SEARCHING FOR BEST MINIMUM DISTANCE CODES UNDER A PAC CODE FRAMEWORK

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Motivation

Recently, Arikan proposed polarization-adjusted convolutional (PAC) codes[1]

- An extension of the capacity-achieving polar codes.
- (128,64) PAC code can perform close to Polyanskiy-Poor-Verdú (PPV) bound in the Gaussian channel in the low-to-moderate signal-to-noise (SNR) regime.

Maximizing minimum distance d_{min} of the code can lead to near optimal performance of the code in the moderate-to-high SNR regime. In this work, we revisit the best d_{min} code search problem under the PAC code framework.

Problem Statement

Two important parameters for the (N, K) PAC codes are:

- **Rate profile $\mathcal{A}$:** Contains the K indices of the bit-channels, $\binom{N}{K}$ possible $\mathcal{A}$.
- **Convolutional Transform $\vec{c}$:** Convolutional transform of the constraint length m, 2^{N-1} possible $\vec{c}$.

Finding $\mathcal{A}$ and $\vec{c}$ to maximize the d_{min} of the PAC code and minimizing $A_{d_{min}}$(# no. of codewords of Hamming weight d_{min}).

Proposed Solution

The stated problem has two difficulties:

- **Search space of the size $\binom{N}{K}2^{N-1}$:** Simulated annealing (SA) is used to efficiently explore part of the search space.
- **d_{min} computation:** Computing the d_{min} of the code is NP-hard in general except for the small (N, K) codes. Graphics processing unit (GPU) uses the inherent parallelism offered by this task to efficiently compute d_{min} for limited N & K.

Proposed Algorithm

Our search algorithm is outlined as follows:

1. Randomly generate $\mathcal{A}$ and $\vec{c}$ with the population size of 20 having 5 neighbors each.
2. Find the generator matrices of the codes depending upon $\mathcal{A}$ and $\vec{c}$, and compute the d_{min} on GPU.
3. Keep all those $\mathcal{A}$ and $\vec{c}$ leading to improved d_{min}.
4. With probability $e^{-(\Delta E/T)}$, keep all those $\mathcal{A}$ and $\vec{c}$ for which d_{min} did not improve, where ΔE is the difference in the energies of the current solution and previous one and T is the temperature used in SA.

Searched Best-d_{min} Codes with Smaller $A_{d_{min}}$

From the truncated union bound of the frame error rate (FER) under maximum-likelihood decoding (MLD), $P_e^{MLD} \approx A_{d_{min}} Q\left(\sqrt{\frac{2 d_{min} R E_b}{N_o}}\right)$ §.

Given the same d_{min}, the code with smaller value of $A_{d_{min}}$ should perform better.

- Newly found codes have significantly smaller values of $A_{d_{min}}$ as illustrated in Table 1.
- (64, 32 12) PAC code shows 0.29 dB gap from PPV bound at the FER of 10^{-4}, which improves over the best known gap by 0.36 dB as shown in Figure 1.

Table 1. Comparison of our newly found codes with best-d_{min} codes[2].

(N, K, d_{min})	Error coefficients $A_{d_{min}}$	
	Best-d_{min} code[2]	Newly found PAC code
$(32, 12, 10)$	237	170
$(32, 18, 6)$	543	110
$(64, 32, 12)$	12878	720

§ $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt$

Simulation Results

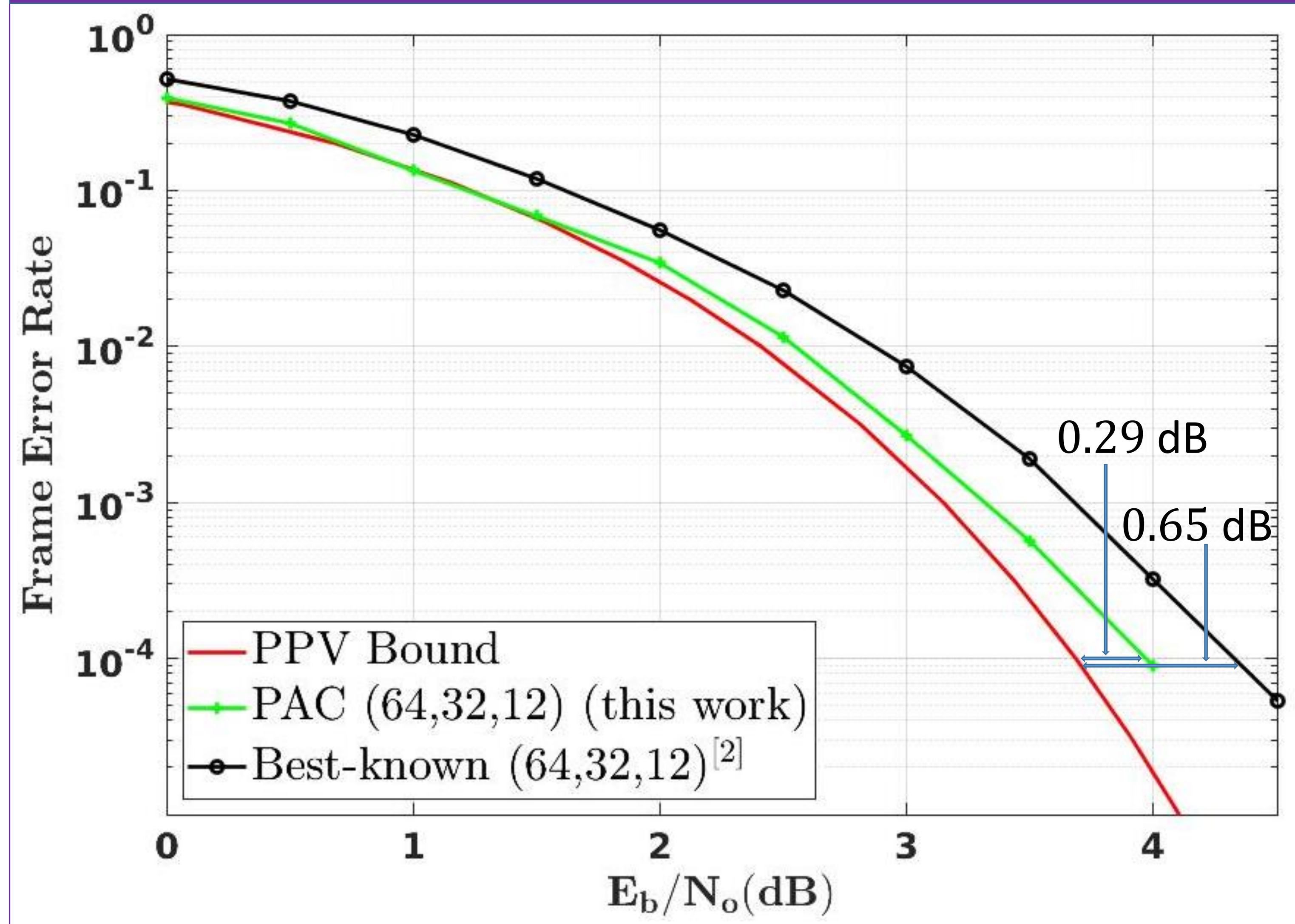

Figure 1. Performance comparison of newly found codes with best-d_{min} codes over the binary input AWGN channel under the A* MLD[3].

Efficiency of the Proposed Algorithm

- For the $(64, 32, 12)$ code the search space is the order of the 10^{37}, while our algorithm found the best solution only by exploring the 10^7 possible solutions.

Table 2. Runtime comparison of the d_{min} computation.

(N, K, d_{min})	GPU†	CPU‡
(64,32,12)	< 2 seconds	> 5 hours

‡Intel(R) Xeon(R) Gold 6242R @ 3.10 GHz, †NVIDIA Tesla V100S-4Q

References

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