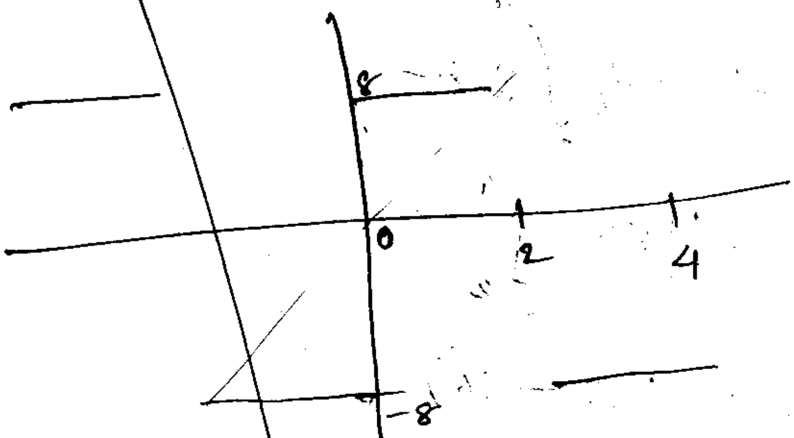
$\boxed{4}$

(a) $f(x) = \begin{cases} 8, & 0<x<2 \\ -8, & 2<x<4 \end{cases}$ period 4 $\qquad 2L = 4$, $L = 2$



This is an odd function

$\therefore a_n = 0$

$$b_n = \frac{1}{L}\int_{0}^{4} f(x) \sin\frac{n\pi x}{L}\, dx$$

$$= \frac{1}{2}\int_0^4 f(x) \sin\frac{n\pi x}{2}\, dx$$

$$= \frac{1}{2}\int_0^2 8 \sin\frac{n\pi x}{2}\, dx + \frac{1}{2}\int_2^4 -8 \sin\frac{n\pi x}{2}\, dx$$

$$= -\frac{1}{2}\times 8 \frac{2}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_0^2 + \frac{1}{2}\times 8\frac{2}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_2^4$$

$$= -\frac{8}{n\pi}(\cos n\pi - 1) + \frac{8}{n\pi}\left(\cos\frac{n\pi 4}{2} - \cos\frac{n\pi 2}{2}\right)$$

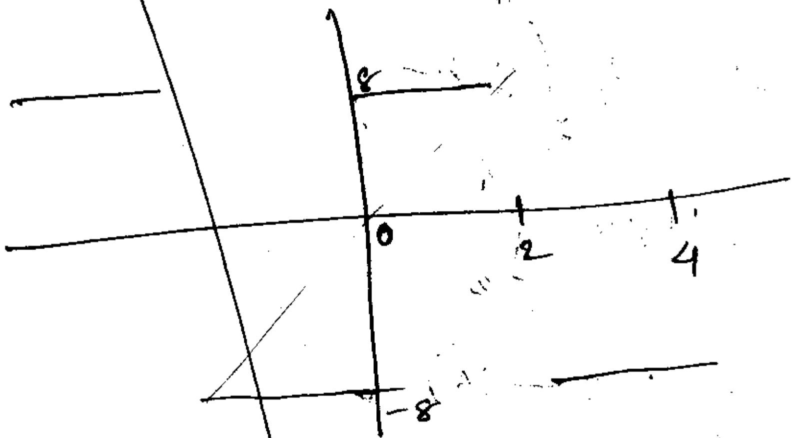
$$= \frac{8}{n\pi}(1-\cos n\pi) + \frac{8}{n\pi}(1-\cos n\pi)$$

$\boxed{4}$

(a) $f(x) = \begin{cases} 8, & 0 < x < 2 \\ -8, & 2 < x < 4 \end{cases}$ period 4

$2L = 4$

$L = 2$



This is an odd function

$\therefore a_n = 0$

$$b_n = \frac{1}{L} \int_{0}^{4} f(x) \sin \frac{n\pi x}{L} \, dx$$

$$= \frac{1}{2} \int_{0}^{4} f(x) \sin \frac{n\pi x}{2} \, dx$$

$$= \frac{1}{2} \int_{0}^{2} 8 \sin \frac{n\pi x}{2} \, dx + \frac{1}{2} \int_{2}^{4} -8 \sin \frac{n\pi x}{2} \, dx$$

$$= -\frac{1}{2} \times 8 \frac{2}{n\pi} \left[\cos \frac{n\pi x}{2} \right]_{0}^{2} + \frac{1}{2} \times 8 \frac{2}{n\pi} \left[\cos \frac{n\pi x}{2} \right]_{2}^{4}$$

$$= -\frac{8}{n\pi} (\cos n\pi - 1) + \frac{8}{n\pi} \left(\cos \frac{n\pi 4}{2} - \cos \frac{n\pi 2}{2} \right)$$

$$= \frac{8}{n\pi} (1 - \cos n\pi) + \frac{8}{n\pi} (1 - \cos n\pi)$$

$$= \frac{16}{n\pi}(1 - \cos n\pi)$$

$$a_0 = \frac{1}{L}\int_0^4 f(x)\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\int_0^2 8\,dx - \frac{1}{2}\int_2^4 8\,dx$$

$$= \frac{1}{2}\left[8x\right]_0^2 - \frac{1}{2}\left[8x\right]_2^4$$

$$= \frac{1}{2}\times 16 - \frac{1}{2}(32 - 16)$$

$$= \frac{1}{2}\times 16 - \frac{1}{2}\times 16$$

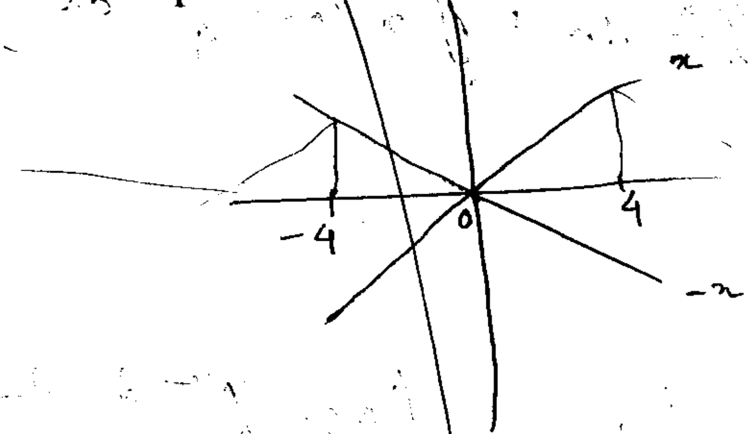
$$= 0$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{16}{n\pi}(1 - \cos n\pi)\sin\frac{n\pi x}{2}$$

(b) $f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases}$ period 8

$2L = 8$

$L = 4$



from the graph we see that the function is even

$\therefore b_n = 0$

$$a_0 = \frac{1}{L}\int_{-4}^{4} f(x)\,dx$$

$$= \frac{1}{4}\left(-\int_{-4}^{0} x\,dx + \int_{0}^{4} x\,dx\right)$$

$$= \frac{1}{4}\left(-\left[\frac{x^2}{2}\right]_{-4}^{0} + \left[\frac{x^2}{2}\right]_{0}^{4}\right)$$

$$= \frac{1}{4}\left(-\left(0 - \frac{16}{2}\right) + \left(\frac{16}{2} - 0\right)\right)$$

$$= \frac{1}{4}(8 + 8) = \frac{1}{4} \times 16 = 4$$

$$a_n = \frac{1}{L}\int_{-4}^{4} f(x)\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{4}\left[-\int_{-4}^{0} x\cos\frac{n\pi x}{4}\,dx + \int_{0}^{4} x\cos\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{4}\left[-\int_{-4}^{0} x\cos\frac{n\pi x}{4}\,dx + \int_{0}^{4} x\cos\frac{n\pi x}{4}\,dx\right]$$

$$\therefore \int x\cos\frac{n\pi x}{4}\,dx$$

$$= x\int\cos\frac{n\pi x}{4}\,dx - \int\frac{d}{dx}(x)\int\cos\frac{n\pi x}{4}\,dx\,dx$$

$$= \frac{4x}{n\pi}\sin\frac{n\pi x}{4} - \frac{4}{n\pi}\int\sin\frac{n\pi x}{4}\,dx$$

$$= \frac{4x}{n\pi}\sin\frac{n\pi x}{4} + \frac{4}{n\pi}\times\frac{4}{n\pi}\cos\frac{n\pi x}{4}$$

$$= \frac{4x}{n\pi}\sin\frac{n\pi x}{4} + \frac{16}{n^2\pi^2}\cos\frac{n\pi x}{4}$$

$$\therefore \int_{-4}^{0} x\cos\frac{n\pi x}{4}\,dx$$

$$= \left(0 + \frac{16}{n^2\pi^2}\right) - \left(\frac{16}{n\pi}\times 0 + \frac{16}{n^2\pi^2}\cos\frac{-n\pi 4}{4}\right)$$

$$= \frac{16}{n^2\pi^2} - \frac{16}{n^2\pi^2}\cos n\pi$$

$$= \frac{16}{n^2\pi^2}(1-\cos n\pi)$$

$$\therefore \int_0^4 x \cos\frac{n\pi x}{4}\,dx$$

$$= \left(\frac{16}{n\pi}\times 0 + \frac{16}{n^2\pi^2}\cos n\pi\right) - \left(0 + \frac{16}{n^2\pi^2}\cos 0\right)$$

$$= \frac{16}{n^2\pi^2}\cos n\pi - \frac{16}{n^2\pi^2}$$

$$= \frac{16}{n^2\pi^2}(\cos n\pi - 1)$$

$$a_n = \frac{1}{4}\left[\frac{16}{n^2\pi^2}(\cos n\pi - 1) + \frac{16}{n^2\pi^2}(\cos n\pi - 1)\right]$$

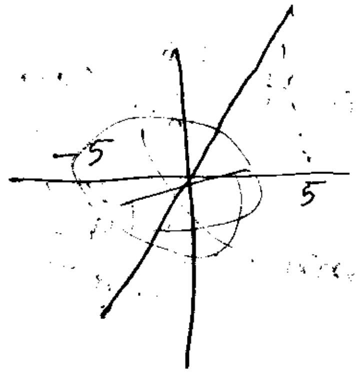
$$= \frac{1}{\cancel{4}}\times 2\times\frac{\cancel{16}^{4}}{n^2\pi^2}(\cos n\pi - 1)$$

$$= \frac{8}{n^2\pi^2}(\cos n\pi - 1)$$

$$f(x) = 2 + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}(\cos n\pi - 1)\cos\frac{n\pi x}{4}$$

(c)

$f(x) = 4x \quad , \quad 0 < x < 10$ period $= 10$



symmetric with respect to ~~axis~~ origin so this is an odd function

$\therefore a_n = 0 \qquad 2L = 10, \ L = 5$

$$a_0 = \frac{1}{L}\int_0^{10} 4x \, dx$$

$$= \frac{1}{5}\int_0^{10} 4x \, dx$$

$$= \frac{4}{5}\left[\frac{x^2}{2}\right]_0^{10}$$

$$= \cancel{\frac{4}{5}\left(\frac{25}{4} - \frac{25}{4}\right)} = \frac{\cancel{4}^{2}}{5} \times \frac{100}{\cancel{4}} = 40$$

$$= \cancel{0}$$

$$b_n = \frac{1}{5}\int_0^{10} 4x \sin\frac{n\pi x}{5} \, dx$$

$$= \frac{4}{5}\int_0^{10} x \sin\frac{n\pi x}{5} \, dx$$

$$\int x \sin \frac{n\pi x}{5}\, dx$$

$$= x \int \sin \frac{n\pi x}{5} - \int \left\{ \frac{d}{dx}(x) \int \sin \frac{n\pi x}{5}\, dx \right\} dx$$

$$= -\frac{5x}{n\pi} \cos \frac{n\pi x}{5} + \frac{5}{n\pi} \int \cos \frac{n\pi x}{5}\, dx$$

$$= -\frac{5x}{n\pi} \cos \frac{n\pi x}{5} + \frac{25}{n^2\pi^2} \sin \frac{n\pi x}{5}$$

$$\int_{0}^{10} x \sin \frac{n\pi x}{4}\, dx = \left(-\frac{5 \times 10}{n\pi} \cos \frac{n\pi \times 10}{5} + \frac{25}{n^2\pi^2} \sin \frac{n\pi \times 10}{5} \right)$$

$$- \left(0 + 0 \right)$$

$$= \left(-\frac{50}{n\pi} + 0 \right)$$

$$= -\frac{50}{n\pi}$$

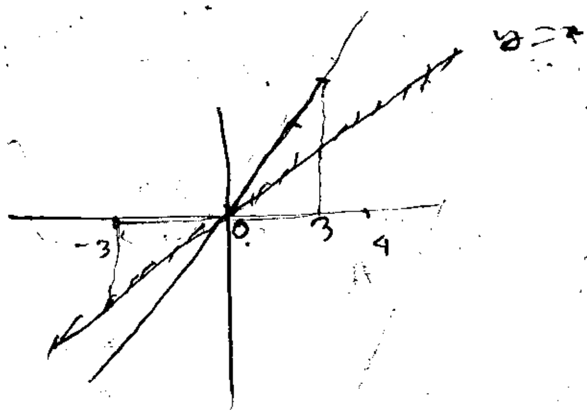
$$\therefore b_n = -\frac{4}{5} \times \frac{50}{n\pi} = -\frac{40}{n\pi}$$

$$f(x) = \frac{40}{2} + \sum_{n}^{\infty} \left(-\frac{40}{n\pi} \right) \sin \frac{n\pi x}{5}$$

$$= 20 - \frac{40}{\pi} \sum_{n}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{5}$$

(d)

$$f(x) = \begin{cases} 2x & 0 \le x \le 3 \\ 0 & -3 \le x \le 4 \end{cases} \quad \text{period } 6$$



$$2L = 6$$

$$L = 3$$

neither odd nor even

$$a_0 = \frac{1}{L}\int_0^4 f(x)\,dx$$

$$= \frac{1}{3}\int_0^3 2x + \frac{1}{3}\int_{-3}^4 0\,dx$$

$$= \frac{1}{3} \times 2\int_0^3 x\,dx$$

$$= 2/3\left[x^2/2\right]_0^3$$

$$= \frac{2}{3} \times \frac{9}{2} = 3$$

$$a_n = \frac{1}{L}\int_0^4 f(x)\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{1}{3}\int_0^3 2x\cos\frac{n\pi x}{2} + \frac{1}{3}\int_{-3}^4 0\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{1}{3}\int_0^3 x\cos\frac{n\pi x}{3}\,dx$$

$$= \frac{2}{3}\int_0^3 x\cos\frac{n\pi x}{3}\,dx$$

$$\therefore \int x\cos\frac{n\pi x}{3}\,dx$$

$$= x\int \cos\frac{n\pi x}{3}\,dx - \int \frac{d}{dx}(x)\int \cos\frac{n\pi x}{3}\,dx\,dx$$

$$= x \times \frac{3}{n\pi}\sin\frac{n\pi x}{3} - \frac{3}{n\pi}\int \sin\frac{n\pi x}{3}\,dx$$

$$= \frac{3x}{n\pi}\sin\frac{n\pi x}{3} + \frac{3}{n\pi}\times\frac{3}{n\pi}\cos\frac{n\pi x}{3}$$

$$= \frac{3x}{n\pi}\sin\frac{n\pi x}{3} + \frac{9}{n^2\pi^2}\cos\frac{n\pi x}{3}$$

$$\int_0^3 x\cos\frac{n\pi x}{3}\,dx$$

$$= \left(\frac{3\times 3}{n\pi}\sin\frac{n\pi 3}{3} + \frac{9}{n^2\pi^2}\cos\frac{n\pi 3}{3}\right) -$$

$$\left(0 + \frac{9}{n^2\pi^2}\right)$$

$$= \frac{9}{n\pi} \sin n\pi + \frac{9}{n^2\pi^2} \cos n\pi - \frac{9}{n^2\pi^2}$$

$$= \frac{9}{n^2\pi^2} \cos n\pi - \frac{9}{n^2\pi^2}$$

$$= \frac{9}{n^2\pi^2} (\cos n\pi - 1)$$

$$\therefore a_n = 2/3 \times \frac{9}{n^2\pi^2} (\cos n\pi - 1)$$

$$= \frac{16}{n^2\pi^2} (\cos n\pi - 1)$$

$$b_n = \frac{1}{L} \int_0^4 f(x) \sin \frac{n\pi x}{L} \, dx$$

$$= \frac{1}{3} \int_0^3 2x \sin \frac{n\pi x}{L} \, dx + \frac{1}{3} \int_{-3}^{4} 0 \sin \frac{n\pi x}{L} \, dx$$

$$= \frac{1}{3} \times 2 \int_0^3 x \sin \frac{n\pi x}{3} \, dx$$

$$= \frac{2}{3} \int_0^3 x \sin \frac{n\pi x}{3} \, dx$$

$$\therefore \int x \sin\frac{n\pi x}{3}\,dx$$

$$= x\int \sin\frac{n\pi x}{3}\,dx - \int\left\{\frac{d}{dx}(x)\int \sin\frac{n\pi x}{3}\,dx\right\}dx$$

$$= -x\,\frac{3}{n\pi}\cos\frac{n\pi x}{3} + \frac{3}{n\pi}\int \cos\frac{n\pi x}{3}\,dx$$

$$= -\frac{3x}{n\pi}\cos\frac{n\pi x}{3} + \frac{3}{n\pi}\times\frac{3}{n\pi}\sin\frac{n\pi x}{3}$$

$$= -\frac{3x}{n\pi}\cos\frac{n\pi x}{3} + \frac{9}{n^2\pi^2}\sin\frac{n\pi x}{3}$$

$$\int_0^3 x\sin\frac{n\pi x}{3}\,dx$$

$$= \left(-\frac{3\times 3}{n\pi}\cos n\pi + \frac{9}{n^2\pi^2}\sin n\pi\right)$$

$$-\ (0+0)$$

$$= -\frac{9}{n\pi}\cos n\pi$$

$$\therefore b_n = -\,{}^{2}/_{3}\times\frac{9}{n\pi}\cos n\pi$$

$$= -\frac{2}{n\pi}\cos n\pi$$

$$f(x) = \frac{3}{2} + \sum \left(\left\{ \frac{6}{n^2\pi^2} (\cos n\pi - 1) \right\} \cos \frac{n\pi x}{3} - \left\{ \frac{2}{n\pi} \cos n\pi \right\} \sin \frac{n\pi x}{3} \right)$$

(5) Expand $f(x) = \cos x \quad 0 < x < \pi$ in fourier sine series.

$\Rightarrow$ As sine series so $a_n = 0$

period $= \pi$, $2L = \pi$, $L = \pi/2$ $\therefore \frac{1}{L} = \frac{2}{\pi}$

$$a_0 = \frac{1}{L} \int_0^\pi f(x)\,dx$$

$$= \frac{1}{\frac{\pi}{2}} \int_0^\pi \cos x\,dx$$

$$= \frac{2}{\pi} \cdot \left[\sin x\right]_0^\pi$$

$$= \frac{2}{\pi} (\sin\pi - \sin 0)$$

$$= \frac{2}{\pi} (0 - 0) = 0$$

$$b_n = \frac{1}{L}\int_0^{\pi} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \cos x \sin\frac{n\pi x}{\frac{\pi}{2}}\,dx$$

$$\int \cos x \sin\frac{n\pi x}{\frac{\pi}{2}}\,dx$$

$$= \int \cos x \sin 2nx\,dx$$

$$= \cos x \int \sin 2nx - \int\left\{\frac{d}{dx}(\cos x)\int \sin 2nx\,dx\right\}dx$$

$$= -\frac{1}{2n}\cos x\cos 2nx - \frac{1}{2n}\int \sin x\cos 2nx\,dx$$

$$= -\frac{\cos x\cos 2nx}{2n} - \frac{1}{2n}\left\{\sin x\int \cos 2nx\,dx - \int \frac{d}{dx}(\sin x)\int \cos 2nx\,dx\,dx\right\}$$

$$= -\frac{\cos x\cos 2nx}{2n} - \frac{1}{2n}\left(\frac{1}{2n}\sin x\sin 2nx - \frac{1}{2n}\int \cos x\sin 2nx\,dx\right.$$

$$= -\frac{\cos x\cos 2nx}{2n} - \frac{1}{4n^2}\sin x\sin 2nx + \frac{1}{4n^2}\int \cos x\sin 2nx\,dx$$

$$\int \cos x \sin 2nx \, dx - \frac{1}{4n^2}\int \cos x \sin 2nx \, dx$$

$$= -\frac{1}{4n^2}\sin x \sin 2nx - \frac{\cos x \cos 2nx}{2n}$$

$$\Rightarrow \left(1-\frac{1}{4n^2}\right)\int \cos x \sin 2nx \, dx = -\frac{1}{2n}\cos x \cos 2nx - \frac{1}{4n^2}\sin x \sin 2nx$$

$$\Rightarrow \frac{4n^2-1}{4n^2}\int \cos x \sin 2nx \, dx = -\left(\frac{1}{2n}\cos x \cos 2nx + \frac{1}{4n^2}\sin x \sin 2nx\right)$$

$$\Rightarrow \frac{1-4n^2}{4n^2}\int \cos x \sin 2nx \, dx = \left(\frac{1}{2n}\cos x \cos 2nx + \frac{1}{4n^2}\sin x \sin 2nx\right)$$

$$\Rightarrow \int \cos x \sin 2nx \, dx = \frac{4n^2}{1-4n^2}\left(\frac{1}{2n}\cos x \cos 2nx + \frac{1}{4n^2}\sin x \sin 2nx\right)$$

$$\int_0^{\pi} \cos x \sin 2nx \, dx =$$

$$\frac{4n^2}{1-4n^2}\left(\left(\frac{1}{2n}\cos\pi \cos 2n\pi + \frac{1}{4n^2}\sin\pi \sin 2n\pi\right) - \left(\frac{1}{2n}\cos 0 \cos 0 + \frac{1}{4n^2}\sin 0 \sin 0\right)\right)$$

$$= \frac{4n^2}{1-4n^2}\left(\left(-\frac{1}{2n}\cos 2n\pi + 0\right) - \left(\frac{1}{2n} + 0\right)\right)$$

$$= \frac{4n^2}{1-4n^2}\left(-\frac{1}{2n} - \frac{1}{2n}\right) \quad [\cos 2n\pi = 1]$$

$$= \frac{4n^2}{1-4n^2}\left(-\frac{1}{n}\right) = \frac{4n}{4n^2-1}$$

$$b_n = \frac{2}{\pi} \times \frac{4n}{4n^2-1}$$

$$= \frac{8n}{4n^2\pi - \pi}$$

$$f(x) = \frac{a_0}{2} + \sum_{n}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$= \sum_{n}^{\infty}\left(\frac{8n}{4n^2\pi - \pi}\right)\sin\frac{n\pi x}{\pi/2}$$

$$f(-x) = \begin{cases} -x, & 0 < -x < 4 \\ 8+x, & 4 < -x < 8 \end{cases}$$

$$= \begin{cases} -x, & -4 < x < 0 \\ 8+x, & -8 < x < 4 \end{cases}$$

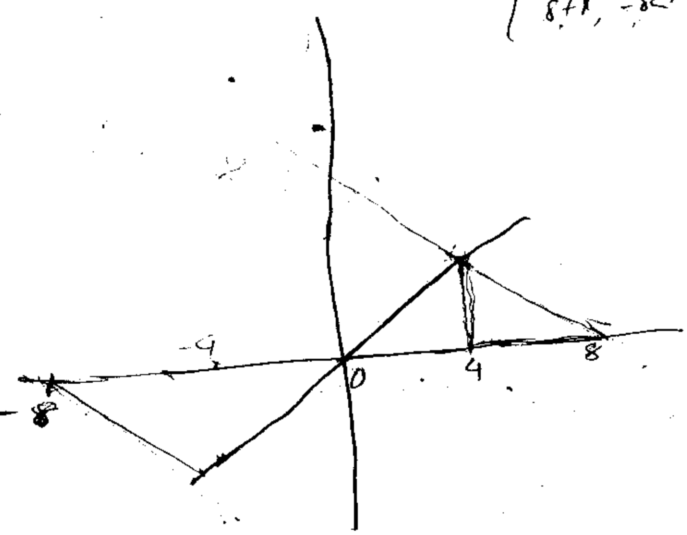
(6)

$$f(x) = \begin{cases} x, & 0 < x < 4 \\ 8-x, & 4 < x < 8 \end{cases}$$

Expand $f(x)$ in fourier

(i) sine series

(ii) Cosine series



$2L = 8$

$L = 4$

$$a_0 = \frac{1}{4}\int_0^8 f(x)\,dx$$

$$= \frac{1}{4}\int_0^4 x\,dx + \int_4^8 8 - x\,dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}\right]_0^4 + \frac{1}{4}\times 8\left[x\right]_4^8 - \frac{1}{4}\times\left[\frac{x^2}{2}\right]_4^8$$

$$+\int_4^8 8x - \int_4^8 \frac{x^2}{2}$$

$$= \frac{1}{4}(8-0) + 2(8-4) - \frac{1}{4}(24)$$

$$64-32 \quad -32+8$$

$$= 8/4 + 2\times 4/2 - 24/4$$

$$= 2 + 8 - 6 = 4$$

4

① Sine series

$a_n = 0$

$$\therefore b_n = \frac{1}{4}\int_0^8 f(x)\sin\frac{n\pi x}{4}\,dx$$

$$= \frac{1}{4}\left[\int_0^4 x \cancel{dx} \sin\frac{n\pi x}{4}\,dx + 8\int_4^8 \sin\frac{n\pi x}{4}\,dx - \int_4^8 x\sin\frac{n\pi x}{4}\,dx \right]$$

$$\int x \cancel{\frac{\sin \pi x}{}} \quad \int x \sin\frac{n\pi x}{4}\,dx$$

$$= x\int \sin\frac{n\pi x}{4} - \int \frac{d}{dx}(x) \int \sin\frac{n\pi x}{4}\,dx\,dx$$

$$= -x\frac{4}{n\pi}\cos\frac{n\pi x}{4} + \frac{4}{n\pi}\int \cos\frac{n\pi x}{4}\,dx$$

$$= -\frac{4x}{n\pi}\cos\frac{n\pi x}{4} + \frac{4\times 4}{n^2\pi^2}\sin\frac{n\pi x}{4}$$

$$= -\frac{4x}{n\pi}\cos\frac{n\pi x}{4} + \frac{16}{n^2\pi^2}\sin\frac{n\pi x}{4}$$

$$\int_0^4 x\sin\frac{n\pi x}{4}\,dx$$

$$= \left(-\frac{16}{n\pi}\cos n\pi + 0\right) - (0+0)$$

$$= -\frac{16}{n\pi}\cos n\pi$$

$$\int_4^8 x \sin\frac{n\pi x}{4}\,dx$$

$$= \left(-\frac{4\times 8}{n\pi}\cos 2n\pi + \frac{16}{n^2\pi^2}\sin 2n\pi\right)$$

$$- \left(-\frac{4\times 4}{n\pi}\cos n\pi + \frac{16}{n^2\pi^2}\sin n\pi\right)$$

$$= \left(-\frac{32}{n\pi}+0\right) - \left(-\frac{16}{n\pi}\cos n\pi + 0\right)$$

$$= -\frac{32}{n\pi} + \frac{16}{n\pi}\cos n\pi$$

$$\therefore b_n = \frac{1}{4}\left(-\frac{16}{n\pi}\cos n\pi - 8\times\frac{4}{n\pi}\left[\cos\frac{n\pi x}{4}\right]_4^8 + \frac{32}{n\pi} - \frac{16}{n\pi}\cos n\pi\right)$$

$$= \frac{1}{4}\left(-\frac{32}{n\pi}\cos n\pi - \frac{32}{n\pi}(1-\cos n\pi) + \frac{32}{n\pi}\right)$$

$$= \frac{1}{4}\left(-\frac{32}{n\pi}\cos n\pi - \frac{32}{n\pi} + \frac{32\pi}{n\pi}\cos + \frac{32}{n\pi}\right)$$