CHAPTER 15
EXERCISE SOLUTIONS

15.1 **a.** Appropriate. Both variables are categorical.
b. Not appropriate. Both variables are quantitative.
c. Not appropriate. Amount willing to spend is quantitative.
d. Appropriate. Both variables are categorical.

15.2 **a.** *p*-value = 0.05
b. 0.10 < *p*-value <0.25 (based on Table A.5). With Excel, *p*-value = CHIDIST(6.7,4) = 0.1526.
c. *p*-value < 0.001 (based on Table A.5). With Excel, CHIDIST(26.23,2) = 0.000002.
d. *p*-value > 0.50 (based on Table A.5). With Excel, CHIDIST(2.28,9) = 0.986

15.3 **a.** 3.84
b. 16.81 [df =(3–1) (4–1) =6]
c. 12.59

15.4 **a.** No, χ^2 = 2.89 < 3.84 (the critical value from Table A.5).
b. Yes, χ^2 = 5.00 > 3.84 (the critical value from Table A.5).
c. Yes, χ^2 = 23.60 > 9.49 (the critical value from Table A.5).
d. No, χ^2 = 23.60 < 25.00 (the critical value from Table A.5).

15.5 **a.** No, χ^2 = 2.89 < 6.63.
b. No, χ^2 = 5.00 < 6.63.
c. Yes, χ^2 = 23.60 > 13.28.
d. No, χ^2 = 23.60 < 30.58.

15.6 **a.** There are 3 equivalent ways to do this. “The relationship between birth order and activity preference is not statistically significant.” or “The proportion of first born or only children who prefer indoor activities is not significantly different from the proportion of non-first born children who do so.” or “There is insufficient evidence to conclude that there is a relationship in the population between birth order and activity preference.”
b. There are 3 equivalent ways to do this. “There is a statistically significant relationship between birth order and activity preference.” or “The proportion of first born or only children in the population who prefer indoor activities is not the same as the proportion of non-first born children who do so.” or “First born or only children have significantly different preferences for outdoor versus indoor activities than non-first born children.”

15.7 **a.** H_0: Gender and type of class taken are not related for the population of students.
H_a: Gender and type of class taken are related for the population of students.
b. df=1.
c. 1.258
d. *p*-value = 0.262. Using Table A.5, p-value is between 0.25 and 0.50.
e. Do not reject the null hypothesis for α=0.05. The relationship between gender and type of class taken is not statistically significant.

15.8 **a.** For females, 90.9% said “most times or always” and 9.1% said “rarely or never.” For males, 78.4% said “most times or always” and 21.6% said “rarely or never.” Females were more likely to wear a seatbelt.
b. H_0: Gender and seatbelt use are independent for the population of 12^{th} graders.
H_a: Gender and seatbelt use are related for the population of 12^{th} graders.

15.8 continued:

c. Expected Count = $\dfrac{\text{Row Total} \times \text{Column Total}}{\text{Total } n} = \dfrac{1178 \times 1888}{2239} = 993.33.$

d. Female, Most times or always: 1888 – 993.33 = 894.67; Male, Rarely or never: 1178 – 993.33 = 184.67; Female, Rarely or never: 1061 – 894.67 = 166.33.

e. $\chi^2 =$

$$\frac{(964 - 894.67)^2}{894.67} + \frac{(97 - 166.33)^2}{166.33} +$$

$$\frac{(924 - 993.33)^2}{993.33} + \frac{(254 - 184.67)^2}{184.67} = 65.14$$

f. The p-value is essentially 0. With Excel, *p*-value = CHIDIST(65.14,1) = 6.98×10^{16}.

g. Using any reasonable á the conclusion is the same. The null hypothesis can be rejected. There is a statistically significant relationship between gender and seat belt use.

15.9 **a.** $c^2 = \dfrac{N(AD - BC)^2}{R_1 R_2 C_1 C_2} = \dfrac{2239(964 \times 254 - 97 \times 924)^2}{1061 \times 1178 \times 1888 \times 351} = 65.137$

b. Degrees of freedom = (2 1) × (2 1) = 1.

c. The p-value is essentially 0. With Excel, *p*-value = CHIDIST(65.137,1) = 6.98×10^{16}.

d. Using any reasonable á the conclusion is the same. The null hypothesis can be rejected. There is a statistically significant relationship between gender and seat belt use.

15.10 **a.** No, there is no obvious choice. Both variables are opinions and neither precedes the other.

b. Row percents: Of those in favor of the death penalty, 21.3% think marijuana should be legal and 78.7% think it should not be legal. Of those opposed to the death penalty, 27.7% think marijuana should be legal and 72.3% think it should not be legal.

Column percents: Of those thinking marijuana should be legal, 71.4% favor the death penalty and 28.6% are opposed. Of those thinking marijuana should not be legal, 77.9% favor the death penalty and 21.1% are opposed.

Both sets of conditional percents are evidence of a possible relationship. For instance, the percent favoring legalization of marijuana is higher for those opposed to the death penalty than it is for those in favor.

b. H_0: Opinion about death penalty and opinion about marijuana legalization are not related.
H_a: Opinion about death penalty and opinion about marijuana legalization are related.

c. Minitab output is below (expected counts are beneath observed counts in each cell). χ^2 =3.92, df = 1, *p*-value = 0.048. If Table A.5 is used, 0.025 < *p*-value < 0.05.
Reject the null hypothesis. Conclude that in the population represented by the sample, there is a relationship between opinion about the death penalty and opinion about marijuana legalization.

Output for Exercise 15.10c

```
          Legal NotLegal      All
Favor       152      561      713
         162.77   550.23   713.00

Oppose       61      159      220
          50.23   169.77   220.00

All         213      720      933
         213.00   720.00   933.00

Chi-Square = 3.920, DF = 1, P-Value = 0.048
```

15.10 continued:

d. $\chi^2 = \dfrac{N(AD-BC)^2}{R_1 R_2 C_1 C_2} = \dfrac{933(152 \times 159 - 561 \times 61)^2}{713 \times 220 \times 213 \times 720} = 3.92$.

15.11 **a.** Either row percents or column percents (or both) can be considered.

Row percents: *Males*: Of males in favor of the death penalty, 25.2% think marijuana should be legal and 74.8% think it should not be legal. Of males opposed to the death penalty, 42.3% think marijuana should be legal and 57.7% think it should not be legal.

Females: Of females in favor of the death penalty, 18.2% think marijuana should be legal and 81.8% think it should not be legal. Of females opposed to the death penalty, 20.8% think marijuana should be legal and 79.2% think it should not be legal.

Column percents: *Males*: Of males thinking marijuana should be legal, 72.7% favors the death penalty and 27.3% are opposed. Of males thinking marijuana should not be legal, 85.3% favors the death penalty and 14.7% are opposed.

Females: Of females thinking marijuana should be legal, 69.9% favors the death penalty and 30.1% are opposed. Of those thinking marijuana should not be legal, 73.2% favors the death penalty and 26.8% are opposed.

Interpretation: The relationship between the two variables is stronger for males than it is for females. For example, row percents differ more for males than for females.

b. Minitab output is below (expected counts are beneath observed counts in each cell).

$\chi^2 = 8.365$, df = 1, p-value = 0.004. If Table A.5 is used, $0.001 < p\text{-value} < 0.005$.

Reject the null hypothesis. Conclude that in the population of men represented by the sample, there is a relationship between opinion about the death penalty and opinion about marijuana legalization.

Output for Exercise 15.11b

```
            Legal NotLegal     All

 Favor         80      238     318
            89.92   228.08  318.00

 Oppose        30       41      71
            20.08    50.92   71.00

 All          110      279     389
           110.00   279.00  389.00

Chi-Square = 8.365, DF = 1, P-Value = 0.004
```

c. Minitab output is below (expected counts are beneath observed counts in each cell).

$\chi^2 = 0.468$, df = 1, p-value = 0.494. If Table A.5 is used, $0.25 < p\text{-value} < 0.50$.

Do not reject the null hypothesis. Conclude that for the population of women represented by the sample, there is not evidence of a relationship between opinion about the death penalty and opinion about marijuana legalization.

Output for Exercise 15.11c

```
            Legal NotLegal     All

 Favor         72      323     395
            74.79   320.21  395.00

 Oppose        31      118     149
            28.21   120.79  149.00

 All          103      441     544
           103.00   441.00  544.00

Chi-Square = 0.468, DF = 1, P-Value = 0.494
```

15.12 **a.** The first sentence should stipulate that the significant relationship was observed for men. The percents given in the second sentence should be changed to 42% and 25%, respectively.
b. The first sentence should say that a relationship was not observed for women. The percents given in the second sentence should be changed to 21% and 18%, respectively.
c. Gender is an interacting variable that affects the strength of the relationship between the two opinions. The statement originally made about significance only applies to men.

15.13 For expected counts, proportion with an ear infection is .7805 within each treatment.
Placebo: 138.93/178 = .7805
Xylitol gum: 139.71/179 = .7805
Xylitol lozenge: 137.37/176 = .7805

15.14 **a.** H_0: Age group and opinion of legal gambling are not related
H_a: Age group and opinion of legal gambling are related
b. We can reject the null hypothesis (in favor of the alternative hypothesis). *p*-value is reported as 0.000. Conclude that age group and opinion of legal gambling are related.
c. Expected Count = $\dfrac{\text{Row Total} \times \text{Column Total}}{\text{Total } n} = \dfrac{501 \times 1220}{2024} = 301.99$

15.15 No. The response variable has 3 categories.

15.16 **a.** Fisher's Exact Test. The counts are too small for the other tests.
b. Chi-square test. The alternative hypothesis is two-sided and the counts are large enough.
c. One-sided *z*- test. The alternative hypothesis is one-sided and the counts are large enough.

15.17 **a.** H_0: Seal performance and temperature are unrelated.
H_a: Seal performance depends on temperature.
b. H_0: Population proportions planning to vote for each candidate were the same in September and June.
H_a: Population proportions planning to vote for each candidate are differed in September and June.
c. H_0: Taking a typing class does not reduce the chance of getting carpal tunnel syndrome.
H_a: Taking a typing class does reduce the chance of getting carpal tunnel syndrome.

15.18 **a.** Given that 2 out of 10 participants have reduced pain, what is the probability that both of them would be in the magnet-treated group?
b. The null hypothesis is that there is no relationship between treatment type and pain reduction, while the alterative hypothesis is that the magnet treatment is more likely to reduce pain. The one-tailed p-value, which is appropriate in this case, is .222. Do not reject the null hypothesis.

15.19 Step 1: H_0: Typical grades and seat belt use are not related for the population of 12^{th} graders.
H_a: Typical grades and seat belt use are related for the population of 12^{th} graders.
Step 2: Expected counts are all greater than 5 so proceed with the chi-square test. Minitab output is shown below. Test statistic is χ^2 = 77.776, df = 2.
Steps 3, 4, and 5: *p*-value = 0.000.
Reject the null hypothesis.
Typical grades and seat belt use are related for the population of 12^{th} graders.

Output for Exercise 15.19

```
Expected counts are printed below observed counts

      Most Times  Rarely    Total
A or B     1354      180     1534
       1292.56   241.44

  C         428      125      553
         465.96    87.04

D or F       65       40      105
          88.47    16.53

Total      1847      345     2192

Chi-Sq =  2.920 + 15.634 +
          3.093 + 16.558 +
          6.228 + 33.343 = 77.776
DF = 2, P-Value = 0.000
```

15.20 **a.** The percent passing differs among the colleges. It is lowest for students in the social sciences and highest for engineering students.

College	***% Pass***	***% Fail***
Business	66% (33/50)	34% (17/50)
Language Arts	64% (32/50)	36% (18/50)
Social Sciences	50% (25/50)	50% (25/50)
Natural Sciences	76% (38/50)	24% (12/50)
Engineering	86% (43/50)	14% (7/50)

b. These hypotheses apply to the population of students similar to those asked.
H_0: Student's college and performance on water level task are not related
H_a: Student's college and performance on water level task are related

c. df = $(r–1)(c–1)= (5–1)(2–1) = 4$.

d. Decide in favor of the alternative hypothesis. The conclusion is that student's college and performance on the water level task are related.
$0.001< p$-value <0.005 (based on Table A.5). With Excel, p-value = CHIDIST(16.915,4) = 0.002.

e. Expected counts are the same in every college.

For pass, $\dfrac{\text{Row Total}\times\text{Column Total}}{\text{Total } n} = \dfrac{50\times171}{250} = 34.2$

For fail, $\dfrac{\text{Row Total}\times\text{Column Total}}{\text{Total } n} = \dfrac{50\times29}{250} = 15.8$

f. We're not telling. Ask an engineer.

15.21 **a.** The researchers probably thought, in advance of collecting data, that short students would be more likely to be bullied. So, a one-sided alternative would be appropriate and a z-test for comparing two proportions should be used.

b. Step 1: H_0: $p_1 - p_2 = 0$ (or $p_1 = p_2$) versus Ha: $p_1 - p_2 > 0$ (or $p_1 > p_2$)
p_1 = proportion ever bullied in population of short students
p_2 = proportion ever bullied in population of students not short

15.21 continued:

Step 2: Sample sizes are sufficiently large and we assume the samples represent random samples. The test statistic is $z = 3.02$. Output for comparing two proportions is given below. For "by hand" calculations, use the method described in Section 13.4. Alternatively, a chi-square test can be done and the relationship $z = \sqrt{\chi^2}$ used to determine the z-statistic.

Steps 3, 4, and 5: p-value = 0.001 (reported in output). Reject the null hypothesis, and conclude that a higher proportion of short students than non-short students have been bullied.

Output for Exercise 15.21

```
Sample      X      N  Sample p
1          42     92  0.456522  (short)
2          30    117  0.256410  (not short)

Estimate for p(1) - p(2):  0.200111
95% lower bound for p(1) - p(2):  0.0919200
Test for p(1) - p(2) = 0 (vs > 0):  Z = 3.02  P-Value = 0.001
```

c. For short students, the 95% C.I. is about .36 to 56. For students not short, the C.I is about .18 to .35. To do "by hand" calculations use $\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Short students: $\hat{p} = 0.457$ and n = 92. Not short: $\hat{p} = 0.256$ and $n = 117$. The confidence intervals do not overlap, further confirming the result in part (b) that the population proportion is higher for the short students.

15.22 **a.** 100 for each of the 3 categories, calculated as 300(1/3) = 100.
b. 250, 250 and 500, respectively, calculated as 1000(1/4), 1000(1/4) and 1000(1/2).
c. 400, 800, 200 and 600, respectively, calculated as 2000(.2), 2000(.4), 2000(.1) and 2000(.3).
d. 500 for each of the 5 categories, calculated as 2500(1/5) = 500.

15.23 No. The expected counts must sum to the same total as the observed counts.

15.24 **a.** No. A chi-square statistic is a sum of non-negative numbers so it would have to be greater than or equal to 0.
b. Yes. This would happen if the observed count equaled the expected count in each cell of the table.
c. Yes. For example, see the solution for exercise 15.28.
d. No. The observed counts will always be whole numbers because they are actual counts.
e. No. The null hypothesis gives hypothesized probabilities for all possible categories so the sum of probabilities must be 1.
f. No. For a goodness of fit test, df = number of categories – 1.

15.25 **a.** H_0: p = $\frac{1}{10}$ for each of the 10 numbers, where p = the probability of a student choosing that number. The alternative hypothesis is that not all of the probabilities are $\frac{1}{10}$.

b. The calculations are as follows, using the expected count of 19 for each category:

$$c^2 = \frac{(2-19)^2}{19} + \frac{(9-19)^2}{19} + \frac{(22-19)^2}{19} + \frac{(21-19)^2}{19} + \frac{(18-19)^2}{19} +$$
$$\frac{(23-19)^2}{19} + \frac{(56-19)^2}{19} + \frac{(19-19)^2}{19} + \frac{(14-19)^2}{19} + \frac{(6-19)^2}{19} = 104.32.$$

c. Degrees of freedom = k – 1 = 10 – 1 = 9.
d. The p-value is essentially 0. From Table A.5, p-value < 0.001 because 104.32 > 27.88.
e. Using any reasonable level of significance the conclusion is the same. At least two of the probabilities are not 1/10. Students are not equally likely to choose each of the digits.

15.26 **a.** H_0: p_1=.3, p_2=.6, p_3=.1 where p_1, p_2 and p_3 are the population proportions who drove, biked and used another way that day, respectively.
b. Expected counts are 300(.3) = 90 drove, 300(.6) = 180 biked, 300(.1) = 30 used another way.
c. $\chi^2 = \frac{(80-90)^2}{90} + \frac{(200-180)^2}{180} + \frac{(20-30)^2}{30} = 6.67$
d. With Excel, p-value = CHIDIST(6.67,2) = 0.0356. From Table A.5, 0.025 < p-value < 0.05. Note that degrees of freedom = 3 – 1 = 2.
e. Reject the null hypothesis for á = 0.05. Conclude that at least two of the proportions using these modes of transportation differed from the norm on the "spare the air" day.

15.27 Step 1: H_0: $p_1 = p_2 = p_3 = \frac{1}{3}$ versus H_a: not all $p_i = \frac{1}{3}$

Step 2: Expected counts = $111 \times \frac{1}{3} = 37$ for all three colors. All expected counts are greater than 5, so we can proceed with the chi-square test.

$\chi^2 = \frac{(59-37)^2}{37} + \frac{(27-37)^2}{37} + \frac{(25-37)^2}{37} = 19.676$; df = k–1= 3–1= 2

Steps 3, 4, and 5: p-value < 0.001 (based on Table A.5).
With Excel, p-value = CHIDIST(19.676,2) = 0.00005.
Reject the null hypothesis. Conclude that in the population represented by the sample, the three colors are not equally preferred.

15.28 Step 1: H_0: $p_1 = .5$, $p_2 = .3$, $p_3 = .2$ (manufacturer's hypothesis)
H_a: not all p_i are as specified in H_0

Step 2: Expected counts: Silver, 111×.5 = 55.5; Blue, 111×.3 = 33.3; Green, 111×.2 = 22.2.
All expected counts are greater than 5 so proceed with the chi-square test.

Test statistic is $\chi^2 = \frac{(59-55.5)^2}{55.5} + \frac{(27-33.3)^2}{33.3} + \frac{(25-22.2)^2}{22.2} = 1.766$; df = k–1= 3–1= 2

Steps 3, 4, and 5: 0.25 < p-value < 0.50 (based on Table A.5).
With Excel, p-value = CHIDIST(1.766,2) = 0.4135.
Do not reject the null hypothesis. Based on this sample, the manufacturer's hypothesis about color preferences is not rejected.

15.29 **a.** Expected counts = 150× .1 = 15 for all digits.
b. All expected counts are greater than 5 so proceed.

Test statistic is $\chi^2 = \frac{(11-15)^2}{15} + \frac{(21-15)^2}{15} + ... + \frac{(9-15)^2}{15} = 12.667$; df = k – 1 = 10 – 1 = 9

c. Do not reject the null hypothesis. There is not statistically significant evidence against the hypothesis that all digits are equally likely to be the last digit of the forecasted high temperature. From Table A.5, 0.10 < p-value < 0.25. With Excel, p-value = CHIDIST(12.667,9) = 0.178.

15.30 **a.** Possible sequences are HH, TH, HT, TT.
Probabilities are: X = 0, p_0 = 1/4 = .25; X = 1, p_1 = 1/2 = .5; X = 2, p_2 = 1/4 = .25
b. H_0: p_0 =.25, p_1 =.50, p_2 =.25
H_a: not all p_i are as specified in H_0

c. Step 2: Expected counts: X = 0, 60×.25 = 15; X = 1, 60×.5 = 30; X = 2, 60×.25 = 15.
All expected counts are greater than 5 so proceed with the chi-square test.

Test statistic is $\chi^2 = \frac{(8-15)^2}{15} + \frac{(40-30)^2}{30} + \frac{(12-15)^2}{15} = 7.2$; df = k–1 = 3–1 =2.

15.30 continued:

Steps 3, 4, and 5: 0.025< p-value < 0.05 (based in Table A.5). With Excel, p-value = CHIDIST(7.2,2) = 0.027.
Reject the null hypothesis. Conclude that observed student results are significantly different from expected results based on theory.

d. Exactly one head occurred more often than expected. Perhaps some students made up their data, and may have had a tendency to claim they got 1 head in the 2 flips.

15.31 Step 1: H_0: p_i = 1/10 = .1 for all digits (equal chance for all digits)
H_a: not all p_i = 1/10

Step 2: Expected counts = 600 ×.1 = 60 for all ten digits. All expected counts are greater than 5, so proceed with the chi-square test.

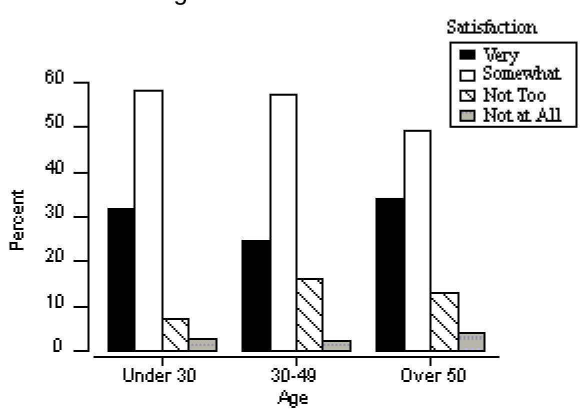
Test statistic is $\chi^2 = \frac{(49-60)^2}{60} + \frac{(61-60)^2}{60} + ... + \frac{(63-60)^2}{60}$ = 5.03 ; df = k–1 = 10–1= 9

Steps 3, 4, and 5: p-value > 0.50 (based on Table A.5).
With Excel, p-value = CHIDIST(5.03,9) = 0.832.
Do not reject the null hypothesis. There is not statistically significant evidence against the hypothesis that all digits are equally likely to be selected.

15.32 **a.** Conditional percents within age groups and a bar chart follow. The age groups are not markedly different. Women in the youngest age group are somewhat less likely to be dissatisfied with their appearance.

	Satisfaction with Appearance			
Age	Very	Somewhat	Not Too	Not at All
Under 30	31.9%	58.2%	7.1%	2.8%
30 – 49	24.8%	57.1%	16.0%	2.0%
Over 50	34.0%	49.0%	13.1%	3.9%

Figure for Exercise 15.32a

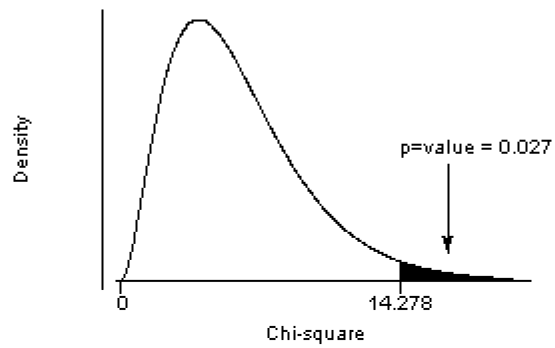


b. df = (r–1)(c–1) = (3–1)(4–1) = 6

c. Yes, the relationship is statistically significant (p-value is reported as 0.027).

d. The p-value is the area to the right of 14.278 in a chi-square distribution with df = 6. The density curve is similar to the one for df = 5 shown in Figure 15.3 on page 534. The p-value represents the probability of observing a relationship as strong or stronger than the one observed in the sample, if there really is no relationship in the population.

Figure for Exercise 15.32d



e. Expected Count = $\dfrac{\text{Row Total} \times \text{Column Total}}{\text{Total n}} = \dfrac{312 \times 98}{747} = 40.93$

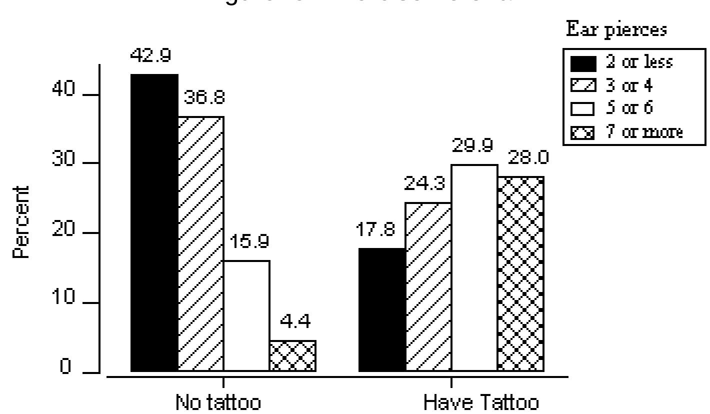
15.33 **a.** The highest value is 3.904, for the cell "18-29" and "Not Too" and the next highest value is 2.607 for the cell "30-49" and "Very."
b. The expected count is higher in both cases. In the first case, the expected count is 18.50 and the observed count is 10. In the second case, the expected count is 88.16 and the observed count is 73.
c. Fewer young women (18-29) than expected were unhappy (not too satisfied) with their appearance. Fewer middle-aged women (30-49) than expected were very satisfied with their appearance. The term "than expected" in this context means "than would be expected if there were no relationship between age and satisfaction."

15.34 **a.** There appears to be a relationship based on the following percents: As the number of ear pierces increases, the percent with a tattoo also increases.

Pierces	**% with Tattoo**
2 or less	7.2% (19/264)
3 or 4	11.0% (26/236)
5 or 6	26.0% (32/123)
7 or more	54.5% (30/55)

The relationship could also be described using the column percents shown in the following bar graph. In the graph, we see that women who have a tattoo tend to have more ear pierces than women who don't have a tattoo.

Figure for Exercise 15.34a



15.34 continued:

b. H_0: Number of ear pierces and having a tattoo (or not) are not related
H_a: Number of ear pierces and having a tattoo (or not) are related

c. Minitab output is below (expected counts are beneath observed counts in each cell).

$\chi^2 = 90.544$, df = 3, *p*-value = 0.000. If Table A.5 is used, *p*-value < 0.001.

Reject the null hypothesis. Conclude that for the population of college women represented by the sample, there is a relationship between number of ear pierces and having a tattoo or not.

Output for Exercise 15.34c

```
              No   Tattoo     All
 2 or le     245       19     264
          222.34    41.66  264.00

 3 or 4      210       26     236
          198.76    37.24  236.00

 5 or 6       91       32     123
          103.59    19.41  123.00

 7 or mo      25       30      55
           46.32     8.68   55.00

 All         571      107     678
          571.00   107.00  678.00

Chi-Square = 90.544, DF = 3, P-Value = 0.000
```

15.35 **a.** The percent with heart disease increases as the severity of vertex baldness increases.

Hair Pattern	**% with Heart Disease**
No Baldness	6.72% (548/8159)
Frontal Baldness	7.55% (333/4408)
Mild Vertex Baldness	8.04% (275/3423)
Moderate Vertex Baldness	9.20% (163/1771)
Severe Vertex Baldness	9.40% (127/1351)

A chi-square test can be done to assess the statistical significance of the relationship between hair pattern and the likelihood of heart disease for men. The hypotheses are:

H_0: Hair pattern and heart disease are not related
H_a: Hair pattern and heart disease are related

Output follows. The test statistic is $\chi^2 = 22.78$ df = 4, *p*-value < 0.001. Reject the null hypothesis in favor of the alternative. The relationship is statistically significant. There is a relationship between hair pattern and heart disease in the population of me similar to the ones in the study.

Output for Exercise 15.35			
	Yes	No	All
No Baldn	548 617.30	7611 7541.70	8159 8159.00
Frontal	333 333.51	4075 4074.49	4408 4408.00
Mild Ver	275 258.98	3148 3164.02	3423 3423.00
Moderate	163 133.99	1608 1637.01	1771 1771.00
Severe V	127 102.22	1224 1248.78	1351 1351.00
All	1446 1446.00	17666 17666.00	19112 19112.00
Chi-Square = 22.875, DF = 4, P-Value = 0.000			

b. This is an observational study. So, there may be confounding factors related to both baldness and heart disease.

15.36 Step 1: H_0: $p_1 = .3,\ p_2 = .2,\ p_3 = .2,\ p_4 = .1,\ p_5 = .1,\ p_6 = .1$ (manufacturer's claim)

H_a: not all p_i are as specified in H_0

Step 2: Expected counts: Brown: 2081×.3 = 624.3; Red and Yellow, 2081×.2 = 416.2; Blue, Orange and Green, 2081×.1 = 208.1.
All expected counts are greater than 5 so proceed with the chi-square test.

Test statistic is $\chi^2 = \frac{(602-624.3)^2}{624.3} + \frac{(396-416.2)^2}{416.2} + \frac{(379-416.2)^2}{416.2} + \frac{(227-208.1)^2}{208.1} + \frac{(242-208.1)^2}{208.1} + \frac{(235-208.1)^2}{208.1}$

= 15.8; df = k–1= 6 – 1 = 5.

Steps 3, 4, and 5 0.005<p-value<0.01; using Excel, CHIDIST(15.8,5) = 0.007. Reject the null hypothesis and conclude that proportions are not as stated at M&M web site for the population from which these bags were drawn.

15.37 **a.** Sheep performed slightly better. For the Sheep, 70.5% were Stars and 29.5% were Duds. For the Goats, 60% were Stars and 40% were Duds. An appropriate graph is a bar graph displaying these percents.

b. The test should be one-sided because the speculation before collecting the data was that Sheep will perform better than Goats. Therefore the hypotheses are:

H_0: $p_1 - p_2 = 0$ (or $p_1 = p_2$) versus Ha: $p_1 - p_2 > 0$ (or $p_1 > p_2$)

p_1 = proportion of Stars in the population of students who are Sheep
p_2 = proportion of Stars in the population of students who are Goats

c. Step 2: Sample sizes are sufficiently large and we assume the samples represent random samples. The test statistic is z = 1.52. Output for comparing two proportions is given below. For "by hand" calculations, use the method described in Section 13.4. Alternatively, a chi-square test can be done and the relationship $z = \sqrt{\chi^2}$ used to determine the z-statistic. Output for the chi-square test is also shown; note that $z = \sqrt{\chi^2} = \sqrt{2.313} = 1.52$.

15.37 continued:

Steps 3, 4, and 5: p-value = 0.064 (reported in output). Do not reject the null hypothesis. Based on these data it cannot be concluded that the proportions of Stars differ in the populations of Sheep and Goats.

Output for Exercise 15.37

```
Test of two proportions:
Sample       X      N  Sample p
1           79    112  0.705357
2           48     80  0.600000

Estimate for p(1) - p(2):  0.105357
95% lower bound for p(1) - p(2):  -0.00925990
Test for p(1) - p(2) = 0 (vs > 0):  Z = 1.52  P-Value = 0.064
```

```
Chi-square test:
           Stars     Duds    Total
Sheep        79       33      112
          74.08    37.92

Goats        48       32       80    Chi-Sq =  0.326 +  0.638 +
          52.92    27.08                        0.457 +  0.893 = 2.313
                                            DF = 1, P-Value = 0.128
Total       127       65      192
```

15.38 **a.** n = 10, p =.5.

b. The following probabilities for a binomial distribution with n = 10, p =.5 were found using Minitab.

Number Correct	Probability
0	0.000977
1	0.009766
2	0.043945
3	0.117188
4	0.205078
5	0.246094
6	0.205078
7	0.117188
8	0.043945
9	0.009766
10	0.000977

c. Use the table in part b; combining 0 to 2 and 8 to 10. Expected counts are shown for use in Parts (d) and (e), and their calculation is explained in answers for those parts.

Number Correct	Sheep	Goats	Null probability	Expected (Sheep)	Expected (Goats)
≤ 2	6	5	0.054688	6.1251	4.3750
3	11	12	0.117188	13.1250	9.3750
4	16	15	0.205078	22.9688	16.4063
5	29	19	0.246094	27.5625	19.6875
6	28	16	0.205078	22.9688	16.4063
7	14	10	0.117188	13.1250	9.3750
≥ 8	8	3	0.054688	6.1251	4.3750

15.38 continued:

d. Step 1: H_0: The probabilities for Sheep are as specified under "Null probability" in part (c)
H_a: The probabilities are not all as specified under "Null probability" in part (c)
Step 2: Expected counts are all greater than 5, so proceed with the test. Total number of Sheep is 112, so expected counts are found by multiplying the "Null probability" by 112. See table in Part(c). $\chi^2 = 4.27$, df = 6.
Steps 3, 4 and 5: p-value =0.64 (from Excel, CHIDIST(4.27,6)), from Table A.5, p-value > .50. Do not reject the null hypothesis. These data are not inconsistent with the null probabilities derived from the binomial distribution with success probability .5.

e. Step 1: H_0: The probabilities for Goats are as specified under "Null probability" in part (c)
H_a: The probabilities are not all as specified under "Null probability" in part (c)
Step 2: Expected counts do not quite fit the relevant criteria; only 5/7 or 71% are greater than 5 (should be at least 80%), but the ones that are less than 5 are just slightly less, so proceed with the test. Total number of Goats is 80, so expected counts are found by multiplying the "Null probability" by 80. See table in Part(c). $\chi^2 = 1.45$, df = 6.
Steps 3, 4 and 5: p-value = 0.96 (from Excel, CHIDIST(1.45,6)), from Table A.5, p-value > .50. Do not reject the null hypothesis. These data are not inconsistent with the null probabilities derived from the binomial distribution with success probability .5. In fact, notice that observed counts are so close to the expected counts that a chi-square statistics as small or smaller than that obtained would occur only about 4% of the time.

15.39 **a.** H_0: p_1= 0.257, p_2=0.322, p_3=0.169, p_4=0.149, p_5=0.103 (U.S. proportions)
H_a: probabilities are not all as specified in the null hypothesis

b. Step 1: Hypotheses given in part (a)
Step 2: χ^2 =10.6, df = 5–1 = 4
Expected counts, calculated as 2904×null p_i are:

Household Size	1	2	3	4	5
Expected	746.328	935.088	490.776	432.696	299.112

Steps 3,4, and 5: p-value = 0.031, from Excel CHIDIST(10.6), or with Table A.5, $0.025<p$-value<0.05. Reject the null hypothes is. Conclude that the observed distribution of household sizes in the GSS survey is inconsistent with the U.S. distribution.
Note: This exercise illustrates the effect of a large sample size on a significance test. Notice that the sample proportions do not differ from the U.S. proportions by very much but due to the large sample size the difference achieves statistical significance.

15.40 **a.** The conditions are met. In particular, only one of the expected counts for the twenty cells is less than 5. Expected counts are shown beneath observed counts in the output for part (b). Also, it is assumed the GSS sample represents a random sample from the population of U.S. adults.

b. Step 1: H_0: Religion and opinion about premarital sex are not related
H_a: Religion and opinion about premarital sex are related
Step 2: χ^2 =157, df = (5–1)(4–1) = 12.
Steps 3,4, and 5: p-value ≈ 0. Reject the null hypothesis. Conclusion is that religion and opinion about premarital sex are related variables. Row percents should be examined to determine how the religions differ.

Output for Exercise 15.40b

```
         Almost A   Always    Never Sometime      All

Catholic       37       62      226      120      445
            42.97   106.14   195.14   100.74   445.00

Jewish          3        0       34       14       51
             4.93    12.16    22.36    11.55    51.00

None           13       20      147       45      225
            21.73    53.67    98.67    50.94   225.00

Other          13       15       40       23       91
             8.79    21.71    39.91    20.60    91.00

Protesta      117      355      384      227     1083
           104.59   258.32   474.92   245.18  1083.00

All           183      452      831      429     1895
           183.00   452.00   831.00   429.00  1895.00

Chi-Square = 157.017, DF = 12, P-Value = 0.000
```

1 cells with expected counts less than 5.0

15.41 **a.** The contingency table is

	Yes	No	Total
Women	80	51	131
Men	27	36	63
Total	107	87	194

b. A chi-square test could be done for

H_0: Gender and response to the question are not related

H_a: Gender and response to the question are related

Note: A one-sided alternative would be appropriate if you think in advance of seeing the data that women will be more likely to say "yes." If so, a *z*-test should be done for

H_0: $p_1 - p_2 = 0$ (or $p_1 = p_2$)

Ha: $p_1 - p_2 > 0$ (or $p_1 > p_2$)

p_1 = proportion who would say "yes" in population of college women

p_2 = proportion who would say "yes" in population of college men

c. There is no indication in the problem statement that this test should be one-sided, so it doesn't matter if a chi-square test or a two-tailed *z*-test is done. (However, see the Note in Part (b).)

d. Step 1: Possible hypotheses are given in part (b).

Step 2: Sample sizes are sufficiently large to proceed (with either test). Assume the sample represents a larger population of college students.

For a chi-square test, $\chi^2 = 5.704$; df = (2–1)(2–1) =1

For a *z*-test of the one-sided alternative, $z = \sqrt{\chi^2} = \sqrt{5.704} = 2.39$. Equivalently, the method of Section 13.4 could be used the find this z-statistic.

Steps 3, 4, and 5: For the chi-square test, $0.01 < p$-value < 0.025 (based on Table A.5). With Excel, *p*-value = CHIDIST(5.704,1) = 0.017. For the *z*-test, *p*-value = $P(z > 2.39) = P(z < -2.39)$= 0.0087 (from Table A.1). In either case, the null hypothesis is rejected in favor of the alternative. Conclude that there is a relationship between gender and response; women are more likely to say "yes" in the population of college students.

15.42 $\chi^2 = \dfrac{N(AD-BC)^2}{R_1R_2C_1C_2} = \dfrac{194(80\times 36 - 51\times 27)^2}{131\times 63\times 107\times 87} = 5.704$

15.43 **a.** H_0: Wearing glasses or not is not related to whether or not boy is juvenile delinquent
H_a: Wearing glasses or not is related to whether or not boy is juvenile delinquent
b. The sample sizes are too small. In particular, in three of the four cells the expected counts are less than 5. The expected counts are:

.

	Wears Glasses	No Glasses
Delinquent	$\dfrac{9\times 6}{16} = 3.375$	9–3.375 =5.625
Nondelinquent	6–3.375 = 2.625	7–2.625 = 4.375

c. *p*-value =0.0349 for a two-sided alternative with the Fisher test. We used a calculator at http://www.matforsk.no/ola/fisher.htm. Reject the null hypothesis and conclude that the wearing glasses or not is related to whether a not a boy is a juvenile delinquent. Notice that delinquents are less likely to wear glasses.
d. The p-value is the probability of observing a difference as large or larger (in either direction) as the one observed if there is no difference in the population. More specifically, note that the difference in proportions wearing glasses is (5/7) – (1/9) = .714–.111=.603. The *p*-value is the probability that the absolute difference in proportions could be .603 or more if there is no difference in the population proportions.
Note: The p-value is the sum of probabilities based on the hypergeometric probability distribution for possible tables with the same row and column totals as observed and with an absolute difference in observed proportions at least as large as observed.

15.44 **a.** Use the data in the first two columns, under "Identified Male Voice?"
The test statistic is χ^2 = 0.267, df = (2–1)(2–1) =1, *p*-value = 0.605.
With Table A.5, *p*-value > 0.50.
Do not reject the null hypothesis. There is not a statistically significant relationship between sex of listener and the ability to identify a male voice. In the following output, expected counts are beneath observed counts.

Output for Exercise 15.44a

```
              Yes       No      All
Male          145       207     352
            148.44    203.56

Female        162       214     376
            158.56    217.44

All           307       421     728

Chi-Sq =   0.080 +   0.058 +
           0.075 +   0.054 = 0.267
DF = 1, P-Value = 0.605
```

b. Use the data in the last two columns, under "Identified Female Voice?"
The test statistic is χ^2 = 13.025, df = (2–1)(2–1) =1, *p*-value = 0.000.
With Table A.5, *p*-value < 0.001.
Reject the null hypothesis. There is a statistically significant relationship between sex of listener and the ability to identify a female voice. In the following output, expected counts are beneath observed counts.

Output for Exercise 15.44b

```
            Yes      No      All
Male        132      220     352
          156.18   195.82

Female      191      185     376
          166.82   209.18

All         323      405     728

Chi-Sq =  3.742 +  2.985 +
          3.504 +  2.794 = 13.025
DF = 1, P-Value = 0.000
```

c. For male listener use the counts under "Identified Male Voice?" (145 and 207).
For female listener use the counts under "Identified Female Voice?" (191 and 185).
The test statistic is $\chi^2 = 6.748$, df = (2–1)(2–1) =1, p-value = 0.009.
With Table A.5, $0.005 < p$-value < 0.01.
Reject the null hypothesis. There is a statistically significant relationship between sex of listener and the ability to identify a voice of the same sex. In the following output, expected counts are beneath observed counts.

Output for Exercise 15.44c

```
            Yes       No      All
Male        145       207     352
          162.46    189.54

Female      191       185      376
          173.54    202.46

All         336       392      728

Chi-Sq =  1.877 +  1.609 +
          1.757 +  1.506 = 6.748
DF = 1, P-Value = 0.009
```

d. For male listener use the counts under "Identified Female Voice?" (132 and 220).
For female listener use the counts under "Identified Male Voice?" (162 and 214).
The test statistic is $\chi^2 = 2.356$, df = (2–1)(2–1) =1, p-value = 0.125.
With Table A.5, $0.10 < p$-value < 0.25.
Do not reject the null hypothesis. There is not evidence of a relationship between sex of listener and the ability to identify a voice of the opposite sex. In the following output, expected counts are beneath observed counts.

Output for Exercise 15.44d

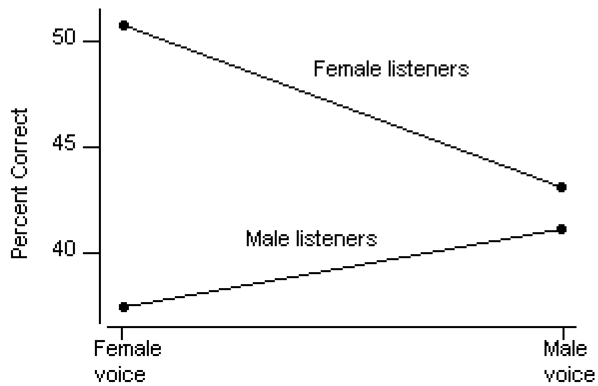
```
            Yes       No      All
Male        132       220     352
          142.15    209.85

Female      162       214      376
          151.85    224.15

All         294       424      728

Chi-Sq =  0.725 +  0.491 +
          0.679 +  0.460 = 2.356
DF = 1, P-Value = 0.125
```

15.44 continued:

e. A statistically significant gender difference was observed in the ability to identify a female voice. Also, a statistically significant gender difference was found in the ability to identify a voice of the same sex. To understand these two findings, and how they are connected it is helpful to find the percent correct identifications for each combination of sex of listener and sex of speaker. A graph of these percents is also presented.

Listener	*Voice*	**% correct**
Male	Male	41.2% (145/352)
Male	Female	37.5% (132/352)
Female	Male	43.1% (162/376)
Female	Female	50.8% (191/376)

Figure for Exercise 15.27e



Notice that female listeners were noticeably more successful at identifying a female voice than male listeners, but males and females had about the same success rate for identifying a male voice. One other fact of note is that the percent of correct identifications is not particularly high in any instance. This has ramifications for court testimony in which a witness identifies somebody by their voice.

15.45 **a.** H_0: Gender and perception of weight are not related

H_a: Gender and perception of weight are related

b. Reject the null hypothesis, p-value = 0.000.There is a statistically significant relationship between gender and weight perception.

c. There are large contributions in the Underweight and Overweight categories for each sex. This reflects a large gender difference in those categories. Relevant condition percents are: for females 30.2% (39/129) said overweight compared to only 3.6% of males (3/83), and for females only 2.3% (3/129) said underweight compared to 19.3% of males (16/83).

15.46 **a.** Step 1: H_0: Gender and opinion about ease of getting a divorce are not related

H_a: Gender and opinion about ease of getting a divorce are related

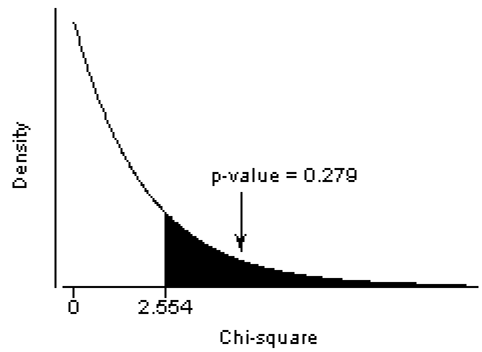
Step 2: Expected counts are all greater than 5 so proceed with the chi-square test.

Test statistic is $\chi^2 = 2.554$, df = 2

Steps 3, 4, and 5: p-value = 0.279. Do not reject the null hypothesis. There is not sufficient evidence to conclude that gender and opinion about ease of getting a divorce are related.

b. p-value is area to the right of 2.554 under a chi-square curve with df = 2. Figure 15.3 and Figure 15.4 on page 534 both show a chi-square density curve for df = 2.

Figure for Exercise 15.46b



15.47 Between 0 and 5.99. In Table A.5, the value for df = 2 under column heading = 0.05 provides the answer. When the null hypothesisis is true, 5% of the time the chi-square statistic will be ≥ 5.99. So, 95% of the time the statistic will be less than this value.

15.48 The numerator and denominator of any $\frac{(O-E)^2}{E}$ component are both positive (always), so the sum of these quantities must also be positive. (The numerator could be zero, so the chi-square test statistic could be zero as well. That would happen if all observed counts were exactly equal to the corresponding expected counts.)

15.49 **a.** Too many cells have expected counts less than 5.
b. Step 1: H_0: Own eye color and eye color attracted to are not related
H_a: Own eye color and eye color attracted to are not related

Step 2: Test statistic is χ^2 = 15.5, df = 4. Output is shown below, with expected counts beneath observed counts.

Steps 3, 4, and 5: p-value = 0.004. With Table A.5, 0.001< p-value < 0.005. Reject the null hypothesis. There is a statistically significant relationship between own eye color and eye color attracted to for the population of college students represented by this sample.

Output for Exercise 15.49b

```
Rows: Own     Columns: Attracted to

          Brown     Blue    HazGr      All

 Brown       30       22       19       71
          20.56    30.65    19.79    71.00

 Blue        15       37       14       66
          19.11    28.49    18.39    66.00

 HazGr        8       20       18       46
          13.32    19.86    12.82    46.00

 All         53       79       51      183
          53.00    79.00    51.00   183.00
             --       --       --       --
Chi-Sq =  4.331 +  2.441 +  0.031 +
          0.886 +  2.541 +  1.049 +
          2.126 +  0.001 +  2.093 = 15.500
DF = 4, P-Value = 0.004
```

15.49 continued:

c. The "contributions" to the chi-square statistic are shown in the calculation of the statistic at the bottom of the output. The largest two contributions are brown attracted to brown (4.331) and blue attracted to blue (2.541). In both cases, the observed count is much higher than the expected count. People appear to be attracted to people with their own eye colors more often than would be expected if the two variables were not related.

15.50 **a.** Step 1: H_0: no relationship between own eye color and whether attracted to own eye color
H_a: relationship between own eye color and whether attracted to own eye color

Step 2: Test statistic is $\chi^2 = 13.193$, df = 3. Output is shown below, with expected counts beneath observed counts.

Steps 3, 4, and 5: p-value = 0.004. With Table A.5, $0.001 < p$-value < 0.005. Reject the null hypothesis. Conclude that in population of college students represented by this sample, there is a statistically significant relationship between own eye color and whether attracted to own eye color.

Output for Exercise 15.50a

```
            No      Yes      All
 Brown      41       30       71
         41.13    29.87    71.00

 Blue       29       37       66
         38.23    27.77    66.00

 Hazel      23        7       30
         17.38    12.62    30.00

 Green      13        3       16
          9.27     6.73    16.00

 All       106       77      183
        106.00    77.00   183.00
            --       --       --
Chi-Sq =  0.000 +  0.001 +
          2.228 +  3.067 +
          1.820 +  2.505 +
          1.503 +  2.069 = 13.193
DF = 3, P-Value = 0.004
```

b. The "contributions" to the chi-square statistic are shown in the calculation of the statistic at the bottom of the output. Those with blue eyes contribute the most, with fewer than expected attracted to other eye colors (29 observed, 38.23 expected, contribution is 2.228) and more than expected attracted to those with same eye color (37 observed, 27.77 expected, contribution is 3.067). Those with brown eyes have almost identical observed and expected counts. In other words, the proportion of brown-eyed people attracted to brown-eyed people is about the same as the overall proportion of people attracted to people with their own eye color (106/183 or about 58% in the sample).

15.51 **a.** Homogeneity. The issue is whether the distribution of responses for satisfaction is the same for the two years.

H_0: Satisfaction with K-12 was the same for the populations of school parents in 1999 and 2000
H_0: Satisfaction with K-12 differed for the populations of school parents in 1999 and 2000

b. Step 2: Test statistic is $\chi^2 = 7.96$, df = 3.

Steps 3,4, and 5: p-value = 0.047. Reject the null hypothesis. Conclude that opinion differed in the two years. Examination of conditional percents for the two years shows that a higher percentage were "completely satisfied" in 1999 than in 2000, while a higher percent were "completely dissatisfied" in 2000 than in 1999.
Output is shown below, with expected counts beneath observed counts.

Output for Exercise 15.51

	1999	2000	All
Com Sat	125	87	212
	116.09	95.91	
Some Sat	155	133	288
	157.70	130.30	
Some Dis	41	34	75
	41.07	33.93	
Com Dis	7	17	24
	13.14	10.86	
All	328	271	599

Chi-Sq = 7.960 DF = 3, P-Value = 0.047

15.52 Percents owning a gun by political party are: Democrat, 35.1%; Independent, 42.6%; Other, 5.9%; Republican, 51.6%. There are only 14 people in the "other" category, so it is reasonable (although not imperative) to drop this category when a chi-square test is done.

Step 1: H_0: Political party and likelihood of gun ownership are not related
H_a: Political party and likelihood of gun ownership are related

Step 2: $\chi^2 = 20.095$, df = 3 if "other" included. Or, $\chi^2 = 18.955$, df = 2 if "other" excluded.

Step 3: p-value ≈ 0 (whether "other" included or excluded)

Steps 4 and 5: Reject the null hypothesis. Conclude that the likelihood of gun ownership is related to political party preference.

Output for Exercise 15.52 (Observed counts and chi-square)

	No	Yes	All
Democrat	242	131	373
Independ	205	152	357
Other	10	4	14
Republic	153	163	316
All	610	450	1060

Chi-Square = 20.095, DF = 3, P-Value = 0.000

15.53 **a.** Step 1: H_0: gender and typical seat location are not related
H_a: gender and typical seat location are related

Step 2: $\chi^2 = 7.112$, df = 2

Step 3: p-value = 0.029 (given in Minitab output)

Steps 4 and 5: Reject the null hypothesis. Conclude that typical seat is related to gender. The conditional percents by sex show that males are more likely than females to sit in the back and females are more likely than males to sit in the front.

Output for Exercise 15.53 (Observed counts and chi-square)

Rows: Sex Columns: Seat

	B	F	M	All
Female	22	38	93	153
Male	24	15	46	85
All	46	53	139	238

Chi-Square = 7.112, DF = 2, P-Value = 0.029

15.53 continued:

Conditional percents for seat location by sex are:

	Back	Front	Middle	All
Female	14.4%	24.8%	60.8%	100%
Male	28.2%	17.7%	54.1%	100%

b. A breakdown of the chi-square calculation is:

```
Chi-Sq =  1.939 +  0.453 +  0.149 +
          3.489 +  0.815 +  0.267 = 7.112
```

The contributions to chi-square are largest for the "Back" location for males (3.489) and females (1.939). This reflects a large difference between the males and females for the likelihood of sitting in the back.

15.54 Step 1: H_0: no relationship between handedness and sex of people it's easiest to make friends with
H_a: there is a relationship between handedness and sex of people it's easiest to make friends with

Step 2: $\chi^2 = 0.375$, df = 1

Step 3: *p*-value = 0.540 (given in Minitab output)

Steps 4 and 5: Do not reject the null hypothesis. There is not evidence of a relationship between the two variables for this exercise.

Output for Exercise 15.54 (Observed counts and chi-square)

```
Rows: Hand     Columns: Friends

      Opposite     Same      All

 Left      13        7       20
 Right    124       90      214
 All      137       97      234

Chi-Square = 0.375, DF = 1, P-Value = 0.540
```

15.55 Answers will vary.