`pybats-detection`: A python package for outlier and structural changes detection in time series analysis

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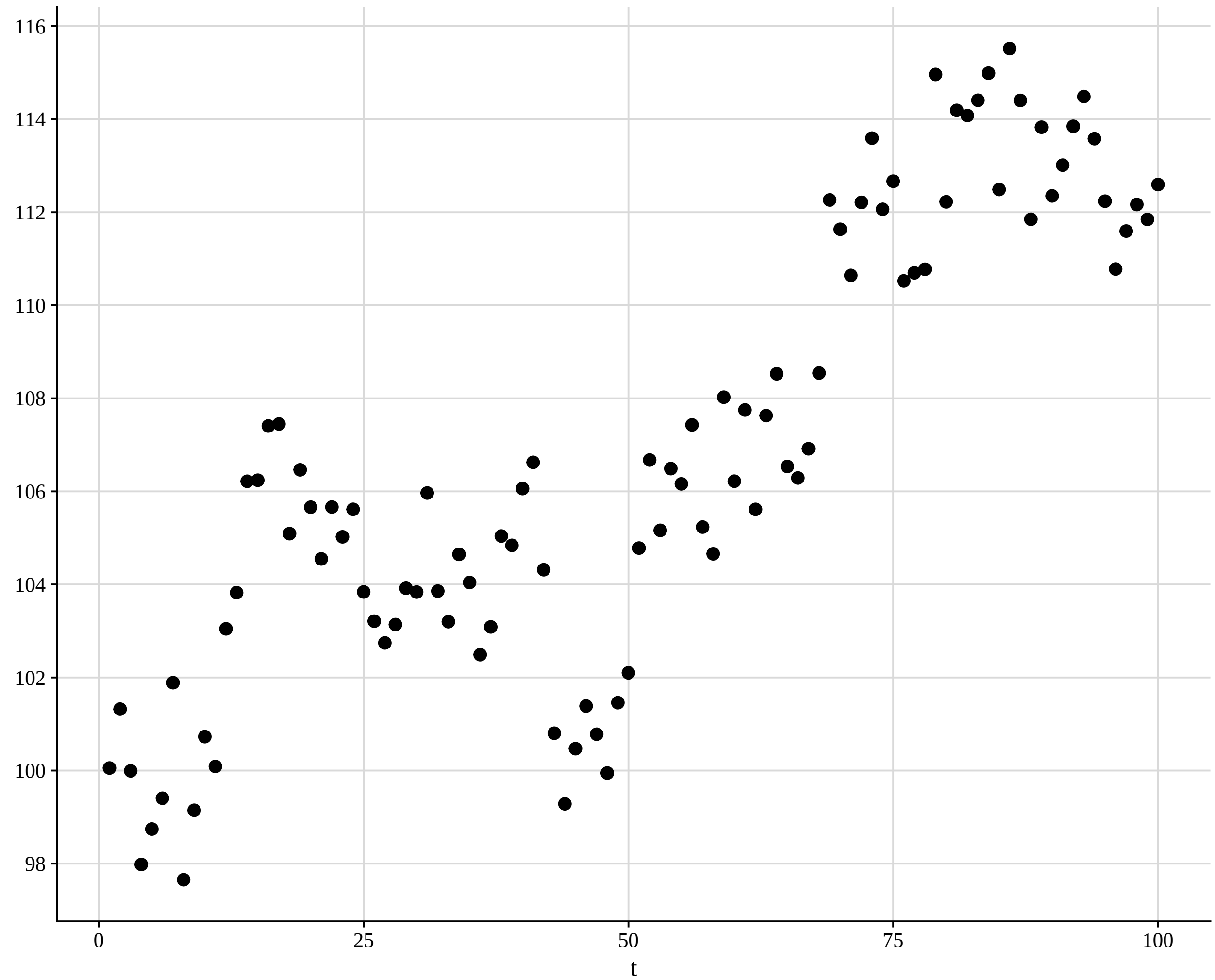
Contents

Smoothing

A brief introduction of the Smooth class in a simulated example.The time series $\mathbf{Y} = (y_1, \ldots, y_n)$ was generated using the *RandomDLM* class, which has as arguments (n, V, W) the number of observations, observational variance and state vector variance. The *.level* method allows you to define the starting level and regime change points.

```
>>> # Generating level data model
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=100, V=1, W=1)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={})
>>> y = df_simulated["y"]
```



The Smooth class allows you to perform a retrospective analysis of $\mathbf{Y}$. First, it is necessary to define the model components with prior values, which is done with the *dlm* class available in the *pybats* package. In this case, it was considered a DLM with level and growth. The prior vector and covariances are defined by $\mathbf{a}$ and $\mathbf{R}$. Lastly the discount factor denoted by *deltrend* is a constant in the interval (0, 1), it's used to coordinate the adaptive capacity of predictions with increasing variance of model components.

Given this, the adjustment will be made considering the declared DLM, in which the moments for prior and posterior distributions for all times will be saved. Subsequently, these moments will be used to obtain the filtered distribution of the state vector, recursively.

```
>>> # Define model components
>>> a = np.array([100, 0])
>>> R = np.eye(2)
>>> np.fill_diagonal(R, val=1)
>>> mod = dlm(a, R, ntrend=2, deltrend=1)
>>>
>>> # Fit with monitoring
>>> smooth = Smoothing(mod=mod)
>>> smooth_fit = smooth.fit(y=y)
```

The Smooth class will return a dictionary with moments for: smoothed predictive and smoothed posterior for model components, and some additional results such as bounds for credibility interval.

```
>>> smooth_fit.get('smooth').get('predictive')
```

```
##              fk    t        qk   df    ci_lower    ci_upper
## 0     99.709689    1  0.258196    1   98.701695  100.717683
## 1     99.849839    2  0.250489    2   98.857004  100.842673
## 2     99.989988    3  0.242938    3   99.012232  100.967745
## 3    100.130138    4  0.235545    4   99.167374  101.092902
## 4    100.270287    5  0.228309    5   99.322427  101.218148
## ..          ...  ...       ...  ...         ...         ...
## 95   113.023888   96  0.228166   96  112.076325  113.971452
## 96   113.164038   97  0.235399   97  112.201573  114.126503
## 97   113.304187   98  0.242789   98  112.326731  114.281643
## 98   113.444337   99  0.250336   99  112.451805  114.436869
## 99   113.584486  100  0.258041  100  112.576796  114.592176
##
## [100 rows x 6 columns]
```

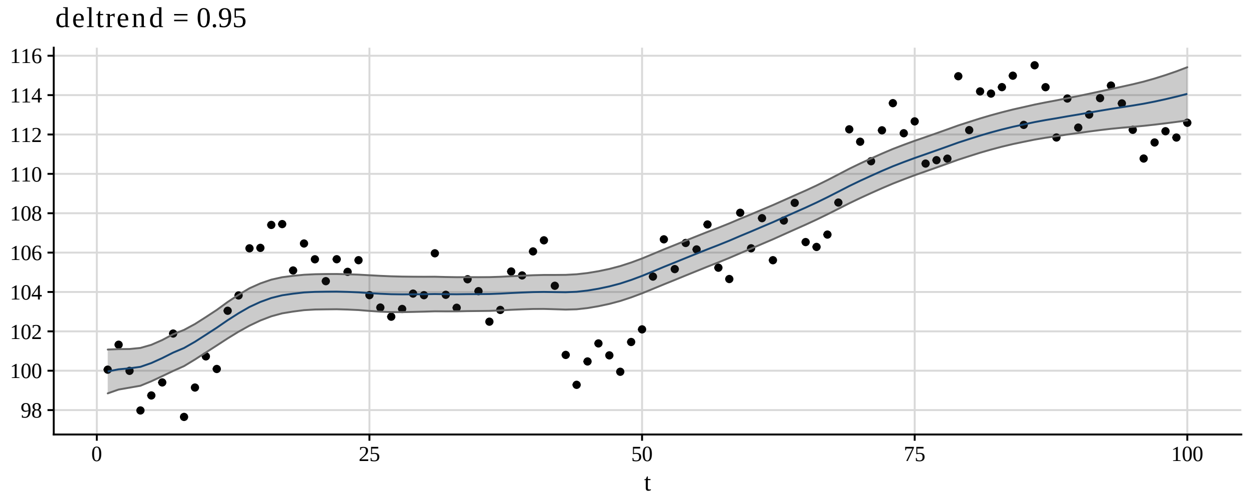
```
>>> smooth_fit.get('smooth').get('posterior')
```

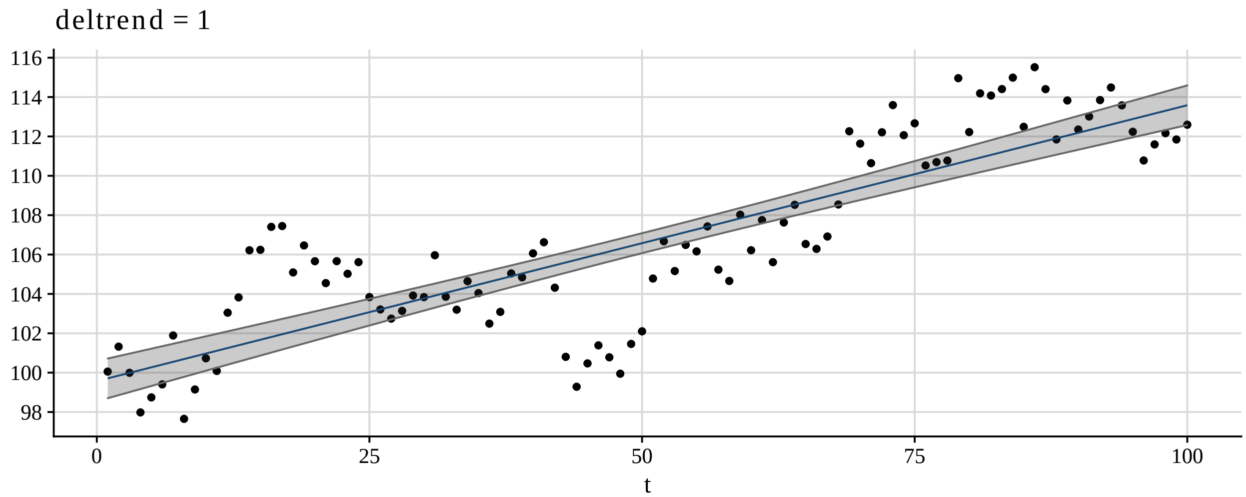
```
##     parameter        mean  variance    t   ci_lower    ci_upper
## 0     theta_1   99.709689  0.258196    1  98.701695  100.717683
## 1     theta_1   99.849839  0.250489    2  98.857004  100.842673
## 2     theta_1   99.989988  0.242938    3  99.012232  100.967745
## 3     theta_1  100.130138  0.235545    4  99.167374  101.092902
## 4     theta_1  100.270287  0.228309    5  99.322427  101.218148
## ..        ...         ...       ...  ...        ...         ...
## 195   theta_2    0.140149  0.000079   96   0.122559    0.157740
## 196   theta_2    0.140149  0.000079   97   0.122559    0.157740
## 197   theta_2    0.140149  0.000079   98   0.122559    0.157740
## 198   theta_2    0.140149  0.000079   99   0.122559    0.157740
## 199   theta_2    0.140149  0.000079  100   0.122559    0.157740
##
## [200 rows x 6 columns]
```

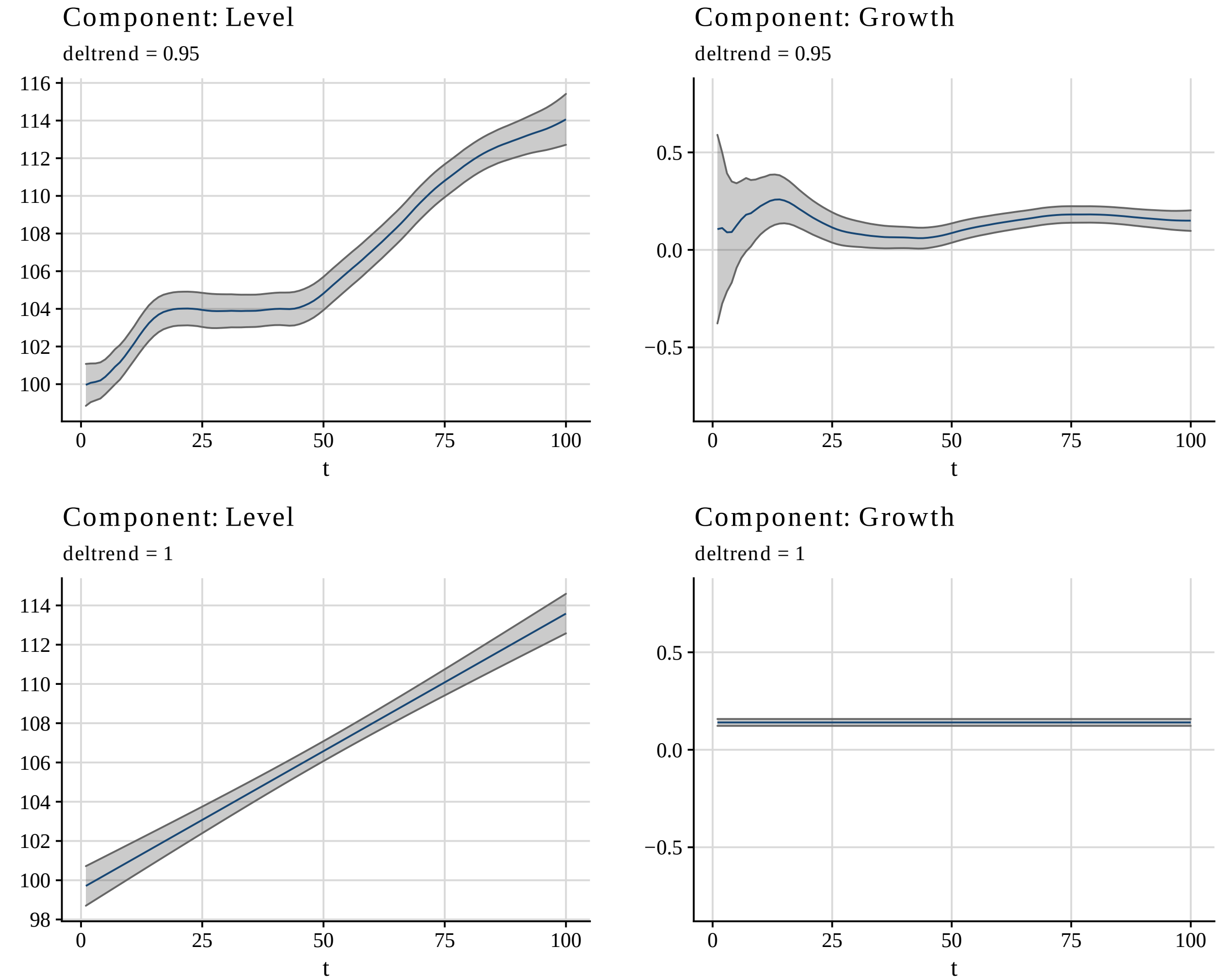
It is also interesting to investigate the recursive analysis considering different discount factors. With the *shortcut_run* auxiliary method, two discount factors were tested.

```
>>> def shortcut_run(deltrend: np.float64):
>>>   """short cut for test smoothing with by deltrend."""
>>>   a = np.array([100, 0])
>>>   R = np.eye(2)
>>>   np.fill_diagonal(R, val=1)
>>>   mod = dlm(a, R, ntrend=2, deltrend=deltrend)
>>>
>>>   # Fit with monitoring
>>>   smooth = Smoothing(mod=mod)
>>>   results_dict = smooth.fit(y=y).get('smooth')
>>>   results_dict.get('predictive').loc[:, 'delta'] = deltrend
>>>   results_dict.get('posterior').loc[:, 'delta'] = deltrend
>>>   results_dict.get('predictive').loc[:, 'real'] = y
>>>
>>>   return results_dict
>>>
>>> deltrends = [0.95, 1]
>>> predictive_smooth_df = pd.concat([shortcut_run(i).get('predictive') for i in deltrends])
```

```
>>> posteriori_smooth_df = pd.concat([shortcut_run(i).get('posterior') for i in deltrends])
```

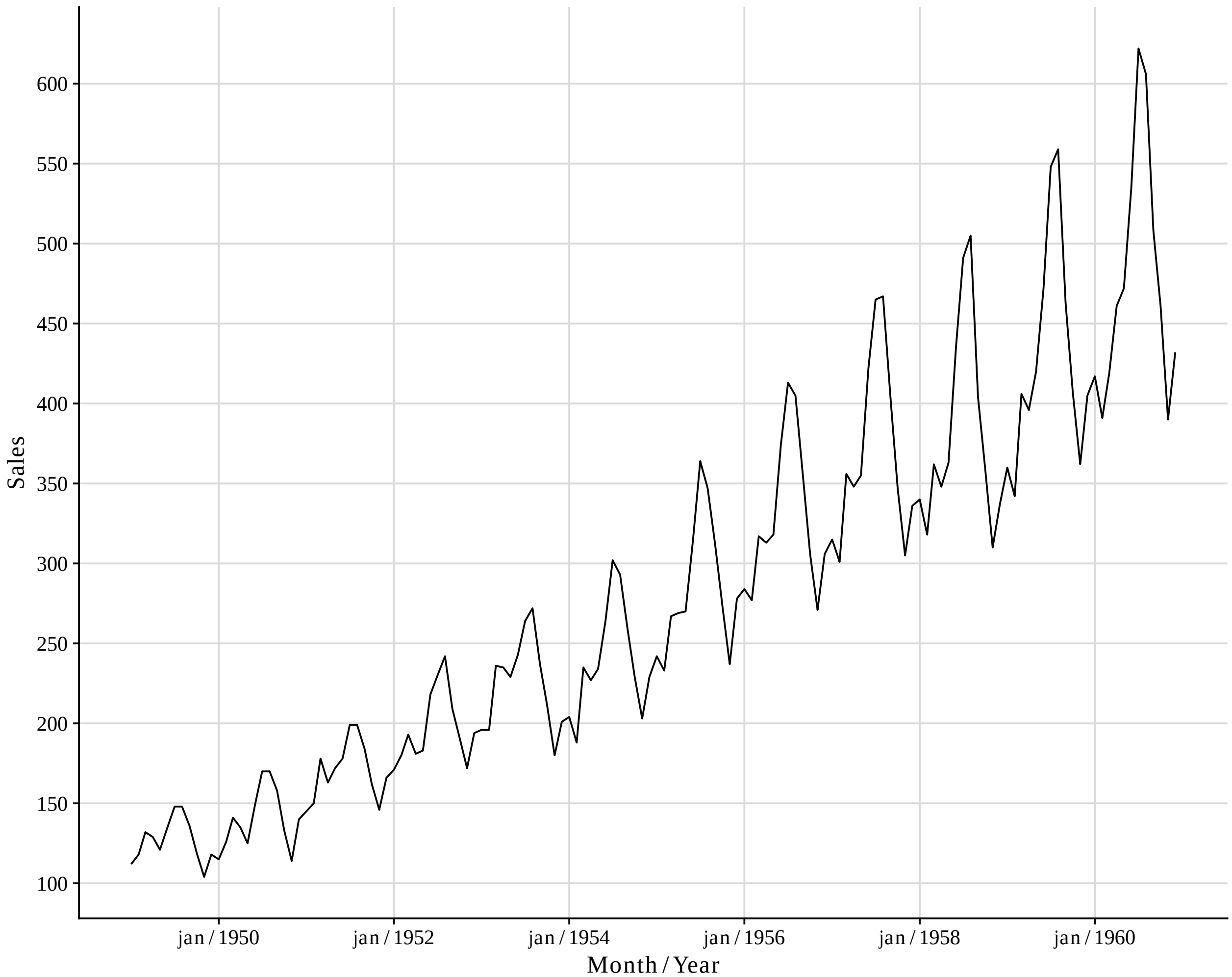






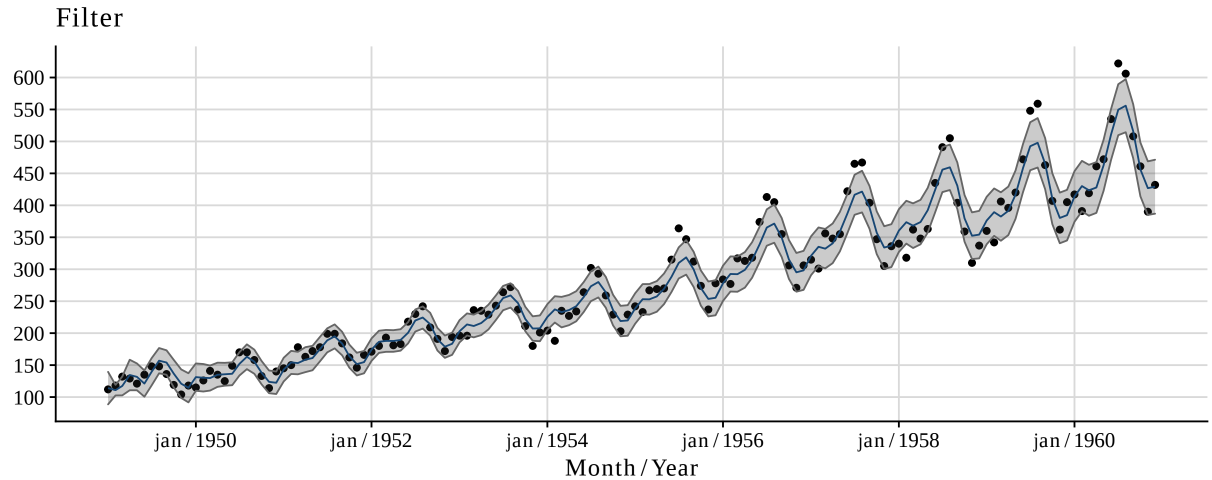
Aplication: AirPassangers dataset

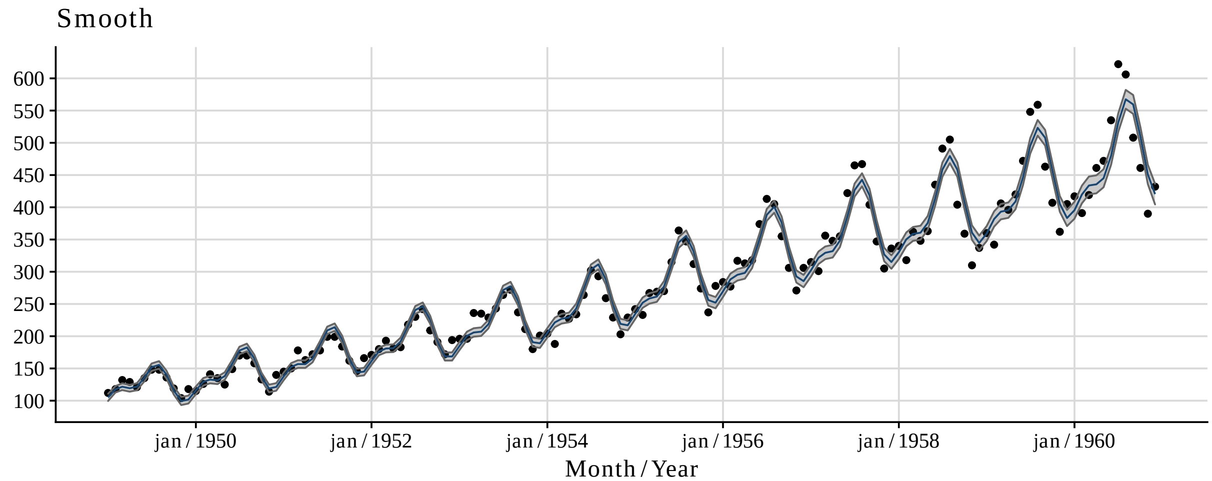
Below is a practical example with the classic Box & Jenkins airline data, Monthly totals of international airline passengers (1949 to 1960), using a normal DLM with three main components: Trend, Growth and Seasonality.



Here it was specified a yearly seasonal effect of period 12, with the first two harmonics. The discount factor for the seasonal components is 0.98.

```
>>> a = np.array([112, 0, 1, -1, 1, -1])
>>> R = np.eye(6)
>>> np.fill_diagonal(R, val=1)
>>> mod = dlm(a, R, ntrend=2, deltrend=.95, delseas=.98,
>>>           seasPeriods=[12], seasHarmComponents=[[1, 2]])
```

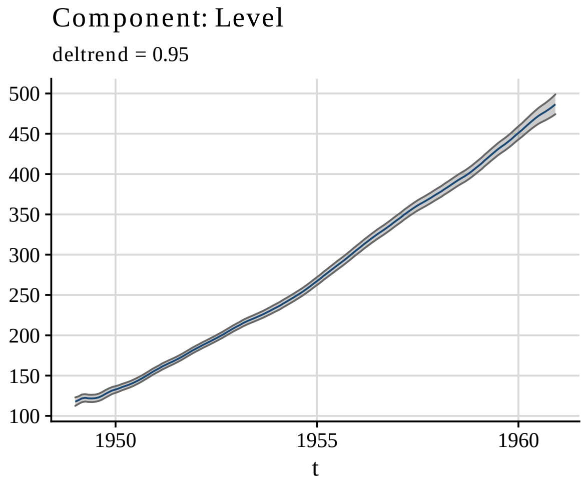


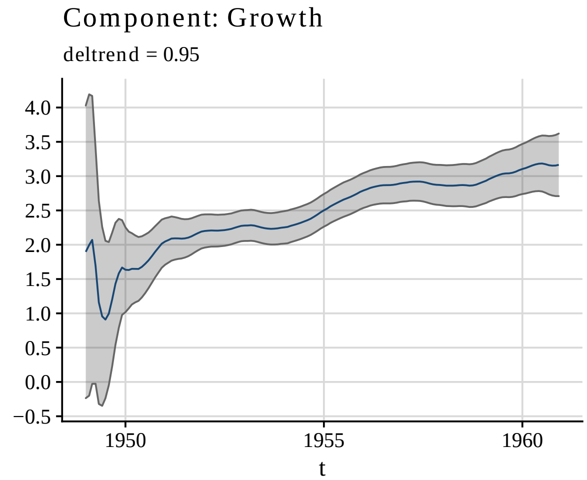


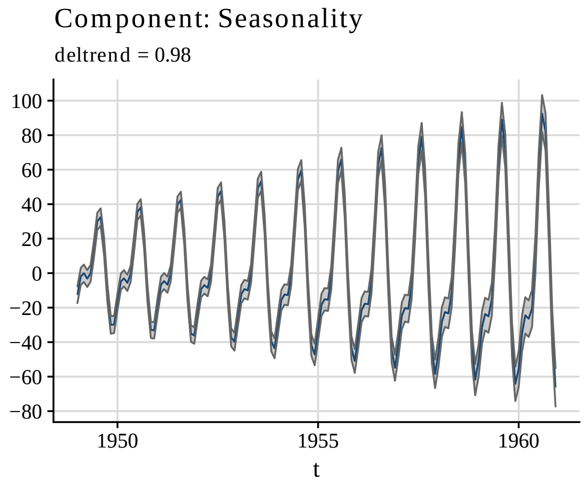
Note that seasonality was included in using harmonic components which totals six parameters; for simplicity the results illustrated for this block of components concern the posterior distribution of their sum, whose mean and variance are given by

$$\mu_{seasonality} = \mathbf{F}_{seasonality}^{\top}\mathbf{a}(-k)_{seasonality}, \sigma_{seasonality} = \mathbf{F}_{seasonality}^{\top}\mathbf{R}(-k)_{seasonality}\mathbf{F}_{seasonality}$$

where $\mathbf{a}(-k)_{seasonality}$ and $\mathbf{R}(-k)_{seasonality}$ are the smoothed mean vector and the smoothed covariance of the posteriori distribution for the seasonality components. $\mathbf{F}$ is the regression vector associated.

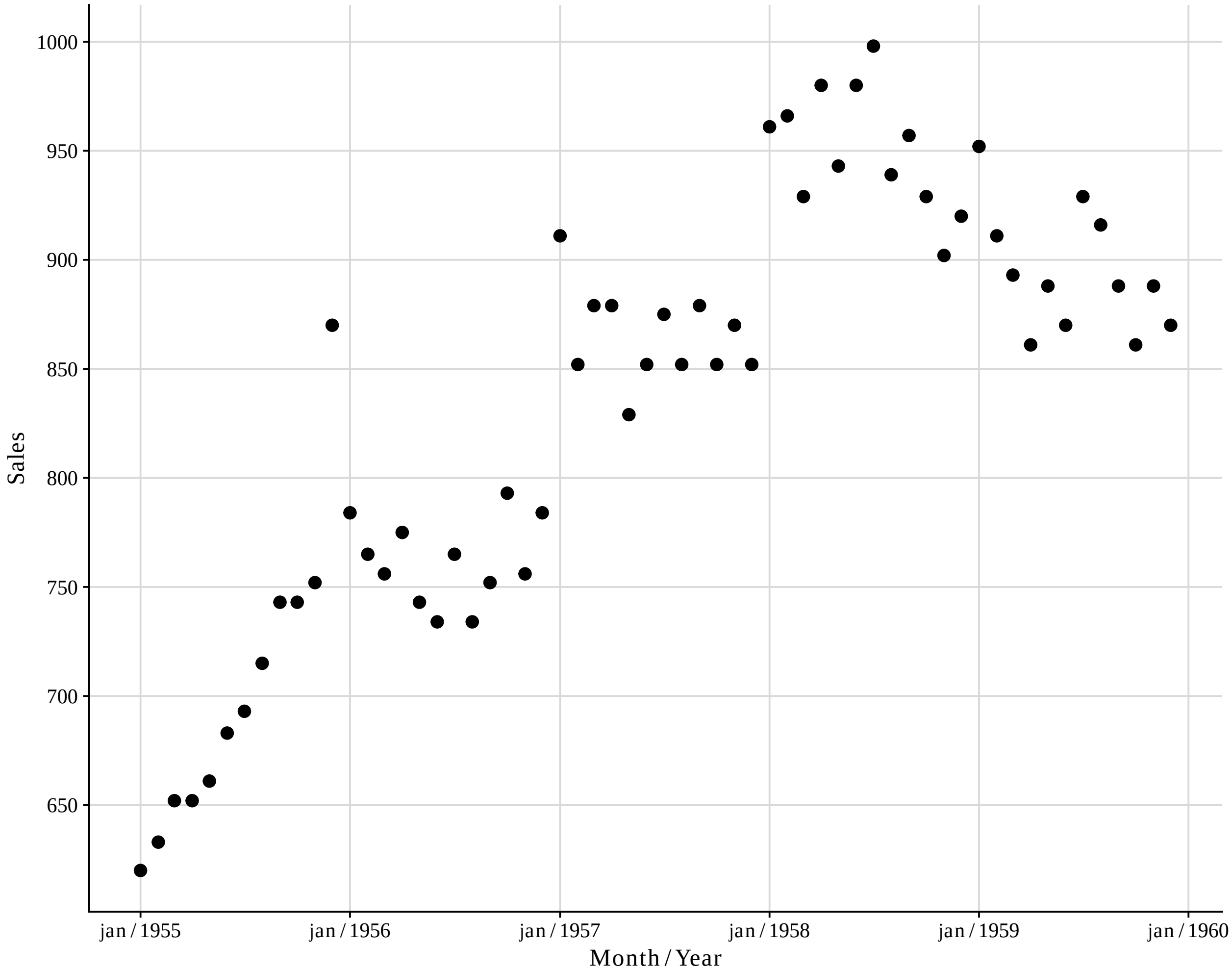

Component: Level
deltrend = 0.95
500
450
400
350
300
250
200
150
100
1950
1955
1960
t


Component: Growth
deltrend = 0.95
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
1950
1955
1960
t


Component: Seasonality
deltrend = 0.98
100
80
60
40
20
0
−20
−40
−60
−80
1950
1955
1960
t

Manual Intervention

CP6



- Performing the fit (filter and smoothing) without interventions.

```
>>> # Define the growth model
>>> a = np.array([600, 1])
>>> R = np.array([[100, 0], [0, 25]])
>>> mod = dlm(a, R, ntrend=2, deltrend=[0.90, 0.98])
>>>
>>> # Filter and Smooth without intervention
>>> smooth = Smoothing(mod=mod)
>>> out_no_int = smooth.fit(y=cp6["sales"])
>>> dict_filter_no_int = out_no_int.get("filter")
>>> dict_smooth_no_int = out_no_int.get("smooth")
```

- Performing the fit (filter and smoothing) with interventions

```
>>> list_interventions = [
>>>     {"time_index": 12, "which": ["variance", "noise"],
>>>      "parameters": [{"v_shift": "ignore"},
>>>                     {"h_shift": np.array([0, 0]),
```

```
>>>                     "H_shift": np.array([[1000, 25], [25, 25]])}]
>>>      },
>>>     {"time_index": 25, "which": ["noise", "variance"],
>>>      "parameters": [{"h_shift": np.array([80, 0]),
>>>                     "H_shift": np.array([[100, 0], [0, 0]])},
>>>                    {"v_shift": "ignore"}]},
>>>     {"time_index": 37, "which": ["subjective"],
>>>      "parameters": [{"a_star": np.array([970, 0]),
>>>                     "R_star": np.array([[50, 0], [0, 5]])}]}
>>> ]
>>> manual_interventions = ManualIntervention(mod=mod)
>>> out_int = manual_interventions.fit(
>>>     y=cp6["sales"], interventions=list_interventions)
>>> dict_filter_int = out_int.get("filter")
>>> dict_smooth_int = out_int.get("smooth")
```

- Organizing the data into two data.frame: `data_predictive` and `data_posterior`.

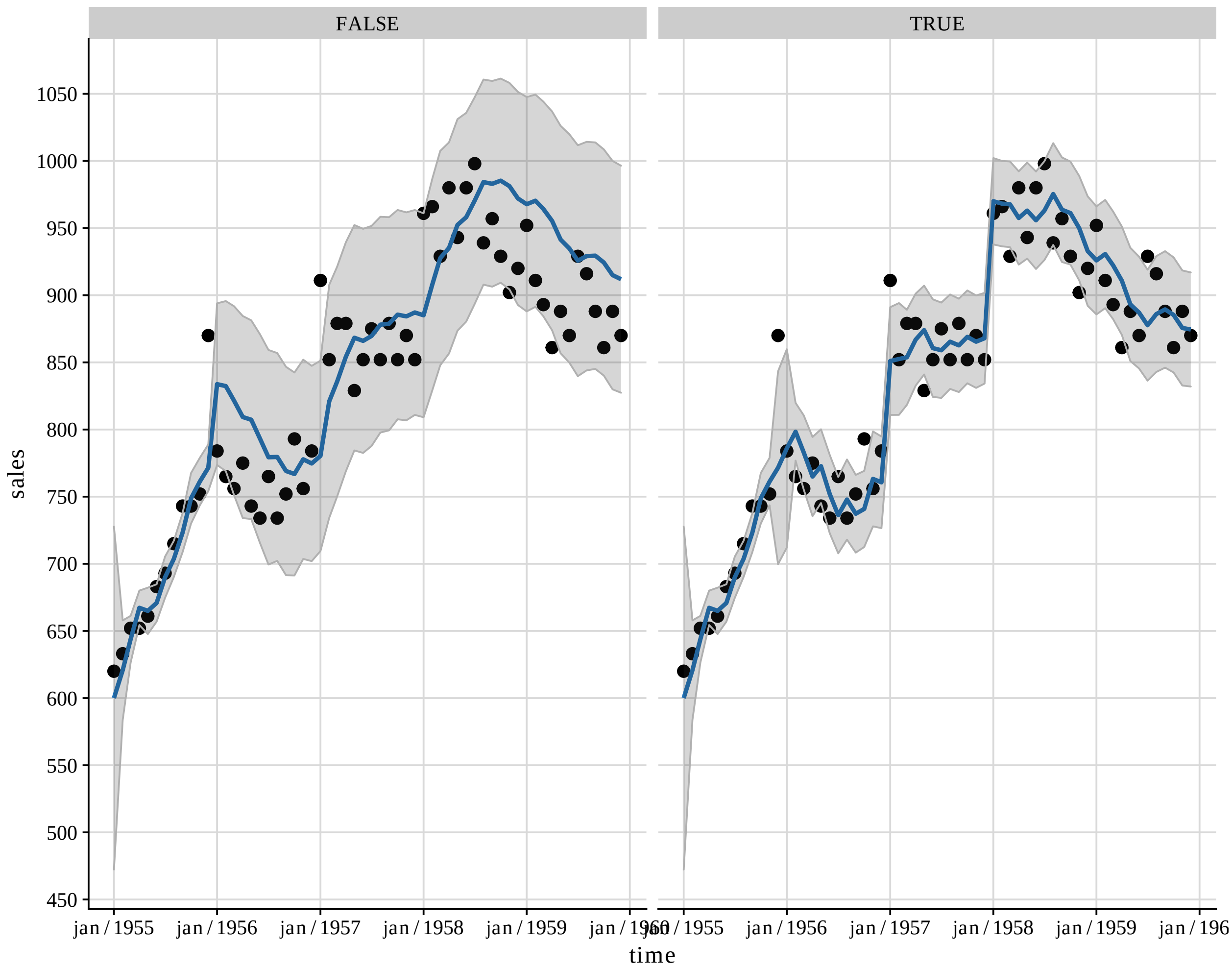
```
>>> # Filter
>>> data_predictive_filter_int = dict_filter_int.get("predictive").copy()
>>> data_predictive_filter_no_int = dict_filter_no_int.get("predictive").copy()
>>> data_predictive_filter_no_int["intervention_type"] = "nothing"
>>> data_predictive_filter_int["intervention_fit"] = True
>>> data_predictive_filter_no_int["intervention_fit"] = False
>>> cols = data_predictive_filter_int.columns
>>> data_predictive_filter = pd.concat(
>>>   [data_predictive_filter_int, data_predictive_filter_no_int[cols]]
>>> )
>>> # Smooth
>>> data_predictive_smooth_int = dict_smooth_int.get("predictive").copy()
>>> data_predictive_smooth_no_int = dict_smooth_no_int.get("predictive").copy()
>>> data_predictive_smooth_no_int["intervention_type"] = "nothing"
>>> data_predictive_smooth_int["intervention_fit"] = True
>>> data_predictive_smooth_no_int["intervention_fit"] = False
>>> cols = data_predictive_smooth_int.columns
>>> data_predictive_smooth = pd.concat(
>>>   [data_predictive_smooth_int, data_predictive_smooth_no_int[cols]]
>>> )
>>> # Append data
>>> data_predictive_smooth["type"] = "smooth"
>>> data_predictive_filter["type"] = "filter"
>>> data_predictive_smooth.rename(columns={"fk": "f", "qk": "q"}, inplace=True)
>>> cols_ord = ["t", "intervention_fit", "type", "f", "q", 'ci_lower', 'ci_upper']
>>> data_predictive = pd.concat(
>>>     [data_predictive_filter[cols_ord], data_predictive_smooth[cols_ord]])
>>> data_predictive = data_predictive.join(cp6)
```

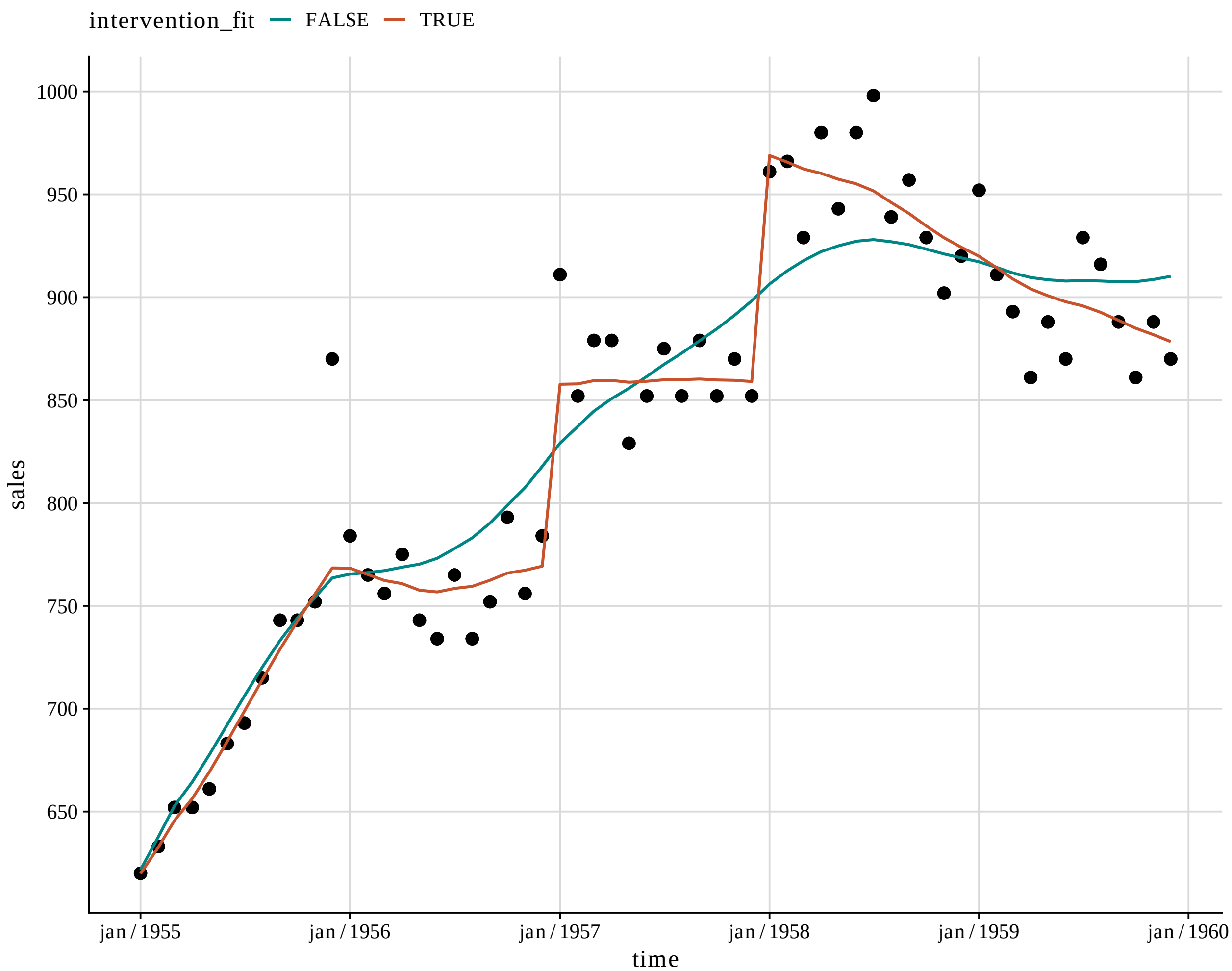
```
>>> # Filter
>>> data_posterior_filter_int = dict_filter_int.get("posterior").copy()
>>> data_posterior_filter_no_int = dict_filter_no_int.get("posterior").copy()
>>> data_posterior_filter_no_int["intervention_type"] = "nothing"
>>> data_posterior_filter_int["intervention_fit"] = True
>>> data_posterior_filter_no_int["intervention_fit"] = False
>>> cols = data_posterior_filter_int.columns
```

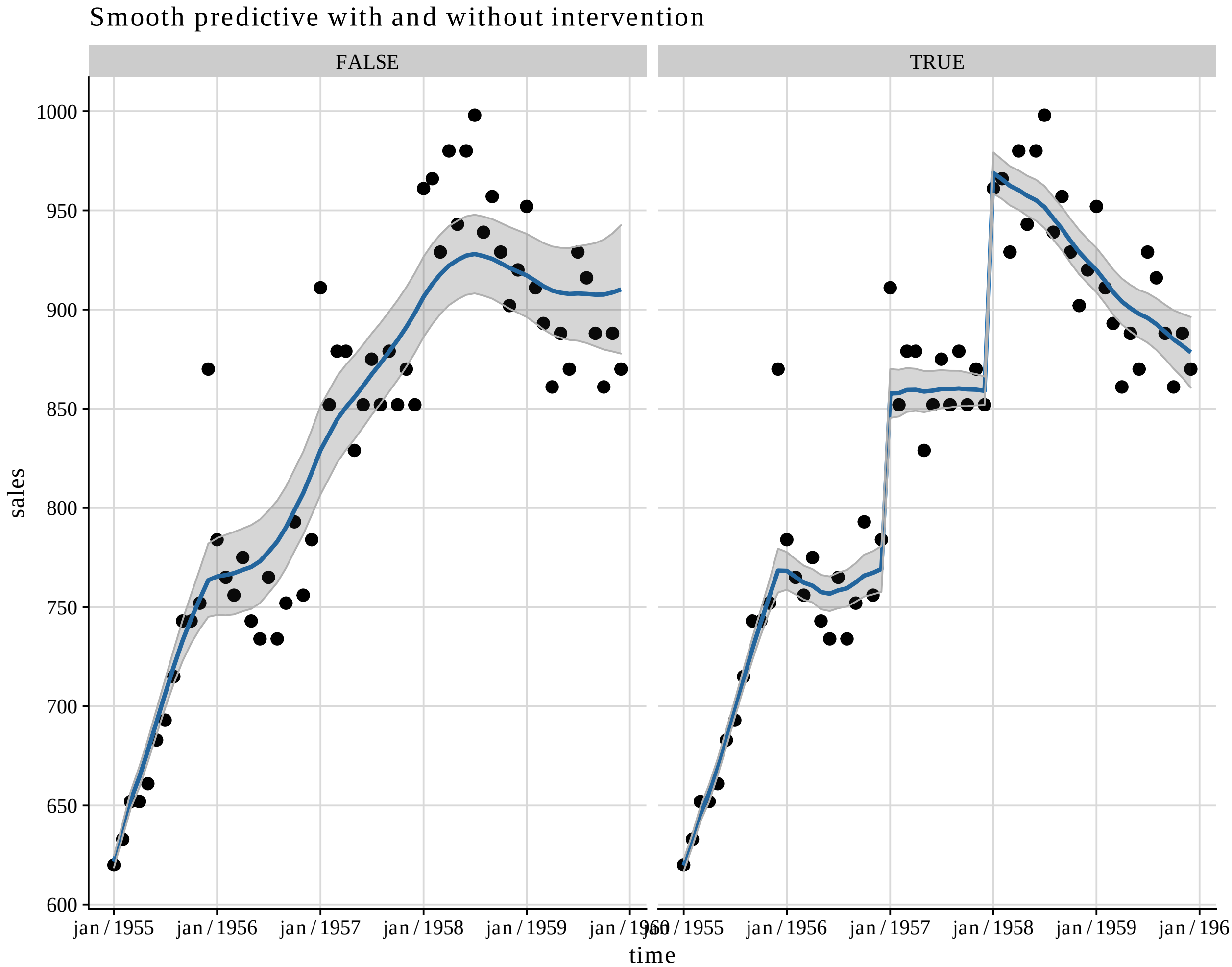
```python
>>> data_posterior_filter = pd.concat(
>>>   [data_posterior_filter_int, data_posterior_filter_no_int[cols]]
>>> )
>>> # Smooth
>>> data_posterior_smooth_int = dict_smooth_int.get("posterior").copy()
>>> data_posterior_smooth_no_int = dict_smooth_no_int.get("posterior").copy()
>>> data_posterior_smooth_no_int["intervention_type"] = "nothing"
>>> data_posterior_smooth_int["intervention_fit"] = True
>>> data_posterior_smooth_no_int["intervention_fit"] = False
>>> cols = data_posterior_smooth_int.columns
>>> data_posterior_smooth = pd.concat(
>>>   [data_posterior_smooth_int, data_posterior_smooth_no_int[cols]]
>>> )
>>> # Append data
>>> data_posterior_smooth["type"] = "smooth"
>>> data_posterior_filter["type"] = "filter"
>>> data_posterior_filter.set_index(data_posterior_filter["t"].values-1, inplace=True)
>>> data_posterior_smooth.set_index(data_posterior_smooth["t"].values-1, inplace=True)
>>>
>>> cols_ord = ["t", "intervention_fit", "type", "parameter", "mean", "variance",
>>>             'ci_lower', 'ci_upper']
>>> data_posterior = pd.concat(
>>>     [data_posterior_filter[cols_ord], data_posterior_smooth[cols_ord]])
>>> data_posterior = data_posterior.join(cp6)
```

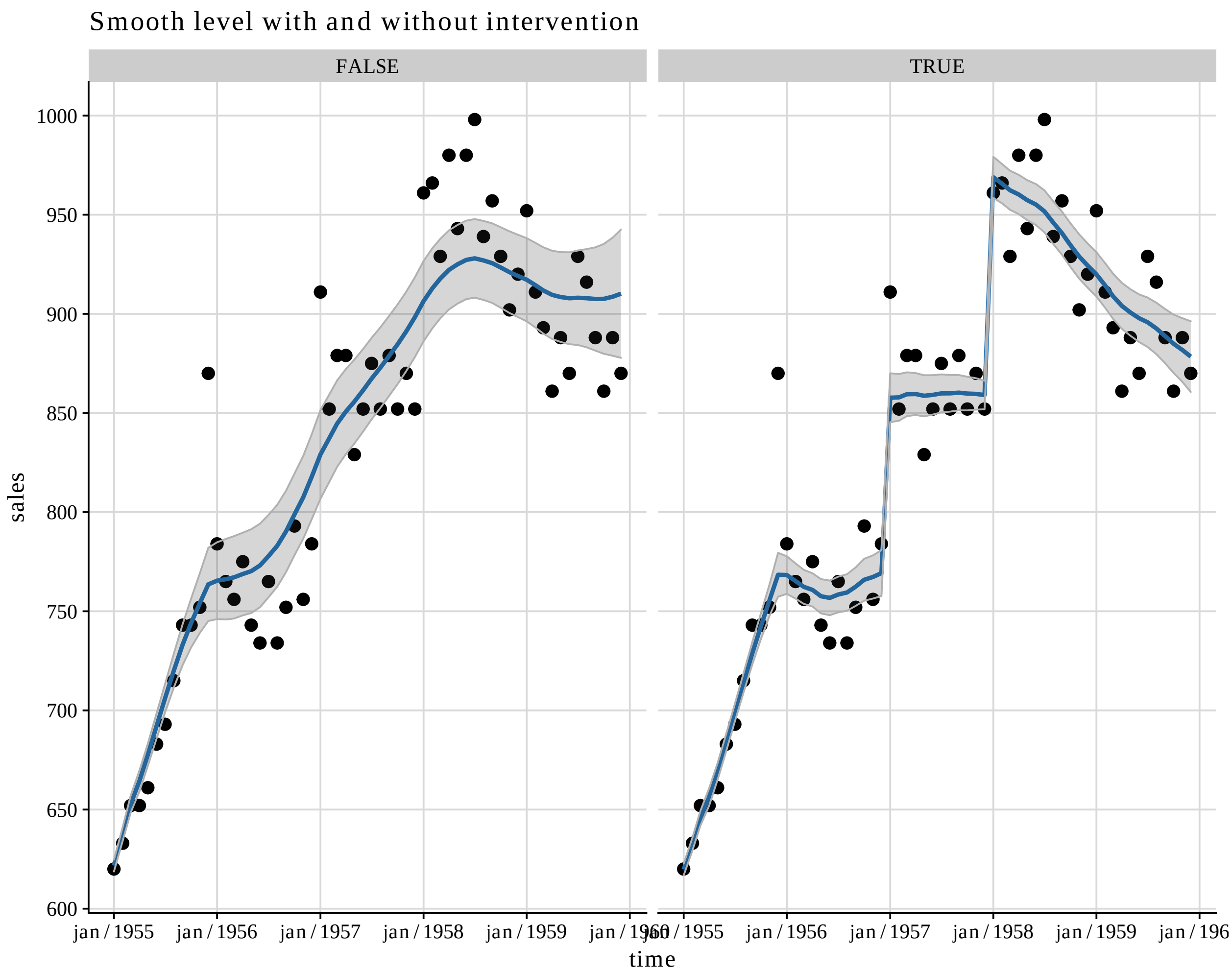
- Plotting some results

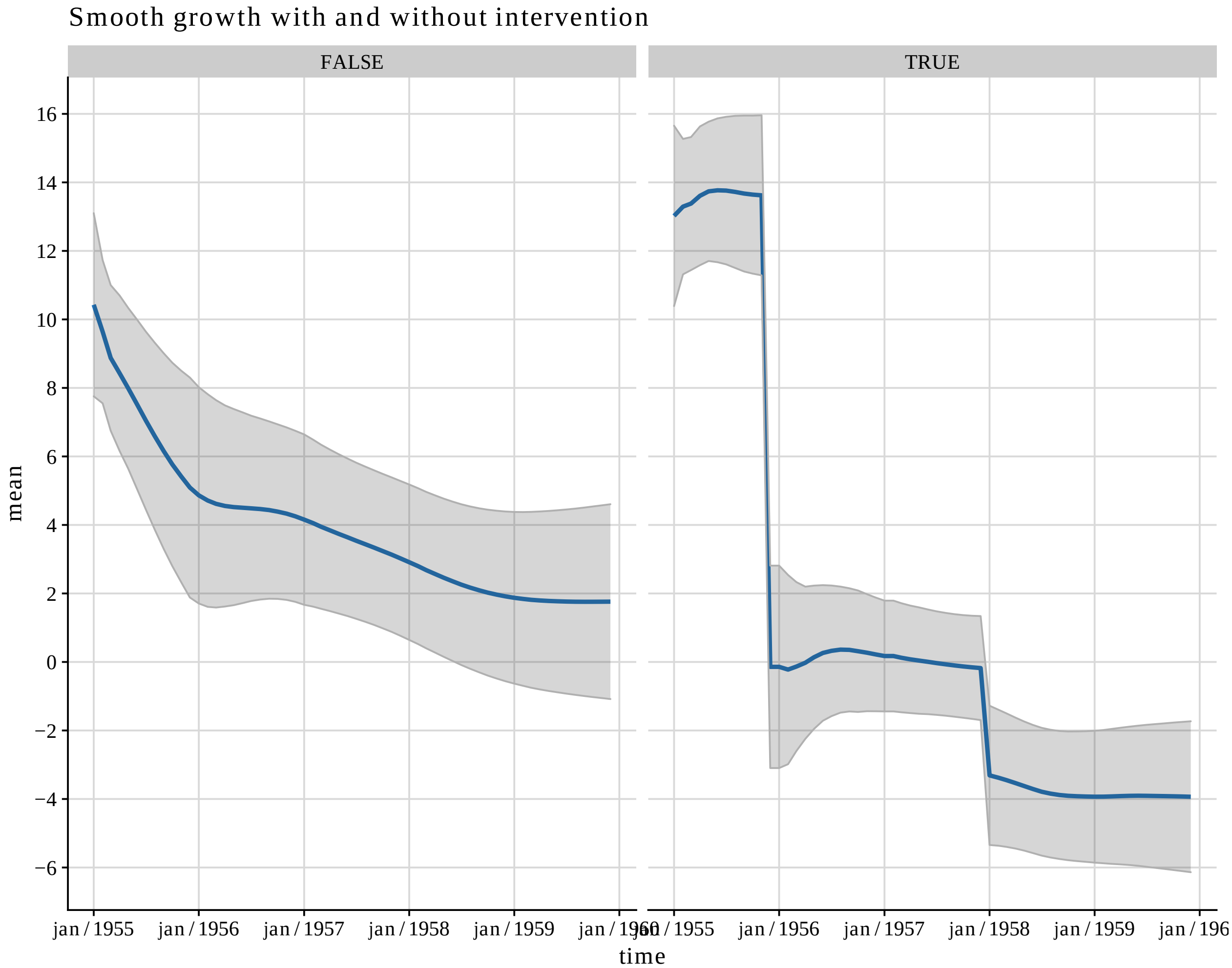

intervention_fit
FALSE
TRUE
1000
950
900
850
800
750
700
650
600
sales
jan / 1955
jan / 1956
jan / 1957
jan / 1958
jan / 1959
jan / 1960
time


FALSE
TRUE
sales
1050
1000
950
900
850
800
750
700
650
600
550
500
450
jan / 1955
jan / 1956
jan / 1957
jan / 1958
jan / 1959
jan / 1960
time

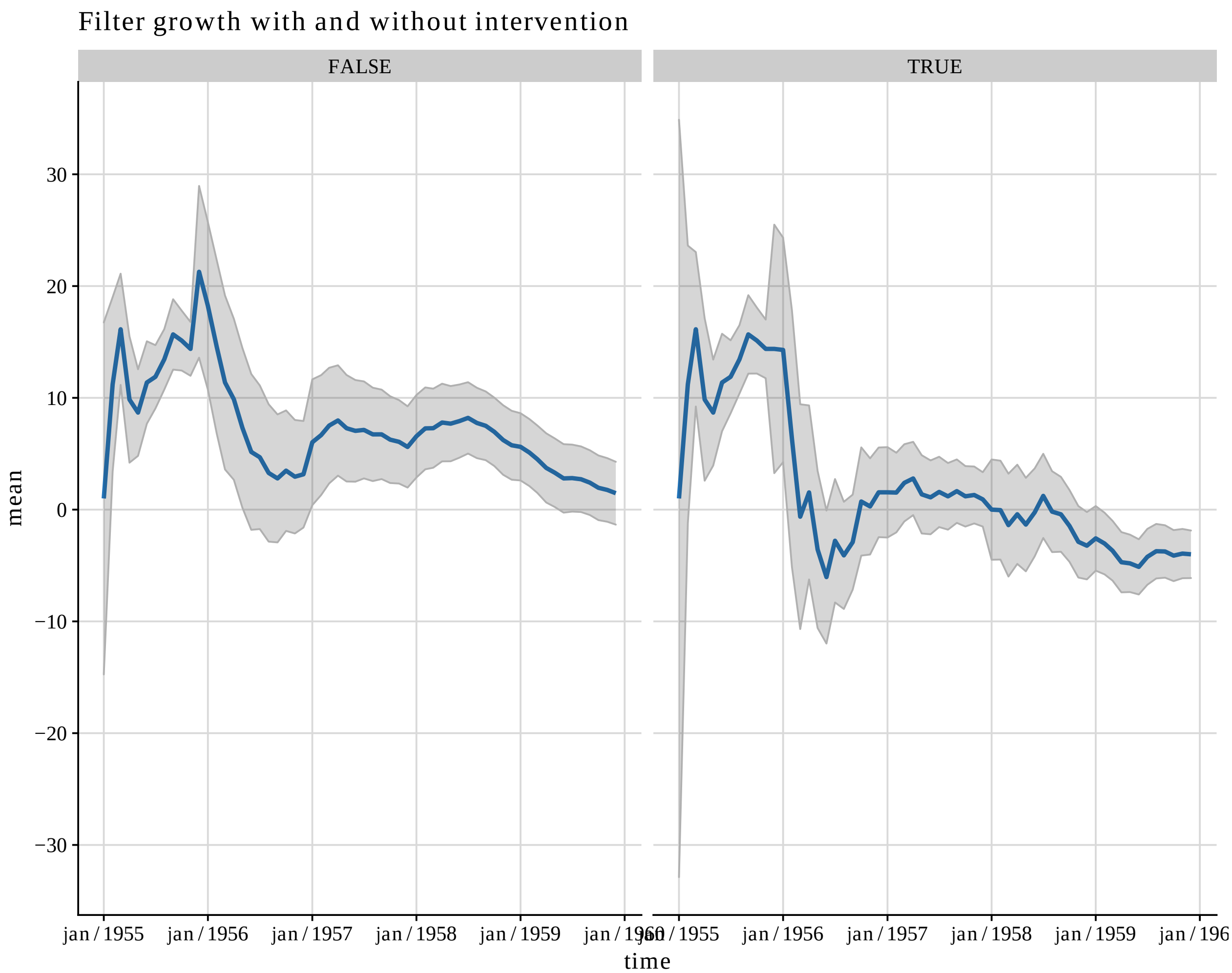

intervention_fit
FALSE
TRUE
1000
950
900
850
800
750
700
650
sales
jan / 1955
jan / 1956
jan / 1957
jan / 1958
jan / 1959
jan / 1960
time


Smooth predictive with and without intervention
FALSE
TRUE
sales
time
1000
950
900
850
800
750
700
650
600
jan / 1955
jan / 1956
jan / 1957
jan / 1958
jan / 1959
jan / 1960


Smooth level with and without intervention
FALSE
TRUE
sales
1000
950
900
850
800
750
700
650
600
time
jan / 1955
jan / 1956
jan / 1957
jan / 1958
jan / 1959
jan / 1960



Smooth growth with and without intervention
FALSE
TRUE
mean
16
14
12
10
8
6
4
2
0
−2
−4
−6
jan / 1955
jan / 1956
jan / 1957
jan / 1958
jan / 1959
jan / 1960
time

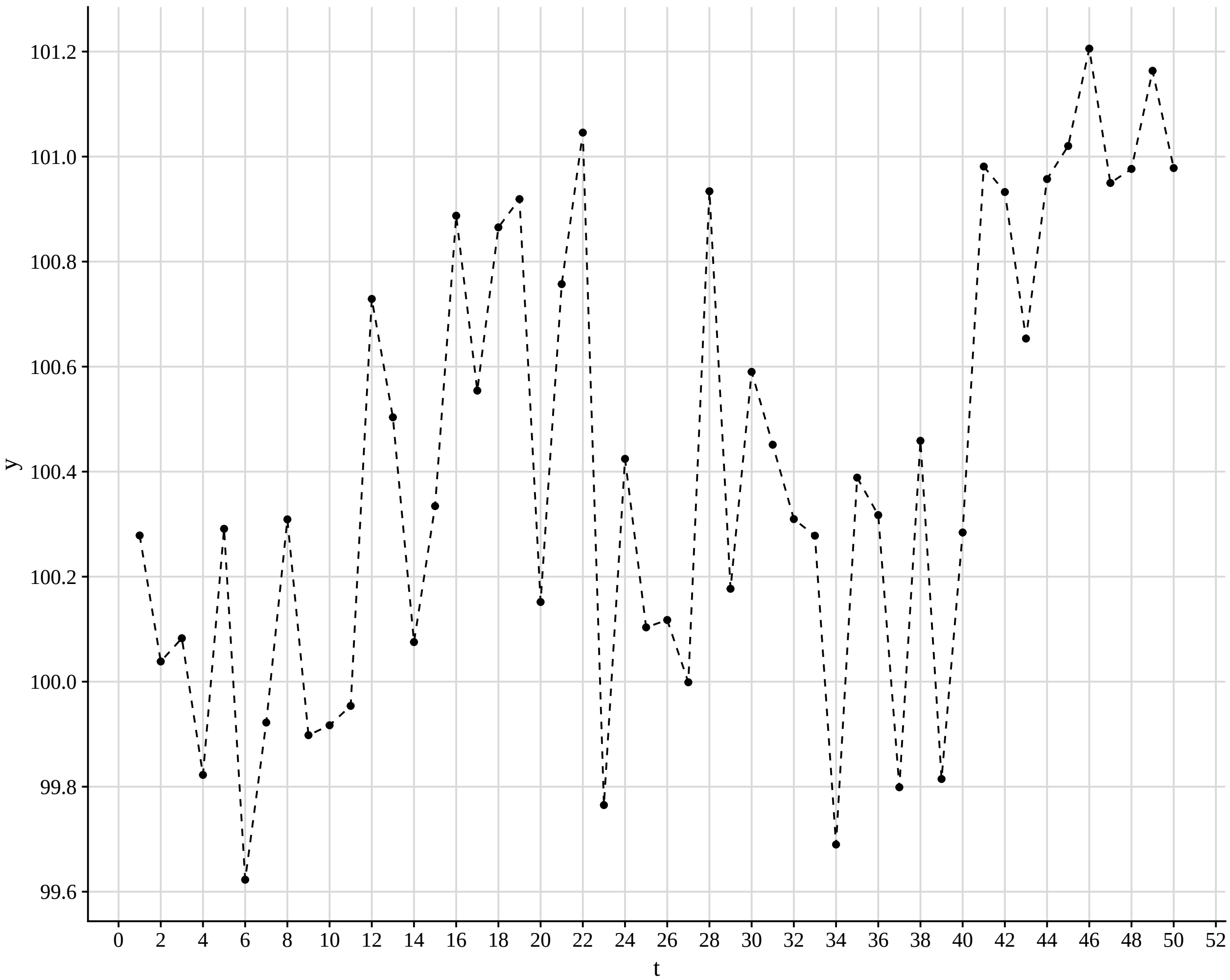


Automatic Monitoring

Simulated examples

Level Change

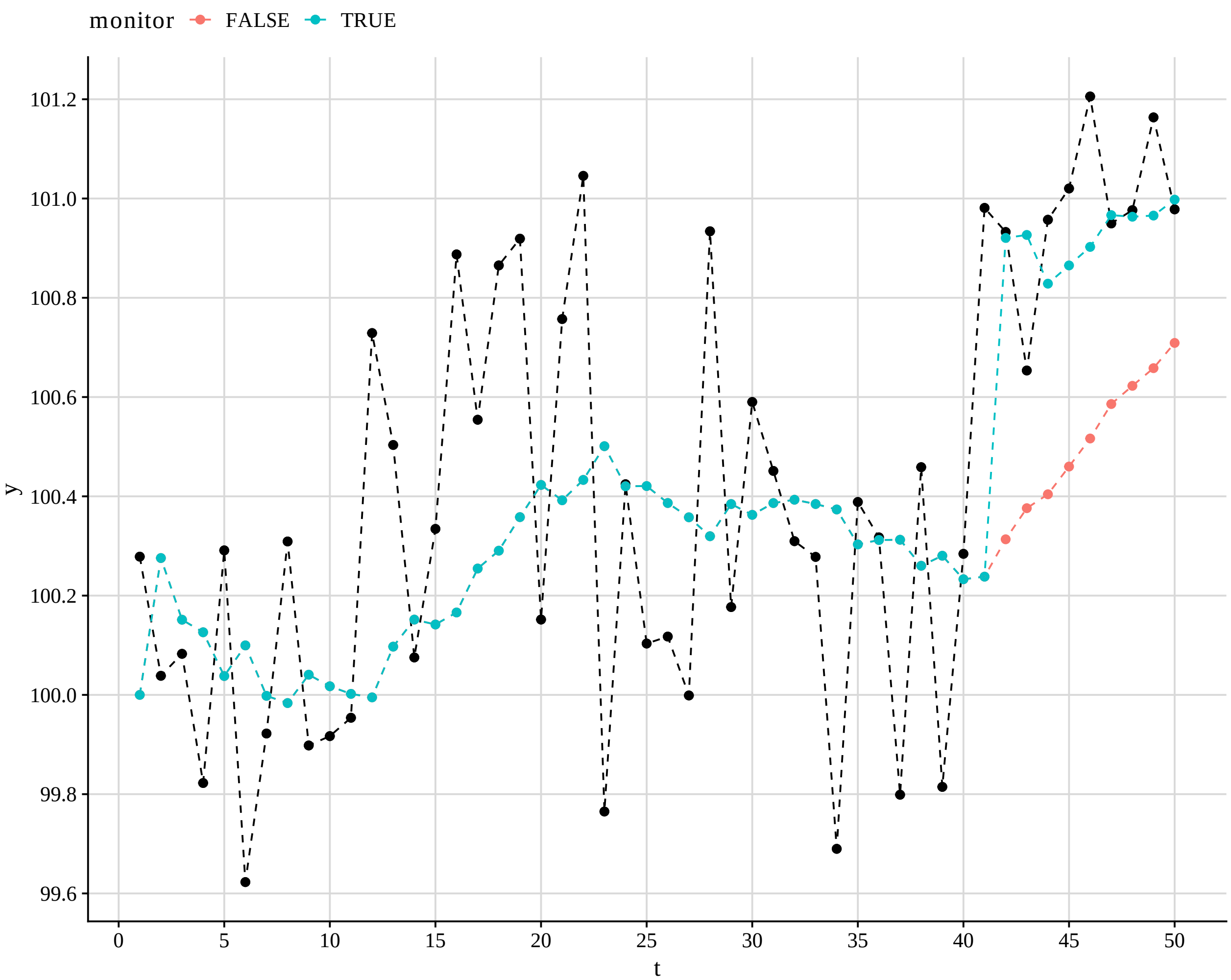
```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=50, V=0.1, W=0.005)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [40],
>>>                 "level_mean_shift": [1],
>>>                 "level_var_shift": [1]})
>>> df_simulated.loc[40:50, "y"] = 101 + np.random.normal(0, 0.2, 10)
```




```
>>> a = np.array([100])
>>> R = np.eye(1)
>>> R[[0]] = 100
>>> mod = dlm(a, R, ntrend=1, deltrend=0.9)
>>>
>>> # Fit without monitoring
>>> fit_without_monitor = Smoothing(mod=mod).fit(y=df_simulated["y"])
>>> df_res = fit_without_monitor.get("filter").get("predictive")
>>> df_res["monitor"] = False
>>>
>>> # Fit with monitoring
>>> monitor = AutomaticMonitoring(mod=mod, bilateral=False)
>>> fit_monitor = monitor.fit(y=df_simulated["y"], h=3, tau=0.135, change_var=[100])
```

```
## Parametric change detected at time 43 with H=12.090162666255477, L=3.769336975935653 and l=3
```

```
>>> df_tmp = fit_monitor.get("filter").get("predictive")
>>> df_tmp["monitor"] = True
>>> cols_ord = ["t", "y", "f", "q", "ci_lower", "ci_upper", "monitor", "e",
>>>             "H", "L", "l"]
>>> df_res = pd.concat([df_res, df_tmp[cols_ord]]).reset_index(drop=True)
```

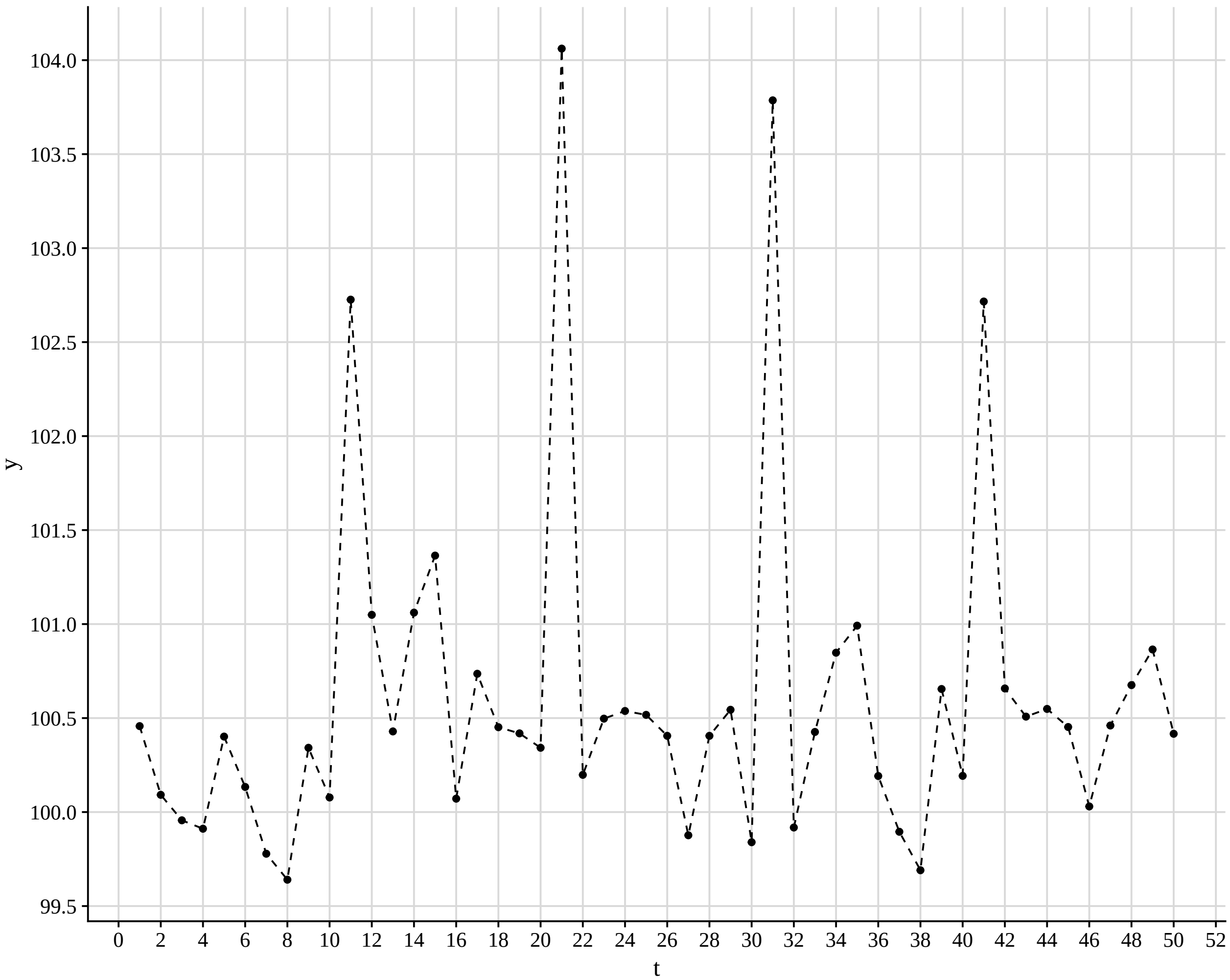



```
##     t      y      f        q         e         H         L l
## 1   1 100.28 100.00 101.0000  0.027708 1.000e+00 1.000e+00 1
## 2   2 100.04 100.28   1.0509 -0.231445 1.000e+00 1.000e+00 1
## 3   3 100.08 100.15   0.5419 -0.093343 1.000e+00 1.000e+00 1
## 4   4  99.82 100.13   0.3630 -0.504038 1.000e+00 1.000e+00 1
## 5   5 100.29 100.04   0.2899  0.470105 1.000e+00 1.000e+00 1
## 6   6  99.62 100.10   0.2424 -0.968590 1.000e+00 1.000e+00 1
## 7   7  99.92 100.00   0.2338 -0.156969 1.000e+00 1.000e+00 1
## 8   8 100.31  99.98   0.2013  0.725492 1.000e+00 1.000e+00 1
## 9   9  99.90 100.04   0.1879 -0.328868 1.000e+00 1.000e+00 1
## 10 10  99.92 100.02   0.1692 -0.244168 1.000e+00 1.000e+00 1
## 11 11  99.95 100.00   0.1533 -0.122789 1.301e+02 1.301e+02 1
## 12 12 100.73  99.99   0.1397  1.963608 2.489e-01 2.489e-01 1
## 13 13 100.50 100.10   0.1694  0.987129 4.658e+00 1.159e+00 2
## 14 14 100.08 100.15   0.1682 -0.185936 1.572e+02 1.572e+02 1
## 15 15 100.33 100.14   0.1567  0.486534 2.091e+01 2.091e+01 1
## 16 16 100.89 100.17   0.1487  1.870796 3.288e-01 3.288e-01 1
## 17 17 100.55 100.25   0.1700  0.726979 1.017e+01 3.342e+00 2
## 18 18 100.87 100.29   0.1651  1.414333 1.293e+00 1.293e+00 1
## 19 19 100.92 100.36   0.1734  1.346991 1.583e+00 1.583e+00 1
## 20 20 100.15 100.42   0.1801 -0.639038 6.122e+02 6.122e+02 1
## 21 21 100.76 100.39   0.1747  0.873260 6.555e+00 6.555e+00 1
```

```
## 22 22 101.05 100.43   0.1726  1.474577 1.079e+00 1.079e+00 1
## 23 23  99.77 100.50   0.1812 -1.729299 1.612e+04 1.612e+04 1
## 24 24 100.42 100.42   0.1959  0.009252 8.755e+01 8.755e+01 1
## 25 25 100.10 100.42   0.1879 -0.731956 8.091e+02 8.091e+02 1
## 26 26 100.12 100.39   0.1844 -0.626588 5.898e+02 5.898e+02 1
## 27 27 100.00 100.36   0.1801 -0.845808 1.138e+03 1.138e+03 1
## 28 28 100.93 100.32   0.1781  1.455528 1.143e+00 1.143e+00 1
## 29 29 100.18 100.38   0.1849 -0.482666 3.830e+02 3.830e+02 1
## 30 30 100.59 100.36   0.1801  0.535852 1.804e+01 1.804e+01 1
## 31 31 100.45 100.39   0.1758  0.154334 5.666e+01 5.666e+01 1
## 32 32 100.31 100.39   0.1704 -0.202337 1.652e+02 1.652e+02 1
## 33 33 100.28 100.38   0.1654 -0.262065 1.976e+02 1.976e+02 1
## 34 34  99.69 100.37   0.1608 -1.705145 1.499e+04 1.499e+04 1
## 35 35 100.39 100.30   0.1695  0.207462 4.831e+01 4.831e+01 1
## 36 36 100.32 100.31   0.1649  0.013050 8.656e+01 8.656e+01 1
## 37 37  99.80 100.31   0.1604 -1.282407 4.218e+03 4.218e+03 1
## 38 38 100.46 100.26   0.1631  0.491867 2.058e+01 2.058e+01 1
## 39 39  99.81 100.28   0.1599 -1.164556 2.962e+03 2.962e+03 1
## 40 40 100.28 100.23   0.1613  0.127470 6.141e+01 6.141e+01 1
## 41 41 100.98 100.24   1.7367  1.872524 3.271e-01 3.271e-01 1
## 42 42 100.93 100.92   0.2812  1.515974 9.532e-01 3.118e-01 2
## 43 43 100.65 100.93   0.2122  0.669203 1.209e+01 1.000e+00 0
## 44 44 100.96 100.83   0.1874  0.297470 3.688e+01 3.688e+01 1
## 45 45 101.02 100.87   0.1728  0.372924 2.941e+01 2.941e+01 1
## 46 46 101.21 100.90   0.1632  0.750340 9.478e+00 9.478e+00 1
## 47 47 100.95 100.97   0.1575 -0.042189 1.022e+02 1.022e+02 1
## 48 48 100.98 100.96   0.1513  0.034045 8.128e+01 8.128e+01 1
## 49 49 101.16 100.97   0.1460  0.517697 1.905e+01 1.905e+01 1
## 50 50 100.98 101.00   0.1423 -0.051842 1.052e+02 1.052e+02 1
```

Outliers

```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=50, V=0.1, W=0.01)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [10, 11, 20, 21, 30, 31, 40, 41],
>>>                 "level_mean_shift": [2, -2, 3, -3, 3.4, -3.4, 3, -3],
>>>                 "level_var_shift": [1, 1, 1, 1, 1, 1, 1, 1]})
```

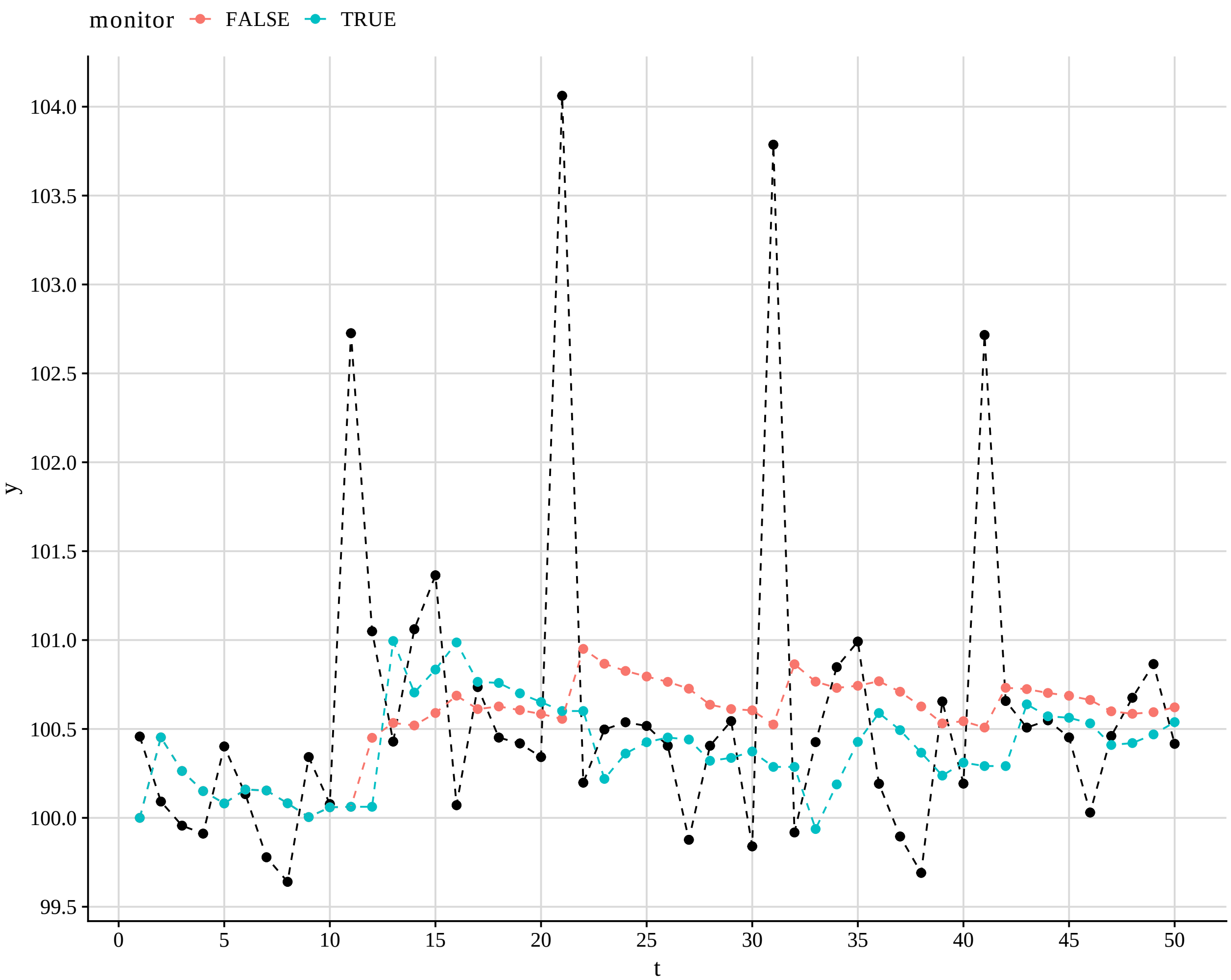



```
>>> a = np.array([100])
>>> R = np.eye(1)
>>> R[[0]] = 100
>>> mod = dlm(a, R, ntrend=1, deltrend=0.9)
>>>
>>> # Fit without monitoring
>>> fit_without_monitor = Smoothing(mod=mod).fit(y=df_simulated["y"])
>>> df_res = fit_without_monitor.get("filter").get("predictive")
>>> df_res["monitor"] = False
>>>
>>> # Fit with monitoring
>>> monitor = AutomaticMonitoring(mod=mod, bilateral=False)
>>> fit_monitor = monitor.fit(y=df_simulated["y"], h=4, tau=0.135, change_var=[100])
```

```
## Potential outlier detected at time 11 with H=2.0200383854441705e-08, L=2.0200383854441705e-08 and  l=
## Potential outlier detected at time 21 with H=1.2517740537692369e-11, L=1.2517740537692369e-11 and  l=
## Potential outlier detected at time 31 with H=3.293965705054862e-13, L=3.293965705054862e-13 and  l=1
## Potential outlier detected at time 41 with H=8.212410369888652e-08, L=8.212410369888652e-08 and  l=1
```

```
>>> df_tmp = fit_monitor.get("filter").get("predictive")
>>> df_tmp["monitor"] = True
>>>
>>> # Append
```

```
>>> cols_ord = ["t", "monitor", "y", "f", "q", "ci_lower", "ci_upper"]
>>> df_res = pd.concat([df_res[cols_ord], df_tmp[cols_ord]]).reset_index(drop=True)
```

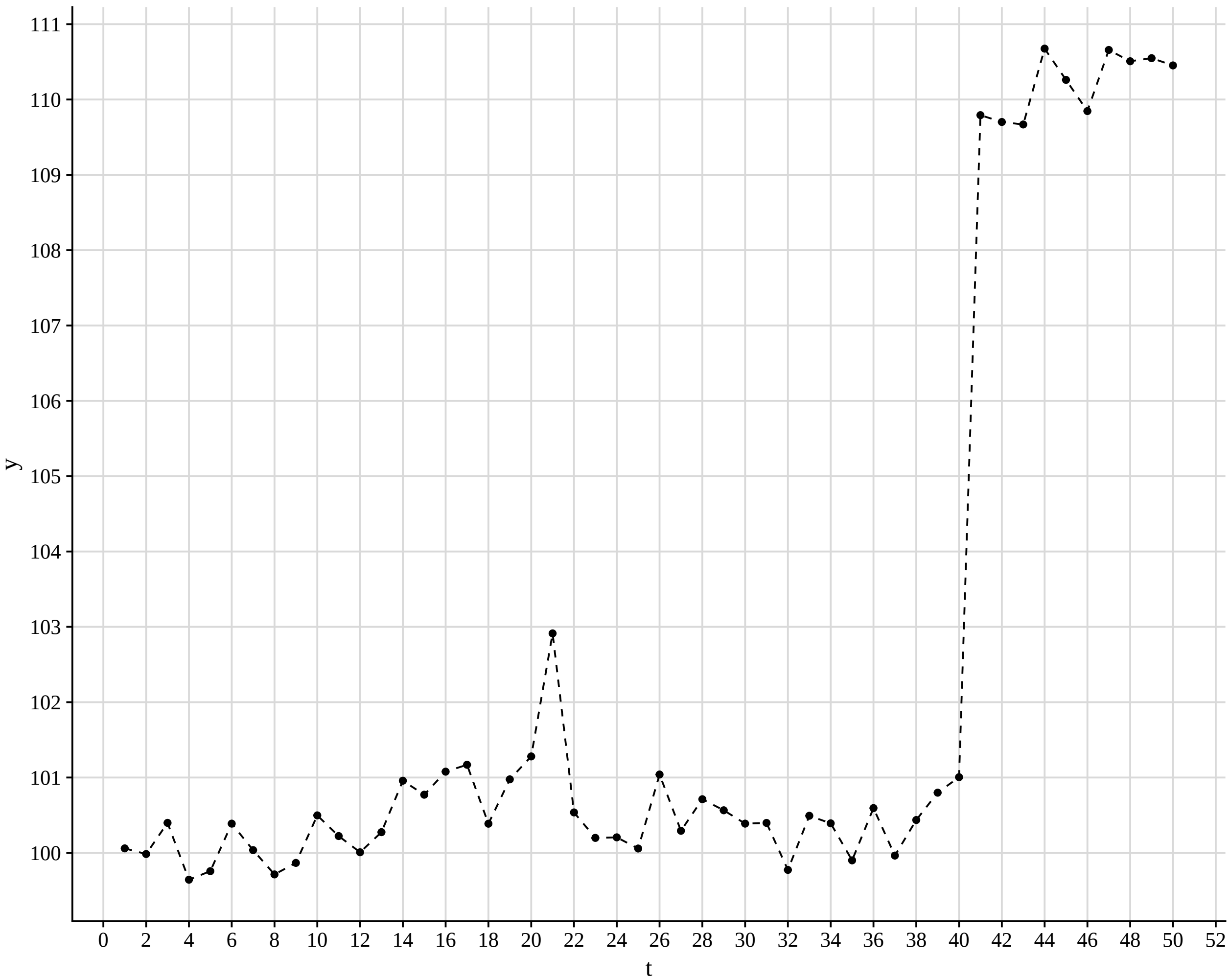



```
##      t      y      f        q        e         H         L l
## 1    1 100.46 100.00 101.0000  0.04550 1.000e+00 1.000e+00 1
## 2    2 100.09 100.45   1.0522 -0.35192 1.000e+00 1.000e+00 1
## 3    3  99.96 100.26   0.5612 -0.41035 1.000e+00 1.000e+00 1
## 4    4  99.91 100.15   0.3958 -0.38023 1.000e+00 1.000e+00 1
## 5    5 100.40 100.08   0.3080  0.57742 1.000e+00 1.000e+00 1
## 6    6 100.13 100.16   0.2631 -0.05013 1.000e+00 1.000e+00 1
## 7    7  99.78 100.15   0.2196 -0.80119 1.000e+00 1.000e+00 1
## 8    8  99.64 100.08   0.2057 -0.97397 1.000e+00 1.000e+00 1
## 9    9 100.34 100.00   0.2015  0.75197 1.000e+00 1.000e+00 1
## 10  10 100.08 100.06   0.1905  0.04164 1.000e+00 1.000e+00 1
## 11  11 102.73 100.06   0.1716  6.42939 2.020e-08 1.000e+00 0
## 12  12 101.05 100.06   2.6468  0.60672 2.633e+02 2.633e+02 1
## 13  13 100.43 100.99   0.2847 -1.06005 2.069e+05 2.069e+05 1
## 14  14 101.06 100.70   0.2200  0.75899 1.432e+02 1.432e+02 1
## 15  15 101.36 100.83   0.1908  1.21403 2.319e+01 2.319e+01 1
## 16  16 100.07 100.99   0.1851 -2.12732 1.479e+07 1.479e+07 1
## 17  17 100.74 100.77   0.2172 -0.06276 3.832e+03 3.832e+03 1
```

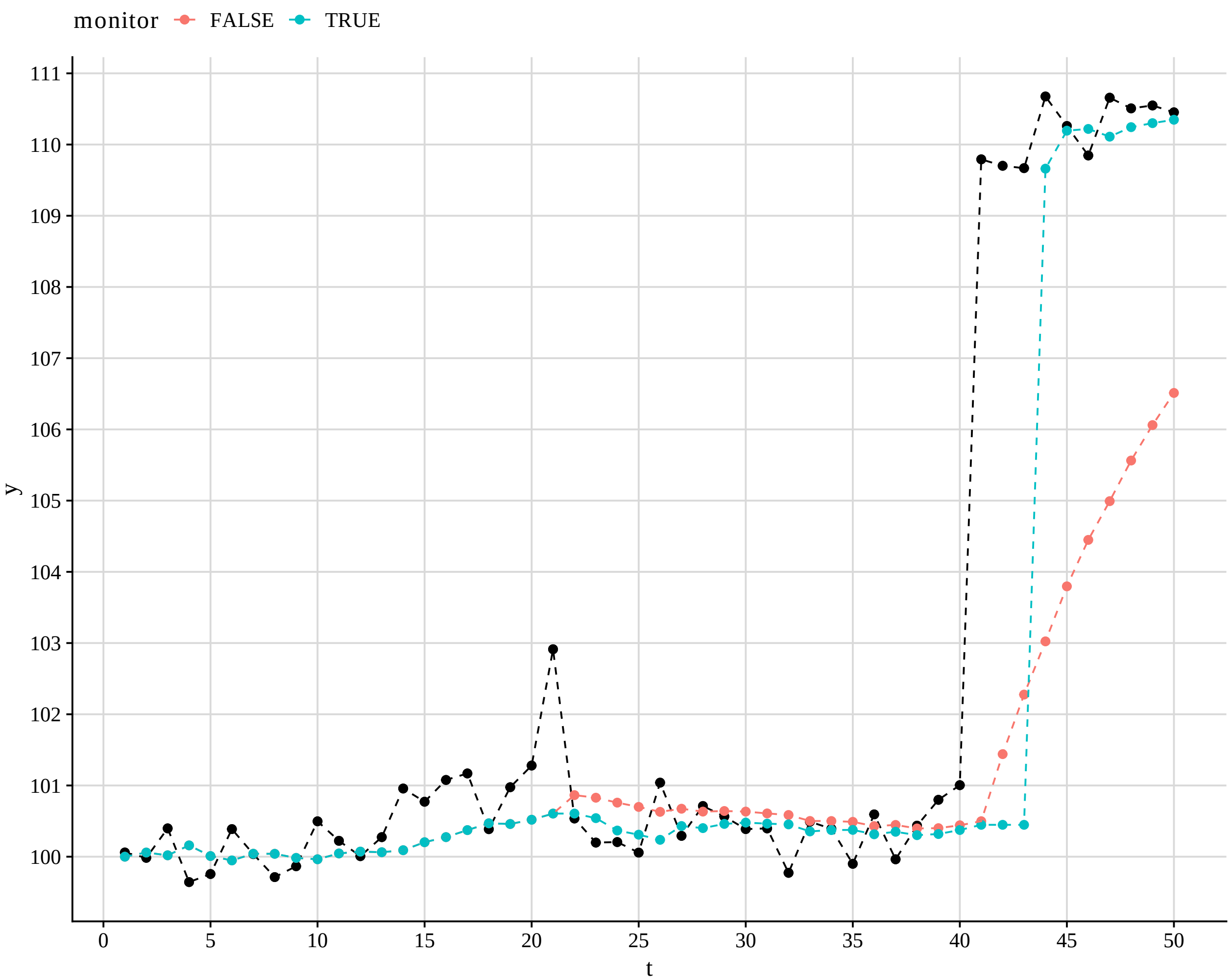
```
## 18 18 100.45 100.76   0.1991 -0.68855 4.683e+04 4.683e+04 1
## 19 19 100.42 100.70   0.1896 -0.64679 3.962e+04 3.962e+04 1
## 20 20 100.34 100.65   0.1811 -0.72604 5.440e+04 5.440e+04 1
## 21 21 104.06 100.60   0.1748  8.27597 1.252e-11 1.000e+00 0
## 22 22 100.20 100.60   2.8231 -0.23975 7.778e+03 7.778e+03 1
## 23 23 100.50 100.22   0.2904  0.51567 3.789e+02 3.789e+02 1
## 24 24 100.54 100.36   0.2146  0.38011 6.517e+02 6.517e+02 1
## 25 25 100.52 100.43   0.1847  0.21290 1.272e+03 1.272e+03 1
## 26 26 100.41 100.45   0.1668 -0.11343 4.692e+03 4.692e+03 1
## 27 27  99.88 100.44   0.1541 -1.43700 9.347e+05 9.347e+05 1
## 28 28 100.41 100.32   0.1561  0.21363 1.268e+03 1.268e+03 1
## 29 29 100.54 100.34   0.1477  0.53839 3.460e+02 3.460e+02 1
## 30 30  99.84 100.37   0.1419 -1.41664 8.616e+05 8.616e+05 1
## 31 31 103.79 100.29   0.1452  9.18538 3.294e-13 1.000e+00 0
## 32 32  99.92 100.29   2.3444 -0.24081 7.811e+03 7.811e+03 1
## 33 33 100.43  99.94   0.2445  0.98890 5.708e+01 5.708e+01 1
## 34 34 100.85 100.19   0.1868  1.52652 6.645e+00 6.645e+00 1
## 35 35 100.99 100.43   0.1739  1.35290 1.331e+01 1.331e+01 1
## 36 36 100.19 100.59   0.1676 -0.97163 1.453e+05 1.453e+05 1
## 37 37  99.90 100.49   0.1610 -1.49086 1.159e+06 1.159e+06 1
## 38 38  99.69 100.37   0.1622 -1.67843 2.455e+06 2.455e+06 1
## 39 39 100.65 100.24   0.1671  1.01996 5.041e+01 5.041e+01 1
## 40 40 100.19 100.31   0.1649 -0.29059 9.532e+03 9.532e+03 1
## 41 41 102.72 100.29   0.1591  6.07876 8.212e-08 1.000e+00 0
## 42 42 100.66 100.29   2.5693  0.22844 1.195e+03 1.195e+03 1
## 43 43 100.51 100.64   0.2699 -0.25147 8.151e+03 8.151e+03 1
## 44 44 100.55 100.57   0.2016 -0.05057 3.649e+03 3.649e+03 1
## 45 45 100.45 100.56   0.1758 -0.26400 8.570e+03 8.570e+03 1
## 46 46 100.03 100.53   0.1616 -1.24712 4.373e+05 4.373e+05 1
## 47 47 100.46 100.41   0.1574  0.12725 1.792e+03 1.792e+03 1
## 48 48 100.68 100.42   0.1499  0.65818 2.143e+02 2.143e+02 1
## 49 49 100.86 100.47   0.1451  1.03864 4.678e+01 4.678e+01 1
## 50 50 100.42 100.54   0.1433 -0.32205 1.081e+04 1.081e+04 1
```

Outlier and Level Change

```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=50, V=0.1, W=0.01)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [20, 21, 40],
>>>                 "level_mean_shift": [3, -3, 10],
>>>                 "level_var_shift": [1, 1, 1]})
```




```
## Potential outlier detected at time 21 with H=3.1218635839427244e-05, L=3.1218635839427244e-05 and  l=
## Potential outlier detected at time 41 with H=2.63690851913607e-34, L=2.63690851913607e-34 and  l=1
## Potential outlier detected at time 42 with H=9.847418238793209e-08, L=9.847418238793209e-08 and  l=1
```

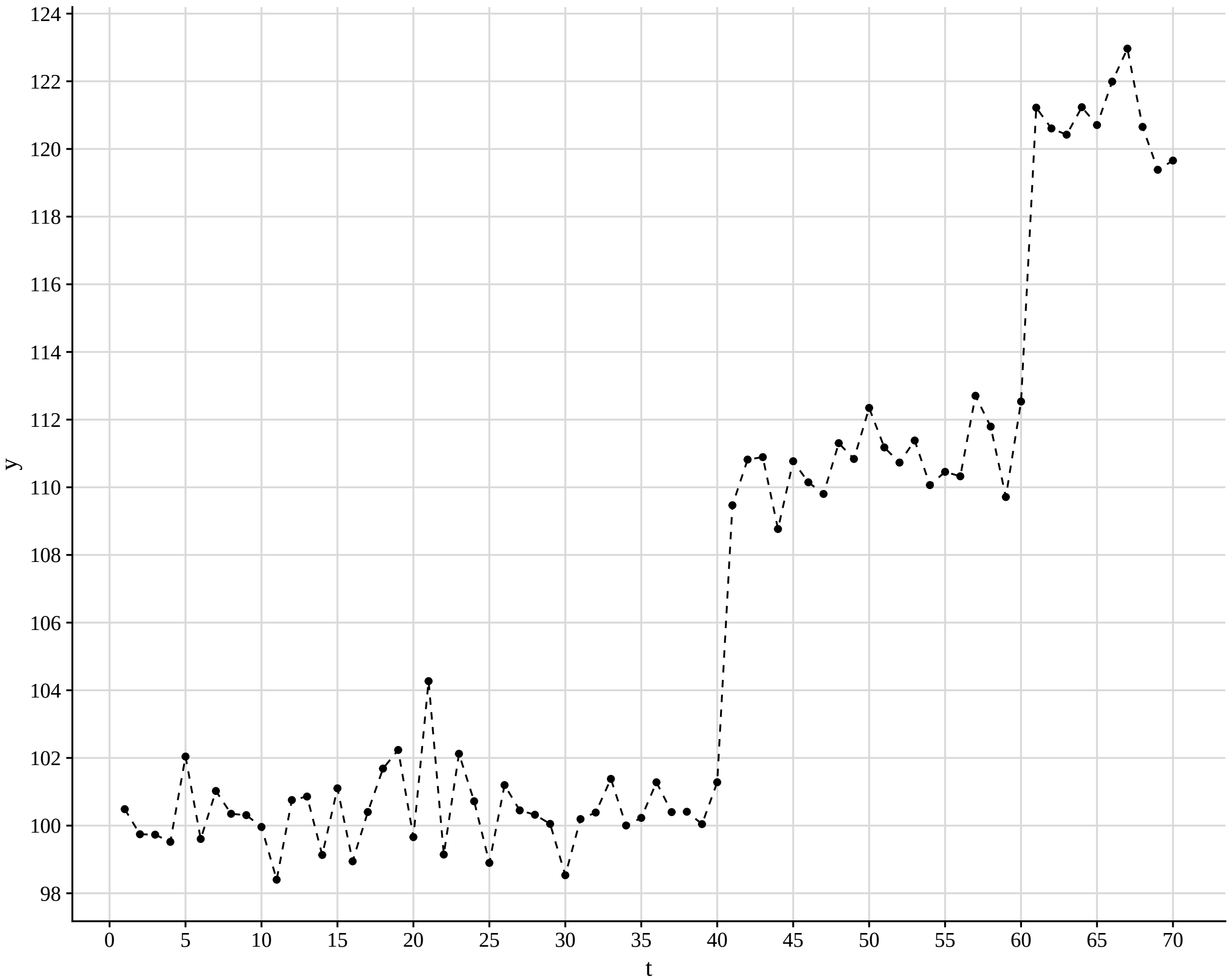



```
##      t      y      f        q         e         H         L l
## 1    1 100.06 100.00 101.0000  0.005906 1.000e+00 1.000e+00 1
## 2    2  99.98 100.06   1.0501 -0.072702 1.000e+00 1.000e+00 1
## 3    3 100.40 100.02   0.5288  0.520070 1.000e+00 1.000e+00 1
## 4    4  99.64 100.16   0.3850 -0.830693 1.000e+00 1.000e+00 1
## 5    5  99.76 100.01   0.3390 -0.433305 1.000e+00 1.000e+00 1
## 6    6 100.39  99.95   0.2817  0.828823 1.000e+00 1.000e+00 1
## 7    7 100.04 100.04   0.2619 -0.010396 1.000e+00 1.000e+00 1
## 8    8  99.71 100.04   0.2247 -0.689692 1.000e+00 1.000e+00 1
## 9    9  99.87  99.98   0.2085 -0.256097 1.000e+00 1.000e+00 1
## 10 10 100.50  99.96   0.1868  1.232935 1.000e+00 1.000e+00 1
## 11 11 100.22 100.05   0.1939  0.402009 5.970e+02 5.970e+02 1
## 12 12 100.01 100.07   0.1790 -0.151071 5.455e+03 5.455e+03 1
## 13 13 100.27 100.06   0.1645  0.521995 3.695e+02 3.695e+02 1
## 14 14 100.96 100.09   0.1552  2.198895 4.513e-01 4.513e-01 1
## 15 15 100.77 100.20   0.1940  1.290263 1.710e+01 7.717e+00 2
## 16 16 101.08 100.27   0.2014  1.788822 2.327e+00 2.327e+00 1
## 17 17 101.17 100.37   0.2267  1.670946 3.729e+00 3.729e+00 1
## 18 18 100.39 100.47   0.2486 -0.164802 5.763e+03 5.763e+03 1
## 19 19 100.98 100.46   0.2354  1.064636 4.216e+01 4.216e+01 1
## 20 20 101.28 100.52   0.2365  1.564744 5.703e+00 5.703e+00 1
## 21 21 102.91 100.61   0.2523  4.593624 3.122e-05 1.000e+00 0
```

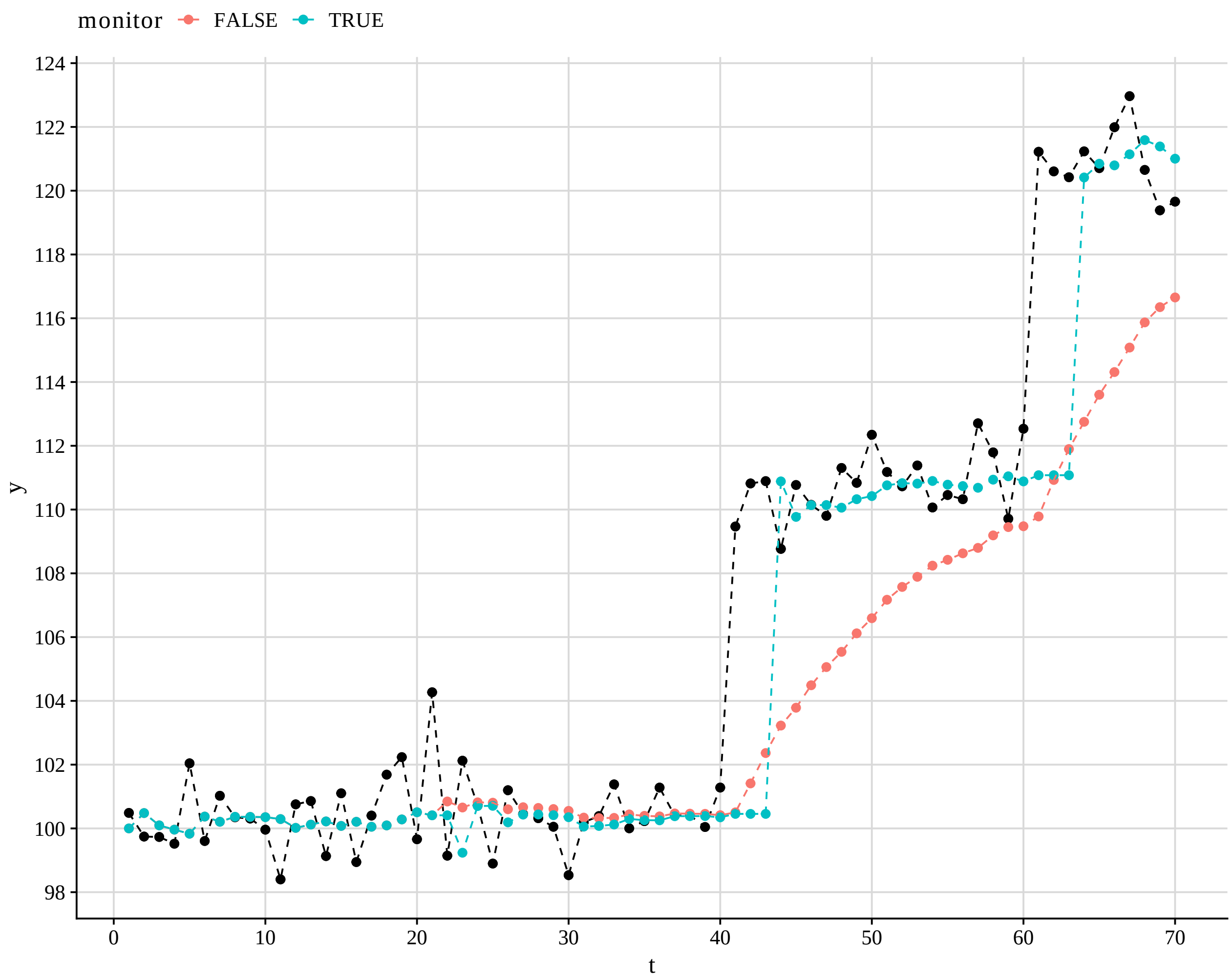
```
## 22 22 100.54 100.61   3.0569 -0.039535 3.492e+03 3.492e+03 1
## 23 23 100.20 100.54   0.4340 -0.521028 2.396e+04 2.396e+04 1
## 24 24 100.21 100.37   0.3238 -0.284362 9.297e+03 9.297e+03 1
## 25 25 100.06 100.31   0.2789 -0.476265 2.003e+04 2.003e+04 1
## 26 26 101.04 100.24   0.2543  1.591679 5.121e+00 5.121e+00 1
## 27 27 100.29 100.43   0.2591 -0.270771 8.805e+03 8.805e+03 1
## 28 28 100.71 100.40   0.2437  0.628259 2.415e+02 2.415e+02 1
## 29 29 100.56 100.46   0.2339  0.215907 1.257e+03 1.257e+03 1
## 30 30 100.39 100.48   0.2229 -0.192873 6.448e+03 6.448e+03 1
## 31 31 100.40 100.46   0.2133 -0.144727 5.318e+03 5.318e+03 1
## 32 32  99.77 100.45   0.2047 -1.503593 1.220e+06 1.220e+06 1
## 33 33 100.49 100.36   0.2113  0.298013 9.050e+02 9.050e+02 1
## 34 34 100.39 100.37   0.2042  0.039400 2.546e+03 2.546e+03 1
## 35 35  99.90 100.38   0.1972 -1.072491 2.175e+05 2.175e+05 1
## 36 36 100.59 100.31   0.1972  0.629152 2.407e+02 2.407e+02 1
## 37 37  99.96 100.35   0.1932 -0.879131 1.004e+05 1.004e+05 1
## 38 38 100.44 100.30   0.1914  0.304634 8.814e+02 8.814e+02 1
## 39 39 100.80 100.32   0.1864  1.111950 3.489e+01 3.489e+01 1
## 40 40 101.00 100.37   0.1871  1.455834 8.817e+00 8.817e+00 1
## 41 41 109.79 100.45   0.1919 21.329571 2.637e-34 1.000e+00 0
## 42 42 109.70 100.45   2.3525  6.033368 9.847e-08 1.000e+00 0
## 43 43 109.67 100.45 218.4051  0.623938 2.457e+02 2.457e+02 1
## 44 44 110.68 109.66   0.3536  1.705067 3.253e+00 3.253e+00 1
## 45 45 110.26 110.19   0.2776  0.124457 1.812e+03 1.812e+03 1
## 46 46 109.85 110.22   0.2413 -0.758511 6.195e+04 6.195e+04 1
## 47 47 110.66 110.11   0.2243  1.154759 2.940e+01 2.940e+01 1
## 48 48 110.51 110.24   0.2171  0.565532 3.104e+02 3.104e+02 1
## 49 49 110.55 110.30   0.2082  0.544045 3.383e+02 3.383e+02 1
## 50 50 110.45 110.35   0.2010  0.232877 1.174e+03 1.174e+03 1
```

Outlier and Two Level Change

```
>>> np.random.seed(66)
>>> rdlm = RandomDLM(n=70, V=1, W=0.01)
>>> df_simulated = rdlm.level(
>>>     start_level=100,
>>>     dict_shift={"t": [20, 21, 40, 60],
>>>                 "level_mean_shift": [5, -5, 10, 10],
>>>                 "level_var_shift": [1, 1, 1, 1]})
```




```
## Potential outlier detected at time 21 with H=0.0010052376448170103, L=0.0010052376448170103 and  l=1
## Potential outlier detected at time 41 with H=2.397219413890933e-13, L=2.397219413890933e-13 and  l=1
## Potential outlier detected at time 42 with H=0.01541991496658125, L=0.01541991496658125 and  l=1
## Potential outlier detected at time 61 with H=5.000422111435014e-15, L=5.000422111435014e-15 and  l=1
## Potential outlier detected at time 62 with H=0.05511185991311692, L=0.05511185991311692 and  l=1
```



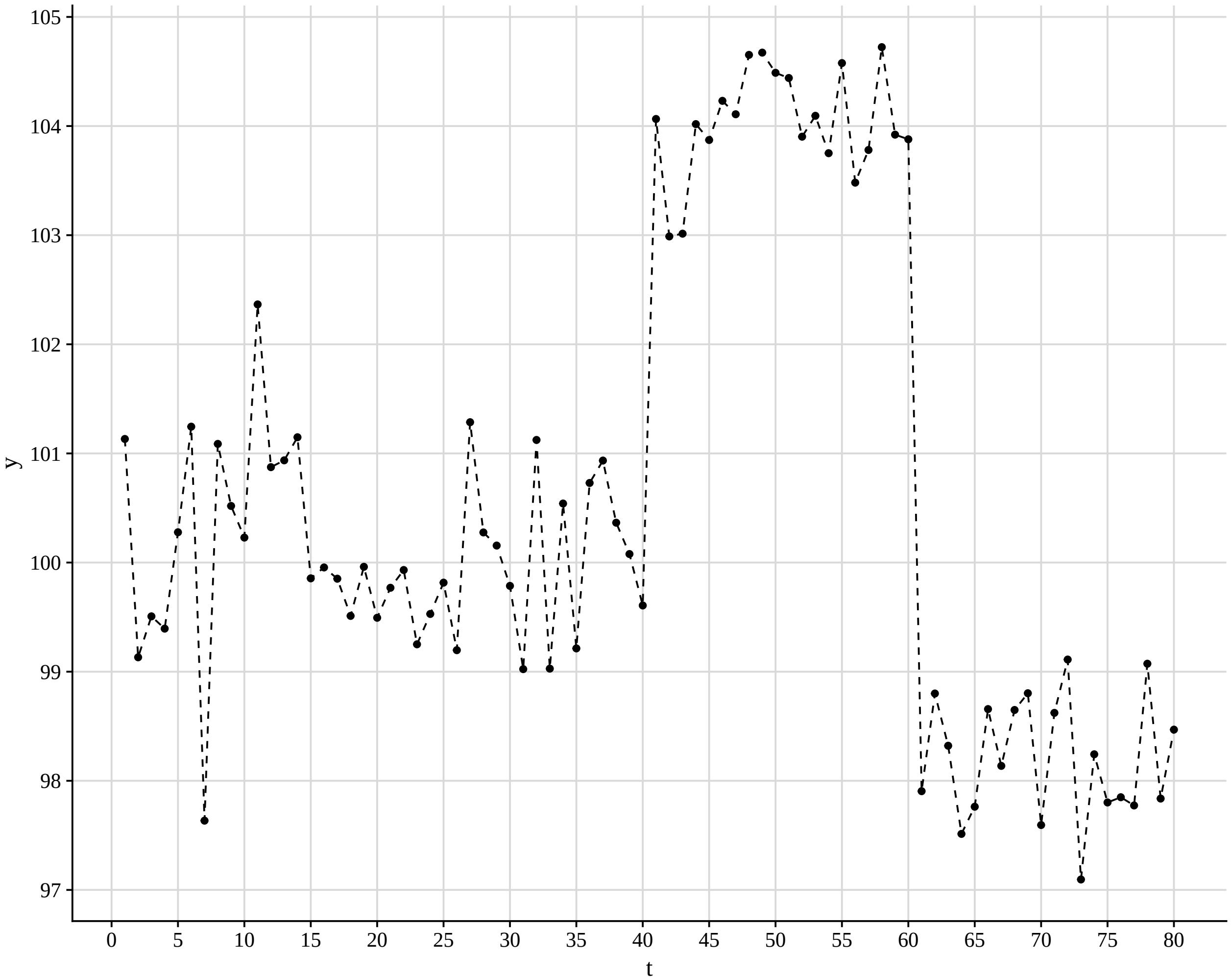

```
##      t      y      f         q         e         H         L l
## 1    1 100.49 100.00  101.0000  0.048367 1.000e+00 1.000e+00 1
## 2    2  99.74 100.48    1.0525 -0.718276 1.000e+00 1.000e+00 1
## 3    3  99.73 100.10    0.6649 -0.445731 1.000e+00 1.000e+00 1
## 4    4  99.52  99.96    0.4735 -0.642503 1.000e+00 1.000e+00 1
## 5    5 102.04  99.83    0.3923  3.525704 1.000e+00 1.000e+00 1
## 6    6  99.61 100.37    1.0952 -0.732493 1.000e+00 1.000e+00 1
## 7    7 101.02 100.21    0.9953  0.817334 1.000e+00 1.000e+00 1
## 8    8 100.35 100.36    0.9354 -0.016446 1.000e+00 1.000e+00 1
## 9    9 100.31 100.36    0.8193 -0.057104 1.000e+00 1.000e+00 1
## 10 10  99.96 100.35    0.7292 -0.459832 1.000e+00 1.000e+00 1
## 11 11  98.40 100.29    0.6707 -2.310043 3.071e+07 3.071e+07 1
## 12 12 100.76 100.02    0.9064  0.775188 1.342e+02 1.342e+02 1
## 13 13 100.86 100.12    0.8732  0.790556 1.262e+02 1.262e+02 1
## 14 14  99.13 100.22    0.8455 -1.181524 3.364e+05 3.364e+05 1
## 15 15 101.10 100.08    0.8641  1.100947 3.646e+01 3.646e+01 1
## 16 16  98.94 100.21    0.8724 -1.351576 6.642e+05 6.642e+05 1
## 17 17 100.40 100.05    0.9120  0.366429 6.883e+02 6.883e+02 1
## 18 18 101.69 100.09    0.8658  1.710841 3.179e+00 3.179e+00 1
## 19 19 102.24 100.28    0.9514  2.003486 9.862e-01 9.862e-01 1
## 20 20  99.66 100.51    1.0926 -0.810314 7.621e+04 7.516e+04 2
## 21 21 104.27 100.41    1.0729  3.725633 1.005e-03 1.000e+00 0
```

```
## 22 22  99.14 100.41   12.9975 -0.351295 1.215e+04 1.215e+04 1
## 23 23 102.12  99.24    1.8561  2.118274 6.231e-01 6.231e-01 1
## 24 24 100.72 100.70    1.6467  0.014348 2.815e+03 1.754e+03 2
## 25 25  98.90 100.71    1.4135 -1.522634 1.317e+06 1.317e+06 1
## 26 26 101.20 100.19    1.4001  0.851742 9.879e+01 9.879e+01 1
## 27 27 100.45 100.43    1.3329  0.014637 2.811e+03 2.811e+03 1
## 28 28 100.32 100.44    1.2500 -0.103953 4.518e+03 4.518e+03 1
## 29 29 100.05 100.41    1.1829 -0.334215 1.135e+04 1.135e+04 1
## 30 30  98.53 100.35    1.1301 -1.709695 2.782e+06 2.782e+06 1
## 31 31 100.19 100.06    1.1890  0.125275 1.806e+03 1.806e+03 1
## 32 32 100.39 100.08    1.1409  0.289497 9.364e+02 9.364e+02 1
## 33 33 101.38 100.12    1.1002  1.200294 2.450e+01 2.450e+01 1
## 34 34 100.00 100.30    1.1082 -0.278882 9.095e+03 9.095e+03 1
## 35 35 100.23 100.26    1.0728 -0.028141 3.336e+03 3.336e+03 1
## 36 36 101.28 100.25    1.0377  1.007153 5.306e+01 5.306e+01 1
## 37 37 100.40 100.38    1.0344  0.015935 2.797e+03 2.797e+03 1
## 38 38 100.41 100.38    1.0034  0.024566 2.702e+03 2.702e+03 1
## 39 39 100.04 100.39    0.9744 -0.347833 1.198e+04 1.198e+04 1
## 40 40 101.28 100.35    0.9503  0.957780 6.464e+01 6.464e+01 1
## 41 41 109.47 100.46    0.9465  9.264824 2.397e-13 1.000e+00 0
## 42 42 110.82 100.46   11.6002  3.043024 1.542e-02 1.000e+00 0
## 43 43 110.89 100.46 1076.9772  0.318020 8.354e+02 8.354e+02 1
## 44 44 108.77 110.88    1.7314 -1.608805 1.858e+06 1.858e+06 1
## 45 45 110.77 109.77    1.3492  0.861384 9.506e+01 9.506e+01 1
## 46 46 110.15 110.14    1.1933  0.007497 2.893e+03 2.893e+03 1
## 47 47 109.80 110.14    1.0943 -0.323459 1.087e+04 1.087e+04 1
## 48 48 111.30 110.06    1.0306  1.227059 2.202e+01 2.202e+01 1
## 49 49 110.84 110.32    1.0139  0.509124 3.890e+02 3.890e+02 1
## 50 50 112.35 110.42    0.9785  1.944294 1.250e+00 1.250e+00 1
## 51 51 111.18 110.76    1.0199  0.413232 5.708e+02 5.708e+02 1
## 52 52 110.73 110.83    0.9911 -0.099277 4.434e+03 4.434e+03 1
## 53 53 111.38 110.81    0.9626  0.579885 2.931e+02 2.931e+02 1
## 54 54 110.07 110.90    0.9431 -0.856058 9.151e+04 9.151e+04 1
## 55 55 110.46 110.78    0.9325 -0.335897 1.143e+04 1.143e+04 1
## 56 56 110.32 110.74    0.9122 -0.431681 1.676e+04 1.676e+04 1
## 57 57 112.71 110.68    0.8946  2.137120 5.778e-01 5.778e-01 1
## 58 58 111.79 110.94    0.9492  0.877814 8.901e+01 5.143e+01 2
## 59 59 109.71 111.04    0.9424 -1.371258 7.186e+05 7.186e+05 1
## 60 60 112.53 110.88    0.9544  1.690431 3.450e+00 3.450e+00 1
## 61 61 121.22 111.08    0.9827 10.232314 5.000e-15 1.000e+00 0
## 62 62 120.61 111.08   12.2306  2.724598 5.511e-02 1.000e+00 0
## 63 63 120.42 111.08 1137.0190  0.277123 9.839e+02 9.839e+02 1
## 64 64 121.23 120.42    1.8053  0.609810 2.600e+02 2.600e+02 1
## 65 65 120.71 120.85    1.3414 -0.119504 4.808e+03 4.808e+03 1
## 66 66 121.99 120.80    1.1742  1.103500 3.609e+01 3.609e+01 1
## 67 67 122.97 121.14    1.1057  1.733631 2.902e+00 2.902e+00 1
## 68 68 120.65 121.59    1.0963 -0.893458 1.063e+05 1.063e+05 1
## 69 69 119.38 121.39    1.0635 -1.942896 7.072e+06 7.072e+06 1
## 70 70 119.66 121.00    1.0872 -1.291934 5.232e+05 5.232e+05 1
```

Bilateral Level Change

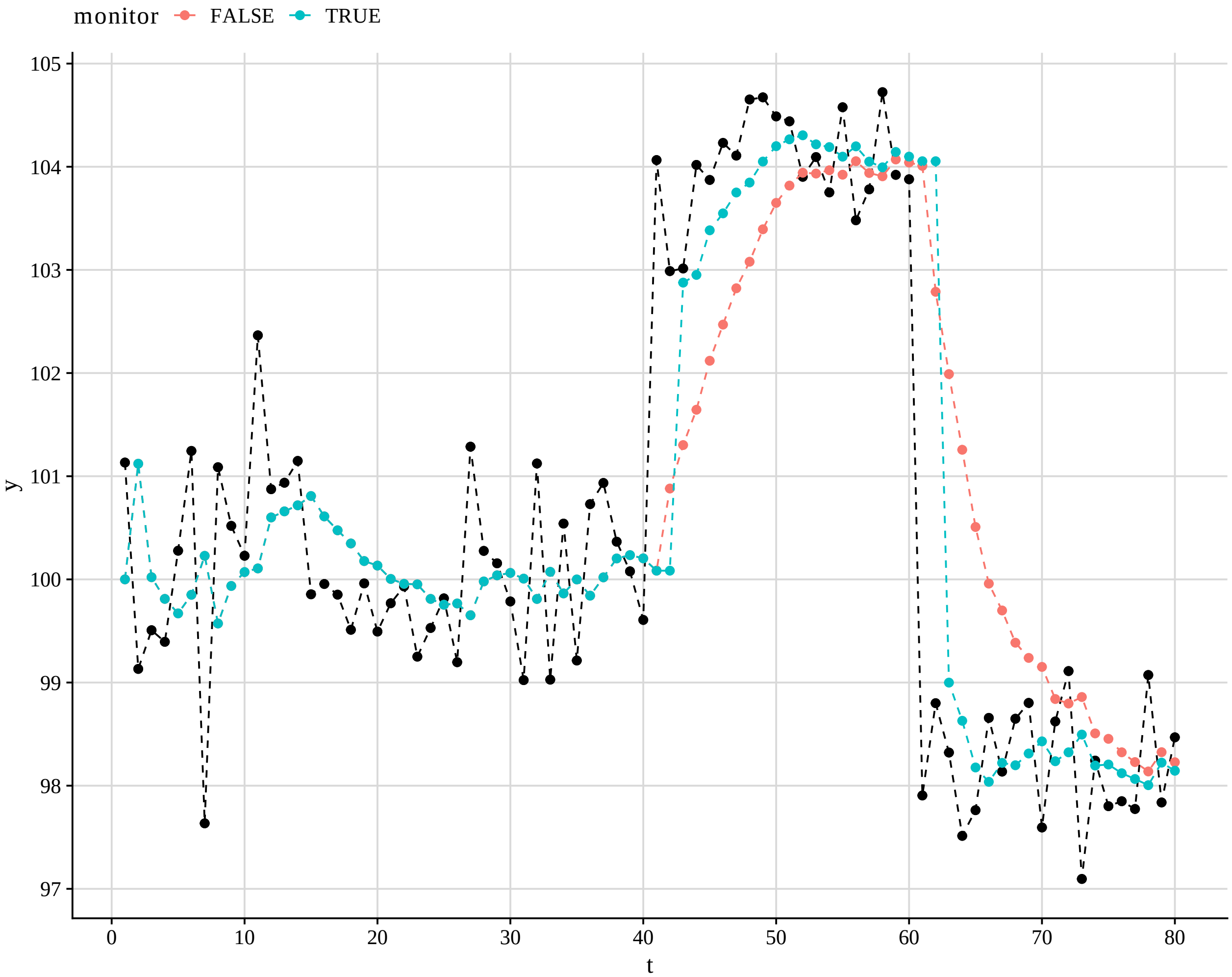
```
>>> np.random.seed(66)
>>> y1 = np.random.normal(loc=100, scale=0.8, size=40)
```

```
>>> y2 = np.random.normal(loc=104, scale=0.5, size=20)
>>> y3 = np.random.normal(loc=98, scale=0.5, size=20)
>>> y = np.concatenate([y1, y2, y3])
>>> t = np.arange(0, len(y)) + 1
>>> df_simulated = pd.DataFrame({"t": t, "y": y})
```




```
## Upper potential outlier detected at time 41 with H=6.852085090801099e-05, L=6.852085090801099e-05and
## Lower potential outlier detected at time 61 with H=1.4223097572368782e-10, L=1.4223097572368782e-10ar
```

```
>>> ggplot(py$df_res, aes(x = t, y = y)) + geom_point(size = 2) + geom_line(linetype = "dashed") +
>>>     geom_point(aes(y = f, col = monitor), size = 2) + geom_line(aes(y = f,
>>>     col = monitor), linetype = "dashed") + scale_x_continuous(breaks = scales::pretty_breaks(10)) +
>>>     scale_y_continuous(breaks = scales::pretty_breaks(10))
```




```
>>> py$df_tmp %>%
>>>     select(t, y, f, q, e, H_lower, L_lower, l_lower, H_upper, L_upper,
>>>         l_upper)
```

```
##      t      y      f        q        e   H_lower   L_lower l_lower
## 1    1 101.13 100.00 101.0000  0.11269 1.000e+00 1.000e+00       1
## 2    2  99.13 101.12   1.1330 -1.86908 1.000e+00 1.000e+00       1
## 3    3  99.51 100.02   1.5682 -0.41039 1.000e+00 1.000e+00       1
## 4    4  99.39  99.81   1.1097 -0.39523 1.000e+00 1.000e+00       1
## 5    5 100.28  99.67   0.8686  0.65198 1.000e+00 1.000e+00       1
## 6    6 101.25  99.85   0.7570  1.60292 1.000e+00 1.000e+00       1
## 7    7  97.63 100.23   0.9045 -2.72678 1.000e+00 1.000e+00       1
## 8    8 101.09  99.57   1.6048  1.19583 1.000e+00 1.000e+00       1
## 9    9 100.52  99.94   1.6612  0.45189 1.000e+00 1.000e+00       1
## 10  10 100.23 100.07   1.5154  0.12842 1.000e+00 1.000e+00       1
## 11  11 102.37 100.11   1.3706  1.92990 6.713e+06 6.713e+06       1
## 12  12 100.87 100.60   1.6732  0.21167 6.951e+03 6.951e+03       1
## 13  13 100.94 100.66   1.5441  0.22333 7.283e+03 7.283e+03       1
## 14  14 101.15 100.72   1.4349  0.35921 1.254e+04 1.254e+04       1
## 15  15  99.86 100.81   1.3483 -0.82044 1.120e+02 1.120e+02       1
## 16  16  99.96 100.61   1.3183 -0.57070 3.041e+02 3.041e+02       1
## 17  17  99.85 100.48   1.2641 -0.55467 3.242e+02 3.242e+02       1
```

```
## 18 18  99.51 100.35   1.2141 -0.75940 1.429e+02 1.429e+02        1
## 19 19  99.96 100.18   1.1859 -0.19892 1.345e+03 1.345e+03        1
## 20 20  99.49 100.13   1.1281 -0.60229 2.680e+02 2.680e+02        1
## 21 21  99.77 100.00   1.0933 -0.22518 1.211e+03 1.211e+03        1
## 22 22  99.93  99.96   1.0456 -0.02454 2.702e+03 2.702e+03        1
## 23 23  99.25  99.95   0.9998 -0.70097 1.806e+02 1.806e+02        1
## 24 24  99.53  99.81   0.9783 -0.28471 9.545e+02 9.545e+02        1
## 25 25  99.82  99.75   0.9421  0.06360 3.845e+03 3.845e+03        1
## 26 26  99.20  99.77   0.9059 -0.59823 2.723e+02 2.723e+02        1
## 27 27 101.29  99.65   0.8842  1.73763 3.111e+06 3.111e+06        1
## 28 28 100.28  99.98   0.9478  0.30431 1.007e+04 1.007e+04        1
## 29 29 100.16 100.04   0.9181  0.12169 4.850e+03 4.850e+03        1
## 30 30  99.79 100.06   0.8879 -0.29325 9.224e+02 9.224e+02        1
## 31 31  99.02 100.01   0.8616 -1.05991 4.296e+01 4.296e+01        1
## 32 32 101.12  99.81   0.8649  1.41205 8.459e+05 8.459e+05        1
## 33 33  99.03 100.07   0.8909 -1.10743 3.553e+01 3.553e+01        1
## 34 34 100.54  99.86   0.8968  0.71473 5.200e+04 5.200e+04        1
## 35 35  99.21 100.00   0.8843 -0.83581 1.053e+02 1.053e+02        1
## 36 36 100.73  99.84   0.8769  0.94767 1.320e+05 1.320e+05        1
## 37 37 100.93 100.02   0.8744  0.97843 1.493e+05 1.493e+05        1
## 38 38 100.37 100.20   0.8734  0.17387 5.976e+03 5.976e+03        1
## 39 39 100.08 100.24   0.8517 -0.17034 1.508e+03 1.508e+03        1
## 40 40  99.61 100.20   0.8310 -0.65477 2.172e+02 2.172e+02        1
## 41 41 104.06 100.08   0.8194  4.39709 1.297e+11 1.297e+11        1
## 42 42 102.99 100.08  17.0461  0.70342 4.970e+04 4.970e+04        1
## 43 43 103.01 102.88   1.4261  0.11467 4.716e+03 4.716e+03        1
## 44 44 104.02 102.95   1.0645  1.03340 1.860e+05 1.860e+05        1
## 45 45 103.87 103.38   0.9551  0.49947 2.198e+04 2.198e+04        1
## 46 46 104.23 103.55   0.8853  0.72574 5.434e+04 5.434e+04        1
## 47 47 104.11 103.75   0.8450  0.38908 1.413e+04 1.413e+04        1
## 48 48 104.65 103.85   0.8101  0.89455 1.067e+05 1.067e+05        1
## 49 49 104.67 104.05   0.7934  0.69911 4.885e+04 4.885e+04        1
## 50 50 104.49 104.20   0.7758  0.32738 1.104e+04 1.104e+04        1
## 51 51 104.44 104.27   0.7552  0.20053 6.648e+03 6.648e+03        1
## 52 52 103.90 104.31   0.7360 -0.46916 4.564e+02 4.564e+02        1
## 53 53 104.09 104.22   0.7213 -0.14506 1.669e+03 1.669e+03        1
## 54 54 103.75 104.19   0.7052 -0.52314 3.678e+02 3.678e+02        1
## 55 55 104.58 104.10   0.6936  0.57543 2.978e+04 2.978e+04        1
## 56 56 103.48 104.20   0.6835 -0.86654 9.312e+01 9.312e+01        1
## 57 57 103.78 104.05   0.6792 -0.32603 8.091e+02 8.091e+02        1
## 58 58 104.72 103.99   0.6676  0.89203 1.057e+05 1.057e+05        1
## 59 59 103.92 104.14   0.6645 -0.27237 1.003e+03 1.003e+03        1
## 60 60 103.88 104.10   0.6534 -0.27149 1.006e+03 1.006e+03        1
## 61 61  97.91 104.05   0.6429 -7.66839 1.422e-10 1.000e+00        0
## 62 62  98.80 104.05  13.5188 -1.42887 9.821e+00 9.821e+00        1
## 63 63  98.32  99.00   1.1488 -0.63253 2.374e+02 2.374e+02        1
## 64 64  97.51  98.63   0.8691 -1.19660 2.487e+01 2.487e+01        1
## 65 65  97.76  98.18   0.7838 -0.46765 4.592e+02 4.592e+02        1
## 66 66  98.66  98.04   0.7298  0.72567 5.432e+04 5.432e+04        1
## 67 67  98.14  98.22   0.6987 -0.10050 1.994e+03 1.994e+03        1
## 68 68  98.65  98.20   0.6719  0.55019 2.692e+04 2.692e+04        1
## 69 69  98.80  98.31   0.6539  0.60663 3.374e+04 3.374e+04        1
## 70 70  97.59  98.43   0.6401 -1.04345 4.589e+01 4.589e+01        1
## 71 71  98.62  98.24   0.6353  0.48382 2.065e+04 2.065e+04        1
```

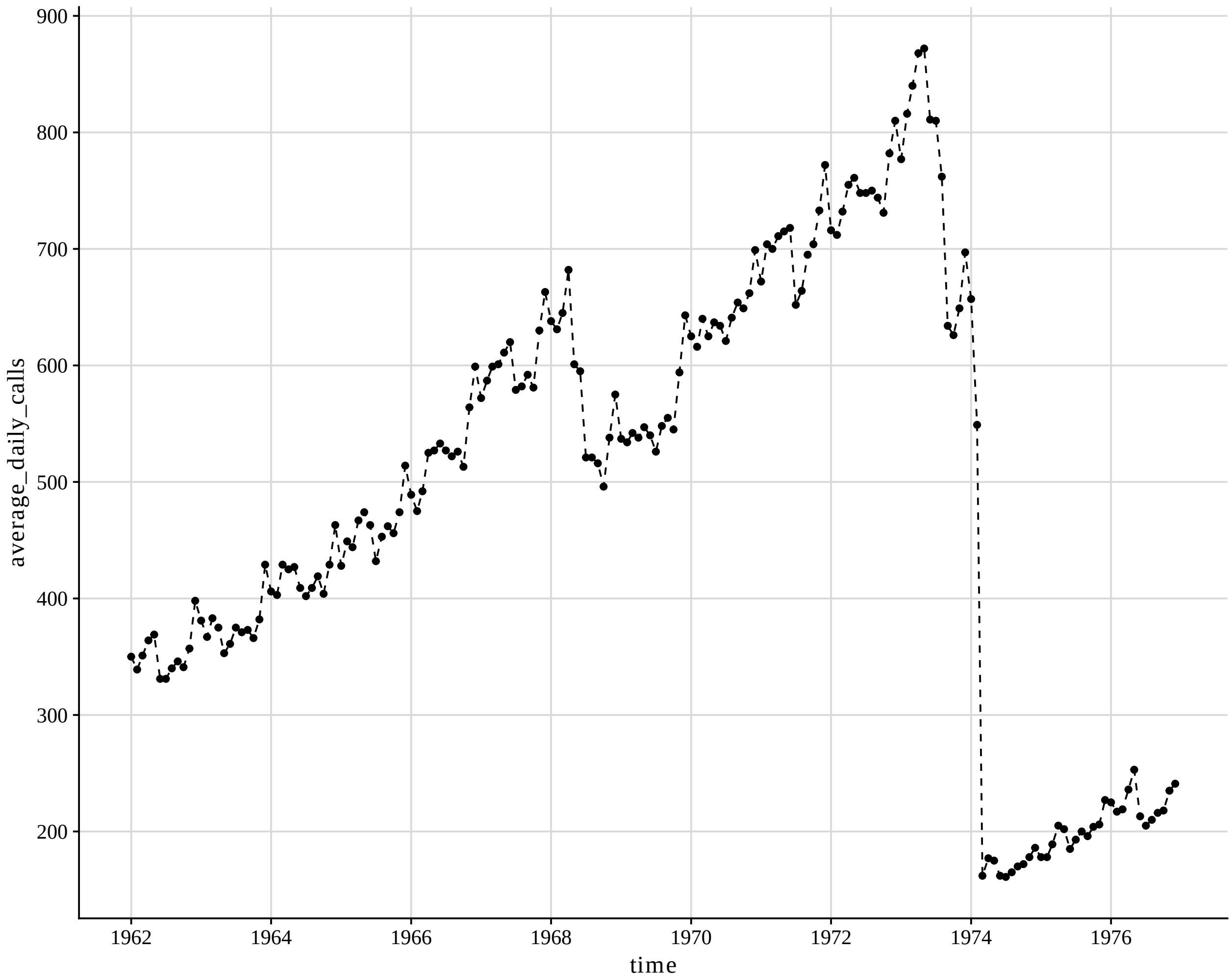
```
## 72 72  99.11  98.32   0.6242  0.99725 1.610e+05 1.610e+05        1
## 73 73  97.10  98.50   0.6209 -1.77631 2.447e+00 2.447e+00        1
## 74 74  98.24  98.20   0.6370  0.06006 3.790e+03 3.790e+03        1
## 75 75  97.80  98.21   0.6264 -0.51024 3.872e+02 3.872e+02        1
## 76 76  97.85  98.12   0.6187 -0.34507 7.497e+02 7.497e+02        1
## 77 77  97.77  98.06   0.6103 -0.37192 6.734e+02 6.734e+02        1
## 78 78  99.07  98.00   0.6025  1.37608 7.326e+05 7.326e+05        1
## 79 79  97.84  98.22   0.6087 -0.49467 4.121e+02 4.121e+02        1
## 80 80  98.47  98.14   0.6023  0.41762 1.584e+04 1.584e+04        1
##      H_upper   L_upper l_upper
## 1  1.000e+00 1.000e+00       1
## 2  1.000e+00 1.000e+00       1
## 3  1.000e+00 1.000e+00       1
## 4  1.000e+00 1.000e+00       1
## 5  1.000e+00 1.000e+00       1
## 6  1.000e+00 1.000e+00       1
## 7  1.000e+00 1.000e+00       1
## 8  1.000e+00 1.000e+00       1
## 9  1.000e+00 1.000e+00       1
## 10 1.000e+00 1.000e+00       1
## 11 1.324e+00 1.324e+00       1
## 12 1.278e+03 1.278e+03       1
## 13 1.220e+03 1.220e+03       1
## 14 7.085e+02 7.085e+02       1
## 15 7.936e+04 7.936e+04       1
## 16 2.923e+04 2.923e+04       1
## 17 2.741e+04 2.741e+04       1
## 18 6.217e+04 6.217e+04       1
## 19 6.606e+03 6.606e+03       1
## 20 3.316e+04 3.316e+04       1
## 21 7.337e+03 7.337e+03       1
## 22 3.288e+03 3.288e+03       1
## 23 4.921e+04 4.921e+04       1
## 24 9.310e+03 9.310e+03       1
## 25 2.311e+03 2.311e+03       1
## 26 3.263e+04 3.263e+04       1
## 27 2.856e+00 2.856e+00       1
## 28 8.825e+02 8.825e+02       1
## 29 1.832e+03 1.832e+03       1
## 30 9.633e+03 9.633e+03       1
## 31 2.068e+05 2.068e+05       1
## 32 1.050e+01 1.050e+01       1
## 33 2.501e+05 2.501e+05       1
## 34 1.709e+02 1.709e+02       1
## 35 8.439e+04 8.439e+04       1
## 36 6.731e+01 6.731e+01       1
## 37 5.952e+01 5.952e+01       1
## 38 1.487e+03 1.487e+03       1
## 39 5.892e+03 5.892e+03       1
## 40 4.091e+04 4.091e+04       1
## 41 6.852e-05 1.000e+00       0
## 42 1.788e+02 1.788e+02       1
## 43 1.884e+03 1.884e+03       1
## 44 4.777e+01 4.777e+01       1
```

```
## 45 4.043e+02 4.043e+02       1
## 46 1.635e+02 1.635e+02       1
## 47 6.287e+02 6.287e+02       1
## 48 8.325e+01 8.325e+01       1
## 49 1.819e+02 1.819e+02       1
## 50 8.047e+02 8.047e+02       1
## 51 1.337e+03 1.337e+03       1
## 52 1.947e+04 1.947e+04       1
## 53 5.325e+03 5.325e+03       1
## 54 2.416e+04 2.416e+04       1
## 55 2.984e+02 2.984e+02       1
## 56 9.543e+04 9.543e+04       1
## 57 1.098e+04 1.098e+04       1
## 58 8.409e+01 8.409e+01       1
## 59 8.861e+03 8.861e+03       1
## 60 8.830e+03 8.830e+03       1
## 61 6.248e+16 6.248e+16       1
## 62 9.048e+05 9.048e+05       1
## 63 3.743e+04 3.743e+04       1
## 64 3.573e+05 3.573e+05       1
## 65 1.935e+04 1.935e+04       1
## 66 1.636e+02 1.636e+02       1
## 67 4.456e+03 4.456e+03       1
## 68 3.300e+02 3.300e+02       1
## 69 2.633e+02 2.633e+02       1
## 70 1.936e+05 1.936e+05       1
## 71 4.304e+02 4.304e+02       1
## 72 5.520e+01 5.520e+01       1
## 73 3.632e+06 3.632e+06       1
## 74 2.344e+03 2.344e+03       1
## 75 2.295e+04 2.295e+04       1
## 76 1.185e+04 1.185e+04       1
## 77 1.320e+04 1.320e+04       1
## 78 1.213e+01 1.213e+01       1
## 79 2.156e+04 2.156e+04       1
## 80 5.609e+02 5.609e+02       1
```

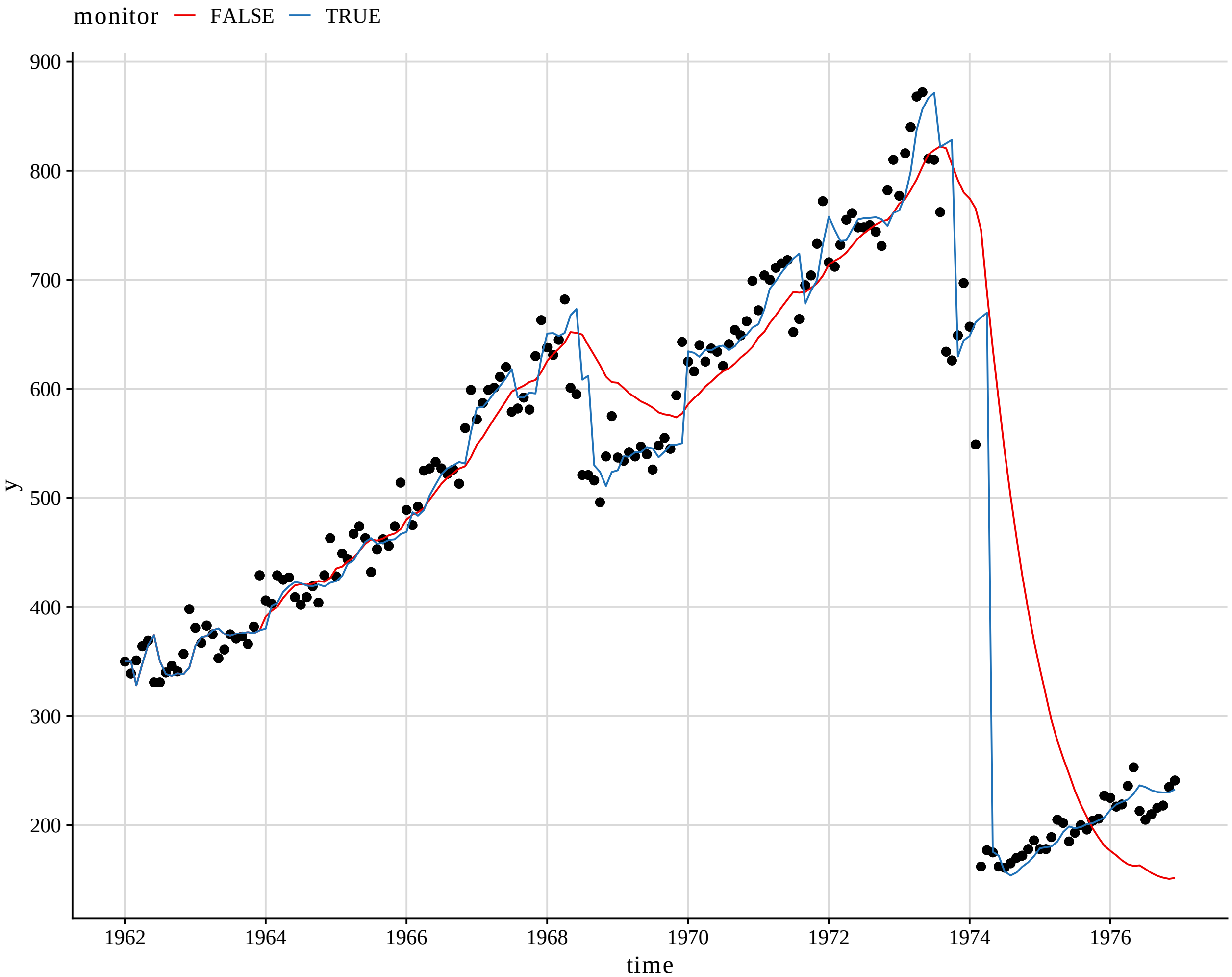
Real data applications

Telephone Calls

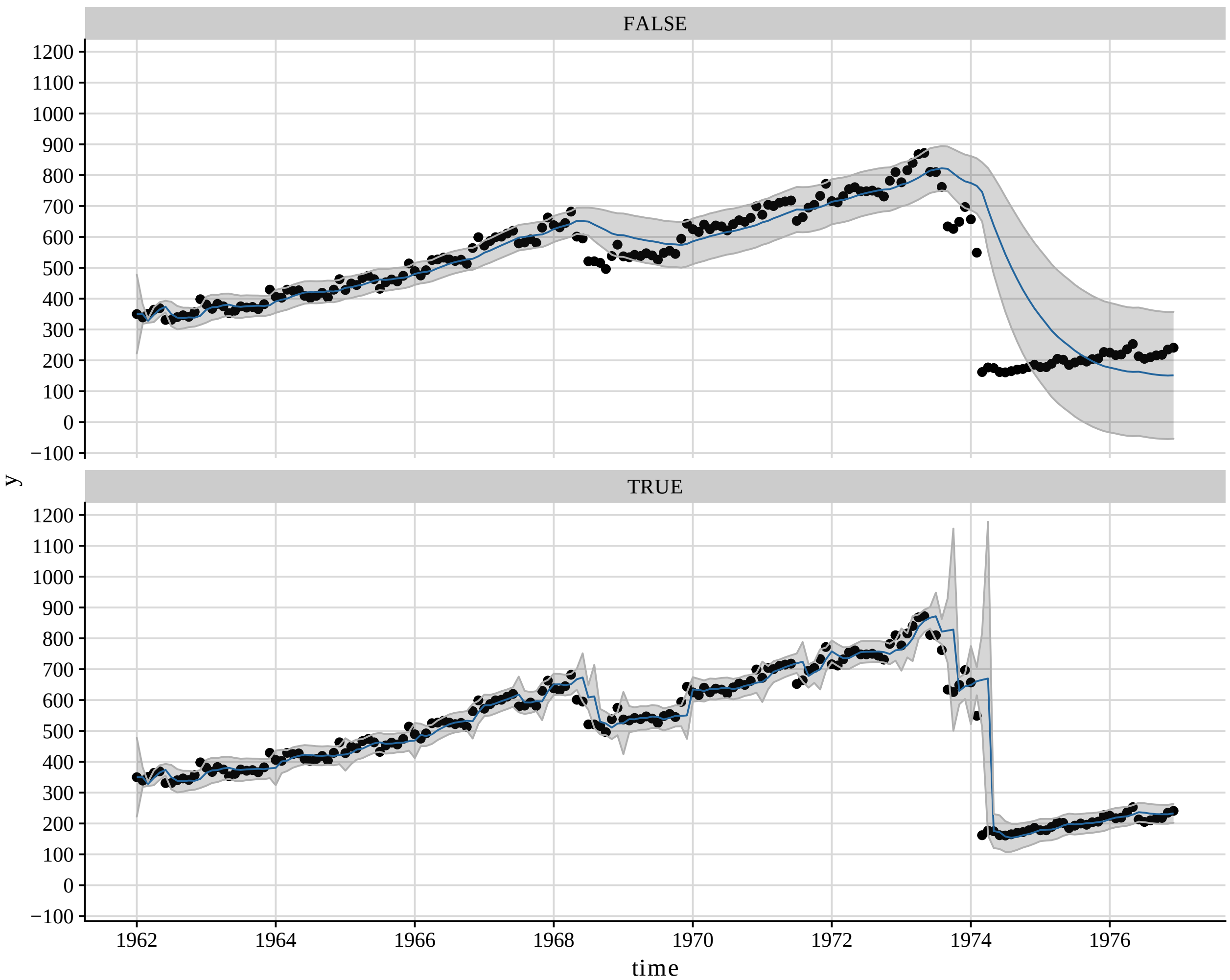
```
## Rows: 180
## Columns: 2
## $ time                <date> 1962-01-01, 1962-02-01, 1962-03-01, 196~
## $ average_daily_calls <dbl> 350, 339, 351, 364, 369, 331, 331, 340, ~
```




```
## Upper potential outlier detected at time 24 with H=0.006182801077763127, L=0.006182801077763127and l=
## Upper potential outlier detected at time 36 with H=0.04695009275309578, L=0.04695009275309578and l=1
## Upper potential outlier detected at time 48 with H=0.010667111715486855, L=0.010667111715486855and l=
## Upper parametric change detected at time 61 with H=371.5075065279889, L=0.8665659319590896 and l=3
## Lower parametric change detected at time 69 with H=73.48957980292538, L=21.11336373558922 and l=3
## Upper parametric change detected at time 73 with H=1017.8894222937022, L=5.103069050925839 and l=3
## Lower potential outlier detected at time 77 with H=0.0004773719349100128, L=0.0004773719349100128and
## Lower potential outlier detected at time 79 with H=8.290756920919369e-05, L=8.290756920919369e-05and
## Upper potential outlier detected at time 84 with H=0.05878813134403288, L=0.05878813134403288and l=1
## Upper potential outlier detected at time 95 with H=0.06892996628785275, L=0.06892996628785275and l=1
## Upper potential outlier detected at time 108 with H=0.07266554972703385, L=0.07266554972703385and l=
## Lower potential outlier detected at time 115 with H=0.00012539263264369665, L=0.00012539263264369665
## Lower parametric change detected at time 121 with H=9.817545249668383, L=9.817545249668383 and l=3
## Upper potential outlier detected at time 132 with H=0.028683109480204753, L=0.028683109480204753and 
## Upper parametric change detected at time 137 with H=1.3781426241191645, L=0.014890903913373155 and l=
## Lower potential outlier detected at time 138 with H=0.009474041226003621, L=0.009474041226003621and 
## Lower potential outlier detected at time 140 with H=0.028300819384206272, L=0.028300819384206272and 
## Lower potential outlier detected at time 141 with H=0.0015973993973896503, L=0.0015973993973896503an
## Upper potential outlier detected at time 144 with H=0.11437026489167935, L=0.11437026489167935and l=
## Lower potential outlier detected at time 146 with H=9.965912212293438e-06, L=9.965912212293438e-06an
## Lower potential outlier detected at time 147 with H=1.3965093239872577e-08, L=1.3965093239872577e-08
```



monitor
FALSE
TRUE
900
800
700
600
500
400
300
200
y
1962
1964
1966
1968
1970
1972
1974
1976
time




```
##                     time   y     f       q        e   H_lower
## 1  1961-12-31 21:00:00 350 350.0 101.000  0.00000 1.000e+00
## 2  1962-01-31 21:00:00 339 350.0  53.653 -1.50175 1.000e+00
## 3  1962-02-28 21:00:00 351 328.3   4.409 10.80510 1.000e+00
## 4  1962-03-31 21:00:00 364 348.0  74.226  1.85430 1.000e+00
## 5  1962-04-30 21:00:00 369 365.0  83.023  0.43429 1.000e+00
## 6  1962-05-31 21:00:00 331 373.9  60.367 -5.52633 1.000e+00
## 7  1962-06-30 21:00:00 331 350.3 280.258 -1.15428 1.000e+00
## 8  1962-07-31 21:00:00 340 338.5 268.177  0.09294 1.000e+00
## 9  1962-08-31 21:00:00 346 336.9 223.792  0.60535 1.000e+00
## 10 1962-09-30 21:00:00 341 339.1 199.316  0.13333 1.000e+00
## 11 1962-10-31 21:00:00 357 338.4 174.300  1.40781 1.000e+00
## 12 1962-11-30 21:00:00 398 344.5 182.370  3.95840 1.000e+00
## 13 1962-12-31 21:00:00 381 364.2 377.451  0.86221 1.000e+00
## 14 1963-01-31 21:00:00 367 372.1 361.838 -0.27018 1.000e+00
## 15 1963-02-28 21:00:00 383 373.1 332.615  0.54294 1.000e+00
## 16 1963-03-31 21:00:00 375 378.7 312.363 -0.20765 1.000e+00
## 17 1963-04-30 21:00:00 353 380.4 290.231 -1.60653 1.000e+00
## 18 1963-05-31 21:00:00 361 375.3 311.412 -0.81209 1.000e+00
## 19 1963-06-30 21:00:00 375 373.5 302.112  0.08850 1.000e+00
## 20 1963-07-31 21:00:00 371 375.5 283.996 -0.26852 1.000e+00
## 21 1963-08-31 21:00:00 373 376.1 268.783 -0.18907 1.399e+03
```

```
## 22 1963-09-30 21:00:00 366 377.0  254.718 -0.68730 1.907e+02
## 23 1963-10-31 22:00:00 382 376.0  246.863  0.38242 1.376e+04
## 24 1963-11-30 22:00:00 429 378.7  236.325  3.27150 1.437e+09
## 25 1963-12-31 22:00:00 406 380.2  737.882  0.95073 1.336e+05
## 26 1964-01-31 22:00:00 403 401.1  339.721  0.10145 4.473e+03
## 27 1964-02-29 22:00:00 429 403.7  269.347  1.54092 1.416e+06
## 28 1964-03-31 21:00:00 425 414.0  259.528  0.68410 4.600e+04
## 29 1964-04-30 21:00:00 427 419.0  243.010  0.51466 2.336e+04
## 30 1964-05-31 21:00:00 409 423.0  230.631 -0.91990 7.522e+01
## 31 1964-06-30 21:00:00 402 422.0  226.004 -1.32784 1.471e+01
## 32 1964-07-31 21:00:00 409 419.7  229.720 -0.70405 1.784e+02
## 33 1964-08-31 21:00:00 419 419.2  225.306 -0.01128 2.849e+03
## 34 1964-09-30 21:00:00 404 420.7  218.311 -1.13070 3.237e+01
## 35 1964-10-31 21:00:00 429 418.8  220.263  0.68522 4.621e+04
## 36 1964-11-30 21:00:00 463 422.3  217.224  2.76467 1.893e+08
## 37 1964-12-31 21:00:00 428 423.7  670.553  0.16498 5.767e+03
## 38 1965-01-31 22:00:00 449 428.4  308.846  1.17159 3.233e+05
## 39 1965-02-28 22:00:00 444 439.4  258.623  0.28767 9.421e+03
## 40 1965-03-31 21:00:00 467 442.7  232.660  1.59164 1.735e+06
## 41 1965-04-30 21:00:00 474 451.6  232.174  1.47039 1.068e+06
## 42 1965-05-31 21:00:00 463 459.6  234.091  0.21942 7.170e+03
## 43 1965-06-30 21:00:00 432 463.0  226.561 -2.05920 7.892e-01
## 44 1965-07-31 21:00:00 453 458.0  243.320 -0.31957 8.302e+02
## 45 1965-08-31 21:00:00 462 458.7  238.287  0.21283 6.984e+03
## 46 1965-09-30 21:00:00 456 461.4  233.433 -0.35057 7.334e+02
## 47 1965-10-31 21:00:00 474 461.9  229.171  0.79614 7.201e+04
## 48 1965-11-30 21:00:00 514 466.7  227.455  3.13515 8.330e+08
## 49 1965-12-31 22:00:00 489 468.8  799.744  0.71489 5.203e+04
## 50 1966-01-31 22:00:00 475 486.9  324.948 -0.66095 2.119e+02
## 51 1966-02-28 22:00:00 492 483.6  263.148  0.51729 2.360e+04
## 52 1966-03-31 21:00:00 525 488.8  238.810  2.33980 3.459e+07
## 53 1966-04-30 21:00:00 527 502.3  250.430  1.56111 1.536e+06
## 54 1966-05-31 21:00:00 533 512.3  253.125  1.30124 5.431e+05
## 55 1966-06-30 21:00:00 527 521.5  254.930  0.34462 1.183e+04
## 56 1966-07-31 21:00:00 522 527.0  250.481 -0.31884 8.327e+02
## 57 1966-08-31 21:00:00 526 529.9  246.572 -0.24769 1.107e+03
## 58 1966-09-30 21:00:00 513 532.9  242.591 -1.27771 1.798e+01
## 59 1966-10-31 21:00:00 564 531.4  784.462  2.07913 1.220e+07
## 60 1966-11-30 22:00:00 599 560.3  345.623  3.43606 2.776e+09
## 61 1966-12-31 22:00:00 572 582.7  302.384  0.52061 2.392e+04
## 62 1967-01-31 22:00:00 587 583.4  277.399  0.21762 7.119e+03
## 63 1967-02-28 22:00:00 599 588.9  264.224  0.61950 3.552e+04
## 64 1967-03-31 21:00:00 601 596.4  258.300  0.28376 9.275e+03
## 65 1967-04-30 21:00:00 611 602.5  253.691  0.53094 2.493e+04
## 66 1967-05-31 21:00:00 620 609.9  250.921  0.63560 3.789e+04
## 67 1967-06-30 21:00:00 579 618.1  836.410 -2.47785 1.479e-01
## 68 1967-07-31 21:00:00 582 592.3  349.730 -1.83396 1.943e+00
## 69 1967-08-31 21:00:00 592 592.0  286.227 -0.92571 7.349e+01
## 70 1967-09-30 21:00:00 581 596.5  262.822 -0.95757 6.470e+01
## 71 1967-10-31 21:00:00 630 595.7  928.883  2.14392 1.580e+07
## 72 1967-11-30 22:00:00 663 628.0  349.863  3.17999 9.967e+08
## 73 1967-12-31 22:00:00 638 650.6  301.960  0.26863 8.730e+03
## 74 1968-01-31 22:00:00 631 651.1  284.202 -1.19203 2.533e+01
## 75 1968-02-29 22:00:00 645 648.4  282.576 -0.20144 1.332e+03
```

```
## 76 1968-03-31 21:00:00 682 651.2  279.036  1.84191 4.722e+06
## 77 1968-04-30 21:00:00 601 667.4  288.028 -3.91180 4.774e-04
## 78 1968-05-31 21:00:00 595 673.2 1542.864 -1.99143 1.035e+00
## 79 1968-06-30 21:00:00 521 608.3  403.266 -4.34945 8.291e-05
## 80 1968-07-31 21:00:00 521 612.0 2631.603 -1.77356 2.474e+00
## 81 1968-08-31 21:00:00 516 529.9  430.229 -0.66807 2.060e+02
## 82 1968-09-30 21:00:00 496 523.8  356.039 -1.47538 8.154e+00
## 83 1968-10-31 21:00:00 538 510.8  352.342  1.44789 9.763e+05
## 84 1968-11-30 21:00:00 575 523.7  358.331  2.70845 1.512e+08
## 85 1968-12-31 21:00:00 537 525.4 2569.300  0.22956 7.467e+03
## 86 1969-01-31 21:00:00 534 538.1  449.195 -0.19559 1.363e+03
## 87 1969-02-28 21:00:00 542 537.7  377.060  0.22238 7.255e+03
## 88 1969-03-31 21:00:00 538 541.9  365.266 -0.20414 1.317e+03
## 89 1969-04-30 21:00:00 547 541.9  359.179  0.26744 8.689e+03
## 90 1969-05-31 21:00:00 540 546.5  348.637 -0.35022 7.344e+02
##      L_lower l_lower   H_upper   L_upper l_upper
## 1  1.000e+00       1 1.000e+00 1.000e+00       1
## 2  1.000e+00       1 1.000e+00 1.000e+00       1
## 3  1.000e+00       1 1.000e+00 1.000e+00       1
## 4  1.000e+00       1 1.000e+00 1.000e+00       1
## 5  1.000e+00       1 1.000e+00 1.000e+00       1
## 6  1.000e+00       1 1.000e+00 1.000e+00       1
## 7  1.000e+00       1 1.000e+00 1.000e+00       1
## 8  1.000e+00       1 1.000e+00 1.000e+00       1
## 9  1.000e+00       1 1.000e+00 1.000e+00       1
## 10 1.000e+00       1 1.000e+00 1.000e+00       1
## 11 1.000e+00       1 1.000e+00 1.000e+00       1
## 12 1.000e+00       1 1.000e+00 1.000e+00       1
## 13 1.000e+00       1 1.000e+00 1.000e+00       1
## 14 1.000e+00       1 1.000e+00 1.000e+00       1
## 15 1.000e+00       1 1.000e+00 1.000e+00       1
## 16 1.000e+00       1 1.000e+00 1.000e+00       1
## 17 1.000e+00       1 1.000e+00 1.000e+00       1
## 18 1.000e+00       1 1.000e+00 1.000e+00       1
## 19 1.000e+00       1 1.000e+00 1.000e+00       1
## 20 1.000e+00       1 1.000e+00 1.000e+00       1
## 21 1.399e+03       1 6.351e+03 6.351e+03       1
## 22 1.907e+02       1 4.659e+04 4.659e+04       1
## 23 1.376e+04       1 6.457e+02 6.457e+02       1
## 24 1.437e+09       1 6.183e-03 1.000e+00       0
## 25 1.336e+05       1 6.649e+01 6.649e+01       1
## 26 4.473e+03       1 1.987e+03 1.987e+03       1
## 27 1.416e+06       1 6.273e+00 6.273e+00       1
## 28 4.600e+04       1 1.932e+02 1.932e+02       1
## 29 2.336e+04       1 3.805e+02 3.805e+02       1
## 30 7.522e+01       1 1.181e+05 1.181e+05       1
## 31 1.471e+01       1 6.040e+05 6.040e+05       1
## 32 1.784e+02       1 4.982e+04 4.982e+04       1
## 33 2.849e+03       1 3.119e+03 3.119e+03       1
## 34 3.237e+01       1 2.745e+05 2.745e+05       1
## 35 4.621e+04       1 1.923e+02 1.923e+02       1
## 36 1.893e+08       1 4.695e-02 1.000e+00       0
## 37 5.767e+03       1 1.541e+03 1.541e+03       1
## 38 3.233e+05       1 2.748e+01 2.748e+01       1
```

```
## 39 9.421e+03       1 9.432e+02 9.432e+02       1
## 40 1.735e+06       1 5.122e+00 5.122e+00       1
## 41 1.068e+06       1 8.318e+00 8.318e+00       1
## 42 7.170e+03       1 1.239e+03 1.239e+03       1
## 43 7.892e-01       1 1.126e+07 1.126e+07       1
## 44 6.552e+02       2 1.070e+04 1.070e+04       1
## 45 6.984e+03       1 1.272e+03 1.272e+03       1
## 46 7.334e+02       1 1.212e+04 1.212e+04       1
## 47 7.201e+04       1 1.234e+02 1.234e+02       1
## 48 8.330e+08       1 1.067e-02 1.000e+00       0
## 49 5.203e+04       1 1.708e+02 1.708e+02       1
## 50 2.119e+02       1 4.193e+04 4.193e+04       1
## 51 2.360e+04       1 3.765e+02 3.765e+02       1
## 52 3.459e+07       1 2.569e-01 2.569e-01       1
## 53 1.536e+06       1 5.787e+00 1.486e+00       2
## 54 5.431e+05       1 1.636e+01 1.636e+01       1
## 55 1.183e+04       1 7.511e+02 7.511e+02       1
## 56 8.327e+02       1 1.067e+04 1.067e+04       1
## 57 1.107e+03       1 8.029e+03 8.029e+03       1
## 58 1.798e+01       1 4.943e+05 4.943e+05       1
## 59 1.220e+07       1 7.287e-01 7.287e-01       1
## 60 2.776e+09       1 3.201e-03 2.333e-03       2
## 61 2.392e+04       1 3.715e+02 1.000e+00       0
## 62 7.119e+03       1 1.248e+03 1.248e+03       1
## 63 3.552e+04       1 2.501e+02 2.501e+02       1
## 64 9.275e+03       1 9.581e+02 9.581e+02       1
## 65 2.493e+04       1 3.565e+02 3.565e+02       1
## 66 3.789e+04       1 2.345e+02 2.345e+02       1
## 67 1.479e-01       1 6.009e+07 6.009e+07       1
## 68 2.873e-01       2 4.574e+06 4.574e+06       1
## 69 1.000e+00       0 1.209e+05 1.209e+05       1
## 70 6.470e+01       1 1.373e+05 1.373e+05       1
## 71 1.580e+07       1 5.623e-01 5.623e-01       1
## 72 9.967e+08       1 8.915e-03 5.013e-03       2
## 73 8.730e+03       1 1.018e+03 1.000e+00       0
## 74 2.533e+01       1 3.509e+05 3.509e+05       1
## 75 1.332e+03       1 6.673e+03 6.673e+03       1
## 76 4.722e+06       1 1.882e+00 1.882e+00       1
## 77 1.000e+00       0 1.861e+10 1.861e+10       1
## 78 1.035e+00       1 8.587e+06 8.587e+06       1
## 79 1.000e+00       0 1.072e+11 1.072e+11       1
## 80 2.474e+00       1 3.592e+06 3.592e+06       1
## 81 2.060e+02       1 4.314e+04 4.314e+04       1
## 82 8.154e+00       1 1.090e+06 1.090e+06       1
## 83 9.763e+05       1 9.101e+00 9.101e+00       1
## 84 1.512e+08       1 5.879e-02 1.000e+00       0
## 85 7.467e+03       1 1.190e+03 1.190e+03       1
## 86 1.363e+03       1 6.518e+03 6.518e+03       1
## 87 7.255e+03       1 1.225e+03 1.225e+03       1
## 88 1.317e+03       1 6.745e+03 6.745e+03       1
## 89 8.689e+03       1 1.023e+03 1.023e+03       1
## 90 7.344e+02       1 1.210e+04 1.210e+04       1
##  [ reached 'max' / getOption("max.print") -- omitted 90 rows ]
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