Take-home Final STAT 515 Xiao Gan

1.

(a) Description of Algorithm:

Derive the posterior density: $\mathrm{h}(\beta_1|\mathrm{Y},\mathrm{X})$, with given $\beta_0, \sigma_i, \lambda$.

$$\mathrm{h}(\beta_1|\mathrm{Y},\mathrm{X}) = \prod_{i=1}^{m} f_{EMG}(\beta_0 + \beta_1 X_i; \mu, \sigma_i, \lambda) \times f_{Normal}(\beta_1, 0{,}10)$$

Sample $\mathrm{h}(\beta_1|\mathrm{Y},\mathrm{X})$ with M-H algorithm:

Start from a random value A[1] that is in the β_1 state space;

Generate y from proposal y=q(A[i]), e.g. q(A[i])~N(A[i], 10);

Accept y with probability = $\min(1, \frac{h(y|\mathrm{Y},\mathrm{X})}{h(\mathrm{A[i]}|\mathrm{Y},\mathrm{X})} \cdot \frac{q(\mathrm{A[i]})}{q(y)}$), .i.e A[i+1]=y;

(If q is "symmetric", e.g. q(A[i])~N(A[i], 10), the second term $\frac{q(\mathrm{A[i]}_i)}{q(y)} = 1$, which can be dropped.)

Else reject y, i.e. A[i+1]= A[i].

Repeat until a satisfactory sample size is reached.

Choice of starting value: A[1] = 5 (I found this value after a few trials)

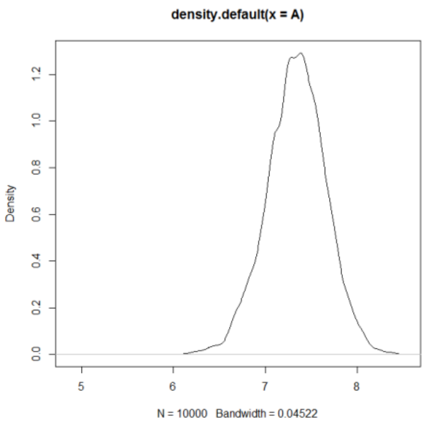
Proposal: q = rnorm(1,A[i-1],tau1), with tau1=1 (acceptance rate 0.3442344)

Sample size: n=10000

(b) point estimate of β_1: 7.334026, MCMCse: 0.007663469

(c) 95% credible interval: (6.676679, 7.937096)

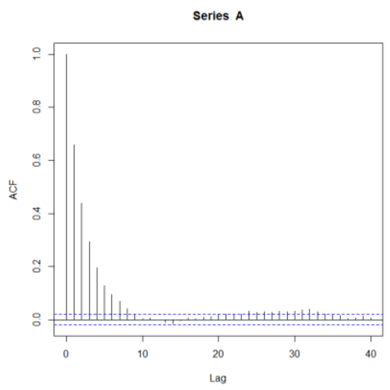
(d) plot of density:



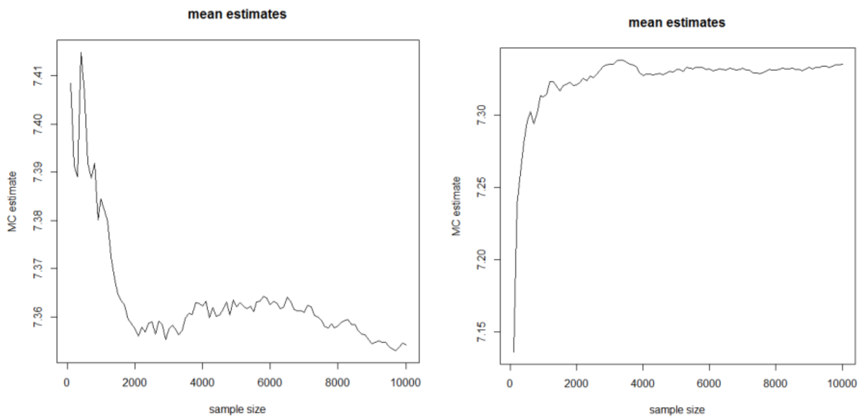
(e) I examined:

(1) how the density plot changes when I increase n. n=1000 and n=10000 has similar shape,

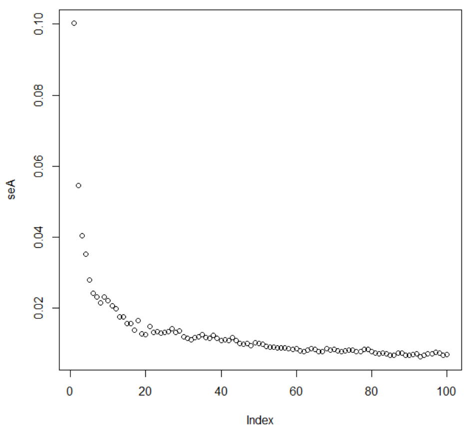
(2) acf function plot: it drops quickly and become small.



(3) mean estimates w.r.t. sample size stabilize at the same level (~7.34), when I start from different values (7 and 5, respectively):



(4) MCMCse estimates w.r.t. sample size: drops quickly without irregular behavior.



2. Note: the program I wrote is not good, and it may take ~10min to get the result of a 10000-sample-size run

(a) Description of Algorithm:

(1) Derive the joint posterior density: $h(\beta_1, \beta_0, \lambda | Y, X)$, with given σ_i.

$$h(\beta_1, \beta_0, \lambda|Y, X) = \prod_{i=1}^{m} f_{EMG}(\beta_0 + \beta_1 X_i; \mu, \sigma_i, \lambda) \times f_{Normal}(\beta_1, 0{,}10) \times f_{Normal}(\beta_0, 0{,}10) \times f_{Gamma}(\lambda, 0.01{,}100)$$

Technically one should then derive full conditional densities, but we know full conditional densities are proportional to joint density, so a lazy way is to just plug in $h(\beta_1, \beta_0, \lambda|Y, X)$

(2) Start from a random vector (A[1],B[1],C[1])=(β_1 , β_0, λ) that is in the state space;

(2-1) Sample β_1 ~$h(\beta_1| \beta_0, \lambda, Y, X)$using M-H with the most up-to-date values:

i. Generate y from proposal y1=q(A[i]|B[[i],C[i]), e.g. q(A[i]|B[[i],C[i])~N(A[i], 10);

ii. Accept y1 with probability = $\min(1, \frac{h(y|Y,X)}{h(A[i]|Y,X)} \cdot \frac{q(A[i]|B[[i],C[i])}{q(y)}$), .i.e A[i+1]=y;

iii. Else reject y, i.e. A[i+1]= A[i].

(2-2) Sample β_0 ~$h(\beta_0| \beta_1, \lambda, Y, X)$ using M-H algorithm with the most up-to-date values in a similar way to above; (note now A[i+1] is the most up-to date value for β_1)

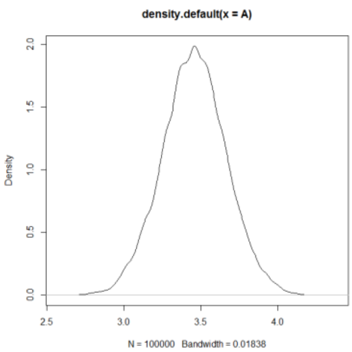
(2-3) Sample λ ~$h(\lambda| \beta_1, \beta_0, Y, X)$ using M-H algorithm with the most up-to-date values in a similar way to above; (note now A[i+1], B[i+1] are the most up-to date value for β_1, β_2)

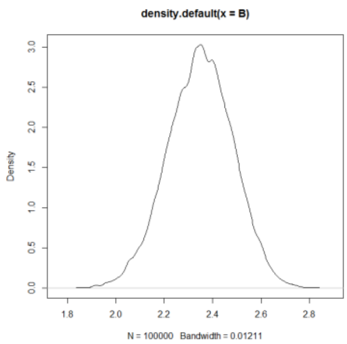
(3) Now we get a new state (A[i+1],B[i+1],C[i+1]). Repeat until a satisfactory sample size is reached.

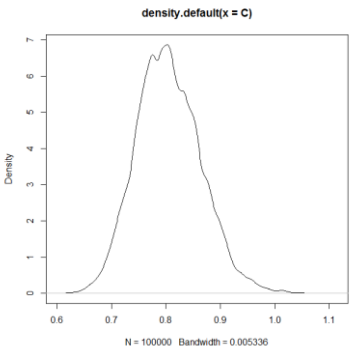
(b) Result summary:

	Mean	MCMCse	95% CI
Beta1	3.459496	0.004523306	3.043983, 3.871376
Beta0	2.348792	0.003337412	2.077541 2.604135
lambda	0.8050606	0.001046896	0.6967748 0.9280380

(c) Density of Beta1, Beta0:, lambda, respectively: (initial=(5,1,0.5))

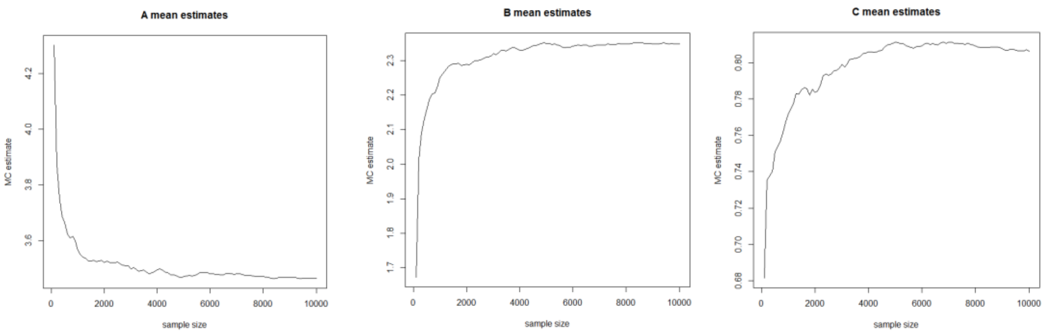




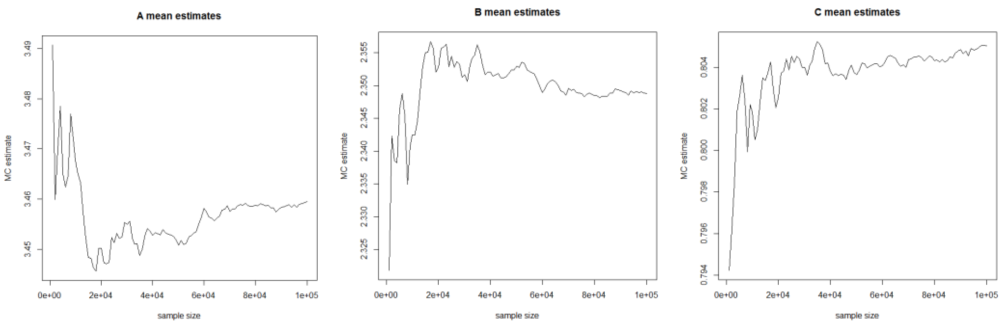


(d) Correlation between the data is -0.7996584.

(e) mean estimate v.s. sample size: initial state=(5,1,0.5)

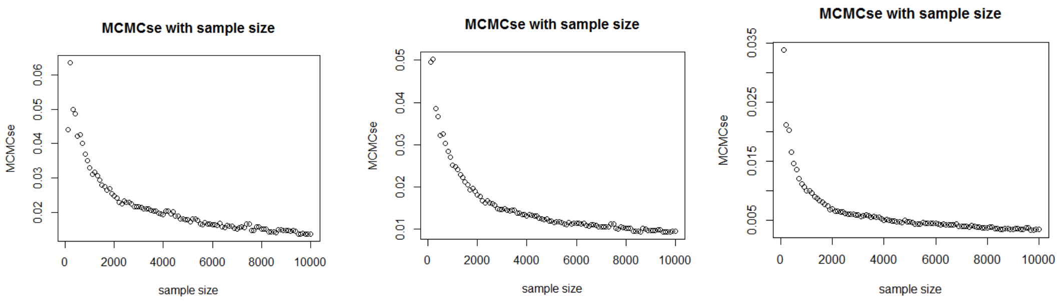


mean estimate v.s. sample size: initial=(4,2,0.7)



Starting from different initial state, the estimated mean converge at the same value.

MCMCse of Beta1, Beta0, lambda, respectively: drop quickly.

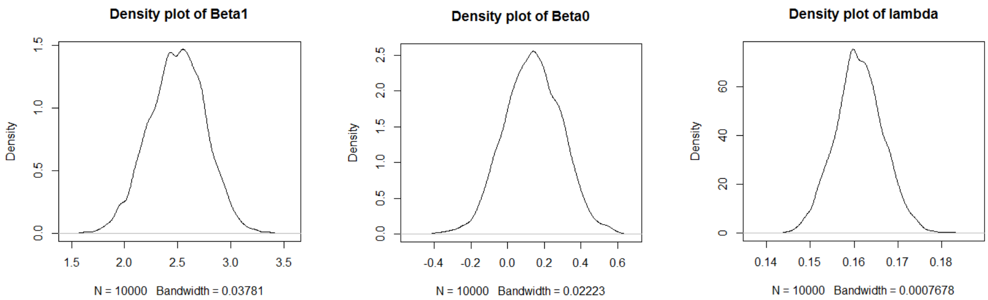


I have also examined the autocorrelations, they drop quickly to a low value, too. However due to limited space I will not present them here.

3. (a) Note: the program I wrote is not good, and it may take ~20min to get the result of a 10000-sample-size run

	Mean	MCMCse	95% CI
Beta1	2.493601	0.01293823	1.962480, 2.994936
Beta0	0.137315	0.008138765	-0.1634433,0.4342377
lambda	0.1617209	0.0001250916	0.1514621, 0.1726969

(b) The density plots:



(c) I have to change proposals q() for Lambda, and adjust parameters of the three V-MH to get reliable approximations.

MCMCse: