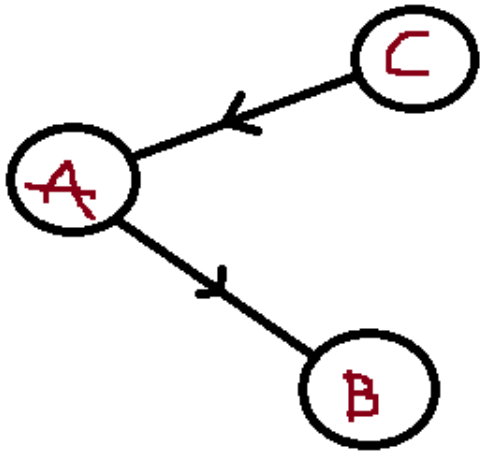
DFS on Directed Graphs

We have looked and understood depth first search (DFS) on an undirected graph. In this lecture we will be running DFS on a directed graph. Let us look at what directed graphs are.
Directed graphs: A graph where the edges specify a direction between a pair of vertices and it must be strictly followed.

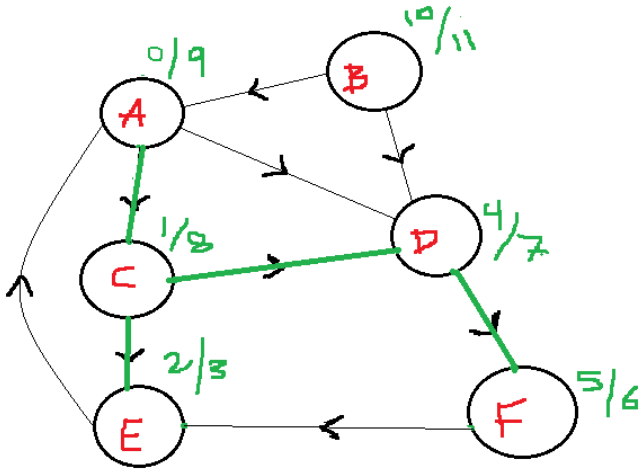
Undirected

Directed

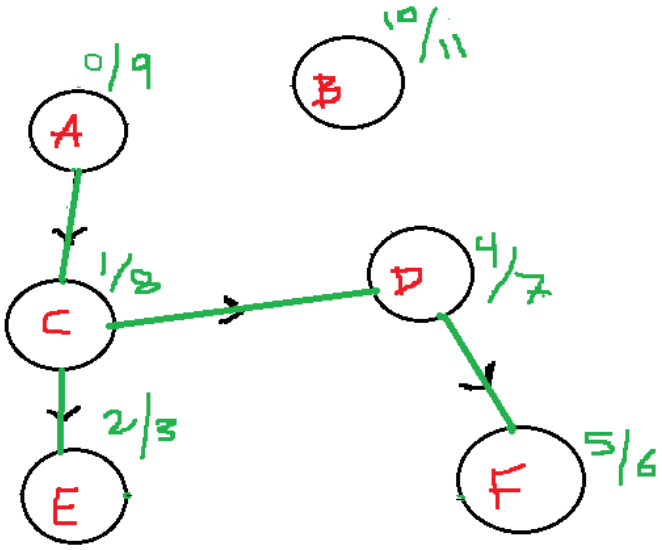
The above diagrams show both directed and undirected graphs. In the undirected one we can move from A to B and also from B to A. But in the directed one we can only move from A to B as directed.
The procedure of running DFS is the same as before except for the fact that we will have to consider the outgoing edged vertices as neighbors.



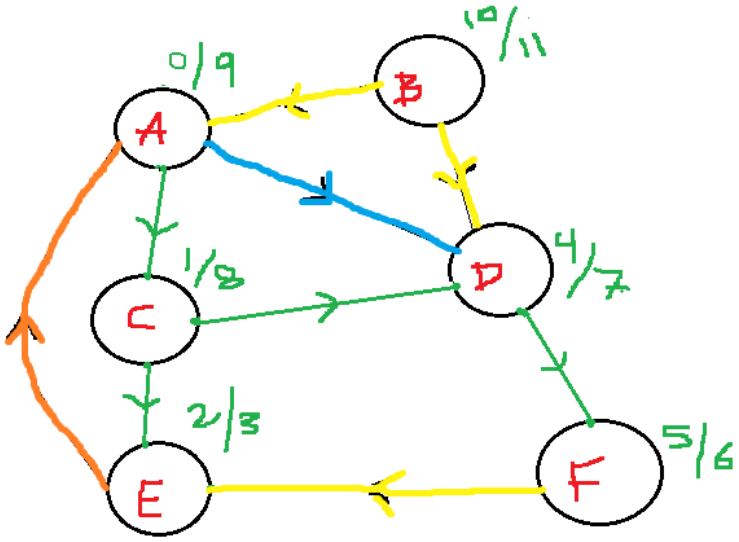
The neighbor of A is B only for the above diagram.
Let us run DFS on a directed graph. The diagram below shows the directed graph after running DFS on it.



The starting time and the ending time of each vertex is clearly shown and the path marked by green are the tree edges. If you don't know what tree edges are please read the previous lectures on graphs. The DFS tree looks as follows.



While studying DFS on undirected graphs we learned tree edges and back edges (edges not used). In case of directed graphs the unused edges are categorized into new types of edges which we will learn shortly.
The picture below shows the edges colored according to types.



The green edges are the **tree edges** (used edges) the rest are unused edges. For an undirected graph all the non used edges were named back edges but in case of directed graphs the case is different. The orange edge from E to A is the **back edge** because the descendant vertex E is going **back** to its ancestor A. E is called the descendant of A because E can be reached from A via C. The blue edge is called the **forward edge** because the ancestor A is going **forward** to its descendant D. Again D is the descendant of A because D can be reached from A via C. The yellow edges are called **cross edge** because the pair of vertices the edge connects contains no ancestor/descendant relationship. The pair of vertices connected through cross edge can belong to the same or different DFS trees. If we look at the DFS tree of the above graph the vertex B is isolated from the rest of the tree means B is itself a tree with one node.