Homework Sheet #2

1

(i) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3 \quad - 7 + 4 \quad = 10$

$$(A|B) = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = R_1 + R_2 \\ R_3' = -3R_1 + R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -5 & -1 & -7 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = (-1)R_2 \\ R_3' = \frac{1}{2} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -26 & -52 \end{array}\right] \longleftarrow R_3' = 5(R_2) + R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right] \longleftarrow R_3' = -\frac{1}{26} R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \longleftarrow R_2' = -2R_3 + R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \longleftarrow R_1' = -R_2 + R_1$$

$\therefore$ $x = 3$, $y = 1$, $z = 2$

$\therefore$ $x_1 = 3$, $x_2 = 1$, $x_3 = 2$

Ans.

(11)

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

$$(A|B) = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \longleftarrow R_1' = \tfrac{1}{2}R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = 2R_1 + R_2 \\ R_3' = -8R_1 + R_3 \end{array}$$

$$= \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -1 & -4/7 & -1/7 \end{array}\right] \longleftarrow \begin{array}{l} R_2' = \tfrac{1}{7}R_2 \\ R_3' = \tfrac{1}{7}R_3 \end{array}$$

$$= \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Free variables $= 3 - 2 = 1$

$\therefore\ x_1 + x_2 + x_3 = 0$ — (i)

$x_2 + \frac{4}{7}x_3 = \frac{1}{7}$ — (ii)

$0.x_3 = 0$

let $x_3 = r$

(ii) —

$x_2 = \frac{1}{7} - \frac{4}{7}r$

(i) $x_1 = -\frac{1}{7} + \frac{4}{7}r$

$= -\frac{1}{7} - \frac{3}{7}r$

Ans.

(iii)

$x - y + 2z - w = -1$

$2x + y - 2z - 2w = -2$

$-x + 2y - 4z + w = 1$

$3x \qquad - 3w = -3$

$$A|B = \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -4 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \leftarrow \begin{array}{l} R_2' = -2R_1 + R_2 \\ R_3' = R_1 + R_3 \end{array}$$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -4 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \leftarrow \begin{array}{l} R_2' = R_3 \\ R_3' = R_2 \end{array}$$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow \begin{array}{l} R_3' = -3(R_2) + R_3 \\ R_4' = -3(R_2) + R_4 \end{array}$$

free Variable $= 4 - 3 = \cancel{2}\ 1$

$x_1 - x_2 + 2x_3 - x_4 = -1$ —(i)

$x_2 - 2x_3 = 0$ —(ii)

$x_3 = 2$ —(iii)

$x_4 = t$ (let)

(ii) $x_2 = 2x_3$
$= 2 \times 2 = 4$

(i) $x_1 = x_2 - 2x_3 + x_4 - 1$
$= 4 - 2 \times 2 + t - 1$
$= 4 - 4 + t - 1$
$= t - 1$

$$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3' = R_3 + R_2$$

free variables $= 4 - 3 = 1$

$x_1 - x_2 + 2x_3 - x_4 = -1$ —(i)

$x_2 - 2x_3 = 0$ —(ii)

$2x_3 = 0$ —(iii)

$x_4 = t$ (let)

(iii) $x_3 = 0$

(ii) $x_2 = 0$

(i) $x_1 = t - 1$

2

(i) $2x_1 + 2x_2 - x_3 + 0 + x_5 = 0$

$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$

$x_1 + x_2 - 2x_3 + 0 - x_5 = 0$

$0 + 0 + x_3 + x_4 + x_5 = 0$

$$\begin{bmatrix} 2 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \leftarrow \begin{matrix} R_1' = R_3 \\ R_3' = R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & -3 & 3 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \leftarrow \begin{matrix} R_2' = R_1 + R_2 \\ R_3' = -2R_1 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & -3 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \end{bmatrix} \longleftarrow \begin{array}{l} R_2' = R_4 \\ R_4' = R_2 \end{array}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \longleftarrow \begin{array}{l} R_3' = R_3 \times \frac{1}{3} \\ R_4' = R_3 \times \left(-\frac{1}{3}\right) \end{array}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = (-1) R_2 + R$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \longleftarrow R_3' = -\frac{1}{2} \times$$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow R_4' = -R_3 + R_4$$

$\therefore\ x_1 + x_2 - 2x_3 + 0 - x_5 = 0$ — ① free variables

$x_3 + x_4 + x_5 = 0$ — ⑪ $= 5 - 3$

$x_4 = 0$ — $= 2$

$x_5 = t$ (let)

$x_2 = p$ (let)

⑪ —

$x_3 = -0 - t$

$= -t$

① —

$x_1 = -p + 2(-t) + t$

$= -p - 2t + t$

$= -p - t$

⑪ $0 + 2x + 2y + 4z = 0$

$w + 0 - y - 3z = 0$

$2w + 3x + y + z = 0$

$-2w + x \quad + 3y - 2z = 0$

$$\begin{bmatrix} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix} \xleftarrow[R_2' = R_1]{R_1' = R_2}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 2 & -5 & 0 \end{bmatrix} \xleftarrow[R_4' = R_1 + R_4]{R_3' = -2(R_1) + R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 2 & -5 & 0 \end{bmatrix} \xleftarrow{R_2' = \frac{1}{2} R_2}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -7 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_2 + R_4]{R_3' = -3R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & \cancel{10} & 1 & 2 & 0 \\ 0 & 0 & 1 & -7 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow[]{R_3' = R_4,\ R_4' = R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow{R_3' = -R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow[R_4' = -R_3 + R_4]{R_3' = -\frac{1}{9} R_3}$$

$x_1 + 0 - x_3 - 3x_4 = 0$ —— (I)

$x_2 + x_3 + 2x_4 = 0$ —— (II)

$x_4 = 0$

$x_3 = t$ (let)

Free
$4 - 3 = 1$

(II) ——

$x_2 = -t - 20$

$= -t$

(I) ——

$x_1 = t + 3 \times 0$

$= t$

Ans.

(2)

$$x + y - z = 1$$
$$2x + 3y + \lambda z = 3$$
$$x + \lambda y + 3z = 2$$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & \lambda & 3 \\ 1 & \lambda & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2+\lambda & 1 \\ 0 & \lambda-1 & 4 & 1 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -2(R_1) + R_2}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2+\lambda & 1 \\ 0 & 0 & -(\lambda+3)(\lambda-2) & -(\lambda-2) \end{bmatrix} \xleftarrow{R_3' = -(\lambda-1)R_2 + R_3}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2+\lambda & 1 \\ 0 & 0 & (\lambda+3)(\lambda-2) & (\lambda-2) \end{bmatrix} \xleftarrow{R_3' = (-1) R_3}$$

$-\lambda + 1 + 1$

$-\lambda + 2$

$-(\lambda - 2)$

$-(\lambda-1)$ $(2+\lambda$

$(-\lambda+1)(2+$

$-2\lambda - \lambda^2 +$
$2 + \lambda$

$-\lambda^2 - 2\lambda + \lambda$
$+2$

$-\lambda^2 - \lambda + 2$
$+4$

$-\lambda^2 - \lambda + 6$

$-(\lambda^2 + \lambda - 6$

$-(\lambda^2 + 3\lambda - 2\lambda$
-6

$-\{\lambda(\lambda+3) - 2(\lambda+3$

$-(\lambda+3)(\lambda-2)$

and $x - 2 \neq 0$

$x \neq 2$

(iii) for unique solution

$(\lambda+3)(\lambda-2) \neq 0$ $\qquad$ $\lambda-2 \neq 0$

$\lambda \neq -3 \quad \lambda \neq 2$ $\qquad$ $\lambda \neq 2$

(ii) for more then one solution

$(\lambda+3)(\lambda-2) = 0$ — (i) $\qquad$ $\lambda-2 = 0$ — (ii)

$\lambda = -3$ $\qquad$ $\lambda = 2$

$\lambda = 2$

$\lambda \neq -3$

$\lambda = 2$ [as it will make both (i) and (ii) valid as $0 = 0$]

(4)

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \xleftarrow[R_3' = -R_1 + R_3]{R_2' = -R_1 + R_2}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \underset{0}{\underline{\lambda-3}} & \underset{\neq 0}{\underline{\mu-10}} \end{bmatrix}$$

$-2 + \lambda - 1$
$= \lambda - 3$
$-4 + \mu - 6$
$\mu - 10$

i) For no solution

$\lambda - 3 = 0 \quad \mu - 10 \neq 0$

$\lambda = 3 \quad \mu \neq 10$

(ii) For more than one solution

$\lambda - 3 = 0 \quad \mu - 10 = 0$

$\lambda = 3 \quad \mu = 10$

(iii) For ~~more~~ unique solution

$\lambda - 3 \neq 0 \quad \mu - 10 \neq 0$

$\lambda \neq 3 \quad \mu \neq 10$

$\mu = 10$