Batch means standard errors for MCMC

Batch means is one method used to compute Monte Carlo standard errors. There is a vast literature on sophisticated methods for computing Monte Carlo standard errors for MCMC output. However batch means has the advantage of being easy to implement and it appears to work reasonably well in practice when implemented carefully.

Suppose we are interested in estimating an expectation $\mu = E(g(X))$ where X is a draw from a distribution f. The batch means method works as follows: Run the Markov chain $\{X_n\}$ for N=ab iterations (we can assume a and b are integers).
Let

$$Y_k = \frac{\sum_{i=(k-1)b+1}^{kb} g(X_i)}{b} \quad k = 1, \ldots, a$$

If we think of the Markov chain as having been divided into "a" batches of size "b" each, Y_k is then the Monte Carlo estimate of μ based on the kth 'batch'. Let

$$\hat{\sigma}^2 = \frac{b}{a-1} \sum_{k=1}^{a} (Y_k - \hat{\mu})^2$$

where $\hat{\mu}$ is simply the sample mean of the chain.

Then the batch means estimate of Monte Carlo standard error is $\frac{\hat{\sigma}}{\sqrt{N}}$.

How do we pick b (and hence a) ?
b (batch size) should be large enough so Y_ks are approximately independent. This is difficult to determine in general and a reasonable heuristic is to use b=$\sqrt{N}$, particularly in the context of running samplers using a "fixed width" approach where the sampler is run until the Monte Carlo standard error is below a certain threshold (see Jones, Haran, Caffo and Neath (2005)).