MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING
ME 310 NUMERICAL METHODS
FALL 2022
PROGRAMMING PROJECT 3

Assignment date : 12.12.2022
Due date : 26.12.2022

The programming project will be submitted through METU-Class, as described in the "Programming Project Assignment Guidelines", which is posted on METU-Class.

Given $n+1$ data points $x_1, x_2, \dots x_{n+1}$ and corresponding function values $y_1, y_2, \dots y_{n+1}$, it is desired to fit a quartic spline $S_k(x)$, where the subscript 'k' designates the spline number in between the two consecutive data points $[x_k, x_{k+1}]$

The equation of the k-th spline is defined by the following polynomial.

$$S_k(x) = s_{k,1} + s_{k,2}(x - x_k) + s_{k,3}(x - x_k)^2 + s_{k,4}(x - x_k)^3 + +s_{k,5}(x - x_k)^4$$

where $\boldsymbol{s} = [s_{k,1} \quad s_{k,2} \quad s_{k,3} \quad s_{k,4} \quad s_{k,5}\ ]^T$ are the unknown coefficients.

- Write down the necessary equations for a sample k[th] internal point and the two end points x_1, x_{n+1}
- Express the problem in matrix form as $\boldsymbol{As} = \boldsymbol{b}$ where $\boldsymbol{s}$ is the unknown coefficients.
- Write a computer program that forms $\boldsymbol{A}$ and $\boldsymbol{b}$ matrices and solves for $\boldsymbol{s}$.
  The input of your program should be $\boldsymbol{x} = [x_1 \quad \cdots \quad x_{n+1}]^T$ and $\boldsymbol{y} = [y_1 \quad \cdots \quad y_{n+1}]^T$ arrays.
  The output should be $n \times 5$ coefficient matrix in the form of

$$\boldsymbol{C} = \begin{bmatrix} s_{1,1} & s_{1,2} & s_{1,3} & s_{1,4} & s_{1,5} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{k,1} & s_{k,2} & s_{k,3} & s_{k,4} & s_{k,5} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{n,1} & s_{n,2} & s_{n,3} & s_{n,4} & s_{n,5} \end{bmatrix}$$

- In your computer program plot the spline obtained and its derivative, in separate figures, in between the data points x_{min} and x_{max}
- Your computer program should ask for and input and print the related spline value and its derivative at that point.

Notes:

You should use the given polynomial description; any other forms will not be graded.

In the first spline $S_1(x)$ take. the second and third derivatives as zero. Assume the n[th] spline $S_n(x)$ has a natural boundary condition with the third derivative equal to zero.

Your program should read the $\boldsymbol{x}$ and $\boldsymbol{y}$ arrays from a given "input.txt" file, output the coefficient matrix C on screen and plot the x-y points and the splines (between the end points).
A sample "input.txt" is attached.

You may use readily available equation solvers.