Exercise : 4.5

Q15 :

$a + 0 + 0 + 0 = 1$
$a + b + 0 + 0 = 0$
$a + b + c + 0 = 1$
$a + b + c + d = 0$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Taking determinant

$= 1 \, (1\{1-0\}) = 0$
$= 1(1)(1) = 0.$
$\Rightarrow 1 \neq 0$

so, it is not trivial solution and linearly independent, and span M_{22} so they form basis for M_{22}. using eqns

$a = 1$
$b = -1$
$c = 1$
$d = -1$

$A_s = (1, -1, 1, -1)$

Q17:

$$a(1+x+x^2)+b(x+x^2)+c(x^2)=(7-x+2x^2)$$

$\Rightarrow a+0+0=7$
$a+b+0=-1$
$a+b+c=2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\Rightarrow 1(1-0)=0$

$\Rightarrow 1 \neq 0$

It is not trivial, so it is linearly independent and ~~span~~ful P_2 and forms basis for P_2.

$\Rightarrow$ using above eqns

$\Rightarrow \boxed{a=7}$

$7+b=-1$

$\Rightarrow \boxed{b=-8}$

$\Rightarrow \boxed{c=3}$

$(P_2)=(7,-8,3)$

b)
(a-

Q11. (a) 'U_3' is the result of u_1 and u_2 so it is linearly dependent that's why it can't be a basis for R_2

b) The two vectors generate a plane in R_3 but doesnot span all of R_3, so it is not a basis for R_3.

c) the polynomial $P = 1$, can not be expressed as a linear combination of given two polynomials, This means these two polynomials do not span P_2. Hence they don't form a basis for P_2.

d) The matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ can't be expressed as a linear combination of the given 4 matrices, this mean these 4 matrices doesn't span M_{22} - thence donot form a basis for M_{22}.

Q21: (a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix}$

(b) $A =$

$T_A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 2 \\ 1 & -3 & 0 \end{bmatrix}$

$T_A =$

$T_A (1,0,0) = a(1,0,-1), T_A(0,1,0) = b(1,1,2)$
$T_A(0,0,1) \; c(1,-3,0)$

$T_A (1,0,$

$a+b+c=0$
$0+b-3c=0$
$-a+2b+0=0$

$a+$
$0+$
$-a+$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix}$

$\det = 1(6)-1(-3)+1(1)$
$= 6+3+1$
$\Rightarrow \boxed{10 \neq 0}$

$= 1$
$=$
$=$

It is linear independent set $\{T_A(e_1), T_A(e_2), T_A(e_3)\}$

The

(b) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$T_A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

$T_A(1,0,0) = (1,0,-1), T_A(0,1,0) = (1,1,2), T_A(0,0,1) = (2,1,1)$

$a + b + 2c = 0$
$0 + b + c = 0$
$-a + b + c = 0$

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$= 1(1-2) - 1(0+1) + 2(0+1)$
$= -1 - 1 + 2$
$\Rightarrow 0 = 0$

The set is linearly ~~in~~dependent.

b)
(a, b)

Q23. (a) $(\sqrt{3}, 1)$

$\frac{\sqrt{3}}{2}$ and $\frac{1}{2} = \cos 30°, \sin 30°$. $u_2 = (0,1)$

$(\sqrt{3}, 1) = 2\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right] + 0(0,1)$

$= (2, 0)$

(b) $(1, 0)$

$(1,0) = c_1\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) + c_2(0,1)$

$\frac{\sqrt{3}}{2} c_1 = 1$

$\frac{1}{2} c_1 + c_2 = 0$

$(w)_s = \left(\frac{2}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

(c) $(0, 1)$

$(0,1) = 0\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) + 1(0,1)$

$(w)_s \quad (0,1)$

d) (a, b)

$$(a, b) = c_1\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) + c_2 (0, 1)$$

$$\frac{\sqrt{3}}{2} c_1 = a$$

$$\frac{1}{2} c_1 + c_2 = b$$

$$c_1 = \frac{2a}{\sqrt{3}} + 2^{nd} = b = \frac{a}{\sqrt{3}}$$

$$(w)_s = \left(\frac{2a}{\sqrt{3}}, b - \frac{a}{\sqrt{3}}\right)$$

Q:27 (a) $w = \begin{bmatrix} 6 \\ -1 \\ 4 \end{bmatrix}$

$$S = \{(3, 1, -4), (2, 5, 6), (1, 4, 8)\}$$

$$w = 6(3, 1, 4) - 1(2, 5, 6) + 4(1, 4, 8)$$

$$= (20, 17, 2)$$

(b) $q = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$

Q1:

$(x^2+1), (x^2-1), (2x-1)$

$q = 3(x^2+1) + 0(x^2-1) + 4(2x-1)$

$= 3x^2 + 8x + 3 - 4$

$= \boxed{3x^2 + 8x - 1}$

$R_2 - 2R$

(c) $B = \begin{bmatrix} -8 \\ 7 \\ 6 \\ 3 \end{bmatrix}$

$R_3 +$

$R_1 +$

$\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$R_3 +$

$-8 \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + 7 \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}$

$+ 3 \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -21 & -103 \\ -106 & 30 \end{bmatrix}$

Exercise : 4.6

Q1:

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + 0 + x_3 = 0$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$R_2 - 2R_3 \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{matrix} R_3 + R_1 \\ R_1 + R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$R_3 + R_2 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2\left(\frac{-1}{1}\right) \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{x_3 = t}$

$\boxed{x_2 = 0}$

$\boxed{x_1 = t}$

It is lineally independent.

Q2:
$$\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

using reduce row echelon form

$$\begin{bmatrix} 1 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 1/4 & 1 & 0 \end{bmatrix}$$

$\boxed{x_4 = t}$

$\boxed{x_3 = s}$

$\boxed{x_2 = -1/4\, s - t}$

$\boxed{x_1 = -1/4\, s}$

lineally independent system having v_1 and v_2 to form basis for sol. set and dimension are "2".

Q3 $2x_1 + x_2 + 3x_3 = 0$
$x_1 + 5x_3 = 0$
$x_2 + x_3 = 0$

By using reduced row echelon

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{x_3 = x_2 = x_1 = 0}$$

It doesn't have any basis. dimension is 0

Q4
$$\begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 2 & -8 & 6 & -2 & 0 \end{bmatrix}$$

By using row echelon.

$$\begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_4 = t$
$x_3 = s$
$x_2 = r$

$$\boxed{x_1 = 4r - 3s + t}$$

It is linearly independent and v_1, v_2, v_3 forms basis for sol. spaces with dimension of '3'.

Q5:

$$\begin{bmatrix} 1 & -3 & 1 & 0 \\ 2 & -6 & 2 & 0 \\ 3 & -9 & 3 & 0 \end{bmatrix}$$

Using reduced row echelon

$$\begin{bmatrix} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = t$

$x_2 = s$

$\boxed{x_1 = -3s - t}$

It is linearly independent. V_1, V_2 form basis for sol. spaces and having dimension of 2

Q7:

Q6:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 3 & 2 & -2 & 0 \\ 4 & 3 & -1 & 0 \\ 6 & 5 & 1 & 0 \end{bmatrix}$$

After reduced row echelon:

$$\begin{bmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t$

$x_2 = -5t$

$x_1 = 4t$

It is linearly independent.

Q 1: $3x - 2y + 5z = 0$

$\boxed{y = s}$

$\boxed{z = t}$

$3x = 2s - 5t$

$\boxed{x = \frac{2}{3}s - \frac{5}{3}t}$

$(x, y, z) = \left(\frac{2}{3}s - \frac{5}{3}t, s, t\right) = s\left(\frac{2}{3}, 1, 0\right) + t\left(-\frac{5}{3}, 0, 1\right)$

They are linearly independent and had dimension of '2'

$\rightarrow$ $(a, b, c) - b = a + c$

$(a, a+c, c)$

$a(1, 1, 0) + c(0, 1, 1)$

The subset of basis in $\{(1,1,0), (0,1,1)\}$
It had dimension of '2'

Q8 : $\rightarrow$ $(a, b, c > 0)$

$= \{(1,0,0,0), (0,1,0,0), (0,0,1,0)\}$ It is linearly independent and had dimension of '3'

$\rightarrow$ (a, b, c, d), where as $d = a+b$ and $c = a-b$

$(a, b, a-b, a+b)$
$a(1, 0, 1, 1) + b(0, 1, -1, 1)$

$S = \{(1, 0, 1, 1), (0, 1, -1, 1)\}$
It has '2' dimensions

$\Rightarrow (a, b, c, d) \quad a = b = c = d$

(a, a, a, a)

$S = \{(1, 1, 1, 1)\}$

dimension = '1'

Q11:

a) $a_0 + a_1 x + a_2 x^2 = 0$

and

$b_0 + b_1 x + b_2 x^2 = 0$

By adding polynomial

$a_0 + b_0 + a_1 x + b_1 x + a_2 x^2 + b_2 x^2 = 0$

$a_0 = b_0 = -a_1 - a_2$ and $b_0 = -b_1 - b_2$

$-a_1 - b_1 - a_2 - b_2 + a_1 x + b_1 x + a_2 x^2 + b_2 x^2 = 0$

$(-a_1 - b_1 - a_2 - b_2) + x(a_1 + b_1) + x^2(a_2 + b_2) = 0$

for multiplication:

$-ka_1 - ka_2 + ka_1 x + ka x^2 = 0$

It is subspace of P_2

c) $-a_1 - a_2 + a_1 x + a_2 x^2 = a_1(-1+x) + a_2(-1+x^2)$

b

$-1+x$ and $-1+x^2$ span w

$a_1 = a_2 = 0$ $(-1+x)$ and $(-1+x^2)$ are

linearly independent form basis for dimension is '2'

Exercise : 4.7

Q1: (a)

$P_{B' \to B} = ?$

$$\left[\begin{array}{cc|cc} 2 & 4 & 1 & -1 \\ 2 & -1 & 3 & -1 \end{array}\right]$$

reduce row echelon

$$\left[\begin{array}{cc|cc} 1 & 0 & 13/10 & -1/2 \\ 0 & 1 & -2/5 & 0 \end{array}\right]$$

c

$$P_{B' \to B} = \begin{bmatrix} +13/10 & -1/2 \\ -2/5 & 0 \end{bmatrix}$$

2)   b) $P_{B \to B'} = ?$

$$\left[\begin{array}{cc|cc} 1 & 1 & 2 & 4 \\ 3 & -1 & 2 & -1 \end{array}\right].$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & -5/2 \\ 0 & 1 & -2 & -13/2 \end{array}\right]$$

$$P_{B \to B'} = \begin{bmatrix} 0 & -5/2 \\ -2 & -13/2 \end{bmatrix}$$

c) $$w = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} c_1 + c_2 \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$2c_1 + 4c_2 = 3$$
$$2c_1 - c_2 = -5$$

$$\begin{bmatrix} 1 & 0 & -17/10 \\ 0 & 1 & 8/5 \end{bmatrix}$$

$$c_1 = -17/10 \quad , c_2 = 8/5$$

$$\begin{bmatrix} -17/5 \\ 8/5 \end{bmatrix} \begin{bmatrix} 0 & -5/2 \\ -2 & -13/5 \end{bmatrix} = \begin{bmatrix} -4 \\ -7 \end{bmatrix}$$

d) $\begin{bmatrix} 3 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$= \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & -7 \end{array}\right]$

$c_1 = -4 \quad , c_2 = -7$

Verified!

Q2: $\left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & 1 & 4 \end{array}\right]$

$P_{B' \to B} = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

$= \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & 0 & 4/11 & 3/11 \\ 0 & 1 & -1/11 & 2/11 \end{array}\right]$

$P_{B \to B'} = \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix}$

c) $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$, $P_{B \to B'} = \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix} \begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} -3/11 \\ -13/11 \end{bmatrix}$

d) $\begin{bmatrix} 3 \\ -5 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

$2c_1 - 3c_2 = 3$

$c_1 + 4c_2 = -5$

$$\left[\begin{array}{cc|c} 1 & 0 & -3/11 \\ 0 & 1 & -13/11 \end{array}\right]$$

$$\begin{bmatrix} -3/11 \\ -13/11 \end{bmatrix}$$

Q3 · (a) $\left[\begin{array}{ccc|ccc} 3 & 1 & -1 & 2 & 2 & 1 \\ 2 & 1 & 0 & 1 & -1 & 2 \\ -5 & -3 & 2 & 1 & 1 & 1 \end{array}\right]$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 5/2 \\ 0 & 1 & 0 & -2 & -3 & -1/2 \\ 0 & 0 & 1 & 5 & 1 & 6 \end{array}\right]$$

$$= \begin{bmatrix} 3 & 2 & 5/2 \\ -2 & -3 & -1/2 \\ 5 & 1 & 6 \end{bmatrix}$$

b) $\begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$(w)_B = \begin{bmatrix} +9 \\ -9 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 5/2 \\ -2 & -3 & -5/2 \\ 5 & 1 & 6 \end{bmatrix} \begin{bmatrix} 9 \\ -9 \\ -5 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -3/2 \\ 6 \end{bmatrix}$$

c) $\begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix} = c_1 \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

$$3c_1 + c_2 - c_3 = -5$$
$$c_1 + c_2 = 8$$
$$-5c_1 \cancel{+} - 3c_2 + 2c_3 = -5$$

$[w]_B =$

Q6 a) $6a_1 +$
$3a_1 +$

b) $2a_1 +$

$$\begin{bmatrix} 1 & 0 & 0 & -7/2 \\ 0 & 1 & 0 & 23/2 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$$[w]_B = \begin{bmatrix} -7/2 \\ 23/2 \\ 6 \end{bmatrix}$$

Q6 : a) $6a_1 + 10a_2 = 2$ $\quad$ $6a_1 + 10a_2 = 0$

$3a_1 + 2a_2 = 0$ $\quad$ $3b_1 + 2b_2 = 2$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} -2/9 & 1/9 \\ 1/3 & -1/6 \end{bmatrix}$$

b) $2a_1 + 3a_2 = 6$ $\quad$ $2b_1 + 3b_2 = 10$

$2a_2 = 3$ $\quad$ $2b_2 = 2$

$$\left[\begin{array}{cc|c} 1 & 0 & 3/4 \\ 0 & 1 & 3/2 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 0 & 7/2 \\ 0 & 1 & 1 \end{array}\right]$$

Q1
$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 1 \\ 2 & 3 & 3 & 4 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & 5 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$$\begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix}$$

b)
$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 2 \\ 3 & 4 & 2 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & 5 \\ 0 & 1 & -1 & -3 \end{array}\right]$$

c)
$$\begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

d)
$$c_1 + 2c_2 = 0$$
$$2c_1 + 3c_2 = 1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$$

4·7

$$W_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$1C_1 + 1C_2 = 2$$
$$3C_1 + 4C_2 = 5$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Q8) (a) $\Rightarrow \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

b) $\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right]$

$$\left[\begin{array}{cc|cc} 1 & 0 & 4/11 & 3/11 \\ 0 & 1 & -1/11 & 2/11 \end{array}\right]$$

$$\begin{bmatrix} 4/11 & -3/11 \\ -1/11 & 2/11 \end{bmatrix}$$

c) $\begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix}$

d) $(5,-3) = (2,1) - (-3,4) \quad [v]_B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

e) $\begin{bmatrix} 3 \\ -5 \end{bmatrix} \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix} = \begin{bmatrix} -3/11 \\ -13/11 \end{bmatrix}$

Q9: a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

b) $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$

$= \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -3 \end{bmatrix}$

c) $\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $c_1 + 2c_2 + 3c_3 = 5$
$2c_1 + 5c_2 + 3c_3 = -3$
$c_1 \quad + 8c_3 = 1$

$\begin{bmatrix} 1 & 0 & 0 & -239 \\ 0 & 1 & 0 & 77 \\ 0 & 0 & 1 & 30 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} -239 \\ 77 \\ 30 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

Q10: a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

b) $P = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad P_{S \to B} = P^{-1}$. In our case $PP = I$

Therefore $P = P^{-1}$, 'P' is symmetric, we also have P^T.

$P_{S \to B} = P^T$

Q11. $V_2 = \left(\cos\left(2\theta - \frac{\pi}{2}\right)\right), \sin\left(2\theta, \frac{\pi}{2}\right) =$

$\sin 2\theta \quad -\cos 2\theta$

$$P_{B \to S} = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$$

b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}, P_{S \to B} = P^{-1}$ in our case

$PP = I$ therefore $P = P^{-1}$, P is symmetric,

$P_{S \to B} = P^T$

Q12. $[V]_{B_2} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} [V]_{B_1}$

$$(V)_{B_3} = \begin{bmatrix} 7 & 2 \\ 4 & -1 \end{bmatrix}$$

$$(V)_{B_3} = \begin{bmatrix} 7 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

$$(V)_{B_1} = \begin{bmatrix} 31 & 11 \\ 7 & 2 \end{bmatrix} (V)_{B_1}$$

$$P_{B1 \to B3} = \begin{bmatrix} 31 & 11 \\ 7 & 2 \end{bmatrix}$$

here $= \begin{bmatrix} -2/15 & 11/15 \\ 7/15 & -31/15 \end{bmatrix}$

Exercise: 4.8

Q5: a) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

b) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 5 \end{bmatrix} + r\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Q6: a) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

b) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 4 \\ -3 \end{bmatrix} + r\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

Q7: b is $\begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = 1 + 3t$, $x_2 = t$

$(x_1, x_2) = (1 + 3t, t) = (1, 0) + t(3, 1)$

$x = 0$ is $(x_1, x_2) = t(3, 1)$

b) $\begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -2-t,\ x_2 = 7-t,\ x_3 = t$

$(x_1, x_2, x_3) = (-2-t, 7-t, t) = (-2, 7, 0) + t(-2, 7, 1)$

$Ax = 0 \quad (x_1, x_2, x_3) = t(-1, -1, 1)$

Q8: a) $\begin{bmatrix} 1 & -2 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -1+2r-s-2t,\ x_2 = r,\ x_3 = s,\ x_4 = t$

$(x_1, x_2, x_3, x_4) = (-1+2r-s-2t, r, s, t) = (-1, 0, 0, 0) + r(2, 1, 0, 0)$

$+ s(-1, 0, 1, 0) + t(-2, 0, 0, 1)$

$x = 0$

$(x_1, x_2, x_3, x_4) = r(2, 1, 0, 0) + s(-1, 0, 1, 0) + t(-2, 0, 0, 1)$

b) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1 = \frac{6}{5} + \frac{7}{5}s$

$x_3 = s,$

(x_1, x_2, x_3, x_4)

Q9: a)

x_1

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & -3/5 & -1/5 & 6/5 \\ 0 & 1 & -4/5 & 3/5 & 7/5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = \frac{6}{5} + \frac{7}{5}s + \frac{7}{5}t$, $x_2 = \frac{7}{5} + \frac{4}{5}s - \frac{3}{5}t$,

$x_3 = s$, $x_4 = t$.

$(x_1, x_2, x_3, x_4) = \left(\frac{6}{5}, \frac{7}{5}, 0, 0\right) + s\left(\frac{1}{5}, \frac{4}{5}, 7, 0\right) +$

$t\left(\frac{1}{5}, \frac{-3}{5}, 0, 7\right)$

<u>Q9</u>: a) $\begin{bmatrix} 1 & 0 & -16 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix}$.

$Ax = 0$

$x_1 = 16t$, $x_2 = 19t$, $x_3 = t$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} + \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$

$[1, 0 - 16]$ $[0 \quad 1 \quad -19]$

b) $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = \frac{1}{2} t$, $x_2 = s$, $x_3 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$[1 \quad 0 \quad -1/2]$

Q10) $A = \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$Ax = 0$

$x_1 = -s + \frac{2}{7} t$, $x_2 = -s - \frac{4}{7} t$,

$x_3 = s$, $x_4 = t$

b) $\begin{bmatrix} 1 & 0 & -3/5 & -1/5 & 6/5 \\ 0 & 1 & -4/5 & 3/5 & 7/5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = \frac{6}{5} + \frac{7}{5}s + \frac{7}{5}t$, $x_2 = \frac{7}{5} + \frac{4}{5}s - \frac{3}{5}t$,

$x_3 = s$, $x_4 = t$.

$(x_1, x_2, x_3, x_4) = \left(\frac{6}{5}, \frac{7}{5}, 0, 0\right) + s\left(\frac{4}{5}, \frac{4}{5}, 7, 0\right) +$

$t\left(\frac{1}{5}, \frac{-3}{5}, 0, 7\right)$

Q9: a) $\begin{bmatrix} 1 & 0 & -16 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix}$

$Ax = 0$

$x_1 = 16t$, $x_2 = 19t$, $x_3 = t$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} + \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$

$[1, 0 -16] \quad [0 \quad 1 \quad -19]$

b) $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = \frac{1}{2} t, \; x_2 = s, \; x_3 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$[1 \quad 0 \quad -1/2]$

Q10) $A = \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$Ax = 0$

$x_1 = -s + \frac{2}{7} t, \; x_2 = -s - \frac{4}{7} t,$

$x_3 = s, \; x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2/7 \\ -4/7 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 & -2/7 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 4/7 \end{bmatrix}$$

b) $$\begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Ax = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ -2 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 & 2 \end{bmatrix}$$

Q14: $$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & -2 \\ -4 & 2 & 3 \\ -3 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\{(1, 1, -4, -3), (2, 0, 2, -2), (2, -2, 3, 12)\}$

R^4 spanned by vectors

Q15:

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\{(1, 1, 0, 0), (0, 0, 1, 1), (-2, 0, 2, 2),$
$(0, -3, 0, 3)\}$

R^4 spanned by these vectors

Q16 v_1, v_2, v_3, v_4

$$A = \begin{bmatrix} 1 & -3 & -1 & -5 \\ 0 & 3 & 3 & 3 \\ 1 & 7 & 9 & 5 \\ 1 & 1 & 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$v_3 = -2w_1 + w_2$ $\quad$ $w_4 = -2w_1 + w_2$

$v_3 = 2v_1 + v_2$ $\quad$ $v_4 = -2v_1 + v_2$

Q17:

$$\begin{bmatrix} 1 & -2 & 4 & 0 & -7 \\ 1 & 3 & -5 & 4 & 18 \\ 5 & 1 & 9 & 2 & 2 \\ 2 & 0 & 4 & -3 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \end{bmatrix}$$

$w_3 = 2w_1 - w_2$

$w_5 = w_1 + 3w_2 + 2w_4$

$V_3 = 2V_1 - V_2 \quad V_5 = -V_1 + 3V_2 + 2V_4$

Q20 $[V_1 \mid V_2] = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & -2 \\ 2 & 4 \end{bmatrix}$

$AB = 0$

$B^T A^T = 0^T$

$\therefore \begin{bmatrix} 1 & -1 & 3 & 2 & 0 \\ 2 & 0 & -2 & 4 & 0 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -1 & 2 & 0 \\ -0 & 1 & -4 & 0 & 0 \end{bmatrix}$

$s \begin{bmatrix} 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & -2 \\ 4 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q 21 $T_A(x) = Ax$

a) $= \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & -1 & 4 & 0 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 8/3 & 0 \\ 0 & 1 & -4/3 & 0 \end{bmatrix}$

$x_1 = -\frac{8}{3}t$, $x_2 = \frac{4}{3}t$, $x_3 = t$

$x = t\left(\frac{-8}{3}, \frac{4}{3}, t\right)$

b) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & -1 & 4 & 3 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 8/3 & 7/3 \\ 0 & 1 & -4/3 & -2/3 \end{bmatrix}$

$x_1 = \frac{7}{3} - \frac{8}{3}t$, $x_2 = \frac{-2}{3} + \frac{4}{3}t$, $x_3 = t$

$x = \left(\frac{7}{3}, \frac{-2}{3}, 0\right) + t\left(-\frac{8}{3}, \frac{4}{3}, 1\right)$

c) $\Rightarrow \begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & -1 & 4 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 8/3 & 1/3 \\ 0 & 1 & -4/3 & -2/3 \end{bmatrix}$

$x_1 = \frac{1}{3} - \frac{8}{3}t$, $x_2 = -\frac{2}{3} + \frac{4}{3}t$, $x_3 = t$

$x = \left(\frac{1}{3}, -\frac{2}{3}, 0\right) + t\left(\frac{-8}{3}, \frac{4}{3}, 1\right)$

Q22: a) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = x_2 = 0 \quad x = (0, 0)$

b) $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

the system has no solution ;
x exist $T_A(x) = b$.