Take home final_STAT 515

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Problem 1

Part a

The posterior of beta1, up to a normalizing constant, can be derived to be as following (for convenience, the ERFC function is written in terms of the pnorm function in R):

$$\Big(exp(\lambda * \beta_1 * \sum_{i=1}^{n} X_i - {\beta_1}^2)\Big) * \prod_{i=1}^{n} \Big(1 - pnorm(\frac{\beta_0 + \beta_1 * X_i + \lambda * {\sigma_i}^2 - Y_i}{\sqrt(2) * \sigma_i})\Big)$$

The plot of this posteior distribution very well resembles a symmetric bell curve. Therefore we propose a normal kernel i.e. use a random walk metropolis hastings algorithm. Which means that the $q(x)$ is symmetric and we have to only keep track of the posterior.
We will use the standard Metropolis-Hastings algorithm i.e. to propose a new sample from a normal distribution centers at the previous sample point and variance τ^2.

We are considering three possible initial values:

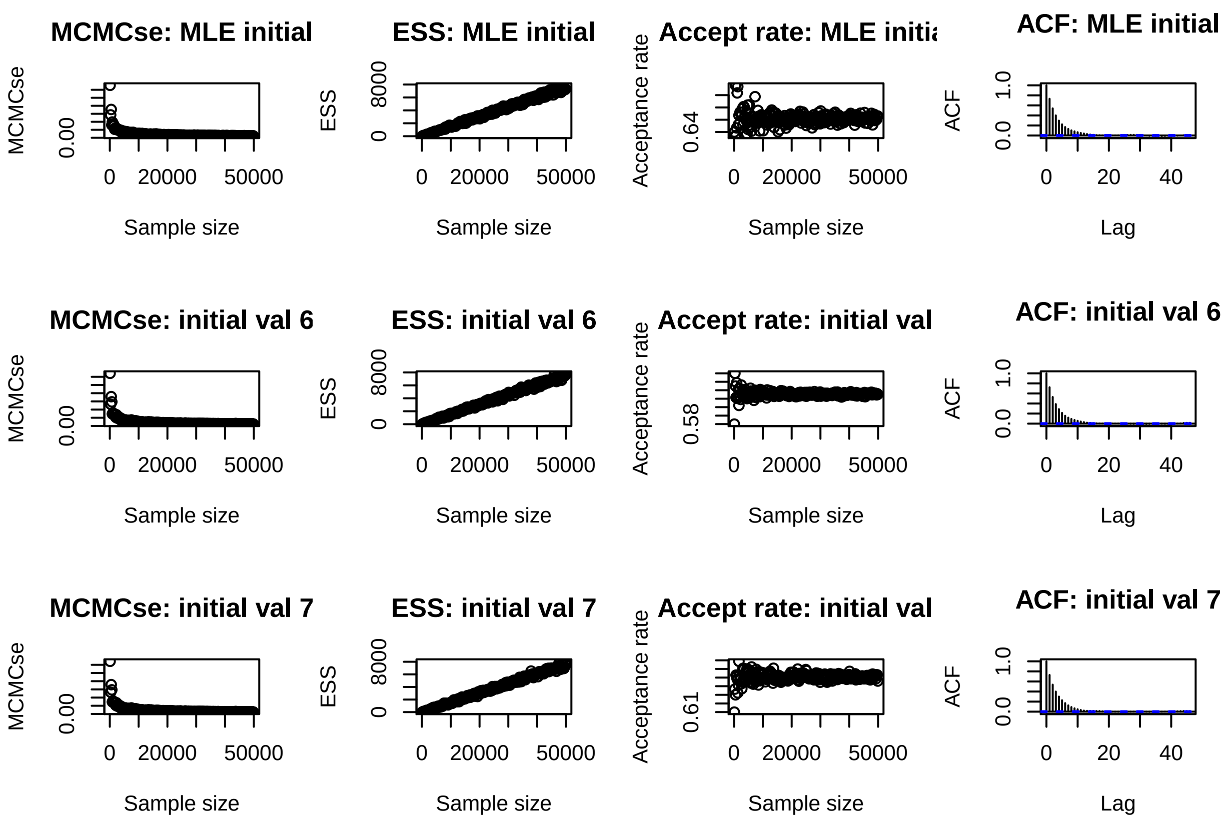
- MLE of $\beta_1 \approx 8.61$
- The value with the highest mass on the plot of joint likelihood function, when plotted against $\beta_1 \approx 6$.
- A value in between the above two (7), to better study the behaviour of MCMC

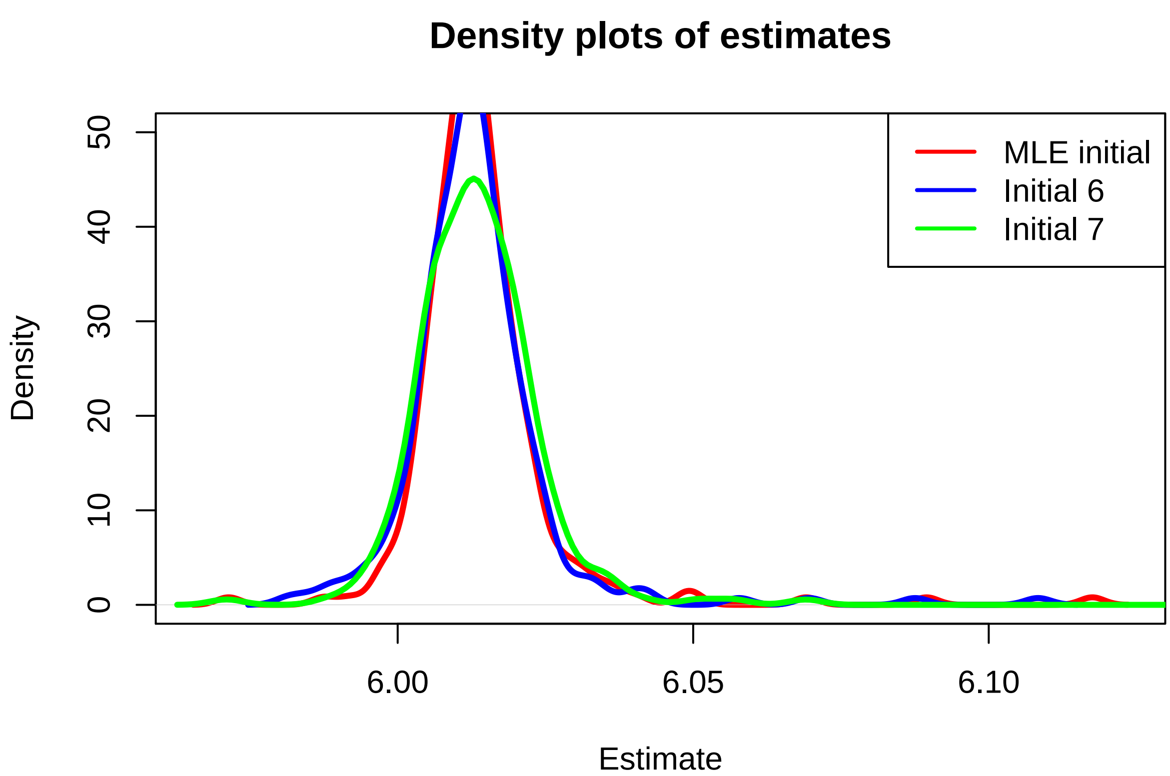
We will now take a look at some of the diagnostics before answering specific answers regarding the estimate. In the following plot matrix we will see each of the following plots calculated with respect to the three initial values we have used:

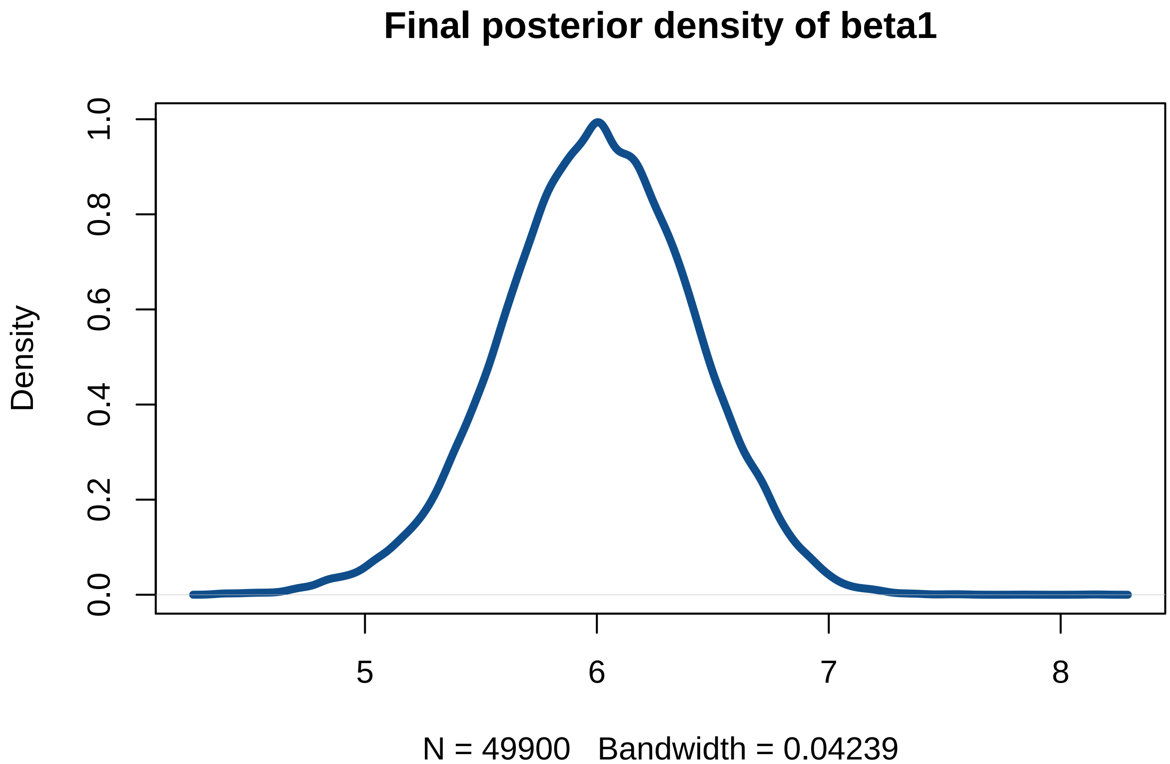
- A plot of MCMCse against sample size
- A plot of ESS against sample size
- A plot of acceptance rate against sample size
- An ACF plot for the sample with highest sample size

We will also see a plot of estimated mean against sample size, for different initial values. This will help us ensure that the tuning parameter chosen here is doing a good job.

The plots below indicate that the initial values are performing almost equally well. However, initial value 7 seems to be doing slightly better than the other two in terms of the acceptance rate and tapering of ACFs. Therefore we will accept that and report the required numbers and the density plot from the largest sample in that set.


MCMCse: MLE initial
ESS: MLE initial
Accept rate: MLE initi
ACF: MLE initial
MCMCse: initial val 6
ESS: initial val 6
Accept rate: initial val
ACF: initial val 6
MCMCse: initial val 7
ESS: initial val 7
Accept rate: initial val
ACF: initial val 7
MCMCse
ESS
Acceptance rate
ACF
Sample size
Lag


Density plots of estimates
MLE initial
Initial 6
Initial 7
Density
Estimate
6.00
6.05
6.10

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Estimated mean= 6.000876 , MCMCse= 0.004511439 , Credible interval= ( 5.160516 6.785571 )
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Problem 2

We will use a variable-at-a-time M-H algorithm to approximate the joint posterior distribution. We can see that the distributions i.e. full conditionals for parameters are:

- β_0 follows EMG or exponentially modified Gaussian with $\mu = 100 * n * \lambda$, $\sigma = 10$ and $\lambda = \lambda$
- β_1 follows EMG or exponentially modified Gaussian with $\mu = 100 * \lambda * \sum X_i$
- The posterior of λ is given as follows:

$$\Big(\lambda^{0.99} * exp\big((\lambda * (100 * n - 1)) + (200 * n * \lambda * \beta_0) + (200 * \lambda * \beta_1 * \sum_{i=1}^{n} X_i) + (200 * n * \lambda)\big)\Big) * \prod_{i=1}^{n} \Big(1 - pnorm(\frac{\beta_0 + \beta_1 * X_i + \lambda * {\sigma_i}^2 - Y_i}{\sqrt(2) * \sigma_i})\Big)$$

Therefore, every proposed sample for the betas will be a Gibbs sampler, generated using the function *rexpgauss* defined in the Professor's code. Each proposed sample for betas will go into the new sample.
Whereas, for λ we propose a random walk M-H algorithm. So we will generate proposed samples using normal distribution with mean at the previous λ value and tuning parameter. Since we are using random walk proposal, q(x) is symmetric and we have to track only the posterior.

As for the initial values, we can once again consider starting at the MLE of each of the parameters or graphically evaluating the same.