

STAT 515

Homework #6, due Friday, Mar. 2 at 2:30pm

This homework may be submitted electronically to ANGEL, though this is not required. I strongly encourage the use of \LaTeX in any case.

Please make every assignment easier to grade by neatly organizing your writeup and clearly labeling your final answers when appropriate. Try using \LaTeX !

1. Let $N(t)$ be a non-homogeneous Poisson process with rate function $\lambda(t)$.
 - (a) Fix some $t > 0$. Derive the conditional distribution of the arrival times $S_1, \dots, S_{N(t)}$, conditional on $N(t) = n$.
 - (b) As you did in Exercise 2(a) of Homework #5, use part (a) to suggest an algorithm for simulating a non-homogeneous Poisson process on $(0, t]$.
 - (c) Use your algorithm to simulate 10,000 realizations of a non-homogeneous Poisson process on $(0, 2]$ where $\lambda(t) = 24t^2$.
 - (d) Using the result of part (c), produce a histogram of the numbers of events in the interval $(0, 1]$. Add to your histogram the true theoretical values, and explain how you found them.
 - (e) Using the result of part (c), produce a histogram of the times until the first event. Add to your histogram the true theoretical density, and explain how you found it. What is the name of the true theoretical distribution?
2. Teams 1 and 2 are playing a game. The teams score points according to independent Poisson processes with rates λ_1 and λ_2 , respectively. The game ends when one team has scored exactly k points more than the other team. What is the probability that team 1 wins?
3. Consider n components with independent lifetimes, where component i has exponentially distributed lifetime with rate λ_i . Suppose that all components are initially in use and remain so until they fail.
 - (a) Find the probability that component 1 is the second component to fail.
 - (b) Find the expected time of the second failure.