# CSE 221: Algorithms

## Introduction to graphs

Mumit Khan

Computer Science and Engineering
BRAC University

**References**

1. Jon Kleinberg and Éva Tardos, *Algorithm Design*. Pearson Education, 2006.
2. T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms, Second Edition*. The MIT Press, September 2001.
3. M. Goodrich and R. Tamassia, *Algorithm Design*. John-Wiley and Sons. 2002.

Last modified: September 10, 2009

# Contents

# Contents

# What is a Graph?

- **Graph** $G$ is a pair $(V, E)$, where $V$ is a finite set (set of vertices) and $E$ is a finite set of pairs from $V$ (set of edges). We will often denote $n = |V|$, $m = |E|$.
- Graph $G$ can be **directed**, if $E$ consists of ordered pairs, or **undirected**, if $E$ consists of unordered pairs. If $(u, v) \in E$, then vertices $u$ and $v$ are **adjacent.**
- We can assign weight function to the edges: $w_G(e)$ is a weight of edge $e \in E$. The graph which has such function assigned is called **weighted**.
- **Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

# Examples of graphs

**Transportation networks** How should you design the highway network in a country? What is the quickest way to drive from Badda to Narayngan j?

**Communication networks** How to send a network packet from Bracu intranet to Yahoo mail server?

**Information networks** Is the World wide a directed or undirected network?

**Social networks** Facebook, Twitter, Instagram, . . . .

**Dependency networks** What courses must you take before you can take CSE-423?

# Examples of graphs

**Transportation networks** How should you design the highway network in a country? What is the quickest way to drive from Badda to Narayngan j?

**Communication networks** How to send a network packet from Bracu intranet to Yahoo mail server?

**Information networks** Is the World wide a directed or undirected network?

**Social networks** Facebook, Twitter, Instagram, ....

**Dependency networks** What courses must you take before you can take CSE-423?

# Examples of graphs

**Transportation networks** How should you design the highway network in a country? What is the quickest way to drive from Badda to Narayngan j?

**Communication networks** How to send a network packet from Bracu intranet to Yahoo mail server?

**Information networks** Is the World wide a directed or undirected network?

**Social networks** Facebook, Twitter, Instagram, ....

**Dependency networks** What courses must you take before you can take CSE-423?

# Examples of graphs

Transportation networks How should you design the highway network in a country? What is the quickest way to drive from Badda to Narayngani?

Communication networks How to send a network packet from Bracu intranet to Yahoo mail server?

Information networks Is the World wide a directed or undirected network?

Social networks Facebook, Twitter, Instagram, . . . .

Dependency networks What courses must you take before you can take CSE-423?

# Examples of graphs

**Transportation networks** How should you design the highway network in a country? What is the quickest way to drive from Badda to Narayngani?

**Communication networks** How to send a network packet from Bracu intranet to Yahoo mail server?

**Information networks** Is the World wide a directed or undirected network?

**Social networks** Facebook, Twitter, Instagram, ....

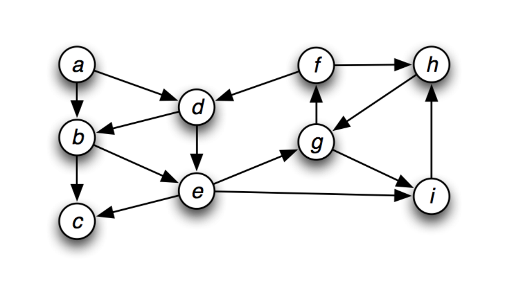
**Dependency networks** What courses must you take before you can take CSE-423?

# What is a Graph?

## Definition (Graph)

**Graph** $G$ is a pair $(V, E)$, where $V$ is a finite set (set of vertices) and $E$ is a finite set of pairs from $V$ (set of edges). We will often denote $n = |V|$, $m = |E|$.

## Example

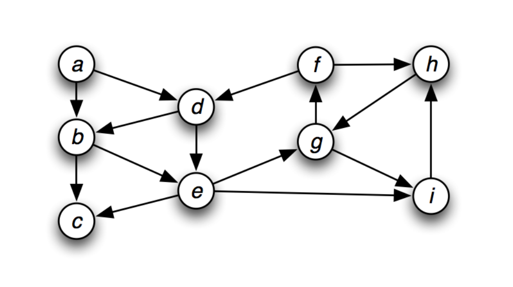


# What is a Graph?

## Definition (Graph)

**Graph** $G$ is a pair $(V, E)$, where $V$ is a finite set (set of vertices) and $E$ is a finite set of pairs from $V$ (set of edges). We will often denote $n = |V|$, $m = |E|$.

## Example



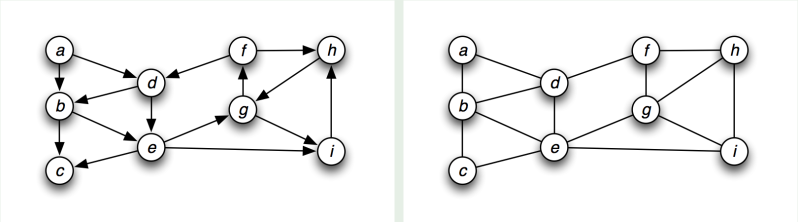

$\boldsymbol{V = \{a, \ldots, i\}, n = 9, m = 15}$.

# Directed vs. undirected graphs

## Definition (Directed vs. undirected graph)

Graph $G$ can be **directed**, if $E$ consists of ordered pairs, or **undirected**, if $E$ consists of unordered pairs. If $(u, v) \in E$, then vertices $u$ and $v$ are **adjacent.**

## Example



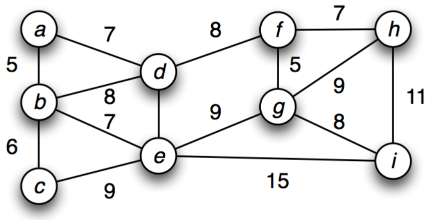

  

# Weighted graphs

## Definition (Weighted graph)

We can assign weight function to the edges: $w_G(e)$ is a weight of edge $e \in E$. The graph which has such function assigned is called **weighted**.

## Example

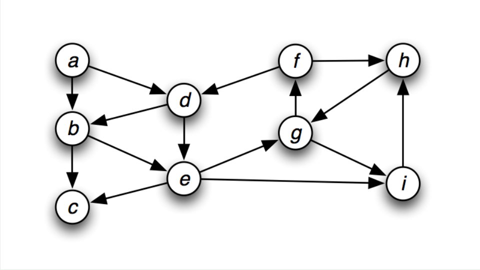


# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Directed graph)



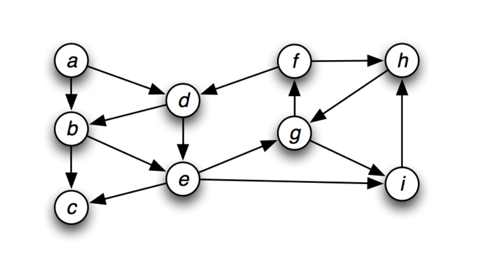

| Vertex | in | out |
|---|---|---|

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Directed graph)



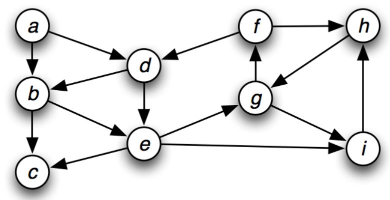

| Vertex | in | out |
|---|---|---|
| $a$ | 0 | 2 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Directed graph)



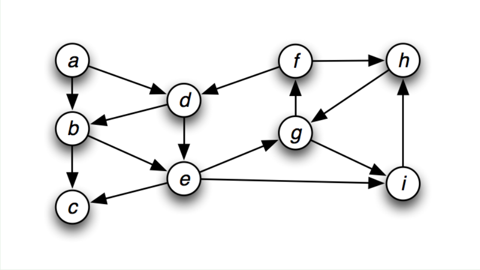

| Vertex | in | out |
|---|---|---|
| $a$ | 0 | 2 |
| $b$ | 2 | 2 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Directed graph)



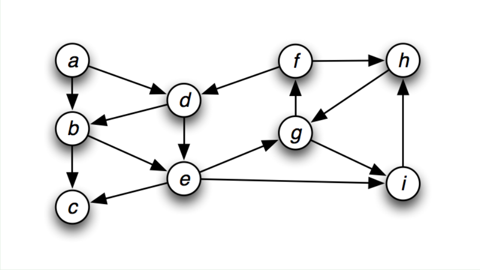

| Vertex | in | out |
|---|---|---|
| $a$ | 0 | 2 |
| $b$ | 2 | 2 |
| $c$ | 2 | 0 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Directed graph)




| Vertex | in | out |
|---|---|---|
| $a$ | 0 | 2 |
| $b$ | 2 | 2 |
| $c$ | 2 | 0 |
| $d$ | 2 | 2 |

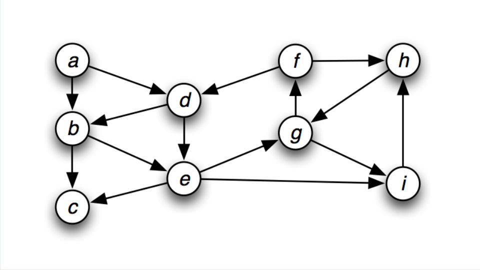
  

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Directed graph)



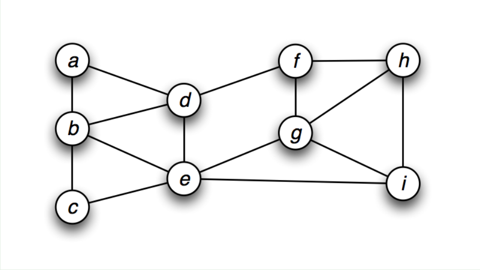

| Vertex | in | out |
|---|---|---|
| $a$ | 0 | 2 |
| $b$ | 2 | 2 |
| $c$ | 2 | 0 |
| $d$ | 2 | 2 |
| $e$ | 2 | 3 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Undirected graph)



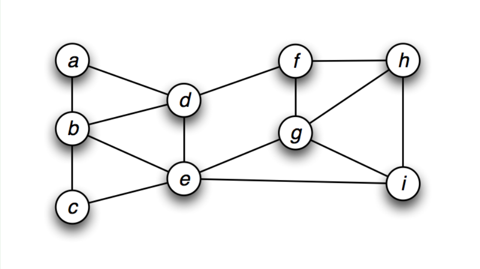

| Vertex | *deg* |
|---|---|

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Undirected graph)



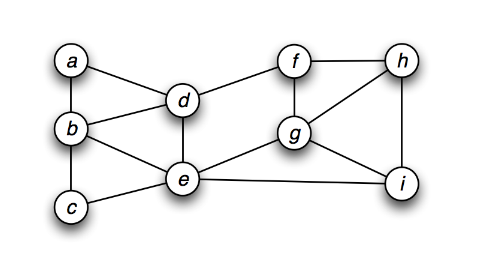

| Vertex | $deg$ |
|---|---|
| $a$ | 2 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Undirected graph)



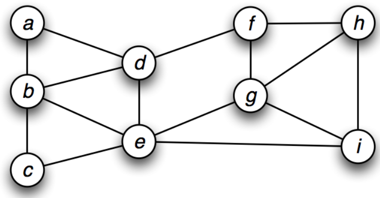

| Vertex | *deg* |
|---|---|
| *a* | 2 |
| *b* | 4 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Undirected graph)



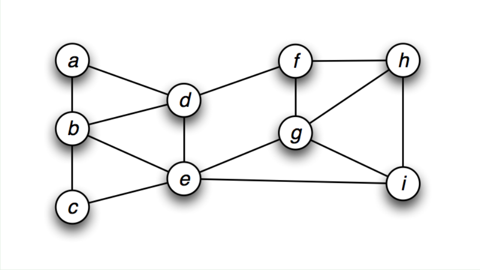

| Vertex | *deg* |
|---|---|
| $a$ | 2 |
| $b$ | 4 |
| $c$ | 2 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Undirected graph)



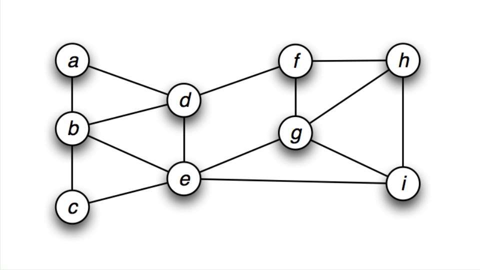

| Vertex | *deg* |
|---|---|
| *a* | 2 |
| *b* | 4 |
| *c* | 2 |
| *d* | 4 |

# Degree of vertices

## Definition (Degree of vertices)

**Degree** of a vertex $v$ is the number of vertices $u$ for which $(u, v) \in E$ or $(v, u) \in E$ (denote $deg(v)$). The number of **incoming edges** to a vertex $v$ is called **in-degree** of the vertex (denote $indeg(v)$). The number of **outgoing edges** from a vertex is called **out-degree** (denote $outdeg(v)$).

## Example (Undirected graph)




| Vertex | *deg* |
|---|---|
| $a$ | 2 |
| $b$ | 4 |
| $c$ | 2 |
| $d$ | 4 |
| $e$ | 5 |

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a graph with $m$ edges, then*

$$\sum_{v \in G} deg(v) = 2m.$$

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a graph with $m$ edges, then*

$$\sum_{v \in G} deg(v) = 2m.$$

**Proof.**

An edge $(u, v)$ is counted twice in the above summation; once by its endpoint $u$ and once by its endpoint $v$. Thus, the total contribution of the edges to the degrees of the vertices is twice the number of edges. □

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a directed graph (digraph) with $m$ edges, then*

$$\sum_{v \in G} indeg(v) = \sum_{v \in G} outdeg(v) = m.$$

# Relationship of $n = |V|$ and $m = |E|$

## Theorem

*If G is a directed graph (digraph) with m edges, then*

$$\sum_{v \in G} indeg(v) = \sum_{v \in G} outdeg(v) = m.$$

## Proof.

In a directed graph, an edge $(u, v)$ contributes one unit to the **out-degree** of its origin $u$ and one unit to the **in-degree** of its destination $v$. Thus, the total contribution of the edges to the out-degrees of the vertices is equal to the number of edges, and similarly for the in-degrees. □

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a simple* ***undirected*** *graph with $n$ vertices and $m$ edges, then $m \leq n(n-1)/2$.*

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a simple **undirected** graph with $n$ vertices and $m$ edges, then $m \leq n(n-1)/2$.*

**Proof.**

Since $G$ is a simple undirected graph without self-loops, the maximum degree of a vertex is $n - 1$. And, since there are $n$ vertices, and each edge is counted twice (once for each end of an edge $(u, v)$, the maximum number of edges is $n(n-1)/2$. Thus, $m \leq n(n-1)/2$, or $\boxed{m = \Theta(n^2).}$ □

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a simple* ***directed*** *graph with $n$ vertices and $m$ edges, then $m \leq n(n-1)$.*

# Relationship of $n = |V|$ and $m = |E|$

**Theorem**

*If $G$ is a simple **directed** graph with $n$ vertices and $m$ edges, then $m \leq n(n-1)$.*

**Proof.**

Since $G$ is a simple graph without self-loops, the maximum in-degree of a vertex is $n - 1$. And, since there are $n$ vertices, and no two edges can have the same origin and destination, $m \leq n(n-1)$, or $\boxed{m = \Theta(n^2).}$ □

# Paths and connectivity

### Definition (Path)

A **path** in a undirected graph $G = (V, E)$ is sequence $P$ of vertices $v1, v_2, \ldots, v_{k-1}, v_k$ with the property that for $i \in \{1, \ldots, k-1\}$, $(v_i, v_{i+1}) \in E$.

# Paths and connectivity

## Definition (Path)

A **path** in a undirected graph $G = (V, E)$ is sequence $P$ of vertices $v1, v_2, \ldots, v_{k-1}, v_k$ with the property that for $i \in \{1, \ldots, k-1\}$, $(v_i, v_{i+1}) \in E$.

## Definition (Simple and cyclic paths)

A path is **simple** if all vertices are distinct. A **cycle** is a path $v_1, v_2, \ldots, v_{k-1}, v_k$ in which $k \geq 2$, and the first $k-1$ nodes are all distinct, and $v_1 = v_k$.

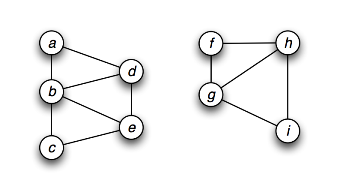
  

# Connected vs. unconnected graphs

## Definition (Connected graph)

The undirected graph $G$ is **connected**, if for every pair of vertices $u, v$ there exists a **path** from $u$ to $v$.
If a graph is not connected, the vertices of the graph can be divided into **connected components**. Two vertices are in the same **connected component** *iff* they are connected by a **path**.

## Example (Connected components)

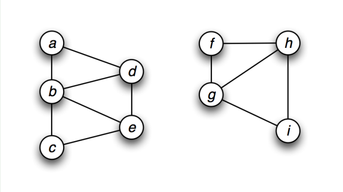


# Connected vs. unconnected graphs

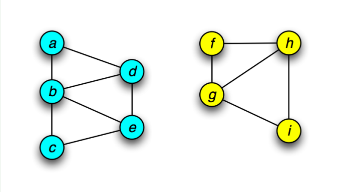
## Definition (Connected graph)

The undirected graph $G$ is **connected**, if for every pair of vertices $u, v$ there exists a **path** from $u$ to $v$.
If a graph is not connected, the vertices of the graph can be divided into **connected components**. Two vertices are in the same **connected component** *iff* they are connected by a **path**.

## Example (Connected components)




$\Longrightarrow$



# Trees

**Definition (Trees)**

A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n - 1$ edges).

# Trees

## Definition (Trees)

A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n - 1$ edges).

## Theorem

*For undirected graph $G$, any two of the following implies the third.*

1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n - 1$ edges.*

# Trees

## Definition (Trees)

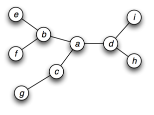
A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n - 1$ edges).

## Theorem

*For undirected graph $G$, any two of the following implies the third.*

1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n - 1$ edges.*

## Example (Trees (or *not*))



# Trees

## Definition (Trees)

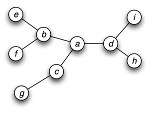
A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n-1$ edges).

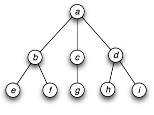
## Theorem

*For undirected graph $G$, any two of the following implies the third.*

1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n-1$ edges.*

## Example (Trees (or *not*))



# Trees

## Definition (Trees)

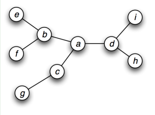
A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n-1$ edges).

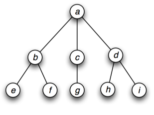
## Theorem

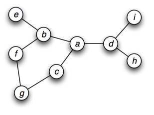
*For undirected graph $G$, any two of the following implies the third.*

1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n-1$ edges.*

## Example (Trees (or *not*))



# Trees

## Definition (Trees)

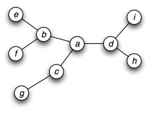
A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n-1$ edges).

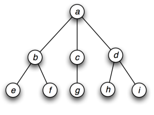
## Theorem

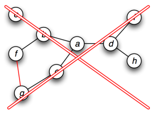
*For undirected graph $G$, any two of the following implies the third.*

1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n-1$ edges.*

## Example (Trees (or *not*))



# Trees

## Definition (Trees)

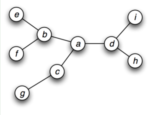
A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n-1$ edges).

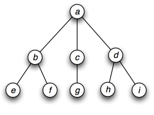
## Theorem

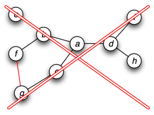
*For undirected graph $G$, any two of the following implies the third.*

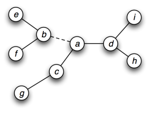
1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n-1$ edges.*

## Example (Trees (or *not*))



# Trees

## Definition (Trees)

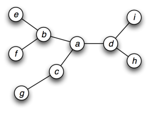
A undirected graph is a *tree* if it is *connected* and does not contain a *cycle* (implies that every $n$-node tree has exactly $n-1$ edges).

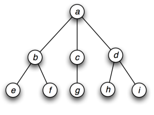
## Theorem

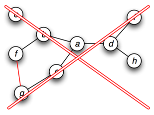
*For undirected graph $G$, any two of the following implies the third.*

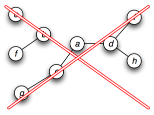
1. *$G$ is connected.*
2. *$G$ does not contain a cycle.*
3. *$G$ has $n-1$ edges.*

## Example (Trees (or *not*))



# Representation of graphs

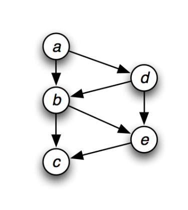
## Definition (Adjacency Matrix)

Represents the graph as an $n \times n$ matrix $A = (a_{i,j})$, where

$$a_{i,j} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E, \\ 0, & \text{otherwise.} \end{cases}$$

The matrix is symmetric in case of undirected graph, while it may be asymmetric if the graph is directed.

## Example (Directed graph)




| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | 0 | 1 | 0 | 1 | 0 |
| $b$ | 0 | 0 | 1 | 0 | 1 |
| $c$ | 0 | 0 | 0 | 0 | 0 |
| $d$ | 0 | 1 | 0 | 0 | 1 |
| $e$ | 0 | 0 | 1 | 0 | 0 |

# Representation of graphs

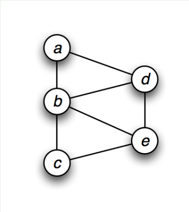
## Definition (Adjacency Matrix)

Represents the graph as an $n \times n$ matrix $A = (a_{i,j})$, where

$$a_{i,j} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E, \\ 0, & \text{otherwise.} \end{cases}$$

The matrix is symmetric in case of undirected graph, while it may be asymmetric if the graph is directed.

## Example (Undirected graph)



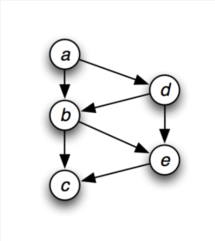

| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | 0 | 1 | 0 | 1 | 0 |
| $b$ | 1 | 0 | 1 | 1 | 1 |
| $c$ | 0 | 1 | 0 | 0 | 1 |
| $d$ | 1 | 1 | 0 | 0 | 1 |
| $e$ | 0 | 1 | 1 | 1 | 0 |

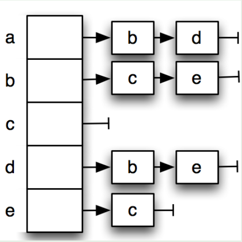
# Representation of graphs

## Definition (Adjacency List)

Represent the graph by listing for each vertex $v_i$ its outgoing vertices in a list $out(v_i)$. (Representation can be linked list, or another appropriate structure.)

## Example (Directed graph)



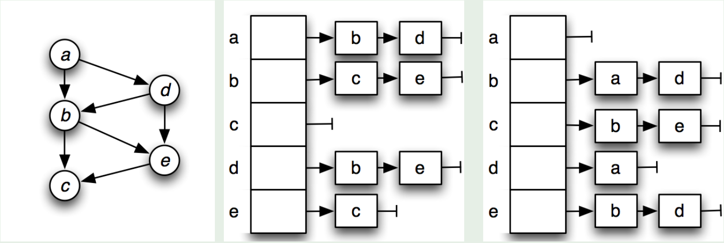
# Representation of graphs

## Definition (Adjacency List)

Represent the graph by listing for each vertex $v_i$ its outgoing vertices in a list $out(v_i)$. (Representation can be linked list, or another appropriate structure.)

If the graph is directed, it makes sense to build for each vertex $v_i$ also list of its incoming vertices $in(v_i)$.

## Example (Directed graph)



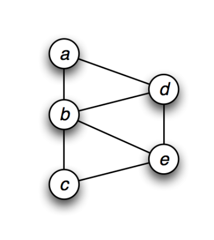
# Representation of graphs

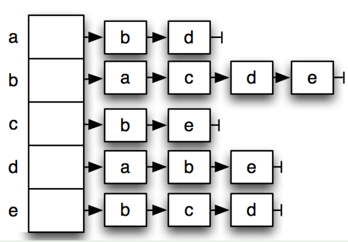
## Definition (Adjacency List)

Represent the graph by listing for each vertex $v_i$ its outgoing vertices in a list $out(v_i)$. (Representation can be linked list, or another appropriate structure.)

If the graph is undirected, all incident edges are listed for each vertex $v_i$.

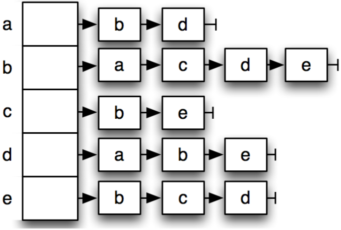
## Example (Undirected graph)



# Comparison of graph representations

| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | 0 | 1 | 0 | 1 | 0 |
| $b$ | 1 | 0 | 1 | 1 | 1 |
| $c$ | 0 | 1 | 0 | 0 | 1 |
| $d$ | 1 | 1 | 0 | 0 | 1 |
| $e$ | 0 | 1 | 1 | 1 | 0 |

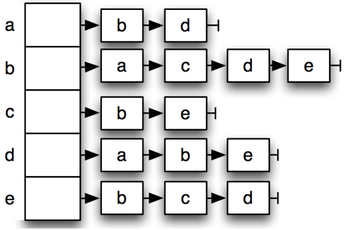



| Operation | Adjacency matrix | Adjacency list |
|---|---|---|
| Is $(u, v) \in E$? | $\Theta(1)$ | $\Theta(outdeg(u))$ |
| List edges outgoing from $u$ | $\Theta(n)$ | $\Theta(outdeg(u))$ |
| Memory | $\Theta(n^2)$ | $\Theta(m + n)$ |

# Comparison of graph representations

| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | 0 | 1 | 0 | 1 | 0 |
| $b$ | 1 | 0 | 1 | 1 | 1 |
| $c$ | 0 | 1 | 0 | 0 | 1 |
| $d$ | 1 | 1 | 0 | 0 | 1 |
| $e$ | 0 | 1 | 1 | 1 | 0 |




| Operation | Adjacency matrix | Adjacency list |
|---|---|---|
| Is $(u, v) \in E$? | $\Theta(1)$ | $\Theta(outdeg(u))$ |
| List edges outgoing from $u$ | $\Theta(n)$ | $\Theta(outdeg(u))$ |
| Memory | $\Theta(n^2)$ | $\Theta(m + n)$ |

**Representation note**

We will be using adjacency lists to represent graphs, unless stated otherwise.

# Some very basic questions for a graph

1. Starting from a given *source* vertex, can I reach a *target* vertex? *s-t connectivity problem*.

# Some very basic questions for a graph

1. Starting from a given *source* vertex, can I reach a *target* vertex? *s-t connectivity problem*.
2. Starting from a given *source* vertex, how can I visit all the other vertices that have a path from it?

# Some very basic questions for a graph

1. Starting from a given *source* vertex, can I reach a *target* vertex? *s-t connectivity problem*.
2. Starting from a given *source* vertex, how can I visit all the other vertices that have a path from it?
3. Is the graph *connected*? If not, what are the *connected components*?

# Some very basic questions for a graph

1. Starting from a given *source* vertex, can I reach a *target* vertex? *s-t connectivity problem*.
2. Starting from a given *source* vertex, how can I visit all the other vertices that have a path from it?
3. Is the graph *connected*? If not, what are the *connected components*?
4. Is there a cycle in this graph?

# Some very basic questions for a graph

1. Starting from a given *source* vertex, can I reach a *target* vertex? *s-t connectivity problem*.
2. Starting from a given *source* vertex, how can I visit all the other vertices that have a path from it?
3. Is the graph *connected*? If not, what are the *connected components*?
4. Is there a cycle in this graph?
5. Given a *source* vertex in weighted graph, what is the *shortest distance* to all the other vertices? *Single-source shortest path problem*.

# Some very basic questions for a graph

1. Starting from a given *source* vertex, can I reach a *target* vertex? *s-t connectivity problem*.
2. Starting from a given *source* vertex, how can I visit all the other vertices that have a path from it?
3. Is the graph *connected*? If not, what are the *connected components*?
4. Is there a cycle in this graph?
5. Given a *source* vertex in weighted graph, what is the *shortest distance* to all the other vertices? *Single-source shortest path problem*.
6. How to compute the *minimum spanning tree*?

# Contents

# Graph traversal techniques

## The graph traversal problem

Given a graph $G = (V, E)$, and a *distinguished vertex* $s$, how do you visit each vertex $v \in V$ exactly once.

# Graph traversal techniques

## The graph traversal problem

Given a graph $G = (V, E)$, and a *distinguished vertex* $s$, how do you visit each vertex $v \in V$ exactly once.

## Breadth-first search (BFS)

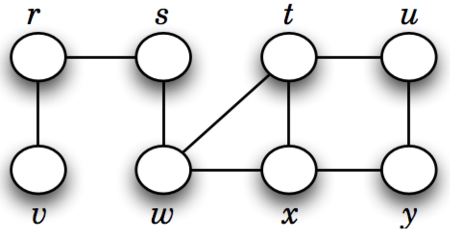
- Start with the source vertex, and *fan out* from there.
- Finds the distance (in terms of edges) of each node from the source.

## Depth-first search (DFS)

- Start with the source vertex, *recursively* visit each neighbor, only backtracking when choices are exhausted.
- Remember *pre-order* tree traversal?

# Basic idea behind breadth first search (BFS)

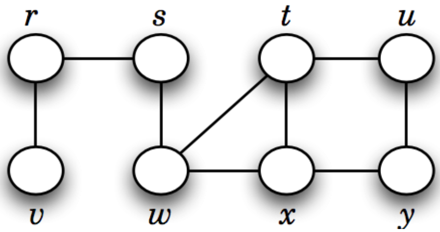
Given a undirected graph $G = (V, E)$, and a distinguished vertex $s$, systematically visit each vertex $v \in V$ that is reachable from $s$.



# Breadth first search (BFS)

$\mathrm{BFS}(G, s) \qquad \triangleright\ G = (V, E)$

```
Q ← {s}
d[v] ← ∞, ∀v ∈ V; d[s] ← 0
while Q ≠ Ø
      do u ← DEQUEUE(Q)
         for each v ∈ Adj[u]
             do if d[v] = ∞
                   then d[v] = d[u] + 1
                        ENQEUEUE(Q, v)
```



# Breadth first search (BFS)

$\mathrm{BFS}(G, s) \qquad \triangleright\ G = (V, E)$

$Q \leftarrow \{s\}$

$d[v] \leftarrow \infty, \forall v \in V;\ d[s] \leftarrow 0$

**while** $Q \neq \emptyset$

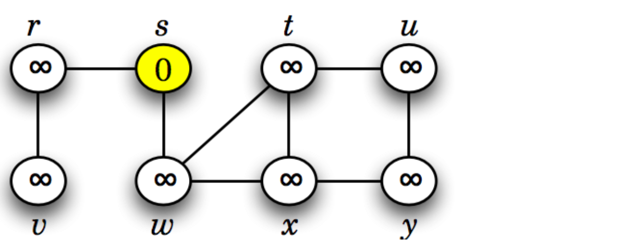
    **do** $u \leftarrow \textsc{Dequeue}(Q)$

        **for** each $v \in Adj[u]$

            **do if** $d[v] = \infty$

                **then** $d[v] = d[u] + 1$

                    $\textsc{Enqeueue}(Q, v)$



# Breadth first search (BFS)

$\text{BFS}(G, s)$ $\quad \triangleright\ G = (V, E)$

$Q \leftarrow \{s\}$

$d[v] \leftarrow \infty, \forall v \in V;\ d[s] \leftarrow 0$

**while** $Q \neq \emptyset$

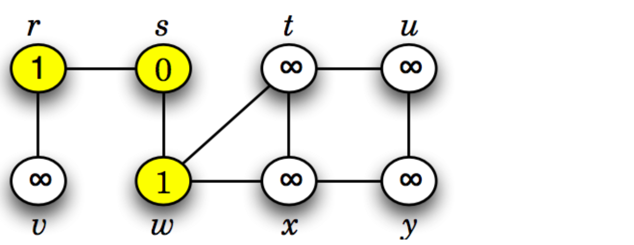
    **do** $u \leftarrow \text{DEQUEUE}(Q)$

        **for** each $v \in Adj[u]$

            **do if** $d[v] = \infty$

                **then** $d[v] = d[u] + 1$

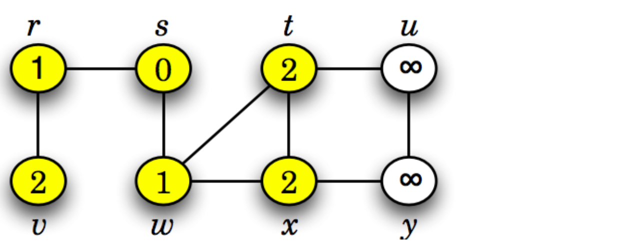
                    $\text{ENQEUEUE}(Q, v)$



# Breadth first search (BFS)

$\text{BFS}(G, s) \qquad \triangleright\ G = (V, E)$

```
Q ← {s}
d[v] ← ∞, ∀v ∈ V; d[s] ← 0
while Q ≠ ∅
    do u ← DEQUEUE(Q)
       for each v ∈ Adj[u]
           do if d[v] = ∞
                 then d[v] = d[u] + 1
                      ENQEUEUE(Q, v)
```



# Breadth first search (BFS)

$\mathrm{BFS}(G, s) \qquad \triangleright\ G = (V, E)$

$Q \leftarrow \{s\}$

$d[v] \leftarrow \infty, \forall v \in V;\ d[s] \leftarrow 0$

**while** $Q \neq \emptyset$

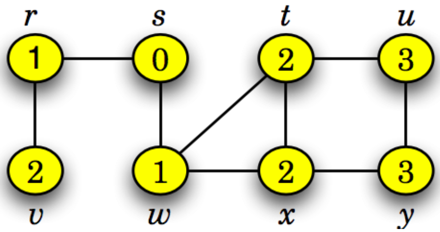
  **do** $u \leftarrow \textsc{Dequeue}(Q)$

    **for** each $v \in Adj[u]$

      **do if** $d[v] = \infty$

        **then** $d[v] = d[u] + 1$

          $\textsc{Enqueue}(Q, v)$



# Breadth first search (BFS)

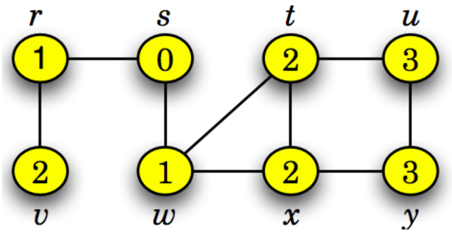
$\mathrm{BFS}(G, s) \qquad \triangleright\ G = (V, E)$

$Q \leftarrow \{s\}$
$d[v] \leftarrow \infty, \forall v \in V;\ d[s] \leftarrow 0$
**while** $Q \neq \emptyset$
  **do** $u \leftarrow \textsc{Dequeue}(Q)$
    **for** each $v \in Adj[u]$
      **do if** $d[v] = \infty$
        **then** $d[v] = d[u] + 1$
          $\textsc{Enqueue}(Q, v)$

## Analysis

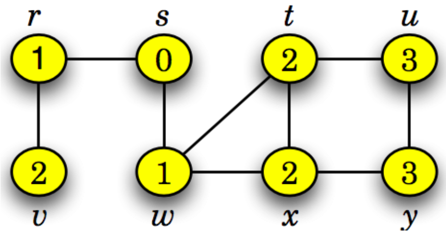
**Time** $= O(V + E)$.

# What does *this* $\mathrm{BFS}(G, s)$ algorithm tell us?



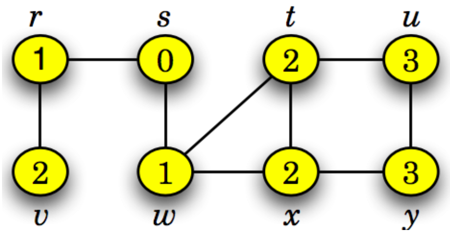

- This graph $G = (V, E)$ is connected since BFS visits all vertices $v \in V$ starting from $s$.

# What does *this* $\mathrm{BFS}(G, s)$ algorithm tell us?



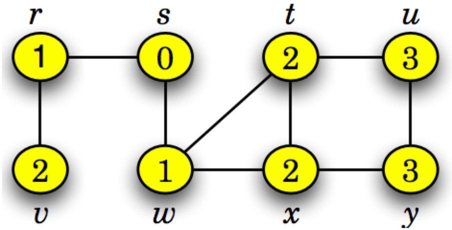

- This graph $G = (V, E)$ is connected since BFS visits all vertices $v \in V$ starting from $s$.
- The minimum number of *hops* or edges between $s$ and any vertex $t \in V$.

# What does *this* $\mathrm{BFS}(G, s)$ algorithm *not* tell us?



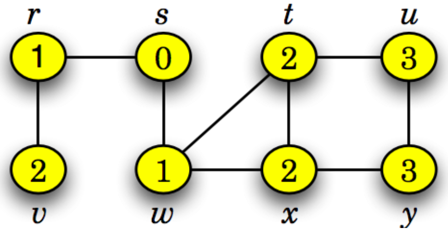

- Is there a cycle in this graph?

# What does *this* $\mathrm{BFS}(G, s)$ algorithm *not* tell us?



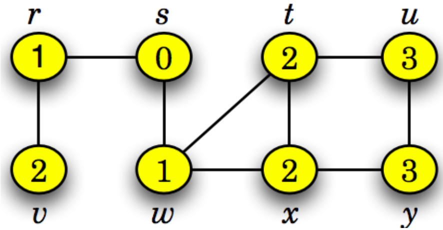

- Is there a cycle in this graph?
- Given a target vertex $t \in V$, what is the path with smallest number of edges back to $s$?

# What does *this* $\mathrm{BFS}(G, s)$ algorithm *not* tell us?




- Is there a cycle in this graph?
- Given a target vertex $t \in V$, what is the path with smallest number of edges back to $s$?
- If this were not a connected graph, how to visit all the vertices?

# What does *this* $\mathrm{BFS}(G, s)$ algorithm *not* tell us?




- Is there a cycle in this graph?
- Given a target vertex $t \in V$, what is the path with smallest number of edges back to $s$?
- If this were not a connected graph, how to visit all the vertices?
- Equivalently, how to compute all the connected components of a graph?

# Computing connected components

## Idea

Since each node is visited no more than once ($d[v] \neq \infty$ after the first visit), simply apply the $\mathrm{BFS}$ algorithm over each vertex $v \in V$, creating a new component each time a vertex is found for which $d[v] = \infty$.

# Computing connected components

## Idea

Since each node is visited no more than once ($d[v] \neq \infty$ after the first visit), simply apply the $\mathrm{BFS}$ algorithm over each vertex $v \in V$, creating a new component each time a vertex is found for which $d[v] = \infty$.

## $\mathrm{BFS}(G)$ $\quad \triangleright\ G = (V, E)$

$d[v] \leftarrow \infty, \forall v \in V$

**for** each $v \in V$

  **do if** $d[v] = \infty$

    **then** $\mathrm{BFS}(G, v)$

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. *color*[$v$]: the color of each vertex visited

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. *color*[*v*]: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing
2. $\pi[v]$: the predecessor pointer

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. *color*$[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing
2. $\pi[v]$: the predecessor pointer
3. $d[v]$: the discovery time (or *hops* from the start)

# A better Breadth first search (BFS) algorithm

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing
2. $\pi[v]$: the predecessor pointer
3. $d[v]$: the discovery time (or *hops* from the start)

```
BFS(G) ▷ G = (V, E)
    ▷ Initialize the vertex colors and predecessors
    for each vertex v ∈ V
        do color[v] = WHITE
           π[v] = NULL
    for each vertex v ∈ V
        do if color[v] = WHITE
              then BFS(G, v)
```

# A better Breadth first search (BFS) algorithm

$\mathrm{BFS}(G, s) \triangleright G = (V, E)$

1 $color[s] = \mathrm{GRAY}; \pi[s] = \mathrm{NULL}; d[s] = 0$

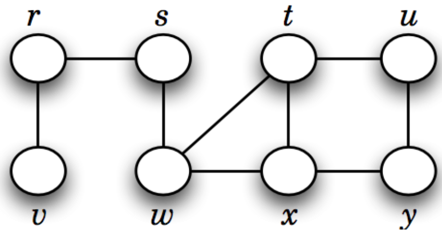
2 $Q = \emptyset$; $\textsc{Enqueue}(Q, s)$

# A better Breadth first search (BFS) algorithm

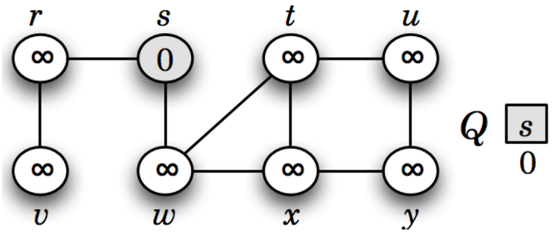
$\mathrm{BFS}(G, s) \triangleright G = (V, E)$

$color[s] = \mathrm{GRAY}; \pi[s] = \mathrm{NULL}; d[s] = 0$
$\mathrm{Q} = \emptyset$; $\textsc{Enqueue}(Q, s)$
**while** $Q \neq \emptyset$
    **do** $u = \textsc{Dequeue}(Q)$
        **for** each vertex $v \in Adj[u]$
            **do if** $color[v] = \mathrm{WHITE}$
                **then** $color[v] = \mathrm{GRAY}$
                    $d[v] = d[u] + 1$
                    $\pi[v] = u$
                    $\textsc{Enqueue}(Q, v)$
        $color[u] = \mathrm{BLACK}$

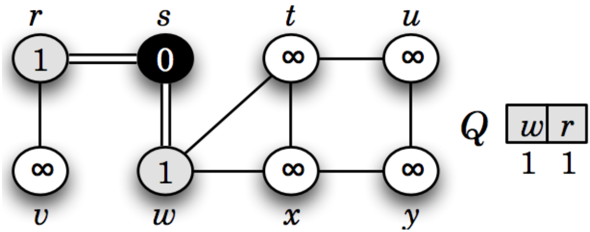
# Application of BFS



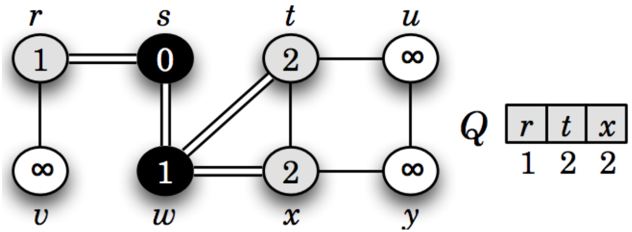
# Application of BFS



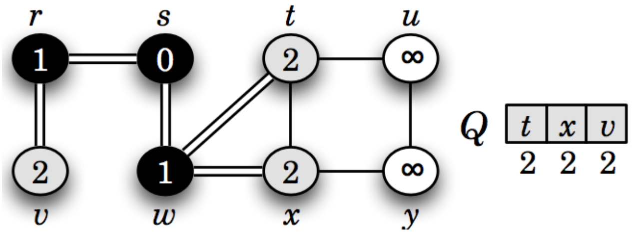
# Application of BFS



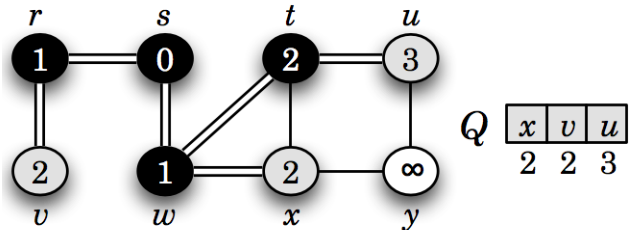
# Application of BFS



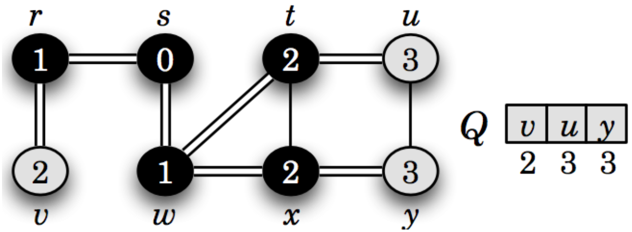
# Application of BFS



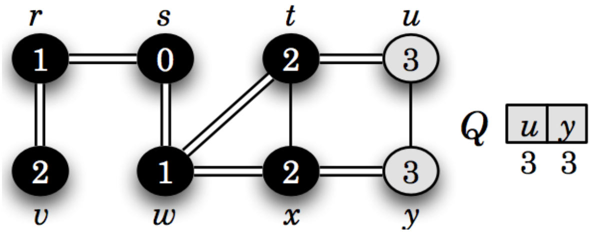
# Application of BFS



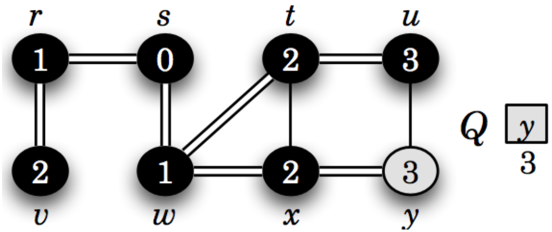
# Application of BFS



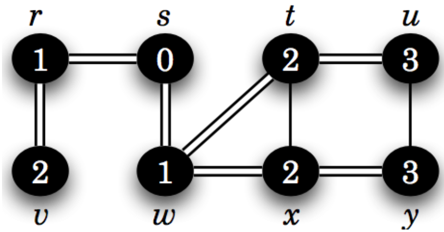
# Application of BFS



# Application of BFS

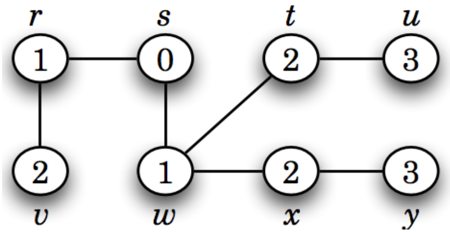


# Application of BFS

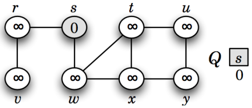



$Q \quad \emptyset$

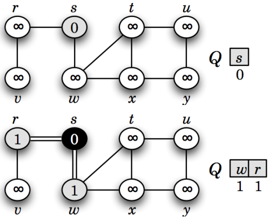
# Application of BFS



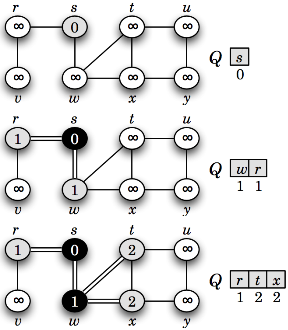
# Application of BFS



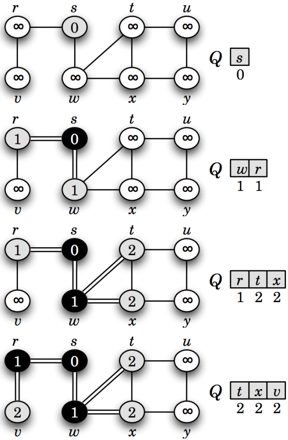
# Application of BFS



# Application of BFS

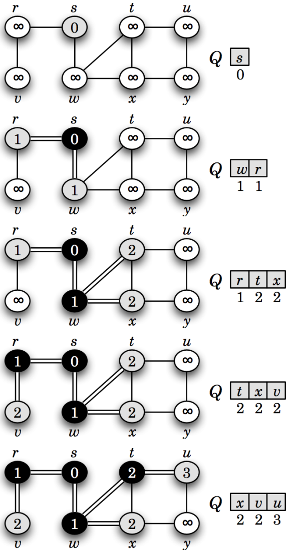


# Application of BFS

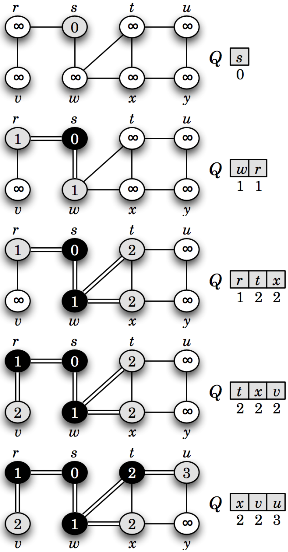


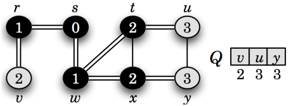

  

# Application of BFS

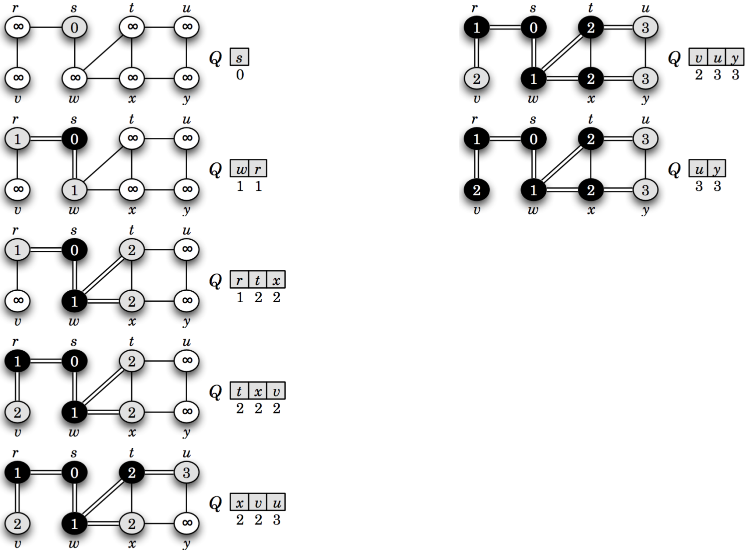


# Application of BFS

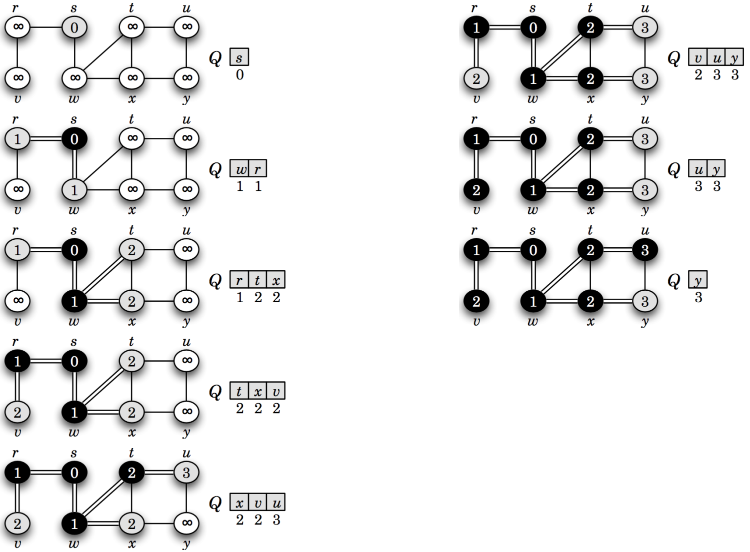


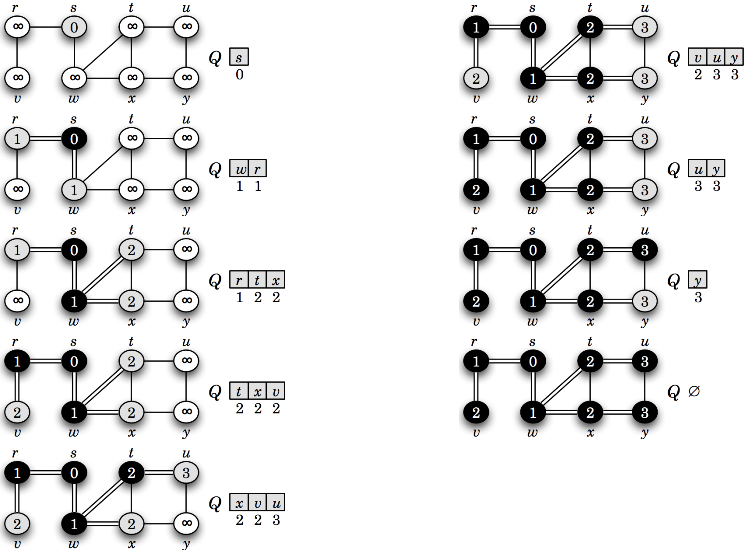
# Application of BFS



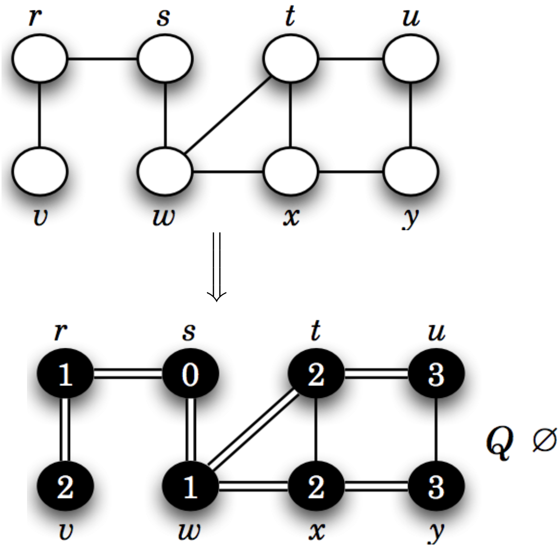
# Application of BFS



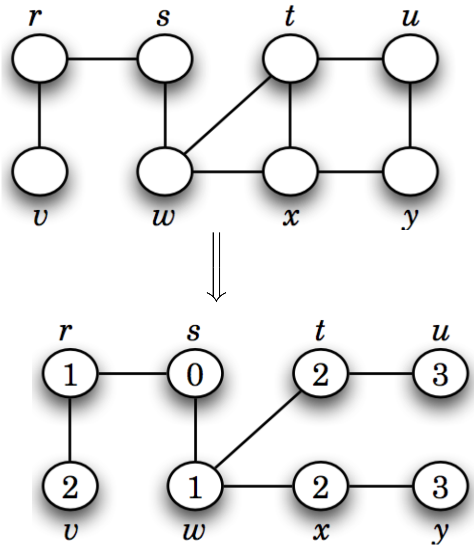
# Application of BFS



# Breadth first search (BFS) tree



# Breadth first search (BFS) tree



# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike $\mathrm{BFS}$ which fans out.

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike $\mathrm{BFS}$ which fans out.

## Information during traversal

1. $color[v]$: the color of each vertex visited

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike BFS which fans out.

## Information during traversal

1. *color*[*v*]: the color of each vertex visited
    - white: undiscovered

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike BFS which fans out.

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike $\mathrm{BFS}$ which fans out.

## Information during traversal

1. $color[v]$: the color of each vertex visited
    - white: undiscovered
    - gray: discovered but not finished processing
    - black: finished processing

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike BFS which fans out.

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing
2. $\pi[v]$: the predecessor pointer

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike $\mathrm{BFS}$ which fans out.

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing
2. $\pi[v]$: the predecessor pointer
3. $d[v]$: the discovery time (or *hops* from the start)

# Depth first search (DFS)

- Starts from an initial vertex.
- Recursively visits each adjacent vertex.
- Strategy to search deeper into the graph, unlike $\mathrm{BFS}$ which fans out.

## Information during traversal

1. $color[v]$: the color of each vertex visited
   - white: undiscovered
   - gray: discovered but not finished processing
   - black: finished processing
2. $\pi[v]$: the predecessor pointer
3. $d[v]$: the discovery time (or *hops* from the start)
4. $f[v]$: the finish time – when processing of $v$ and all its descendants have finished.

# DFS algorithm

$\mathrm{DFS}(G) \triangleright G = (V, E)$

- **for** each vertex $u \in V[G]$
    - **do** $color[u] \leftarrow \mathrm{WHITE}$
        - $\pi[u] \leftarrow \mathrm{NIL}$
- $time \leftarrow 0$
- **for** each vertex $u \in V[G]$
    - **do if** $color[u] = \mathrm{WHITE}$
        - **then** $\mathrm{DFS\text{-}VISIT}(\mathrm{G}, u)$

$\mathrm{DFS\text{-}VISIT}(G, u) \triangleright G = (V, E), u \in V$

- $color[u] \leftarrow \mathrm{GRAY}$
- $time \leftarrow time + 1$
- $d[u] \leftarrow time$
- **for** each vertex $v \in Adj[u]$
    - **do if** $color[v] = \mathrm{WHITE}$
        - **then** $\pi[v] \leftarrow u$
            - $\mathrm{DFS\text{-}VISIT}(\mathrm{G}, v)$
- $color[u] \leftarrow \mathrm{BLACK}$
- $f[u] \leftarrow time + 1$

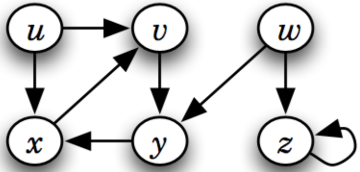
# Application of DFS

$\mathrm{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$<br>
    **do** $color[u] \leftarrow \text{WHITE}$<br>
        $\pi[u] \leftarrow \text{NIL}$<br>
$time \leftarrow 0$<br>
**for** each vertex $u \in V[G]$<br>
    **do if** $color[u] = \text{WHITE}$<br>
        **then** $\text{DFS-VISIT}(\mathrm{G}, u)$

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow \text{GRAY}$<br>
$time \leftarrow time + 1$<br>
$d[u] \leftarrow time$<br>
**for** each vertex $v \in Adj[u]$<br>
    **do if** $color[v] = \text{WHITE}$<br>
        **then** $\pi[v] \leftarrow u$<br>
            $\text{DFS-VISIT}(\mathrm{G}, v)$<br>
$color[u] \leftarrow \text{BLACK}$<br>
$f[u] \leftarrow time + 1$



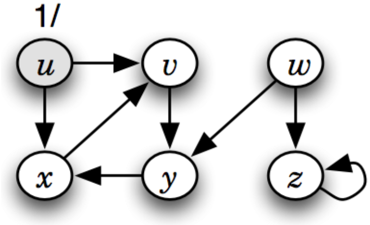
# Application of DFS

$\mathrm{DFS}(G) \triangleright G = (V, E)$

```
for each vertex u ∈ V[G]
    do color[u] ← WHITE
       π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
    do if color[u] = WHITE
          then DFS-VISIT(G, u)
```

$\mathrm{DFS\text{-}VISIT}(G, u)$

```
color[u] ← GRAY
time ← time + 1
d[u] ← time
for each vertex v ∈ Adj[u]
    do if color[v] = WHITE
          then π[v] ← u
               DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time + 1
```



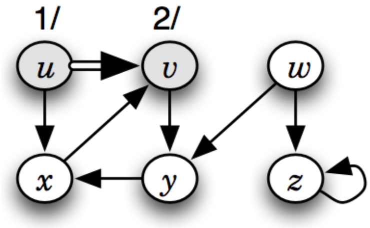
# Application of DFS

DFS($G$) $\triangleright$ $G = (V, E)$

for each vertex $u \in V[G]$
    do $color[u] \leftarrow$ WHITE
        $\pi[u] \leftarrow$ NIL
$time \leftarrow 0$
for each vertex $u \in V[G]$
    do if $color[u] =$ WHITE
        then DFS-VISIT(G, $u$)

DFS-VISIT($G, u$)

$color[u] \leftarrow$ GRAY
$time \leftarrow time + 1$
$d[u] \leftarrow time$
for each vertex $v \in Adj[u]$
    do if $color[v] =$ WHITE
        then $\pi[v] \leftarrow u$
            DFS-VISIT(G, $v$)
$color[u] \leftarrow$ BLACK
$f[u] \leftarrow time + 1$



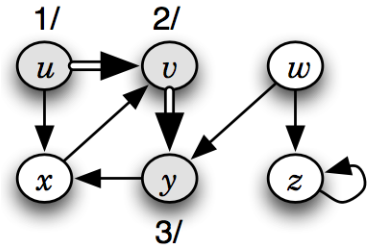
# Application of DFS

$\text{DFS}(G) \rhd G = (V, E)$

**for** each vertex $u \in V[G]$<br>
  **do** $color[u] \leftarrow \text{WHITE}$<br>
   $\pi[u] \leftarrow \text{NIL}$<br>
$time \leftarrow 0$<br>
**for** each vertex $u \in V[G]$<br>
  **do if** $color[u] = \text{WHITE}$<br>
    **then** $\text{DFS-VISIT}(\text{G}, u)$

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow \text{GRAY}$<br>
$time \leftarrow time + 1$<br>
$d[u] \leftarrow time$<br>
**for** each vertex $v \in Adj[u]$<br>
  **do if** $color[v] = \text{WHITE}$<br>
    **then** $\pi[v] \leftarrow u$<br>
      $\text{DFS-VISIT}(\text{G}, v)$<br>
$color[u] \leftarrow \text{BLACK}$<br>
$f[u] \leftarrow time + 1$



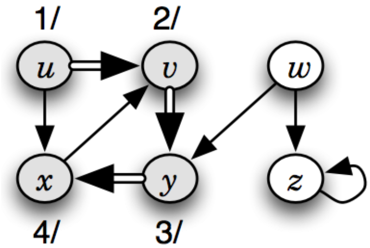
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$
  **do** $color[u] \leftarrow$ WHITE
    $\pi[u] \leftarrow$ NIL
$time \leftarrow 0$
**for** each vertex $u \in V[G]$
  **do if** $color[u] =$ WHITE
    **then** DFS-VISIT(G, $u$)

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow$ GRAY
$time \leftarrow time + 1$
$d[u] \leftarrow time$
**for** each vertex $v \in Adj[u]$
  **do if** $color[v] =$ WHITE
    **then** $\pi[v] \leftarrow u$
      DFS-VISIT(G, $v$)
$color[u] \leftarrow$ BLACK
$f[u] \leftarrow time + 1$



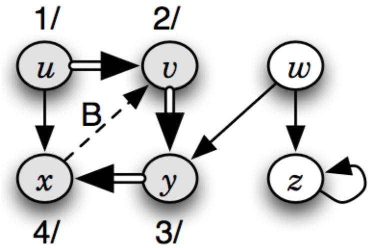
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$
    **do** $color[u] \leftarrow$ WHITE
        $\pi[u] \leftarrow$ NIL
$time \leftarrow 0$
**for** each vertex $u \in V[G]$
    **do if** $color[u] =$ WHITE
        **then** DFS-VISIT(G, $u$)

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow$ GRAY
$time \leftarrow time + 1$
$d[u] \leftarrow time$
**for** each vertex $v \in Adj[u]$
    **do if** $color[v] =$ WHITE
        **then** $\pi[v] \leftarrow u$
            DFS-VISIT(G, $v$)
$color[u] \leftarrow$ BLACK
$f[u] \leftarrow time + 1$



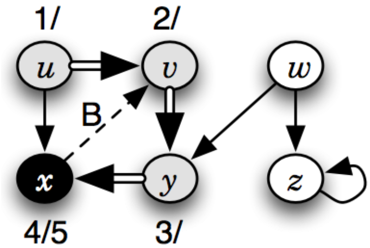
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$
  **do** $color[u] \leftarrow$ WHITE
    $\pi[u] \leftarrow$ NIL
$time \leftarrow 0$
**for** each vertex $u \in V[G]$
  **do if** $color[u] =$ WHITE
    **then** DFS-VISIT(G, $u$)

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow$ GRAY
$time \leftarrow time + 1$
$d[u] \leftarrow time$
**for** each vertex $v \in Adj[u]$
  **do if** $color[v] =$ WHITE
    **then** $\pi[v] \leftarrow u$
      DFS-VISIT(G, $v$)
$color[u] \leftarrow$ BLACK
$f[u] \leftarrow time + 1$



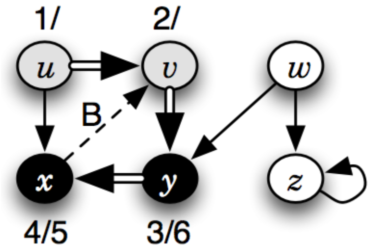
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$
    **do** $color[u] \leftarrow$ WHITE
        $\pi[u] \leftarrow$ NIL
$time \leftarrow 0$
**for** each vertex $u \in V[G]$
    **do if** $color[u] =$ WHITE
        **then** DFS-VISIT(G, $u$)

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow$ GRAY
$time \leftarrow time + 1$
$d[u] \leftarrow time$
**for** each vertex $v \in Adj[u]$
    **do if** $color[v] =$ WHITE
        **then** $\pi[v] \leftarrow u$
            DFS-VISIT(G, $v$)
$color[u] \leftarrow$ BLACK
$f[u] \leftarrow time + 1$



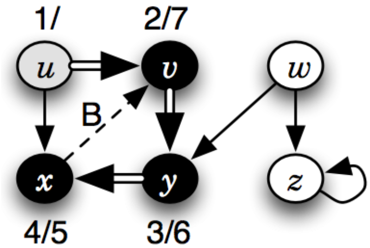
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$
    **do** $color[u] \leftarrow$ WHITE
        $\pi[u] \leftarrow$ NIL
$time \leftarrow 0$
**for** each vertex $u \in V[G]$
    **do if** $color[u] =$ WHITE
        **then** DFS-VISIT(G, $u$)

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow$ GRAY
$time \leftarrow time + 1$
$d[u] \leftarrow time$
**for** each vertex $v \in Adj[u]$
    **do if** $color[v] =$ WHITE
        **then** $\pi[v] \leftarrow u$
            DFS-VISIT(G, $v$)
$color[u] \leftarrow$ BLACK
$f[u] \leftarrow time + 1$



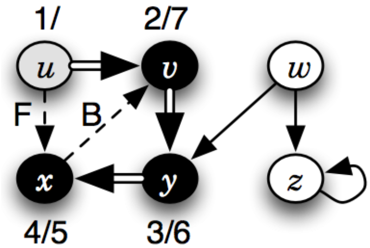
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

```
for each vertex u ∈ V[G]
      do color[u] ← WHITE
         π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
      do if color[u] = WHITE
            then DFS-VISIT(G, u)
```

$\text{DFS-VISIT}(G, u)$

```
color[u] ← GRAY
time ← time + 1
d[u] ← time
for each vertex v ∈ Adj[u]
      do if color[v] = WHITE
            then π[v] ← u
                 DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time + 1
```



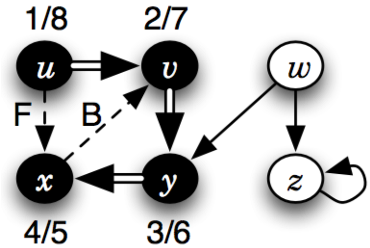
# Application of DFS

DFS$(G)$ $\triangleright$ $G = (V, E)$

```
for each vertex u ∈ V[G]
     do color[u] ← WHITE
        π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
     do if color[u] = WHITE
           then DFS-VISIT(G, u)
```

DFS-VISIT$(G, u)$

```
color[u] ← GRAY
time ← time +1
d[u] ← time
for each vertex v ∈ Adj[u]
     do if color[v] = WHITE
           then π[v] ← u
                DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time +1
```



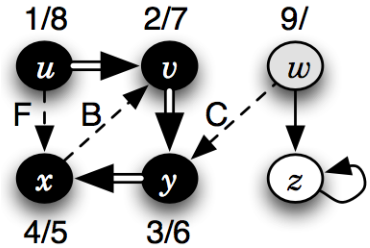
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

```
for each vertex u ∈ V[G]
        do color[u] ← WHITE
           π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
        do if color[u] = WHITE
              then DFS-VISIT(G, u)
```

$\text{DFS-VISIT}(G, u)$

```
color[u] ← GRAY
time ← time + 1
d[u] ← time
for each vertex v ∈ Adj[u]
        do if color[v] = WHITE
              then π[v] ← u
                   DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time + 1
```



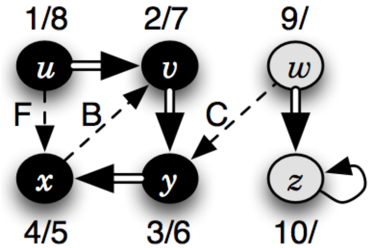
# Application of DFS

DFS($G$) ▷ $G = (V, E)$

```
for each vertex u ∈ V[G]
        do color[u] ← WHITE
           π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
        do if color[u] = WHITE
              then DFS-VISIT(G, u)
```

DFS-VISIT($G, u$)

```
color[u] ← GRAY
time ← time + 1
d[u] ← time
for each vertex v ∈ Adj[u]
        do if color[v] = WHITE
              then π[v] ← u
                   DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time + 1
```



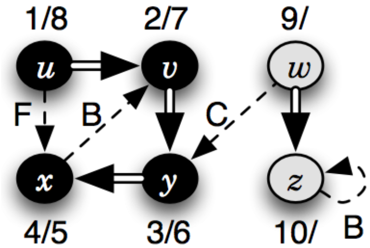
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

```
for each vertex u ∈ V[G]
        do color[u] ← WHITE
           π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
        do if color[u] = WHITE
              then DFS-VISIT(G, u)
```

$\text{DFS-VISIT}(G, u)$

```
color[u] ← GRAY
time ← time + 1
d[u] ← time
for each vertex v ∈ Adj[u]
        do if color[v] = WHITE
              then π[v] ← u
                   DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time + 1
```



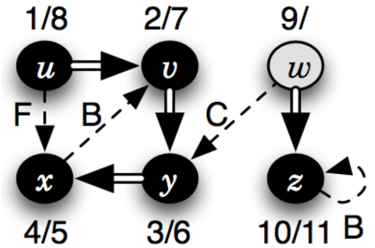
# Application of DFS

$\text{DFS}(G) \triangleright G = (V, E)$

**for** each vertex $u \in V[G]$<br>
  **do** $color[u] \leftarrow$ WHITE<br>
    $\pi[u] \leftarrow$ NIL<br>
$time \leftarrow 0$<br>
**for** each vertex $u \in V[G]$<br>
  **do if** $color[u] =$ WHITE<br>
    **then** DFS-VISIT(G, $u$)

$\text{DFS-VISIT}(G, u)$

$color[u] \leftarrow$ GRAY<br>
$time \leftarrow time + 1$<br>
$d[u] \leftarrow time$<br>
**for** each vertex $v \in Adj[u]$<br>
  **do if** $color[v] =$ WHITE<br>
    **then** $\pi[v] \leftarrow u$<br>
      DFS-VISIT(G, $v$)<br>
$color[u] \leftarrow$ BLACK<br>
$f[u] \leftarrow time + 1$



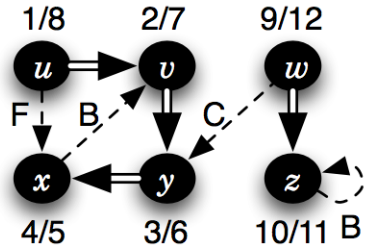
# Application of DFS

DFS($G$) $\triangleright$ $G = (V, E)$

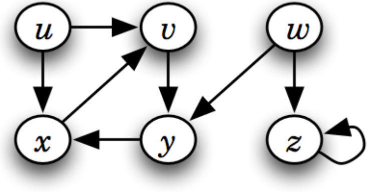
```
for each vertex u ∈ V[G]
    do color[u] ← WHITE
       π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
    do if color[u] = WHITE
          then DFS-VISIT(G, u)
```

DFS-VISIT($G, u$)

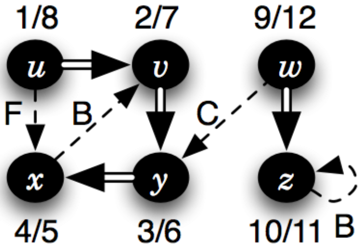
```
color[u] ← GRAY
time ← time + 1
d[u] ← time
for each vertex v ∈ Adj[u]
    do if color[v] = WHITE
          then π[v] ← u
               DFS-VISIT(G, v)
color[u] ← BLACK
f[u] ← time + 1
```



# Depth first search (DFS) tree (or, forest)

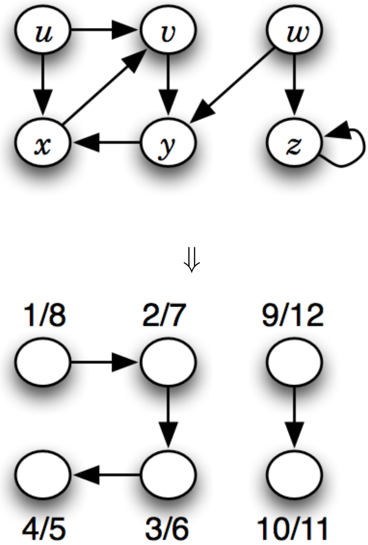



$\Downarrow$

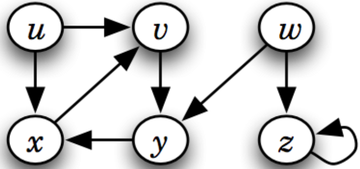


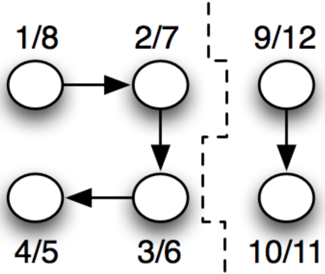
# Depth first search (DFS) tree (or, forest)



# Depth first search (DFS) tree (or, forest)




$\Downarrow$



# Questions to answer using DFS

- How to detect a cycle in the graph?

# Questions to answer using DFS

- How to detect a cycle in the graph?
- How to compute the connected components? (Hint: line 7 in the $\mathrm{DFS}(G)$ starts a new tree, with each tree representing a connected component)

# Questions to answer using DFS

- How to detect a cycle in the graph?
- How to compute the connected components? (Hint: line 7 in the $\mathrm{DFS}(G)$ starts a new tree, with each tree representing a connected component)
- Of what use are the *discovery* and *finish* times?

# Questions to answer using DFS

- How to detect a cycle in the graph?
- How to compute the connected components? (Hint: line 7 in the $\mathrm{DFS}(G)$ starts a new tree, with each tree representing a connected component)
- Of what use are the *discovery* and *finish* times?
- What are the different types of *non-tree* edges in a BFS or DFS tree of an undirected graph? What about a directed graph?

# Questions to answer using DFS

- How to detect a cycle in the graph?
- How to compute the connected components? (Hint: line 7 in the $\mathrm{DFS}(G)$ starts a new tree, with each tree representing a connected component)
- Of what use are the *discovery* and *finish* times?
- What are the different types of *non-tree* edges in a BFS or DFS tree of an undirected graph? What about a directed graph?
- How to use DFS to *topologically sort* a directed acyclic graph (DAG)?

# Questions to answer using DFS

- How to detect a cycle in the graph?
- How to compute the connected components? (Hint: line 7 in the $\mathrm{DFS}(G)$ starts a new tree, with each tree representing a connected component)
- Of what use are the *discovery* and *finish* times?
- What are the different types of *non-tree* edges in a BFS or DFS tree of an undirected graph? What about a directed graph?
- How to use DFS to *topologically sort* a directed acyclic graph (DAG)?
- ...

1. DAG: Directed Acyclic Graph
2. Topological Sort
3. Back Edges, Tree Edges, Cross Edge
4. Strongly Connected Components

Note: Study from class notes and book