BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences
Final Examination
Semester: Summer 2015
Course Title: Linear Algebra and Fourier Analysis
Course No.: MAT216

Time: 3 hours
Total Marks: 50

Date: August 17, 2015

Note: Question 1 is compulsory. Answer any TWO from Part A, any TWO from Part B, and any ONE from Part C.

1. Answer all of the following:
   (a) "A homogeneous linear system is always consistent."- explain. [1]
   (b) What do you mean by nonsingularity of a matrix? Relate the concept of nonsingularity to the solution of a system of linear equations. [1]
   (c) Mention the relation between the rank and nullity of a matrix A. [1]
   (d) Express $\iint_R f(x, y)dA$ using Riemann sum. [1]
   (e) Sketch the odd extension of the function $f(x) = x$, $0 < x < 2$ and find it's period. [1]

Part A

2. (a) Define eigenvalue and eigenvector. [2]
   (b) Find the eigenvalues of the matrix [3]

$$A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}.$$

   (c) Find the eigenvectors of the matrix A and also find the matrix P that diagonalizes A. [4]

3. (a) State the elementary row operations. [2]
   (b) Find the rank of the matrix [3]

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{pmatrix}.$$

   (c) Write $w = (1, 1, 1)$ as a linear combination of vectors in the set S. [4]

$$S = \{(1, 2, 3), (0, 1, 2), (-1, 0, 1)\}.$$

   Are the vectors in S linearly independent?

4. (a) Define basis and dimension of a vector space with example. [2]

   (b) Solve the following system of equations [3]

$$\begin{aligned} x_1 - x_2 + 2x_3 &= -3 \\ -x_1 + 2x_2 + 3x_3 &= 11 \\ 3x_1 - 7x_2 + 4x_3 &= -23 \\ 2x_1 - x_2 + 9x_3 &= 2. \end{aligned}$$

   (c) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation defined by [4]

$$T(x_1, x_2, x_3) = (x_1 + 4x_2 + 5x_3, 2x_1 + x_2 + 3x_3, -x_1 + 3x_2 + 2x_3, x_1 + 11x_2 + 12x_3)\,.$$

   Find the standard matrix for this transformation. Also find the basis and dimension for $\ker(T)$.

Part B

5. (a) Formulate the volume of a sphere with centre at the origin and radius r using double and triple integrals. [2]

   (b) Calculate $\nabla \times \vec{F}$, where $\vec{F} = 2xz\hat{\imath} + 3z^2\hat{\jmath} + (x^2 + 6yz)\hat{k}$. [3]

   (c) Use the transformation $u = x + y$ and $v = x - y$ to find [4]

$$\iint_R (x - y)e^{x^2 - y^2}\, dA$$

   over the rectangular region R enclosed by the lines $x + y = 0$, $x + y = 1$, $x - y = 1$, and $x - y = 4$.

6. (a) Express the double integral $\displaystyle\iint_R x\, dA$, where R is bounded by the curves $y = x^2$ and $y = \sqrt{x}$, as an iterated integral and evaluate. [2]

   (b) Use triple integral to find the volume of the solid enclosed by the plane $z = y$, the xy-plane, and the parabolic cylinder $y = 1 - x^2$. [3]

   (c) Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (2y+3)\hat{\imath} + xz\hat{\jmath} + (yz - x)\hat{k}$ and C is the curve from $(0, 0, 0)$ to $(2, 1, 1)$ defined by $x = 2t^2$, $y = t$, $z = t^3$. [4]

7. (a) State Green's theorem. [2]

   (b) Use Green's theorem to evaluate $\displaystyle\oint_C (x + y^2)dx + (3x + 2xy)dy$, where C is the circle $x^2 + y^2 = 4$ oriented counterclockwise. [3]

   (c) Evaluate $\displaystyle\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dzdydx.$ [4]

Part C

8. (a) Define periodic, even, and odd functions with examples. Graph the following function [4]

$$f(x) = \begin{cases} 0 & -3 < x < 0 \\ 2x & 0 \le x < 3. \end{cases} \quad \text{Period} = 6.$$

   (b) Expand the function in 8(a) in Fourier series. [5]

9. (a) Define Fourier series for a periodic function f defined over $[-L, L]$ with period $2L$. Write the Dirichlet conditions for convergence of Fourier series. [4]

   (b) Expand the following function in a Fourier cosine series. [5]

$$f(x) = \begin{cases} x & 0 < x < 4 \\ 8 - x & 4 < x < 8. \end{cases}$$