

1) Prove $L_2 = \{0^r 1^s 0^t : r, s, t \text{ are integers; } r > 0, t > 0, s \geq 0; s < r+t\}$ is not a regular language.

→ we have to use pumping lemma for regular languages for prove that this language is not a regular language.

Assume that L_2 is a regular language

then we have a constant c and string $w \in L_2$

we can break w into 3 pieces such as $x y z$

then $x y^i z$ is also $\in L_2 \quad i \geq 0$

$|xy| \leq c \quad \&\& \quad |y| > 0 (\text{pumping part})$

lets assume that we have a string

$s = 000111100 \quad (r=3) \&\& (s=4) \&\& (t=2)$

case1: If we chose $y=1$ pumping part the following pumping number of 1's is larger than sum of all 0's and it is not a member of L_2

case2: If we chose pumping part y any 0's than other pieces x and z has to take 2 elements at the same time and it is not possible for language.

According to all cases L_2 is not a regular language.

2) Use the CFL pumping lemma to show the given language is not to be context-free.

$L = \{a^n b^n c^m \mid n \leq m \leq 2n\}$

→ Assume that L is CFL

then we have pumping length p

and any string $z \in L$ and its length greater or equal to P

w can be splitted 5 pieces such as $u v w x y \in L$

and also $u (v^i) w (x^i) y \in L$

$v x \neq \epsilon \quad \&\& \quad |v w x| \leq p$

now we have rule $L = a^n b^n c^n c^{(m-n)} \quad \&\& \quad m < 2n$

lets assume that we have a string

$s = aaaabbbbcccccc \quad n=4 \quad m=6$

case 1: If we chose pumping part v and x as both 'a' every pumping number of a's greater than b' and continue greater than c and it is not a member of L {for ex($vwx=aaaa \& v=a \ w=a \ x=aa$)}

case 2: If we chose pumping part v and x both 'a' and 'b' (for ex $abbb \ v=a \ w=bb \ x=b$) the following pumping number of a's and b's will be greater than c. However number of c's must be greater than a' and b's according to rule and it is not a member of rule. Also v and x can not be hold two element like b's and c's too.

Case 3: If we chose pumping part v and x both 'c' the following pumping number of c's will be larger than $2n$ and it is not a member of L too

According to all cases L is not a CFL.