Differential Calculus

*Home work sheet #1

Domain & Range

1. Sketch the following functions and find the domain and range:

   i. $f(x) = \dfrac{1}{x-2}$

   ii. $g(x) = \dfrac{|x|}{x}$

   iii. $h(x) = x^2 - 2$

   iv. $k(x) = \sqrt{4 - x^2}$

Find out the domain and range of the following functions:

1. $f(x) = \dfrac{1}{x-3}$

2. $f(x) = \sqrt{x^2 - 9}$

3. $f(x) = \sqrt{9 - x^2}$

4. $f(x) = \begin{cases} 2x+6 & ,-3 \le x \le 0 \\ 6 & ,0 < x < 2 \\ 2x-6 & ,2 \le x \le 5 \end{cases}$

5. $f(x) = \dfrac{x}{|x|}$

6. $f(x) = x^3 + 2$

7. $f(x) = \begin{cases} x+2 & ,x \le -1 \\ x^3 & ,|x| < 1 \\ -x+3 & ,x \ge 1 \end{cases}$

8. $f(x) = e^x$

9. $f(x) = 3Sinx$

10. $f(x) = \sqrt{2x+4}$

11. $f(x) = \dfrac{1}{5x+7}$

12. $f(x) = -\sqrt{x^2 - 7x + 10}$

13. $f(x) = \begin{cases} x^2 & ,x < 0 \\ x & ,0 \le x \le 1 \\ \dfrac{1}{x} & ,x > 1 \end{cases}$

14. $f(x) = \sqrt{x^2 - 5x + 6}$

15. $f(x) = Sin^2 x$

16. $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & ,x \ne 1 \\ 2 & ,x = 1 \end{cases}$

17. $f(x) = \log x$

18. $f(x) = \ln(x^2 + 1)$

Sketch the above functions except 16 and 18 .

******These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

*Home work sheet #2

<u>Limit</u>

Find the limit of the following functions:

1. $\lim_{x\to 0}\dfrac{x}{\sqrt{x+1}-1}$

2. $\lim_{x\to 2}\dfrac{2x^2-5x+2}{5x^2-7x-6}$

3. $f(x)=\begin{cases} x^2+1 & ,x>0 \\ 1 & ,x=0 \\ 1+x & ,x<0 \end{cases}$

   Find $\lim_{x\to 0} f(x)$

4. $f(x)=\begin{cases} 3x-1 & ,x<1 \\ 3-x & ,x>1 \end{cases}$

   Find $\lim_{x\to 1} f(x)$

5. $\lim_{x\to 0}\dfrac{x}{|x|}$

6. $\lim_{x\to \infty}\dfrac{3x+5}{6x-8}$

7. $f(x)=\begin{cases} 2-x & ,x<1 \\ x^2+1 & ,x>1 \end{cases}$

   Find $\lim_{x\to 1} f(x)$

8. $f(x)=\begin{cases} e^{\frac{-|x|}{2}} & ,-1<x<0 \\ x^2 & ,0<x<2 \end{cases}$

   Find $\lim_{x\to 0} f(x)$

9. $f(x)=\begin{cases} \dfrac{1}{x+2} & ,x<-2 \\ x^2-5 & ,-2<x<3 \\ \sqrt{x+13} & ,x>3 \end{cases}$

   Find $\lim_{x\to -2} f(x)$ and $\lim_{x\to 3} f(x)$

10. $f(x)=\begin{cases} x^2 & ,x<1 \\ 2.4 & ,x=1 \\ x^2+1 & ,x>1 \end{cases}$

    Does $\lim_{x\to 1} f(x)$ exist?

11. $\lim_{x\to \infty}\left(\sqrt{x^6+5x^3}-x^3\right)$

12. Prove that $\lim_{x\to \infty}\left(1+\dfrac{1}{x}\right)^x=e$

13. $\lim_{x\to \infty}\sqrt[3]{\dfrac{3x+5}{6x-8}}$

14. $\lim_{x\to -\infty}\dfrac{4x^2-x}{2x^3-5}$

15. Let $f(x)=\begin{cases} 2x+1 & ,x<1 \\ 3-x & ,x>1 \end{cases}$, find $\lim_{x\to 1} f(x)$.

**These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

*Home work sheet # 3

<u>Continuity and Differentiability</u>

a) Test the continuity of the following functions:

1. $f(x)=\begin{cases} \cos x, & x \ge 0 \\ -\cos x, & x<0 \end{cases}$ at $x=0$.

2. $f(x)=\begin{cases} x\cos(1/x), & x \ne 0 \\ 0 & , x=0 \end{cases}$ at $x=0$.

3. $f(x)=\begin{cases} e^{1/x} & , x \ne 0 \\ 1 & , x=0 \end{cases}$ at $x=0$.

4. $f(x)=\begin{cases} e^{\frac{-|x|}{2}} & , -1<x<0 \\ x^2 & , 0 \le x < 2 \end{cases}$ at $x=0$.

5. $f(x)=\begin{cases} (x-a)\sin\left(\dfrac{1}{x-a}\right) & , x \ne a \\ 0 & , x=a \end{cases}$ at $x=a$.

6. $f(x)=\begin{cases} 1 & , x<0 \\ 1+\sin x & , 0 \le x < \pi/2 \\ 2+(x-\pi/2)^2 & , x \ge \pi/2 \end{cases}$
at $x=0$ and $x=\pi/2$.

7. $f(x)=\begin{cases} x\sin(1/x) & , x \ne 0 \\ 0 & , x=0 \end{cases}$ at $x=0$.

8. $f(x)=\begin{cases} \dfrac{|x-3|}{x-3} & , x \ne 3 \\ 0 & , x=3 \end{cases}$ at $x=3$.

9. $f(x)=|x|+|x-1|$ at $x=0$ and $x=1$.

10. $f(x)=\begin{cases} (1+x)^{1/x} & , x \ne 0 \\ 1 & , x=0 \end{cases}$ at $x=0$.

b) Test the differentiability of the following functions:

2. $f(x)=\begin{cases} x^2\cos(1/x) & , x \ne 0 \\ 0 & , x=0 \end{cases}$ at $x=0$.

3. $f(x)=|x|$ at $x=0$.

2. $f(x)=\begin{cases} x^2\sin(1/x) & , x \ne 0 \\ 0 & , x=0 \end{cases}$ at $x=0$.

(*c*) Let $f(x)=\begin{cases} x^2-16x & , x<9 \\ 12\sqrt{x} & , x \ge 9 \end{cases}$. Is *f*(*x*) continuous at $x=9$? Determine whether *f*(*x*) is differentiable at $x=9$.

(*d*) Let $f(x)=\begin{cases} x^2 & , x \le 1 \\ \sqrt{x} & , x>1 \end{cases}$. Is *f*(*x*) continuous at $x=1$? Determine whether *f*(*x*) is differentiable at $x=1$.

(*e*) Show that $f(x)=\begin{cases} x^2+1 & , x \le 1 \\ x & , x>1 \end{cases}$ is not continuous and differentiable at $x=1$. Sketch the graph of *f*(*x*).

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Differential Calculus

*Home work sheet # 4

Techniques of Differentiation

1. Find the differential coefficients of the following functions with respect to x. (i.e. $\frac{dy}{dx}$).

$(i)\, y = \sin x \sin 2x \sin 3x,\ \ (ii)\, y = \operatorname{cosec}^3 x,\ \ (iii)\, y = \cos 2x \cos 3x,\ \ (iv)\, y = \sin^{-1}(x^2),$

$(v)\, y = \tan(\sin^{-1} x),\quad (vi)\, \cot^{-1}\left(\frac{1+x}{1-x}\right),\quad (vii)\, \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\quad (viii)\, \sin^{-1}\left(\frac{2x}{1+x^2}\right),$

$(ix)\, \tan^{-1}\left(\frac{2x}{1-x^2}\right),\quad (x)\, \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right),\quad (xi)\, \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right),\quad (xii)\, \ln\sqrt{\frac{1-\cos x}{1+\cos x}}.$

2. Find the differential coefficients of:

$(i)(\sin x)^{\ln x},\qquad (ii)\, (\sin x)^{\cos x} + (\cos x)^{\sin x}.$

3. Find $\frac{dy}{dx}$ in the following cases:

$(i)\, 3x^4 - x^2 y + 2y^3 = 0,\quad (ii)\, x^3 + y^3 + 4x^2 y - 25 = 0,\quad (iii)\, x^y = y^x.$

4. Find $\frac{dy}{dx}$ when

$(i)\, x = a\cos^3\theta,\ y = a\sin^3\theta,\quad (ii)\, x = \sin^2\theta, y = \tan\theta,\quad (iii)\, x = a\sec^2\theta, y = a\tan^2\theta.$

5. Differentiate the left-side functions with respect to the right-side ones:

$(i)\, \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $\quad (ii)\, x^{\sin^{-1}(x)}$ with respect to $\sin^{-1} x.$

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Differential Calculus

*Home work sheet # 5

Maxima and minima

1. Find (a) the open intervals on which f is increasing , (b) the open intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d)) the open intervals on which f is concave down and (e) the x- coordinate of all inflection points.

   $(i)\ f(x) = x^2 - 5x + 6 \qquad (ii)\ f(x) = 5 + 12x - x^3$

   $(iii)\ f(x) = x^4 - 8x^2 + 16 \qquad (iv)\ f(x) = \dfrac{x^2}{x^2 + 2} \qquad (v)\ f(x) = \sqrt[3]{x + 2}.$

2. Locate the critical numbers and identify which critical numbers correspond to stationary points.

   $(i)\ f(x) = x^3 + 3x^2 - 9x + 1 \quad (ii)\ f(x) = x^4 - 6x^2 - 3$

   $(iii)\ f(x) = \dfrac{x}{x^2 + 2} \qquad (iv)\ f(x) = x^{2/3}$

   $(v)\ f(x) = x^{1/3}(x + 4) \qquad (vi)\ f(x) = \cos 3x.$

3. Find the relative extrema (maxima/ minima) using both the first and second derivative tests.

   $(i) f(x) = 2x^3 - 9x^2 + 12x \qquad (ii) f(x) = \dfrac{x}{2} - \sin x, \quad 0 < x < 2\pi.$

4. Use the given derivative to find all critical numbers of f and determine whether a relative maximum, relative minimum, or neither occurs there.

   $(i)\ f'(x) = x^3(x^2 - 5) \qquad (ii)\ f'(x) = \dfrac{x^2 - 1}{x^2 + 1}.$

**These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

*Home work sheet # 6

Successive Differentiation

a. Find the *n* th derivative of the following functions:

1.$y = x^n$
2.$y = (ax+b)^n$
3.$y = \ln(ax+b)$
4.$y = \dfrac{1}{x+a}$
5.$y = e^{ax}$
6.$y = \sin(ax+b)$
7.$y = \cos(ax+b)$

b. If $y = e^{ax}\sin bx,$ then show that $y_2 - 2ay_1 + (a^2+b^2)y = 0.$

c. If $y = e^{x}\sin x,$ then show that $y_4 + 4y = 0.$

Rolle's and Mean Value Theorem

a. Verify the hypothesis of Rolle's Theorem for the following functions:

1. $f(x) = x^2 - 6x + 8; \quad [2,4]$
2. $f(x) = \cos x; \quad [\pi/2, 3\pi/2]$
3. $f(x) = \dfrac{x}{2} - \sqrt{x}; \quad [0, 4].$

b. Verify the hypothesis of Mean Value Theorem for the following functions:

1. $f(x) = x^3 + x - 4; \quad [-1,2]$
2. $f(x) = \sqrt{x+1}; \quad [0,3]$
3. $f(x) = \sqrt{25 - x^2}; \quad [0,5].$

Maclaurin and Taylor Series

1. Find the Taylor series for the following functions:

   (i) $\sin x$, at $x_0 = \dfrac{\pi}{2}$. $\qquad (ii)$ $\ln x$, at $x_0 = 2$.
2. Expand $y = \ln x$ in the power of $x-2$ and $y = e^{ax}$ in the power of $x-1$.
3. Find the Maclaurin series for the function e^{ax} and $\cos x$.
4. Find the Maclaurin polynomial p_0, p_1, p_2, p_3 for $e^x \cos x$.
5. Expand $y = \ln(x+1)$, $y = \sin x$, $y = \cos x$ in the power of x.

**These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

*Home work sheet # 7

Leibnitz's Theorem

1. If $y=\tan^{-1}x$, then show that $(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0.$
2. If $y=\cot^{-1}x$, then show that $(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0.$
3. If $y\sqrt{1-x^2}=\sin^{-1}x$, then show that $(1-x^2)y_{n+1}-(2n+1)xy_n-n^2y_{n-1}=0.$
4. If $y=e^{\tan^{-1}x}$, then show that $(1+x^2)y_{n+2}+(2nx+2x-1)y_{n+1}+n(n+1)y_n=0.$
5. If $y=e^{m\sin^{-1}x}$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0.$
6. If $y=(\sin^{-1}x)^2$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0.$
7. If $\log_e y=a\sin^{-1}x$ then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0.$
8. If $y=e^{m\cos^{-1}x}$ then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0.$
9. If $\log_e y=\tan^{-1}x$ then show that $(1+x^2)y_{n+2}+(2nx+2x-1)y_{n+1}+n(n+1)y_n=0.$
10. If $y=\left(\cos^{-1}x\right)^2$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0.$
11. If $\ln y=m\cos^{-1}x$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0.$
12. If $x=\tan(\ln y)$, then show that $(1+x^2)y_{n+2}+(2nx+2x-1)y_{n+1}+n(n+1)y_n=0.$

Indeterminate Forms

Find the limit using L' Hospital's rule:

$$1.\lim_{x\to1}\frac{\ln x}{x-1},\quad 2.\lim_{x\to3}\frac{x-3}{3x^2-13x+12},\quad 3.\lim_{x\to0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right),\quad 4.\lim_{x\to\pi}\frac{\sin x}{x-\pi}$$

$$5.\lim_{x\to0}\frac{x-\tan^{-1}x}{x^3},\quad 6.\lim_{x\to+\infty}\frac{e^{3x}}{x^2},\quad 7.\lim_{x\to0}\frac{a^x-1-x\log a}{x^2},\quad 8.\lim_{x\to0}(e^x+x)^{\frac{1}{x}}$$

$$9.\lim_{x\to0}\left(\frac{1}{x}-\frac{1}{\sin x}\right),\quad 10.\lim_{x\to\pi}(x-\pi)\cot x,\quad 11.\lim_{x\to0}\frac{\ln(\sin x)}{\ln(\tan x)},\quad 12.\lim_{x\to\infty}xe^{-x}$$

$$13.\lim_{x\to0}\frac{\sin 2x}{x},\quad 14.\lim_{x\to0}\frac{\sin x}{x^2},\quad 15.\lim_{x\to\infty}\frac{x}{e^x},\quad 16.\lim_{x\to0}\left(\frac{1}{x}-\frac{1}{xe^x}\right),\quad 17.\lim_{x\to0}\frac{\sin 2x}{\sin 5x}.$$

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Differential Calculus

*Home work sheet # 8

Partial derivative

1. Let $f(x,y) = 3x^3y^2$. Find
   (a) $f_x(x,y)$ (b) $f_y(x,y)$ (c) $f_x(x,1)$ (d) $f_y(1,y)$ (f) $f_x(1,2)$ (g) $f_y(1,2)$.
2. Let $f(x,y) = xe^{-y} + 5y$ or $f(x,y) = \sqrt{3x+2y}$
   (a) Find the Slope of the surface $z = f(x,y)$ in the x- direction at the point (3,0)/(4,2)
   (b) Find the slope of the surface $z = f(x,y)$ in the y direction of the point (3,0)/(4,2)
3. Let $f(x,y) = 4x^2 - 2y + 7x^4y^5$, Find (a) f_{xx} (b) f_{yy} (c) f_{xy} (d) f_{yx}.
4. Let $Z = Sin(y^2 - 4x) or Z = (x+y)^{-1}$
   (a) Find the rate of Change of z w. r. to x at the point (2,1)/(-2, 4) with y kept fixed.
   (b) Find the rate of Change of z w. r. to y at the point (2,1)/(-2, 4) with x kept fixed.
5. Let $f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z$, find
   (a) f_{xy} (b) f_{yz} (c) f_{xz} (d) f_{zz} (e) f_{zyy} (f) f_{zxy} (g) f_{zyx} (h) f_{xxyz}.
6. Let $f(x,y,z) = \sqrt{xy} + \ln\left(x^2z^3\right) - x\tan(z)$. Compute f_x , f_z , f_{xy} , f_{xyz}

Maxima Minima of function of several variable

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9–20 Locate all relative maxima, relative minima, and saddle points, if any.

9. $f(x,y) = y^2 + xy + 3y + 2x + 3$
10. $f(x,y) = x^2 + xy - 2y - 2x + 1$
11. $f(x,y) = x^2 + xy + y^2 - 3x$
12. $f(x,y) = xy - x^3 - y^2$
13. $f(x,y) = x^2 + y^2 + \dfrac{2}{xy}$
14. $f(x,y) = xe^y$
15. $f(x,y) = x^2 + y - e^y$
16. $f(x,y) = xy + \dfrac{2}{x} + \dfrac{4}{y}$
17. $f(x,y) = e^x \sin y$
18. $f(x,y) = y \sin x$
19. $f(x,y) = e^{-(x^2+y^2+2x)}$
20. $f(x,y) = xy + \dfrac{a^3}{x} + \dfrac{b^3}{y} \quad (a \neq 0, b \neq 0)$

Lagrange Multiplier

5–12 Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint. Also, find the points at which these extreme values occur.

5. $f(x,y) = xy;\ 4x^2 + 8y^2 = 16$
6. $f(x,y) = x^2 - y^2;\ x^2 + y^2 = 25$
7. $f(x,y) = 4x^3 + y^2;\ 2x^2 + y^2 = 1$
8. $f(x,y) = x - 3y - 1;\ x^2 + 3y^2 = 16$
9. $f(x,y,z) = 2x + y - 2z;\ x^2 + y^2 + z^2 = 4$
10. $f(x,y,z) = 3x + 6y + 2z;\ 2x^2 + 4y^2 + z^2 = 70$
11. $f(x,y,z) = xyz;\ x^2 + y^2 + z^2 = 1$
12. $f(x,y,z) = x^4 + y^4 + z^4;\ x^2 + y^2 + z^2 = 1$

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Differential Calculus

*Home work sheet # 9

Transformation of Co-ordinates

1. Find the polar co-ordinates of the points $(2\sqrt{3},-2),\ (0,-2),\ (1,1)$.
2. Find the rectangular co-ordinates of the points $(7, 2\pi/3),\ (8, 9\pi/4),\ (0, \pi)$.
3. Change to Cartesian coordinates the equations $(i)\, r = a\sin\theta,\ (ii)\sqrt{r} = \sqrt{a}\cos\left(\frac{\theta}{2}\right)$.
4. Transform to polar coordinates the equations $(i) 9x^2 + 4y^2 = 36,\ (ii) x^3 = y^2(2a - x)$.
5. Transform to parallel axes through the new origin (1,-2) of the equation $2x^2 + y^2 - 4x + 4y = 0$.
6. Transform the equation $x^2 + y^2 - 8x + 14y + 5 = 0$ to parallel axes through (4,-7).
7. Transform the equation $7x^2 - 2xy + y^2 + 1 = 0$ to axes turned through the angle $\tan^{-1}\left(\frac{1}{2}\right)$.
8. Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ to rectangular axes through the point (2,-1) and inclined at an angle $\tan^{-1}\left(\frac{4}{3}\right)$.
9. Transform the equation $9x^2 + 15xy + y^2 + 12x - 11y - 5 = 0$, so as to remove the terms x and y.
10. Transform the equation $11x^2 + 3xy + 7y^2 + 19 = 0$, so as to remove the term xy.
11. Determine the equation of the curve $2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$ when the origin is transferred to the point $(-2, 3)$.
12. Remove the xy term from the equation $9x^2 + 24xy + 2y^2 + 54 = 0$.
13. Determine the equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ after rotating of axes through 30°.
14. Transform the equation $9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0$ so as to remove the terms in x and y.

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Differential Calculus

*Home work sheet # 10

Pair of straight lines

1. Show that the following equations represents a pair of straight lines; find also their point of intersection and the angle between them:

   $(i)\ 2y^2 - xy - x^2 + y + 2x - 1 = 0, \quad (ii) 2x^2 - 2xy + x + 2y - 3 = 0,$
   $(iii)\ x^2 + 3xy + 2y^2 + \frac{1}{8}x - \frac{1}{32} = 0, \quad (iv) 21x^2 + 40xy - 21y^2 + 44x + 122y - 17 = 0.$

2. Find the value of λ or k so that the following equations may represent pairs of straight lines:

   $(i)\ 2\lambda xy - y^2 + 4x + 2y + 8 = 0, \qquad (ii) 2x^2 + xy - y^2 - 2x - 5y + k = 0$
   $(iii)\ x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0, \qquad (iv) 12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0.$

3. Find the equations of the bisectors of the angles between the following pairs of straight lines:

   $(i) x^2 + xy - 6y^2 - x - 8y - 2 = 0, \quad (ii) 8x^2 - 14xy + 6y^2 + 2x - y - 1 = 0,$
   $(iii) 2x^2 + xy - y^2 - 3x + 6y - 9 = 0, \quad (iv) 2x^2 + 7xy + 6y^2 + 13x + 22y + 20 = 0$

Circle

1. Find the equation of the circle with

   (i) centre (-2, -1) and radius 4, (ii) centre (9, 0) and radius 1,

   (iii) centre (0, 0) and radius 5.

2. Find the centre and radius of the following circles:

   $(i) 5x^2 + 5y^2 - 11x - 9y - 12 = 0, \quad (ii) x^2 + y^2 + 2x + 2y + 1 = 0$
   $(iii) x^2 + y^2 + 2x - 4y - 8 = 0.$

3. Find the equation of the circle passing through the points:

   $(i)\ (1, 3), (2, -1), (-1, 1), \quad (ii)\ (-4, -3), (-1, -7), (0, 0), \quad (iii)\ (3, 1), (4, -3), (1, -1).$

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Differential Calculus

*Home work sheet # 11

Tangent and Normal of Circle

1. Show that the circles $x^2 + y^2 - 2x + 4y + 3 = 0$ and $x^2 + y^2 - 8x - 2y + 9 = 0$ touch one another at (2, -1).
2. Find the equation of the circle through the intersection of the circles $x^2 + y^2 - 9x + 14y - 7 = 0$ and $x^2 + y^2 + 15x + 14 = 0$ and passes through the point (2, 5).
3. Find the equation of the circle through the intersection of the circles $x^2 + y^2 = 1$ and $x^2 + y^2 + 2x + 4y + 1 = 0$ which touches the straight line $x + 2y + 5 = 0$.
4. Find the radical centre of the three circles $x^2 + y^2 + x + 2y + 3 = 0$, $x^2 + y^2 + 2x + 4y + 5 = 0$ and $x^2 + y^2 - 7x - 8y - 9 = 0$.

Conic Section

1. Reduce the following equations to their standard forms:

   (i) $x^2 - 6xy + 9y^2 - 2x - 3y + 1 = 0$
   (ii) $x^2 - 4xy + y^2 + 8x + 2y - 5 = 0$
   (iii) $4x^2 - 24xy - 6y^2 + 4x - 12y + 1 = 0$
   (iv) $9x^2 - 4xy + 6y^2 - 10x - 7 = 0$
   (v) $x^2 - 4xy - 2y^2 + 10x + 4y = 0$
   (vi) $x^2 + 4y^2 - 2x - 16y + 1 = 0$
   (vii) $9x^2 + 24xy + 16y^2 + 22x + 46y + 9 = 0$
   $(viii)$ $3x^2 + 2xy + 3y^2 + 2x - 6y + \frac{25}{2} = 0.$

2. Find the centre of the following conics:

   (i) $x^2 - 4xy + y^2 + 8x + 2y - 5 = 0$
   (ii) $x^2 - 2xy + 2y^2 - 3x + 7y - 1 = 0$
   (iii) $3x^2 - 7xy - 6y^2 + 3x - 9y + 5 = 0.$

**These problems are for the students only as home work. Search the reference books for more examples.*