(11) $\int_{(0,0)}^{(3,2)} 2xe^y\,dx + x^2e^y\,dy$

$\frac{\partial f(x,y)}{\partial y} = 2xe^y$

$\frac{\partial f(x,y)}{\partial x} = 2xe^y$

~~conservative~~ Independent

$\frac{\partial \varphi}{\partial x} = 2xe^y \qquad \frac{\partial \varphi}{\partial y} = x^2e^y$

$\varphi = \int 2xe^y\,dx + K(y)$

$\varphi = x^2e^y + K(y)$

$\frac{\partial \varphi}{\partial y} = x^2e^y + K'(y)$

$x^2e^y = x^2e^y + K'(y)$

$K'(y) = x^2e^y - e^y$

$\therefore K(y) = x^2e^y - e^y + C$

$\varphi = x^2e^y + C$

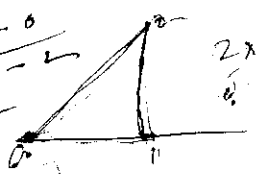
$\int_{(0,0)}^{(3,2)} 2xe^y\,dx + x^2e^y\,dy$

$= 9e^2$

* Find

$\frac{x-0}{0-1} = \frac{y-0}{0-2}$

$y = 2x$



$\left(\frac{\partial}{\partial x} \text{ & } \frac{\partial}{\partial y}\right)$

$$\int (xy + z^3)\, ds$$

$x = \cos t$ $\quad \frac{dx}{dt} = \sin t$

$y = \sin t$ $\quad \frac{dy}{dt} = -\cos t$

$z = t$ $\quad \frac{dz}{dt} = 1$

$$\sqrt{2}\int_0^{\pi} (\cos t \sin t + t^3)\, dt \qquad \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$= \sqrt{2}\int_0^{\pi} \cos t \sin t\, dt + \sqrt{2}\int_0^{\pi} t^3\, dt$$

$$= \frac{\sqrt{2}}{2}\left[\sin^2 t\right]_0^{\pi} + \frac{\sqrt{2}}{4}\left[t^4\right]_0^{\pi}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2}(t - \cos 2t) + \frac{\sqrt{2}}{4} t^4$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \qquad \frac{\sqrt{2}}{4}\pi^4$$

(4)
$$\int_{(1,1)}^{(3,3)} \left(e^x \ln y - \frac{e^y}{x}\right) dx + \left(\frac{e^x}{y} - e^y \ln x\right) dy$$

$$\therefore \frac{\partial f}{\partial y} = \frac{e^x}{y} - \frac{e^y}{x}$$

$$\frac{\partial g}{\partial y} = \frac{e^x}{y} - \frac{e^y}{x}$$

Independent

$$\frac{\partial \varphi}{\partial x} = e^x \ln y - \frac{e^y}{x}$$

$$\frac{\partial \varphi}{\partial y} = \frac{e^x}{y} - e^y \ln x$$

$$\varphi = \int e^x \ln y - \frac{e^y}{x}\, dx + K(y)$$

$$= e^x \ln y - \ln x\, e^y + K(y)$$

$$\frac{\partial \varphi}{\partial y} = \frac{e^x}{y} - e^y \ln x + K'(y)$$

$x(t) = 0 + t$

$x(y) = y$

$\varphi = e^x \ln y - e^y \ln x + k$

$$\int_{(1,1)}^{(3,3)} \left(e^x \ln y - \frac{e^y}{x}\right) dx + \left(\frac{e^x}{y} - e^y \ln x\right) dy$$

$= \varphi(3,3) - \varphi(1,1)$

$= e^3 \ln 3 - e^3 \ln 3 - e \ln 1 + e \ln 1$

$= 0$ Ans.

(10)

$$\int_{(0,0)}^{(1,\pi/2)} e^x \sin y \, dx + e^x \cos y \, dy$$

$$\frac{\partial f}{\partial y} = e^x \cos y$$

$$\frac{\partial g}{\partial x} = e^x \cos y$$

independent.

$$\frac{\partial \varphi}{\partial x} = e^x \sin y \qquad \frac{\partial \varphi}{\partial y} = e^x \cos y$$

$$\varphi = \int e^x \sin y \, dx + K(y)$$

$$= e^x \sin y + K(y)$$

$$\frac{\partial \varphi}{\partial y} = e^x \cos y + K'(y)$$

$$e^x \cos y = e^x \cos y + K'(y)$$

$$K'(y) = 0$$

$$K(y) = C$$

$$\varphi = e^x \sin y + C$$

$\varphi(1, \pi/2) - \varphi(0,0)$

$= e \sin \pi/2$

$= e$ Ans.