BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences

Final Examination

Semester: Fall 2017

Course Title: Linear Algebra and Fourier Analysis

Course No: MAT216

Time: 2 hours
Total Marks: 45

Date: December 18 , 2017

SECTION A

Answer question 1 and any three from question 2 to question 5.

1. (a) "An $n \times n$ matrix A does have n distinct eigenvalues, so matrix A is diagonalizable", Is the statement true or false? If any of its eigenvalue is zero then is it invertible? [1

(b) Suppose that a system of linear equations can be put into the form $AX = B$ where $\det A = 0$. Then the system has: [1
i) no solution ii) infinitely many solutions iii) an unique solution.
Find the possible answer(s).

(c) Do not evaluate. Tell what does the following integral represent. [

$$\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\, dx; \quad a > 0.$$

(d) If the system has non-zero solutions, then the vectors are linearly independent—Is the statement true or false? [

(e) If the even extension of the function $f(x) = x$, $0 < x < 2$ is periodic, then find its period. [

2. (a) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by: [

$$T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t).$$

Find the basis and dimension of $N(T)$ (nullspace of T) .

(b) Find the eigenvalues of the following matrix: [

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}.$$

3. (a) Use double integral to find the area of the region R enclosed between the parabola $y = \sqrt{2x}$ and the line $y = x$. [5]

(b) Convert the integration into cylindrical coordinates: [5]

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 \, dz \, dy \, dx.$$

4. (a) Evaluate: [5]

$$\iint_R \sqrt{x^2+y^2} \, dA,$$

where R is the region bounded by the unit circle centered at the origin.

(b) Evaluate the following line integral by using Green's theorem [5]
where $\mathbf{F}(x,y) = (x^3 - y)\mathbf{i} + (x + y^3)\mathbf{j}$ in a simple closed contour, defined by the equations: $y = x$ and $y = x^2$, oriented counterclockwise.

5. (a) Convert the following triple integral to spherical coordinates: [5]

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2+z^2} \, dz \, dy \, dx.$$

(b) Use the transformation $u = x - y$ and $v = x + y$ to find [5]

$$\iint_R \frac{e^{x-y}}{x+y} \, dA$$

over the rectangular region R enclosed by the lines $y = x$, $y = 5 + x$, $y = 2 - x$, and $y = 4 - x$.

SECTION B

Answer any <u>ONE</u>.

6. Define Fourier series for a periodic function. Graph the following function: [10]

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 \le x < \pi. \end{cases}$$

Expand $f(x)$ in a Fourier series, assuming that f is periodic outside the open interval $(-\pi, \pi)$.

7. Define Fourier sine series. Graph and expand the following function: [10]

$$f(x) = x^2, \quad 0 \le x < 1$$

in a Fourier sine series.