

8 CFG 0

①  $\{w \in \{0,1\}^* \mid w \text{ contains at least three 1s}\}$

$$S \rightarrow \cancel{0} A 1 A 1 A 1 A$$

$$A \rightarrow 0A \mid 1A \mid \cancel{0} \epsilon$$

②  $\{w \in \{0,1\}^* \mid w = w^R \text{ and } |w| \text{ is even}\}$

$$S = 0S \mid 1S \mid \epsilon$$

③  $\{w \in \{0,1\}^* \mid \text{the length of } w \text{ is odd and the middle symbol is } 0\}$

$$S \rightarrow \cancel{S} 0 S \mid 0S \mid 1S \mid 0$$

④  $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } i=k\}$

$$S = X \mid Y$$

$$Y = aYc \mid N$$

$$X = aXb \mid M$$

$$N = bN \mid \epsilon$$

$$M = cM \mid \epsilon$$

$$e) \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i+j=k\}$$

$$S = aSc \mid \epsilon$$

$$X = bXc \mid \epsilon$$

f) CFG for even palindrome.

$$P = OPQ \mid 1PQ1 \mid \epsilon$$

g) CFG for odd palindrome.

$$P = OPQ \mid 1P1 \mid 0 \mid 1 \mid \epsilon$$

$$h) \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$$

$$S = ABC \mid CD$$

$$A = aA \mid \epsilon$$

$$B = bBc \mid \epsilon \mid cD$$

$$C = acb \mid \epsilon \mid aA$$

$$D = cD \mid \epsilon \quad E = bE \mid b$$

i) set of all strings of a's and b's that are not of the form  $ww$ , that is, not equal to any string repeated.

$$S = AB \mid BA \mid A \mid B$$

$$A = aAa \mid aAb \mid bAa \mid bAb \mid a$$

$$B = aBa \mid aBb \mid bBa \mid bBb \mid b$$

j) set of all strings with twice as many 0's ~~as~~ 1's

$$S = \cancel{0A0} \mid AB \mid BA$$

$$A = \cancel{0A0} \mid \cancel{00}$$

$$B = \cancel{1B1} \mid \cancel{11}$$

$$S = 0S0S \mid 0S1S0 \mid 1S0S0$$

$$S = 001 \mid 010 \mid 100 \mid \epsilon$$

W) All strings with exactly one occurrence of substring bbb.  
 $\Sigma = \{a, b\}$ .

$$S = A b b b B A$$

$$A = \cancel{aA} | \cancel{bB} | \cancel{abA} | \cancel{baA} | Bb |$$

$$A = aA | a | \epsilon | Aa | \epsilon$$

$$B = b | b$$

$$C = aA$$

Y) All string with more a than b.

$$S \rightarrow Aa | M S | S M A$$

$$A \rightarrow Aa | \epsilon$$

$$M \rightarrow M M | b M a | a M b | \epsilon$$

m) All strings with a number of a's divisible by 4.

$$S \rightarrow aaaaS | bS | aaaa$$

n) All strings in the language  
 $L: \{a^n b^m a^{2n} \mid n, m \geq 0\}$

$$S \rightarrow aSaa \mid B$$

$$B \rightarrow bB \mid \epsilon$$

o)  $a^n b^m$ ,  $n > m$  or  $m > n$

~~$$S \rightarrow aSb \mid$$~~

$$S \rightarrow aXb \mid aYb$$

$$X \rightarrow aXb \mid aX \mid a$$

$$Y \rightarrow aYb \mid Yb \mid b$$