# HW 2

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1)

a) $w_i - w_0$ = Volume change, and cross-section area is a function of height $A(h)$

$$V(t) = \int_0^{h(t)} A(h')\,dh' , \quad \frac{dV(t)}{dt} = A(h(t))\,\dot{h}(t)$$

$$A(h(t))\,\dot{h}(t) = w_i - k\sqrt{\rho g h(t)}$$

$$\dot{h}(t) = \left[w_i - k\sqrt{\rho g h(t)}\right]\frac{1}{A(h(t))} = f(w_i, h)$$

$$\tilde{w}_i = w_i - w_i^{op}$$

$$\tilde{h}(t) = h(t) - h^{op}$$

$$\frac{d\tilde{h}(t)}{dt} = \left[\tilde{w}_i - k\sqrt{\rho g}\,\frac{1}{2h^{op}}\,\tilde{h}(t)\right]\frac{1}{A(h^{op})}$$

b) $\Delta p = k\sqrt{\rho g h(t)}$
$$\frac{d\Delta p}{dt} = \frac{1}{2}\,\frac{k\sqrt{\rho g}}{\sqrt{h(t)}}\,\frac{dh(t)}{dt}$$

$$\frac{2A(h(t))\sqrt{h(t)}}{k\sqrt{\rho g}}\,\frac{d\Delta p}{dt} = w_i - k\sqrt{\Delta p} \quad , \quad \tilde{\Delta} p = \Delta p - \Delta p^{op}$$

$$\frac{d\tilde{\Delta} p}{dt} = \left(\frac{k^2 \rho g\, \Delta p^{op}}{2A(h^{op})}\right)\tilde{w}_i - \frac{1}{2}\left(\frac{k^2 \rho g\, \Delta p^{op}}{2A(h^{op})\sqrt[2]{\Delta p^{3\,op}}}\right)\tilde{\Delta} p$$

c) $\dot{h}(t) = \left[ w_i - k\sqrt{\rho g h(t)} \right] \frac{1}{2A(h)}$

$$0 = w_i^{op} - k\sqrt{\rho g h^{op}}$$

$$u_{ss} = w_i^{op} = k\sqrt{\rho g h^{op}}$$

2) $\dot{x}_1 V = F x_{1f} - F x_1 + r_1 V \Rightarrow \dot{x}_1 = d x_{1f} - d x_1 + r_1$

$\dot{x}_2 V = F x_{2f} - F x_2 - r_2 V \Rightarrow \dot{x}_2 = d x_{2f} - d x_2 - r_2$

$x_{1f} = 0 \quad , \quad \frac{r_1}{r_2} = Y \quad , \quad r_1 = \mu(x_2) x_1$

$\dot{x}_1 = -d x_1 + \mu(x_2) x_1$

$\dot{x}_2 = d x_{2f} - d x_2 - \frac{\mu(x_2) x_1}{Y}$

b) $\mu = (\mu_m x_2)/(k_m + x_2)$

$0 = -d x_1 + \mu(x_2) x_1 = f_1(x_1, x_2) \quad , \quad 0 = d x_{2f} - d x_2 - \frac{\mu(x_2) x_1}{Y} = f_2(x_1, x_2, x_{2f})$

$\mu(x_2) = d \Rightarrow \frac{\mu_m x_2}{k_m + x_2} = d \Rightarrow x_2^{op} = \frac{d \cdot k_m}{\mu_m - d}$

$x_{2f}^{op} - x_2^{op} - \frac{x_1^{op}}{Y} = 0 \Rightarrow x_{2f}^{op} = x_2^{op} + \frac{x_1^{op}}{Y}$

c) $\tilde{x}_1(t) = x_1(t) - x_1^{op} \quad , \quad \tilde{x}_2(t) = x_2(t) - x_2^{op} \quad , \quad \tilde{x}_{2f}(t) = x_{2f} - x_{2f}^{op}$

$$\frac{d}{dt}\tilde{x}_1(t) = \left.\frac{\partial f_1(x_1, x_2)}{\partial x_1}\right|_{\substack{x_1 = x_1^{op} \\ x_2 = x_2^{op}}} (x_1(t) - x_1^{op}) + \left.\frac{\partial f_1(x_1, x_2)}{\partial x_2}\right|_{\substack{x_1 = x_1^{op} \\ x_2 = x_2^{op}}} (x_2(t) - x_2^{op})$$

$$\frac{d}{dt}\tilde{x}_1(t) = \left[-d + \frac{\mu_m x_2^{op}}{k_m + x_2^{op}}\right] \tilde{x}_1(t) + \frac{\mu_m k_m x_1^{op}}{(k_m + x_2^{op})^2} \tilde{x}_2(t)$$

$$\frac{d}{dt}\tilde{x}_2(t) = d\,\tilde{x}_{2f}(t) - \frac{\mu_m x_2^{op}}{(k_m + x_2^{op})Y}\tilde{x}_1(t) - \frac{\mu_m k_m x_1^{op}}{(k_m + x_2^{op})^2 Y}\tilde{x}_2(t)$$

d)

$$0 = -d x_1^{op} + \frac{\mu_m x_2^{op} x_1^{op}}{k_m + x_2^{op} + k_1 x_2^{2op}}$$

$$d = \frac{\mu_m x_2^{op}}{k_m + x_2^{op} + k_1 x_2^{2op}}$$

$$x_2^{op} = \frac{-\left(\frac{d - \mu_m}{d k_1}\right) \pm \sqrt{\left(\frac{d - \mu_m}{d k_1}\right)^2 - \frac{4 k_1}{k_m}}}{2}$$

$$x_{2f}^{op} = x_2^{op} + \frac{x_1^{op}}{Y}$$

3)

c) Two waveforms are periodic. If number of preys increasing number of predator also increasing. However, after some point prey population begin to decrease because of the number of the predator. When prey population begin to decrease number of predator starts to decrease after some time. Biochemical reactor question can be one of the example.

e) $0 = y_1^{op} - \frac{y_1^{op} y_2^{op}}{n_2}$

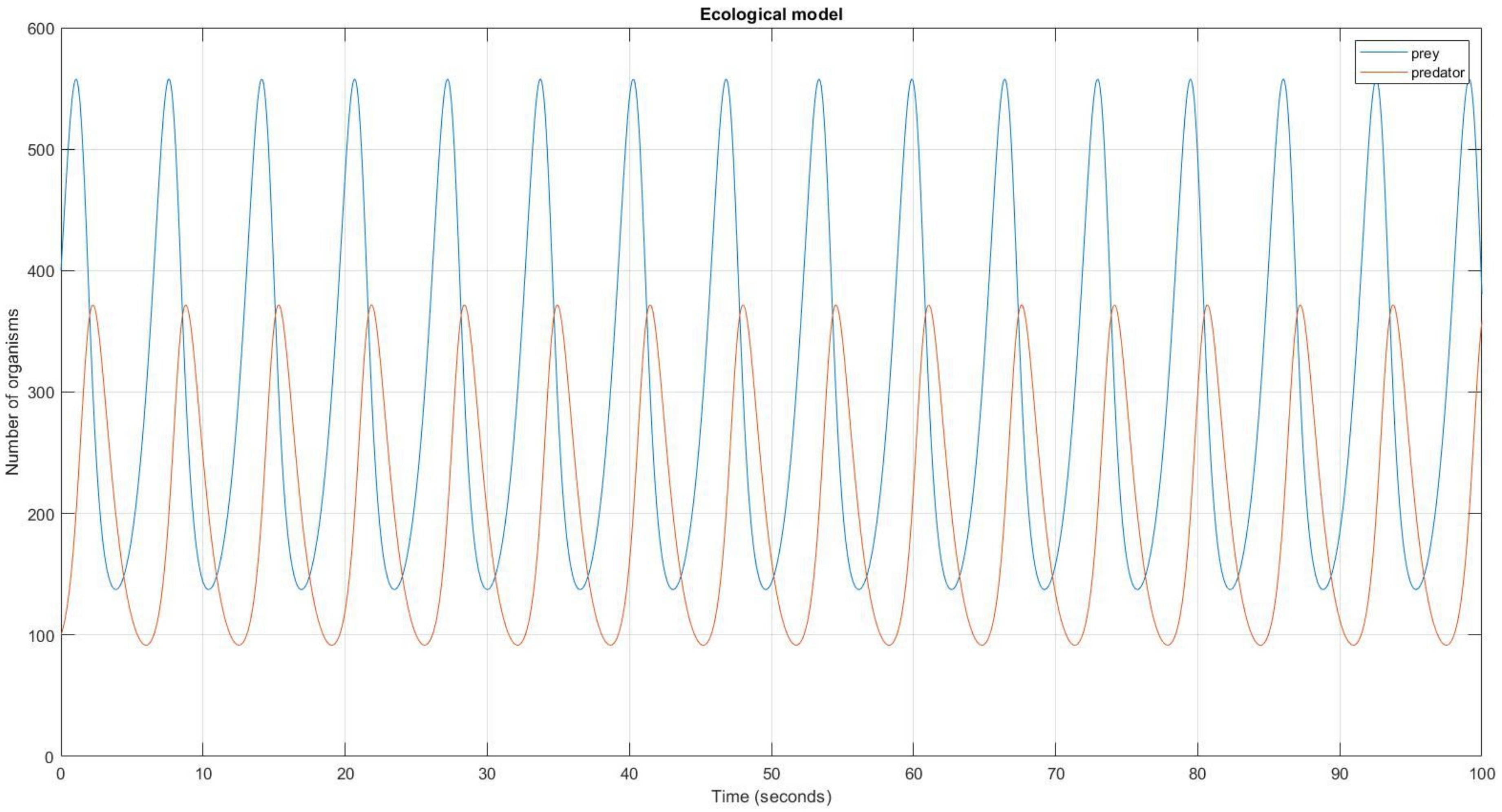
$y_2^{op} = n_2$

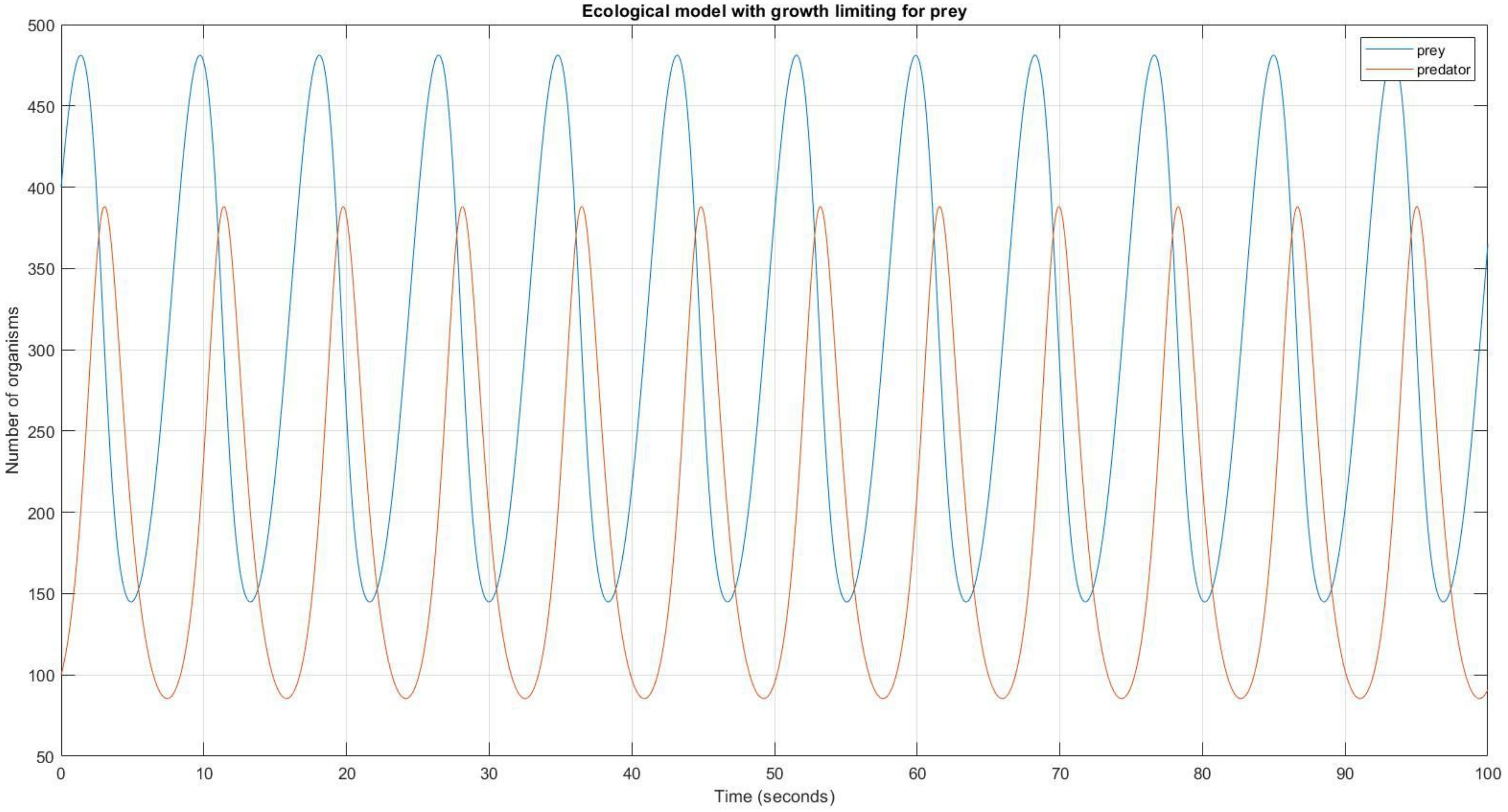
$y_1^{op} = n_1$

$0 = -y_2^{op} + \frac{y_1^{op} y_2^{op}}{n_1}$

If we set our initial conditions as equilibrium points number of both preys and predators stays constant.

g) They are still periodic but length of the period increased and maximum number of preys decreased.


Ecological model
prey
predator
Number of organisms
Time (seconds)
0
100
200
300
400
500
600
0
10
20
30
40
50
60
70
80
90
100



Ecological model with growth limiting for prey
prey
predator
Number of organisms
Time (seconds)
500
450
400
350
300
250
200
150
100
50
0
10
20
30
40
50
60
70
80
90
100