BRAC University

Home Work sheet # 5

MAT – 216

1. Evaluate the line integral $\int_C (xy + z^3)\, ds$ from (1,0,0) to (-1,0,0) along the helix C that is represented by the parametric equation x = cost, y = sin t , z = t $(0 \le t \le \pi)$.

2. Evaluate $\int_C xy\,dx + x^2\,dy$ if

   (a) C consists of line segments from (2,1) to (4,1) and from (4,1) to (4,5).

   (b) C is the line segment from (2,1) and (4,5).

   (c) Parametric equation for C are x = 3t – 1, y = $3t^2$ – 2t ; $1 \le t \le 5/3$.

3. Show that (a) $\int (6x^2 y - 3xy^2)\,dy + (6xy^2 - y^3)\,dx$ is independent of the path joining the points (1,2) and (3,4) (b) hence evaluate the integral.

4. Let $F(x, y) = (3x^2 y + 2)\, i + (x^3 + 4y^3)\, j$ represents a force field.

   Determine if $\int_C F.\,dr$ is independent of path if it is, find a potential function ϕ .

5. Let $F(x, y) = 2\,x\,y^3\; i + (1 + 3x^2 y^2)\; j$

   (a) Show that F is a Conservative Vector field on the entire xy – plane ,

   (b) find f by first integrating $\dfrac{\partial f}{\partial x}$,

   (c) find f by first integrating $\dfrac{\partial f}{\partial y}$.

6. Use the potential function obtained in example (5) to evaluate the integral

$$\int_{(1,4)}^{(3,1)} 2xy^3\,dx + (1 + 3x^2 y^2)\,dy .$$

From Book :- (Calculus, Howard Anton 10th edition, soft copy)

Exercise set 15.3 - (1-6), (9-14)

EXERCISE SET 15.3 $\boxed{C}$ CAS

1–6 Determine whether $\mathbf{F}$ is a conservative vector field. If so, find a potential function for it.

1. $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$

2. $\mathbf{F}(x, y) = 3y^2\mathbf{i} + 6xy\mathbf{j}$

3. $\mathbf{F}(x, y) = x^2y\mathbf{i} + 5xy^2\mathbf{j}$

4. $\mathbf{F}(x, y) = e^x \cos y\mathbf{i} - e^x \sin y\mathbf{j}$

5. $\mathbf{F}(x, y) = (\cos y + y\cos x)\mathbf{i} + (\sin x - x\sin y)\mathbf{j}$

6. $\mathbf{F}(x, y) = x\ln y\mathbf{i} + y\ln x\mathbf{j}$

7. In each part, evaluate $\int_C 2xy^3\,dx + (1 + 3x^2y^2)\,dy$ over the curve C, and compare your answer with the result of Example 5.

(a) C is the line segment from $(1, 4)$ to $(3, 1)$.

(b) C consists of the line segment from $(1, 4)$ to $(1, 1)$, followed by the line segment from $(1, 1)$ to $(3, 1)$.

8. (a) Show that the line integral $\int_C y\sin x\,dx - \cos x\,dy$ is independent of the path.

(b) Evaluate the integral in part (a) along the line segment from $(0, 1)$ to $(\pi, -1)$.

(c) Evaluate the integral $\int_{(0,1)}^{(\pi,-1)} y\sin x\,dx - \cos x\,dy$ using Theorem 15.3.1, and confirm that the value is the same as that obtained in part (b).

9–14 Show that the integral is independent of the path, and use Theorem 15.3.1 to find its value.

9. $\displaystyle\int_{(1,2)}^{(4,0)} 3y\,dx + 3x\,dy$

10. $\displaystyle\int_{(0,0)}^{(1,\pi/2)} e^x \sin y\,dx + e^x \cos y\,dy$

11. $\displaystyle\int_{(0,0)}^{(3,2)} 2xe^y\,dx + x^2e^y\,dy$

12. $\displaystyle\int_{(-1,2)}^{(0,1)} (3x - y + 1)\,dx - (x + 4y + 2)\,dy$

13. $\displaystyle\int_{(2,-2)}^{(-1,0)} 2xy^3\,dx + 3y^2x^2\,dy$

14. $\displaystyle\int_{(1,1)}^{(3,3)} \left(e^x \ln y - \frac{e^y}{x}\right)dx + \left(\frac{e^x}{y} - e^y \ln x\right)dy$, where x and y are positive.

Green's theorem

Exercise set 15.4 - 1-14

EXERCISE SET 15.4 [C] CAS

1–2 Evaluate the line integral using Green's Theorem and check the answer by evaluating it directly. ■

1. $\oint_C y^2\,dx + x^2\,dy$, where C is the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$ oriented counterclockwise.

2. $\oint_C y\,dx + x\,dy$, where C is the unit circle oriented counterclockwise.

3–13 Use Green's Theorem to evaluate the integral. In each exercise, assume that the curve C is oriented counterclockwise. ■

3. $\oint_C 3xy\,dx + 2xy\,dy$, where C is the rectangle bounded by $x = -2$, $x = 4$, $y = 1$, and $y = 2$.

4. $\oint_C (x^2 - y^2)\,dx + x\,dy$, where C is the circle $x^2 + y^2 = 9$.

5. $\oint_C x\cos y\,dx - y\sin x\,dy$, where C is the square with vertices $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$, and $(0, \pi/2)$.

6. $\oint_C y\tan^2 x\,dx + \tan x\,dy$, where C is the circle $x^2 + (y+1)^2 = 1$.

7. $\oint_C (x^2 - y)\,dx + x\,dy$, where C is the circle $x^2 + y^2 = 4$.

8. $\oint_C (e^x + y^2)\,dx + (e^y + x^2)\,dy$, where C is the boundary of the region between $y = x^2$ and $y = x$.

9. $\oint_C \ln(1+y)\,dx - \frac{xy}{1+y}\,dy$, where C is the triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 4)$.

10. $\oint_C x^2 y\,dx - y^2 x\,dy$, where C is the boundary of the region in the first quadrant, enclosed between the coordinate axes and the circle $x^2 + y^2 = 16$.

11. $\oint_C \tan^{-1} y\,dx - \frac{y^2 x}{1+y^2}\,dy$, where C is the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$.

12. $\oint_C \cos x \sin y\,dx + \sin x \cos y\,dy$, where C is the triangle with vertices $(0, 0)$, $(3, 3)$, and $(0, 3)$.

13. $\oint_C x^2 y\,dx + (y + xy^2)\,dy$, where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$.

14. Let C be the boundary of the region enclosed between $y = x^2$ and $y = 2x$. Assuming that C is oriented counterclockwise, evaluate the following integrals by Green's Theorem:

    (a) $\oint_C (6xy - y^2)\,dx$ (b) $\oint_C (6xy - y^2)\,dy$.