Homework 8, Stat 515, Spring 2015

Due Wednesday, April 15, 2015 **beginning of class**

Make sure you submit R code to the Angel drop folder, using the same naming conventions as in the last homework, e.g. your name should appear in the name of the R program. Please pay attention to good programming style in order to receive full credit.

1. Define the univariate Poisson kernel density function (Yang, 2004; or see "wrapped Cauchy" in Levy (1939) and Wintner (1947)) as follows:

$$f(\theta;\mu,\rho) = \frac{1}{2\pi}\frac{1-\rho^2}{1-2\rho\cos(\theta-\mu)+\rho^2}, \quad \mu-\pi \le \theta \le \mu+\pi$$

   (a) Simulate 10,000 draws from this density for $\mu = 3, \rho = 0.7$ using a rejection sampler. Report the envelope density (q) and bounding constant (K) used in the rejection sampler. You may use any envelope density from which you are easily able to produce draws in `R` (or the statistical programming language you are using). Plot the smoothed approximate density.

   (b) Approximate the expectation $E(\theta^2)$ using samples from (a) and report associated Monte Carlo standard errors.

   (c) Approximate $P(\theta > 4)$ and report associated Monte Carlo standard errors.

2. Write a program to simulate a non-homogeneous Poisson process on the interval $(0, 10)$ where the intensity function is $\lambda(t) = \max(0.2, 3\sin(t))$. Simulate the process by first simulating the total number of counts on the interval, then use rejection sampling to sample from the conditional distribution of the location of the points given the total number of points on the interval.

   (a) Provide pseudocode for your algorithm. (A clear description of your algorithm without using specific programming language. Fully specify any distributions you use or derive.)

   (b) Plot 2 realizations of this non-homogeneous Poisson process in separate figures. Make sure you plot the events clearly, for example if the vector `arrivals` has all the arrival times of the events, you may choose to use `plot(arrivals, rep(0.2, length(arrivals)), pch="x")`. Of course you are free to choose better ways to draw this.

3. Let $\{X(t), t > 0\}$ be Brownian motion with 0 drift and variance parameter σ^2. Suppose $X(1) = B$.

   (a) What is the joint distribution of $X(s_1), X(s_2), \ldots, X(s_n)$ for $s_1, \ldots, s_n \in (0, 1)$? Clearly explain the main steps in your justification.

   (b) Suppose $\sigma^2 = 3$, $B = 10$, $s_1 = 0.1, s_2 = 0.3$. What is the correlation between $X(s_1)$ and $X(s_2)$?

   (c) Let $0 < s_1 < s_2 < 1$ and $\sigma^2 = 3$, $B = 10$. Plot the correlation between $X(s_1), X(s_2)$ as a function of the distance, $d = \mid s_1 - s_2 \mid$.

4. For the problem above, assume $\sigma^2 = 3$, $B = 10$. Now assume that s_1, s_2 are the order statistics of two Uniform(0,1) random variates. Use Monte Carlo to approximate the probability that the correlation between $X(s_1)$ and $X(s_2)$ is greater than 0.1.

   (a) Provide pseudocode for your algorithm.

   (b) Report your Monte Carlo sample size and your Monte Carlo standard error.