Date______20___

Exercise #4.1:

Q1 a) $\vec{u}+\vec{v} = (-1,2)+(3,4) = (-1+3, 2+4) = (2,6)$; $k(\vec{u}) = 3(-1,2) = (0,6)$

b) Since V is set of real numbers in ordered pair form therefore the results of scalar multiplication and addition is also real numbers. Hence, V is 'closed' under scalar multiplication and addition

c) Axiom #1 to #5

d) Axiom #7

$k(u+v) = ku+kv$
$= k(0, k(u_2+v_2))$
$= (0, (ku_2+kv_2))$
$= (0, ku_2)+(0, kv_2)$
$= ku+kv$ proved

Axiom #8

$(k+m)u = ku+mu$
$= (0, (k+m)u_2)$
$= (0, (ku_2+mu_2))$
$= (0, ku_2)+(0, mu_2)$
$= ku+mu$ proved

Axiom #9

$k(mu) = (km)u$
$= (0, k(mu_2))$
$= (0, kmu_2)$
$= (0, (km)u_2)$
$= (km)u$ proved

e) Axiom #10

$1u = u \rightarrow 1(u_1, u_2) \Rightarrow (0, u_2) \neq (u_1, u_2)$ $\therefore$ not proved

Q3 vector space

Q5 not a vector space because axiom 5 and 6 fails

Q7 not a vector space because axiom 8 fails

Q9 vector space

Q11 vector space

Q13 A normal 2x2 matrix

Axiom # 3

$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}, v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}, w = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$

$u+(v+w) = (u+v)+w$

$= \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \left(\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \right)$

$= \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11}+w_{11} & v_{12}+w_{12} \\ v_{21}+w_{21} & v_{22}+w_{22} \end{bmatrix}$

$= \begin{bmatrix} u_{11}+v_{11}+w_{11} & u_{12}+v_{12}+w_{12} \\ u_{21}+v_{21}+w_{21} & u_{22}+v_{22}+w_{22} \end{bmatrix}$

$= \begin{bmatrix} (u_{11}+v_{11})+w_{11} & (u_{12}+v_{12})+w_{12} \\ (u_{21}+v_{21})+w_{21} & (u_{22}+v_{22})+w_{22} \end{bmatrix}$

$= \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$

$(u+v)+w$ shown

Axiom # 7

$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}, v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$

$k(u+v) = ku+kv$

$= k\left(\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \right)$

$= k \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix}$

$= \begin{bmatrix} ku_{11}+kv_{11} & ku_{12}+kv_{12} \\ ku_{21}+kv_{21} & ku_{22}+kv_{22} \end{bmatrix}$

$= \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} + \begin{bmatrix} kv_{11} & kv_{12} \\ kv_{21} & kv_{22} \end{bmatrix}$

$= k\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + k\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$

$ku+kv$ shown

Date______ 20___

Axiom #8

$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$

$(k+m)u = ku + mu$

$= (k+m)\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$

$= \begin{bmatrix} (k+m)u_{11} & (k+m)u_{12} \\ (k+m)u_{21} & (k+m)u_{22} \end{bmatrix}$

$= \begin{bmatrix} ku_{11}+mu_{11} & ku_{12}+mu_{12} \\ ku_{21}+mu_{21} & ku_{22}+mu_{22} \end{bmatrix}$

$= \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} + \begin{bmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{bmatrix}$

$= k\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + m\begin{bmatrix} u_{11} & u_{21} \\ u_{21} & u_{22} \end{bmatrix}$

$ku + mu$ shown

Axiom # 9

$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$

$k(mu) = (km)u$

$= k\left(m\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}\right)$

$= k\begin{bmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{bmatrix}$

$= \begin{bmatrix} kmu_{11} & kmu_{12} \\ kmu_{21} & kmu_{22} \end{bmatrix}$

$= \begin{bmatrix} (km)u_{11} & (km)u_{12} \\ (km)u_{21} & (km)u_{22} \end{bmatrix}$

$= (km)\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$

$= (km)\,u$ shown

Exercise #4.2:

Q1 Let $\vec{u} = (u_1, u_2, u_3)$; $\vec{v} = (v_1, v_2, v_3)$

a) $(a,0,0) \to \vec{u}+\vec{v} = (u_1,0,0) + (v_1,0,0) = (u_1+v_1, 0, 0)$

$\to k\vec{u} = k(u_1,0,0) = (ku_1,0,0)$ ∴ it is a subspace

b) $(a,1,1) \to \vec{u}+\vec{v} = (u_1,1,1) + (v_1,1,1) = (u_1+v_1, 2, 2)$ ∴ not a subspace

c) $(a,b,c) \to \vec{u}+\vec{v} = (u_1, u_1+u_3, u_3) + (v_1, v_1+v_3, v_3) = (u_1+v_1, u_1+u_3+v_1+v_3, v_3+u_3)$

$\to k\vec{u} = k(u_1, u_1+u_3, u_3) = (ku_1, ku_1+ku_3, ku_3)$ ∴ it is a subspace

Q3 a) Let $\vec{u} = \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & \vdots \\ 0 & 0 & u_{nn} \end{bmatrix}$ $\vec{v} = \begin{bmatrix} v_{11} & 0 & 0 \\ & v_{22} & \\ 0 & 0 & v_{nn} \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & \\ 0 & 0 & u_{nn} \end{bmatrix} + \begin{bmatrix} v_{11} & 0 & 0 \\ & v_{22} & \\ 0 & 0 & v_{nn} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & 0 & 0 \\ \vdots & u_{22}+v_{22} & \\ 0 & 0 & v_{nn}+u_{nn} \end{bmatrix}$

$k\vec{u} = k\begin{bmatrix} u_{11} & 0 & 0 \\ \vdots & u_{22} & \vdots \\ 0 & 0 & u_{nn} \end{bmatrix} = \begin{bmatrix} ku_{11} & 0 & 0 \\ \vdots & \cancel{ku_{22}} & \cancel{0} \\ 0 & 0 & ku_{nn} \end{bmatrix}$ ∴ it is a subspace

b) Let $\vec{u} = \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} v_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_{nn} \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} v_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_{nn} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & 0 & 0 \\ 0 & u_{22} & \\ 0 & 0 & v_{nn} \end{bmatrix}$

∴ not a subspace

c) Let $\vec{u} = \begin{bmatrix} u_{11} & 0 & 0 \\ & 0 & \\ 0 & 0 & -u_{nn} \end{bmatrix}$ $\vec{v} = \begin{bmatrix} v_{11} & 0 & 0 \\ & 0 & \\ 0 & 0 & -v_{nn} \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 0 & 0 \\ & 0 & \\ 0 & 0 & -u_{nn} \end{bmatrix} + \begin{bmatrix} v_{11} & 0 & 0 \\ & & \\ 0 & 0 & -v_{nn} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & 0 & 0 \\ & 0 & \\ 0 & 0 & -u_{nn}+v_{nn} \end{bmatrix}$

$ku = k\begin{bmatrix} u_{11} & 0 & 0 \\ & 0 & \\ 0 & 0 & u_{nn} \end{bmatrix} = \begin{bmatrix} ku_{11} & 0 & 0 \\ & & \\ 0 & 0 & -ku_{nn} \end{bmatrix}$ $\therefore$ it is a subspace

d) Let $\vec{u} = \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & \\ 0 & 0 & u_{nn} \end{bmatrix}$ $\vec{v} = \begin{bmatrix} v_{11} & 1 & 1 \\ 1 & v_{22} & \\ 1 & 1 & v_{nn} \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & \\ 0 & 0 & u_{nn} \end{bmatrix} + \begin{bmatrix} v_{11} & \phi & \phi \\ \phi & v_{22} & \\ 1 & 1 & v_{nn} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & 1 & 1 \\ 1 & u_{22}+v_{22} & \\ 1 & 1 & u_{nn}+v_{nn} \end{bmatrix}$

$ku = k\begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & \\ 0 & 0 & u_{nn} \end{bmatrix} = \begin{bmatrix} ku_{11} & 0 & 0 \\ & ku_{22} & 0 \\ 0 & 0 & ku_{nn} \end{bmatrix}$ $\therefore$ it is a subspace

Q5 a) Let $u = 0+u_1x+u_2x^2+u_3x^3$, $v = 0+v_1x+v_2x^2+v_3x^3$

$u+v = (0+u_1x+u_2x^2+u_3x^3)+(0+v_1x+v_2x^2+v_3x^3) = (0+u_1x+v_1x+u_2x^2+v_2x^2 + u_3x^3+v_3x^3)$

$ku = k(0+u_1x+u_2x^2+u_3x^3) = (0+ku_1x+ku_2x^2+ku_3x^3)$ $\therefore$ it is a subspace

b) Let $u = -u_1+u_1x-u_1x^2+u_1x^3$, $v = v_1-v_1x+v_1x^2-v_1x^3$

$u+v = (-u_1+u_1x-u_1x^2+u_1x^3)+(v_1-v_1x+v_1x^2-v_1x^3) = (-u_1+v_1+u_1x-v_1x-u_1x^2+v_1x^2 + u_1x^3-v_1x^3)$

$ku = k(-u_1+u_1x-u_1x^2+u_1x^3) = (-ku_1+ku_1x-ku_1x^2+ku_1x^3)$ $\therefore$ it is a subspace

Q7 skipping because of the involvement of functions

Q9 a) Let $x = (a,0,a,0 \cdots)$, $y = (b,0,b,0 \cdots)$

$x+y = (a+b,0,a+b,0 \cdots)$ $\therefore x+y \in V$

$kx = (ka,0,ka,0 \cdots)$ $\therefore kx \in V$

$\}$ V is subspace of R^∞

b) Let $x = (a,1,a,1 \cdots)$, $y = (b,1,b,1 \cdots)$

$x+y = (a+b,2,a+b,2 \cdots)$ $\therefore x+y \notin V$

Hence, V is not a subspace of R^∞

Date ______ 20___

Q11 a) Let $u = \begin{bmatrix} u_{11} & 0 \\ u_{21} & 0 \end{bmatrix}$ $\quad v = \begin{bmatrix} v_{11} & 0 \\ v_{21} & 0 \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 0 \\ u_{21} & 0 \end{bmatrix} + \begin{bmatrix} v_{11} & 0 \\ v_{21} & 0 \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & 0 \\ u_{21}+v_{21} & 0 \end{bmatrix}$

$ku = k\begin{bmatrix} u_{11} & 0 \\ u_{21} & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} ku_{11} & 0 \\ ku_{21} & 0 \end{bmatrix}$ $\therefore$ it is a subspace.

b) Let $u = \begin{bmatrix} u_{11} & 1 \\ u_{21} & 1 \end{bmatrix}$ $\quad v = \begin{bmatrix} v_{11} & 1 \\ v_{21} & 1 \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 1 \\ u_{21} & 1 \end{bmatrix} + \begin{bmatrix} v_{11} & 1 \\ v_{21} & 1 \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & 2 \\ u_{21}+v_{21} & 2 \end{bmatrix}$ not a subspace.

c) Let $u = \begin{bmatrix} u_{11} & 2+u_{22} \\ u_{21} & u_{22} \end{bmatrix}$ $\quad v = \begin{bmatrix} v_{11}-2 & v_{12} \\ -v_{21} & -v_{22} \end{bmatrix}$

$u+v = \begin{bmatrix} u_{11} & 2+u_{22} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11}-2 & v_{12} \\ -v_{21} & -v_{22} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11}-2 & v_{12}+2+u_{22} \\ u_{21}-v_{21} & u_{22}-v_{22} \end{bmatrix}$ $\therefore$ not a subspace.

Q13. a) Let $u = (u_1, u_1^2, u_1^3, u_1^4)$ $\quad v = (v_1, v_1^2, v_1^3, v_1^4)$

$u+v = (u_1, u_1^2, u_1^3, u_1^4) + (v_1, v_1^2, v_1^3, v_1^4) = (u_1+v_1,\ u_1^2+v_1^2,\ u_1^3+v_1^3,\ u_1^4+v_1^4)$

$\therefore$ not a subspace

b) Let $u = (u_1, 0, u_2, 0)$ $\quad v = (v_1, 0, v_2, 0)$

$u+v = (u_1, 0, u_2, 0) + (v_1, 0, v_2, 0) = (u_1+v_1, 0, u_2+v_2, 0)$

$ku = k(u_1, 0, u_2, 0) = (ku_1 + 0, ku_2, 0)$ $\therefore$ it is a subspace

Q15 Let W be a subspace for all polynomials with degree ≤ 6.

- Scalar addition and multiplication for all polynomials with degree ≤ 6 will satisfy W and for degree > 6 it will not

Hence option a and b will span subspace W but option c will not span it

Q19 a) $A \cdot \begin{bmatrix} -1 & 1 & 1 & 0 \\ 3 & -1 & 0 & 0 \\ 2 & -4 & -5 & 0 \end{bmatrix} \xrightarrow[\substack{R_2-3R_1 \\ R_3-2R_1}]{-R_1} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & -2 & -3 & 0 \end{bmatrix} \xrightarrow[R_3+2R_2]{\frac{1}{2}R_2} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

let $z = a$, $\quad y = -3/2\,a$, $\quad x = -1/2\,a$

S.set $= (x, y, z) = \{(-1/2\,a, -3/2\,a, a)\} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + a\begin{pmatrix} -1/2 \\ -3/2 \\ 1 \end{pmatrix} \right\}$

$\therefore$ it is a line

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Date________ 20___

b) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 5 & 3 & 0 \\ 1 & 0 & 8 & 0 \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -2 & 5 & 0 \end{bmatrix} \xrightarrow{R_3 + 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$z = 0, \ y = 0, \ x = 0$

S·sol: $\{(x, y, z)\} = \{(0, 0, 0)\}$ ∴ Origin

c) $A = \begin{bmatrix} 1 & -3 & 1 & 0 \\ 2 & -6 & 2 & 0 \\ 3 & -9 & 3 & 0 \end{bmatrix} \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ∴ plane

$x - 3y + z = 0$

d) $A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & -1 & 4 & 0 \\ 3 & 1 & 11 & 0 \end{bmatrix} \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 4 & 8 & 0 \end{bmatrix} \xrightarrow{R_3 - 4R_2} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Let $z = t$, $y = -2t$, $x = -3t$

S set $\{(x, y, z)\} = \{(-3t, -2t, -t)\} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} \right\}$ ∴ line

Date______20___

Exercise # 4.3

Q1 a) $u=(0,-2,2)$; $v=(1,3,-1)$

$k_1(0,-2,2)+k_2(1,3,-1)=(2,2,2)$

$$\begin{bmatrix}0&1&2\\-2&3&2\\2&-1&2\end{bmatrix}\xrightarrow{R_3\leftrightarrow R_1}\begin{bmatrix}2&-1&2\\-2&3&2\\0&1&2\end{bmatrix}\xrightarrow[R_2+2R_1]{\frac12 R_1}\begin{bmatrix}1&-1/2&1\\0&2&4\\0&1&2\end{bmatrix}\xrightarrow[R_3-R_2]{\frac12 R_2}\begin{bmatrix}1&-1/2&1\\0&1&2\\0&0&0\end{bmatrix}$$

$\therefore\ k_2=2$ and $k_1=2$ $\Rightarrow$ $2u+2v=(2,2,2)$ Hence, it is linear combination

$2(0,-2,2)+2(1,3,-1)=(2,2,2)$

b) $u=(0,-2,2)$, $v=(1,3,-1)$.

$k_1(0,-2,2)+k_2(1,3,-1)=(0,4,5)$

$$\begin{bmatrix}0&1&0\\-2&3&4\\2&-1&5\end{bmatrix}\xrightarrow{R_3\leftrightarrow R_1}\begin{bmatrix}2&-1&5\\-2&3&4\\0&1&0\end{bmatrix}\xrightarrow[R_2+2R_1]{\frac12 R_1}\begin{bmatrix}1&-1/2&5/2\\0&2&9\\0&1&0\end{bmatrix}\xrightarrow[R_3-R_2]{\frac12 R_2}\begin{bmatrix}1&-1/2&5/2\\0&2&9\\0&0&-9\end{bmatrix}$$

No solution hence not a linear combination

c) $u=(0,-2,2)$, $v=(1,3,-1)$

$k_1(0,-2,2)+k_2(1,3,-1)=(0,0,0)$

$$\begin{bmatrix}0&1&0\\-2&3&0\\2&-1&0\end{bmatrix}\xrightarrow{R_3\leftrightarrow R_1}\begin{bmatrix}2&-1&0\\-2&3&0\\0&1&0\end{bmatrix}\xrightarrow[R_2+2R_1]{\frac12 R_1}\begin{bmatrix}1&-1/2&0\\0&2&0\\0&1&0\end{bmatrix}\xrightarrow[R_3-R_2]{\frac12 R_2}\begin{bmatrix}1&-1/2&0\\0&1&0\\0&0&0\end{bmatrix}$$

$\therefore\ k_2=0$ and $k_1=0$ $\Rightarrow$ $0u+0v=(0,0,0)$ Hence, it is a linear combination

Q3 a) $A=\begin{bmatrix}4&0\\-2&-2\end{bmatrix}$, $B=\begin{bmatrix}1&-1\\2&3\end{bmatrix}$, $C=\begin{bmatrix}0&2\\1&4\end{bmatrix}$

$k_1\begin{bmatrix}4&0\\-2&-2\end{bmatrix}+k_2\begin{bmatrix}1&-1\\2&3\end{bmatrix}+k_3\begin{bmatrix}0&2\\1&4\end{bmatrix}=\begin{bmatrix}6&-8\\-1&-8\end{bmatrix}$

$$\begin{bmatrix}4&1&0&6\\0&-1&2&-8\\-2&2&1&-1\\-2&3&4&-8\end{bmatrix}\xrightarrow[\substack{R_3+2R_1\\R_4+2R_1}]{\frac14 R_1}\begin{bmatrix}1&1/4&0&6/4\\0&-1&2&-8\\0&2.5&1&2\\0&3.5&4&-5\end{bmatrix}\xrightarrow[\substack{R_3-2.5R_2\\R_4-3.5R_2}]{-R_2}\begin{bmatrix}1&1/4&0&6/4\\0&1&-2&8\\0&0&6&-18\\0&0&11&-33\end{bmatrix}$$

$$\xrightarrow[R_4-11R_3]{\frac13 R_3}\begin{bmatrix}1&1/4&0&6/4\\0&1&-2&8\\0&0&1&-3\\0&0&0&0\end{bmatrix}$$

$\therefore\ k_3=-3,\ k_2=2,\ k_1=1$

$A+2B-3C=\begin{bmatrix}6&-8\\-1&-8\end{bmatrix}$ Hence, it is a linear combination

b) $A=\begin{bmatrix}4&0\\-2&-2\end{bmatrix}$, $B=\begin{bmatrix}1&-1\\2&3\end{bmatrix}$ $C=\begin{bmatrix}0&2\\1&4\end{bmatrix}$

$k_1\begin{bmatrix}4&0\\-2&-2\end{bmatrix}+k_2\begin{bmatrix}1&-1\\2&3\end{bmatrix}+k_3\begin{bmatrix}0&2\\1&4\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

By inspection $k_1=k_2=k_3=0$

$\therefore$ $0A+0B+0C=\begin{bmatrix}0&0\\0&0\end{bmatrix}$ Hence, it is a linear combination

Date ______ 20__

c) $A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$

$k_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + k_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 7 & 1 \end{bmatrix}$

$$\begin{bmatrix} 4 & 1 & 0 & -1 \\ 0 & -1 & 2 & 5 \\ -2 & 2 & 1 & 7 \\ -2 & 3 & 4 & 1 \end{bmatrix} \xrightarrow[\substack{R_3+2R_1 \\ R_4+2R_1}]{1/4 R_1} \begin{bmatrix} 1 & 1/4 & 0 & -1/4 \\ 0 & -1 & 2 & 5 \\ 0 & 2.5 & 1 & 6.5 \\ 0 & 3.5 & 4 & 0.5 \end{bmatrix} \xrightarrow[\substack{R_3-2.5R_2 \\ R_4-3.5R_2}]{-R_2} \begin{bmatrix} 1 & 1/4 & 0 & -1/4 \\ 0 & -1 & 2 & 5 \\ 0 & 0 & 6 & -6 \\ 0 & 0 & 11 & -6 \end{bmatrix}$$

$$\begin{matrix} 1/6 R_3 \\ R_4 - 11R_3 \end{matrix} \begin{bmatrix} 1 & 1/4 & 0 & -1/4 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 60 \end{bmatrix}$$

No solution. Hence, not a linear combination

Q5 $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$

a) $k_1 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + k_2 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + k_4 \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

b) $k_1 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + k_2 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + k_4 \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$$\left[\begin{array}{cccc|cc} 1 & 0 & 0 & 2 & 1 & 3 \\ -1 & 1 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & -1 & 4 & 2 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2+R_1} \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & -5 & 2 & -4 \end{array}\right] \xrightarrow{R_4-R_2} \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & -7 & -1 & -8 \end{array}\right]$$

$$\begin{matrix} R_3 \leftrightarrow R_4 \\ -R_3 \end{matrix} \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 7 & 1 & 8 \\ 0 & 0 & 0 & 1 & 2 & 1 \end{array}\right]$$

$\therefore$ a) $-3A + 12B - 13C + 2D = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

b) $A + B + C + D = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

Q7 a) $v_1 = (2,2,2)$, $v_2 = (0,0,3)$, $v_3 = (0,1,1)$

$$\left[\begin{array}{ccc} 2 & 2 & 2 \\ 0 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{cc} 2 & 2 \\ 0 & 0 \\ 0 & 1 \end{array} \Rightarrow (0+0+0)-(0+6+0) \Rightarrow Det(A) \neq 0 \therefore \text{span}$$

b) $v_1 = (2,-1,3)$, $v_2 = (4,1,2)$, $v_3 = (8,-1,8)$

$$\left[\begin{array}{ccc} 2 & -1 & 3 \\ 4 & 1 & 2 \\ 8 & -1 & 8 \end{array}\right] \begin{array}{cc} 2 & -1 \\ 4 & 1 \\ 8 & -1 \end{array} \Rightarrow (16-16-12)-(24-4-32) \Rightarrow Det(A) = 0 \therefore \text{do not span}$$

Q9 $P_1 = 1 - x + 2x^2$, $P_2 = 3 + x$, $P_3 = 5 - x + 4x^2$, $P_4 = -2 - 2x + 2x^2$

$$\left[\begin{array}{cccc} 1 & -1 & 2 & 0 \\ 3 & 1 & 0 & 0 \\ 5 & -1 & 4 & 0 \\ -2 & -2 & 2 & 0 \end{array}\right] \begin{array}{ccc} 1 & -1 & 2 \\ 3 & 1 & 0 \\ 5 & -1 & 4 \\ -2 & -2 & 2 \end{array} \Rightarrow (0+0+0+0) - (0+0+0+0) \Rightarrow Det(A) = 0$$

$\therefore$ do not span

Date ______ 20 ___

Q11 a) $k_1\begin{bmatrix}1&0\\1&0\end{bmatrix} + k_2\begin{bmatrix}1&1\\0&0\end{bmatrix} + k_3\begin{bmatrix}0&1\\0&1\end{bmatrix} + k_4\begin{bmatrix}0&0\\1&1\end{bmatrix} = \begin{bmatrix}a_1&b_1\\c_1&d_1\end{bmatrix}$

$\begin{bmatrix}1&1&0&0\\0&1&1&0\\1&0&0&1\\0&0&1&1\end{bmatrix}\begin{matrix}1&1&0\\0&1&1\\1&0&0\\0&0&1\end{matrix} \Rightarrow (0+0+0+0)-(0+0+0+0) \Rightarrow \det(A)=0 \therefore$ do not span

b) $k_1\begin{bmatrix}1&-1\\0&1\end{bmatrix} + k_2\begin{bmatrix}0&1\\0&0\end{bmatrix} + k_3\begin{bmatrix}1&1\\1&0\end{bmatrix} + k_4\begin{bmatrix}1&0\\0&1\end{bmatrix} = \begin{bmatrix}a_1&b_1\\c_1&d_1\end{bmatrix}$

$\begin{bmatrix}1&0&1&1\\-1&1&1&0\\0&0&1&0\\1&0&0&1\end{bmatrix}\begin{matrix}1&0&1\\-1&1&1\\0&0&1\\1&0&0\end{matrix} \Rightarrow (0+0+0+0)-(0+0+0+0) \Rightarrow \det(A)=0 \therefore$ do not span

c) $k_1\begin{bmatrix}1&0\\0&0\end{bmatrix} + k_2\begin{bmatrix}1&1\\0&0\end{bmatrix} + k_3\begin{bmatrix}1&1\\1&0\end{bmatrix} + k_4\begin{bmatrix}1&1\\1&1\end{bmatrix}$

$\begin{bmatrix}1&1&1&1\\0&1&1&1\\0&0&1&1\\0&0&0&1\end{bmatrix}\begin{matrix}1&1&1\\0&1&1\\0&0&1\\0&0&0\end{matrix} \Rightarrow (1+0+0+0)-(0+0+0+0) \Rightarrow \det(A)\neq 0 \therefore$ span

Q13 a) $\begin{bmatrix}0&2\\1&-2\\1&0\end{bmatrix}=A \quad u=(1,1,1)$

$T_A(e_1) = \begin{bmatrix}0&2\\1&-2\\1&0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix}$ $\quad k_1(0,1,1)+k_2(2,-2,0)=(1,1,1)$

$T_A(e_2) = \begin{bmatrix}0&2\\1&-2\\1&0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\-2\\0\end{bmatrix}$

$0k_1+2k_2=1 \quad \therefore k_1=1$

$1k_1-2k_2=1 \quad k_2=1/2 \ \& \ k_2=0$

$k_1+0k_2=1$

Since we obtain 2 diff values for k_2 $\therefore$ it doesn't span

b) $\begin{bmatrix}0&2\\1&1\\2&0\end{bmatrix}=A \quad u=(1,1,1)$

$T_A(e_1): \begin{bmatrix}0&2\\1&1\\2&0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\2\end{bmatrix}$ $\quad k_1(0,1,2)+k_2(2,1,0) = (1,1,1)$

$T_A(e_2) = \begin{bmatrix}0&2\\1&1\\2&0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\1\\0\end{bmatrix}$

$0k_1+2k_2=1 \quad \therefore k_1=1/2$

$\cancel{k_1+k_2=1} \quad k_2=1/2$

$2k_1+0k_2=1$

Since we obtain same values of k_2 $\therefore$ it spans.

Q15 a) $A=\begin{bmatrix}1&1&1&1&0\\1&0&1&0&0\\0&1&0&1&0\end{bmatrix} \xrightarrow{R_2-R_1} \begin{bmatrix}1&1&1&1&0\\0&-1&0&-1&0\\0&1&0&1&0\end{bmatrix} \xrightarrow{-R_2,\ R_1-R_2,\ R_3-R_2} \begin{array}{c}w\ \ x\ \ y\ \ z\quad\\ \begin{bmatrix}1&0&1&0&0\\0&1&0&1&0\\0&0&0&0&0\end{bmatrix}\end{array}$

$w+y=0 \ \& \ x+z=0$

let $w=a,\ x=b \quad \therefore y=-a,\ z=-b$

S set: $(w,x,y,z)=(a,b,-a,-b) = (a+0, b+0, -a+0, -b+0)$

$(a,0,-a,0)+(0,b,0,-b) \rightarrow$ represents u & v

$\therefore$ set $\{u,v\}$ spans W

b) using RREF from part a $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$w + y = 0$ let $w = a$ $y = -a$

$x + z = 0$ let $x = b$ $z = -b$

S set: $(w, x, y, z) = (a, b, -a, -b) = (a+0, 0+b, -a+0, 0-b)$

$= (a, 0, -a, 0) + (0, b, 0, -b)$

↳ represents u ↳ represents u+v

$\therefore$ set $\{u, v\}$ spans W

Q17 $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ $u_1 = (1, 2)$ $u_2 = (-1, 1)$

a) $T_A(u_1) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ $\Big\}$ $k_1(-1, 4) + k_2(-2, 2) = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$T_A(u_2) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} -1 & -2 \\ 4 & 2 \end{bmatrix} \rightarrow \det(A) \neq 0$ $\therefore$ spans

b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$ $u_1 = (1, 2)$ $u_2 = (-1, 1)$

$T_A(u_1) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $\Big\}$ $k_1(-1, 2) + k_2(-2, 4) = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$T_A(u_2) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ $\begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix} \rightarrow \det(A) = 0$ $\therefore$ does not span

Q19 $p_1 = 1 + x^2$ $q_1 = 2x$

$p_2 = 1 + x + x^2$ $q_2 = 1 + x^2$

$q_2 = p_1$

$q_1 = -2(p_1) + 2(p_2)$ $\Big\}$ Since q_1 and q_2 are present in the span of p_1 and p_2 so

span $\{p_1, p_2\}$ = span $\{q_1, q_2\}$

Exercise # 4.4:

Q1 a) u_2 is scalar multiple of u_1 ($u_2 = 5u_1$)

b) A set of 3 vectors in $\mathbb{R}^2$ is linearly dependent

c) P_2 is scalar multiple of P_1 ($P_2 = 2P_1$)

d) B is a scalar multiple of A ($B = -A$)

Q3 a) $k_1(3,8,7,-3) + k_2(1,5,3,-1) + k_3(2,-1,2,6) + k_4(4,2,6,4) = (0,0,0,0)$

$(3k_1 + k_2 + 2k_3 + 4k_4) = 0$ $\quad$ $(7k_1 + 3k_2 + 2k_3 + 6k_4) = 0$

$(8k_1 + 5k_2 - k_3 + 2k_4) = 0$ $\quad$ $(-3k_1 - k_2 + 6k_3 + 4k_4) = 0$

$$\begin{bmatrix} 3 & 1 & 2 & 4 & 0 \\ 8 & 5 & -1 & 2 & 0 \\ 7 & 3 & 2 & 6 & 0 \\ -3 & -1 & 6 & 4 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \tfrac{1}{3}R_1 \\ R_2 - 8R_1 \\ \cancel{R_3 - 7R_1} \\ R_4 + 3R_1 \end{array} \begin{bmatrix} 1 & 1/3 & 2/3 & 4/3 & 0 \\ 0 & 7/3 & -19/3 & -26/3 & 0 \\ \cancel{0} & \cancel{2/3} & \cancel{-8/3} & \cancel{-10/3} & \cancel{0} \\ 0 & 0 & 8 & 8 & 0 \end{bmatrix} \rightarrow \begin{array}{c} 3/7R_2 \\ R_3 - 2/3R_2 \end{array} \begin{bmatrix} 1 & 1/3 & 2/3 & 4/3 & 0 \\ 0 & 1 & -19/7 & -26/7 & 0 \\ 0 & 0 & -6/7 & -6/7 & 0 \\ 0 & 0 & 8 & 8 & 0 \end{bmatrix}$$

$$\begin{array}{c} -7/6R_3 \\ R_4 - 8R_3 \end{array} \begin{bmatrix} 1 & 1/3 & 2/3 & 4/3 & 0 \\ 0 & 1 & -19/7 & -26/7 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $k_4 = t$, $k_3 = -t$, $k_2 = t$, $k_1 = -t$

$\therefore$ linearly dependent (infinitely many sol)

b) $k_1(3,0,-3,6) + k_2(0,2,3,1) + k_3(0,-2,-2,0) + k_4(-2,1,2,1) = (0,0,0,0)$

$$\begin{bmatrix} 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ -3 & 3 & -2 & 2 & 0 \\ 6 & 1 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{array}{c} \tfrac{1}{3}R_1 \\ R_3 + 3R_1 \\ R_4 - 6R_1 \end{array} \begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 1 & 0 & 13 & 0 \end{bmatrix} \rightarrow \begin{array}{c} \tfrac{1}{2}R_2 \\ R_3 - 3R_2 \\ R_4 - R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 1 & 25/2 & 0 \end{bmatrix}$$

$$R_4 - R_3 \begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 14 & 0 \end{bmatrix}$$

$\therefore k_1 = k_2 = k_3 = k_4 = 0$

Hence, it is linearly independent

Q5 a) $k_1\begin{bmatrix}1 & 0\\1 & 2\end{bmatrix} + k_2\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} + k_3\begin{bmatrix}0 & 1\\2 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$$\begin{bmatrix}1 & 1 & 0 & 0\\0 & 2 & 1 & 0\\1 & 2 & 2 & 0\\2 & 1 & 1 & 0\end{bmatrix} \xrightarrow{\substack{R_3-R_1\\R_4-2R_1}} \begin{bmatrix}1 & 1 & 0 & 0\\0 & 2 & 1 & 0\\0 & 1 & 2 & 0\\0 & -1 & 1 & 0\end{bmatrix} \xrightarrow{\substack{1/2 R_2\\R_3-R_2\\R_4+R_2}} \begin{bmatrix}\boxed{1} & 1 & 0 & 0\\0 & \boxed{1} & 1/2 & 0\\0 & 0 & 3/2 & 0\\0 & 0 & 3/2 & 0\end{bmatrix}$$

$$\xrightarrow{\substack{2/3 R_3\\R_4-3/2 R_3}} \begin{bmatrix}1 & 1 & 0 & 0\\0 & 1 & 1/2 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

$\therefore k_1 = k_2 = k_3 = 0$

Hence, it is linearly independent

b) $k_1\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\end{bmatrix} + k_2\begin{bmatrix}0 & 0 & 1\\0 & 0 & 0\end{bmatrix} + k_3\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix} \rightarrow k_1 = k_2 = k_3 = 0$$

Hence, it is linearly independent

Q7 a) $k_1(2,-2,0) + k_2(6,1,4) + k_3(2,0,-4) = (0,0,0)$

$$\begin{bmatrix}2 & -2 & 0\\6 & 1 & 4\\2 & 0 & -4\end{bmatrix}\begin{matrix}2 & -2\\6 & 1\\2 & 0\end{matrix}$$

$|A| = (-8-16+0) - (0+0+24) = 0$

$\therefore$ vectors do not lie on plane R^3

b) $k_1(-6,7,2) + k_2(3,2,4) + k_3(4,-1,2) = (0,0,0)$

$$\begin{bmatrix}-6 & 7 & 2\\3 & 2 & 4\\4 & -1 & 2\end{bmatrix}\begin{matrix}-6 & 7\\3 & 2\\4 & -1\end{matrix}$$

$|A| = (-24+56-6) - (16+24+42) = -56$

$\therefore$ vectors lie on plane R^3

Q9 a) $k_1(0,3,1,-1) + k_2(6,0,5,1) + k_3(4,-7,1,3) = (0,0,0,0)$

$$\begin{bmatrix}0 & 6 & 4 & 0\\3 & 0 & -7 & 0\\1 & 5 & 1 & 0\\-1 & 1 & 3 & 0\end{bmatrix} \rightarrow \substack{R_3 \leftrightarrow R_1\\R_2-3R_1\\R_4+R_1} \begin{bmatrix}1 & 5 & 1 & 0\\0 & -15 & -10 & 0\\0 & 6 & 4 & 0\\0 & 6 & 4 & 0\end{bmatrix} \rightarrow \substack{-1/5 R_2\\R_3-2R_2\\R_4-2R_2} \begin{bmatrix}1 & 5 & 1 & 0\\0 & 3 & 2 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

Let $k_3 = t$, $k_2 = -2t/3$, $k_1 = 7/3\,t$

$\therefore$ vectors are linearly dependent

b) $V_3 = -7/3\, V_1 + 2/3\, V_2$ $\qquad$ $3V_3 + 7V_1 = V_2$

$V_2 = 7/2\, V_1 + 3/2\, V_3$

$V_1 = \frac{2}{7} V_1 - \frac{3}{7} V_3$

Date________20____

Q11 $k_1(\lambda, -1/2, -1/2) + k_2(-1/2, \lambda, -1/2) + k_3(-1/2, -1/2, \lambda) = (0,0,0)$

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix} \begin{matrix} \lambda & -1/2 \\ -1/2 & \lambda \\ -1/2 & -1/2 \end{matrix}$$

$|A| = 0$

$(\lambda^3 - 1/8 - 1/8) - (1/4\lambda \times 3) = 0$

$\therefore \lambda = 1$ or $\lambda = -1/2$

Q13 a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ $u_2 = (-1,1)$ $u_1 = (1,2)$

$T_A(u_1) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ $k(-1,4) + k(-2,2) = (0,0)$

$T_A(u_2) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} -1 & -2 \\ 4 & 2 \end{bmatrix} \rightarrow \det(A) \neq 0 \therefore$ linearly independent

b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$ $u_2 = (-1,1)$ $u_1 = (1,2)$

$T_A(u_1) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $k(-1,2) + k(-2,4) = (0,0)$

$T_A(u_2) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ $\begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix} \rightarrow \det(A) = 0 \therefore$ linearly dependent.

Q15 a) Linearly Independent

b) Linearly Dependent

Date______ 20__

Exercise # 4.5:

Q1) $\{(2,1), (3,0)\}$ basis vector for $\mathbb{R}^2$.

$c_1v_1 + c_2v_2 = 0 \rightarrow c_1(2,1) + c_2(3,0) = 0$

$c_1v_1 + c_2v_2 = b \rightarrow c_1(2,1) + c_2(3,0) = b$

} ~~give same~~ coefficient matrix

$A \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ $\det(A) = 0-3$

$\det(A) \neq 0 \therefore \{(2,1),(3,0)\}$ are basis vector for $\mathbb{R}^2$

Q3) $\{(x^2+1), (x^2-1), (2x-1)\}$ basis for P^2

$c_1v_1 + c_2v_2 + c_3v_3 = 0 \rightarrow c_1(1,1,0) + c_2(0,0,2) + c_3(1,-1,-1) = 0x^2 - 0x - 0$

$c_1v_1 + c_2v_2 + c_3v_3 = b \rightarrow c_1(1,1,0) + c_2(0,0,2) + c_3(1,-1,-1) = ax^2 + bx + c$

$A \left[\begin{array}{ccc} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 2 & -1 \end{array}\right] \begin{array}{cc} 1 & 0 \\ 1 & 0 \\ 0 & 2 \end{array}$ $\det(A) = (0+0+2) - (0-2)$

$\det(A) \neq 0 \therefore \{(x^2+1), (x^2-1), (2x-1)\}$ are basis for P^2

Q5) $\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ basis for M_{22}.

$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0 \rightarrow c_1\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + c_2\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix} + c_4\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = b \rightarrow c_1\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + c_2\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix} + c_4\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A = \left[\begin{array}{cccc} 3 & 0 & 0 & 1 \\ 6 & -1 & -8 & 0 \\ -3 & -1 & -12 & -1 \\ -6 & 0 & -4 & 2 \end{array}\right] \begin{array}{ccc} 3 & 0 & 0 \\ 6 & -1 & -8 \\ -3 & -1 & 12 \\ -6 & 0 & -4 \end{array}$ $\det(A) = (72+0+0+24) - (-48+0+0+0)$

$\det(A) \neq 0 \therefore \left\{\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\right\}$ are basis for M_{22}

Q7 a) $\{(2,-3,1), (4,1,1), (0,-7,1)\}$

$c_1v_1 + c_2v_2 + c_3v_3 = 0 \rightarrow c_1(2,-3,1) + c_2(4,1,1) + c_3(0,-7,1) = 0$

$c_1v_1 + c_2v_2 + c_3v_3 = b \rightarrow c_1(2,-3,1) + c_2(4,1,1) + c_3(0,-7,1) = b$

$A \left[\begin{array}{ccc} 2 & 4 & 0 \\ -3 & 1 & -7 \\ 1 & 1 & 1 \end{array}\right] \begin{array}{cc} 2 & 4 \\ -3 & 1 \\ 1 & 1 \end{array}$ $\det(A) = (2-28+0) - (0-14-12)$

$\det(A) = 0 \therefore$ these are not the basis vectors

b) $\{(1,6,4), (2,4,-1), (-1,2,5)\}$

$c_1v_1 + c_2v_2 + c_3v_3 = 0 \rightarrow c_1(1,6,4) + c_2(2,4,-1) + c_3(-1,2,5) = 0$

$c_1v_1 + c_2v_2 + c_3v_3 = b \rightarrow c_1(1,6,4) + c_2(2,4,-1) + c_3(-1,2,5) = b$

$A = \left[\begin{array}{ccc} 1 & 2 & -1 \\ 6 & 4 & 2 \\ 4 & -1 & 5 \end{array}\right] \begin{array}{cc} 1 & 2 \\ 6 & 4 \\ 4 & -1 \end{array}$ $\det(A) = (20+16+6) - (-16-2+60)$

$\det(A) = 0 \therefore$ these are not the basis vectors

Date ______ 20 ___

Q9 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

$C_1V_1 + C_2V_2 + C_3V_3 + C_4V_4 = 0 \rightarrow C_1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + C_2\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} + C_3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + C_4\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$C_1V_1 + C_2V_2 + C_3V_3 + C_4V_4 = b \rightarrow C_1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + C_2\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} + C_3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + C_4\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A = \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & -2 & -1 & -1 & 0 \\ 1 & 3 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 \end{array}\right] \xrightarrow{\substack{R_3 - R_1 \\ R_4 - R_1}} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & -2 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right] \xrightarrow{\substack{-R_2 \\ R_3 - R_2}} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right] \xrightarrow{\substack{-R_3 \\ R_4 - R_3}} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$C_1 = C_2 = C_3 = C_4 = 0$ $\quad$ $\det(A) = 0$ $\quad \therefore$ not the basis

Q11 a) $u_1 = (2,-4)$, $u_2 = (3,8)$, $w = (1,1)$

$C_1V_1 + C_2V_2 = (1,1) \rightarrow C_1(2,-4) + C_2(3,8) = (1,1)$

$2C_1 + 3C_2 = 1$ $\quad \therefore C_1 = 5/28$; $C_2 = 3/14$

$-4C_1 + 8C_2 = 1$ $\quad$ Co-ordinate vector $= (5/28, 3/14)$

b) $u_1 = (1,1)$, $u_2 = (0,2)$, $w = (a,b)$

$C_1V_1 + C_2V_2 = (a,b) \rightarrow C_1(1,1) + C_2(0,2) = (a,b)$

$C_1 + 0C_2 = a$ $\quad \therefore C_1 = a$; $C_2 = (b-a)/2$

$C_1 + 2C_2 = b$ $\quad$ Co-ordinate vector: $(a, (b-a)/2)$.

Q13 a) $v_1 = (1,0,0)$, $v_2 = (2,2,0)$, $v_3 = (3,3,3)$, $v = (2,-1,3)$

$C_1V_1 + C_2V_2 + C_3V_3 = (2,-1,3) \rightarrow C_1(1,0,0) + C_2(2,2,0) + C_3(3,3,3) = (2,-1,3)$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 2 & 3 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix} \xrightarrow{\frac{1}{3}R_3} \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 2 & 3 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\therefore C_3 = 1$, $C_2 = -2$, $C_1 = 3$

Coordinate vector $= (3,-2,1)$

b) $v_1 = (1,2,3)$, $v_2 = (-4,5,6)$, $v_3 = (7,-8,9)$, $v = (5,-12,3)$

$C_1V_1 + C_2V_2 + C_3V_3 = (5,-12,3) \rightarrow C_1(1,2,3) + C_2(-4,5,6) + C_3(7,-8,9) = (5,-12,3)$

$$\begin{bmatrix} 1 & -4 & 7 & 5 \\ 2 & 5 & -8 & -12 \\ 3 & 6 & 9 & 3 \end{bmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - 3R_1}} \begin{bmatrix} 1 & -4 & 7 & 5 \\ 0 & 13 & -22 & -22 \\ 0 & 18 & -12 & -12 \end{bmatrix} \xrightarrow{\substack{\frac{1}{13}R_2 \\ R_3 - 18R_2}} \begin{bmatrix} 1 & -4 & 7 & 5 \\ 0 & 1 & -22/13 & -22/13 \\ 0 & 0 & 240/13 & 240/13 \end{bmatrix}$$

$\therefore C_3 = 1$, $C_2 = 0$, $C_1 = -2$

Co-ordinate vector $= (-2,0,1)$

Date______ 20___

Q15 $A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ $A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. $A \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0 \rightarrow c_1\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + c_2\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + c_4\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = b \rightarrow c_1\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + c_2\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + c_4\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{matrix}$ $\det(A) = (1+0+0+0) - 0$

$\det(A) \neq 0$ $\therefore$ basis of M_{22}

$k_1\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + k_2\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + k_3\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + k_4\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - R_1} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_4 - R_2]{R_3 - R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & +1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & +1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$

$k_1 = 1$, $k_2 = -1$, $k_3 = 1$, $k_4 = -1$ $\therefore (A)_S = (1, -1, 1, -1)$

Q17 $p_1 = 1 + x + x^2$, $p_2 = x + x^2$, $p_3 = x^2$; $p = 7 - x + 2x^2$

$c_1p_1 + c_2p_2 + c_3p_3 = 0 \rightarrow c_1(1 + x + x^2) + c_2(x + x^2) + c_3(x^2) = 0x^2 + 0x + 0$

$c_1p_1 + c_2p_2 + c_3p_3 = b \rightarrow c_1(1 + x + x^2) + c_2(x + x^2) + c_3(x^2) = ax^2 + bx + c$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{matrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{matrix}$ $\det(A) = (0+0+0) - (1+0+0)$

$\det(A) \neq 0$ $\therefore$ basis of P_2

$k_1(1 + x + x^2) + k_2(x + x^2) + k_3(x^2) = 7 - x + 2x^2$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 7 \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & -1 & -1 & 5 \end{bmatrix} \xrightarrow[-R_2]{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & -1 & -3 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$k_1 = 7$, $k_2 = -8$, $k_3 = 3$ $\therefore (P)_S = (7, -8, 3)$

Q19 a) Too many vectors for the given dimensional space

b) Too few vectors for the given dimensional space

c) Too few vectors for the given dimensional space

d) No error

Q21 a) $T_A(e_1) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$T_A(e_2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$T_A(e_3) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$

$k_1(1, 0, -1) + k_2(1, 1, 2) + k_3(1, -3, 0)$

$A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{matrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{matrix} \rightarrow \det(A) \neq 0$ $\therefore$ linearly independent

Date ______ 20___

b) $T_A(e_1) = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$T_A(e_2) = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$T_A(e_3) = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$k_1(1,0,-1) + k_2(1,1,2) + k_3(2,1,1)$

$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{matrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{matrix}$

$\det(A) = 0$ $\therefore$ linearly dependent

Q23 Let $\vec{v} = x\vec{i} + y\vec{j}$; $\vec{v} = x'\vec{u_1} + y'\vec{u_2}$

Taking dot products

$\vec{v}\cdot\vec{i} = x = x'\vec{u_1}\cdot\vec{i} + y'\vec{u_2}\cdot\vec{i} = x'\cos 30 \Rightarrow x' = x/\cos 30$

$\vec{v}\cdot\vec{j} = y = x'\vec{u_1}\cdot\vec{j} + y'\vec{u_2}\cdot\vec{j} = x'\cos 60 + y' \Rightarrow y' = y - x\dfrac{\cos 60}{\cos 30}$

a) $(\sqrt{3},1) \rightarrow x' = \sqrt{3}/\cos 30 = 2$; $y' = 1 - \sqrt{3}\dfrac{\cos 60}{\cos 30} = 0 \Rightarrow (2,0)$

b) $(1,0) \rightarrow x' = 1/\cos 30 = 2\sqrt{3}/3$; $y' = 0 - 1\dfrac{\cos 60}{\cos 30} = -1/\sqrt{3} \Rightarrow (2/\sqrt{3}, -1/\sqrt{3})$

c) $(0,1) \rightarrow x' = 0/\cos 30 = 0$; $y' = 1 - 0\dfrac{\cos 60}{\cos 30} = 1 \Rightarrow (0,1)$

d) $(a,b) \rightarrow x' = a/\cos 30 = 2a\sqrt{3}/3$; $y' = b - a\dfrac{\cos 60}{\cos 30} = b - \dfrac{a}{\sqrt{3}} \Rightarrow \left(\dfrac{2a}{\sqrt{3}}, b - \dfrac{a}{\sqrt{3}}\right)$

Q25 $1,\ 2t,\ -2+4t^2,\ -12t+8t^3$

$C_1P_1 + C_2P_2 + C_3P_3 + C_4P_4 = 0 \rightarrow C_1(1) + C_2(2t) + C_3(-2+4t^2) + C_4(-12t+8t^3) = 0$

a) $C_1P_1 + C_2P_2 + C_3P_3 + C_4P_4 = b \rightarrow C_1(1) + C_2(2t) + C_3(-2+4t^2) + C_4(-12t+8t^3) = b$

$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -2 & 0 & 4 & 0 \\ 0 & -12 & 0 & 8 \end{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ -2 & 0 & 4 \\ 0 & -12 & 0 \end{matrix}$

$\det(A) = (64+0+0+0) - 0$

$\det(A) \neq 0$ $\therefore$ basis of P_3

b) $\begin{bmatrix} 1 & 0 & -2 & 0 & -1 \\ 0 & 2 & 0 & -12 & -4 \\ -2 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 8 & 8 \end{bmatrix}$

$C_4 = 1,\ C_3 = 2,\ C_2 = 4,\ C_1 = 3$

Coordinate vector : $(3,4,2,1)$

Q27 a) $w = (6)(3,1,-4) + (-1)(2,5,6) + (4)(1,4,8) = (20,17,2)$

b) $w = (3)(x^2+1) + (0)(x^2-1) + (4)(2x-1) = 3x^2 + 8x - 1$

c) $w = -8\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + 7\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -21 & -103 \\ -106 & 30 \end{bmatrix}$