# Homework 4, Stat 515, Spring 2015

Due Wednesday, February 18, 2015 **beginning of class**

1. Ehrenfest diffusion:

   (a) Simulate the simple Ehrenfest diffusion process (following the description in the Ross book) with total number of particles, $m = 50$. Start the process at $X_1 = 5$. Run the process for 10,000 iterations and draw a histogram of the resulting values of $X_t$ (in `R`, use the `hist` command). You can also obtain an approximation to the density based on these samples by using the command `plot(density(SampleVectorName), main="Approximate Density of Ehrenfest Diffusion Samples")`.

   (b) Read through the argument provided for deriving the stationary distribution of the Ehrenfest diffusion. State the stationary distribution $\pi$ for the particular Ehrenfest diffusion you are considering above. Now simulate 10,000 draws from the stationary distribution of the Ehrenfest diffusion (you may need to use the command `rbinom`) and overlay a density plot of its samples on top of the previous plot by using `lines(density(BinomialSampleVectorName),lty=2)`. Make sure you provide a clear heading for the plot this plot clearly

   (c) State the results from class that explains what you observe from your comparisons above.

   (d) Compare the proportion of values greater than 15 for the binomial distribution using `pbinom`, and estimate this quantity using the two sets of samples you just generated. Report the true value along with your two estimates. Notice that you can use both sets of samples, the Markov chain as well as the iid sampler, to approximate this probability (expectation with respect to Binomial).

   (e) Start the process at $X_1 = 5$ and obtain $X_3$ for the Ehrenfest diffusion. Repeat this 10,000 times and obtain a histogram for $X_3$. Now start the process at $X_1 \sim \pi$ where $\pi$ is the stationary distribution above and again obtain $X_3$ for the Ehrenfest diffusion 10,000 times. Compare these two histograms to the Binomial distribution above. Briefly explain what you observe and why.

2. Consider a branching process where $X_0 = 1$ and every individual at the $i$th generation has probability $p_j$ of having $j$ offspring, with $p_0 = 1/2, p_1 = 1/6, p_2 = 1/3$. Let $X_t$ be the state of the process at time $t$.

   (a) Let the probability this process will go extinct be $\pi_0$. Calculate $\pi_0$.

   (b) What are the expected value and variance of $X_2$?

   (c) Now simulate 10,000 realizations of the process up to $t = 2$, i.e., $X_0, X_1, X_2$. Plot the histogram of $X_2$, and report the sample mean and variance. Compare them to the theoretical mean and variance obtained from the previous problem. The code available here might be useful for this problem:`http://www.stat.psu.edu/~mharan/515/hwdir/hw04ex.R` .

   (d) Now simulate 10,000 realizations of the branching process. Let $L$ be the time taken before extinction. Show a histogram of $L$. Note that since a particular realization of the branching process may be extremely long or, for some processes, be infinite (since it may never go extinct), it is useful to impose (artificially) a very large upper limit for the length of the branching process when simulating realizations.