



BRAC University

FUNCTION: LIMIT, CONTINUITY, & DIFFERENTIABILITY

PROBLEMS ON LIMIT

Find the limits of the following functions:

1. $\lim_{x \rightarrow 1} \frac{x}{\sqrt{x+1} - 1}$

2. $\lim_{x \rightarrow 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$

3. $\lim_{x \rightarrow 0} \frac{|x|}{x}$

4. $\lim_{x \rightarrow \infty} \frac{3x + 5}{6x - 7}$

5. $\lim_{x \rightarrow \infty} \sqrt{\frac{3x + 5}{6x + 3}}$

6. $\lim_{x \rightarrow \infty} \frac{4x^2 - x}{2x^3 - 5}$

7. $f(x) = \begin{cases} 2 - x & x < 1 \\ x^2 + 1 & x > 1 \end{cases}$

Find $\lim_{x \rightarrow 1} f(x)$.

8. $f(x) = \begin{cases} 3x - 1 & x < 1 \\ 3 - x & x > 1 \end{cases}$

Does $\lim_{x \rightarrow 1} f(x)$ exist?

9. $f(x) = \begin{cases} 2x + 1 & x < 1 \\ 3 - x & x > 1 \end{cases}$

Does $\lim_{x \rightarrow 1} f(x)$ exist?

10. $f(x) = \begin{cases} \frac{1}{x+2} & x < -2 \\ x^2 - 5 & -2 \leq x \leq 3 \\ \sqrt{x+13} & x > 3 \end{cases}$

Find $\lim_{x \rightarrow -2} f(x)$ and $\lim_{x \rightarrow 3} f(x)$

11. $f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 1 & x = 0 \\ 1 + x & x > 0 \end{cases}$

Find $\lim_{x \rightarrow 0} f(x)$

12. $f(x) = \begin{cases} x + 2 & x \leq -1 \\ 6 & -1 < x \leq 1 \\ 2x - 6 & x > 1 \end{cases}$

Find $\lim_{x \rightarrow -1} f(x)$ and $\lim_{x \rightarrow 1} f(x)$

13. $f(x) = \begin{cases} e^{-\frac{|x|}{2}} & -1 < x < 0 \\ 1 & x > 0 \end{cases}$

Does $\lim_{x \rightarrow 0} f(x)$ exist?

14. $f(x) = \begin{cases} e^{-\frac{|x|}{2}} & -1 < x < 0 \\ x & x > 0 \end{cases}$

Does $\lim_{x \rightarrow 0} f(x)$ exist?

PROBLEMS ON CONTINUITY & DIFFERENTIABILITY

Test the continuity of the following functions:

$$1. f(x) = \begin{cases} \frac{x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$2. f(x) = \begin{cases} \frac{x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$3. f(x) = \begin{cases} \frac{x^2}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$4. f(x) = \begin{cases} \cos x & x \geq 0 \\ -\cos x & x < 0 \end{cases}$$

$$5. f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$6. f(x) = \begin{cases} x \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$7. f(x) = \begin{cases} \sqrt{|x|} & x \geq 0 \\ -\sqrt{|x|} & x < 0 \end{cases}$$

$$8. f(x) = \begin{cases} e^{\frac{1}{x}} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$9. f(x) = \begin{cases} e^{-\frac{|x|}{2}} & -1 < x < 0 \\ x^2 & 0 \leq x < 2 \end{cases}$$

$$10. f(x) = \begin{cases} (x-a) \sin\left(\frac{1}{x-a}\right) & x \neq a \\ 0 & x = a \end{cases}$$

$$11. f(x) = |x| + |x-1| \text{ at } x = 0, 1$$

$$12. f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$13. f(x) = \begin{cases} \frac{|x-2|}{x-2} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$14. f(x) = \begin{cases} (1+x)^{\frac{1}{x}} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Try to sketch the above functions. You are instructed to study worked out examples and problems from reference books as well to improve your concept.