1.5 Continuity
Solutions to the Selected Problems

11–22. Find values of x, if any, at which f is not continuous.

11.

$$f(x) = 5x^4 - 3x + 7$$

Solution

f is a polynomial function. A polynomial function is everywhere continuous.

14.

$$f(x) = \frac{x+2}{x^2-4}$$

Solution

f is a rational function (ratio of two polynomial functions). A rational function is not continuous at where the denominator function becomes zero.

Solving the equation

$$x^2 - 4 = 0$$

yields the discontinuities at

$$x = \pm 2$$

17.

$$f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$$

Solution

$$x = 0, \pm 1$$

21.

$$f(x) = \begin{cases} 2x+3, & x \le 4 \\ 7 + \dfrac{16}{x}, & x > 4 \end{cases}$$

Solution

f has no discontinuity.

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Exercise

$$f(x) = \begin{cases} 2 + \dfrac{3}{x}, & x \le 4 \\ 7 + 16x, & x > 4 \end{cases}$$

29–30. Find a value of the constant k, if possible, that will make the function continuous everywhere.

29.

$$f(x) = \begin{cases} 7x - 2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$$

Solution

$$f(1) = 5$$

Left-sided limit at $x = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (7x - 2) = 5$$

Right-sided limit at $x = 1$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (kx^2) = k$$

For continuity, we require,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1) \Rightarrow \boxed{k = 5}$$

35–36 Find the values of x (if any) at which f is not continuous, and determine whether each such value is a removable discontinuity.

35. (a)

$$f(x) = \frac{|x|}{x}$$

Solution

The function f is undefined at $x = 0$. Therefore, f has a discontinuity at $x = 0$.

Left-sided limit at $x = 0$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -\lim_{x \to 0^-} 1 = -1.$$

Right-sided limit at $x = 0$

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$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^-} 1 = 1.$$

Since

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

The discontinuity at $x = 0$ is not removable.

35. (b)

$$f(x) = \frac{x^2 + 3x}{x + 3}$$

Solution

The function f is undefined at $x = -3$. Therefore, f has a discontinuity at $x = -3$.

Left-sided limit at $x = -3$

$$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \frac{x^2 + 3x}{x + 3} = \lim_{x\to -3^-} \frac{x(x + 3)}{(x + 3)} = \lim_{x\to -3^-} x = -3.$$

Right-sided limit at $x = -3$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \frac{x^2 + 3x}{x + 3} = \lim_{x\to -3^+} \frac{x(x + 3)}{(x + 3)} = \lim_{x\to -3^+} x = -3.$$

Since

$$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^+} f(x)$$

The discontinuity at $x = -3$ is removable.

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