STAT540 Stat. Comp. Project Presentation

A Comparison between Likelihood-free MCMC & Likelihood-free Sequential Monte Carlo

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Pseudocode of ABC MCMC

1. Initialize $\theta_i, i = 0$
2. Propose θ^* according to a proposal distribution $q(\theta|\theta_i)$
3. Simulate a dataset x^* from $f(x|\theta^*)$
4. If $\rho(x_0, x^*) \leq \epsilon$, go to 5, otherwise set $\theta_{i+1} = \theta_i$ and go to 6
5. Set $\theta_{i+1} = \theta^*$ with probability

   $$\alpha = \min\left\{1, \frac{\pi(\theta^*)q(\theta_i|\theta^*)}{\pi(\theta_i)q(\theta^*|\theta_i)}\right\}$$

   and $\theta_{i+1} = \theta_i$ with probability $1 - \alpha$
6. Set $i = i + 1$, go to 2.

Potential problem of ABC MCMC

Simulation may get stuck in low probability region and the Markov chain stops moving for a really long time. For example,

proposal and prior are both chosen to be multivariate normal

- For threshold $\epsilon = 2$, the chain may stops moving for 500 steps
- For threshold $\epsilon = 0.5$, the chain may stops moving for 10,000 steps

Pseudocode of ABC SMC

1. Initialize threshold schedule $\epsilon_1 > \cdots > \epsilon_T$
2. Set $t = 1$

   For $i = 1, 2, \cdots, N$

   - Simulate $\theta_i^{(1)} \sim p(\theta)$ and $x \sim p(x|\theta_i^{(1)})$ until $\rho(x, x_{\text{obs}}) < \epsilon_1$
   - Set $w_i = 1/N$

3. For $t = 2, \cdots, T$

   For $i = 1, 2, \cdots, N$

   - Repeat:

     Pick θ_i^* from the $\theta_j^{(t-1)}$'s with probabilities $w_j^{(t-1)}$,draw $\theta_i^{(t)} \sim K_t(\theta_i^{(t)}|\theta_i^*)$ and $x \sim p(x|\theta_i^{(t)})$; until $\rho(x, x_{\text{obs}}) < \epsilon_t$
   - Compute new weights as

   $$w_i^{(t)} \propto \frac{p(\theta_i^{(t)})}{\sum_j w_j^{(t-1)} K_t(\theta_i^{(t)}|\theta_j^{(t-1)})}$$

   Normalize $w_i^{(t)}$ over $i = 1, 2, \cdots, N$

Pseudocode of ABC SMC with Adaptive Weights

1. Initialize threshold schedule $\epsilon_1 > \cdots > \epsilon_T$
2. Set $t = 1$

   For $i = 1, 2, \cdots, N$
   - Simulate $\theta_i^{(1)} \sim p(\theta)$ and $x \sim p(x|\theta_i^{(1)})$ until $\rho(x, x_{\text{obs}}) < \epsilon_1$
   - Set $w_i = 1/N$
3. For $t = 2, \cdots, T$

   Compute data based weights $v_i^{(t-1)} \propto w_i^{(t-1)} K_{x,t}(x_{\text{obs}}|x_i^{(t-1)})$

   Normalize weights $v_i^{(t-1)}$ over $i = 1, 2, \cdots, N$

   For $i = 1, 2, \cdots, N$
   - Repeat:

     Pick θ_i^* from the $\theta_j^{(t-1)}$'s with probabilities $v_j^{(t-1)}$,draw $\theta_i^{(t)} \sim K_{\theta,t}(\theta_i^{(t)}|\theta_i^*)$ and $x \sim p(x|\theta_i^{(t)})$; until $\rho(x, x_{\text{obs}}) < \epsilon_t$
   - Compute new weights as

$$w_i^{(t)} \propto \frac{p(\theta_i^{(t)})}{\sum_j v_j^{(t-1)} K_{\theta,t}(\theta_i^{(t)}|\theta_j^{(t-1)})}$$

   Normalize $w_i^{(t)}$ over $i = 1, 2, \cdots, N$

Different Choices of Transition Kernel

- Component-wise perturbation kernels
- Multivariate normal perturbation kernels
- local perturbation kernels
  - Multivariate normal kernel with M nearest neighbors
  - Multivariate normal kernel with optimal covariance matrix
  - Perturbation kernel based on the Fisher information for model defined by ODE/SDE

Simulation Results

Average Number of Simulation Steps per Accepted Particle:

	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	Total
ABC SMC	54	103	1436	738	524	2855
ABC SMC AW	52	53	123	364	542	1134
ABC SMC OLCM	1	153	1479	897	467	2997

$$N = 100, \quad \epsilon = (200, 100, 10, 2, 1)^T$$

Simulation Results

Average Number of Simulation Steps per Accepted Particle:

	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	Total
ABC SMC	265	490	9726	4112	2408	17001
ABC SMC AW	262	259	818	2356	2120	5815
ABC SMC OLCM	1	728	7482	3931	2016	14158

$$N = 500, \quad \epsilon = (200, 100, 10, 2, 1)^T$$

Simulation Results

Change the parameter setting and evaluate the performance in terms of L^2 norm error:

- True parameter: $\theta = (1, -3, 5)^T$
- Threshold schedule: $\epsilon = (2, 0.5, 0.025)^T$
- Simulate $n = 50$ independent runs with particle number $N = 100$ and compute the average L^2-norm errors

The L^2 differences of three algorithms are:

$$\text{diff}_{SMC} = 0.862, \quad \text{diff}_{AW} = 0.853, \quad \text{diff}_{OLCM} = 0.851$$

Boxplots for Estimated Parameter Moments