Systems of Linear Equations and Row Reduction

1. For each augmented matrix, find all solutions to the system of linear equations that it represents.

   (a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 3 & 6 & 1 \\ 0 & 2 & 1 & 7 \\ 0 & 0 & 3 & 9 \end{array}\right]$

2. Use row reduction to find solutions to each of the following systems of linear equations.

   (a)
   $$\begin{aligned} 3x_1 + 6x_2 + 3x_3 &= -3 \\ 5x_1 - 3x_2 + 18x_3 &= 8 \\ 7x_1 + 2x_2 + 19x_3 &= 5 \end{aligned}$$

   (b)
   $$\begin{aligned} x_1 + 2x_2 &= 3 \\ 3x_1 - 6x_2 &= 9 \\ x_1 + x_2 &= 10 \end{aligned}$$

3. For what values of h is the following system consistent?

   $$\begin{aligned} x_1 + hx_2 &= 1 \\ 2x_2 &= 2 \\ 3x_1 - x_3 &= 3 \end{aligned}$$

4. When doing row reduction, we are allowed to perform three types of operations: multiply any row by a nonzero scalar, swap two rows, and add a multiple of one row to another. In the first operation, why did we have to specify that the scalar is nonzero?

5. How many solutions does a system of linear equations have if the coefficient matrix in REF has:

   (a) A pivot in every row?

   (b) A pivot in every column?

   (c) A free variable (i.e. a column with no pivot)?

   (d) More columns than rows?

   (e) More rows than columns?

6. For what values of c are the following augmented matrices consistent?

   (a) $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & c \end{array}\right]$ (b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ c & 3 & -2 \\ 0 & 0 & 0 \end{array}\right]$