Combining a Glacier Dynamics Model with Multiple Surface Datasets

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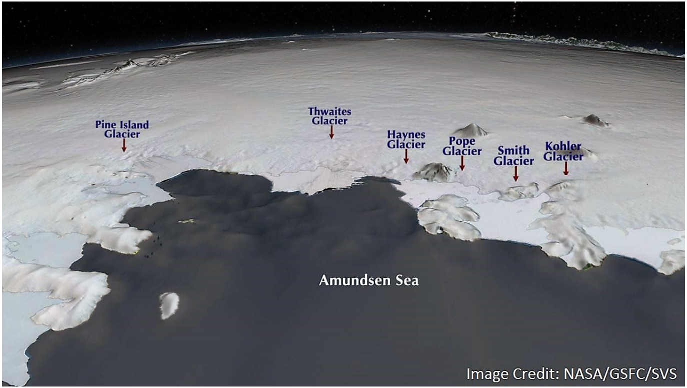
Ice Streams

- Ice sheets: Enormous mass of glacial land ice.
- Antarctic ice sheet is over 14 million km^2.
  - $\approx$ Italy$\times$ 46
  - $\approx$ United States + Mexico combined.
- Melting entire Antarctic ice sheet: sea level rise $\approx$ 57 m.
- Even a modest contribution to sea level rise can have a major impact, e.g. low-lying areas, areas prone to storm surges.
- **Ice stream**:
  - Corridors of fast flow within an ice sheet
  - Discharge most of the ice and sediment from the ice sheets
  - Flow orders of magnitude faster than their surrounding ice.
  - Their behaviour and stability is important to overall ice sheet dynamics and mass balance.
    (cf. Bennett, 2003)
- This talk will focus on ice streams.

The West Antarctic Ice Sheet



Talk Preview

- The West Antarctic Ice Sheet is drained by fast-flowing ice streams
- These are major contributors to ice loss
- Key components for understanding ice stream's stability and dynamics: ice thickness and bedrock topography.

Of interest:

- Interpolate ice thickness while obeying the underlying physics
- Estimate unknown quantities in a mathematical model that we use to understand ice stream dynamics.

Proposal:

- Bayesian approach that incorporates information from multiple data sets combined with a mathematical model that describes the physics of the ice stream.
- Focus on Thwaites glacier, covers an area of 182,000 km^2 ($\approx$ Italy/2). Estimated ice loss has doubled since the 1990s.
- This is preliminary work toward developing more widely applicable methodology.

Ice Sheets Versus Ice Streams

- Our research group has worked on physical models for the West Antarctic ice sheet based on Pollard and De Conto (2009, 2011).
  - Treat the physics like a **black box** and estimate parameters. Computer model emulation-calibration for high-dimensional binary spatial fields. Series of papers: Chang, Haran, Pollard, Applegate (2016a, 2016b, 2017)
- With ice streams: we work **directly** with much simpler model. Are interested in parameter estimation as well as interpolation of ice thickness/bedrock topography.

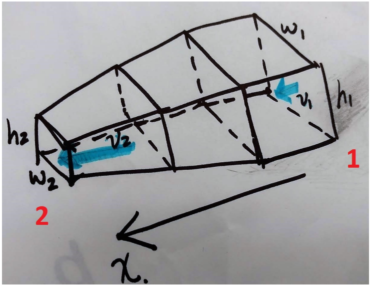
Basic Idea

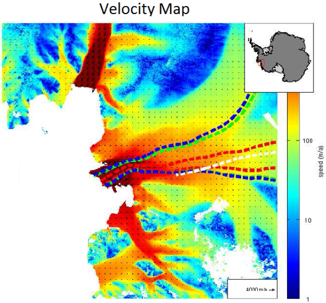
- Principal physics commonly applied is conservation of ice mass
- Given observations of surface elevation, ice velocity and surface mass balance, we can deduce the ice thickness assuming the ice sheet is in a steady state. Examples:
  - Using coarse grid in Antarctica (Warner and Budd, 2000)
  - Higher resolution in Greenland (Morlighem et al., 2011, 2013, 2014)
- Here we add to the physics by
  1. Adding a new component to dynamics model: shallow ice approximation (SIA).
  2. Including the varying glacier width to account for tributaries, which contribute to mass flux
- Model is still simple enough that we can solve it quickly.
- Our statistical model accounts for errors/uncertainties.

Mathematical Flowline Model

- Model ice as an incompressible material (constant density)
- Flux: the action or process of flowing or flowing out
  =Velocity Field ($\bar{V}$) $\times$ Surface Area ($H\omega$)
- Mass conservation along the flowline
  $\implies$ Flux at 1 - Flux at 2 = 0
- For an open rectangular channel: snow accumulation

$$\frac{\partial(\bar{V}_x H_x \omega_x)}{\partial x} = a_x \omega_x$$



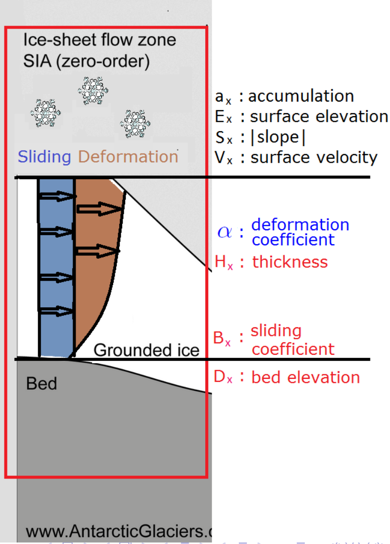


Flowline Model Adjustment

- Depth-averaged velocity can not be observed
- We use surface velocity with adjustment as an approximation
- Adjustment accounts for ice deformation based on the Shallow Ice Approximation

$$\frac{\partial}{\partial x}\left(\overbrace{\left(V_x - \boxed{\frac{\alpha}{20}\left(\rho g|S_x|\right)^3 H_x^4}\right)}^{U_x} H_x\omega_x\right)$$

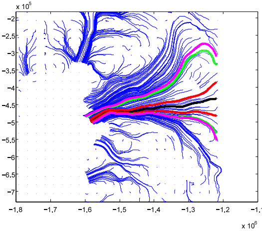
$$= a_x\omega_x$$

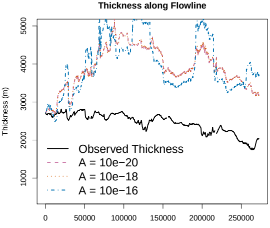


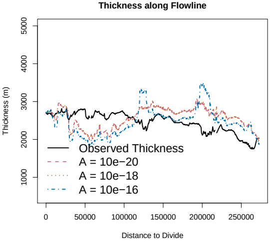
Non-Statistical Approach

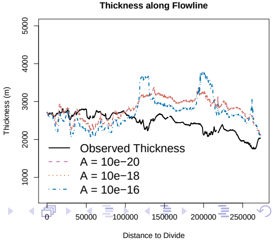
For each flow width, we can predict thickness according to the flowline model.

Finite differences + solving fifth order polynomials.









Uncertainties in Modeling an Ice Stream

- Spatially sparse thickness observations, $H_{x_i}^{obs}, i = 1, \ldots, m$, subject to observational errors
  - Model errors are normally distributed
- Unknown quantities in the mathematical model (**parameters**)
  - Deformation coefficient α
  - Flow width $\boldsymbol{\omega}_x$, modeled with a latent Gaussian Process
- Discrepancy between mathematical model and true underlying process
- Observational errors in the input processes (surface velocity, surface slope and snow accumulation rate)

Hierarchical Bayesian Model

Goal: predict thickness and inference for deformation coefficient α

Nuisance flow width $\boldsymbol{\omega}_x$ needed for thickness prediction

- Ice Thickness Model:
  - observations model: $\boldsymbol{H}^{\text{obs}} \mid \boldsymbol{h}, \boldsymbol{\theta} \sim N(\boldsymbol{h}, \sigma_H^2 \boldsymbol{I})$
  - physics (deterministic) model:
    $h \mid v, s, a, \omega, \boldsymbol{\theta} = M(v, s, a, \omega, \alpha)$
- Flowline Width Model: $\boldsymbol{\omega} \mid \boldsymbol{\theta} \sim \text{GP}(0, C(\boldsymbol{\theta}_\omega))$
- Input Process Model:
  $v, s, a \mid \theta, \boldsymbol{V}^{\text{obs}}, \boldsymbol{S}^{\text{obs}}, \boldsymbol{a}^{\text{obs}}$
  $\sim f(v \mid \boldsymbol{V}^{\text{obs}}, \theta_v) f(s \mid \boldsymbol{S}^{\text{obs}}, \theta_s) f(a \mid \boldsymbol{a}^{\text{obs}}, \theta_a)$
  - velocity model: $f(v \mid \boldsymbol{V}^{\text{obs}}, \theta_v)$
  - slope model: $f(s \mid \boldsymbol{S}^{\text{obs}}, \theta_s)$
  - accumulation rate model: $f(a \mid \boldsymbol{a}^{\text{obs}}, \theta_a)$
- Prior: $p(\boldsymbol{\theta}) = p(\sigma_H^2) p(\alpha) p(\boldsymbol{\theta}_\omega) p(\theta_v) p(\theta_s) p(\theta_a)$

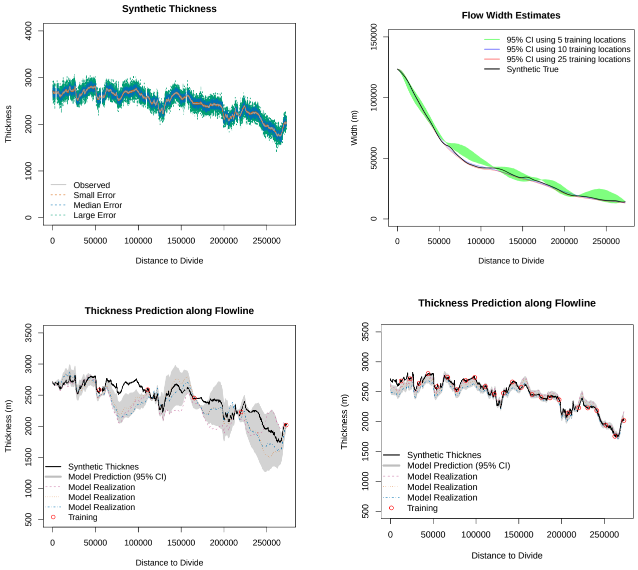
	Thickness	Velocity	Slope	Accum.rate
Observation:	$\boldsymbol{H}^{obs}$	$\boldsymbol{V}^{obs}$	$\boldsymbol{S}^{obs}$	$\boldsymbol{a}^{obs}$
True processes:	h	v	s	a

Simulated Example

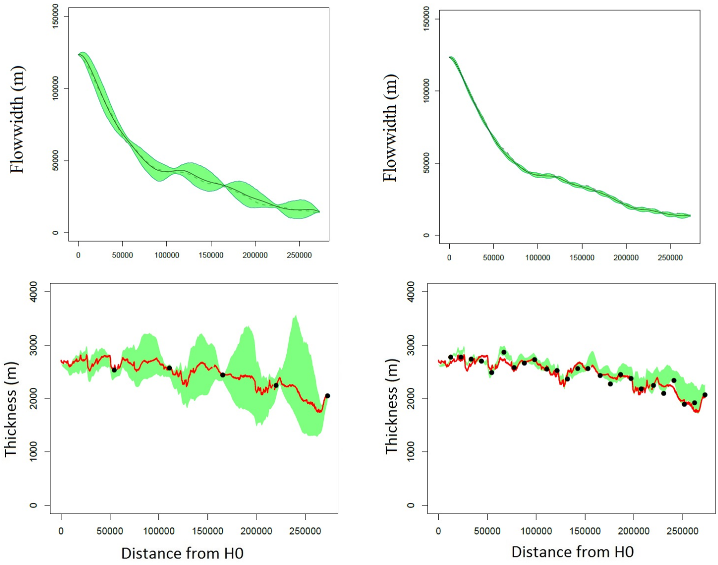
data sets

- Simulate synthetic data using the followings:
  - $A = 10^{-17}$
  - Smooth surface elevation, accumulation rate
  - Observed flowwidth
  - Observed ice thickness
- Simulate velocity from flowline model
- Add small noise to observed ice thickness to create synthetic data

Simulation Results



Thwaites Glacier

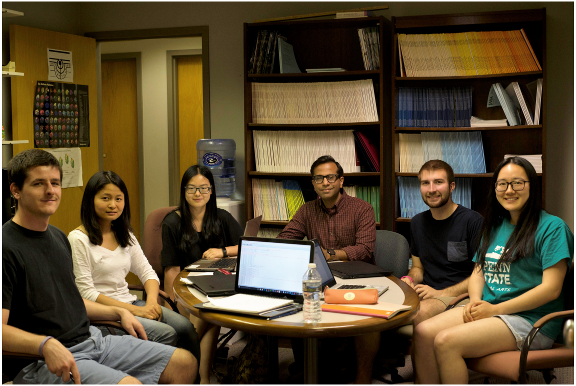


Conclusions

- Our approach can potentially be used for other glaciers in Antarctica with:
  - reliable surface data
  - sparse thickness observation
- Bayesian methods and computational tools allow us to combine:
  - multiple data sets
  - glacier dynamics model
  - estimate thickness while accounting for uncertainties
- Caveats:
  - Unable to account for known errors in the input processes
  - Model discrepancy is absent in hierarchical model

Manuscript to appear in Environmetrics, draft available online
`https://arxiv.org/abs/1612.01454`

Acknowledgments



Research group consisting of grads and undergraduates
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Ice Sheet Data

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