- Read Sections 4.6 through 4.8 (both editions)
- HW #3 should go up later today; it will be due Wednesday, Feb. 8
- Solutions for HW #2 use Sweave (source file is provided)
- I will not be in class on:
  Friday, Feb. 10; Wednesday, Mar. 14; Monday, Mar. 12 (probably)

Notes: I highly recommend checking out Sweave! To try it, see if you can convert the .Rnw file for solution set #2 to a pdf.

State i is recurrent if and only if, conditional on $X_0 = i$,

- $P(\text{ever revisiting } i) = 1$
- $E(\#\{T > 0 : X_T = i\}) = \infty$.
- $\sum_{n=1}^{\infty} P_{ii}^n = \infty$.

Notes: This is a slide from a previous lecture. We'll use the third fact in the following proof.

Theorem: Recurrence (or transience) is a class property.
How do we prove this?

Notes: This is Corollary 4.2, and we followed the proof given in the textbook quite closely. It is instructive to walk through the steps, since none of them is too difficult.

Theorem: All states of a finite, irreducible Markov chain are recurrent.
How do we prove this?

Notes: First, irreducibility means only a single communicating class, which means we must merely show that a single state is recurrent (since recurrence is a class property). If not, then every state is transient, which means that with probability one, every state has a finite "last time visited". But this is impossible, since this means that the entire chain has a "last time visited" equal to the maximum of the last times for every state.

In a Markov chain, is impossible to move:

A from a transient state to a transient state

B from a recurrent state to a transient state

C from a transient state to a recurrent state

D from a recurrent state to a recurrent state

Notes: Correct answer: B. This is good practice thinking about some of the properties of Markov chain states. As part of our discussion of this question, we noted that once you enter a recurrent class, you never leave it. (This was proved in homework #2.) Also, it may be possible to move from one transient class to another, but then it is never possible to move back.

Suppose you have \$2 and you bet on fair games of chance until you either go broke or have \$5.

- What is the expected number of time steps that you have \$2?
- What is the probability that you will at some point have \$1?

Notes: These are interesting questions that are answered in Section 4.6. Because we have time, and because it gives us some more practice with important concepts like recurrence, transience, and conditioning, we will explore them.

Let $s_{ij} \stackrel{\text{def}}{=} E(\#\{T : X_T = i \mid X_0 = j\})$

- For which i and j is s_{ij} meaningful?
- Let us show $S = I + P_T S$ for correctly defined S and P_T.

Notes: We argued that the answer to the first question is "only the transient states". The second equation is proven by a conditioning argument, and it leads to a simple-to-solve matrix equation that will lead us to the matrix S, which immediately answers questions like "What is the expected number of time steps that you have \$2?"