Exploration of Computationally Efficient ABC-SMC Algorithms

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What is Approximate Bayesian Computation (ABC)?

Motivating problem:
The likelihood function is intractable, unknown or computationally costly.

ABC is used as an approximate method for Bayesian inference.

<u>General form of the ABC Algorithm</u>

1. Draw θ* from $\pi(\theta)$
2. Simulate **x*** from f(x|θ*)
3. Accept θ* if d($\mathbf{x}_O$, $\mathbf{x}^*$) $\leq \varepsilon$

ABC-SMC Algorithm

Basic Idea:
Improve the acceptance rate by decomposing the problem of sampling into sub-problems.

<ins>General form of the ABC-SMC Algorithm</ins>

1. Define a tolerance schedule $\varepsilon_1 > \varepsilon_2 > ... > \varepsilon_T \geq 0$
2. For t=1
   Sample θ^* from $\pi(\theta)$. Repeat until N particles are accepted
3. For t=2
   Sample θ^* from Population 1 and perturb it.
   Repeat until N particles are accepted
   Calculate weights for accepted particles
4. Repeat Step 3 until N particles in Population T have been accepted.

Challenges of ABC-SMC

Computing problem:

- This algorithm has a computational complexity that is quadratic in number of particles (N).
- Determination of tolerance levels can also be difficult.

One solution:

Del Moral et al. (2012) proposes an adaptive ABC-SMC algorithm that determines the tolerance levels and has a computational complexity that is linear in the number of particles.

How does this algorithm differ from ABC-SMC?

Basic idea of the adaptive ABC-SMC Algorithm

1. For t=1

    Sample θ^* from $\pi(\theta)$. Repeat until N particles are accepted

2. For t=2........

    Determine $\boldsymbol{\varepsilon}_t$ such that ESS is "controlled"

    If $\boldsymbol{\varepsilon}_t < \boldsymbol{\varepsilon}$, set $\boldsymbol{\varepsilon}_t = \boldsymbol{\varepsilon}$

    If the ESS < N_T resample N particles from the previous population
    Else for i = 1....N, if the i^{th} weight > 0,

    Sample $(\theta_{n-1}^{(i)}, X_{1:M,n-1}^{(i)}) \sim K_n((\theta_{n-1}^{(i)}, X_{1:M,n-1}^{(i)}), .)$

    Repeat until $\boldsymbol{\varepsilon}_{t-1} = \boldsymbol{\varepsilon}$

Toy Example

Consider a model of the following form:

$$\theta \sim \text{Unif}(-10,10)$$
$$f(x|\theta) = 0.5\phi(x;\, \theta,\, 1) + 0.5\phi(x;\, \theta,\, 1/100)$$

The posterior density associated with the y=0 observation is given by

$$\pi(\theta \mid x) \propto (\phi(\theta;\, 0,\, 1) + \phi(\theta;\, 0,\, 1/100))I_{[-10,10]}(\theta)$$

Toy Example Results

The ABC-SMC and Adaptive ABC-SMC algorithm were both run in R for example.

For both, target $\varepsilon = 0.01$. The number of particles used were 1000. For the ABC-SMC algorithm, $T = 100$.

For the adaptive ABC-SMC algorithm, a normal random walk MH kernel was used and $N_T = N/2$.

Wall Time

ABC-SMC	Adaptive ABC-SMC
2.574 hrs	0.426 sec

Potential challenges:

- There are still parameters that need to be fixed (N, α)
- Can result in particle duplication
- It will perform poorly when $f(x|\theta)$ has high relative variance

My contribution:

I will run an both ABC-SMC algorithms discussed here on data relating to PPRV transmission.

Summary

ABC are a class of methods that can be used when the likelihood function is intractable or expensive to compute

ABC SMC is a less computationally expensive ABC method

The main advantage of the algorithm proposed in this paper is the determination of the sequence of tolerance levels

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