Penn State STAT 540
Homework #2, due Tuesday, November 6, 2018

Please submit separate files for your R code and your writeup (check with the TA, but do not zip the files together!). As before, (i) your `R` code in a file titled PSUemailidHW2.R (e.g. muh10HW2.R), (ii) pdf file that contains a clear writeup for the questions below named PSUemailidHW2.pdf (e.g. muh10HW2.pdf). Note that your code should be readily usable, without any modifications. Provide enough details about your algorithms – make it brief, but provide enough information so anyone who reads it would be able to construct the algorithm (provide pseudocode).

(1) The pdf of a logistic distribution is

$$f(x) = \frac{\exp(-(x-\mu)/\gamma)}{\gamma(1+\exp(-(x-\mu)/\gamma))^2}, \; -\infty < \mu < \infty, \; \gamma > 0.$$

Suppose $X_1, \ldots, X_n$ are iid from the logistic distribution with parameters μ, γ. The data are `https://personal.psu.edu/muh10/540/hw2B.dat` . Assume that the prior distributions are $\mu \sim N(0,5)$, and $\gamma \sim \text{Gamma}(\alpha = 1, \beta = 10)$ where the parameterization of the Gamma pdf is $\frac{1}{\Gamma(\alpha)\beta^\alpha}\gamma^{\alpha-1}e^{-\gamma/\beta}$. Write a Metropolis-Hastings algorithm to simulate from the posterior pdf $\pi(\mu, \gamma \mid X_1, \ldots, X_n)$. Plot your approximation to the posterior pdf of γ and μ. Report the posterior expectation of each of the following four random variables, $\mu, \gamma, \gamma^2, \mu/\gamma$, and report the Monte Carlo standard errors for each (in `R` you can use the mcse command in the mcmcse package; if you have to write your own code, you can translate it from here `https://personal.psu.edu/muh10/batchmeans.R` or `https://personal.psu.edu/muh10/cbm.c`). Also provide a 95% credible interval for μ and γ. Provide pseudocode for the algorithm you implemented. Make sure you report (i) how you obtained the starting value, (ii) how you determined the run length of the Markov chain and how you decided that your approximations were reliable (convergence diagnostics, including relevant plots), (iii) how you tuned the algorithm (adjusted your proposal).

(2) Assume that data, $X_1, \ldots, X_n$ are independent and identical draws from a logistic distribution.

(a) 1D optimization: Write a Newton-Raphson algorithm to obtain the maximum likelihood estimate of the parameter μ of a logistic distribution. Assume that $\gamma = 1$ (fixed and known). Use the following data set: `https://personal.psu.edu/muh10/540/hw2A.dat`

i. Provide your pseudocode including how you obtained a starting value and what stopping criteria you used. Also include your `R` code.

ii. Discuss your choice of step size s, and compare your algorithm's behavior for at least two different choices (you can show a plot of the trajectory of your iterations, for instance). Does your algorithm have the ascent property?

iii. Report your MLE, standard error estimates and a 95% confidence interval for μ. For standard error estimates, you can use standard mathematical statistics theory. That is, you can approximate variance by using either observed or expected Fisher information evaluated at the MLE. Note that the Newton-Raphson algorithm already makes use of the observed Fisher information at each update.

(b) 2D optimization: Write a Newton-Raphson algorithm to obtain the maximum likelihood estimates of the parameters μ, γ of a logistic distribution. Note that the Newton-Raphson algorithm in higher dimensions is just the multivariate analogue of the 1D univariate Newton-Raphson. Use the following data set (different from above data set): `https://personal.psu.edu/muh10/540/hw2B.dat`

i. Provide your pseudocode including how you obtained a starting value and what stopping criteria you used. Also include your `R` code.

ii. Discuss your choice of step size s, and compare your algorithm's behavior for at least two different choices (you can show a plot of the trajectory of your iterations, for instance). Does your algorithm have the ascent property?

iii. Report your MLE, standard error estimates and 95% confidence intervals for μ, γ. Again, you should use observed or expected Fisher information at the MLE to approximate standard errors.

(4) Conduct a simulation study where you compare the Mean Squared Error (MSE) for two estimators: the maximum likelihood estimator (MLE) and the posterior mean for the parameter γ in the logistic distribution above. Do this for $\mu = 3$ (assume μ is known so it does not have to be estimated) and for $\gamma = 1$ and $\gamma = 10$. Consider two different sample sizes in each case, $n = 10$ and $n = 500$.

Extra: You do not have to turn this in, but you can also consider a simulation study comparing coverage of the 95% confidence intervals with 95% credible intervals based on the maximum likelihood and Bayesian approaches for each case above.