- For Monday, read Section 5.4.1
- HW#5 will be due next Friday. *It must be submitted electronically!*
- The midterm will be . . . ??? (Wednesday evening?)

Notes: HW#5 is somewhat shorter than normal because it must be done electronically.

A Poisson process is a special type of counting process:

- Counts of events in disjoint time intervals are independent.
- For any $0 \le s < t$, the number of events in $(s, s + t]$ is Poisson with parameter λt.
- Technically, we should also say $N(0) = 0$.

Notes: This is the "Poisson-based" definition of the Poisson process. It's the one we discussed on Wednesday.

A Poisson process is a special type of counting process (equivalent re-definition):

- Counts of events in disjoint time intervals are independent.
- $P[N(s + h) - N(s) = 1] = \lambda h + o(h)$ as $h \to 0$.
- $P[N(s + h) - N(s) \ge 2] = o(h)$ as $h \to 0$.
- Technically, we should also say $N(0) = 0$.

Notes: This is the "first-principles" definition of the Poisson process. To understand it, we had to define the little-o notation: If $f(x) = o(x)$ as $x \to 0$, this means that $f(x)/x \to 0$ as $x \to 0$. Intuitively, $f(x) = o(x)$ as $x \to 0$ means that $f(x)$ goes to zero faster than x does. An important special case of this notation is this: If $f(x) = o(1)$ as $x \to 0$, then $f(x) \to 0$ as $x \to 0$.

Why are the two definitions equivalent?

- How can we check that the first definition ("Poisson") implies the second ("first principles")?
- How about the other way around (i.e., first principles implies Poisson)?

Notes: The first proof may be done directly (using "brute force") and uses the fact that $e^{-\lambda h} = 1 - \lambda h + o(h)$ as $h \to 0$. The second proof requires some type of moment-generating function approach (the book uses the Laplace transform, which is similar to the MGF). However, it is possible to understand the intuition by dividing a time interval of length t into k equal subintervals, then studying what happens as k goes to infinity. Basically, the distribution of the number of events can be shown to be roughly binomial with parameters $(k, \lambda t/k)$, and therefore the Poisson approximation to the binomial (with parameter $t\lambda$) gets more and more accurate as $k \to \infty$.