Pset2A

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Question 1:

a.

Proof by Contradiction. We know that $Y \subseteq Z$ therefore, we can define Z as the union of elements in the set Y, and its difference with Z. That is

$$Z = Y \cup (Z \setminus Y)$$

We know from the axioms that $P(\mathcal{E}) = 1$. Therefore, we can assume for the set Z,

$$P(Z) = 1$$

Therefore,

$$P(Z) = P(Y) \cup P(Z \setminus Y)$$

It can be seen that if $P(Y) > P(Z)$ then the axiom that the total probability of an event space equals 1 does not hold. This is a contradiction. Therefore we can conlude that,

$$P(Y) \leq P(Z)$$

b.

Proof by Induction. By definition, we know that $P(X|Z) = \frac{P(X \cap Z)}{P(Z)}$. From the previous question, we also know that $P(X) \leq P(Z)$ since $X \subseteq Z$. Therefore it can be seen that $P(X \cap Z) \leq P(Z)$. Since Z is the total event space, we can see that

$$0 < P(Z) \leq 1$$

and as a result

$$0 < P(X \cap Z) \leq P(Z) \leq 1$$

. Therefore

$$0 < P(X|Z) = \frac{P(X \cap Z)}{P(Z)} \leq 1$$

c.

Proof by Induction. From the axioms, we know that $P(\mathcal{E}) = 1$ and $P(\mathcal{E} \cup \emptyset) = \mathcal{P}(\mathcal{E}) + \mathcal{P}(\emptyset)$. Therefore, it can be seen that,

$$1 = 1 + P(\emptyset)$$

We can conclude that

$$P(\emptyset) = 0$$

d.

Proof by Induction. From the axioms, we know that $P(\mathcal{E}) = 1$. Also, from the question, we note that $\bar{X} = \mathcal{E} - \mathcal{X}$. Therefore

$$P(\bar{X}) = P(\mathcal{E} - \mathcal{X})$$

From the axioms, we know that $P(\mathcal{E} - \mathcal{X}) = \mathcal{P}(\mathcal{E}) - \mathcal{P}(\mathcal{X})$. Therefore,

$$P(\bar{X}) = P(\mathcal{E}) - \mathcal{P}(\mathcal{X})$$

This equals

$$P(\bar{X}) = 1 - P(X)$$

and by associativity

$$P(X) = 1 - P(\bar{X})$$

e.

Proof by Induction. From the axioms, we know that

$$P(\text{singing and rainy}|\text{rainy}) = \frac{\text{P(singing and rainy and rainy)}}{P(rainy)}$$

We also know that (rainy and rainy) = rainy

Therefore,

$$P(\text{singing and rainy}|\text{rainy}) = \frac{\text{P(singing and rainy)}}{P(rainy)}$$

and finally,

$$P(\text{singing and rainy}|\text{rainy}) = P(\text{singing} \mid \text{rainy})$$

f.

Proof by Induction. From the axioms, we know that

$$P(X|Y) = \frac{P(X\, and\, Y)}{P(Y)}$$

Therefore,

$$P(\bar{X}|Y) = \frac{P(\bar{X}\, and\, Y)}{P(Y)}$$

$$= \frac{P(\bar{X})\ and\ P(Y)}{P(Y)}$$

also $P(\bar{X}) = P(1 - X)$. Therefore

$$P(\bar{X}|Y) = \frac{P(1 - X)\ and\ P(Y)}{P(Y)}$$

By distributivity

$$P(\bar{X}|Y) = \frac{(P(1)\ and\ P(Y)) - (P(X)\ and\ P(Y))}{P(Y)}$$

$$= \frac{P(Y) - P(X)\ and\ P(Y)}{P(Y)}$$

$$= \frac{P(Y)}{P(Y)} - \frac{P(X)\ and\ P(Y)}{P(Y)}$$

$$= 1 - \frac{P(X)\ and\ P(Y)}{P(Y)}$$

Therefore

$$P(\bar{X}|Y) = 1 - P(X|Y)$$

and finally,

$$P(X|Y) = 1 - P(\bar{X}|Y)$$

g.

$$= \frac{P(X \cap Y \cap Y)}{P(Y)} + \frac{P(X \cap \bar{Y} \cap \bar{Y})}{P(\bar{Y})} \cap \frac{P(\bar{Z} \cap X \cap \bar{Z})}{P(X)}$$

$$= \frac{P(X \cap Y)}{P(Y)} + \frac{P(X \cap \bar{Y})}{P(\bar{Y})} \cap \frac{P(\bar{Z} \cap X)}{P(X)}$$

$$= P(X|Y) + P(X|\bar{Y}) \cdot P(\bar{Z}|X)$$

$$= P(X|Y) + P(\bar{Z}|\bar{Y})$$

h.

From the axioms we know that

$$P(A, B) = P(A) \cdot P(B)$$

Therefore

$$P(X|Y, Z) = \frac{P(X\, and\, Y)}{P(Y)} \cdot Z$$

Since $P(X|Y) = 0$ then

$$\begin{aligned} P(X|Y, Z) &= 0 \cdot Z \\ &= 0 \end{aligned}$$

Question 2:

a.

$$\text{For all situations } z \sum_{n=0} p(X = cry_{\mathrm{n}}|Y = z) = 1 \tag{1}$$

b.

Table 1: Joint Probability Table

p(cry, situation)	Predator!	Timber!	I need help!	TOTAL
bwa	0	0	0.64	0.64
bwee	0	0	0.08	0.08
kiki	0.2	0	0.08	0.28
TOTAL	0.2	0	0.8	1

c.

1.

`p(Predator!  | kiki)`

2.

$$\frac{\texttt{p(kiki and Predator!)}}{\texttt{p(kiki)}}$$

3.

$$= \frac{0.2}{0.28} = 0.7143$$

4.

$$\frac{\texttt{p(kiki|Predator!)}\cdot\texttt{p(Predator!)}}{\texttt{p(Predator!|kiki)}\cdot\texttt{p(kiki)}+\texttt{p(Predator!|bwa)}\cdot\texttt{p(bwa)}+\texttt{p(Predator!!|bwee)}\cdot\texttt{p(bwee)}}$$

5.

$$= \frac{1\cdot 0.2}{1\cdot 0.28+0\cdot 0.064+0\cdot 0.08} = 0.7143$$

Question 3:

Absolute discounting proof

$$\text{For all situations } F_c^P \sum_{i=0}^{F_c^P} \frac{(F - F_0^c)\delta}{F_0^c - C_{\mathrm{c}}} \tag{2}$$

Taking $\alpha = \frac{(F-F_0^c)\delta}{F_0^c - C_{\mathrm{c}}}$, then we can express this sum as

$$\sum \alpha_{\mathrm{i}} = \alpha_0 + \alpha_1 + ... + \alpha_{\mathrm{n}}$$

$$\text{For all situations } \beta = \frac{r-\delta}{C_{\mathrm{c}}} \tag{3}$$

$$\sum \beta_{\mathrm{i}} = \beta_0 + \beta_1 + ... + \beta_{\mathrm{n}}$$

From the definition of absolute discounting, it can be seen that

$$\sum \alpha_{\mathrm{i}} + \sum \beta_{\mathrm{i}} = \alpha_0 + \alpha_1 + ... + \alpha_{\mathrm{n}} + \beta_0 + \beta_1 + ... + \beta_{\mathrm{n}}$$

Therefore

$$\sum \alpha_{\mathrm{i}} + \sum \beta_{\mathrm{i}} = 1$$

Linear Discounting proof

By the definition of linear discounting, it can be seen that

$$\text{for } \chi = \frac{(1-\alpha)r}{C_{\mathrm{c}}},\ \delta = \frac{\alpha}{F_0^c}$$

where α is a constant between 0 and 1. Then

$$\sum \chi + \sum \delta = \chi_0 + \chi_1 + ... + \chi_{\mathrm{n}} + \delta_0 + \delta_1 + ... + \delta_{\mathrm{n}} = 1$$