Probability and Statistics

Exercise 4

1. $E[2X_1+3] = 2 \cdot E[X_1] + 3 = 2 \cdot 2 + 3 = 7$

2. $V[2X_1+3] = 2V[X_1] + 3 = 2 \cdot 1 + 3 = 5$

3. Since the values are random, corellation between each combination is 0
And corellation between the values themselves is obviously 1
So the corellation matrix is:

	X_1	X_2	X_3
X_1	1	0	0
X_2	0	1	0
X_3	0	0	1

4. $E[X_1+X_2+X_3] = E[X_1] + E[X_2] + E[X_3] = 2+3+0 = 5$

5. $V[X_1+X_2+X_3] = V[X_1] + V[X_2] + V[X_3] = 1+2+3 = 6$

6. $E\left[\frac{X_1+X_2+X_3}{3}\right] = \frac{E[X_1+X_2+X_3]}{3} = 5/3 = 1\frac{2}{3}$

7. $V\left[\frac{X_1+X_2+X_3}{3}\right] = \frac{V[X_1+X_2+X_3]}{3} = 6/3 = 2$

Exercise 5

1. $Y = X^2;\ y \geq 0$

$Y \leq y \Rightarrow X^2 \leq y$

$-\sqrt{y} \leq X \leq \sqrt{y}$

2. $F_Y(y) := P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq x \leq \sqrt{y})$

3. $F_Y(y) = P(-\sqrt{y} \leq x \leq \sqrt{y}) = \int_{-\sqrt{y}}^{\sqrt{y}} f(x)\,dx = F_X(\sqrt{y}) - F_X(-\sqrt{y})$

4. $F_Y(y) = F_X(\sqrt{y}) - F_X(-\sqrt{y}) = F_X(\sqrt{y}) - (1 - F_X(\sqrt{y}))$

$F_Y(y) = 2F_X(\sqrt{y}) - 1$

5. $P(Y < 3) = 2F_X(\sqrt{3}) - 1 \approx 2F_X(1.73) - 1 = 2 \cdot 0.9582 - 1 = 0.9164$

$P(Y < 5) = 2F_X(\sqrt{5}) - 1 \approx 2 \cdot F_X(2.24) - 1 = 2 \cdot 0.9875 - 1 = 0.975$

6. $f_Y(y) = \frac{dF_Y(y)}{dy}$

7. $f_Y(y) = \frac{d\,2F_X(\sqrt{y}) - 1}{dy} = \frac{1}{\sqrt{y}} \frac{dF_X(\sqrt{y})}{dy} = \frac{1}{\sqrt{y}} f_X(\sqrt{y})$