Fourier Series and application

1.(a) Determine the Fourier series for

$$f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases},$$

Period = 8

(b) Find the Fourier coefficients for

$$f(x) = \begin{cases} 0, & -5 < x < 0 \\ 3, & 0 < x < 5 \end{cases}$$

Period = 10 .

2. Expand $f(x) = x$, $0 < x < 2$ in a half – range $f(x) = \begin{cases} x, & 0 < x < 4 \\ 8 - x, & 4 < x < 8 \end{cases}$

(i) Sine series   (ii) Cosine series.

3. Expand $f(x) = \begin{cases} \dfrac{1}{4} - x, & 0 < x < \dfrac{1}{2} \\ x - \dfrac{3}{4}, & \dfrac{1}{2} < x < 1 \end{cases}$,   in a Fourier series of Sine terms only.

4. Graph each of the following functions and find its corresponding Fourier series, using properties of even and odd function wherever applicable.

(a) $f(x) = \begin{cases} 8, & 0 < x < 2 \\ -8, & 2 < x < 4 \end{cases}$, *Period* 4

(b) $f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases}$, *Period* 8

(c) $f(x) = 4x$, $0 < x < 10$, *Period* 10

(d) $f(x) = \begin{cases} 2x, & 0 \le x \le 3 \\ x, & -3 \le x \le 4 \end{cases}$, *Period* 6

5. Expand $f(x) = \cos x$, $0 < x < \pi$ in a Fourier sine series.

6. Expand $f(x) = \begin{cases} x, & 0 < x < 4 \\ 8 - x, & 4 < x < 8 \end{cases}$ in (a) Sine series (b) Cosine series.

7. Solve the following boundary value problem

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2} \qquad u(0,t) = u(4,t) = 0 \qquad u(x,0) = 25x, \quad where\ 0 < x < 4, t > 0$$

8. Show that the solution of the boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad u_x(0,t) = u_x(\pi,t) = 0 \qquad u(x,0) = f(x), \quad where\ 0 < x < \pi, t > 0$$

is $u(x,t) = \dfrac{1}{\pi} \int_0^\pi f(x) dx + \dfrac{2}{\pi} \sum_{m=1}^\infty e^{-m^2 t} \cos mx \int_0^\pi f(x) \cos mx dx$