November 16, 2023

MATH 230 Homework 4

Due on Thursday, November 23, 2023

(Submit your answers to Moodle in a pdf file)

- There are 5 questions in this homework. Each is 20 points.
- You should show your work to get full credit. Correct answers without sufficient explanation may not get full credit.

Questions:

1. At a certain location on Eskisehir road, the number of cars exceeding the speed limit by more than 10 kilometres per hour, during a half hour period is a Poisson random variable with mean of 8.4 per half hour. What is the probability that waiting time for the first car exceeding the speed limit by more than 10 kilometres per hour, will be less than 5 minutes?

2. The article " On Assessing the Accuracy of Offshore Wind Turbine Reliability - Based Design Loads from the Environmental Counter Method" (*Int. Journal of Offshore and Polar Engr.*, 2005: 132-140) proposes the Weibull distribution with $\alpha = 1.307$ and $\beta = 1.817$ as a model for 1-hour significant wave height (m) at a certain site.

a) What is the probability that wave height is at most 0.5 m?

b) What is the median of the wave height distribution?

3. The time (in hours) required to repair a machine is a gamma random variable with parameters $\alpha = 2$ and $\beta = 2$. Find the probability that the repair time of a machine exceeds 2 hours.

4. The joint density of X and Y is

$$f(x,y)=\begin{cases} cye^{-x} & x>0,\quad 0<y<2 \\ 0 & otherwise \end{cases}$$

a) Find the value of constant c.

b) Find marginal density functions of X and Y. Are X and Y independent?

5. Joint density of (X,Y) is given by

$$f(x,y)=\begin{cases} x+y & 0<x<1 \quad 0<y<1 \\ 0 & elsewhere \end{cases}$$

a) Find the density of Z = XY.

b) Find *Cov* (X,Y).