Take home exam for 515
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1.a First, we need to find the posterior function of $\Pi(\beta_1/Y,X)$

$\Pi(\beta_1/Y,X)=f(Y;\beta_0+\beta_1*X,\sigma,\lambda)*\exp(-\beta_1*\beta_1/200)/C$

($f(Y;\beta_0+\beta_1*X,\sigma,\lambda)$ is the pdf of EMG, and Y,X can be vectors. C is a normalizing constant.)

We can apply a Metropolis-Hastings algorithm for sampling from the posterior distribution using a normal proposal distribution centered at the current value of the Markov chain and with variance T^2.

(i) Choose a start value. Based on the plot of data1, β_1 is the slope of the regression. So I choose 8 as the start value.

(ii) Propose for a next value of β_1* based on the normal distribution with mean β_1 and variance T^2.

(iii) Accept β_1* (replacing β_1 by β_1* as long as a standard uniform random variable is less than $\Pi(\beta_1*)/\Pi(\beta_1)$. (Since normal proposal is symmetric, that is $q(\beta_1/\beta_1*)=q(\beta_1*/\beta_1)$

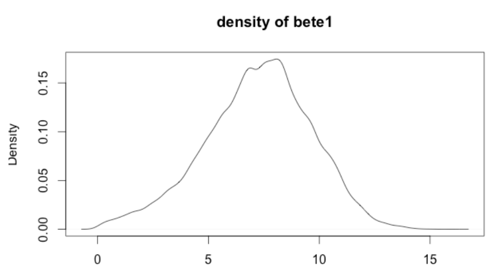
Here I use a log ratio, so the formula will be $\log U<\log\Pi(\beta_1*)-\log\Pi(\beta_1)$ if we accept β_1*. When using EMG pdf choose log=true.

(iiii) Go back to step (ii)

b Expectation of β_1= 7.138139, MCMCse= 0.02820279

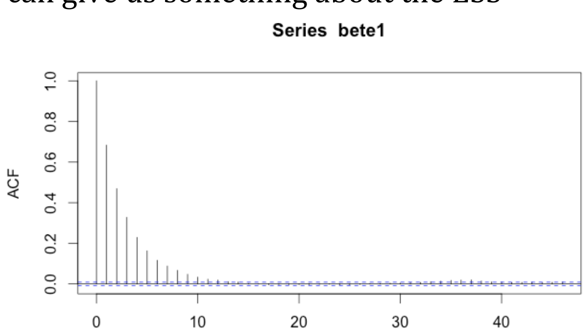
c (2.5% ,97.5%)=(1.838434 , 11.535300)

d

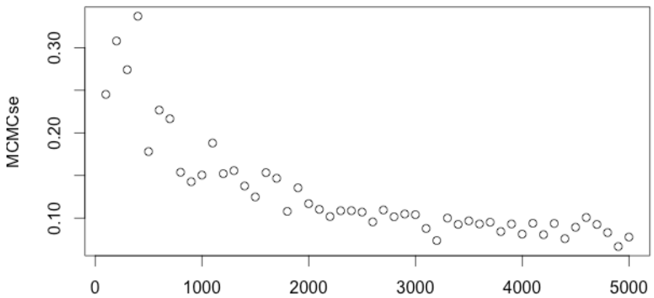


e

First, check the auto-correlation of samples. By trying different T^2 values, I find T =10 has good results. Here's the plot. I didn't calculate the ESS, for I think this plot can give us something about the ESS



Next, plot the MCMCse according to different number of realizations



Also, I got the acceptance rate as 0.28728 (acceptable).

2a First, the likelihood function is

$\pi(\beta_0, \beta_1, \lambda / Y, X) = f(Y; \beta_0+\beta_1*X, \sigma, \lambda)*\lambda^{(-0.99)}*\exp(-\lambda/100)*\exp(-\beta_1*\beta_1/200)*\exp(-\beta_0*\beta_0/200)$

Next, get marginal distribution of λ

$\pi(\lambda / \beta_0, \beta_1, Y, X) = f(Y; \beta_0+\beta_1*X, \sigma, \lambda)*\lambda^{(-0.99)}*\exp(-\lambda/100)$

for log level $\pi(\lambda / \beta_0, \beta_1, Y, X) = \log f(Y; \beta_0+\beta_1*X, \sigma, \lambda) - 0.99\log\lambda - \lambda/100$

Next, get marginal distribution of β_1

$\pi(\beta_1 / \lambda, \beta_0, Y, X) = f(Y; \beta_0+\beta_1*X, \sigma, \lambda)*\exp(-\beta_1*\beta_1/200)$

for log level $\pi(\lambda / \beta_0, \beta_1, Y, X) = \log f(Y; \beta_0+\beta_1*X, \sigma, \lambda) - \beta_1*\beta_1/200$

Next, get marginal distribution of β_0

$\pi(\beta_0, \beta_1, \lambda / Y, X) = f(Y; \beta_0+\beta_1*X, \sigma, \lambda)*\exp(-\beta_0*\beta_0/200)$

for log level $\pi(\lambda / \beta_0, \beta_1, Y, X) = \log f(Y; \beta_0+\beta_1*X, \sigma, \lambda) - \beta_0*\beta_0/200$

(i) choose starting values based on the X, Y plot from data 2: $\beta_0=0$, $\beta_1=0$, and $\lambda=0.1$

(ii)Update parameters

For λ:

1)Use M-H to update λ. The proposal distribution should be U(0,1), propose a λ^*. (I tried gamma and exponential distribution as proposal, however, they did not work well for the autocorrelation. For normal distribution there will be half of the data <=0, so, I choose a U(0,1). It seems good)

2)accept λ if $\pi(\lambda^* / \beta_0, \beta_1, Y, X) / \pi(\lambda / \beta_0, \beta_1, Y, X) > U$. Still can transform to log values. Note: here, the β_0, β_1 should be the most up to date values. And $q(\lambda / \lambda^*) = q(\lambda^* / \lambda)$

For β_1

1)Use M-H to update λ. The proposal distribution should be normal with mean=current β_1, and sd=T, propose a β^*, adjust T to get better results.

2)accept β^* if $\pi(\beta_1^* / \beta_0, \lambda, Y, X) / \pi(\beta_1 / \beta_0, \lambda, Y, X) > U$. Still can transform to log values. Note: here, the β_0, λ should be the most up to date values.

For β_0

Use the same method as β_1

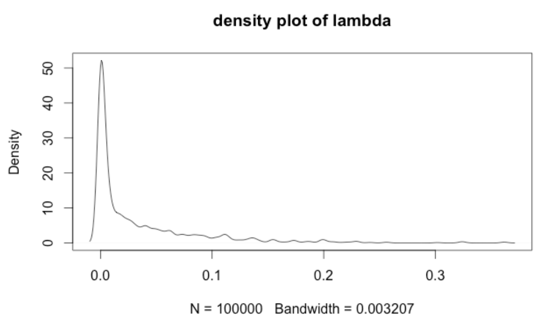
b

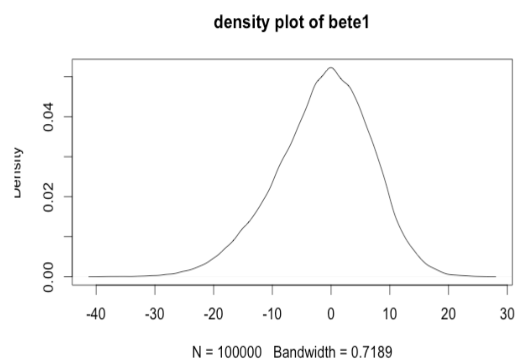
	mean	MCMCse	(0.025,0.975)
λ	0.05452495	0.0008154688	0.001122283 0.221680106
β_1	-0.9769537	**0.0717041**	-17.61843 12.50166
β_0	-6.312205	0.05553525	-20.047835 1.412603

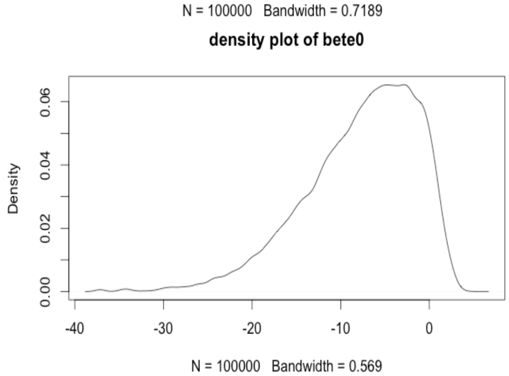
c cor(β_1, β_0)= -0.1955171

d

density plots

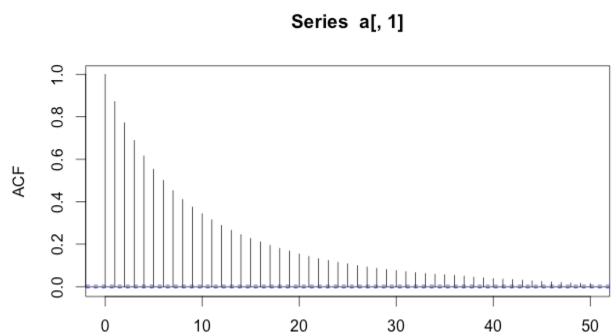


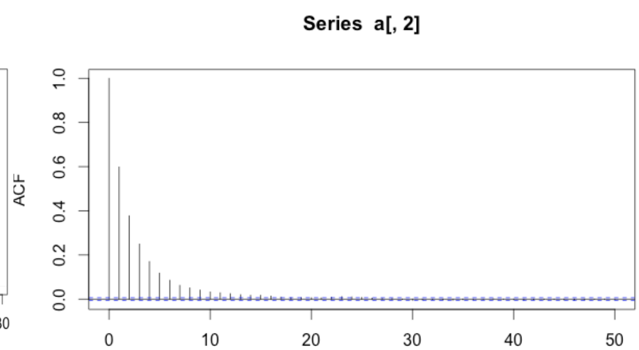


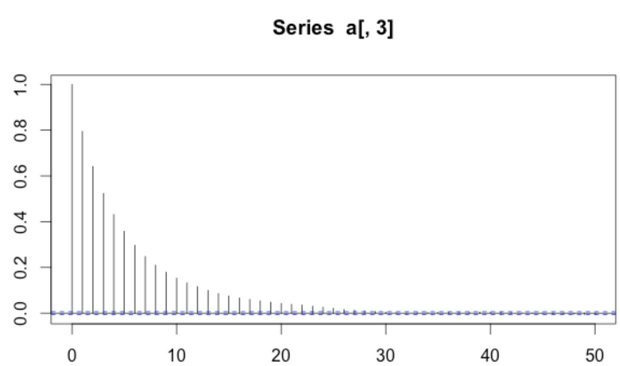


e Auto-correlation check

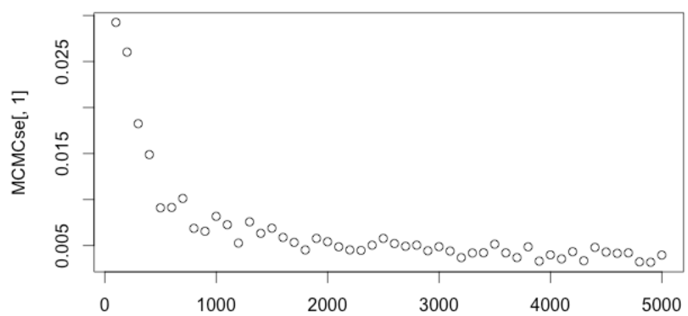
for λ, β_1, β_0

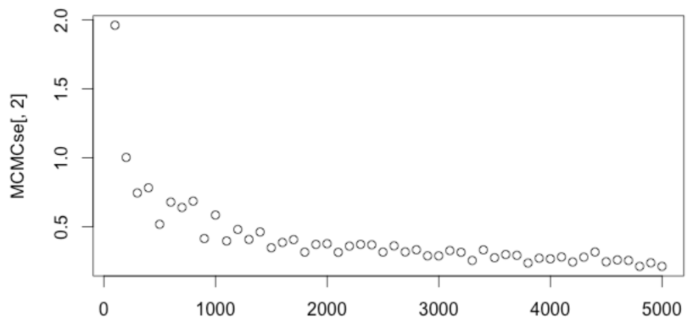


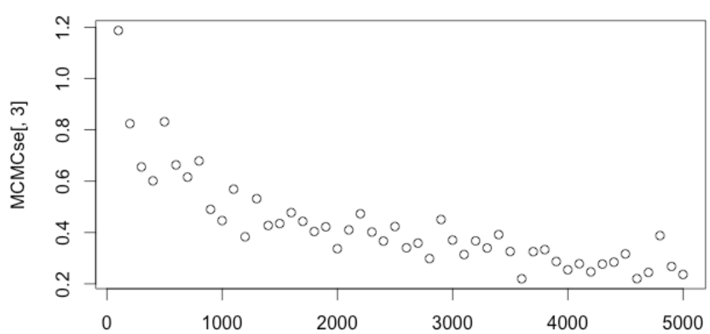




plot the MCMCse for 100-5000 Realizations for λ , β_1, β_0





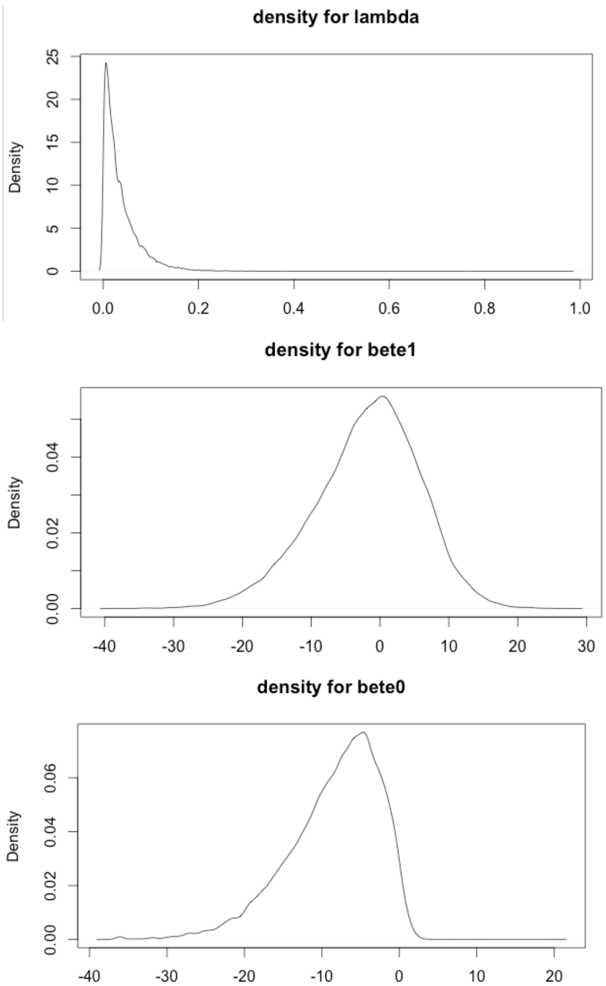


Markov chain algorithm ran for 1e+05 iterations:
(accept.rate for lambda 0.1095111)
(accept.rate for bete1 0.5607956)
(accpet.rate for bete0 0.4116141)

3a

	mean	MCMCse	(0.025,0.975)
λ	0.03622615	0.0006705153	0.000827586 0.142940637
β_1	-2.04731	**0.07532999**	-18.64347 ,11.82401
β_0	-7.839889	0.05529316	-21.49537806 0.02105507

b



C The only change I made is to change the start values. From the X/Y plot, $\beta_0=3$, $\beta_1=0$, $\lambda=1$. (The error term seems to be bi-norm as plot below, however, I can not make any change for the model)