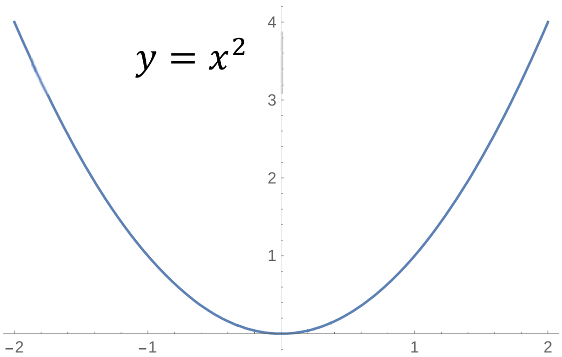
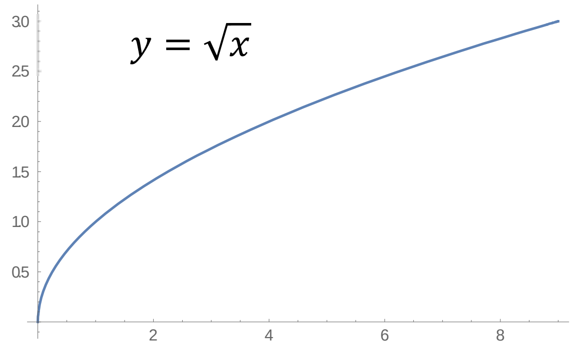
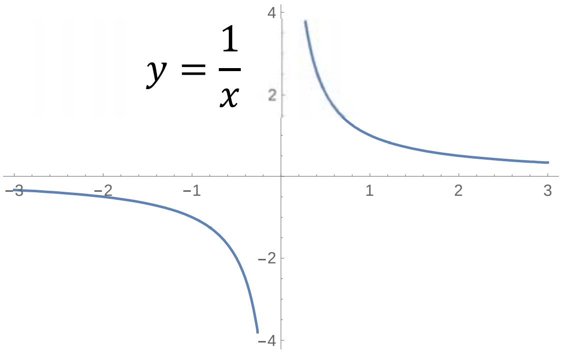
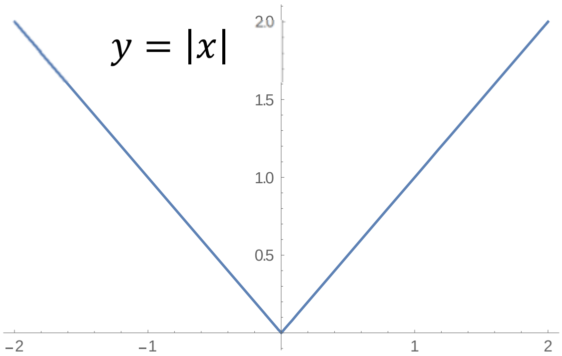
0.2 Solutions to the Selected Problems

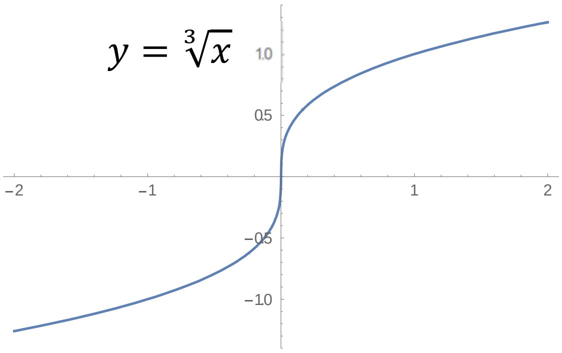
5–24. Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = x^2, y = \sqrt{x}, y = 1/x, y = |x|$ or, $y = \sqrt[3]{x}$ appropriately.











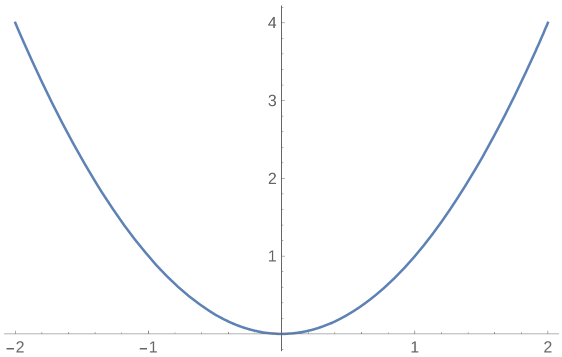
Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

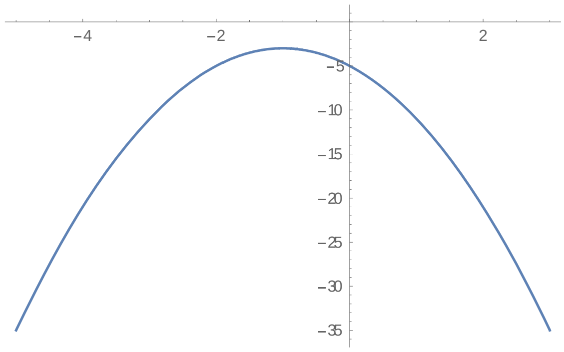
0.2 Solutions to the Selected Problems

5.

$$y = -2(x+1)^2 - 3$$

Solution

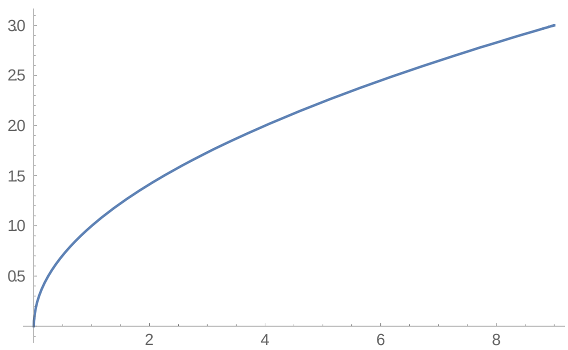


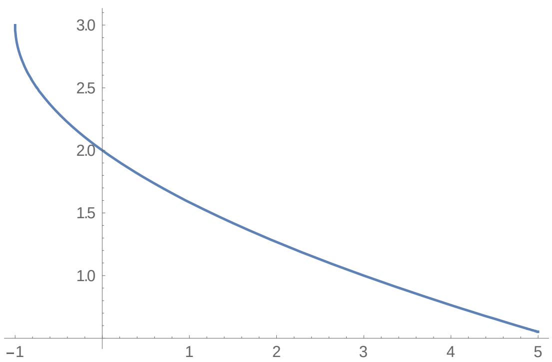


9.

$$y = 3 - \sqrt{x+1}$$

Solution





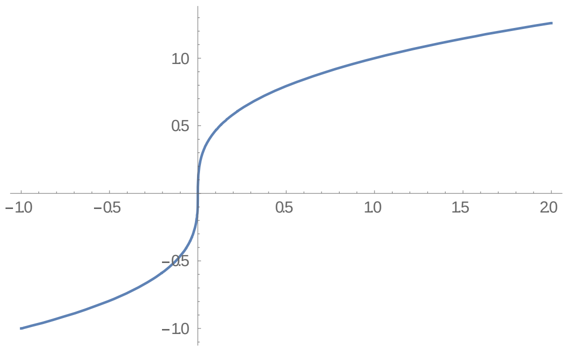
Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

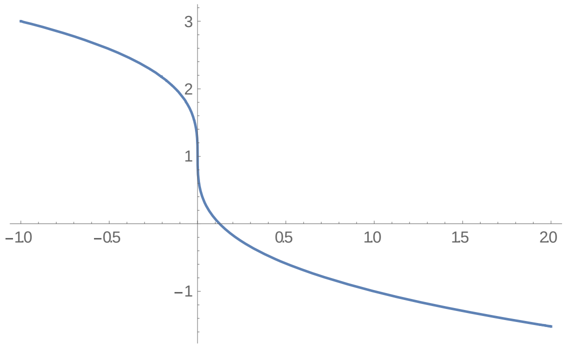
0.2 Solutions to the Selected Problems

21.

$$y = 1 - 2\sqrt[3]{x}$$

Solution

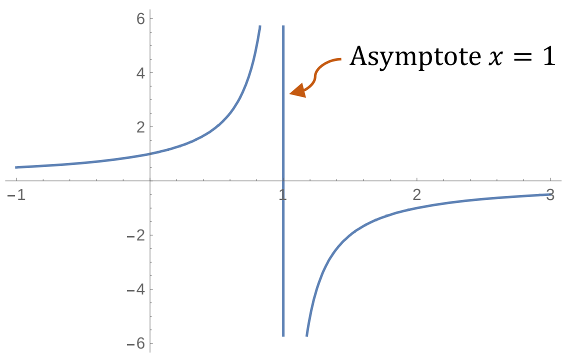




14.

$$y = \frac{1}{1 - x}$$

Solution



Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

0.2 Solutions to the Selected Problems

27. Find the formulas for $f+g, f-g, fg$, and f/g and state the domain of the functions.

$$f(x)=2\sqrt{x-1}, \qquad g(x)=\sqrt{x-1}$$

Solution

Domain of f is $D_f=[1,+\infty)$.

Domain of g is $D_g=[1,+\infty)$.

$$(f+g)(x)\stackrel{\text{def}}{=}f(x)+g(x)=2\sqrt{x-1}+\sqrt{x-1}$$

$$(f+g)(x)=3\sqrt{x-1}$$

Domain of $f+g$ is $D_{f+g}=[1,+\infty)$.

$$(f-g)(x)\stackrel{\text{def}}{=}f(x)-g(x)=2\sqrt{x-1}-\sqrt{x-1}$$

$$(f-g)(x)=\sqrt{x-1}$$

Domain of $f-g$ is $D_{f-g}=[1,+\infty)$.

$$(fg)(x)\stackrel{\text{def}}{=}f(x)g(x)=2\sqrt{x-1}\times\sqrt{x-1}$$

$$(fg)(x)=2(x-1)$$

Domain of fg is $D_{fg}=[1,+\infty)$.

$$\left(\frac{f}{g}\right)(x)\stackrel{\text{def}}{=}\frac{f(x)}{g(x)}=\frac{2\sqrt{x-1}}{\sqrt{x-1}}=2; \quad x\neq 1$$

$$(fg)(x)=2$$

Domain of f/g is $D_{f/g}=(1,+\infty)$.

28. Find the formulas for $f+g, f-g, fg$, and f/g and state the domain of the functions.

$$f(x)=\frac{x}{1+x^2}, \qquad g(x)=\frac{1}{x}$$

Solution

Domain of f is $D_f=(-\infty,+\infty)$.

Domain of g is $D_g=(-\infty,0)\cup(0,+\infty)$.

$$(f+g)(x)\stackrel{\text{def}}{=}f(x)+g(x)=\frac{x}{1+x^2}+\frac{1}{x}$$

$$(f+g)(x)=\frac{1+2x^2}{x(1+x^2)}$$

Domain of $f+g$ is $D_{f+g}=(-\infty,0)\cup(0,+\infty)$.

$$(f-g)(x)\stackrel{\text{def}}{=}f(x)-g(x)=\frac{x}{1+x^2}-\frac{1}{x}$$

$$(f-g)(x)=\frac{-1}{x(1+x^2)}$$

Domain of $f-g$ is $D_{f-g}=(-\infty,0)\cup(0,+\infty)$.

$$(fg)(x)\stackrel{\text{def}}{=}f(x)g(x)=\frac{x}{1+x^2}\times\frac{1}{x}$$

$$(fg)(x)=\frac{1}{1+x^2}$$

Domain of fg is $D_{fg}=(-\infty,0)\cup(0,+\infty)$.

$$\left(\frac{f}{g}\right)(x)\stackrel{\text{def}}{=}\frac{f(x)}{g(x)}=\frac{\frac{x}{1+x^2}}{\frac{1}{x}}=\frac{x^2}{1+x^2}$$

$$(fg)(x)=\frac{x^2}{1+x^2}$$

Domain of f/g is $D_{f/g}=(-\infty,0)\cup(0,+\infty)$.

31–34. Find formulas for $f\circ g, g\circ f, f\circ f$ and $g\circ g$, and state the domain of the compositions.

31.

$$f(x)=x^2,\qquad g(x)=\sqrt{1-x}$$

Domain of f is $D_f=(-\infty,+\infty)$.

Domain of g is $D_g=(-\infty,1]$.

$$(f\circ g)(x)\stackrel{\text{def}}{=}f(g(x))=\left(g(x)\right)^2=\left(\sqrt{1-x}\right)^2=1-x$$

Domain of $f\circ g$ is $D_{f\circ g}=(-\infty,1]$.

$$(g\circ f)(x)\stackrel{\text{def}}{=}g(f(x))=\sqrt{1-f(x)}=\sqrt{1-x^2}$$

Domain of $g\circ f$ is $D_{g\circ f}=[-1,1]$.

$$(f\circ f)(x)\stackrel{\text{def}}{=}f(f(x))=\left(f(x)\right)^2=(x^2)^2=x^4$$

Domain of $f\circ f$ is $D_{f\circ f}=(-\infty,+\infty)$.

$$(g\circ g)(x) \stackrel{\text{def}}{=} g\big(g(x)\big) = \sqrt{1-g(x)} = \sqrt{1-\sqrt{1-x}}$$

Domain of $g\circ g$ is $D_{g\circ g} = [0,1]$.

33.

$$f(x) = \frac{1+x}{1-x}, \qquad g(x) = \frac{x}{1-x}$$

Domain of f is $D_f = (-\infty,1)\cup(1,+\infty)$.

Domain of g is $D_g = (-\infty,1)\cup(1,+\infty)$.

$$(f\circ g)(x) \stackrel{\text{def}}{=} f\big(g(x)\big) = \frac{1+g(x)}{1-g(x)} = \frac{1+\dfrac{x}{1-x}}{1-\dfrac{x}{1-x}} = \frac{\dfrac{1}{1-x}}{\dfrac{1-2x}{1-x}} = \frac{1}{1-2x}$$

Domain of $f\circ g$ is $D_{f\circ g} = (-\infty,1/2)\cup(1/2,1)\cup(1,+\infty)$.

$$(g\circ f)(x) \stackrel{\text{def}}{=} g\big(f(x)\big) = \frac{f(x)}{1-f(x)} = \frac{\dfrac{1+x}{1-x}}{1-\dfrac{1+x}{1-x}} = -\frac{1+x}{2x}$$

Domain of $f\circ g$ is $D_{f\circ g} = (-\infty,0)\cup(0,1)\cup(1,+\infty)$.

$$(f\circ f)(x) \stackrel{\text{def}}{=} f\big(f(x)\big) = \frac{1+f(x)}{1-f(x)} = \frac{1+\dfrac{1+x}{1-x}}{1-\dfrac{1+x}{1-x}} = \frac{\dfrac{2}{1-x}}{\dfrac{-2x}{1-x}} = -\frac{1}{x}$$

Domain of $f\circ f$ is $D_{f\circ f} = (-\infty,0)\cup(0,1)\cup(1,+\infty)$.

$$(g\circ g)(x) \stackrel{\text{def}}{=} g\big(g(x)\big) = \frac{g(x)}{1-g(x)} = \frac{\dfrac{x}{1-x}}{1-\dfrac{x}{1-x}} = \frac{\dfrac{x}{1-x}}{\dfrac{1-2x}{1-x}} = \frac{x}{1-2x}$$

Domain of $g\circ g$ is $D_{g\circ g} = (-\infty,1/2)\cup(1/2,1)\cup(1,+\infty)$.

35–36. Find formulas for $f\circ g\circ h, g\circ f\circ h, h\circ g\circ f$ and state the domain of the compositions.

35.

$$f(x) = x^2+1, \qquad g(x) = \frac{1}{x}, \qquad h(x) = x^3$$

Domain of f is $D_f = (-\infty,+\infty)$.

Domain of g is $D_g = (-\infty,0)\cup(0,+\infty)$.

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

Domain of h is $D_h = (-\infty, +\infty)$.

$$(f \circ g \circ h)(x) \stackrel{\text{def}}{=} (f \circ g)\big(h(x)\big) \stackrel{\text{def}}{=} f\left(g\big(h(x)\big)\right)$$

$$g\big(h(x)\big) = \frac{1}{h(x)} = \frac{1}{x^3}$$

$$f\left(g\big(h(x)\big)\right) = f\left(\frac{1}{x^3}\right) = \left(\frac{1}{x^3}\right)^2 + 1 = \frac{1}{x^6} + 1$$

Domain of $f \circ g \circ h$ is $D_{f \circ g \circ h} = (-\infty, 0) \cup (0, +\infty)$.

37–42. Express f as a composition of two functions; that is, find g and h such that

$$f = g \circ h.$$

37 (a)

$$f(x) = \sqrt{x+2}$$

Solution

$$g(x) = \sqrt{x}, \qquad h(x) = x + 2.$$

37 (b)

$$f(x) = |x^2 - 3x + 5|$$

Solution

$$g(x) = |x|, \qquad h(x) = x^2 - 3x + 5.$$

39 (a)

$$f(x) = \sin^2 x$$

Solution

$$g(x) = x^2, \qquad h(x) = \sin x.$$

40 (a)

$$f(x) = \sin(x^2)$$

Solution

$$g(x) = \sin x\,, \qquad h(x) = x^2.$$

41 (a)

$$f(x) = (1 + \sin(x^2))^3$$

Solution

$$g(x) = x^3, \qquad h(x) = 1 + \sin(x^2).$$

0.2 Solutions to the Selected Problems

Alternative

$$g(x) = x^3, \qquad h(x) = 1 + x, \qquad k(x) = \sin x\,, \qquad p(x) = x^2$$

Show that,

$$f = g \circ h \circ k \circ p.$$

42 (a)

$$f(x) = \frac{1}{1 - x^2}$$

Solution

$$g(x) = \frac{1}{1 - x}, \qquad h(x) = x^2.$$

Alternative

$$g(x) = \frac{1}{x}, \qquad h(x) = 1 - x^2.$$

53–56. Find

$$\frac{f(x+h) - f(x)}{h} \text{ and } \frac{f(w) - f(x)}{w - x}$$

Simplify as much as possible.

53.

$$f(x) = 3x^2 - 5$$

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{(3(x+h)^2 - 5) - (3x^2 - 5)}{h} \\
&= \frac{3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5}{h} \\
&= \frac{6xh + 3h^2}{h} \\
&= 6x + 3h
\end{aligned}$$

$$\begin{aligned}
\frac{f(w) - f(x)}{w - x} &= \frac{(3w^2 - 5) - (3x^2 - 5)}{w - x} \\
&= \frac{3w^2 - 3x^2}{w - x} \\
&= \frac{3(w + x)(w - x)}{(w - x)} \\
&= 3(w + x)
\end{aligned}$$

0.2 Solutions to the Selected Problems

55.

$$f(x) = \frac{1}{x}$$

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{x+h}-\frac{1}{x}}{h} \\
&= \frac{x-(x+h)}{h} \\
&= \frac{-h}{h} \\
&= -1
\end{aligned}$$

$$\begin{aligned}
\frac{f(w)-f(x)}{w-x} &= \frac{\frac{1}{w}-\frac{1}{x}}{w-x} \\
&= \frac{x-w}{w-x} \\
&= -\frac{(w-x)}{(w-x)} \\
&= -1
\end{aligned}$$

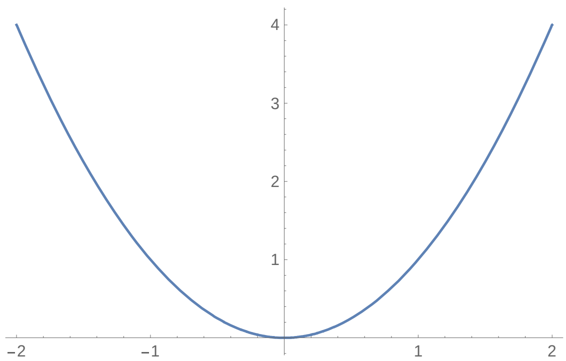
63. Classify the function as even, odd, or neither.

(a) $f(x) = x^2$

Solution

$$f(-x) = (-x)^2 = x^2 = f(x)$$

f is an even function.



The graph of f is symmetric about the y-axis.

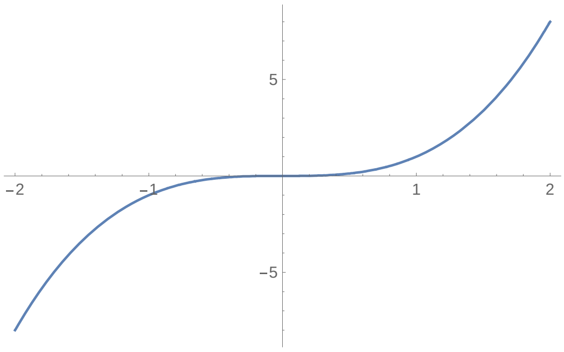
Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

(b) $f(x) = x^3$

Solution

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

f is an odd function.



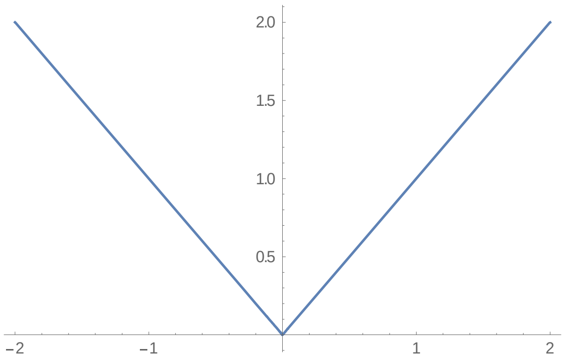
The graph of f is symmetric about the origin.

(c) $f(x) = |x|$

Solution

$$f(-x) = |-x| = |x| = f(x)$$

f is an even function.



The graph of f is symmetric about the y-axis.

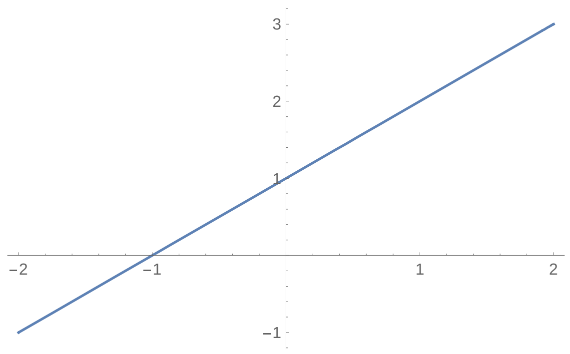
(d) $f(x) = x + 1$

Solution

$$f(-x) = -x + 1 \neq \pm f(x)$$

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

f is neither even nor an even function.



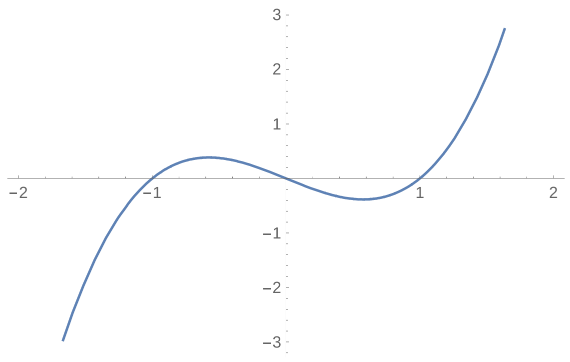
The graph of f has no symmetry about the y-axis or about the origin.

(e) $f(x) = \dfrac{x^5 - x}{1 + x^2}$

Solution

$$f(-x) = \frac{(-x)^5 - (-x)}{1 + (-x)^2} = \frac{-x^5 + x}{1 + x^2} = -\frac{x^5 - x}{1 + x^2} = -f(x)$$

f is an odd function.



The graph of f is symmetric about the origin.

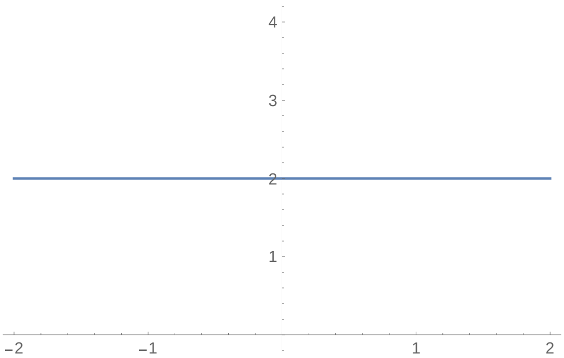
(f) $f(x) = 2$

Solution

$$f(-x) = 2 = f(x)$$

f is an even function.

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University



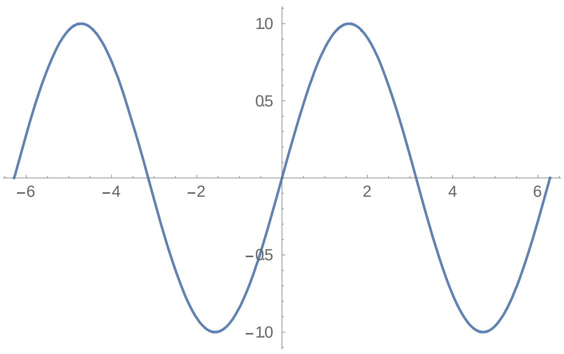
The graph of f is symmetric about the y-axis.

(g) $f(x) = \sin x$

Solution

$$f(-x) = \sin(-x) = -\sin x = -f(x)$$

f is an odd function.



The graph of f is symmetric about the origin.

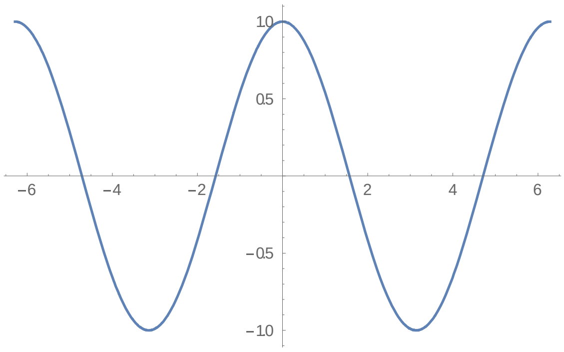
(h) $f(x) = \cos x$

Solution

$$f(-x) = \cos(-x) = \cos x = f(x)$$

f is an even function.

0.2 Solutions to the Selected Problems



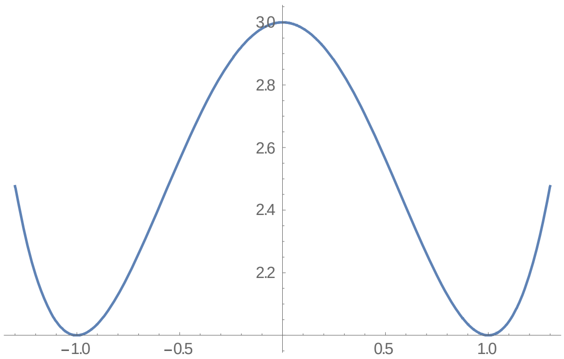
The graph of f is symmetric about the y-axis.

(i) $f(x) = x^4 - 2x^2 + 3$

Solution

$$f(-x) = (-x)^4 - 2(-x)^2 + 3 = x^4 - 2x^2 + 3 = f(x)$$

f is an even function.



The graph of f is symmetric about the y-axis.

(j) $f(x) = x^3 - 2x$

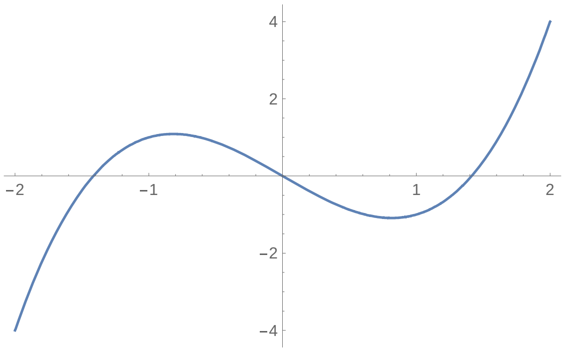
Solution

$$f(-x) = (-x)^3 - 2(-x) = -x^3 + 2x = -(x^3 - 2x) = -f(x)$$

f is an odd function.

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

0.2 Solutions to the Selected Problems



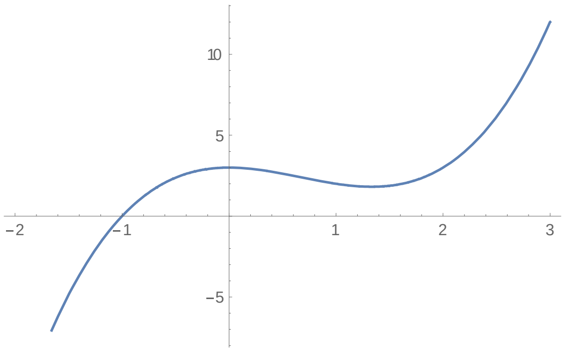
The graph of f is symmetric about the origin.

(k) $f(x) = x^3 - 2x^2 + 3$

Solution

$$f(-x) = (-x)^3 - 2(-x)^2 + 3 = -x^3 - 2x^2 + 3 \neq \pm f(x)$$

f is neither even nor an even function.



The graph of f has no symmetry about the y-axis or about the origin.

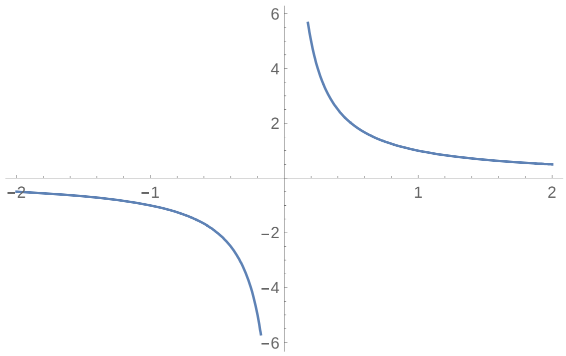
(l) $f(x) = \dfrac{1}{x}$

Solution

$$f(-x) = \frac{1}{(-x)} = -\frac{1}{x} = -f(x)$$

f is an odd function.

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University



The graph of f is symmetric about the origin.

66–67. Use Theorem 0.2.3 to determine whether the graph has symmetries about the x-axis, the y-axis, or the origin.

66 (a)

$$x = 5y^2 + 9$$

Solution

Symmetry about the x-axis

Replacing y by $-y$ yields

$$x = 5(-y)^2 + 9$$

which simplifies to the original equation

$$x = 5y^2 + 9.$$

Thus, the graph is symmetric about the x-axis.

Symmetry about the y-axis

Replacing x by $-x$ we get

$$(-x) = 5y^2 + 9$$

which does not simplify to the original equation. So the graph is not symmetric about the y-axis.

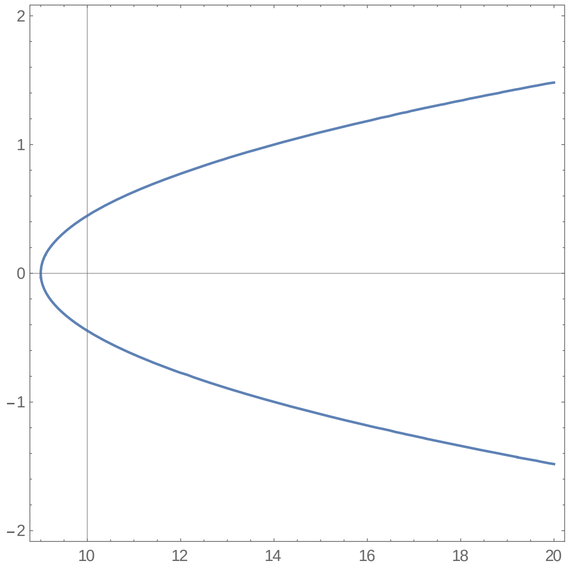
Symmetry about the origin

Replacing x by $-x$ and y by $-y$ we get

$$(-x) = 5(-y)^2 + 9$$

which does not yield the origin equation. So the graph is not symmetric about the origin

Graph:



66 (b)

$$x^2 - 2y^2 = 3$$

Solution

Symmetry about the x-axis

Replacing y by $-y$ yields

$$x^2 - 2(-y)^2 = 3$$

which simplifies to the original equation

$$x^2 - 2y^2 = 3.$$

Thus, the graph is symmetric about the x-axis.

Symmetry about the y-axis

Replacing x by $-x$ we get

$$(-x)^2 - 2y^2 = 3 \Rightarrow x^2 - 2y^2 = 3$$

which simplifies to the original equation. So the graph is also symmetric about the y-axis.

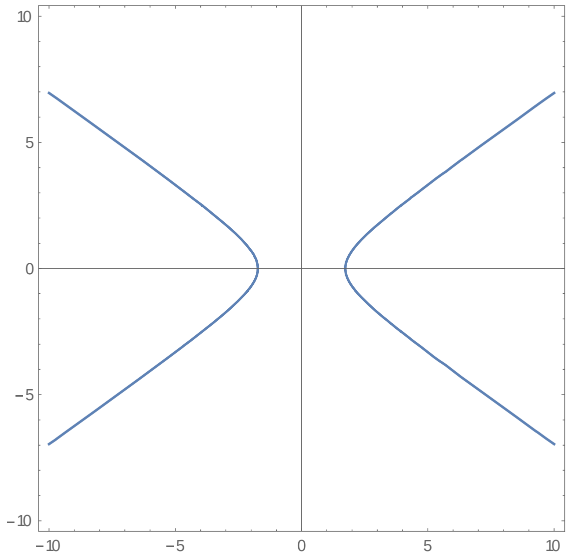
Symmetry about the origin

Replacing x by $-x$ and y by $-y$ we get

$$(-x)^2 - 2(-y)^2 = 3 \Rightarrow x^2 - 2y^2 = 3$$

the original equation back. So the graph is symmetric about the origin.

Graph:



67 (b)

$$y = \frac{x}{3 + x^2}$$

Solution

Symmetry about the x-axis

Replacing y by $-y$ yields

$$-y = \frac{x}{3 + x^2}$$

which does not simplify to the original equation. So the graph is not symmetric about the x-axis.

Symmetry about the y-axis

Replacing x by $-x$ yields

$$y = \frac{-x}{3 + (-x)^2} = -\frac{x}{3 + x^2}.$$

which does not simplify to the original equation. So the graph is not symmetric about the y-axis.

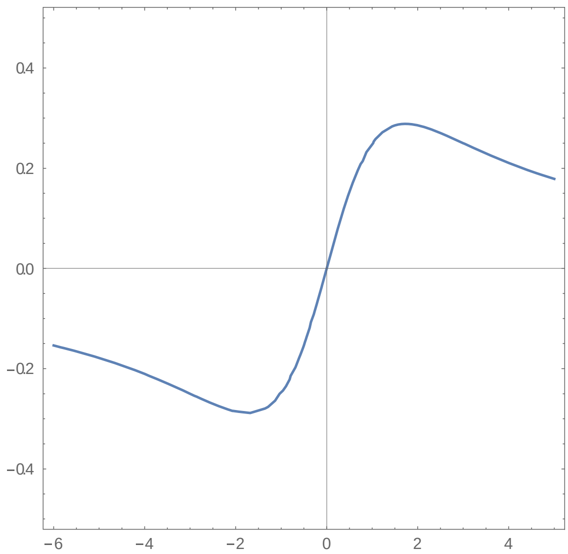
Symmetry about the origin

Replacing x by $-x$ we get

$$(-y) = \frac{(-x)}{3+(-x)^2} = -\frac{x}{3+x^2} \Rightarrow y = \frac{x}{3+x^2}$$

the original equation back. So the graph is symmetric about the origin.

Graph:



Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University