \frac{4-1}{8} - 3/4 \times 1/2 \right]$

$= 2 \left[\frac{1}{8} - \frac{1}{8} + 3/8 - 3/8 \right] = 0$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{1}\left\{\int_0^{1/2}\left(\frac{1}{4} - x\right)\sin\frac{n\pi x}{1} + \int_{1/2}^{1}\left(x - \frac{3}{4}\right)\sin\frac{n\pi x}{1}\right\}$$

$$2\left\{\frac{1}{4}\int_0^{1/2}\sin\frac{n\pi x}{1}\,dx \quad \cancel{\frac{1}{4}\left[x\right]_0^{1/2}} - \int_0^{1/2} x\sin\frac{n\pi x}{1}\,dx + \int_{1/2}^{1} x\sin\frac{n\pi x}{1}\,dx - \frac{3}{4}\int_{1/2}^{1}\sin\frac{n\pi x}{1}\,dx\right\}$$

$$= 2\left\{-\frac{1}{4}\times\frac{1}{n\pi}\left[\cos n\pi x\right]_0^{1/2} - \int_0^{1/2} x\sin n\pi x\,dx + \int_{1/2}^{1} x\sin n\pi x\,dx + \frac{3}{4}\times\frac{1}{n\pi}\left[\cos n\pi x\right]_{1/2}^{1}\right\}$$

$$\int x\sin n\pi x\,dx$$

$$= x\int\sin n\pi x\,dx - \int\frac{d}{dx}(x)\int\sin n\pi x\,dx\,dx$$

$$= -x\frac{1}{n\pi}\cos n\pi x + \frac{1}{n\pi}\int\cos n\pi x\,dx$$

$$= -\frac{x}{n\pi}\cos n\pi x + \frac{1}{n^2\pi^2}\sin n\pi x$$

$$\int_0^{1/2} x \sin n\pi x$$

$$= \left(-\frac{x}{n\pi} \cos n\pi x + \frac{1}{n^2\pi^2} \sin n\pi x \right)_0^{1/2}$$

$$= \left(-\frac{1}{2n\pi} \cos \frac{n\pi}{2} + \frac{1}{n^2\pi^2} \sin \frac{n\pi}{2} \right)$$

$$- (0 + 0)$$

$$= -\frac{1}{2n\pi} \cos \frac{n\pi}{2} + \frac{1}{n^2\pi^2} \sin \frac{n\pi}{2}$$

$$\int_{1/2}^{1} x \sin n\pi x = \left[-\frac{x}{n\pi} \cos n\pi x + \frac{1}{n^2\pi^2} \sin n\pi x \right]_{1/2}^{1}$$

$$= \left(-\frac{1}{n\pi} \cos n\pi + \frac{1}{n^2\pi^2} \sin n\pi \right)$$

$$- \left(-\frac{1}{2\pi n} \cos \frac{n\pi}{2} + \frac{1}{n^2\pi^2} \sin \frac{n\pi}{2} \right)$$

$$= -\frac{1}{n\pi} \cos n\pi + \cancel{\frac{}{n\pi^2}} + \frac{1}{2n\pi} \cos \frac{n\pi}{2} - \frac{1}{n^2\pi^2} \sin \frac{n\pi}{2}$$

$$- \int_0^{1/2} x \sin n\pi x \, dx + \int_{1/2}^{1} x \sin n\pi x \, dx$$

$$= \frac{1}{2n\pi} \cos \frac{n\pi}{2} - \frac{1}{n^2\pi^2} \sin \frac{n\pi}{2} - \frac{1}{n\pi} \cos n\pi$$

$$+ \frac{1}{2n\pi} \cos \frac{n\pi}{2} - \frac{1}{n^2\pi^2} \sin \frac{n\pi}{2}$$

$$= \frac{1}{n\pi}\cos\frac{n\pi}{2} - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{1}{n\pi}\cos n\pi$$

$$b_n = 2\Big\{ -\frac{1}{4n\pi}\left(\cos\frac{n\pi}{2} - 1\right) +$$

$$\frac{1}{n\pi}\cos\frac{n\pi}{2} - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$- \frac{1}{n\pi}\cos n\pi + \frac{3}{4n\pi}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)\Big\}$$

$$= 2\Big(-\frac{1}{4n\pi}\cos\frac{n\pi}{2} + \frac{1}{4n\pi} +$$

$$\frac{1}{n\pi}\cos\frac{n\pi}{2} - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$- \frac{1}{n\pi}\cos n\pi + \frac{3}{4n\pi}\cos n\pi -$$

$$\frac{3}{4n\pi}\cos\frac{n\pi}{2}\Big)$$

$$= 2\Big(\left(-\frac{1}{4n\pi} + \frac{1}{n\pi} - \frac{3}{4n\pi}\right)\cos\frac{n\pi}{2}$$

$$+ \frac{1}{4n\pi} - \left(\frac{1}{n\pi} - \frac{3}{4n\pi}\right)\cos n\pi$$

$$\cancel{+\frac{1}{4n\pi}} - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$= 2\left[\left(\frac{-1+4-3}{4n\pi}\right)\cos\frac{n\pi}{2} + \frac{1}{4n\pi}\right.$$

$$\left. - \left(\frac{4-3}{4n\pi}\right)\cos n\pi - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$= 2 \times \frac{0}{4n\pi}\cos\frac{n\pi}{2} + \frac{1\times 2}{4n\pi}$$

$$- \frac{2\times 1}{4n\pi}\cos n\pi - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$= \frac{1}{2n\pi} - \frac{1}{2n\pi}\cos n\pi - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

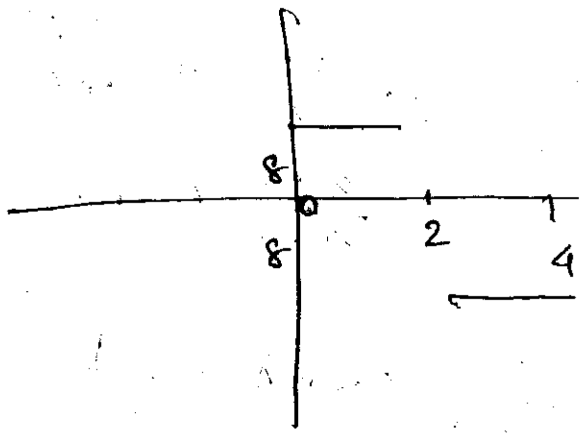
$$f(x) = \sum_{n=1}^{\infty}\left(\frac{1}{2n\pi} - \frac{1}{2n\pi}\cos n\pi - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}\right)\sin\frac{n\pi x}{1}$$

(4)

(a) $f(x) = \begin{cases} 8, & 0 < x < 2 \\ -8, & 2 < x < 4 \end{cases}$ period 4

$2L = 4$

$L = 2$



Neither Odd Nor even

$a_0 = \frac{1}{2}\int_0^4 f(x)\,dx$

$= \frac{1}{2}\int_0^2 8\,dx - \frac{1}{2}\int_2^4 8\,dx$

$= 4\,[x]_0^2 - 4\,[x]_2^4$

$= 4(2-0) - 4(4-2)$

$= 4\times2 - 4\times2 = 0$

$$a_n = \frac{1}{2} \int_0^4 f(x) \cos \frac{n\pi x}{2} \, dx$$

$$= \frac{1}{2} \left[\int_0^2 8 \cos \frac{n\pi x}{2} - \int_2^4 8 \cos \frac{n\pi x}{2} \, dx \right]$$

$$= \frac{1}{2} \left[8 \times \frac{2}{n\pi} \left[\sin \frac{n\pi x}{2} \right]_0^2 - 8 \times \frac{2}{n\pi} \left[\sin \frac{n\pi x}{2} \right]_2^4 \right]$$

$$= \frac{1}{2} \left[\frac{16}{n\pi} (\sin n\pi - \sin 0) - \frac{16}{n\pi} (\sin 2n\pi - \sin n\pi) \right]$$

$$= \frac{1}{2} [0 - 0] = 0$$

$$b_n = \frac{1}{2} \int_0^4 f(x) \sin \frac{n\pi x}{2} \, dx$$

$$= \frac{1}{2} \left[8 \int_0^2 \sin \frac{n\pi x}{2} \, dx - 8 \int_2^4 \sin \frac{n\pi x}{2} \, dx \right]$$

$$= \frac{1}{2} \left[-8 \times \frac{2}{n\pi} \left[\cos \frac{n\pi x}{2} \right]_0^2 + 8 \times \frac{2}{n\pi} \left[\cos \frac{n\pi x}{2} \right]_2^4 \right]$$

$$= \frac{1}{2} \left[-\frac{16}{n\pi} (\cos n\pi - 1) + \frac{16}{n\pi} (1 - \cos n\pi) \right]$$

$$= \frac{1}{2} \left[\frac{16}{n\pi} (1 - \cos n\pi) + \frac{16}{n\pi} (1 - \cos n\pi) \right]$$

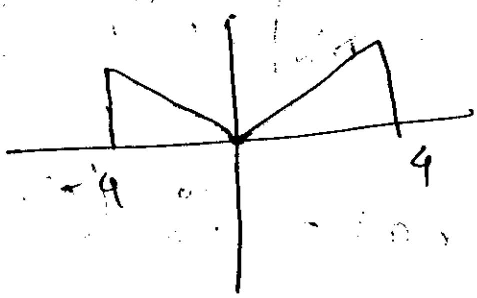
$$= \frac{16}{n\pi} (1 - \cos n\pi)$$

$$f(x) = \frac{16}{\pi} \sum_{n=1}^{\infty} \frac{1-\cos n\pi}{n} \sin \frac{n\pi x}{2}$$

(b)

$$f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases}$$

$L = 4$



From the function we can see that this is an even function

so $b_n = 0$

$$a_0 = \frac{2}{L} \int_0^4 x \, dx$$

$$= 2/4 \int_0^4 x \, dx$$

$$= \frac{1}{2} \left[\frac{x^2}{2}\right]_0^4$$

$$= 1/2 \times 8$$

$$= 4$$

$$a_n = \frac{2}{4}\int_0^4 x \cos\frac{n\pi x}{4}\,dx$$

$$\therefore \int x\cos\frac{n\pi x}{4}\,dx$$

$$= x\int \cos\frac{n\pi x}{4} - \int \frac{d}{dx}(x)\int \cos\frac{n\pi x}{4}\,dx\,dx$$

$$= \frac{4x}{n\pi}\sin\frac{n\pi x}{4} - \frac{4}{n\pi}\int \sin\frac{n\pi x}{4}\,dx$$

$$= \frac{4x}{n\pi}\sin\frac{n\pi x}{4} + \frac{16}{n^2\pi^2}\cos\frac{n\pi x}{4}\,dx$$

$$\int_0^4 x\cos\frac{n\pi x}{4}\,dx$$

$$= \left(\frac{16}{n\pi}\sin n\pi + \frac{16}{n^2\pi^2}\cos n\pi\right) - \left(0 + \frac{16}{n^2\pi^2}\right)$$

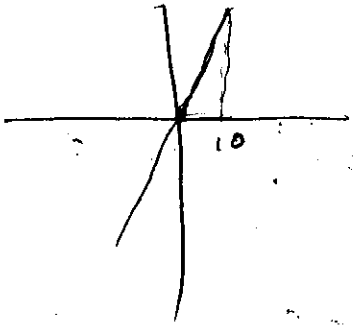
$$= \frac{16}{n^2\pi^2}(\cos n\pi - 1)$$

$$f(x) = 2\,\cancel{\frac{16}{2\pi^2}}\sum \frac{\cos n\pi - 1}{\cancel{k}}\cos\frac{n\pi x}{4}$$

$$= 2 + \frac{8}{\pi^2}\sum \frac{\cos n\pi - 1}{n^2}\cos\frac{n\pi x}{4}$$

(c)

$f(x) = 4x, \quad 0 < x < 10$



$2L = 10$

$L = 10/2$

$= 5$

Neither odd nor even

$a_0 = \frac{1}{5}\int_0^{10} 4x \, dx$

$= \frac{4}{5}\left[\frac{x^2}{2}\right]_0^{10}$

$= \frac{4}{5} \times \frac{100}{2}$ 20

$= 40$

$a_n = \frac{4}{5}\int_0^{10} x \cos\frac{n\pi x}{5} \, dx$

$\cancel{= 4/5}$

$\int x \cos\frac{n\pi x}{5} = x\int \cos\frac{n\pi x}{5} - \int \frac{d}{dx}(x) \int \cos\frac{n\pi x}{5} \, dx \, dx$

$= \frac{5x}{n\pi} \sin\frac{n\pi x}{5} - \frac{5}{n\pi}\int \sin\frac{n\pi x}{5} \, dx$

$= \frac{5x}{n\pi} \sin\frac{n\pi x}{5} + \frac{25}{n^2\pi^2} \cos\frac{n\pi x}{5} \, dx$

π

$$\int_0^{10} x\cos\frac{n\pi x}{5}\,dx$$

$$= \cancel{\left(\frac{50}{n\pi}\times 0 + \frac{25}{n^2\pi^2}\right) - \left(0 + \frac{25}{n^2\pi^2}\right)}$$

$$= \frac{\cancel{50}}{n^2\pi^2}$$

$$= \left(\frac{50}{n\pi}\sin 2n\pi + \frac{25}{n^2\pi^2}\cos 2n\pi\right) - \left(0 + \frac{25}{n^2\pi^2}\right)$$

$$= \frac{50}{n\pi}\sin 2n\pi + \frac{25}{n^2\pi^2}\cos 2n\pi - \frac{25}{n^2\pi^2}$$

$$= \frac{50n\pi\sin 2n\pi + 25\cos 2n\pi - 25}{n^2\pi^2}$$

$$= \frac{25(2n\pi\sin 2n\pi + \cos 2n\pi - 1)}{n^2\pi^2}$$

$\therefore a_n = 4/5 \times$

$$b_n = \frac{4}{5}\int_0^{10} x\sin\frac{n\pi x}{5}\,dx$$

$$\int x\sin\frac{n\pi x}{5}\,dx = x\int\sin\frac{n\pi x}{5} - \int\frac{d}{dx}(x)\int\sin\frac{n\pi x}{5}\,dx$$

$$= -x\frac{5}{n\pi}\cos\frac{n\pi x}{5} + \frac{5}{n\pi}\int\cos\frac{n\pi x}{5}$$

$$= -\frac{5x}{n\pi}\cos\frac{n\pi x}{5} + \frac{25}{n\pi}\sin\frac{x}{5}$$

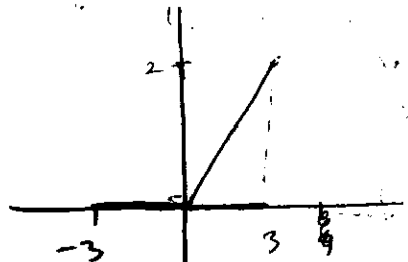
$$\int_0^{10} x \sin \frac{n\pi x}{5}$$

$$= \left(-\frac{\cancel{25}\, 50}{n\pi} \cos 2n\pi + \frac{25}{n\pi} \sin 2n\pi \right)$$

$$- \left(- 0 + 0 \right)$$

$$= -\frac{50}{n\pi} \cos 2n\pi + \frac{25}{n\pi} \sin 2n\pi$$

(c) $f(x) = \begin{cases} 2x, & 0 \le x \le 3 \\ 0, & -3 \le x \le 0 \end{cases}$ period = 6



Neighter odd nor even

$2L = 6$

$L = 3$

$$a_0 = \frac{1}{3}\int_{-3}^{3} f(x)\,dx$$

$$= \frac{1}{3}\int_{-3}^{0} 0\,dx + \frac{1}{3}\int_{0}^{3} 2x\,dx$$

$$= \frac{1}{3} \times 2 \left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3} \times \frac{9}{2} = 3$$

$$a_n = \frac{1}{3}\int_{-3}^{3} f(x)\cos\frac{n\pi x}{3}\,dx$$

$$= \frac{1}{3}\int_{-3}^{0} 0\cos\frac{n\pi x}{3}\,dx + \frac{1}{3}\int_{0}^{3} 2x\cos\frac{n\pi x}{3}\,dx$$

$$= \frac{2}{3}\int_{0}^{3} x\cos\frac{n\pi x}{3}\,dx$$

$$\int x\cos\frac{n\pi x}{3}\,dx = x\int\cos\frac{n\pi x}{3} - \int\frac{d}{dx}(x)\int\cos\frac{n\pi x}{3}\,dx\,dx$$

$$= \frac{3x}{n\pi}\sin\frac{n\pi x}{3} - \frac{3}{n\pi}\int\sin\frac{n\pi x}{3}\,dx$$

$$= \frac{3x}{n\pi}\sin\frac{n\pi x}{3} + \frac{9}{n^2\pi^2}\cos\frac{n\pi x}{3}$$

$$\int_{0}^{3} x\cos\frac{n\pi x}{3}\,dx = \left(0 + \frac{9}{n^2\pi^2}\cos n\pi\right) - \left(0 + \frac{9}{n^2\pi^2}\right)$$

$$= \frac{9}{n^2\pi^2}(\cos n\pi - 1)$$

$$\therefore a_n = 2/3 \times \frac{9}{n^2\pi^2}(\cos n\pi - 1)$$

$$= \frac{6}{n^2\pi^2}(\cos n\pi - 1)$$

$$b_n = \frac{1}{3}\int_{-3}^{3} f(x) \sin \frac{n\pi x}{3}\, dx$$

$$= \frac{1}{3}\int_{-3}^{0} 0 \sin \frac{n\pi x}{3}\, dx + \frac{1}{3}\int_{0}^{3} 2x \sin \frac{n\pi x}{3}\, dx$$

$$= \frac{2}{3}\int_{0}^{3} x \sin \frac{n\pi x}{3}\, dx$$

$$\int x \sin \frac{n\pi x}{3}\, dx = x\int \sin \frac{n\pi x}{3} - \int \frac{d}{dx}(x) \int \sin \frac{n\pi x}{3}\, dx\, dx$$

$$= -\frac{3x}{n\pi} \cos \frac{n\pi x}{3} + \frac{3}{n\pi}\int \cos \frac{n\pi x}{3}\, dx$$

$$= -\frac{3x}{n\pi} \cos \frac{n\pi x}{3} + \frac{9}{n^2\pi^2} \sin \frac{n\pi x}{3}$$

$\therefore$ ~~$b_n = \frac{2}{3}$~~

$$\int_{0}^{3} x \sin \frac{n\pi x}{3}\, dx = \left(-\frac{9}{n\pi} \cos n\pi + 0\right) - (0 + 0)$$

$$= -\frac{9}{n\pi} \cos n\pi$$

$$\therefore\ b_n = \frac{2}{3}\left(-\frac{9}{n\pi} \cos n\pi\right)$$

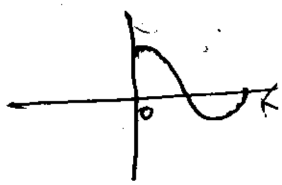
$$= -\frac{6}{n\pi} \cos n\pi$$

$$f(x) = \frac{3}{2} + \sum_{n}^{\infty} \left\{ \left(\frac{6}{n^2\pi^2} (\cos n\pi - 1) \right) \cos \frac{n\pi x}{2} + -\frac{6}{n\pi} \cos n\pi \sin \frac{n\pi x}{2} \right\}$$

(5)

$f(x) = \cos x \ , 0 < x < \pi$

$L = \pi$



book ans

$\frac{8}{\pi}\sum \frac{n\sin 2nx}{4n^2-1}$

As sine series

$a_n = 0.$

$$b_n = \frac{2}{\pi}\int_0^{\pi} \cos x \sin \frac{n\pi x}{\pi}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \cos x \sin nx\,dx$$

$$\int \cos x \sin nx\,dx = \cos x \int \sin nx\,dx - \int \frac{d}{dx}(\cos x)\int \sin nx\,dx\,dx$$

$$= -\frac{\cos x}{n}\overset{\cos nx}{\cancel{\sin nx}} + \frac{1}{n}\int -\sin x \cos nx\,dx$$

$$= -\frac{\cos x\,\overset{\cos nx}{\cancel{\sin nx}}}{n} - \frac{1}{n}\int \sin x \cos nx\,dx$$

$$= -\frac{\cos x\,\overset{\cos nx}{\cancel{\sin nx}}}{n} - \frac{1}{n}\left\{\sin x \int \cos nx\,dx - \int \frac{d}{dx}(\sin x)\int \cos nx\,dx\,dx\right\}$$

$$= -\frac{\cos x\,\overset{\cos nx}{\cancel{\sin nx}}}{n} - \frac{1}{n}\left\{\frac{\sin x \sin nx}{n} - \frac{1}{n}\int \cos x \sin nx\,dx\right.$$

$$= -\frac{\cos x \cos nx}{n} - \frac{1}{n}\times\frac{\sin x \sin nx}{n} + \frac{1}{n^2}\int \cos x \sin nx\,dx$$

II.

$$\int \cos x \sin nx\,dx - \frac{1}{n^2}\int \cos x \sin nx\,dx$$

$$= -\frac{\cos x \cos nx}{n} - \frac{\sin x \sin nx}{n^2}$$

$$\left(1-\frac{1}{n^2}\right)\int \cos x \sin nx\,dx = -\frac{\cos x \cos nx}{n} - \frac{\sin x \sin nx}{n^2}$$

$$\int \cos x \sin nx\,dx = -\left(\frac{\cancel{n^2-1}}{\cancel{n^2}}\right)\left(\frac{\cos x \cos nx}{n} + \frac{\sin x \sin nx}{n^2}\right)$$

$$= \frac{\cancel{1-n^2}\ n^2}{\cancel{n^2}\ 1-n^2} \times \left(\frac{\cos x \cos nx}{n} + \frac{\sin x \sin nx}{n^2}\right)$$

$$\int_0^{\pi} \cos x \sin nx = \left(\left(\frac{\cancel{1-n^2}\ n^2}{\cancel{n^2}\ 1-n^2}\right)\left(\frac{\cos\pi \cos n\pi}{n} + \frac{\sin\pi \sin n\pi}{n^2}\right)\right.$$

$$- \left(\frac{\cancel{1-n^2}\ n^2}{\cancel{n^2}\ 1-n^2}\right)\left(\frac{\cos 0 . \cos 0}{n} + \frac{\sin 0 \sin 0}{n^2}\right)$$

$$= \left(\frac{\cancel{1-n^2}\ n^2}{\cancel{n^2}\ 1-n^2}\right)\left(\frac{-\cos n\pi}{n}\right) - \left(\frac{1-n^2}{n^2}\right)\left(\frac{1}{n}\right)$$

$$= \left(\frac{\cancel{1-n^2}\ n^2}{\cancel{n^2}\ 1-n^2}\right)\left(-\frac{\cos n\pi}{n} - \frac{1}{n}\right)$$

$$= \left(\frac{\cancel{1-n^2}}{\cancel{n^2}} \right) \left(\frac{-\cos n\pi - n}{n} \right)$$

$$= \frac{n^2 - 1}{n^2} \times \frac{\cos n\pi + n}{n^2}$$

$\cos(x)$

$$= \frac{\cancel{(n^2-1)}(\cos n\pi + n)}{n^4}$$

$$= \frac{-n(\cos n\pi + 1)}{n(1-n^2)}$$

$$= \frac{n(\cos n\pi + 1)}{n^2 - 1}$$

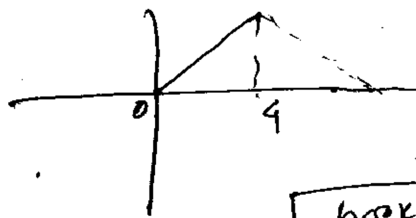
$$b_n = \frac{2n(\cos n\pi + 1)}{\pi(n^2-1)}$$

$$a_0 = \frac{2}{\pi} \int_0^\pi \cos x\,dx = \frac{2}{\pi} \left[\sin x \right]_0^\pi$$

$$= \frac{2}{\pi}(\sin\pi - \sin 0)$$

$$= 0$$

(6) $f(x) = \begin{cases} x, & 0 < x < 4 \\ 8-x, & 4 < x < 8 \end{cases}$



book ans

$\frac{32}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin\frac{n\pi}{2} \sin\frac{n\pi x}{8}$

$\frac{16}{\pi^2} \sum \left(\frac{2\cos n\pi/2 - \cos n\pi - 1}{n^2} \right) \cos\frac{n\pi x}{8}$

a $\cancel{2L = 8}$

$L = \cancel{4}\ 8$

$a_0 = \frac{\cancel{2}}{\cancel{8}} \int_0^4 x\,dx + \frac{1}{4}\int_4^8 8-x\,dx$

$= \frac{1}{\cancel{8}}\left[\frac{x^2}{2}\right]_0^4 + \frac{1}{\cancel{8}} 8\left[x\right]_4^8 - \frac{1}{4}\left[\frac{x^2}{2}\right]_4^8$

$= \frac{1}{4}(8-0) + 2(8-4) - \frac{1}{4}(32-8)$

$= \frac{1}{4} \times 8 + 2 \times 4 - \frac{1}{4} \times 24 = 2 + 8 - 6 = 4$

$a_n = \frac{2}{8}\int_0^4 x\cos\frac{n\pi x}{8}\,dx + \frac{2}{8}\int_4^8 8\cos\frac{n\pi x}{8}\,dx - \frac{2}{8}\int_4^8 x\cos\frac{n\pi x}{8}\,dx$

$= \frac{1}{4}\int_0^4 x\cos\frac{n\pi x}{8}\,dx + \frac{1}{4} \times 8 \int_4^8 \cos\frac{n\pi x}{8}\,dx - \frac{1}{4}\int_4^8 x\cos\frac{n\pi x}{8}\,dx$

$\int x\cos\frac{n\pi x}{8}\,dx = x\int \cos\frac{n\pi x}{8}\,dx - \int \frac{d}{dx}(x) \int \cos\frac{n\pi x}{8}\,dx\,dx$

$= \frac{8x}{n\pi}\sin\frac{n\pi x}{8} - \frac{8}{n\pi}\int \sin\frac{n\pi x}{8}\,dx$

$= \frac{8x}{n\pi}\sin\frac{n\pi x}{8} + \frac{64}{n^2\pi^2}\cos\frac{n\pi x}{8}$