1.2 Computing Limits
Solutions to the Selected Problems

3–30. Find the limits.

3.

$$\lim_{x\to 2} x(x-1)(x+1)$$

Solution

$$\begin{aligned}\lim_{x\to 2} x(x-1)(x+1) &= \lim_{x\to 2} x \times \lim_{x\to 2}(x-1) \times \lim_{x\to 2}(x+1)\\ &= 2\times(2-1)\times(2+1)\\ &= 6\end{aligned}$$

$$\lim_{x\to 2} x(x-1)(x+1) = 6$$

5.

$$\lim_{x\to 3}\frac{x^2-2x}{x+1}$$

Solution

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-2x}{x+1} &= \frac{\lim_{x\to 3}(x^2-2x)}{\lim_{x\to 3}(x+1)}\\ &= \frac{\lim_{x\to 3}(x^2)-\lim_{x\to 3}(2x)}{\lim_{x\to 3}x+\lim_{x\to 3}1}\\ &= \frac{3^2-2\times 3}{3+1}\\ &= \frac{3}{4}\end{aligned}$$

$$\boxed{\lim_{x\to 3}\frac{x^2-2x}{x+1} = \frac{3}{4}}$$

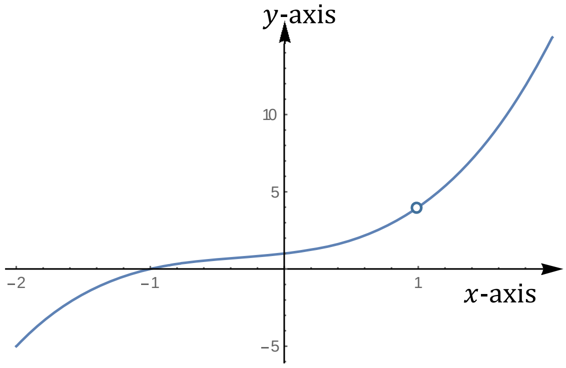
Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

1.2 Computing Limits
Solutions to the Selected Problems

7.

$$\lim_{x\to 1}\frac{x^4-1}{x-1}$$

Solution



$$\begin{aligned}\lim_{x\to 1}\frac{x^4-1}{x-1} &= \lim_{x\to 1}\frac{(x-1)(x^3+x^2+x+1)}{(x-1)}\\ &= \lim_{x\to 1}\frac{(x^3+x^2+x+1)}{1}\\ &= \lim_{x\to 1}x^3+\lim_{x\to 1}x^2+\lim_{x\to 1}x+\lim_{x\to 1}1\\ &= 4\end{aligned}$$

$$\boxed{\lim_{x\to 1}\frac{x^4-1}{x-1}=4}$$

9.

$$\lim_{x\to -1}\frac{x^2+6x+5}{x^2-3x-4}$$

Solution

$$\begin{aligned}\lim_{x\to -1}\frac{x^2+6x+5}{x^2-3x-4} &= \lim_{x\to -1}\frac{(x+1)(x+5)}{(x+1)(x-4)}\\ &= \lim_{x\to -1}\frac{(x+5)}{(x-4)}\\ &= \frac{\lim_{x\to -1}(x+5)}{\lim_{x\to -1}(x-4)}\\ &= \frac{4}{-5}\end{aligned}$$

$$\boxed{\lim_{x\to -1}\frac{x^2+6x+5}{x^2-3x-4}=-\frac{4}{5}}$$

13.

$$\lim_{t\to 2}\frac{t^3+3t^2-12t+4}{t^3-4t}$$

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

Solution

$$\lim_{t\to 2}\frac{t^3+3t^2-12t+4}{t^3-4t}=\lim_{t\to 2}\frac{(t-2)(t^2+5t-2)}{t(t-2)(t+2)}$$

$$=\lim_{t\to 2}\frac{(t^2+5t-2)}{t(t+2)}$$

$$=\frac{\lim\limits_{t\to 2}(t^2+5t-2)}{\lim\limits_{t\to 2}t(t+2)}$$

$$=\frac{4+10-2}{2\times 4}$$

$$=\frac{3}{2}$$

$$\boxed{\lim_{t\to 2}\frac{t^3+3t^2-12t+4}{t^3-4t}=\frac{3}{2}}$$

15.

$$\lim_{x\to 3^+}\frac{x}{x-3}$$

Solution

$$\lim_{x\to 3^+}\frac{x}{x-3}=\lim_{x\to 3^+}\frac{x-3+3}{x-3}$$

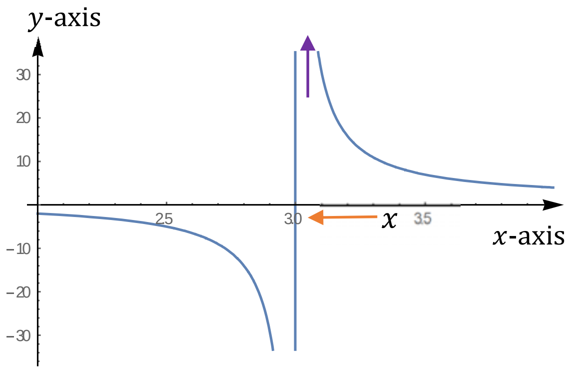
$$=\lim_{x\to 3^+}\left(\frac{x-3}{x-3}+\frac{3}{x-3}\right)$$

$$=\lim_{x\to 3^+}\left(1+\frac{3}{x-3}\right)$$

$$=\lim_{x\to 3^+}1+3\times\lim_{x\to 3^+}\left(\frac{1}{x-3}\right)$$

$$=1+3\times(+\infty)$$

$$=+\infty$$



$$\boxed{\lim_{x\to 3^+}\frac{x}{x-3}=+\infty}$$

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

1.2 Computing Limits
Solutions to the Selected Problems

27.

$$\lim_{x\to 2^+}\frac{1}{|2-x|}$$

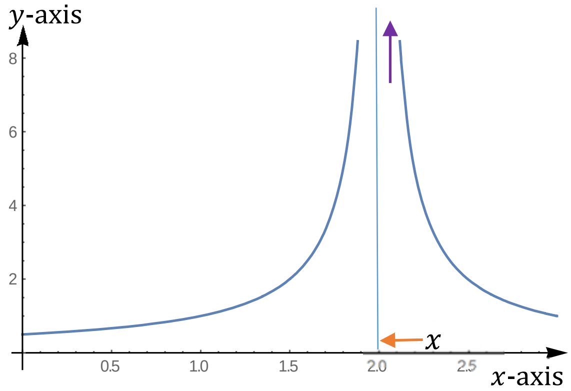
Solution

$$\lim_{x\to 2^+}\frac{1}{|2-x|}=\lim_{x\to 2^+}\frac{1}{-(2-x)}$$

$$=\lim_{x\to 2^+}\frac{1}{(x-2)}$$

$$=+\infty$$

$$\boxed{\lim_{x\to 2^+}\frac{1}{|2-x|}=+\infty}$$



29.

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$$

Solution

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}=\lim_{x\to 9}\frac{\left(\sqrt{x}\right)^2-(3)^2}{\sqrt{x}-3}$$

$$=\lim_{x\to 9}\frac{\left(\sqrt{x}+3\right)\left(\sqrt{x}-3\right)}{\left(\sqrt{x}-3\right)}$$

$$=\lim_{x\to 9}\left(\sqrt{x}+3\right)$$

$$=6$$

$$\boxed{\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}=6}$$

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

1.2 Computing Limits
Solutions to the Selected Problems

31. Let

$$f(x)=\begin{cases} x-1, & x\le 3 \\ 3x-4, & x>3 \end{cases}$$

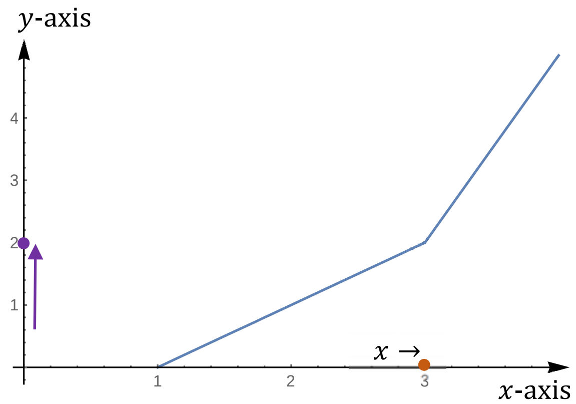
Find

$(a)\quad \lim_{x\to 3^-} f(x) \qquad\qquad (b)\quad \lim_{x\to 3^+} f(x) \qquad\qquad (c)\quad \lim_{x\to 3} f(x)$

Solution

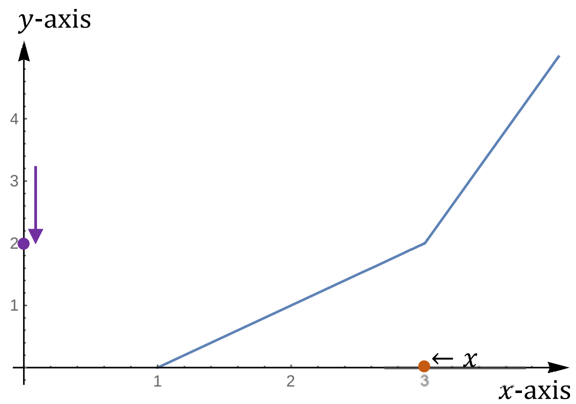
(a)

$$\begin{aligned}\lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-}(x-1)\\ &= 3-1\\ &= 2\end{aligned}$$



(b)

$$\begin{aligned}\lim_{x\to 3^+} f(x) &= \lim_{x\to 3^+}(3x-7)\\ &= 9-7\\ &= 2\end{aligned}$$



(c)

Since $\lim_{x\to 3^-} f(x) = 2 = \lim_{x\to 3^+} f(x)$

therefore

$$\boxed{\lim_{x\to 3} f(x) = 2}$$

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University

1.2 Computing Limits
Solutions to the Selected Problems

37–38. Rationalize the numerator then find the limit.

37.

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$$

Solution

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}=\lim_{x\to 0}\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}-2\right)}{x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{x+4}\right)^2-(2)^2}{x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{x-4-4}{x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{4}+2\right)}$$

$$=\frac{1}{4}$$

$$\boxed{\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}=\frac{1}{4}}$$

Mohammad Hassan Murad
Department of Mathematics and Natural Sciences
BRAC University