Statistical Methods for Complex Models in Climate Science

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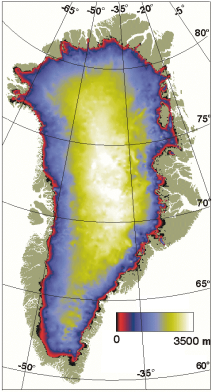
This Talk

- Complex models are often used to make projections about future climate. E.g. Ice models are often used to make projections about future ice sheet behavior.
- Model input parameters are like knobs/dials on the climate model. They greatly influence how the model behaves.
- What values should these parameters be set to? How sure/uncertain are we about their values?
- Use data! E.g. recent data on the ice sheet
- Lots of challenges: large spatial data, complicated errors.
- A SCRiM research group is developing statistical methods for parameter inference for climate models.

Greenland Ice Sheet

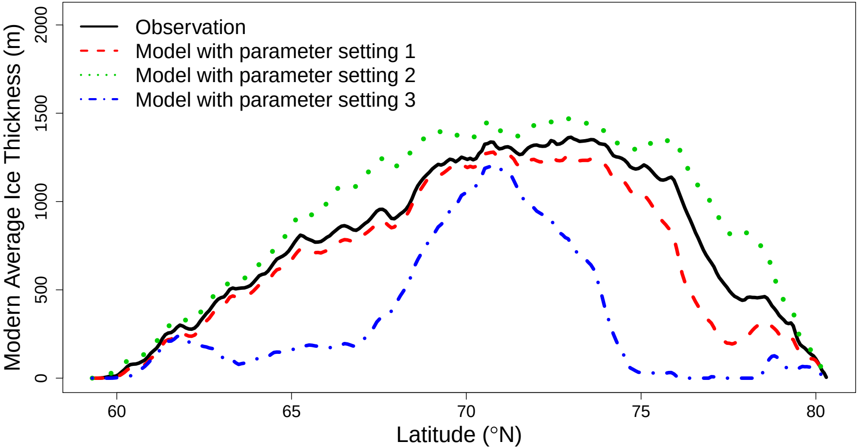
Important contributor to sea level rise: Total melting results in sea level rise of 7m.



Bamber et al. (2001)

Calibration Problem

Which parameter settings best match observations?

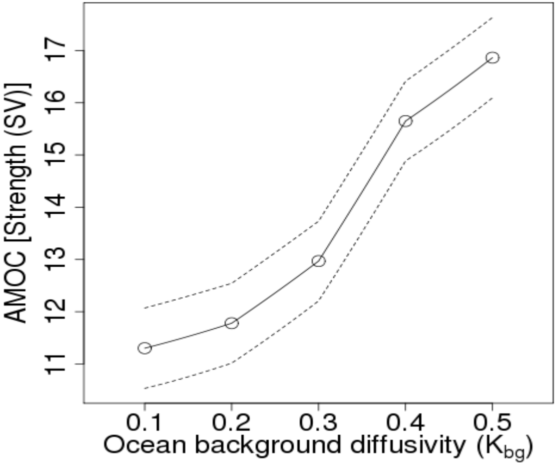


The AMOC and Climate Change

- Atlantic Meridional Overturning Circulation (AMOC): AMOC heat transport makes a substantial contribution to the moderate climate of Europe (cf. Bryden et al., 2005)
- Any slowdown in the overturning circulation may have major implications for climate change
- AMOC projections from climate models.

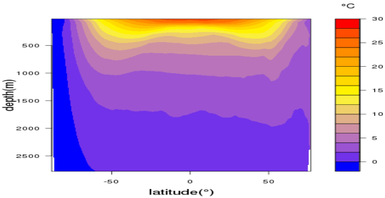
A major source of uncertainty about the AMOC is due to uncertainty about $\mathbf{K_{bg}}$: model parameter that quantifies the intensity of vertical mixing in the ocean.

AMOC and Model Parameter K_{bg}

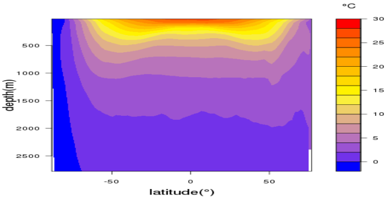


Ocean Temperatures

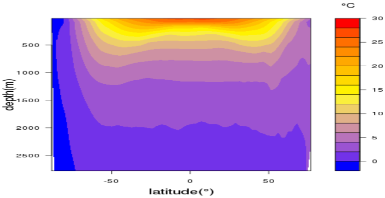
K_{bg} of 0.1



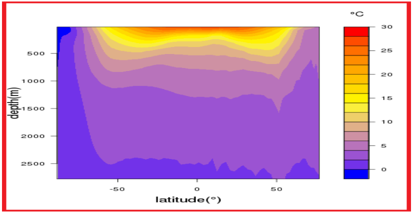
K_{bg} of 0.2



K_{bg} of 0.3



Observations



(2D versions of 3D data)

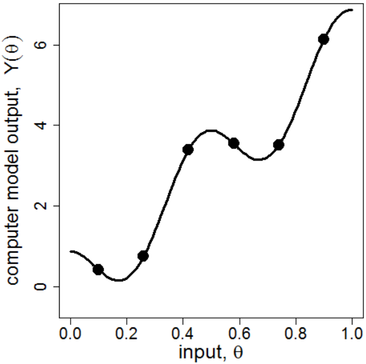
Two-stage Approach to Emulation-Calibration

1. Emulation step: Find fast approximation for climate model using Gaussian process (GP).
   Information used: climate model runs at various parameter settings.
2. Calibration step: Infer climate parameter using emulator and observations. Important: account for errors in the data, and data-model discrepancy (model is an imperfect representation of reality).
   Information used: climate observations and emulator.

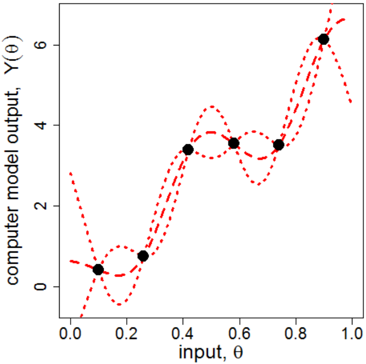
References: Bhat, Haran, Olson, Keller (2012); Chang, Haran, Olson, Keller (2014); Liu, Bayarri and Berger (2009)

Emulation Step: A Simple Example

We use a statistical model called a **Gaussian process.** This model is a fast emulator (approximation) of the computer model.



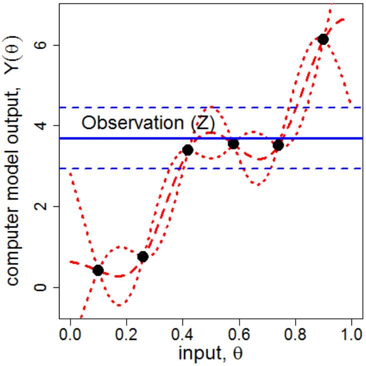
Computer model output (y-axis) vs. input (x-axis)



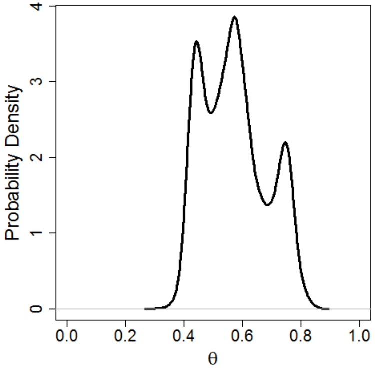
Emulation (approximation) of computer model using GP

Calibration Step: A Simple Example

We use statistical methods called **Bayesian inference and Markov chain Monte Carlo**: Use emulator (from before) and observations to learn about parameters.

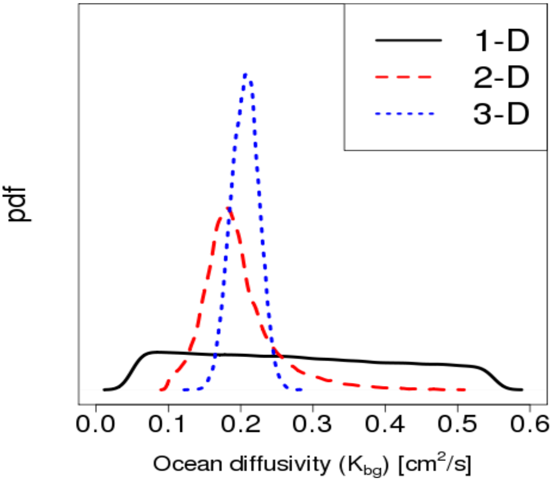


Combining observation and emulator



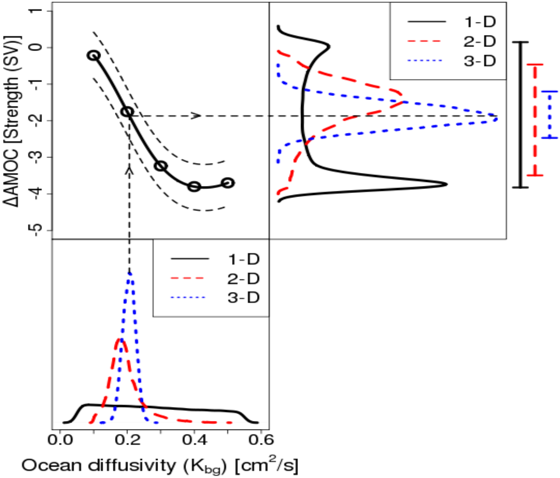
Posterior PDF of θ given model output and observation

Results for K_{bg} Inference



(from Chang, Haran, Olson and Keller, 2014)

MOC Projections for 2100 Using Inferred K_{bg}



(from Chang, Haran, Olson and Keller, 2014)

Concluding Thoughts

- Without probability and statistics, it is not possible to quantify risk.
- Advanced statistical methods allow us to
  - Utilize all (*large*) data sets which can often help reduce uncertainties about projections.
  - Account for errors, uncertainties carefully.
  - Learn about various sources of error, e.g. discrepancy between data and model.
  - Learn about complicated interactions among model parameters.
- This work requires expertise (people with M.S. and Ph.D.s!) in statistics and geosciences. We work together very closely.

Statistical Methods: Details

BEGIN: FANCY STATS...

Summary of Statistical Problem

- **Goal**: Learning about $\boldsymbol{\theta}$ based on two sources of information:
  - **Observations**: Mean ice thickness profile† $\mathbf{Z} = (Z(\mathbf{s}_1), \ldots, Z(\mathbf{s}_n))^T$, where $\mathbf{s}_1, \ldots, \mathbf{s}_n$ are latitude points.
  - **Ice model output*** for mean ice thickness $\mathbf{Y}(\boldsymbol{\theta}_1), \ldots, \mathbf{Y}(\boldsymbol{\theta}_p)$, where each $\mathbf{Y}(\boldsymbol{\theta}_i) = (Y(\mathbf{s}_1, \boldsymbol{\theta}_i), \ldots, Y(\mathbf{s}_n, \boldsymbol{\theta}_i))^T$ is spatial process (Applegate et al 2012).

  $\mathbf{Z}$ and $\mathbf{Y}(\boldsymbol{\theta}_i)$'s are n-dimensional vectors

- Important: output at each $\boldsymbol{\theta}_i$ is a spatial process. $n = 264$ locations, $p = 100$ runs.

†Averaged over longitude

*SICOPOLIS (Greve, 1997; Greve et al., 2011)

Step 1: Dimension Reduction

- Consider model outputs at $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_p$ as replicates and obtain PCs

$$\begin{pmatrix} Y(\mathbf{s}_1, \boldsymbol{\theta}_1) & \ldots & Y(\mathbf{s}_n, \boldsymbol{\theta}_1) \\ \vdots & \ddots & \vdots \\ Y(\mathbf{s}_1, \boldsymbol{\theta}_p) & \ldots & Y(\mathbf{s}_n, \boldsymbol{\theta}_p) \end{pmatrix}_{p \times n} \Rightarrow \begin{pmatrix} Y_1^R(\boldsymbol{\theta}_1) & \ldots & Y_{J_y}^R(\boldsymbol{\theta}_1) \\ \vdots & \ddots & \vdots \\ Y_1^R(\boldsymbol{\theta}_p) & \ldots & Y_{J_y}^R(\boldsymbol{\theta}_p) \end{pmatrix}_{p \times J_y}$$

- PCs pick up characteristics of model output that vary most across input parameters $\theta_1, \ldots, \theta_p$.

Step 2: Emulation Using PCs

- Fit 1-dimensional GP for each series $Y_j^R(\boldsymbol{\theta}_1), \ldots, Y_j^R(\boldsymbol{\theta}_p)$
- $\boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R)$: J_y-dimensional emulation process for PCs, $\mathbf{Y}^R$ is collection of PCs
- Computation reduces from $\mathcal{O}(n^3p^3)$ to $\mathcal{O}(J_yp^3)$ (6.13×10^{12} to 3.33×10^6 flops).
- Emulation for original output: compute $\mathbf{K}_y\boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R)$ where $\mathbf{K}_y$ is matrix of scaled eignvectors

Dimension Reduction for Discrepancy Process

- Kernel convolution: Specifying n-dimensional discrepancy process δ using J_d-dimensional knot process $\boldsymbol{\nu}$ ($J_d < n$) and kernel functions

- Kernel basis matrix $\mathbf{K}_d$ links grid locations $\mathbf{s}_1, \ldots, \mathbf{s}_n$ to knot locations $\mathbf{a}_1, \ldots, \mathbf{a}_{J_d}$;

$$\{\mathbf{K}_d\}_{ij} = \exp\left(-\frac{\|\mathbf{s}_i - \mathbf{a}_j\|}{\phi_d}\right)$$

  with $\phi_d > 0$. Fix ϕ_d at large value determined by expert judgment

- Results in better identifiability: Overly flexible discrepancy process may be confounded with emulator

Calibration in Reduced Dimensions

- Probability model for dimension-reduced observation $\mathbf{Z}^R$:

$$\mathbf{Z} = \underbrace{\mathbf{K}_y \boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R)}_{\text{emulator}} + \underbrace{\mathbf{K}_d \boldsymbol{\nu}}_{\text{discrepancy}} + \underbrace{\boldsymbol{\epsilon}}_{\text{observation error}},$$

$$\Rightarrow \mathbf{Z}^R = (\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\mathbf{Z} = \begin{pmatrix} \boldsymbol{\eta}(\boldsymbol{\theta}, \mathbf{Y}^R) \\ \boldsymbol{\nu} \end{pmatrix} + (\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\boldsymbol{\epsilon},$$

  with combined basis $[\mathbf{K}_y\ \mathbf{K}_d]$, knot process $\boldsymbol{\nu} \sim N(\mathbf{0}, \kappa_d \mathbf{I})$, and observational error $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$.

- Infer $\boldsymbol{\theta}$ through posterior distribution

$$\pi(\boldsymbol{\theta}, \kappa_d, \sigma^2 | \mathbf{Z}^R, \mathbf{Y}^R) \propto \underbrace{L(\mathbf{Z}^R | \mathbf{Y}^R, \boldsymbol{\theta}, \kappa_d, \sigma^2)}_{\text{likelihood given by above}} \underbrace{p(\boldsymbol{\theta})p(\kappa_d)p(\sigma^2)}_{\text{priors}}$$