PDA
Pushdown Automata

① $a^n b^n$; $n \geq 1$

Push all the a's. Whenever we get a b, we will pop a.

$(a, z_0 / a z_0)$ $(b, a / \epsilon)$

$(b, a / \epsilon)$

$\epsilon, z_0 / z_0$
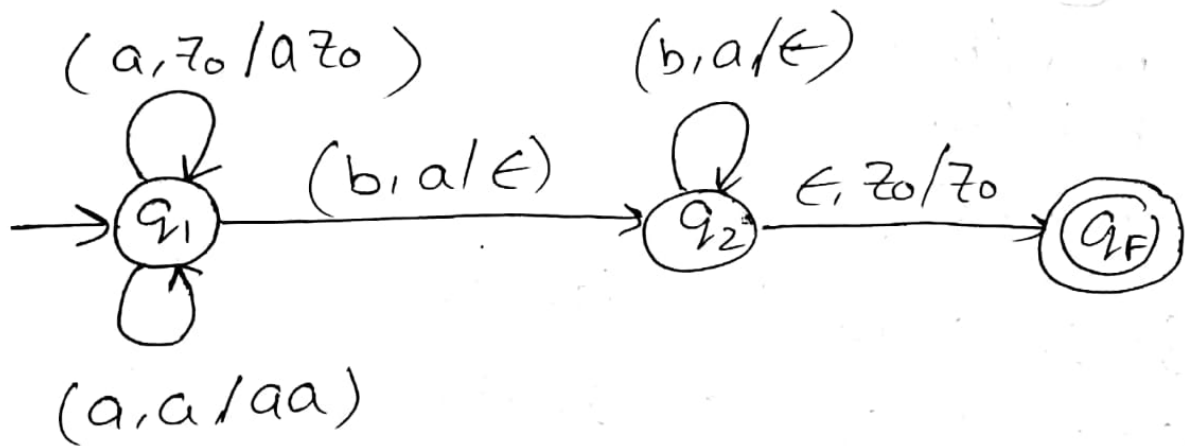
$(a, a / aa)$

Transition Function

$\delta(q_1, a, z_0) \rightarrow (q_1, a z_0)$

$\delta(q_1, a, a) \rightarrow (q_1, aa)$

$\delta(q_1, b, a) \rightarrow (q_2, \epsilon)$

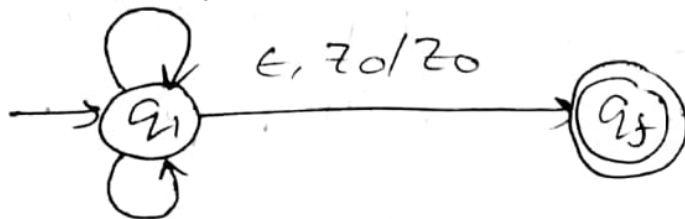$\delta(q_2, b, a) \rightarrow (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) \rightarrow (q_f, \epsilon)$

② Number of a = number of b

$a, b / \epsilon$
$a, a / aa$
$a, z_0 / a z_0$

$b, z_0 / b z_0$
$b, b / bb$
$b, a / \epsilon$

$\delta(q_1, a, z_0) \rightarrow (q_1, a)$

$\delta(q_1, a, a) \rightarrow (q_1, a)$

$\delta(q_1, b, z_0) \rightarrow (q_1, b)$

$\delta(q_1, b, b) \rightarrow (q_1, b)$
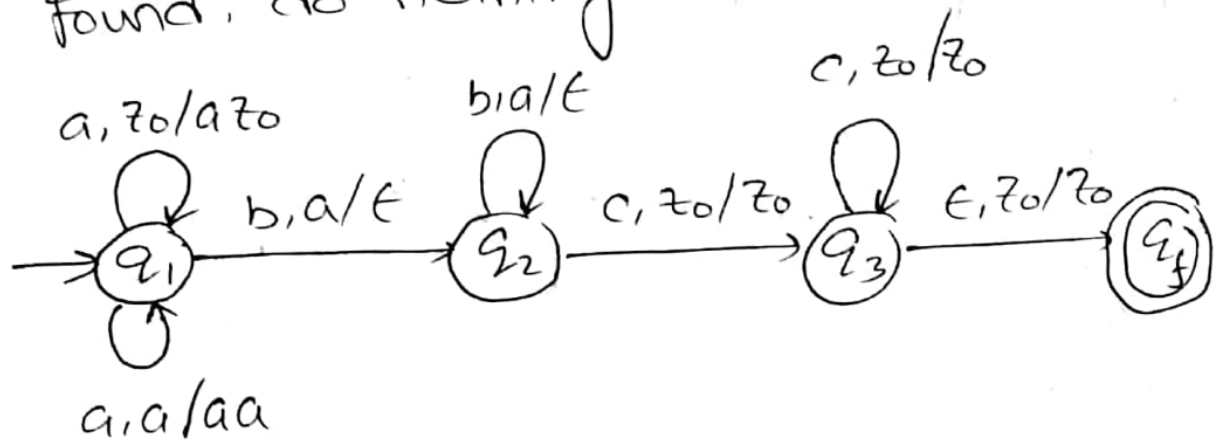
$\delta(q_1, a, b) \rightarrow (q_1, \epsilon)$

$\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) \rightarrow (q_f, z_0)$

③ $a^n b^n c^m$ / $n, m \geq 1$

Push a's. If b is found, pop a. If c is found, do nothing.

$a, z_0/a z_0$ $b, a/\epsilon$ $c, z_0/z_0$

$a, a/aa$

$$\delta(q_1, a, z_0) \rightarrow (q_1, a)$$
$$\delta(q_1, a, a) \rightarrow (q_1, a)$$
$$\delta(q_1, b, a) \rightarrow (q_2, \epsilon)$$
$$\delta(q_2, b, a) \rightarrow (q_2, \epsilon)$$
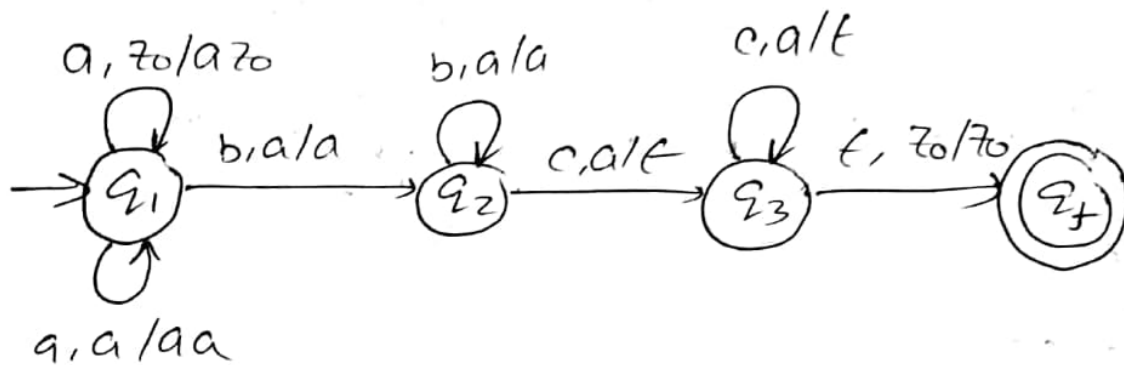$$\delta(q_2, c, z_0) \rightarrow (q_3, z_0)$$
$$\delta(q_3, c, z_0) \rightarrow (q_3, z_0)$$
$$\delta(q_3, \epsilon, z_0) \rightarrow (q_f, z_0)$$

④ $a^n b^m c^n \; / \; n, m \geqslant 1$

for a, push a, for b, do nothing.

for c, pop a.

$$a, z_0/a z_0 \qquad\qquad b, a/a \qquad\qquad c, a/\epsilon$$

$$b, a/a \qquad\qquad c, a/\epsilon \qquad\qquad \epsilon, z_0/z_0$$

$q_1 \longrightarrow q_2 \longrightarrow q_3 \longrightarrow q_f$

$a, a/aa$

⑤ $a^{m+n} b^m c^n$

for a, push a, for b and c, pop a.

$$b, a/\epsilon$$

$$a, z_0/a z_0$$
$$b, a/\epsilon \qquad\qquad \epsilon, z_0/z_0$$

$q_1 \longrightarrow q_1 \longrightarrow q_f$

$a, a/aa \qquad\qquad c, a/\epsilon$
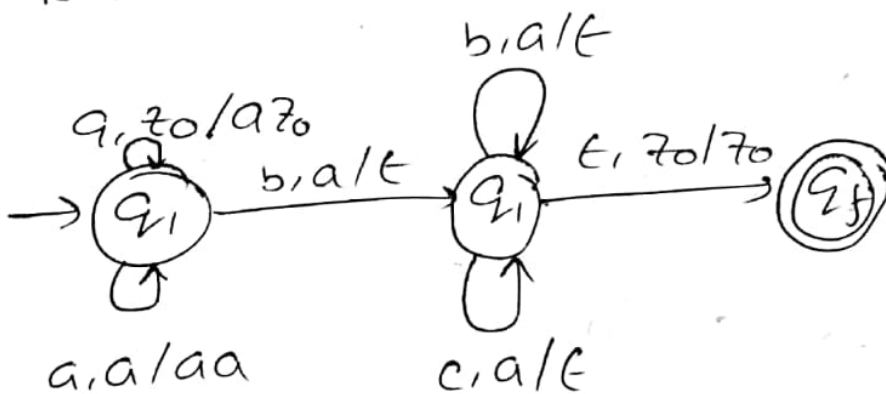
(6) $a^n b^{m+n} c^m$
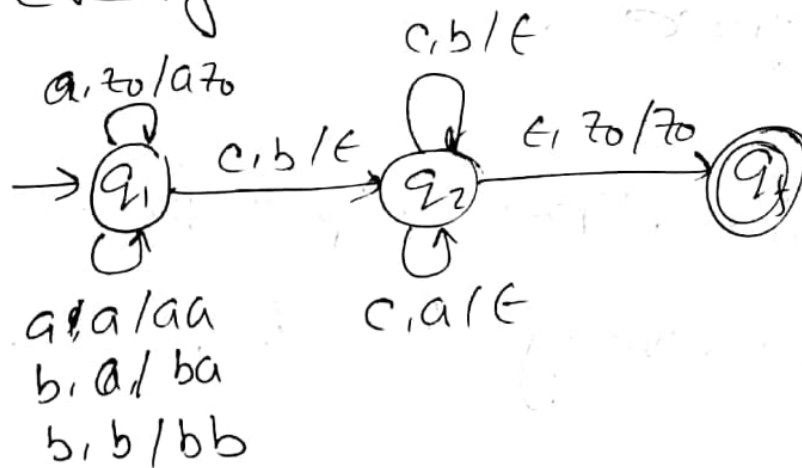
~~for a~~ Treat it like $a^n b^n b^m c^m$.

Push a. for b, pop a. when a will

be fished, push @b. Then for c,

pop b.

a, a/aa
a, z_0/a z_0

b,b/bb
~~b,z_0~~

c,b/ϵ

b, z_0/b z_0 → q_2 c,b/ϵ → q_3 ϵ, z_0/z_0 → q_4

q_1

c,b/ϵ

ϵ, z_0/z_0

b, a/ϵ
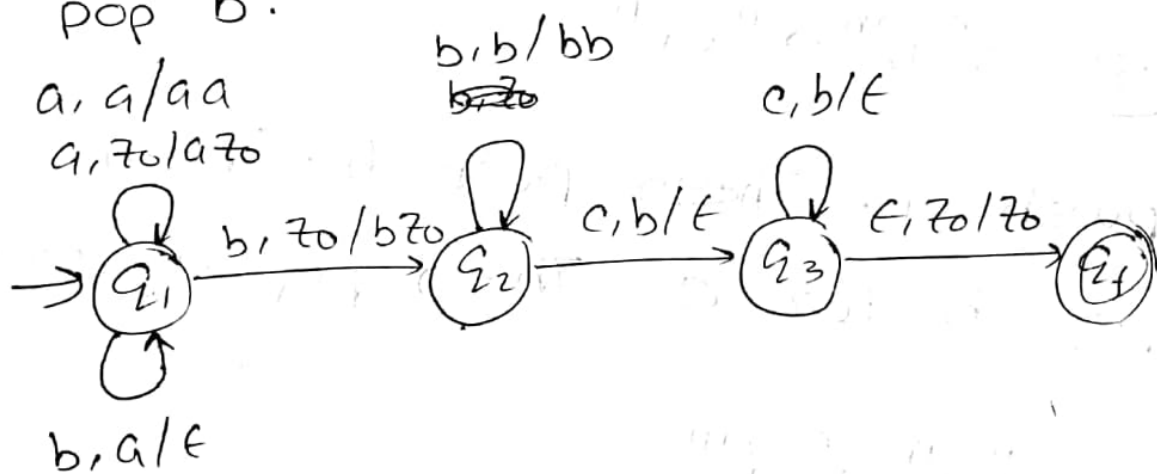
(7) $a^n b^m c^{n+m}$

Push a and b, and pop b and a for

every c.

a, z_0/a z_0

c,b/ϵ

q_1 c,b/ϵ → q_2 ϵ, z_0/z_0 → q

a⸱a/aa
b, a/ ba
b, b/bb

c, a/ϵ

⑧ $a^n b^n c^m d^m$

push a. pop a for b.
push c. pop c for d

⑨ $a^n b^m c^m d^n$

push a. Then push b. Then
pop b for c. Then pop a for d.

⑩ $a^n b^m c^n d^m$

Here, there is way to check or
save n or m. So this is not
context free.

⑪ $a^n b^{2n}$

push a twice. Then pop a for b.
or, push a, for every two b, pop a.
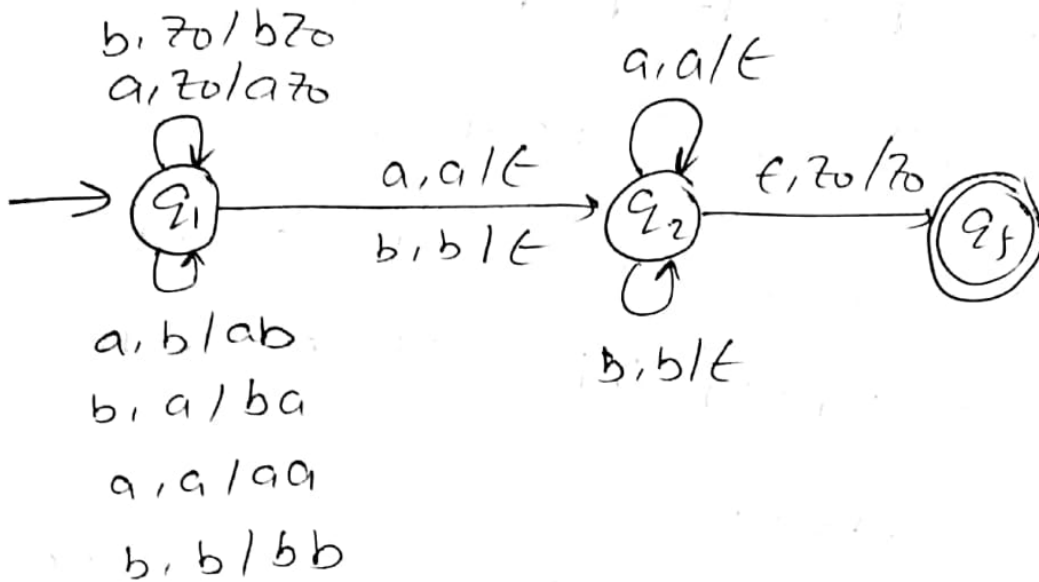
(12) $a^n b^n c^n$

It is not context free.

(13) WW^R (Palindrome)

In this case, we will start popping when we will reach the centre.
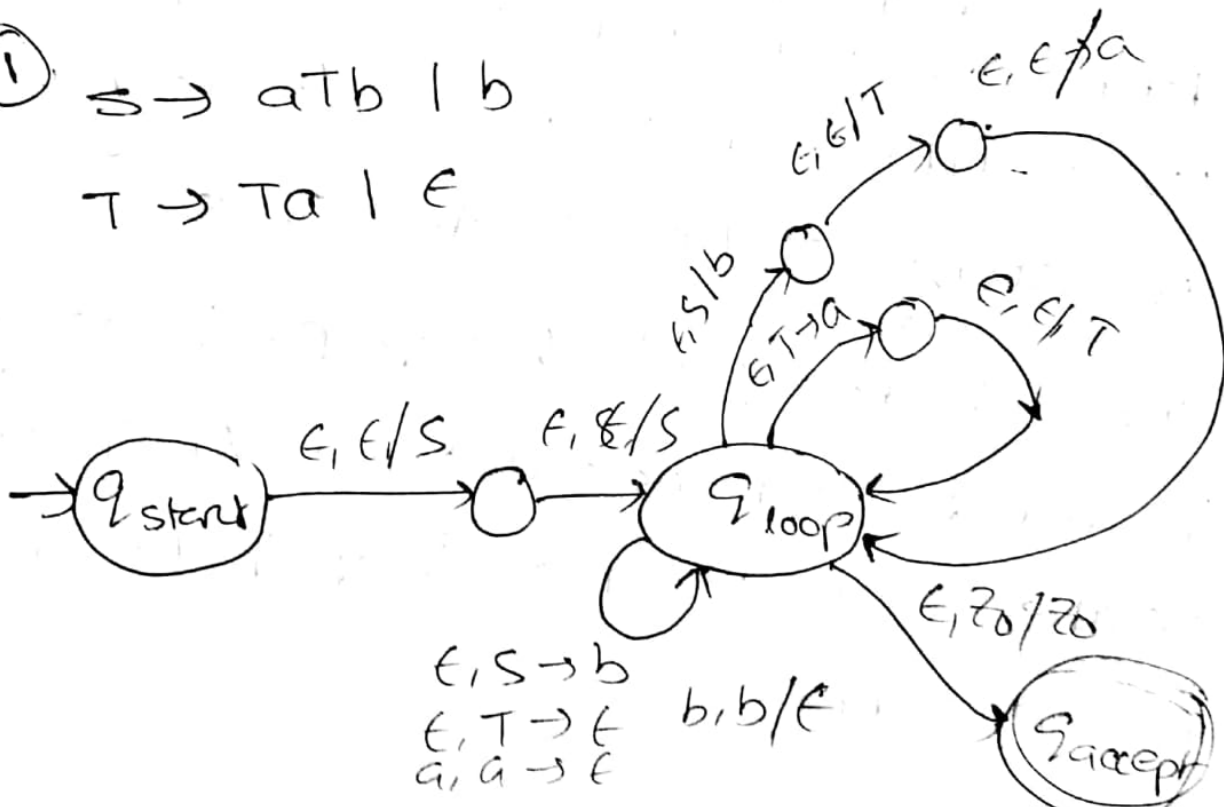
But how do we know whether the centre arrived?

PDA . which is is non deterministic. It works like NFA where simultaneously we can stay in two state. Thus one state will keep push as if it didn't get the centre. Another state will assume that it reached the centre.

Top diagram labels:

q_1 self-loop (top): $b, z_0/bz_0$; $a, z_0/az_0$

q_1 self-loop (bottom): $a, b/ab$; $b, a/ba$; $a, a/aa$; $b, b/bb$

$q_1 \to q_2$: $a, a/\epsilon$; $b, b/\epsilon$

q_2 self-loop (top): $a, a/\epsilon$

q_2 self-loop (bottom): $b, b/\epsilon$

$q_2 \to q_f$: $\epsilon, z_0/z_0$

CFG to PDA)

① $S \to aTb \mid b$

$T \to Ta \mid \epsilon$

Bottom PDA labels:

q_{start}

$\epsilon, \epsilon/S$

$\epsilon, \&/S$

q_{loop}

$\epsilon, S \to b$
$\epsilon, T \to \epsilon$
$a, a \to \epsilon$

$b, b/\epsilon$

$\epsilon, \&/S$

$\epsilon, S/b$

$\epsilon, T \to a$

$\epsilon, \&/T$

$\epsilon, \epsilon/a$

$\epsilon, \&/T$

$\epsilon, z_0/z_0$

q_{accept}

② E → aAa | bbB | B+B

A → C   AB

B → aBb | aBD | ab | ε

C → E | ε

D → abD | D

ε, A / C
ε, E / B+B
ε, E / bbB
ε, E / aAa

ε, ε / ε

q_1

ε, B / aBb
ε, B / abD
ε, B / AB
ε, C / E
ε, D / abd
ε, D / D
a, a / ε
b, b / ε
+, + / ε
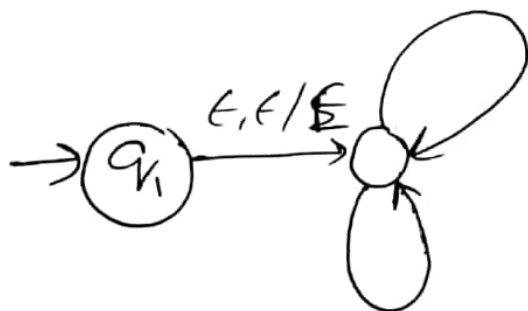
Question taken
from prev Question
where D → abD | D
D will call D and
again D will call D

Qiyamat is the
final state !