`pybats-detection`: A python package for outlier and structural changes detection in time series analysis

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Contents

Smoothing

A brief introduction of the Smoothing class in a simulated example. A time series $\mathbf{Y} = (y_1, \ldots, y_T)$ was generated using the `RandomDLM` class which has the arguments (n, V, W): the number of observations, observational variance and state vector variance. This class has three methods that simulate data using different mechanisms:

- `.level`: dynamic level model;
- `.growth`: dynamic growth model;
- `.level_with_covariates`: dynamic level model where Y is simulated given X, a matrix of fixed covariates.

For now, we stick with `.level`, simulating $n = 100$ observations with both observational and state vector variance equals to one 1, the starting level is set to 100. The simulated data is plotted below.

```
>>> # Generating level data model
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=100, V=1, W=1)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={})
>>> y = df_simulated["y"]
```

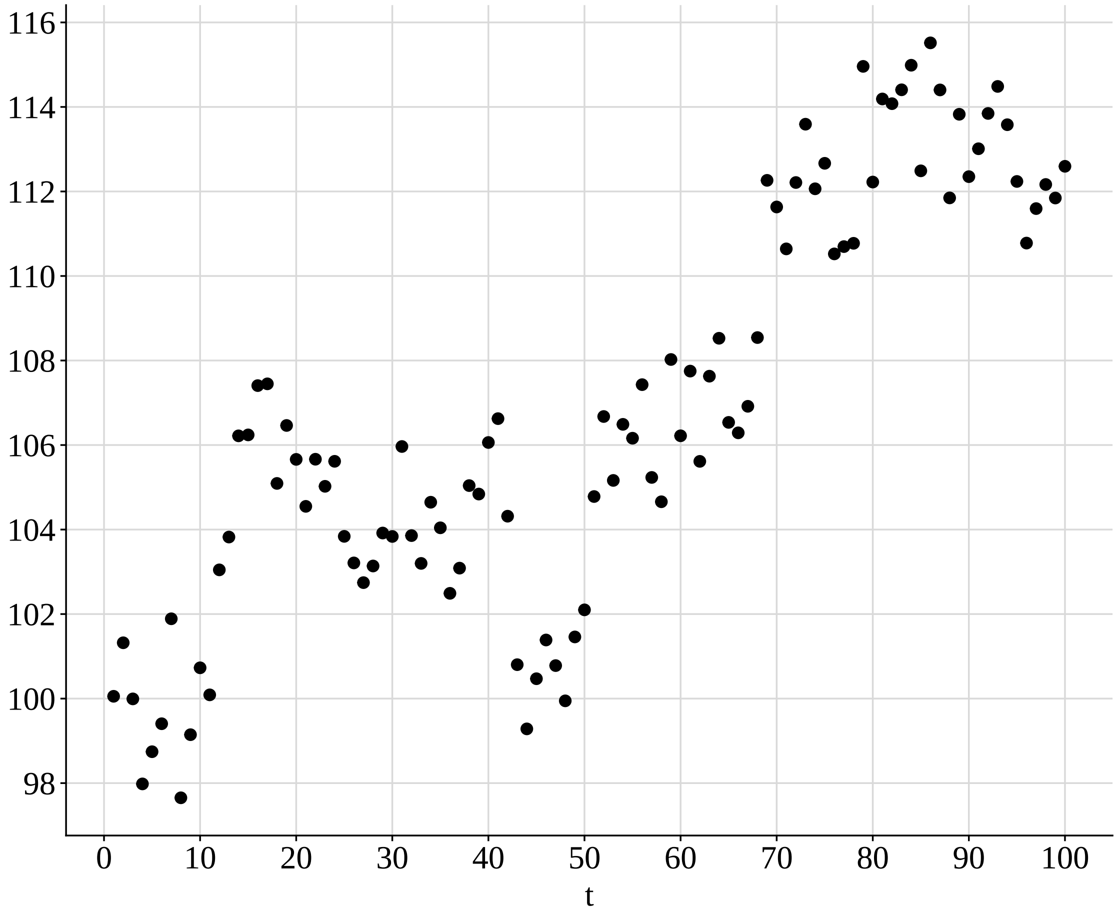


Figure 1: Simulated data

The Smoothing class allows you to perform a retrospective analysis for $\mathbf{Y}$, obtaining the distribution of $(\boldsymbol{\theta}_{T-k}|D_T)$, for $k \geq 1$, the k-step smoothed distribution for the state vector at time T, which is analogous to the k-step ahead forecast distribution $(\boldsymbol{\theta}_{t+k}|D_t)$.

To use Smoothing, first it is necessary to define the model components with prior values, which is done with the `dlm` class available in the `pybats` package. In this case, it was considered a DLM with level and growth. The prior vector and covariances are defined by $\mathbf{a}$ and $\mathbf{R}$. Lastly, the discount factor denoted by `deltrend` is a constant in the interval [0, 1], which is used to coordinate the adaptive capacity of predictions with increasing variance of model components.

```
>>> # Define model components
>>> a = np.array([100, 0])
>>> R = np.eye(2)
>>> np.fill_diagonal(R, val=1)
>>> mod = dlm(a, R, ntrend=2, deltrend=.95)
```

Given this, the method `.fit` will initialize the model and the loop forecast, observe and update begin. The prior and posterior moments $(\mathbf{a}_t, \mathbf{m}_t, \mathbf{C}_t, \mathbf{R}_t)$ will be computed for all t and saved. Subsequently, these moments will be used to obtain the moments for $(\boldsymbol{\theta}_{T-k}|D_T)$, recursively with $k \geq 1$, and denoted by $(\mathbf{a}_T(-k), \mathbf{m}_T(-k), \mathbf{C}_T(-k), \mathbf{R}_T(-k))$.

```
>>> # Fit with monitoring
>>> smooth = Smoothing(mod=mod)
>>> smooth_fit = smooth.fit(y=y)
```

This will return a dictionary with moments for: smoothed and filtered predictive distributions and for the posterior distributions of the model components. Each one can be obtained using the respective key

```
>>> smooth_fit.get('smooth').get('predictive')
```

```
>>> smooth_fit.get('smooth').get('posterior')
```

```
>>> smooth_fit.get('filter').get('predictive')
```

```
>>> smooth_fit.get('filter').get('posterior')
```

Below the results for the predictive and posterior smoothed distributions

smoothed predictive

The results for the smoothed predictive distribution consists of: $f_T(-k), q_T(-k)$ and the bounds for the credibility interval (`ci_lower`, `ci_upper`). Given by

$$f_T(-k) = \mathbf{F}^{'}\mathbf{a}_T(-k), \qquad q_T(-k) = \mathbf{F}^{'}\mathbf{R}_T(-k)\mathbf{F}$$

The credibility interval is is obtained from the corresponding smoothed distributions for the mean response of the series. Since V is considered unknown, then

$$(\mu_T(-k)|D_T) \sim T_{n_T}[f_T(-k), q_T(-k)]$$

For this simulated example, the results for the smoothed predictive distribution for the mean response are

```
>>> smooth_fit.get('smooth').get('predictive').round(2).head()
```

Table 1: Smothed predictive distribution results

fk	t	qk	df	ci_lower	ci_upper
99.97	1	0.31	1	98.85	101.1
100.07	2	0.27	2	99.05	101.1
100.12	3	0.24	3	99.14	101.1
100.20	4	0.23	4	99.24	101.2
100.39	5	0.22	5	99.47	101.3

as for the filtered distribution

```
>>> smooth_fit.get('smooth').get('predictive').round(2).head()
```

Table 2: Filtered predictive distribution results

parameter	mean	variance	t	ci_lower	ci_upper
Intercept	99.97	0.31	1	98.85	101.1
Intercept	100.07	0.27	2	99.05	101.1

parameter	mean	variance	t	ci_lower	ci_upper
Intercept	100.12	0.24	3	99.14	101.1
Intercept	100.20	0.23	4	99.24	101.2
Intercept	100.39	0.22	5	99.47	101.3

Plotting the filtered vs smoothed predictive distributions results is possible to see difference, primarily in the length of the credibility interval.

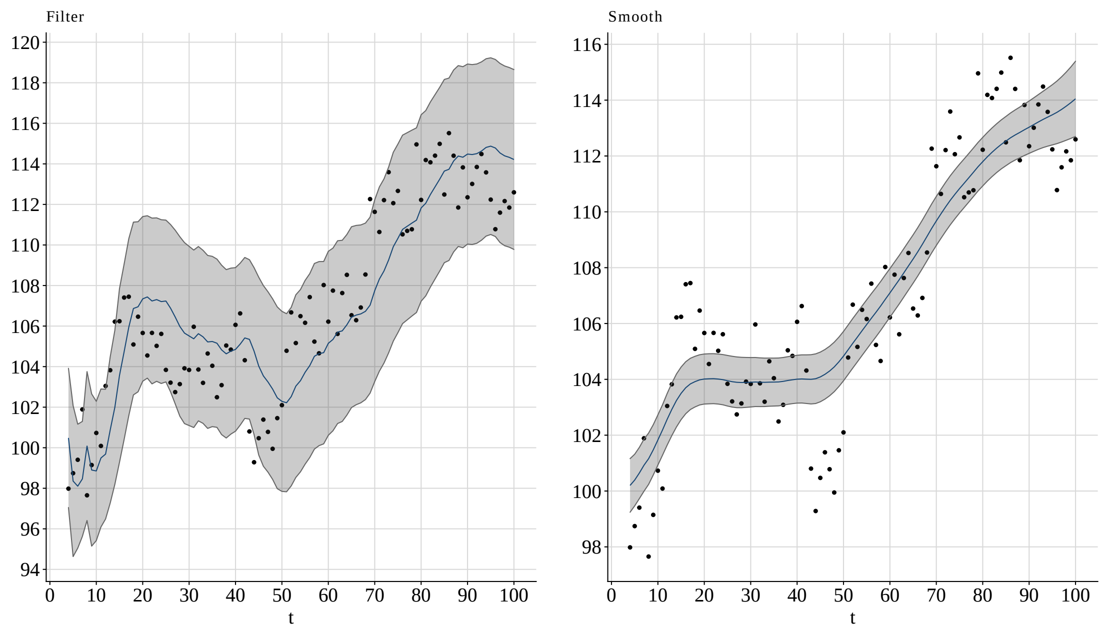


Figure 2: Mean response for Filtered and Smoothed predictive distributions with 95% credibility intervals.

smoothed posterior

The results for the posterior distributions are analogous, where

- parameter: Indicator for the respective state space parameter in $\boldsymbol{\theta}$;
- mean: The smoothed posterior distribution mean for time $t = T - k$ ($\mathbf{m}(-k)$);
- variance: The smoothed posterior distribution variance for time t ($\mathbf{C}(-k)$).
- credibility interval (`ci_lower`, `ci_upper`): The credibility interval obtained from the corresponding smoothed posterior distributions. Since V is considered unknown, then

$$(\boldsymbol{\theta}_{T-k}|D_T) \sim T_{n_T}[\mathbf{a}_T(-k), \mathbf{R}_T(-k)].$$

```
>>> smooth_fit.get('smooth').get('posterior').round(2).head()
```

Table 3: Smothed posterior distribution results

parameter	mean	variance	t	ci_lower	ci_upper
Intercept	99.97	0.31	1	98.85	101.1
Intercept	100.07	0.27	2	99.05	101.1
Intercept	100.12	0.24	3	99.14	101.1
Intercept	100.20	0.23	4	99.24	101.2

parameter	mean	variance	t	ci_lower	ci_upper
Intercept	100.39	0.22	5	99.47	101.3

As before we plot the results for filtered and smoothed distributions, in this case for each state space parameter. As expected, the smoothed posterior distributions show a less erratic behavior with shorter credibility intervals.

Aplication: AirPassangers dataset

Below is a practical example with the classic Box & Jenkins airline data, Monthly totals of international airline passengers (1949 to 1960). This data has a clear multiplicative seasonality, using a linear model (with additive seasonality) may be a naive approximation for this data. But, just for the sake of comparison between filtered and smoothing we stick with the linear model.

Using a normal DLM with three main components: Trend, Growth and Seasonality. The seasonality is modeled using the Fourier form representation, which depends on the parity of a period p and the number of harmonics components. Formally, the $\mathbf{r}^{th}$ harmonic component is given by

$$S_r(.) = a_r \cos(\alpha r) + b_r \sin(\alpha r), \quad r = 1, \ldots, h, \quad a_r = 2\pi/p, \quad h <= p/2$$

Here it was specified a yearly seasonal effect with period $p = 12$ and the first two harmonics. The discount factor for the level and growth components is set to 0.95, and 0.98 for the seasonal components. The results are plotted below.

```
>>> a = np.array([112, 0, 0, 0, 0, 0])
>>> R = np.eye(6)
>>> np.fill_diagonal(R, val=100)
>>> mod = dlm(a, R, ntrend=2, deltrend=.95, delseas=.98,
>>>           seasPeriods=[12], seasHarmComponents=[[1, 2]])
```

Since the seasonality was modeled using harmonic components, the model has a total of six parameters: level, growth and four for seasonality (a_1, b_1, a_2, b_2). For simplicity, the results for de posterior distributions considered the sum of the harmonic components, whose moments are given by

$$\mu_{seas} = \mathbf{F}'_{seas}\mathbf{a}_T(-k), \qquad \sigma^2_{seas} = \mathbf{F}'_{seas}\mathbf{R}_T(-k)\mathbf{F}_{seas}$$

where $\mathbf{F}'_{seas} = [0, 0, 1, 0, 1, 0]$. The results are illustrated below.

Manual Intervention

CP6

To illustrate the subjective intervention class we use the CP6 data graphed below. This time series runs from January 1955 to December 1959, providing monthly total sales, in monetary terms on a standard scale, of a product by a major company in UK. Note that the use of standard time series models may not wield satisfactory results as there a some points that need some attention:

1. During 1955 the market grows fast at a fast but steady rate,
2. A jump in December 1955.
3. The sales flattens off for 1956.
4. There is a major jump in the sales level in early 1957.
5. Another jump in early 1958.
6. Throughout the final two years, there is a steady decline back to late 1957.

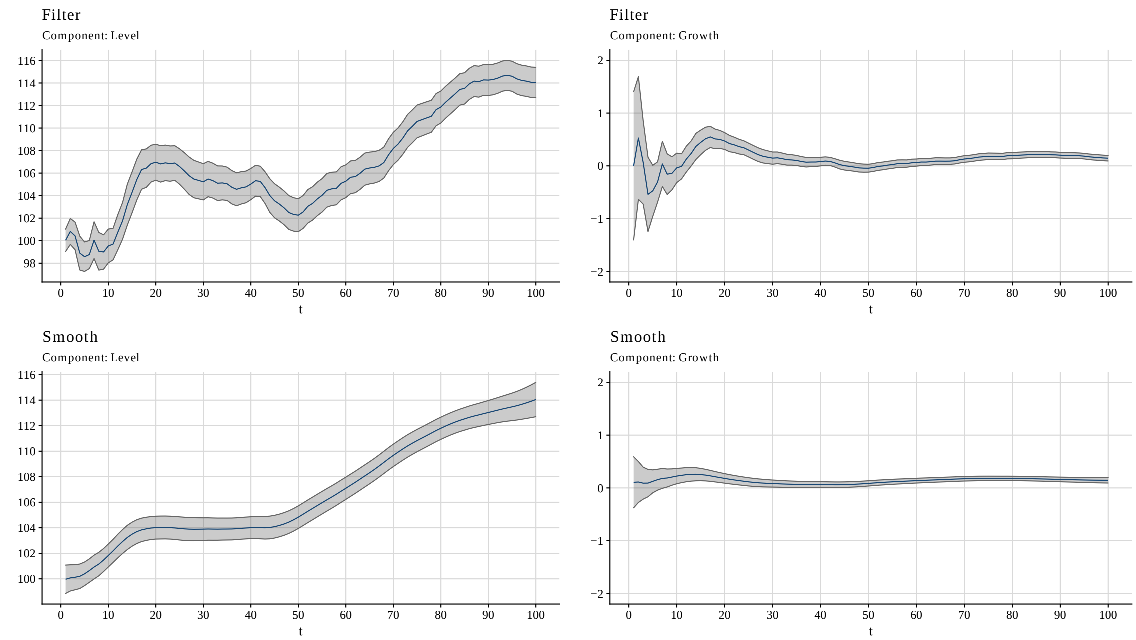


Figure 3: Mean response for Filtered and Smoothed posterior distributions for each model component with 95% credibility intervals.

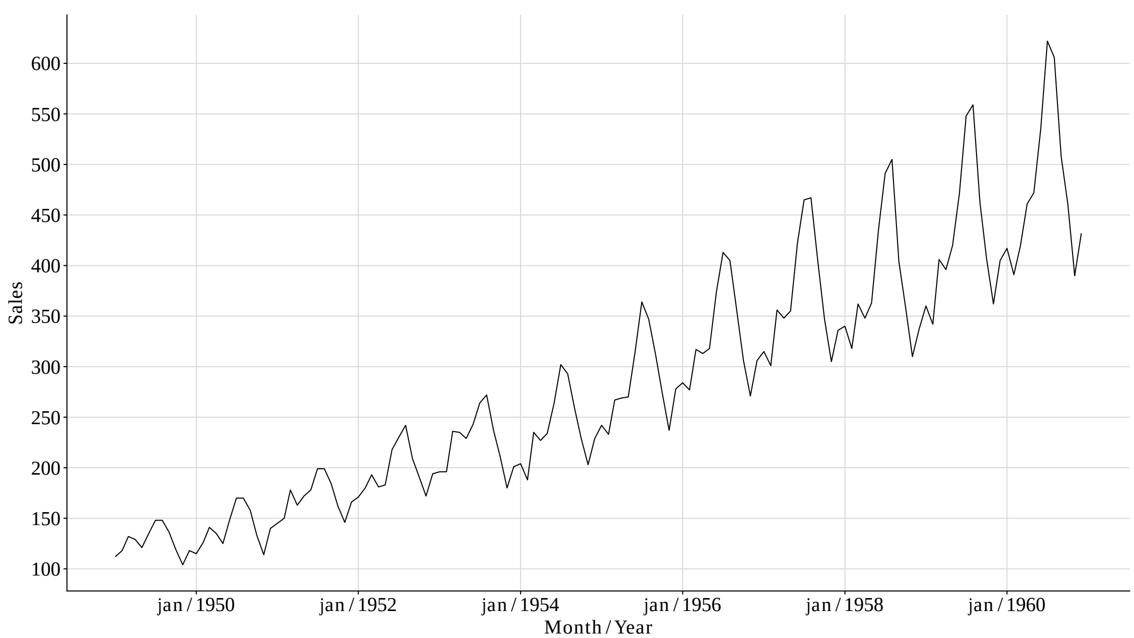


Figure 4: Monthly totals of international airline passengers, 1949 to 1960.

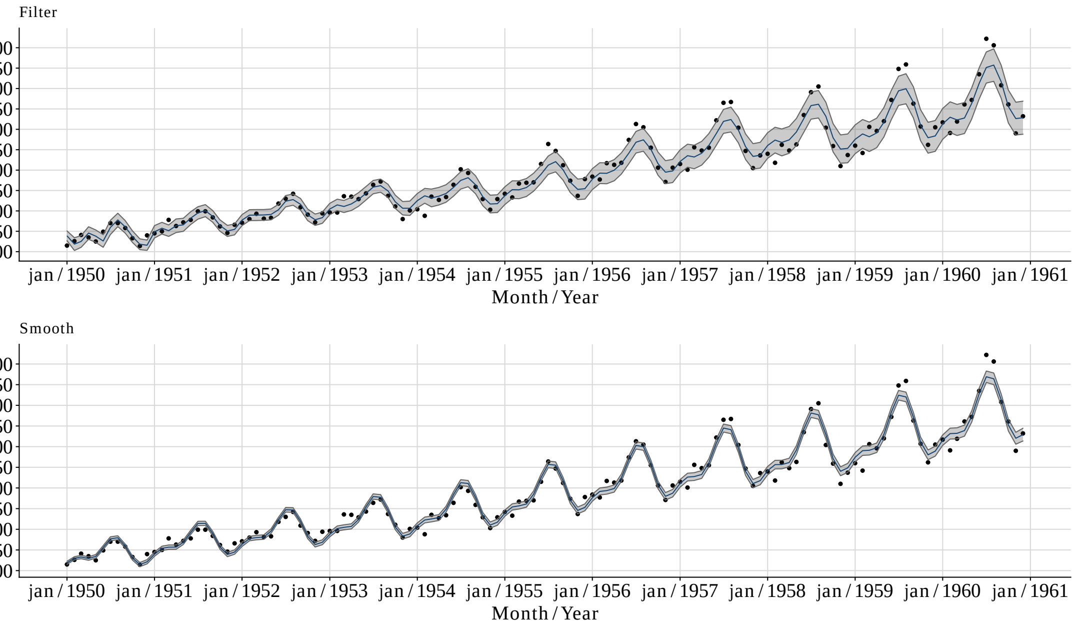


Figure 5: Mean response for Filtered and Smoothed predictive distributions with 95% credibility intervals.

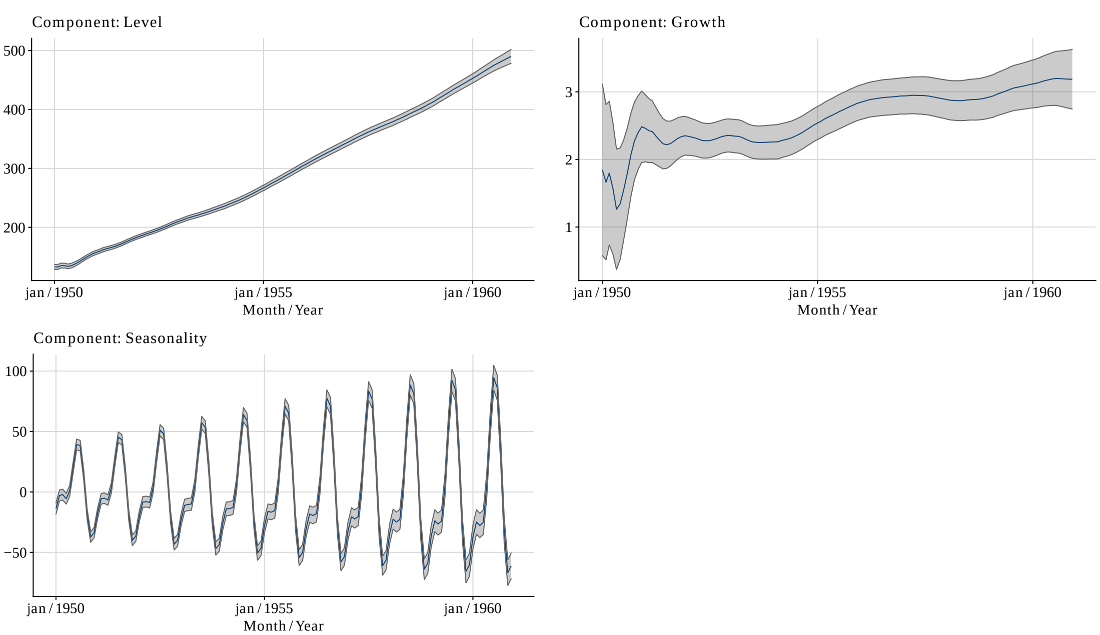


Figure 6: Mean response for Filtered and Smoothed posterior distributions for each model component with 95% credibility intervals.

```
>>> cp6 = load_cp6()
```

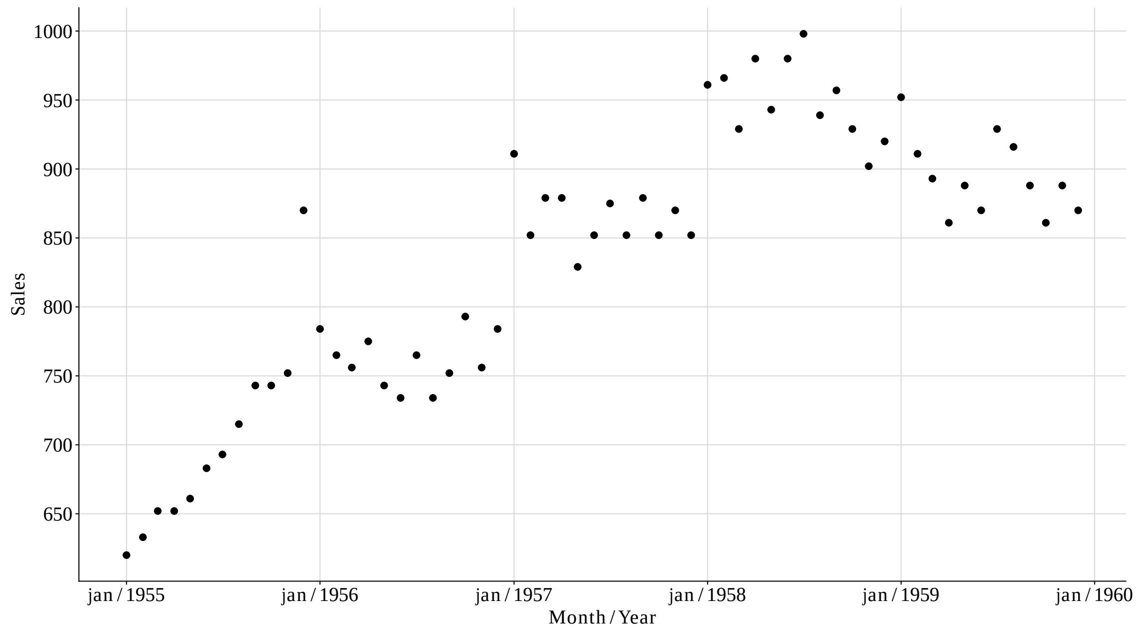


Figure 7: CP6 sales series

Fit Without Intervention

Given this, let's see how a standard dlm performs. The model used is defined below.

```
>>> # Define the growth model
>>> a = np.array([600, 1])
>>> R = np.array([[100, 0], [0, 25]])
>>> mod = dlm(a, R, ntrend=2, deltrend=[0.90, 0.98])
```

```
>>> # Filter and Smooth without intervention
>>> smooth = Smoothing(mod=mod)
>>> out_no_int = smooth.fit(y=cp6["sales"])
>>> dict_filter_no_int = out_no_int.get("filter").get("predictive")
```

Note that until November 1955 the forecast distribution was quite acceptable, the credibility interval was relatively small and the errors was were distributed around zero and inside the interval. But with the jump in December 1955 the level rises dramatically, the biggest problem is not the model's inability to efficiently predict this point, but the influence it has on future predictions. Note that for most of the year 1956 the predicted sales overestimation the real sales, giving a cluster of negative errors $(y_t - f_t)$. In early 1957 another jump was observed, but in this case, it was accompanied by a regime change. And this has great impact in the amplitude of the credibility intervals. In early 1958 another regime change, followed by a change in grow, that is not properly modeled since from August 1958 to January 1960 all errors were negative with the exception of July 1959.

Fit With Intervention

With the intervention class it is possible to consider outside information to define the prior distribution at the time t. This can be done in two ways: noise or subjective. Which must be provided in a list of dictionaries containing the time the intervention will be carried out and the type. Lets start with a empty list

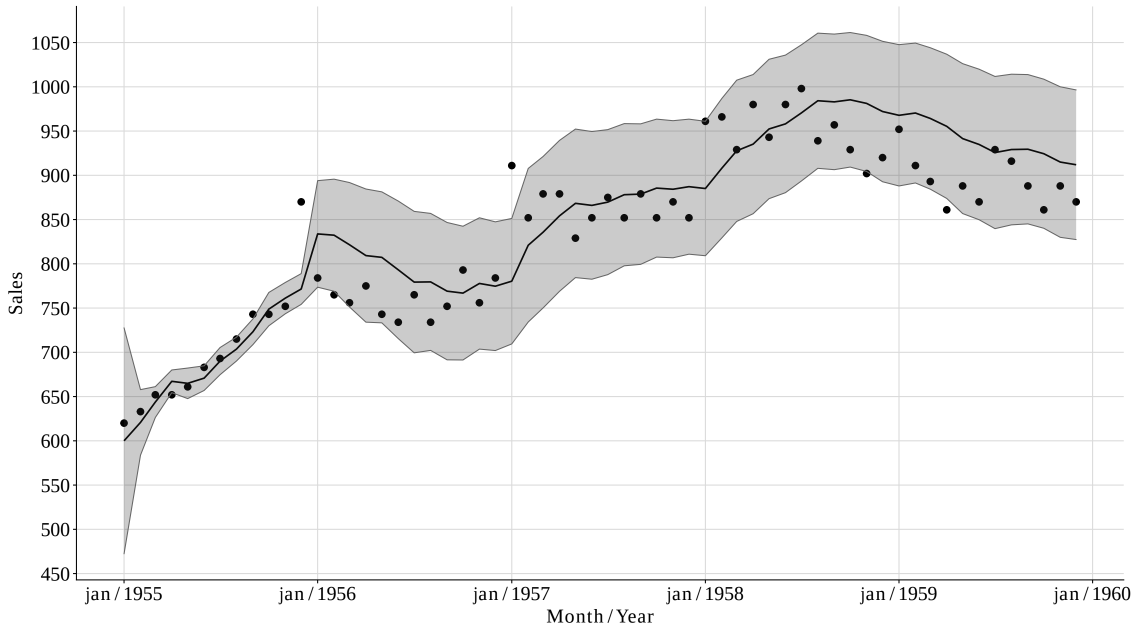


Figure 8: Mean response for Filtered predictive distribution with 95% credibility interval

```
>>> intervention_list = []
```

Noise Intervention in Prior Variance

In our example, suppose that a change in growth for the year 1956 was anticipated. An increase in uncertainty about level and growth can be done by the addition of a matrix H_t to R_t at time $t = 12$ given by

$$H_t = \begin{bmatrix} 100 & 25 \\ 25 & 25 \end{bmatrix}$$

Thus, there is an increase (a positive shift) in the prior variance of the components. In our list of interventions we have

```
>>> intervention_list = [{
>>>   "time_index": 12, "which": ["noise"],
>>>   "parameters": [{
>>>     "h_shift": np.array([0, 0]),
>>>     "H_shift": np.array([[100, 25], [25, 25]])}]
>>> }]
```

where

- `time_index`: time of intervention;
- `which`: type of intervention (in this case, a noise intervention);
- `parameters`: the values for the intervention.
  - `h_shit`: Shift in mean (we'll see more about that later).
  - `H_shift`: Shift in variance.

Noise Intervention in Prior Mean and Variance

It is also possible to intervene in the prior mean. Suppose an increase in the market level is expected for the year 1957, we can add a change in level of 80 units and increase the variance by 100 at January ($t = 25$)

$$\mathbf{h}_{25} = \begin{bmatrix} 80 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{H}_{25} = \begin{bmatrix} 100 & 0 \\ 0 & 0 \end{bmatrix}$$

now, updating our intervention list

```
>>> intervention_list = [{
>>>   "time_index": 12, "which": ["noise"],
>>>   "parameters": [{
>>>     "h_shift": np.array([0, 0]),
>>>     "H_shift": np.array([[100, 25], [25, 25]])}],
>>>
>>>   "time_index": 25, "which": ["noise"],
>>>   "parameters": [{
>>>     "h_shift": np.array([80, 0]),
>>>     "H_shift": np.array([[100, 0], [0, 0]])}]
>>> }]
```

In January 1958 ($t = 37$) another jump in level is anticipated, this time of about 100 units with a feeling of increased certainly about the new level and also a anticipated uncertainly for the growth. At this time, the prior mean and variance given by

$$\mathbf{a}_{37} = \begin{bmatrix} 864.5 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{R}_{37} = \begin{bmatrix} 91.7 & 9.2 \\ 9.2 & 1.56 \end{bmatrix}$$

are simply altered to

$$\mathbf{a}^*_{37} = \begin{bmatrix} 970 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{R}^*_{37} = \begin{bmatrix} 50 & 0 \\ 0 & 5 \end{bmatrix}$$

Performing the fit (filter and smoothing) with interventions

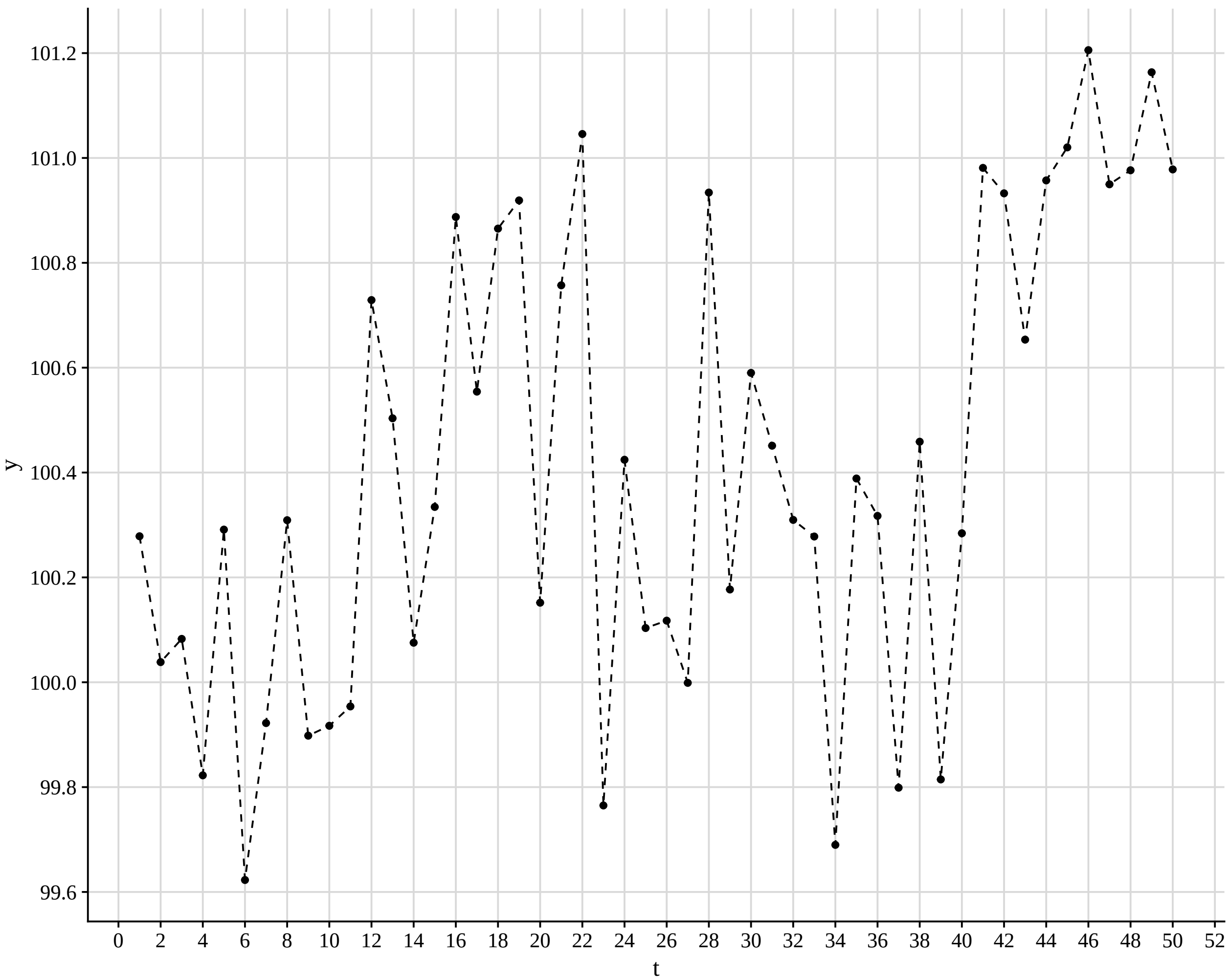
```
>>> list_interventions = [
>>>     {"time_index": 12, "which": ["variance", "noise"],
>>>      "parameters": [{"v_shift": "ignore"},
>>>                     {"h_shift": np.array([0, 0]),
>>>                      "H_shift": np.array([[1000, 25], [25, 25]])}]
>>>      },
>>>     {"time_index": 25, "which": ["noise", "variance"],
>>>      "parameters": [{"h_shift": np.array([80, 0]),
>>>                      "H_shift": np.array([[100, 0], [0, 0]])},
>>>                     {"v_shift": "ignore"}]},
>>>     {"time_index": 37, "which": ["subjective"],
>>>      "parameters": [{"a_star": np.array([970, 0]),
>>>                      "R_star": np.array([[50, 0], [0, 5]])}]}
>>> ]
>>> manual_interventions = Intervention(mod=mod)
>>> out_int = manual_interventions.fit(
>>>     y=cp6["sales"], interventions=list_interventions)
>>> dict_filter_int = out_int.get("filter")
>>> dict_smooth_int = out_int.get("smooth")
```

Automatic Monitoring

Simulated examples

Level Change

```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=50, V=0.1, W=0.005)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [40],
>>>                 "level_mean_shift": [1],
>>>                 "level_var_shift": [1]})
>>> df_simulated.loc[40:50, "y"] = 101 + np.random.normal(0, 0.2, 10)
```

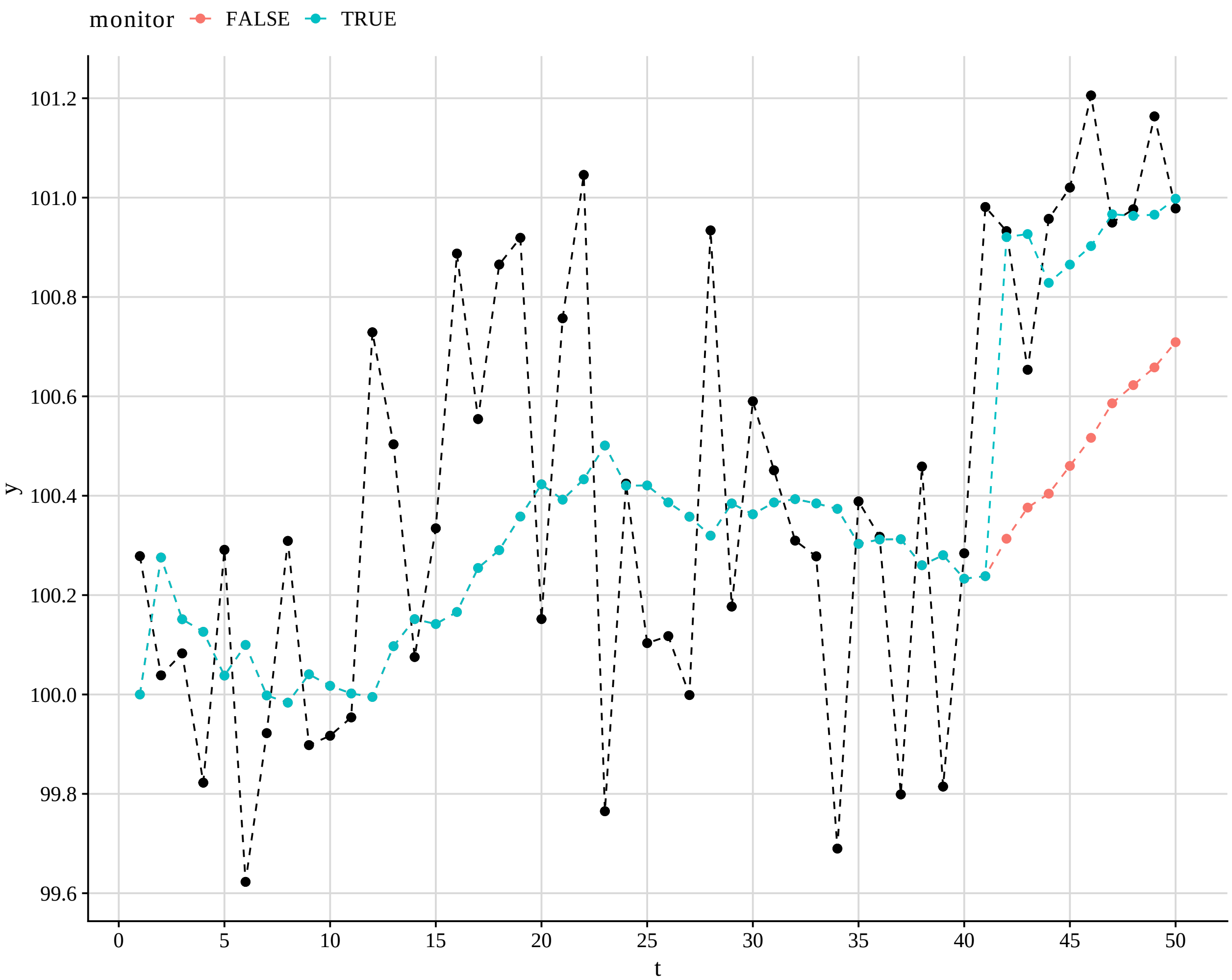



```
>>> a = np.array([100])
>>> R = np.eye(1)
>>> R[[0]] = 100
>>> mod = dlm(a, R, ntrend=1, deltrend=0.9)
>>>
>>> # Fit without monitoring
>>> fit_without_monitor = Smoothing(mod=mod).fit(y=df_simulated["y"])
```

```
>>> df_res = fit_without_monitor.get("filter").get("predictive")
>>> df_res["monitor"] = False
>>>
>>> # Fit with monitoring
>>> monitor = Monitoring(mod=mod, bilateral=False)
>>> fit_monitor = monitor.fit(y=df_simulated["y"], h=3, tau=0.135, change_var=[100])
```

```
## Parametric change detected at time 43 with H=1.2090e+01, L=3.7693e+00 and l=3
```

```
>>> df_tmp = fit_monitor.get("filter").get("predictive")
>>> df_tmp["monitor"] = True
>>> cols_ord = ["t", "y", "f", "q", "ci_lower", "ci_upper", "monitor", "e",
>>>             "H", "L", "l"]
>>> df_res = pd.concat([df_res, df_tmp[cols_ord]]).reset_index(drop=True)
```

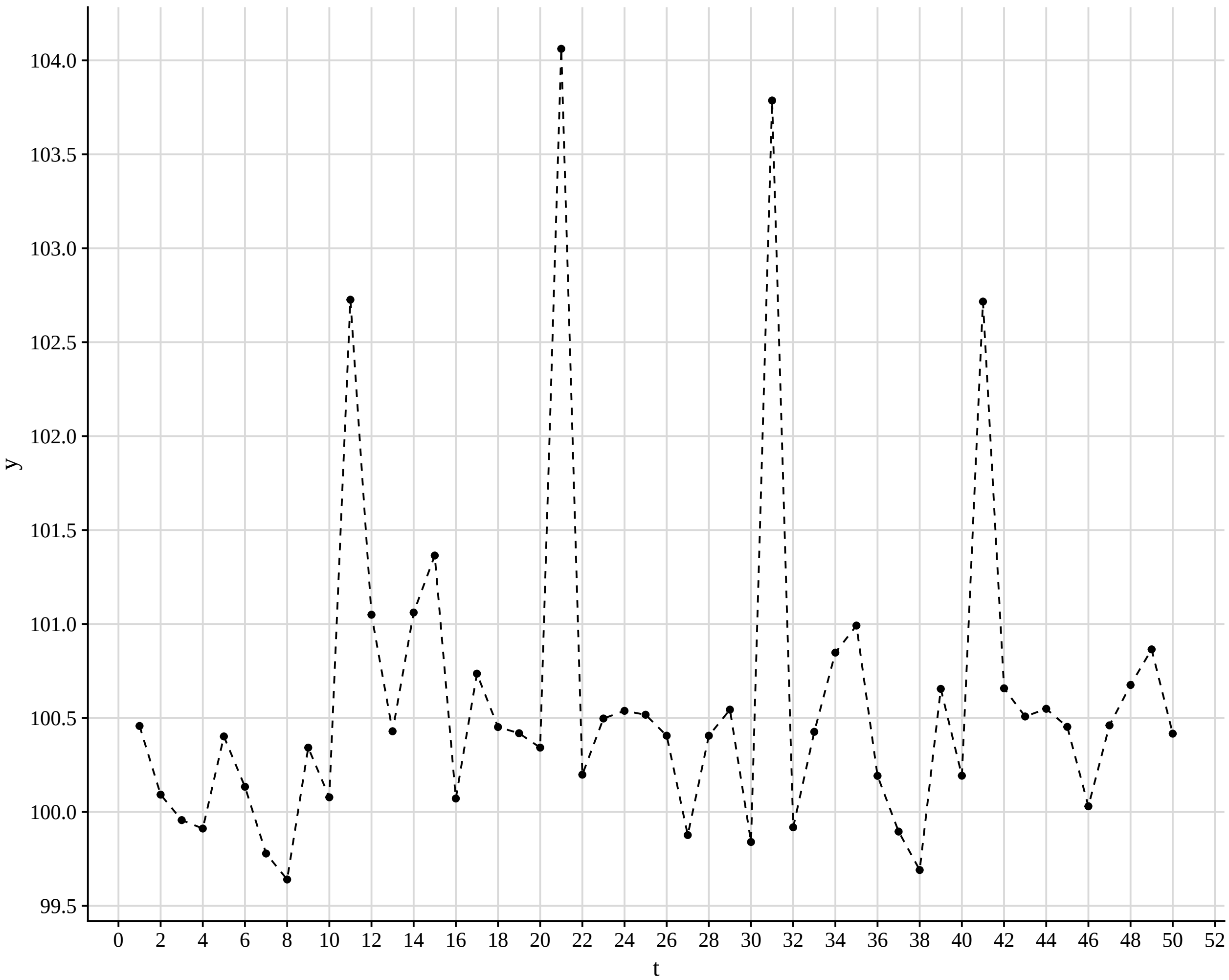



```
##      t      y      f        q         e         H         L l
## 1    1 100.28 100.00 101.0000  0.027708 1.000e+00 1.000e+00 1
## 2    2 100.04 100.28   1.0509 -0.231445 1.000e+00 1.000e+00 1
## 3    3 100.08 100.15   0.5419 -0.093343 1.000e+00 1.000e+00 1
## 4    4  99.82 100.13   0.3630 -0.504038 1.000e+00 1.000e+00 1
## 5    5 100.29 100.04   0.2899  0.470105 1.000e+00 1.000e+00 1
## 6    6  99.62 100.10   0.2424 -0.968590 1.000e+00 1.000e+00 1
```

```
## 7   7  99.92 100.00   0.2338 -0.156969 1.000e+00 1.000e+00 1
## 8   8 100.31  99.98   0.2013  0.725492 1.000e+00 1.000e+00 1
## 9   9  99.90 100.04   0.1879 -0.328868 1.000e+00 1.000e+00 1
## 10 10  99.92 100.02   0.1692 -0.244168 1.000e+00 1.000e+00 1
## 11 11  99.95 100.00   0.1533 -0.122789 1.301e+02 1.301e+02 1
## 12 12 100.73  99.99   0.1397  1.963608 2.489e-01 2.489e-01 1
## 13 13 100.50 100.10   0.1694  0.987129 4.658e+00 1.159e+00 2
## 14 14 100.08 100.15   0.1682 -0.185936 1.572e+02 1.572e+02 1
## 15 15 100.33 100.14   0.1567  0.486534 2.091e+01 2.091e+01 1
## 16 16 100.89 100.17   0.1487  1.870796 3.288e-01 3.288e-01 1
## 17 17 100.55 100.25   0.1700  0.726979 1.017e+01 3.342e+00 2
## 18 18 100.87 100.29   0.1651  1.414333 1.293e+00 1.293e+00 1
## 19 19 100.92 100.36   0.1734  1.346991 1.583e+00 1.583e+00 1
## 20 20 100.15 100.42   0.1801 -0.639038 6.122e+02 6.122e+02 1
## 21 21 100.76 100.39   0.1747  0.873260 6.555e+00 6.555e+00 1
## 22 22 101.05 100.43   0.1726  1.474577 1.079e+00 1.079e+00 1
## 23 23  99.77 100.50   0.1812 -1.729299 1.612e+04 1.612e+04 1
## 24 24 100.42 100.42   0.1959  0.009252 8.755e+01 8.755e+01 1
## 25 25 100.10 100.42   0.1879 -0.731956 8.091e+02 8.091e+02 1
## 26 26 100.12 100.39   0.1844 -0.626588 5.898e+02 5.898e+02 1
## 27 27 100.00 100.36   0.1801 -0.845808 1.138e+03 1.138e+03 1
## 28 28 100.93 100.32   0.1781  1.455528 1.143e+00 1.143e+00 1
## 29 29 100.18 100.38   0.1849 -0.482666 3.830e+02 3.830e+02 1
## 30 30 100.59 100.36   0.1801  0.535852 1.804e+01 1.804e+01 1
## 31 31 100.45 100.39   0.1758  0.154334 5.666e+01 5.666e+01 1
## 32 32 100.31 100.39   0.1704 -0.202337 1.652e+02 1.652e+02 1
## 33 33 100.28 100.38   0.1654 -0.262065 1.976e+02 1.976e+02 1
## 34 34  99.69 100.37   0.1608 -1.705145 1.499e+04 1.499e+04 1
## 35 35 100.39 100.30   0.1695  0.207462 4.831e+01 4.831e+01 1
## 36 36 100.32 100.31   0.1649  0.013050 8.656e+01 8.656e+01 1
## 37 37  99.80 100.31   0.1604 -1.282407 4.218e+03 4.218e+03 1
## 38 38 100.46 100.26   0.1631  0.491867 2.058e+01 2.058e+01 1
## 39 39  99.81 100.28   0.1599 -1.164556 2.962e+03 2.962e+03 1
## 40 40 100.28 100.23   0.1613  0.127470 6.141e+01 6.141e+01 1
## 41 41 100.98 100.24   1.7367  1.872524 3.271e-01 3.271e-01 1
## 42 42 100.93 100.92   0.2812  1.515974 9.532e-01 3.118e-01 2
## 43 43 100.65 100.93   0.2122  0.669203 1.209e+01 1.000e+00 0
## 44 44 100.96 100.83   0.1874  0.297470 3.688e+01 3.688e+01 1
## 45 45 101.02 100.87   0.1728  0.372924 2.941e+01 2.941e+01 1
## 46 46 101.21 100.90   0.1632  0.750340 9.478e+00 9.478e+00 1
## 47 47 100.95 100.97   0.1575 -0.042189 1.022e+02 1.022e+02 1
## 48 48 100.98 100.96   0.1513  0.034045 8.128e+01 8.128e+01 1
## 49 49 101.16 100.97   0.1460  0.517697 1.905e+01 1.905e+01 1
## 50 50 100.98 101.00   0.1423 -0.051842 1.052e+02 1.052e+02 1
```

Outliers

```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=50, V=0.1, W=0.01)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [10, 11, 20, 21, 30, 31, 40, 41],
>>>                 "level_mean_shift": [2, -2, 3, -3, 3.4, -3.4, 3, -3],
>>>                 "level_var_shift": [1, 1, 1, 1, 1, 1, 1, 1]})
```

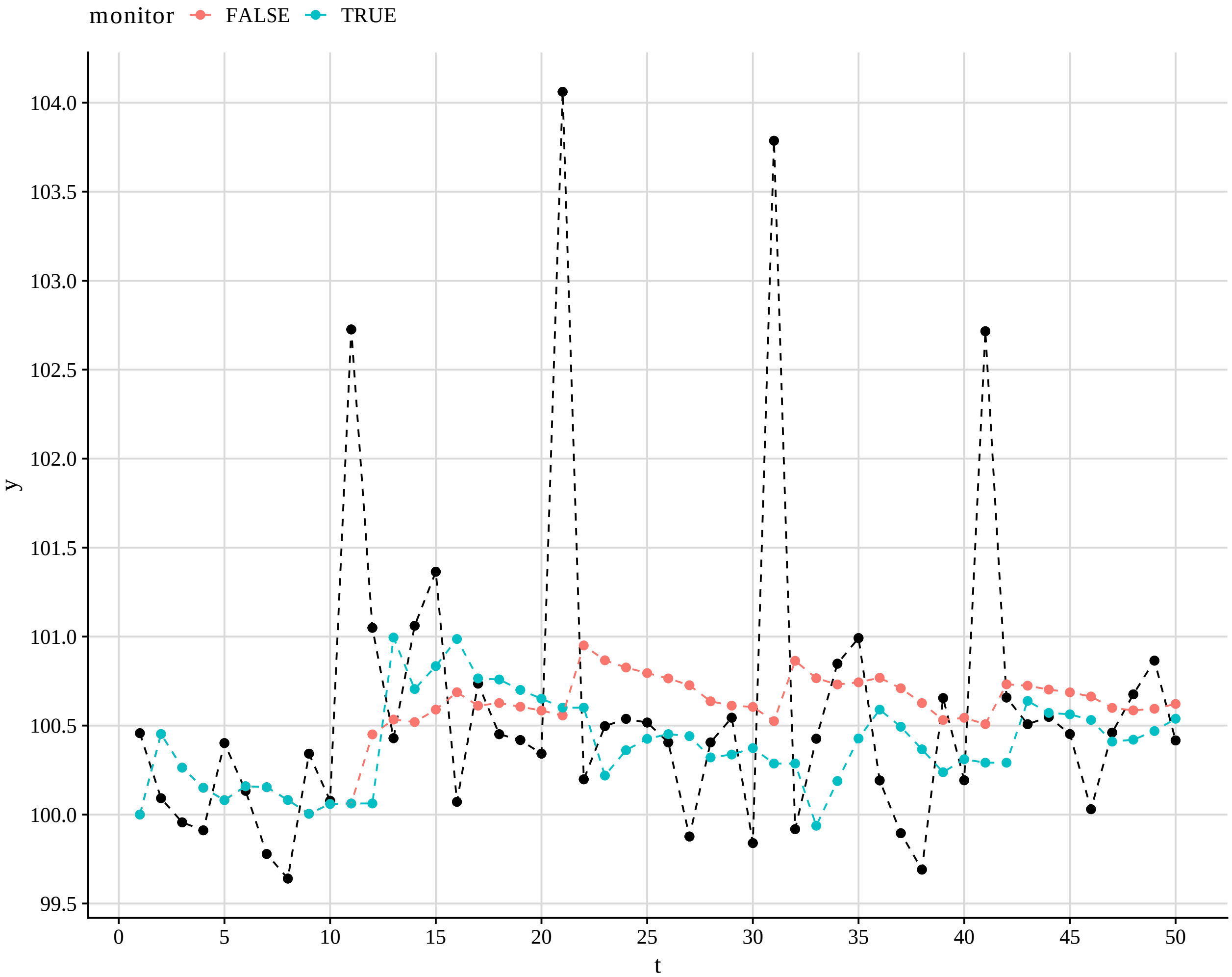



```
>>> a = np.array([100])
>>> R = np.eye(1)
>>> R[[0]] = 100
>>> mod = dlm(a, R, ntrend=1, deltrend=0.9)
>>>
>>> # Fit without monitoring
>>> fit_without_monitor = Smoothing(mod=mod).fit(y=df_simulated["y"])
>>> df_res = fit_without_monitor.get("filter").get("predictive")
>>> df_res["monitor"] = False
>>>
>>> # Fit with monitoring
>>> monitor = Monitoring(mod=mod, bilateral=False)
>>> fit_monitor = monitor.fit(y=df_simulated["y"], h=4, tau=0.135, change_var=[100])
```

```
## Potential outlier detected at time 11 with H=2.0200e-08, L=2.0200e-08 and l=1
## Potential outlier detected at time 21 with H=1.2518e-11, L=1.2518e-11 and l=1
## Potential outlier detected at time 31 with H=3.2940e-13, L=3.2940e-13 and l=1
## Potential outlier detected at time 41 with H=8.2124e-08, L=8.2124e-08 and l=1
```

```
>>> df_tmp = fit_monitor.get("filter").get("predictive")
>>> df_tmp["monitor"] = True
>>>
```

```
>>> # Append
>>> cols_ord = ["t", "monitor", "y", "f", "q", "ci_lower", "ci_upper"]
>>> df_res = pd.concat([df_res[cols_ord], df_tmp[cols_ord]]).reset_index(drop=True)
```

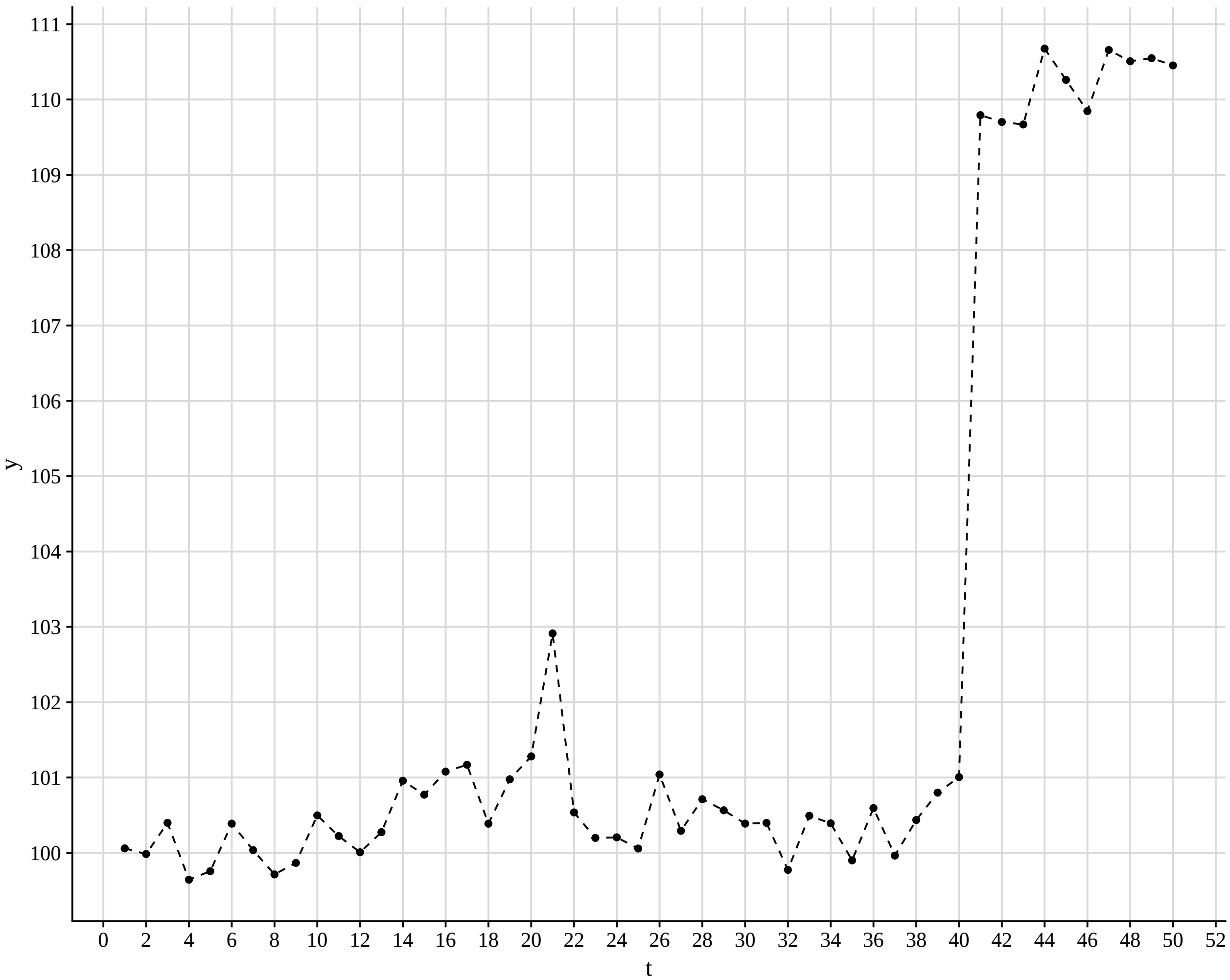



```
##        t      y      f        q        e         H         L l
## 1      1 100.46 100.00 101.0000  0.04550 1.000e+00 1.000e+00 1
## 2      2 100.09 100.45   1.0522 -0.35192 1.000e+00 1.000e+00 1
## 3      3  99.96 100.26   0.5612 -0.41035 1.000e+00 1.000e+00 1
## 4      4  99.91 100.15   0.3958 -0.38023 1.000e+00 1.000e+00 1
## 5      5 100.40 100.08   0.3080  0.57742 1.000e+00 1.000e+00 1
## 6      6 100.13 100.16   0.2631 -0.05013 1.000e+00 1.000e+00 1
## 7      7  99.78 100.15   0.2196 -0.80119 1.000e+00 1.000e+00 1
## 8      8  99.64 100.08   0.2057 -0.97397 1.000e+00 1.000e+00 1
## 9      9 100.34 100.00   0.2015  0.75197 1.000e+00 1.000e+00 1
## 10    10 100.08 100.06   0.1905  0.04164 1.000e+00 1.000e+00 1
## 11    11 102.73 100.06   0.1716  6.42939 2.020e-08 1.000e+00 0
## 12    12 101.05 100.06   2.6468  0.60672 2.633e+02 2.633e+02 1
## 13    13 100.43 100.99   0.2847 -1.06005 2.069e+05 2.069e+05 1
## 14    14 101.06 100.70   0.2200  0.75899 1.432e+02 1.432e+02 1
## 15    15 101.36 100.83   0.1908  1.21403 2.319e+01 2.319e+01 1
## 16    16 100.07 100.99   0.1851 -2.12732 1.479e+07 1.479e+07 1
```

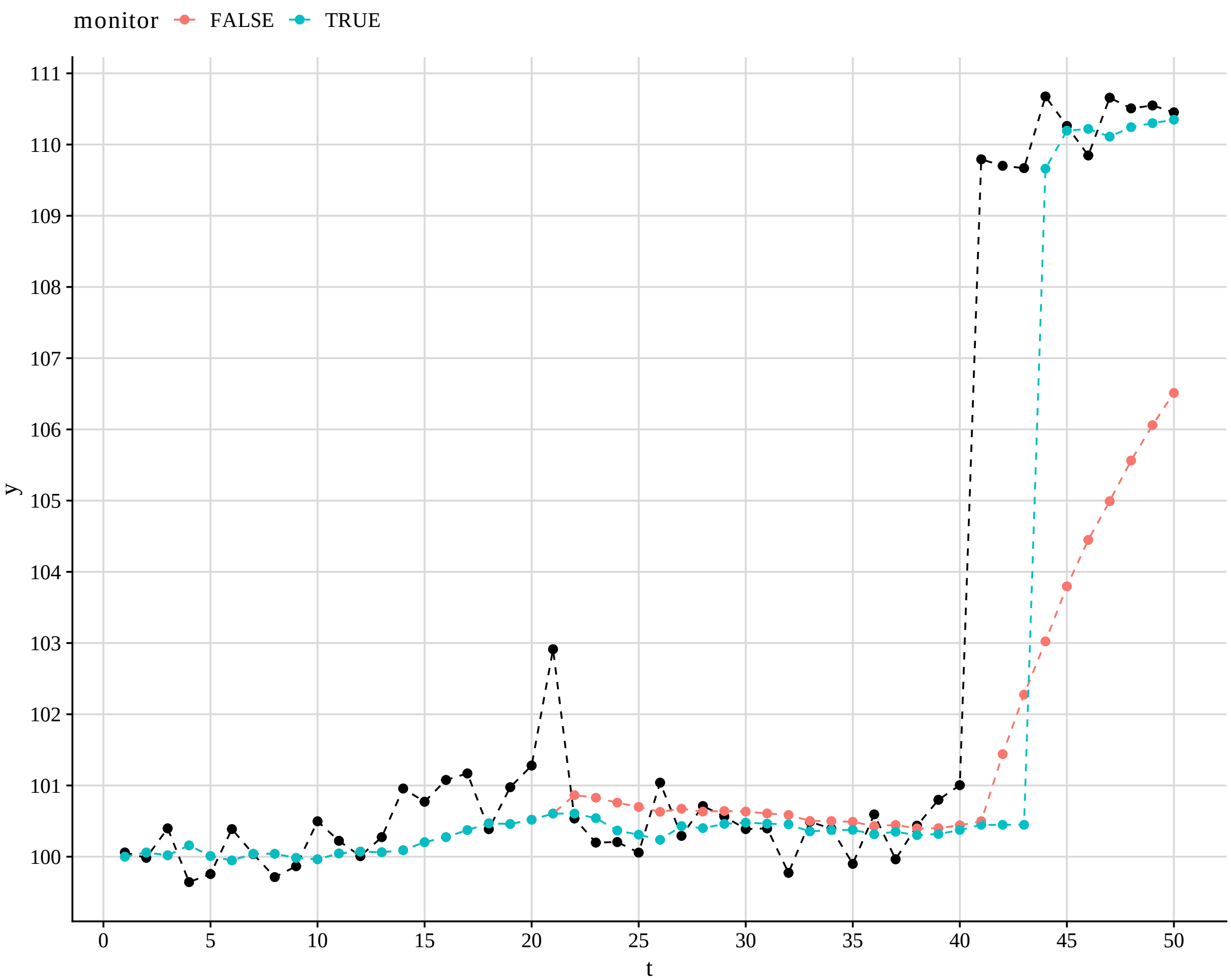
```
## 17 17 100.74 100.77   0.2172 -0.06276 3.832e+03 3.832e+03 1
## 18 18 100.45 100.76   0.1991 -0.68855 4.683e+04 4.683e+04 1
## 19 19 100.42 100.70   0.1896 -0.64679 3.962e+04 3.962e+04 1
## 20 20 100.34 100.65   0.1811 -0.72604 5.440e+04 5.440e+04 1
## 21 21 104.06 100.60   0.1748  8.27597 1.252e-11 1.000e+00 0
## 22 22 100.20 100.60   2.8231 -0.23975 7.778e+03 7.778e+03 1
## 23 23 100.50 100.22   0.2904  0.51567 3.789e+02 3.789e+02 1
## 24 24 100.54 100.36   0.2146  0.38011 6.517e+02 6.517e+02 1
## 25 25 100.52 100.43   0.1847  0.21290 1.272e+03 1.272e+03 1
## 26 26 100.41 100.45   0.1668 -0.11343 4.692e+03 4.692e+03 1
## 27 27  99.88 100.44   0.1541 -1.43700 9.347e+05 9.347e+05 1
## 28 28 100.41 100.32   0.1561  0.21363 1.268e+03 1.268e+03 1
## 29 29 100.54 100.34   0.1477  0.53839 3.460e+02 3.460e+02 1
## 30 30  99.84 100.37   0.1419 -1.41664 8.616e+05 8.616e+05 1
## 31 31 103.79 100.29   0.1452  9.18538 3.294e-13 1.000e+00 0
## 32 32  99.92 100.29   2.3444 -0.24081 7.811e+03 7.811e+03 1
## 33 33 100.43  99.94   0.2445  0.98890 5.708e+01 5.708e+01 1
## 34 34 100.85 100.19   0.1868  1.52652 6.645e+00 6.645e+00 1
## 35 35 100.99 100.43   0.1739  1.35290 1.331e+01 1.331e+01 1
## 36 36 100.19 100.59   0.1676 -0.97163 1.453e+05 1.453e+05 1
## 37 37  99.90 100.49   0.1610 -1.49086 1.159e+06 1.159e+06 1
## 38 38  99.69 100.37   0.1622 -1.67843 2.455e+06 2.455e+06 1
## 39 39 100.65 100.24   0.1671  1.01996 5.041e+01 5.041e+01 1
## 40 40 100.19 100.31   0.1649 -0.29059 9.532e+03 9.532e+03 1
## 41 41 102.72 100.29   0.1591  6.07876 8.212e-08 1.000e+00 0
## 42 42 100.66 100.29   2.5693  0.22844 1.195e+03 1.195e+03 1
## 43 43 100.51 100.64   0.2699 -0.25147 8.151e+03 8.151e+03 1
## 44 44 100.55 100.57   0.2016 -0.05057 3.649e+03 3.649e+03 1
## 45 45 100.45 100.56   0.1758 -0.26400 8.570e+03 8.570e+03 1
## 46 46 100.03 100.53   0.1616 -1.24712 4.373e+05 4.373e+05 1
## 47 47 100.46 100.41   0.1574  0.12725 1.792e+03 1.792e+03 1
## 48 48 100.68 100.42   0.1499  0.65818 2.143e+02 2.143e+02 1
## 49 49 100.86 100.47   0.1451  1.03864 4.678e+01 4.678e+01 1
## 50 50 100.42 100.54   0.1433 -0.32205 1.081e+04 1.081e+04 1
```

Outlier and Level Change

```python
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=50, V=0.1, W=0.01)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [20, 21, 40],
>>>                 "level_mean_shift": [3, -3, 10],
>>>                 "level_var_shift": [1, 1, 1]})
```




```
## Potential outlier detected at time 21 with H=3.1219e-05, L=3.1219e-05 and l=1
## Potential outlier detected at time 41 with H=2.6369e-34, L=2.6369e-34 and l=1
## Potential outlier detected at time 42 with H=9.8474e-08, L=9.8474e-08 and l=1
```

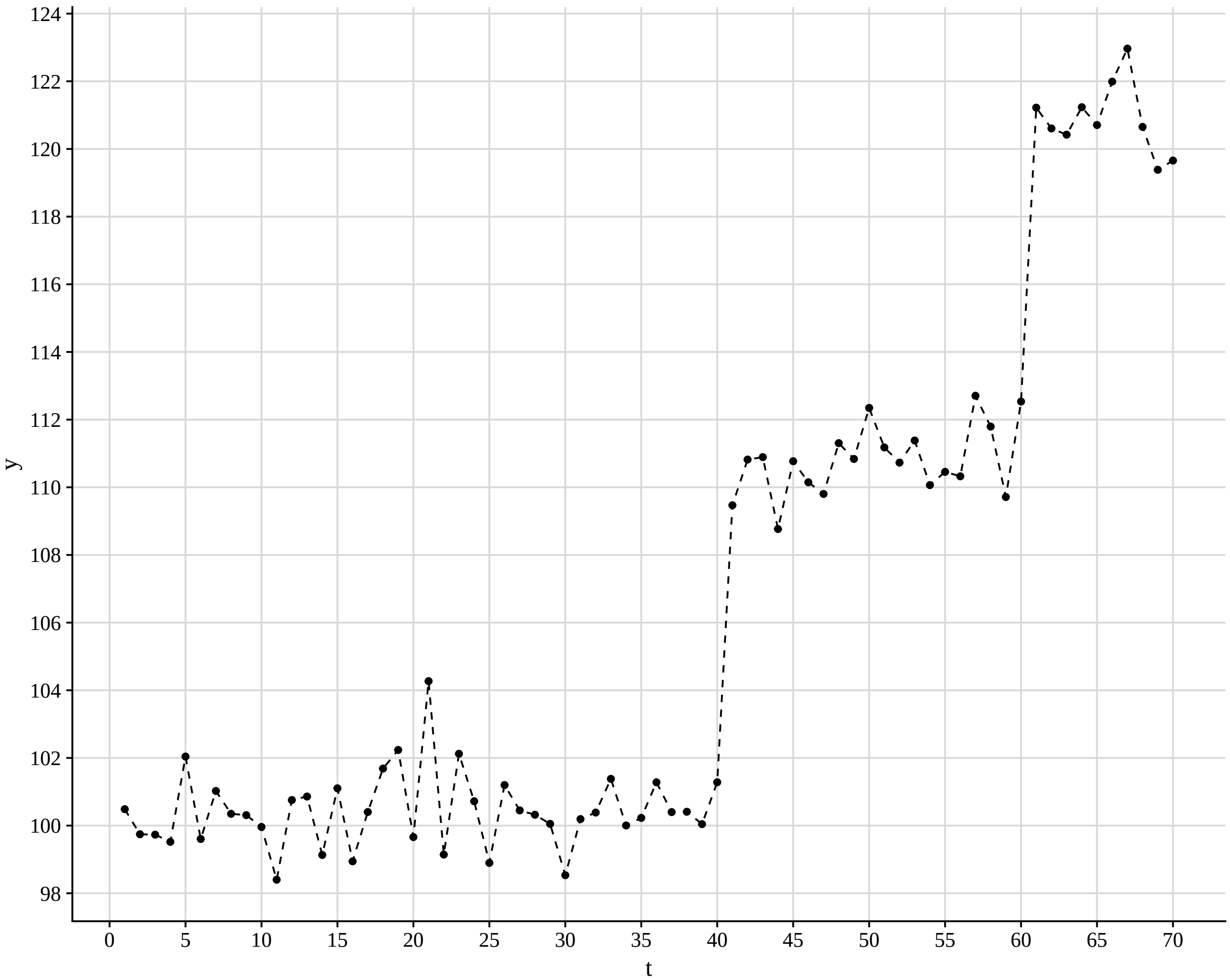



```
##     t      y      f        q         e         H         L l
## 1   1 100.06 100.00 101.0000  0.005906 1.000e+00 1.000e+00 1
## 2   2  99.98 100.06   1.0501 -0.072702 1.000e+00 1.000e+00 1
## 3   3 100.40 100.02   0.5288  0.520070 1.000e+00 1.000e+00 1
## 4   4  99.64 100.16   0.3850 -0.830693 1.000e+00 1.000e+00 1
## 5   5  99.76 100.01   0.3390 -0.433305 1.000e+00 1.000e+00 1
## 6   6 100.39  99.95   0.2817  0.828823 1.000e+00 1.000e+00 1
## 7   7 100.04 100.04   0.2619 -0.010396 1.000e+00 1.000e+00 1
## 8   8  99.71 100.04   0.2247 -0.689692 1.000e+00 1.000e+00 1
## 9   9  99.87  99.98   0.2085 -0.256097 1.000e+00 1.000e+00 1
## 10 10 100.50  99.96   0.1868  1.232935 1.000e+00 1.000e+00 1
## 11 11 100.22 100.05   0.1939  0.402009 5.970e+02 5.970e+02 1
## 12 12 100.01 100.07   0.1790 -0.151071 5.455e+03 5.455e+03 1
## 13 13 100.27 100.06   0.1645  0.521995 3.695e+02 3.695e+02 1
## 14 14 100.96 100.09   0.1552  2.198895 4.513e-01 4.513e-01 1
## 15 15 100.77 100.20   0.1940  1.290263 1.710e+01 7.717e+00 2
## 16 16 101.08 100.27   0.2014  1.788822 2.327e+00 2.327e+00 1
## 17 17 101.17 100.37   0.2267  1.670946 3.729e+00 3.729e+00 1
## 18 18 100.39 100.47   0.2486 -0.164802 5.763e+03 5.763e+03 1
## 19 19 100.98 100.46   0.2354  1.064636 4.216e+01 4.216e+01 1
## 20 20 101.28 100.52   0.2365  1.564744 5.703e+00 5.703e+00 1
## 21 21 102.91 100.61   0.2523  4.593624 3.122e-05 1.000e+00 0
```

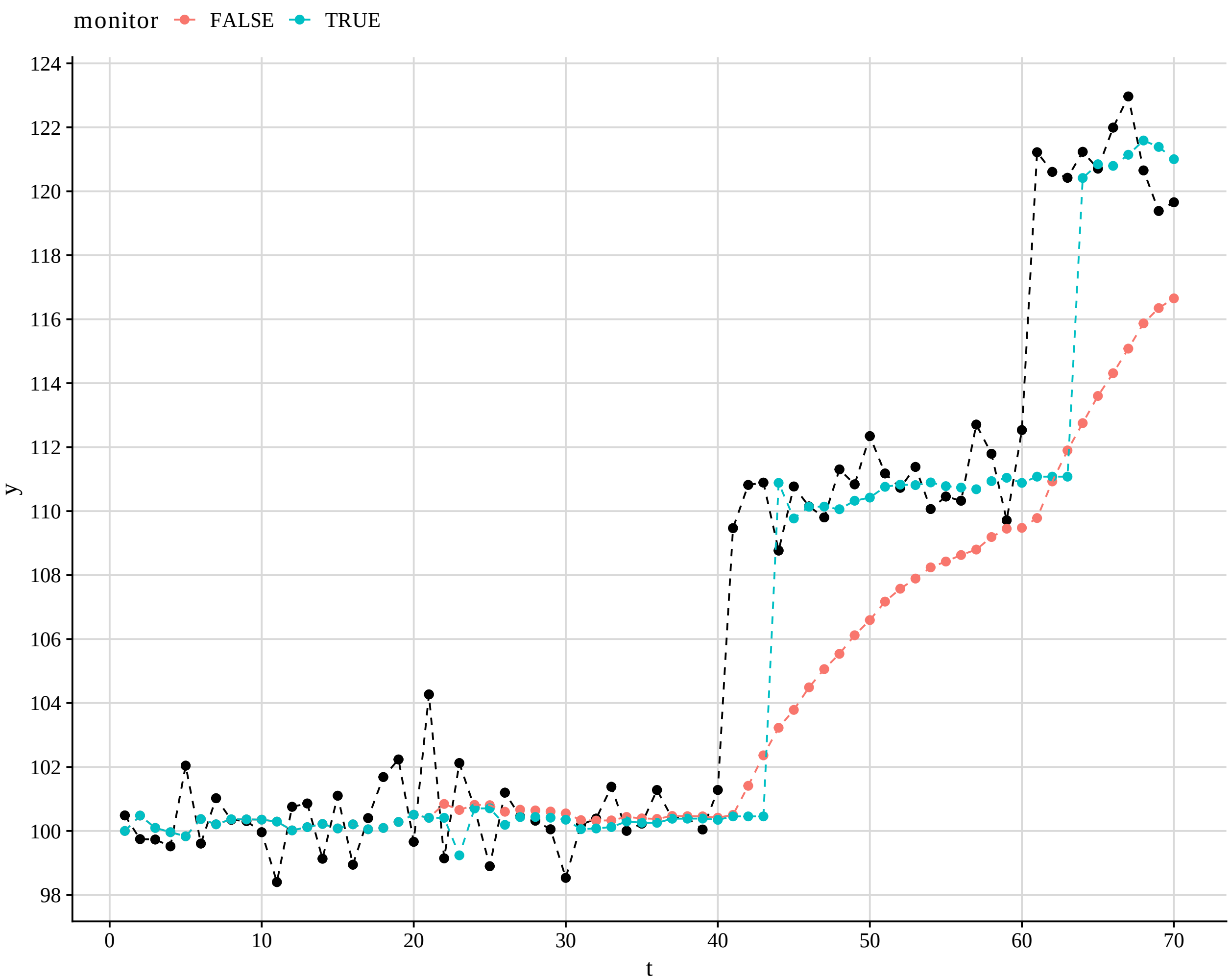
```
## 22 22 100.54 100.61   3.0569 -0.039535 3.492e+03 3.492e+03 1
## 23 23 100.20 100.54   0.4340 -0.521028 2.396e+04 2.396e+04 1
## 24 24 100.21 100.37   0.3238 -0.284362 9.297e+03 9.297e+03 1
## 25 25 100.06 100.31   0.2789 -0.476265 2.003e+04 2.003e+04 1
## 26 26 101.04 100.24   0.2543  1.591679 5.121e+00 5.121e+00 1
## 27 27 100.29 100.43   0.2591 -0.270771 8.805e+03 8.805e+03 1
## 28 28 100.71 100.40   0.2437  0.628259 2.415e+02 2.415e+02 1
## 29 29 100.56 100.46   0.2339  0.215907 1.257e+03 1.257e+03 1
## 30 30 100.39 100.48   0.2229 -0.192873 6.448e+03 6.448e+03 1
## 31 31 100.40 100.46   0.2133 -0.144727 5.318e+03 5.318e+03 1
## 32 32  99.77 100.45   0.2047 -1.503593 1.220e+06 1.220e+06 1
## 33 33 100.49 100.36   0.2113  0.298013 9.050e+02 9.050e+02 1
## 34 34 100.39 100.37   0.2042  0.039400 2.546e+03 2.546e+03 1
## 35 35  99.90 100.38   0.1972 -1.072491 2.175e+05 2.175e+05 1
## 36 36 100.59 100.31   0.1972  0.629152 2.407e+02 2.407e+02 1
## 37 37  99.96 100.35   0.1932 -0.879131 1.004e+05 1.004e+05 1
## 38 38 100.44 100.30   0.1914  0.304634 8.814e+02 8.814e+02 1
## 39 39 100.80 100.32   0.1864  1.111950 3.489e+01 3.489e+01 1
## 40 40 101.00 100.37   0.1871  1.455834 8.817e+00 8.817e+00 1
## 41 41 109.79 100.45   0.1919 21.329571 2.637e-34 1.000e+00 0
## 42 42 109.70 100.45   2.3525  6.033368 9.847e-08 1.000e+00 0
## 43 43 109.67 100.45 218.4051  0.623938 2.457e+02 2.457e+02 1
## 44 44 110.68 109.66   0.3536  1.705067 3.253e+00 3.253e+00 1
## 45 45 110.26 110.19   0.2776  0.124457 1.812e+03 1.812e+03 1
## 46 46 109.85 110.22   0.2413 -0.758511 6.195e+04 6.195e+04 1
## 47 47 110.66 110.11   0.2243  1.154759 2.940e+01 2.940e+01 1
## 48 48 110.51 110.24   0.2171  0.565532 3.104e+02 3.104e+02 1
## 49 49 110.55 110.30   0.2082  0.544045 3.383e+02 3.383e+02 1
## 50 50 110.45 110.35   0.2010  0.232877 1.174e+03 1.174e+03 1
```

Outlier and Two Level Change

```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=70, V=1, W=0.01)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [20, 21, 40, 60],
>>>                 "level_mean_shift": [5, -5, 10, 10],
>>>                 "level_var_shift": [1, 1, 1, 1]})
```




```
## Potential outlier detected at time 21 with H=1.0052e-03, L=1.0052e-03 and l=1
## Potential outlier detected at time 41 with H=2.3972e-13, L=2.3972e-13 and l=1
## Potential outlier detected at time 42 with H=1.5420e-02, L=1.5420e-02 and l=1
## Potential outlier detected at time 61 with H=5.0004e-15, L=5.0004e-15 and l=1
## Potential outlier detected at time 62 with H=5.5112e-02, L=5.5112e-02 and l=1
```



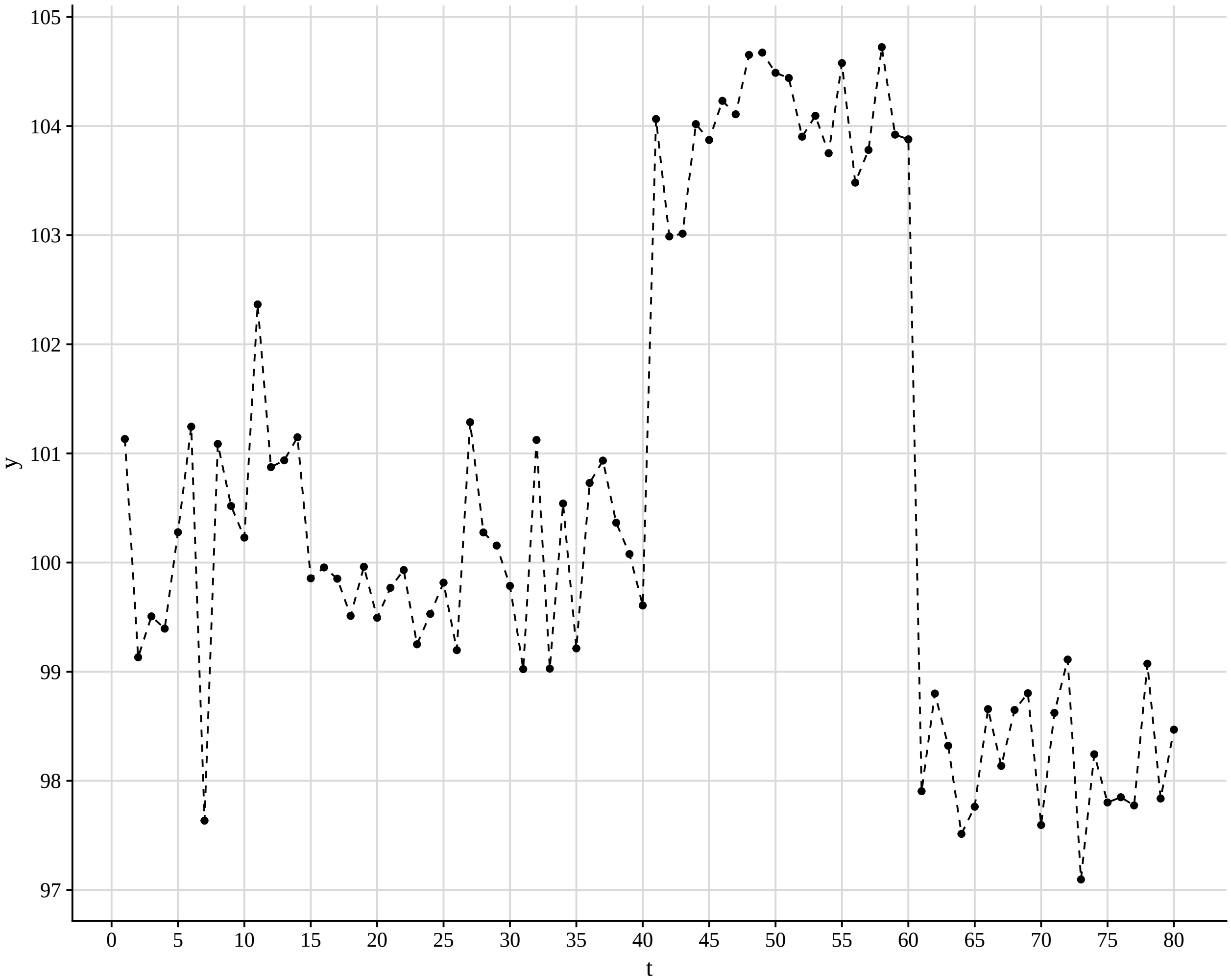

```
##      t      y      f        q         e         H         L l
## 1    1 100.49 100.00 101.0000  0.048367 1.000e+00 1.000e+00 1
## 2    2  99.74 100.48   1.0525 -0.718276 1.000e+00 1.000e+00 1
## 3    3  99.73 100.10   0.6649 -0.445731 1.000e+00 1.000e+00 1
## 4    4  99.52  99.96   0.4735 -0.642503 1.000e+00 1.000e+00 1
## 5    5 102.04  99.83   0.3923  3.525704 1.000e+00 1.000e+00 1
## 6    6  99.61 100.37   1.0952 -0.732493 1.000e+00 1.000e+00 1
## 7    7 101.02 100.21   0.9953  0.817334 1.000e+00 1.000e+00 1
## 8    8 100.35 100.36   0.9354 -0.016446 1.000e+00 1.000e+00 1
## 9    9 100.31 100.36   0.8193 -0.057104 1.000e+00 1.000e+00 1
## 10 10  99.96 100.35   0.7292 -0.459832 1.000e+00 1.000e+00 1
## 11 11  98.40 100.29   0.6707 -2.310043 3.071e+07 3.071e+07 1
## 12 12 100.76 100.02   0.9064  0.775188 1.342e+02 1.342e+02 1
## 13 13 100.86 100.12   0.8732  0.790556 1.262e+02 1.262e+02 1
## 14 14  99.13 100.22   0.8455 -1.181524 3.364e+05 3.364e+05 1
## 15 15 101.10 100.08   0.8641  1.100947 3.646e+01 3.646e+01 1
## 16 16  98.94 100.21   0.8724 -1.351576 6.642e+05 6.642e+05 1
## 17 17 100.40 100.05   0.9120  0.366429 6.883e+02 6.883e+02 1
## 18 18 101.69 100.09   0.8658  1.710841 3.179e+00 3.179e+00 1
## 19 19 102.24 100.28   0.9514  2.003486 9.862e-01 9.862e-01 1
## 20 20  99.66 100.51   1.0926 -0.810314 7.621e+04 7.516e+04 2
## 21 21 104.27 100.41   1.0729  3.725633 1.005e-03 1.000e+00 0
```

```
## 22 22  99.14 100.41   12.9975 -0.351295 1.215e+04 1.215e+04 1
## 23 23 102.12  99.24    1.8561  2.118274 6.231e-01 6.231e-01 1
## 24 24 100.72 100.70    1.6467  0.014348 2.815e+03 1.754e+03 2
## 25 25  98.90 100.71    1.4135 -1.522634 1.317e+06 1.317e+06 1
## 26 26 101.20 100.19    1.4001  0.851742 9.879e+01 9.879e+01 1
## 27 27 100.45 100.43    1.3329  0.014637 2.811e+03 2.811e+03 1
## 28 28 100.32 100.44    1.2500 -0.103953 4.518e+03 4.518e+03 1
## 29 29 100.05 100.41    1.1829 -0.334215 1.135e+04 1.135e+04 1
## 30 30  98.53 100.35    1.1301 -1.709695 2.782e+06 2.782e+06 1
## 31 31 100.19 100.06    1.1890  0.125275 1.806e+03 1.806e+03 1
## 32 32 100.39 100.08    1.1409  0.289497 9.364e+02 9.364e+02 1
## 33 33 101.38 100.12    1.1002  1.200294 2.450e+01 2.450e+01 1
## 34 34 100.00 100.30    1.1082 -0.278882 9.095e+03 9.095e+03 1
## 35 35 100.23 100.26    1.0728 -0.028141 3.336e+03 3.336e+03 1
## 36 36 101.28 100.25    1.0377  1.007153 5.306e+01 5.306e+01 1
## 37 37 100.40 100.38    1.0344  0.015935 2.797e+03 2.797e+03 1
## 38 38 100.41 100.38    1.0034  0.024566 2.702e+03 2.702e+03 1
## 39 39 100.04 100.39    0.9744 -0.347833 1.198e+04 1.198e+04 1
## 40 40 101.28 100.35    0.9503  0.957780 6.464e+01 6.464e+01 1
## 41 41 109.47 100.46    0.9465  9.264824 2.397e-13 1.000e+00 0
## 42 42 110.82 100.46   11.6002  3.043024 1.542e-02 1.000e+00 0
## 43 43 110.89 100.46 1076.9772  0.318020 8.354e+02 8.354e+02 1
## 44 44 108.77 110.88    1.7314 -1.608805 1.858e+06 1.858e+06 1
## 45 45 110.77 109.77    1.3492  0.861384 9.506e+01 9.506e+01 1
## 46 46 110.15 110.14    1.1933  0.007497 2.893e+03 2.893e+03 1
## 47 47 109.80 110.14    1.0943 -0.323459 1.087e+04 1.087e+04 1
## 48 48 111.30 110.06    1.0306  1.227059 2.202e+01 2.202e+01 1
## 49 49 110.84 110.32    1.0139  0.509124 3.890e+02 3.890e+02 1
## 50 50 112.35 110.42    0.9785  1.944294 1.250e+00 1.250e+00 1
## 51 51 111.18 110.76    1.0199  0.413232 5.708e+02 5.708e+02 1
## 52 52 110.73 110.83    0.9911 -0.099277 4.434e+03 4.434e+03 1
## 53 53 111.38 110.81    0.9626  0.579885 2.931e+02 2.931e+02 1
## 54 54 110.07 110.90    0.9431 -0.856058 9.151e+04 9.151e+04 1
## 55 55 110.46 110.78    0.9325 -0.335897 1.143e+04 1.143e+04 1
## 56 56 110.32 110.74    0.9122 -0.431681 1.676e+04 1.676e+04 1
## 57 57 112.71 110.68    0.8946  2.137120 5.778e-01 5.778e-01 1
## 58 58 111.79 110.94    0.9492  0.877814 8.901e+01 5.143e+01 2
## 59 59 109.71 111.04    0.9424 -1.371258 7.186e+05 7.186e+05 1
## 60 60 112.53 110.88    0.9544  1.690431 3.450e+00 3.450e+00 1
## 61 61 121.22 111.08    0.9827 10.232314 5.000e-15 1.000e+00 0
## 62 62 120.61 111.08   12.2306  2.724598 5.511e-02 1.000e+00 0
## 63 63 120.42 111.08 1137.0190  0.277123 9.839e+02 9.839e+02 1
## 64 64 121.23 120.42    1.8053  0.609810 2.600e+02 2.600e+02 1
## 65 65 120.71 120.85    1.3414 -0.119504 4.808e+03 4.808e+03 1
## 66 66 121.99 120.80    1.1742  1.103500 3.609e+01 3.609e+01 1
## 67 67 122.97 121.14    1.1057  1.733631 2.902e+00 2.902e+00 1
## 68 68 120.65 121.59    1.0963 -0.893458 1.063e+05 1.063e+05 1
## 69 69 119.38 121.39    1.0635 -1.942896 7.072e+06 7.072e+06 1
## 70 70 119.66 121.00    1.0872 -1.291934 5.232e+05 5.232e+05 1
```

Bilateral Level Change

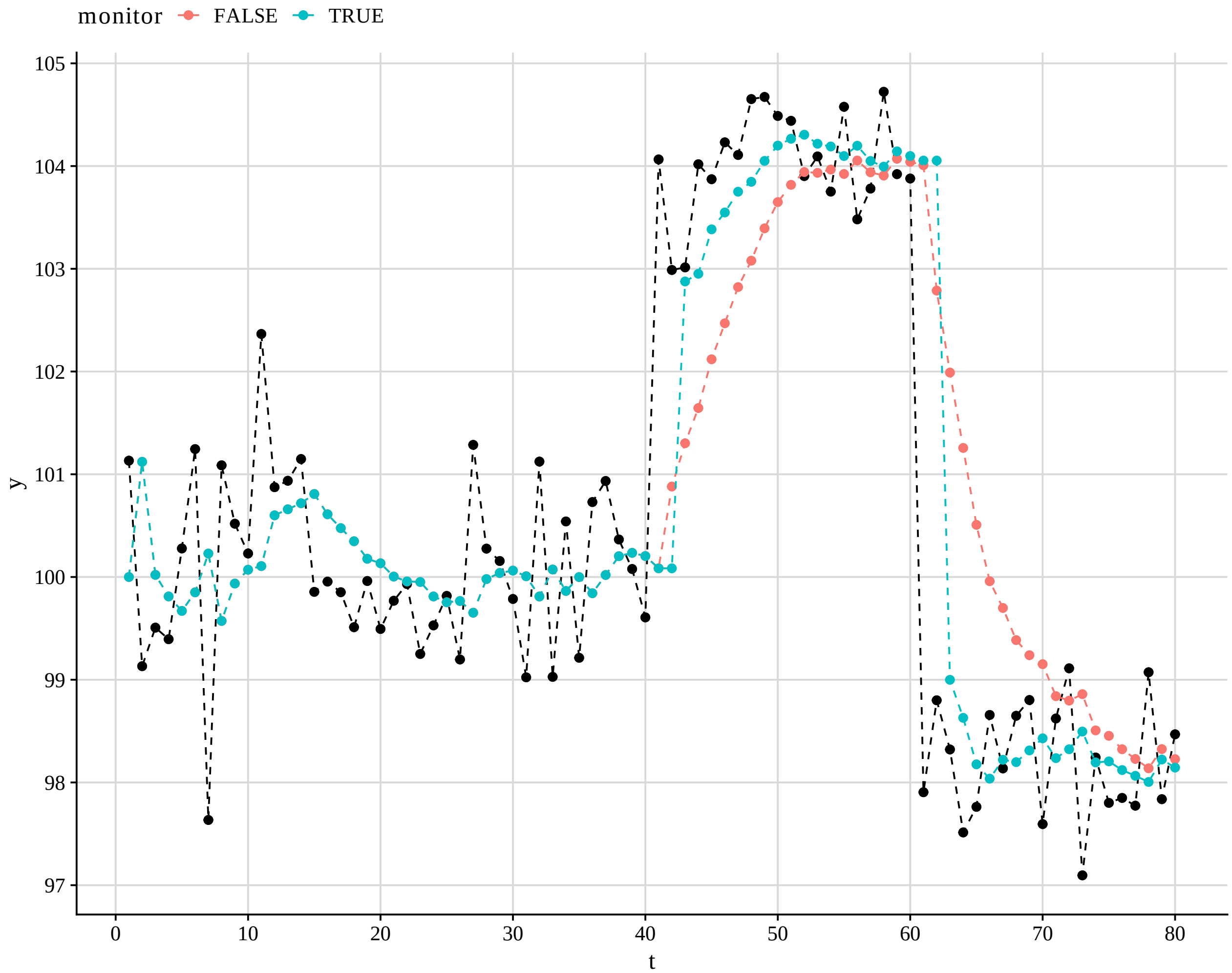
```
>>> np.random.seed(66)
>>> y1 = np.random.normal(loc=100, scale=0.8, size=40)
```

```
>>> y2 = np.random.normal(loc=104, scale=0.5, size=20)
>>> y3 = np.random.normal(loc=98, scale=0.5, size=20)
>>> y = np.concatenate([y1, y2, y3])
>>> t = np.arange(0, len(y)) + 1
>>> df_simulated = pd.DataFrame({"t": t, "y": y})
```




```
## Upper potential outlier detected at time 41 with H=6.8521e-05, L=6.8521e-05 and l=1
## Lower potential outlier detected at time 61 with H=1.4223e-10, L=1.4223e-10 and l=1
```

```
>>> ggplot(py$df_res, aes(x = t, y = y)) +
>>>   geom_point(size = 2) +
>>>   geom_line(linetype = "dashed") +
>>>   geom_point(aes(y = f, col = monitor), size = 2) +
>>>   geom_line(aes(y = f, col = monitor), linetype = "dashed") +
>>>   scale_x_continuous(breaks = scales::pretty_breaks(10)) +
>>>   scale_y_continuous(breaks = scales::pretty_breaks(10))
```




```
>>> py$df_tmp %>%
>>>   select(t, y, f, q, e, H_lower, L_lower, l_lower,
>>>          H_upper, L_upper, l_upper)
```

```
##       t      y      f        q        e   H_lower   L_lower l_lower
## 1     1 101.13 100.00 101.0000  0.11269 1.000e+00 1.000e+00       1
## 2     2  99.13 101.12   1.1330 -1.86908 1.000e+00 1.000e+00       1
## 3     3  99.51 100.02   1.5682 -0.41039 1.000e+00 1.000e+00       1
## 4     4  99.39  99.81   1.1097 -0.39523 1.000e+00 1.000e+00       1
## 5     5 100.28  99.67   0.8686  0.65198 1.000e+00 1.000e+00       1
## 6     6 101.25  99.85   0.7570  1.60292 1.000e+00 1.000e+00       1
## 7     7  97.63 100.23   0.9045 -2.72678 1.000e+00 1.000e+00       1
## 8     8 101.09  99.57   1.6048  1.19583 1.000e+00 1.000e+00       1
## 9     9 100.52  99.94   1.6612  0.45189 1.000e+00 1.000e+00       1
## 10   10 100.23 100.07   1.5154  0.12842 1.000e+00 1.000e+00       1
## 11   11 102.37 100.11   1.3706  1.92990 6.713e+06 6.713e+06       1
## 12   12 100.87 100.60   1.6732  0.21167 6.951e+03 6.951e+03       1
## 13   13 100.94 100.66   1.5441  0.22333 7.283e+03 7.283e+03       1
## 14   14 101.15 100.72   1.4349  0.35921 1.254e+04 1.254e+04       1
## 15   15  99.86 100.81   1.3483 -0.82044 1.120e+02 1.120e+02       1
## 16   16  99.96 100.61   1.3183 -0.57070 3.041e+02 3.041e+02       1
## 17   17  99.85 100.48   1.2641 -0.55467 3.242e+02 3.242e+02       1
```

```
## 18 18  99.51 100.35   1.2141 -0.75940 1.429e+02 1.429e+02        1
## 19 19  99.96 100.18   1.1859 -0.19892 1.345e+03 1.345e+03        1
## 20 20  99.49 100.13   1.1281 -0.60229 2.680e+02 2.680e+02        1
## 21 21  99.77 100.00   1.0933 -0.22518 1.211e+03 1.211e+03        1
## 22 22  99.93  99.96   1.0456 -0.02454 2.702e+03 2.702e+03        1
## 23 23  99.25  99.95   0.9998 -0.70097 1.806e+02 1.806e+02        1
## 24 24  99.53  99.81   0.9783 -0.28471 9.545e+02 9.545e+02        1
## 25 25  99.82  99.75   0.9421  0.06360 3.845e+03 3.845e+03        1
## 26 26  99.20  99.77   0.9059 -0.59823 2.723e+02 2.723e+02        1
## 27 27 101.29  99.65   0.8842  1.73763 3.111e+06 3.111e+06        1
## 28 28 100.28  99.98   0.9478  0.30431 1.007e+04 1.007e+04        1
## 29 29 100.16 100.04   0.9181  0.12169 4.850e+03 4.850e+03        1
## 30 30  99.79 100.06   0.8879 -0.29325 9.224e+02 9.224e+02        1
## 31 31  99.02 100.01   0.8616 -1.05991 4.296e+01 4.296e+01        1
## 32 32 101.12  99.81   0.8649  1.41205 8.459e+05 8.459e+05        1
## 33 33  99.03 100.07   0.8909 -1.10743 3.553e+01 3.553e+01        1
## 34 34 100.54  99.86   0.8968  0.71473 5.200e+04 5.200e+04        1
## 35 35  99.21 100.00   0.8843 -0.83581 1.053e+02 1.053e+02        1
## 36 36 100.73  99.84   0.8769  0.94767 1.320e+05 1.320e+05        1
## 37 37 100.93 100.02   0.8744  0.97843 1.493e+05 1.493e+05        1
## 38 38 100.37 100.20   0.8734  0.17387 5.976e+03 5.976e+03        1
## 39 39 100.08 100.24   0.8517 -0.17034 1.508e+03 1.508e+03        1
## 40 40  99.61 100.20   0.8310 -0.65477 2.172e+02 2.172e+02        1
## 41 41 104.06 100.08   0.8194  4.39709 1.297e+11 1.297e+11        1
## 42 42 102.99 100.08  17.0461  0.70342 4.970e+04 4.970e+04        1
## 43 43 103.01 102.88   1.4261  0.11467 4.716e+03 4.716e+03        1
## 44 44 104.02 102.95   1.0645  1.03340 1.860e+05 1.860e+05        1
## 45 45 103.87 103.38   0.9551  0.49947 2.198e+04 2.198e+04        1
## 46 46 104.23 103.55   0.8853  0.72574 5.434e+04 5.434e+04        1
## 47 47 104.11 103.75   0.8450  0.38908 1.413e+04 1.413e+04        1
## 48 48 104.65 103.85   0.8101  0.89455 1.067e+05 1.067e+05        1
## 49 49 104.67 104.05   0.7934  0.69911 4.885e+04 4.885e+04        1
## 50 50 104.49 104.20   0.7758  0.32738 1.104e+04 1.104e+04        1
## 51 51 104.44 104.27   0.7552  0.20053 6.648e+03 6.648e+03        1
## 52 52 103.90 104.31   0.7360 -0.46916 4.564e+02 4.564e+02        1
## 53 53 104.09 104.22   0.7213 -0.14506 1.669e+03 1.669e+03        1
## 54 54 103.75 104.19   0.7052 -0.52314 3.678e+02 3.678e+02        1
## 55 55 104.58 104.10   0.6936  0.57543 2.978e+04 2.978e+04        1
## 56 56 103.48 104.20   0.6835 -0.86654 9.312e+01 9.312e+01        1
## 57 57 103.78 104.05   0.6792 -0.32603 8.091e+02 8.091e+02        1
## 58 58 104.72 103.99   0.6676  0.89203 1.057e+05 1.057e+05        1
## 59 59 103.92 104.14   0.6645 -0.27237 1.003e+03 1.003e+03        1
## 60 60 103.88 104.10   0.6534 -0.27149 1.006e+03 1.006e+03        1
## 61 61  97.91 104.05   0.6429 -7.66839 1.422e-10 1.000e+00        0
## 62 62  98.80 104.05  13.5188 -1.42887 9.821e+00 9.821e+00        1
## 63 63  98.32  99.00   1.1488 -0.63253 2.374e+02 2.374e+02        1
## 64 64  97.51  98.63   0.8691 -1.19660 2.487e+01 2.487e+01        1
## 65 65  97.76  98.18   0.7838 -0.46765 4.592e+02 4.592e+02        1
## 66 66  98.66  98.04   0.7298  0.72567 5.432e+04 5.432e+04        1
## 67 67  98.14  98.22   0.6987 -0.10050 1.994e+03 1.994e+03        1
## 68 68  98.65  98.20   0.6719  0.55019 2.692e+04 2.692e+04        1
## 69 69  98.80  98.31   0.6539  0.60663 3.374e+04 3.374e+04        1
## 70 70  97.59  98.43   0.6401 -1.04345 4.589e+01 4.589e+01        1
## 71 71  98.62  98.24   0.6353  0.48382 2.065e+04 2.065e+04        1
```

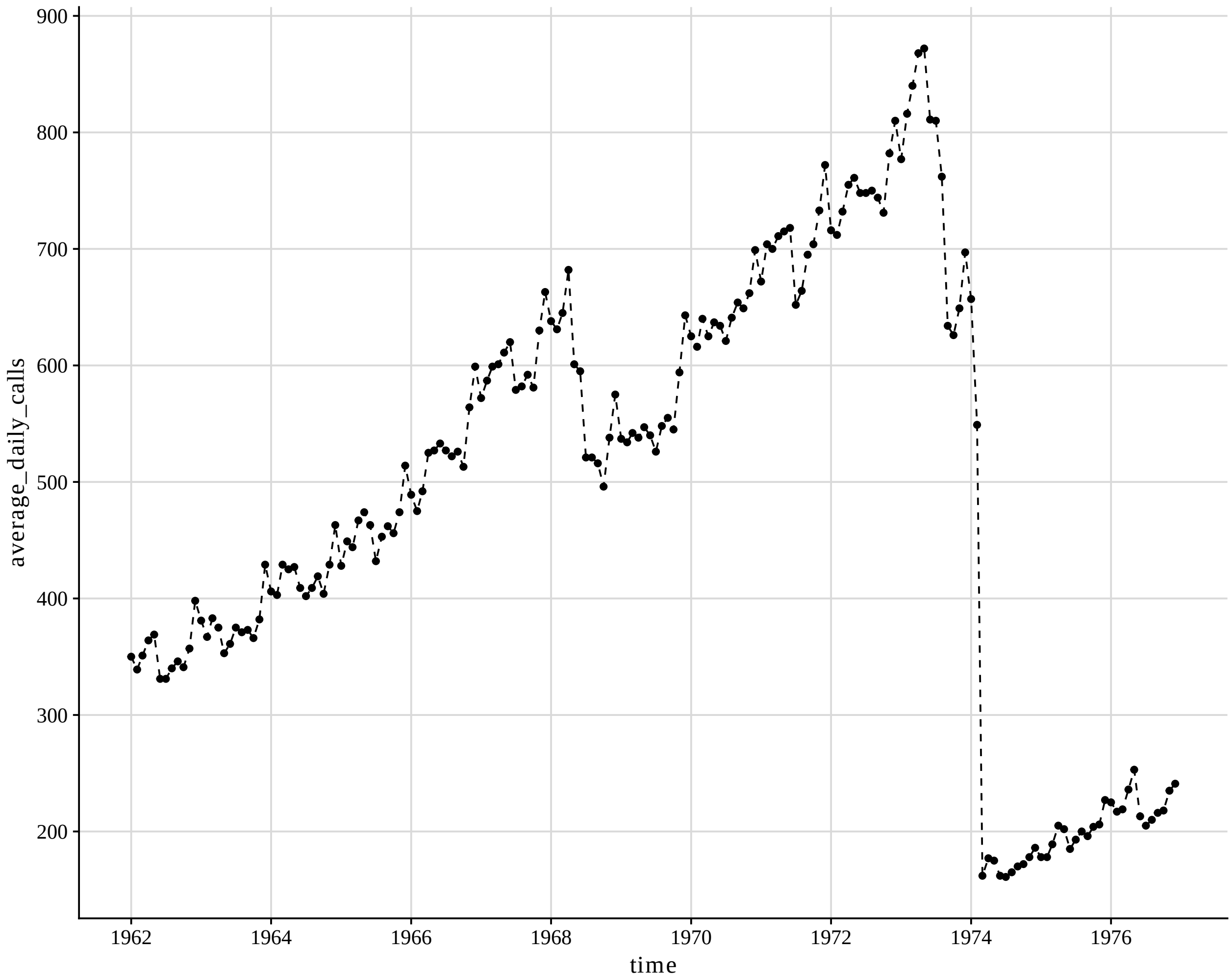
```
## 72 72  99.11  98.32   0.6242  0.99725 1.610e+05 1.610e+05        1
## 73 73  97.10  98.50   0.6209 -1.77631 2.447e+00 2.447e+00        1
## 74 74  98.24  98.20   0.6370  0.06006 3.790e+03 3.790e+03        1
## 75 75  97.80  98.21   0.6264 -0.51024 3.872e+02 3.872e+02        1
## 76 76  97.85  98.12   0.6187 -0.34507 7.497e+02 7.497e+02        1
## 77 77  97.77  98.06   0.6103 -0.37192 6.734e+02 6.734e+02        1
## 78 78  99.07  98.00   0.6025  1.37608 7.326e+05 7.326e+05        1
## 79 79  97.84  98.22   0.6087 -0.49467 4.121e+02 4.121e+02        1
## 80 80  98.47  98.14   0.6023  0.41762 1.584e+04 1.584e+04        1
##      H_upper   L_upper l_upper
## 1  1.000e+00 1.000e+00       1
## 2  1.000e+00 1.000e+00       1
## 3  1.000e+00 1.000e+00       1
## 4  1.000e+00 1.000e+00       1
## 5  1.000e+00 1.000e+00       1
## 6  1.000e+00 1.000e+00       1
## 7  1.000e+00 1.000e+00       1
## 8  1.000e+00 1.000e+00       1
## 9  1.000e+00 1.000e+00       1
## 10 1.000e+00 1.000e+00       1
## 11 1.324e+00 1.324e+00       1
## 12 1.278e+03 1.278e+03       1
## 13 1.220e+03 1.220e+03       1
## 14 7.085e+02 7.085e+02       1
## 15 7.936e+04 7.936e+04       1
## 16 2.923e+04 2.923e+04       1
## 17 2.741e+04 2.741e+04       1
## 18 6.217e+04 6.217e+04       1
## 19 6.606e+03 6.606e+03       1
## 20 3.316e+04 3.316e+04       1
## 21 7.337e+03 7.337e+03       1
## 22 3.288e+03 3.288e+03       1
## 23 4.921e+04 4.921e+04       1
## 24 9.310e+03 9.310e+03       1
## 25 2.311e+03 2.311e+03       1
## 26 3.263e+04 3.263e+04       1
## 27 2.856e+00 2.856e+00       1
## 28 8.825e+02 8.825e+02       1
## 29 1.832e+03 1.832e+03       1
## 30 9.633e+03 9.633e+03       1
## 31 2.068e+05 2.068e+05       1
## 32 1.050e+01 1.050e+01       1
## 33 2.501e+05 2.501e+05       1
## 34 1.709e+02 1.709e+02       1
## 35 8.439e+04 8.439e+04       1
## 36 6.731e+01 6.731e+01       1
## 37 5.952e+01 5.952e+01       1
## 38 1.487e+03 1.487e+03       1
## 39 5.892e+03 5.892e+03       1
## 40 4.091e+04 4.091e+04       1
## 41 6.852e-05 1.000e+00       0
## 42 1.788e+02 1.788e+02       1
## 43 1.884e+03 1.884e+03       1
## 44 4.777e+01 4.777e+01       1
```

```
## 45 4.043e+02 4.043e+02       1
## 46 1.635e+02 1.635e+02       1
## 47 6.287e+02 6.287e+02       1
## 48 8.325e+01 8.325e+01       1
## 49 1.819e+02 1.819e+02       1
## 50 8.047e+02 8.047e+02       1
## 51 1.337e+03 1.337e+03       1
## 52 1.947e+04 1.947e+04       1
## 53 5.325e+03 5.325e+03       1
## 54 2.416e+04 2.416e+04       1
## 55 2.984e+02 2.984e+02       1
## 56 9.543e+04 9.543e+04       1
## 57 1.098e+04 1.098e+04       1
## 58 8.409e+01 8.409e+01       1
## 59 8.861e+03 8.861e+03       1
## 60 8.830e+03 8.830e+03       1
## 61 6.248e+16 6.248e+16       1
## 62 9.048e+05 9.048e+05       1
## 63 3.743e+04 3.743e+04       1
## 64 3.573e+05 3.573e+05       1
## 65 1.935e+04 1.935e+04       1
## 66 1.636e+02 1.636e+02       1
## 67 4.456e+03 4.456e+03       1
## 68 3.300e+02 3.300e+02       1
## 69 2.633e+02 2.633e+02       1
## 70 1.936e+05 1.936e+05       1
## 71 4.304e+02 4.304e+02       1
## 72 5.520e+01 5.520e+01       1
## 73 3.632e+06 3.632e+06       1
## 74 2.344e+03 2.344e+03       1
## 75 2.295e+04 2.295e+04       1
## 76 1.185e+04 1.185e+04       1
## 77 1.320e+04 1.320e+04       1
## 78 1.213e+01 1.213e+01       1
## 79 2.156e+04 2.156e+04       1
## 80 5.609e+02 5.609e+02       1
```

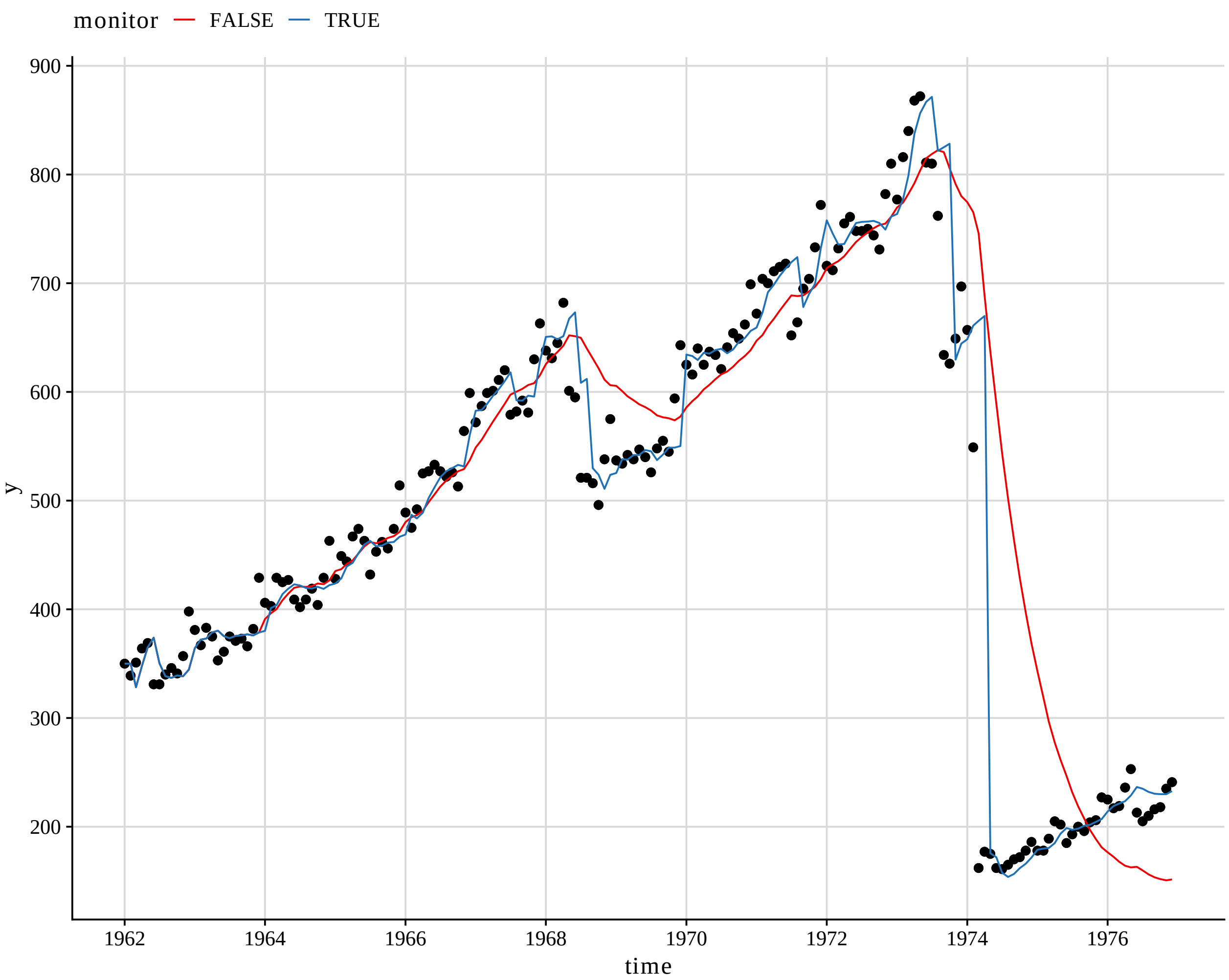
Real data applications

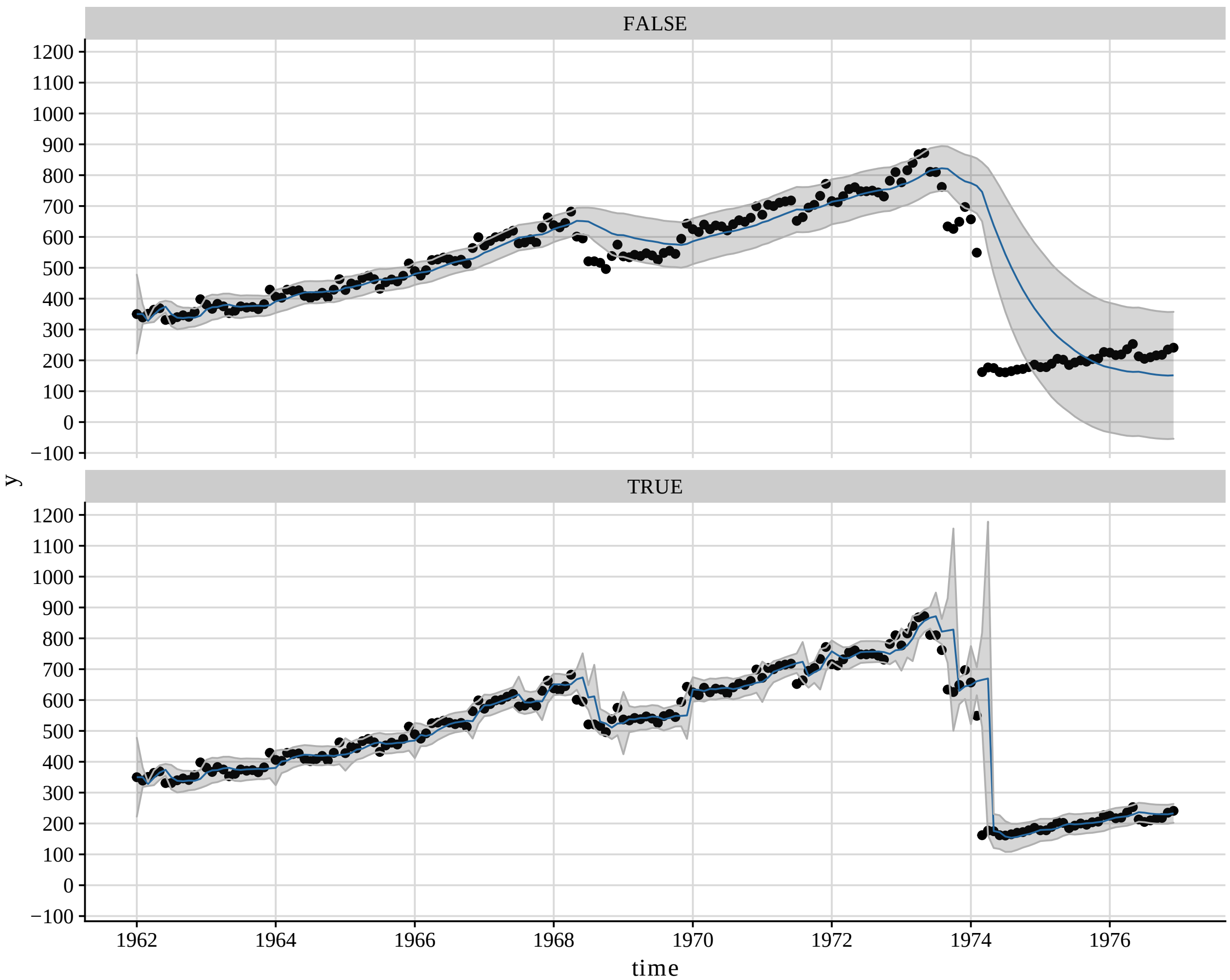
Telephone Calls

```
## Rows: 180
## Columns: 2
## $ time                <date> 1962-01-01, 1962-02-01, 1962-03-01, 196~
## $ average_daily_calls <dbl> 350, 339, 351, 364, 369, 331, 331, 340, ~
```




```
## Upper potential outlier detected at time 24 with H=6.1828e-03, L=6.1828e-03 and l=1
## Upper potential outlier detected at time 36 with H=4.6950e-02, L=4.6950e-02 and l=1
## Upper potential outlier detected at time 48 with H=1.0667e-02, L=1.0667e-02 and l=1
## Upper parametric change detected at time 61 with H=3.7151e+02, L=8.6657e-01 and l=3
## Lower parametric change detected at time 69 with H=7.3490e+01, L=2.1113e+01 and l=3
## Upper parametric change detected at time 73 with H=1.0179e+03, L=5.1031e+00 and l=3
## Lower potential outlier detected at time 77 with H=4.7737e-04, L=4.7737e-04 and l=1
## Lower potential outlier detected at time 79 with H=8.2908e-05, L=8.2908e-05 and l=1
## Upper potential outlier detected at time 84 with H=5.8788e-02, L=5.8788e-02 and l=1
## Upper potential outlier detected at time 95 with H=6.8930e-02, L=6.8930e-02 and l=1
## Upper potential outlier detected at time 108 with H=7.2666e-02, L=7.2666e-02 and l=1
## Lower potential outlier detected at time 115 with H=1.2539e-04, L=1.2539e-04 and l=1
## Lower parametric change detected at time 121 with H=9.8175e+00, L=9.8175e+00 and l=3
## Upper potential outlier detected at time 132 with H=2.8683e-02, L=2.8683e-02 and l=1
## Upper parametric change detected at time 137 with H=1.3781e+00, L=1.4891e-02 and l=3
## Lower potential outlier detected at time 138 with H=9.4740e-03, L=9.4740e-03 and l=1
## Lower potential outlier detected at time 140 with H=2.8301e-02, L=2.8301e-02 and l=1
## Lower potential outlier detected at time 141 with H=1.5974e-03, L=1.5974e-03 and l=1
## Upper potential outlier detected at time 144 with H=1.1437e-01, L=1.1437e-01 and l=1
## Lower potential outlier detected at time 146 with H=9.9659e-06, L=9.9659e-06 and l=1
## Lower potential outlier detected at time 147 with H=1.3965e-08, L=1.3965e-08 and l=1
```


monitor FALSE TRUE
900
800
700
600
500
400
300
200
y
1962
1964
1966
1968
1970
1972
1974
1976
time




```
##                    time   y     f       q        e   H_lower
## 1  1961-12-31 21:00:00 350 350.0 101.000  0.00000 1.000e+00
## 2  1962-01-31 21:00:00 339 350.0  53.653 -1.50175 1.000e+00
## 3  1962-02-28 21:00:00 351 328.3   4.409 10.80510 1.000e+00
## 4  1962-03-31 21:00:00 364 348.0  74.226  1.85430 1.000e+00
## 5  1962-04-30 21:00:00 369 365.0  83.023  0.43429 1.000e+00
## 6  1962-05-31 21:00:00 331 373.9  60.367 -5.52633 1.000e+00
## 7  1962-06-30 21:00:00 331 350.3 280.258 -1.15428 1.000e+00
## 8  1962-07-31 21:00:00 340 338.5 268.177  0.09294 1.000e+00
## 9  1962-08-31 21:00:00 346 336.9 223.792  0.60535 1.000e+00
## 10 1962-09-30 21:00:00 341 339.1 199.316  0.13333 1.000e+00
## 11 1962-10-31 21:00:00 357 338.4 174.300  1.40781 1.000e+00
## 12 1962-11-30 21:00:00 398 344.5 182.370  3.95840 1.000e+00
## 13 1962-12-31 21:00:00 381 364.2 377.451  0.86221 1.000e+00
## 14 1963-01-31 21:00:00 367 372.1 361.838 -0.27018 1.000e+00
## 15 1963-02-28 21:00:00 383 373.1 332.615  0.54294 1.000e+00
## 16 1963-03-31 21:00:00 375 378.7 312.363 -0.20765 1.000e+00
## 17 1963-04-30 21:00:00 353 380.4 290.231 -1.60653 1.000e+00
## 18 1963-05-31 21:00:00 361 375.3 311.412 -0.81209 1.000e+00
## 19 1963-06-30 21:00:00 375 373.5 302.112  0.08850 1.000e+00
## 20 1963-07-31 21:00:00 371 375.5 283.996 -0.26852 1.000e+00
## 21 1963-08-31 21:00:00 373 376.1 268.783 -0.18907 1.399e+03
```

```
## 22 1963-09-30 21:00:00 366 377.0  254.718 -0.68730 1.907e+02
## 23 1963-10-31 22:00:00 382 376.0  246.863  0.38242 1.376e+04
## 24 1963-11-30 22:00:00 429 378.7  236.325  3.27150 1.437e+09
## 25 1963-12-31 22:00:00 406 380.2  737.882  0.95073 1.336e+05
## 26 1964-01-31 22:00:00 403 401.1  339.721  0.10145 4.473e+03
## 27 1964-02-29 22:00:00 429 403.7  269.347  1.54092 1.416e+06
## 28 1964-03-31 21:00:00 425 414.0  259.528  0.68410 4.600e+04
## 29 1964-04-30 21:00:00 427 419.0  243.010  0.51466 2.336e+04
## 30 1964-05-31 21:00:00 409 423.0  230.631 -0.91990 7.522e+01
## 31 1964-06-30 21:00:00 402 422.0  226.004 -1.32784 1.471e+01
## 32 1964-07-31 21:00:00 409 419.7  229.720 -0.70405 1.784e+02
## 33 1964-08-31 21:00:00 419 419.2  225.306 -0.01128 2.849e+03
## 34 1964-09-30 21:00:00 404 420.7  218.311 -1.13070 3.237e+01
## 35 1964-10-31 21:00:00 429 418.8  220.263  0.68522 4.621e+04
## 36 1964-11-30 21:00:00 463 422.3  217.224  2.76467 1.893e+08
## 37 1964-12-31 21:00:00 428 423.7  670.553  0.16498 5.767e+03
## 38 1965-01-31 22:00:00 449 428.4  308.846  1.17159 3.233e+05
## 39 1965-02-28 22:00:00 444 439.4  258.623  0.28767 9.421e+03
## 40 1965-03-31 21:00:00 467 442.7  232.660  1.59164 1.735e+06
## 41 1965-04-30 21:00:00 474 451.6  232.174  1.47039 1.068e+06
## 42 1965-05-31 21:00:00 463 459.6  234.091  0.21942 7.170e+03
## 43 1965-06-30 21:00:00 432 463.0  226.561 -2.05920 7.892e-01
## 44 1965-07-31 21:00:00 453 458.0  243.320 -0.31957 8.302e+02
## 45 1965-08-31 21:00:00 462 458.7  238.287  0.21283 6.984e+03
## 46 1965-09-30 21:00:00 456 461.4  233.433 -0.35057 7.334e+02
## 47 1965-10-31 21:00:00 474 461.9  229.171  0.79614 7.201e+04
## 48 1965-11-30 21:00:00 514 466.7  227.455  3.13515 8.330e+08
## 49 1965-12-31 22:00:00 489 468.8  799.744  0.71489 5.203e+04
## 50 1966-01-31 22:00:00 475 486.9  324.948 -0.66095 2.119e+02
## 51 1966-02-28 22:00:00 492 483.6  263.148  0.51729 2.360e+04
## 52 1966-03-31 21:00:00 525 488.8  238.810  2.33980 3.459e+07
## 53 1966-04-30 21:00:00 527 502.3  250.430  1.56111 1.536e+06
## 54 1966-05-31 21:00:00 533 512.3  253.125  1.30124 5.431e+05
## 55 1966-06-30 21:00:00 527 521.5  254.930  0.34462 1.183e+04
## 56 1966-07-31 21:00:00 522 527.0  250.481 -0.31884 8.327e+02
## 57 1966-08-31 21:00:00 526 529.9  246.572 -0.24769 1.107e+03
## 58 1966-09-30 21:00:00 513 532.9  242.591 -1.27771 1.798e+01
## 59 1966-10-31 21:00:00 564 531.4  784.462  2.07913 1.220e+07
## 60 1966-11-30 22:00:00 599 560.3  345.623  3.43606 2.776e+09
## 61 1966-12-31 22:00:00 572 582.7  302.384  0.52061 2.392e+04
## 62 1967-01-31 22:00:00 587 583.4  277.399  0.21762 7.119e+03
## 63 1967-02-28 22:00:00 599 588.9  264.224  0.61950 3.552e+04
## 64 1967-03-31 21:00:00 601 596.4  258.300  0.28376 9.275e+03
## 65 1967-04-30 21:00:00 611 602.5  253.691  0.53094 2.493e+04
## 66 1967-05-31 21:00:00 620 609.9  250.921  0.63560 3.789e+04
## 67 1967-06-30 21:00:00 579 618.1  836.410 -2.47785 1.479e-01
## 68 1967-07-31 21:00:00 582 592.3  349.730 -1.83396 1.943e+00
## 69 1967-08-31 21:00:00 592 592.0  286.227 -0.92571 7.349e+01
## 70 1967-09-30 21:00:00 581 596.5  262.822 -0.95757 6.470e+01
## 71 1967-10-31 21:00:00 630 595.7  928.883  2.14392 1.580e+07
## 72 1967-11-30 22:00:00 663 628.0  349.863  3.17999 9.967e+08
## 73 1967-12-31 22:00:00 638 650.6  301.960  0.26863 8.730e+03
## 74 1968-01-31 22:00:00 631 651.1  284.202 -1.19203 2.533e+01
## 75 1968-02-29 22:00:00 645 648.4  282.576 -0.20144 1.332e+03
```

```
## 76 1968-03-31 21:00:00 682 651.2  279.036  1.84191 4.722e+06
## 77 1968-04-30 21:00:00 601 667.4  288.028 -3.91180 4.774e-04
## 78 1968-05-31 21:00:00 595 673.2 1542.864 -1.99143 1.035e+00
## 79 1968-06-30 21:00:00 521 608.3  403.266 -4.34945 8.291e-05
## 80 1968-07-31 21:00:00 521 612.0 2631.603 -1.77356 2.474e+00
## 81 1968-08-31 21:00:00 516 529.9  430.229 -0.66807 2.060e+02
## 82 1968-09-30 21:00:00 496 523.8  356.039 -1.47538 8.154e+00
## 83 1968-10-31 21:00:00 538 510.8  352.342  1.44789 9.763e+05
## 84 1968-11-30 21:00:00 575 523.7  358.331  2.70845 1.512e+08
## 85 1968-12-31 21:00:00 537 525.4 2569.300  0.22956 7.467e+03
## 86 1969-01-31 21:00:00 534 538.1  449.195 -0.19559 1.363e+03
## 87 1969-02-28 21:00:00 542 537.7  377.060  0.22238 7.255e+03
## 88 1969-03-31 21:00:00 538 541.9  365.266 -0.20414 1.317e+03
## 89 1969-04-30 21:00:00 547 541.9  359.179  0.26744 8.689e+03
## 90 1969-05-31 21:00:00 540 546.5  348.637 -0.35022 7.344e+02
##      L_lower l_lower   H_upper   L_upper l_upper
## 1  1.000e+00       1 1.000e+00 1.000e+00       1
## 2  1.000e+00       1 1.000e+00 1.000e+00       1
## 3  1.000e+00       1 1.000e+00 1.000e+00       1
## 4  1.000e+00       1 1.000e+00 1.000e+00       1
## 5  1.000e+00       1 1.000e+00 1.000e+00       1
## 6  1.000e+00       1 1.000e+00 1.000e+00       1
## 7  1.000e+00       1 1.000e+00 1.000e+00       1
## 8  1.000e+00       1 1.000e+00 1.000e+00       1
## 9  1.000e+00       1 1.000e+00 1.000e+00       1
## 10 1.000e+00       1 1.000e+00 1.000e+00       1
## 11 1.000e+00       1 1.000e+00 1.000e+00       1
## 12 1.000e+00       1 1.000e+00 1.000e+00       1
## 13 1.000e+00       1 1.000e+00 1.000e+00       1
## 14 1.000e+00       1 1.000e+00 1.000e+00       1
## 15 1.000e+00       1 1.000e+00 1.000e+00       1
## 16 1.000e+00       1 1.000e+00 1.000e+00       1
## 17 1.000e+00       1 1.000e+00 1.000e+00       1
## 18 1.000e+00       1 1.000e+00 1.000e+00       1
## 19 1.000e+00       1 1.000e+00 1.000e+00       1
## 20 1.000e+00       1 1.000e+00 1.000e+00       1
## 21 1.399e+03       1 6.351e+03 6.351e+03       1
## 22 1.907e+02       1 4.659e+04 4.659e+04       1
## 23 1.376e+04       1 6.457e+02 6.457e+02       1
## 24 1.437e+09       1 6.183e-03 1.000e+00       0
## 25 1.336e+05       1 6.649e+01 6.649e+01       1
## 26 4.473e+03       1 1.987e+03 1.987e+03       1
## 27 1.416e+06       1 6.273e+00 6.273e+00       1
## 28 4.600e+04       1 1.932e+02 1.932e+02       1
## 29 2.336e+04       1 3.805e+02 3.805e+02       1
## 30 7.522e+01       1 1.181e+05 1.181e+05       1
## 31 1.471e+01       1 6.040e+05 6.040e+05       1
## 32 1.784e+02       1 4.982e+04 4.982e+04       1
## 33 2.849e+03       1 3.119e+03 3.119e+03       1
## 34 3.237e+01       1 2.745e+05 2.745e+05       1
## 35 4.621e+04       1 1.923e+02 1.923e+02       1
## 36 1.893e+08       1 4.695e-02 1.000e+00       0
## 37 5.767e+03       1 1.541e+03 1.541e+03       1
## 38 3.233e+05       1 2.748e+01 2.748e+01       1
```

```
## 39 9.421e+03       1 9.432e+02 9.432e+02       1
## 40 1.735e+06       1 5.122e+00 5.122e+00       1
## 41 1.068e+06       1 8.318e+00 8.318e+00       1
## 42 7.170e+03       1 1.239e+03 1.239e+03       1
## 43 7.892e-01       1 1.126e+07 1.126e+07       1
## 44 6.552e+02       2 1.070e+04 1.070e+04       1
## 45 6.984e+03       1 1.272e+03 1.272e+03       1
## 46 7.334e+02       1 1.212e+04 1.212e+04       1
## 47 7.201e+04       1 1.234e+02 1.234e+02       1
## 48 8.330e+08       1 1.067e-02 1.000e+00       0
## 49 5.203e+04       1 1.708e+02 1.708e+02       1
## 50 2.119e+02       1 4.193e+04 4.193e+04       1
## 51 2.360e+04       1 3.765e+02 3.765e+02       1
## 52 3.459e+07       1 2.569e-01 2.569e-01       1
## 53 1.536e+06       1 5.787e+00 1.486e+00       2
## 54 5.431e+05       1 1.636e+01 1.636e+01       1
## 55 1.183e+04       1 7.511e+02 7.511e+02       1
## 56 8.327e+02       1 1.067e+04 1.067e+04       1
## 57 1.107e+03       1 8.029e+03 8.029e+03       1
## 58 1.798e+01       1 4.943e+05 4.943e+05       1
## 59 1.220e+07       1 7.287e-01 7.287e-01       1
## 60 2.776e+09       1 3.201e-03 2.333e-03       2
## 61 2.392e+04       1 3.715e+02 1.000e+00       0
## 62 7.119e+03       1 1.248e+03 1.248e+03       1
## 63 3.552e+04       1 2.501e+02 2.501e+02       1
## 64 9.275e+03       1 9.581e+02 9.581e+02       1
## 65 2.493e+04       1 3.565e+02 3.565e+02       1
## 66 3.789e+04       1 2.345e+02 2.345e+02       1
## 67 1.479e-01       1 6.009e+07 6.009e+07       1
## 68 2.873e-01       2 4.574e+06 4.574e+06       1
## 69 1.000e+00       0 1.209e+05 1.209e+05       1
## 70 6.470e+01       1 1.373e+05 1.373e+05       1
## 71 1.580e+07       1 5.623e-01 5.623e-01       1
## 72 9.967e+08       1 8.915e-03 5.013e-03       2
## 73 8.730e+03       1 1.018e+03 1.000e+00       0
## 74 2.533e+01       1 3.509e+05 3.509e+05       1
## 75 1.332e+03       1 6.673e+03 6.673e+03       1
## 76 4.722e+06       1 1.882e+00 1.882e+00       1
## 77 1.000e+00       0 1.861e+10 1.861e+10       1
## 78 1.035e+00       1 8.587e+06 8.587e+06       1
## 79 1.000e+00       0 1.072e+11 1.072e+11       1
## 80 2.474e+00       1 3.592e+06 3.592e+06       1
## 81 2.060e+02       1 4.314e+04 4.314e+04       1
## 82 8.154e+00       1 1.090e+06 1.090e+06       1
## 83 9.763e+05       1 9.101e+00 9.101e+00       1
## 84 1.512e+08       1 5.879e-02 1.000e+00       0
## 85 7.467e+03       1 1.190e+03 1.190e+03       1
## 86 1.363e+03       1 6.518e+03 6.518e+03       1
## 87 7.255e+03       1 1.225e+03 1.225e+03       1
## 88 1.317e+03       1 6.745e+03 6.745e+03       1
## 89 8.689e+03       1 1.023e+03 1.023e+03       1
## 90 7.344e+02       1 1.210e+04 1.210e+04       1
##  [ reached 'max' / getOption("max.print") -- omitted 90 rows ]
```