CSE331: Automata and Computability
Worksheet 3 (CFG)

2. Give context-free grammars that generate the following languages.

a) L= {w | w contains at least three 1's}.

b) L={w | w starts and ends with the same symbol}.

c) L= {w | the length of w is odd}

d) L={w | the length of w is odd and its middle is 0}

e) L={w | w contains twice as many 1s as 0s}

1. Give a context-free grammar for each of the following languages.

a) $L(G) = \{ 0^n 1^m 0^m \mid n, m \geq 0 \}$ over the terminals $\{0,1\}$

b) $L(G) = \{ a^n b^m c^k \mid n, m, k \geq 0$ and $n=2m+3k \}$ over $\sum=\{a,b,c\}$

c) $L(G) = \{ a^n b^m \mid 0 < n < m < 3n \}.\sum=\{a,b\}$

d) $L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0$ and $i=j$ or $j=k \}.\sum=\{a,b,c\}$

e) $L(G) = \{ a^i b^j c^k \mid j \neq i+k \}.\sum=\{a,b,c\}$

f) $L(G) = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m > 0 \}$.

3. Consider the following context-free grammar $\Sigma=\{0,1\}$.

$S \rightarrow A\ 1\ B$

$A \rightarrow 0A \mid \varepsilon$

$B \rightarrow 0B \mid 1B \mid \varepsilon$

Give leftmost and rightmost derivations and parse tree for the following strings

a) 0010101

b) 10100

c) 00011

4. Which language generates the grammar G given by the productions.

$S \rightarrow aSa \mid aBa$

$B \rightarrow bB \mid b$

5. Explain/Prove why the grammar below is ambiguous.

$S \rightarrow 0A \mid 1B$

$A \rightarrow 0AA \mid 1S \mid 1$

$B \rightarrow 1BB \mid 0S \mid 0$

6. Given the following ambiguous context free grammar

$S \rightarrow Ab \mid aaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$

a) Find leftmost and rightmost derivations for aaaaab, aabb, ab.

b) Show the parse trees for the above strings in (a).

c) Find an equivalent unambiguous context-free grammar.

d) Give the unique leftmost derivation and parse tree for the above strings generated

from the unambiguous grammar you designed in (c).

6. a) Show that the following grammar is ambiguous.

$S \rightarrow aEbS$

$S \rightarrow aEbScS \mid \epsilon$

$E \rightarrow d$

b) Consider the grammar with start symbol D, $\sum=\{c, a, b, . , 0, 1\}$

$D \rightarrow TL$

$T \rightarrow c \mid Tc$

$L \rightarrow L.V \mid V$

$V \rightarrow a \mid b \mid 0 \mid 1 \mid Va \mid Vb \mid V0 \mid V1$

i) Derive ccab.01 using leftmost derivation
ii) Derive cabb0011.ab1 using rightmost derivation