Homework 1, Stat 515, Spring 2015

Due Wednesday, January 21, 2015 **beginning of class**

Please read through Chapter 2 to review basic ideas about random variables. Make sure you read relevant sections in Chapter 3. For `R` code examples that might be helpful for these problems, take a look at `http://www.stat.psu.edu/~mharan/515/hwdir/hw1ex.R`

1. Assume $X \sim \text{Beta}(\alpha, \beta)$. Find $E(X \mid X < t)$ where $t \in (0, 1)$.

2. A gambler wins each game with probability p. In each of the following cases, determine the expected total number of wins.

   (a) The gambler will play n games; if he wins X of these games, then he will play an additional X games before stopping.

   (b) The gambler will play until he wins; if it takes him Y games to get this win, then he will play an additional Y games.

3. (a) Show that $\text{Cov}(X, Y) = \text{Cov}(X, E(Y \mid X))$.

   (b) Suppose that for constants a and b, $E(Y \mid X) = a + bX$. Show that $b = \text{Cov}(X, Y)/\text{Var}(X)$.

4. Suppose that you arrive at a party, along with a random number of additional people. The number of additional people, $X \sim \text{Poisson}(10)$. The times at which people (including you) arrive at the party are independent uniform(0,1) random variables.

   (a) Find the expected number of people who arrive before you.

   (b) Find the variance of the number of people who arrive before you.

5. A coin that comes up heads with probability p is flipped n consecutive times. What is the probability that starting with the first flip there are always more heads than tails that have appeared ? Hint: use the solution to the ballot problem (in Chapter 3.)

6. Toy drug trial. Consider n trials, each with probability of success p. Assume the trials are independent given p.
   Now, suppose $p \sim \text{Beta}(\alpha, \beta)$, $i = 1, \ldots, n$. Recall that if X is a Beta r.v.:

   $$f_X(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} I(0 < x < 1), \ \ \alpha > 0, \beta > 0$$

   $$\text{E}(X) = \frac{\alpha}{\alpha + \beta}, \quad \text{Var}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}.$$

   (a) Compute the expected value of the total number of successes.

   (b) Compute the variance of the total number of successes.

7. A short simulation exercise: Estimate the answers you obtained for Problem 4 above via simulation. You already have the answer so you can compare your estimates with the answer. Use 1000 replications of the process (one replicate of the process = one randomly sampled party.)

   - First download, install `R`; see the course webpage `http://www.stat.psu.edu/~mharan/515/Rlinks.html` for useful `R` links.
   - You can find a simple example for random variate simulation here: `http://www.stat.psu.edu/~mharan/515/hwdir/hw1ex.R`. You can adapt this example to estimate the expectation and variance for this problem.

   Note: Ideally, you should also be reporting simulation (Monte Carlo) standard errors for your estimates; we will discuss this later in the course.

Because this is your first assignment and your `R` code here will be quite short, please include a print out of your `R` code with the assignment. You do not have to type up this assignment, but if you want to start getting familiar with LaTeX, try looking at `http://www.stat.psu.edu/~mharan/515/Latexlinks.html`