Version 2: Full conditional distributions for a Bayesian chain point model with Gamma hyperpriors

Our goal is to draw samples from the 5-dimensional **posterior** distribution $f(k, \theta, \lambda, b_1, b_2|\mathbf{Y})$ The posterior distribution is

$$
\begin{aligned}
f(k, \theta, \lambda, b_1, b_2|\mathbf{Y}) \propto & \prod_{i=1}^{k} \frac{\theta^{Y_i} e^{-\theta}}{Y_i!} \prod_{i=k+1}^{n} \frac{\lambda^{Y_i} e^{-\lambda}}{Y_i!} \\
& \times \frac{1}{\Gamma(0.5) b_1^{0.5}} \theta^{-0.5} e^{-\theta/b_1} \times \frac{1}{\Gamma(0.5) b_2^{0.5}} \lambda^{-0.5} e^{-\lambda/b_2} \\
& \times e^{-b_1} e^{-b_2} \frac{1}{n}
\end{aligned} \tag{1}
$$

From 1 we can obtain full conditional distributions for each parameter by ignoring all terms that are constant with respect to the parameter.
For θ:

$$
f(\theta|k, \lambda, b_1, b_2, \mathbf{Y}) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i} e^{-\theta}}{Y_i!} \times \frac{1}{\Gamma(0.5) b_1^{0.5}} \theta^{-0.5} e^{-\theta/b_1} \tag{2}
$$

For λ:

$$
f(\lambda|k, \theta, b_1, b_2, \mathbf{Y}) \propto \prod_{i=k+1}^{n} \frac{\lambda^{Y_i} e^{-\lambda}}{Y_i!} \times \frac{1}{\Gamma(0.5) b_2^{0.5}} \lambda^{-0.5} e^{-\lambda/b_2} \tag{3}
$$

For k:

$$
f(k|\theta, \lambda, b_1, b_2, \mathbf{Y}) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i} e^{-\theta}}{Y_i!} \prod_{i=k+1}^{n} \frac{\lambda^{Y_i} e^{-\lambda}}{Y_i!} \tag{4}
$$

For b_1:

$$
f(b_1|k, \theta, \lambda, b_2, \mathbf{Y}) \propto \frac{1}{b_1^{0.5}} e^{-\theta/b_1} \times e^{-b_1} \tag{5}
$$

For b_2:

$$
f(b_2|k, \theta, \lambda, b_1|\mathbf{Y}) \propto \times \frac{1}{b_2^{0.5}} e^{-\lambda/b_2} e^{-b_2} \tag{6}
$$

$f(b_1|k, \theta, \lambda, b_2, \mathbf{Y})$ and $f(b_2|k, \theta, \lambda, b_1|\mathbf{Y})$ are not any recognizable densities. We can use a Metropolis-Hastings accept-reject step to sample from their full conditionals. For example for the full conditional distribution of b_1, suppose the current value of b_1 is b_1^{curr}

(1) Draw a proposed value b_1^* $\sim$Normal(b_1,2)
(2) Accept the proposed value with probability $\alpha = \min\{1, \frac{f(b_1^*|\theta^{curr})}{f(b_1^{curr}|\theta^{curr})}\}$,i.e. draw U $\sim Unif(0,1)$ and set the update of the chain to b_1^* if $U < \alpha$ else set the update of the chain to b_1^{curr}