BRAC University

Course Code: MAT 216

Homeworks SHEET - 3

Q - 1 Determine whether each of the following sets are linearly independent / dependent:

(i){(2 ,1 ,2) , (0 , 1 , - 1) , (4 , 3 , 3)} .
(ii) {(3 , 0 , 1 , -1) , (2 , -1, 0 , 1) , (1 , 1 , 1 , -2)} .
(iii) {(1 , - 4 , 2) , (3 , - 5 , 1) , (2 , 7 , 8) , (- 1 ,1 , 1)} .
(iv) {(0 , 1 , 0 , 1) , (1 , 2 , 3 , -1) , (8 , 4 , 3 , 2) , (0 , 3 , 2 , 0)} .
(v) {(1 , 3 , 2) , (1 , -7 , - 8) , (2 , 1 , - 1)} .
(vi) {(3 , 0 , 4 , 1) , (6 , 2 , -1 , 2) , (-1 , 3 , 5 , 1) , (- 3 , 7 , 8 , 3)}
(vii) {(4 , -4 , 8 , 0) , (2 , 2 , 4 , 0) , (6 , 0 , 0 , 2) , (6 , 3 , -3 , 0)} .

Q – 2 Prove that the following vectors form a basis for R^3 / R^4 :

(i) (1 , 2 , 0) , (0 , 5 , 7) & (-1 , 1 , 3) . (ii) {(2 , 0 , 1) , (1 , 1 , 1)} .
(iii) {(1 , 1 , 1 , 1) , (0 , 1 , 1 , 1) , (0 , 0 , 1 , 1) , (0 , 0 , 0 , 1)} .

Q – 3 Find the dimension of the subspace generated by the set
{(1 , 2 , 1) , (3 , 1 , 2) , (1 , -3 , 4)} of $V_3(R)$.

Q – 4 Let U be the subspace of R^3 spanned (generated) by the vectors
(1 , 2 , 1) , (0 , - 1 , 0) & (2 , 0 , 2) , find a basis and dimension of **U .**

Q – 5 Let W be the subspace of R^4 generated by the vectors
(1 ,- 2 , 5 ,-3) , (2 , 3 , 1 , - 4) & (3 , 8 , - 3 , - 5) , find a basis and dimension of **W**.

Q – 6 Find the rank of the following matrices :

(i) $A = \begin{bmatrix} 6 & 2 & 0 & 4 \\ -2 & -1 & 3 & 4 \\ -1 & -1 & 6 & 10 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ **(iii)** $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

(iv) $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **(v)** $A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$.

Q – 7 Find the rank and nullity of the following matrices :

(i) $A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$ **(iii)** $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

(iv) $A = \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$

Q – 8 Which of the following define linear transformation from R^3 to R^2 :

(i) $T(x, y, z) = (x - y, x - z)$ (ii) $T(x, y, z) = (3x - 2y + z, x - 3y - 2z)$

(iii) $T(x, y, z) = (x + 1, y + z)$.

Q – 9 Let T : $R^4 \to R^3$ be the linear transformation defined by

$T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$,

find a basis & dimension of range space of (T) & null space of (T) .

Q – 10 Let T : $R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$,

find a basis & dimension of (i) Image (T) & (ii) Ker (T) .

Q – 11 Let T : $R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (3x - y, y - z, 3x - 2y + z)$,

find a basis & dimension of (i) Image (T) & (ii) Ker (T) .

Q – 12 Let T : $R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (x + 2y - 3z, 2x - y + 4z, 4x + 3y - 2z)$,

find a basis & dimension of (i) Image (T) & (ii) Ker (T) .

Q – 13 Find all eigenvalues and the corresponding eigenvectors of the following matrices :

(i) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ **(iii)** $A = \begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}$

Q- 14 Find a matrix P that diagonalizes the following matrices , also find $P^{-1}AP$:

(i) $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ **(iii)** $A = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix}$