

Markov chain Monte Carlo exercise

Consider the following hierarchical changepoint model for the number of occurrences Y_i of some event during time interval i with change point k .

$$Y_i|k, \theta, \lambda \sim \text{Poisson}(\theta) \text{ for } i = 1, \dots, k$$

$$Y_i|k, \theta, \lambda \sim \text{Poisson}(\lambda) \text{ for } i = k + 1, \dots, n$$

Assume the following prior distributions:

$$\theta|b_1 \sim G(0.5, b_1), \lambda|b_2 \sim G(0.5, b_2)$$

$$b_1 \sim G(1, 1), b_2 \sim G(1, 1)$$

$$k \sim \text{Discrete Uniform}(1, \dots, n)$$

k, θ, λ are independent and b_1, b_2 are independent.

Assume the Gamma density parameterization $G(\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

Inference for this model is therefore based on the 5-dimensional posterior distribution $f(k, \theta, \lambda, b_1, b_2|\mathbf{Y})$ where $\mathbf{Y}=(Y_1, \dots, Y_n)$. Apply this model to the data set available from the webpage [INSERT WEBPAGE](#).

- (1) Use the Metropolis-Hastings algorithm for simulating from the posterior distribution $f(k, \theta, \lambda, b_1, b_2|\mathbf{Y})$. Include an estimated density plot based on your samples for each of the 5 parameters.
- (2) The parameters of interest here are k, θ, λ : report your MCMC estimates of their means.
- (3) Compute Monte Carlo standard errors for your mean estimates for k, θ, λ using the batch means method (this will be covered on 4/15/05).

As before, you should use heuristics such as plots of your estimates, auto-correlations, and acceptance rates to help guide you through the simulation process. Include the most relevant plots in your report.