Astrostatistics Short Course
Markov chain Monte Carlo exercise

Consider the following hierarchical changepoint model for the number of occurrences Y_i of some event during time interval i with change point k.

$$Y_i|k,\theta,\lambda \sim \text{Poisson}(\theta) \text{ for } i = 1,\ldots,k$$
$$Y_i|k,\theta,\lambda \sim \text{Poisson}(\lambda) \text{ for } i = k+1,\ldots,n$$

Assume the following prior distributions:

$$\theta|b_1 \sim G(0.5, b_1), \lambda|b_2 \sim G(0.5, b_2)$$

$$b_1 \sim G(1,1), b_2 \sim G(1,1)$$

$$k \sim \text{Discrete Uniform}(1,\ldots,n)$$

k, θ, λ are independent and b_1, b_2 are independent.
Assume the Gamma density parameterization $\text{G}(\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

Inference for this model is therefore based on the 5-dimensional posterior distribution $f(k,\theta,\lambda,b_1,b_2|\mathbf{Y})$ where $\mathbf{Y}=(Y_1,\ldots,Y_n)$. Apply this model to the data set available from the webpage INSERT WEBPAGE.

(1) We will use the Metropolis-Hastings algorithm for simulating from the posterior distribution $f(k,\theta,\lambda,b_1,b_2|\mathbf{Y})$.
Provide joint posterior distributions here:

The full conditional distributions are (filled in for all but one of the parameters):

Using heuristics such as plots of your estimates, autocorrelations, and acceptance rates to help guide you through the simulation process.

II. What to do with the samples produced:
(1) Include an estimated density plot based on your samples for each of the 5 parameters. (plot(density(x))
(2) The parameters of interest here are k, θ, λ: report your MCMC estimates of their means.
(3) Compute Monte Carlo standard errors for your mean estimates for k, θ, λ using the batch means method (download from CODA?)