Ice Model Calibration Using Spatial Processes

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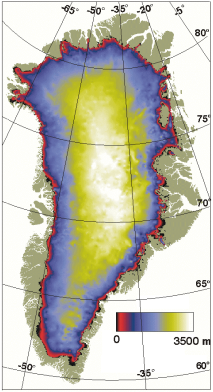
This Talk

- Ice models are often used to make projections about future ice sheet behavior.
- A major source of uncertainty about these projections is due to uncertainty about climate model input parameters.
- We propose a method for learning about ice model parameters from ice model outputs and observations.
- Challenges: Data in the form of high-dimensional spatial processes. Complicated error structures.
- I will describe novel computationally efficient approaches based on Principal components (PC) and kernel convolution.

Greenland Ice Sheet

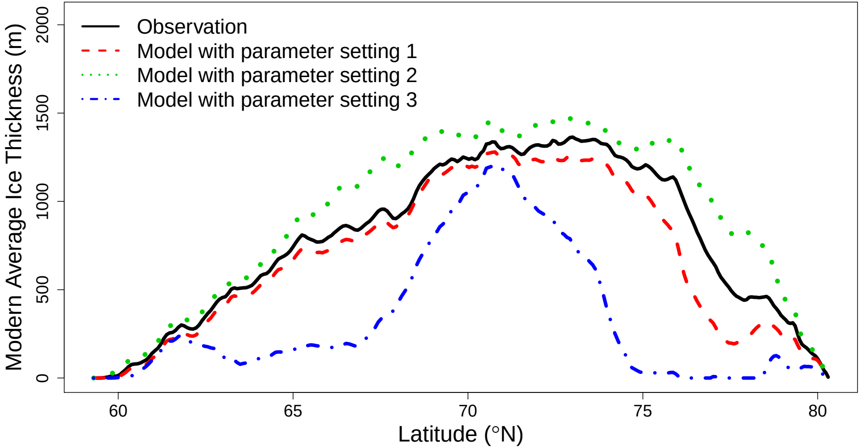
Important contributor to sea level rise: Total melting results in sea level rise of 7m.



Bamber et al. (2001)

Calibration Problem

Which parameter settings best match observations?

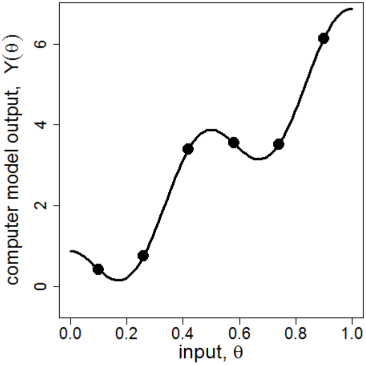


Two-stage Approach to Emulation-Calibration

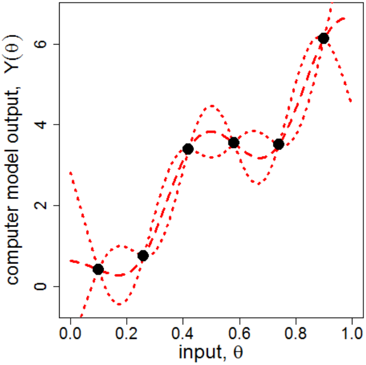
1. Emulation step: Find fast approximation for climate model using Gaussian process (GP)
2. Calibration step: Infer climate parameter using emulator and observations, while accounting for data-model discrepancy

   (Bhat, Haran, Olson, Keller, 2012; Liu, Bayarri and Berger, 2009)

Emulation Step

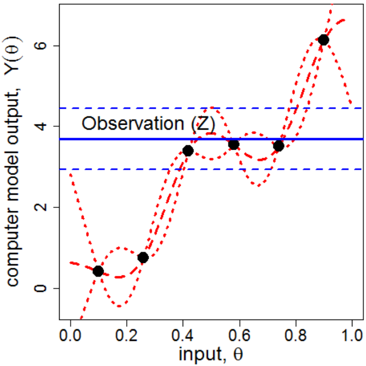


Computer model output (y-axis) vs. input (x-axis)

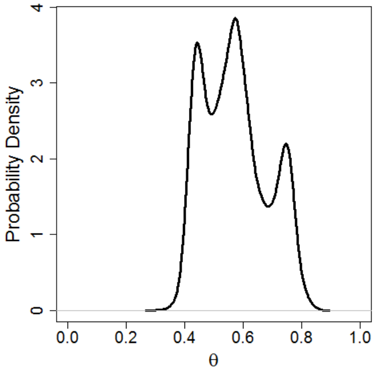


Emulation (approximation) of computer model using GP

Calibration Step



Combining observation and emulator



Posterior PDF of θ given model output and observations

Summary of Statistical Problem

- **Goal**: Learning about $\boldsymbol{\theta}$ based on two sources of information:
  - **Observations**: Mean ice thickness profile† $\mathbf{Z} = (Z(\mathbf{s}_1), \ldots, Z(\mathbf{s}_n))^T$, where $\mathbf{s}_1, \ldots, \mathbf{s}_n$ are latitude points.
  - **Ice model output*** for mean ice thickness $\mathbf{Y}(\boldsymbol{\theta}_1), \ldots, \mathbf{Y}(\boldsymbol{\theta}_p)$, where each $\mathbf{Y}(\boldsymbol{\theta}_i) = (Y(\mathbf{s}_1, \boldsymbol{\theta}_i), \ldots, Y(\mathbf{s}_n, \boldsymbol{\theta}_i))^T$ is spatial process (Applegate et al 2012).

$\mathbf{Z}$ and $\mathbf{Y}(\boldsymbol{\theta}_i)$'s are n-dimensional vectors

- Important: output at each $\boldsymbol{\theta}_i$ is a spatial process. $n = 264$ locations, $p = 100$ runs.

†Averaged over longitude
*SICOPOLIS (Greve, 1997; Greve et al., 2011)

Step 1: Dimension Reduction

- Consider model outputs at $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_p$ as replicates and obtain PCs

$$\begin{pmatrix} Y(\mathbf{s}_1, \boldsymbol{\theta}_1) & \ldots & Y(\mathbf{s}_n, \boldsymbol{\theta}_1) \\ \vdots & \ddots & \vdots \\ Y(\mathbf{s}_1, \boldsymbol{\theta}_p) & \ldots & Y(\mathbf{s}_n, \boldsymbol{\theta}_p) \end{pmatrix}_{p \times n} \Rightarrow \begin{pmatrix} Y_1^R(\boldsymbol{\theta}_1) & \ldots & Y_{J_y}^R(\boldsymbol{\theta}_1) \\ \vdots & \ddots & \vdots \\ Y_1^R(\boldsymbol{\theta}_p) & \ldots & Y_{J_y}^R(\boldsymbol{\theta}_p) \end{pmatrix}_{p \times J_y}$$

- PCs pick up characteristics of model output that vary most across input parameters $\theta_1, \ldots, \theta_p$.

Step 2: Emulation Using PCs

- Fit 1-dimensional GP for each series $Y_j^R(\boldsymbol{\theta}_1), \ldots, Y_j^R(\boldsymbol{\theta}_p)$
- $\boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R)$: J_y-dimensional emulation process for PCs, $\mathbf{Y}^R$ is collection of PCs
- Computation reduces from $\mathcal{O}(n^3 p^3)$ to $\mathcal{O}(J_y p^3)$ (6.13×10^{12} to 3.33×10^6 flops).
- Emulation for original output: compute $\mathbf{K}_y \boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R)$ where $\mathbf{K}_y$ is matrix of scaled eignvectors

Dimension Reduction for Discrepancy Process

- Kernel convolution: Specifying n-dimensional discrepancy process $\boldsymbol{\delta}$ using J_d-dimensional knot process $\boldsymbol{\nu}$ ($J_d < n$) and kernel functions
- Kernel basis matrix $\mathbf{K}_d$ links grid locations $\mathbf{s}_1, \ldots, \mathbf{s}_n$ to knot locations $\mathbf{a}_1, \ldots, \mathbf{a}_{J_d}$;

$$\{\mathbf{K}_d\}_{ij} = \exp\left(-\frac{\|\mathbf{s}_i - \mathbf{a}_j\|}{\phi_d}\right)$$

  with $\phi_d > 0$. Fix ϕ_d at large value determined by expert judgment
- Results in better identifiability: Overly flexible discrepancy process may be confounded with emulator

Calibration in Reduced Dimensions

- Probability model for dimension-reduced observation $\mathbf{Z}^R$:

$$\mathbf{Z} = \underbrace{\mathbf{K}_y \boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R)}_{\text{emulator}} + \underbrace{\mathbf{K}_d \boldsymbol{\nu}}_{\text{discrepancy}} + \underbrace{\boldsymbol{\epsilon}}_{\text{observation error}},$$

$$\Rightarrow \mathbf{Z}^R = (\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\mathbf{Z} = \begin{pmatrix} \boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R) \\ \boldsymbol{\nu} \end{pmatrix} + (\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\boldsymbol{\epsilon},$$

with combined basis $[\mathbf{K}_y \; \mathbf{K}_d]$, knot process $\boldsymbol{\nu} \sim N(\mathbf{0}, \kappa_d \mathbf{I})$, and observational error $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I})$.

- Infer $\boldsymbol{\theta}$ through posterior distribution

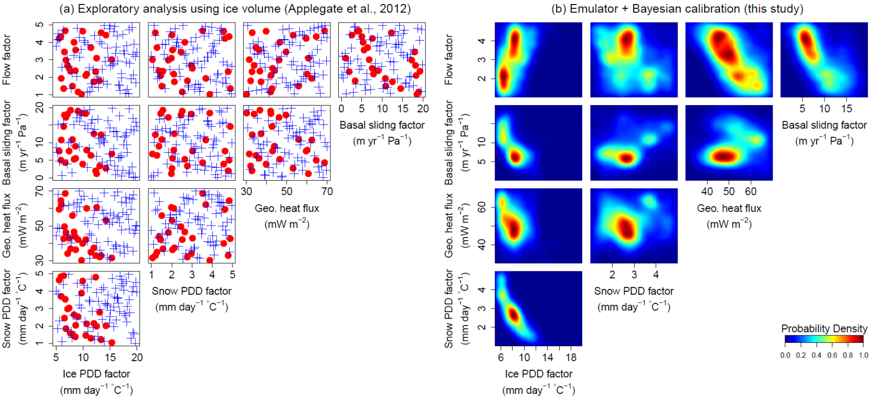
$$\pi(\boldsymbol{\theta}, \kappa_d, \sigma^2 | \mathbf{Z}^R, \mathbf{Y}^R) \propto \underbrace{L(\mathbf{Z}^R | \mathbf{Y}^R, \boldsymbol{\theta}, \kappa_d, \sigma^2)}_{\text{likelihood given by above}} \underbrace{p(\boldsymbol{\theta}) p(\kappa_d) p(\sigma^2)}_{\text{priors}}$$

Perfect Model Experiment

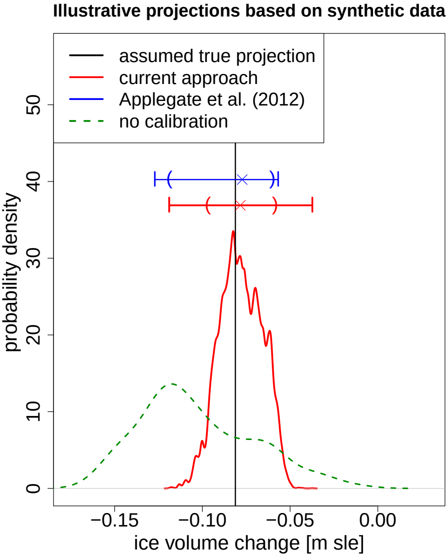
Test if our calibration method can recover the "truth".

1. Pick one model output as synthetic truth.
2. Generate observational data by adding structural error.
3. Calibrate the parameters using remaining model outputs.
4. See if we get back parameter setting for synthetic truth.

Parameter Densities



Ice Volume Change Projection



Discussion

- Dimension reduction-based approach:
  - Very fast, scales well with n, number of spatial locations: Potential to apply for bigger data set
  - Very easy to use: Automatic emulation step
- Ice model calibration:
  - Provides sharper posterior densities for input parameters and sea level rise projections
  - Shows clear interaction between parameters

References

- Chang, W., Applegate, P., Haran, M. and Keller, K. (2014) Probabilistic calibration of a Greenland Ice Sheet model using spatially-resolved synthetic observations: toward projections of ice mass loss with uncertainties, *submitted to Geophysical Model Development Discussion*
- Chang, W., Haran, M., Olson, R., and Keller, K. (2013) Fast dimension-reduced climate model calibration, *accepted for publication in the Annals of Applied Statistics, arXiv:1303.1382*.
- Applegate, P. J., Kirchner, N., Stone, E. J., Keller, K., and Greve, R., 2012, An assessment of key model parametric uncertainties in projections of Greenland Ice Sheet behavior: *The Cryosphere* 6, 589-606.

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