Middle East Technical University
Department of Mechanical Engineering
ME 310 Numerical Methods
Fall 2022
Programming Project 2

The programming project will be submitted through METU-Class, as described in the "Programming Project Assignment Guidelines", which is posted on METU-Class.

Assignment date:	23.11.2022
Due date:	07.12.2022

Suppose that N particles are placed at equally spaced points, $x_0, x_1, x_2, \ldots, x_{N-1}$ in the bounded domain $[0, 1]$. Any particle with index i is located at x_i and denoted by p_i. The particles p_{i-1} and p_{i+1} are always left and right neighbors of the particle p_i, respectively.

The position of the particle located on the left-most location (i.e. p_0 at point $x_0 = 0$) moved with the function $f(t) = 0.25 \times sin(\pi t)$. As an example, the particle p_0 is moved from 0 to 0.25 at $t = 0.5$ and from 0 to -0.25 at $t = 1.5$. The particle's position at the right-most location stays fixed (i.e. $x_{N-1} = 1.0$ does not change).

If the neighbors of any particle **do not** change after moving the particle p_0, distribution of the internal particles (i.e. $i = 1, 2, 3, \ldots, N-2$) can be obtained with the following relation,

$$x'_{i+1} - 2x'_i + x'_{i-1} = 0 \tag{1}$$

a) By writing the first and the last 3 equations show that

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ 1 & -2 & 1 & 0 & \ldots & 0 \\ 0 & 1 & -2 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & 1 & -2 & 1 \\ 0 & \ldots & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'_0 \\ x'_1 \\ x'_2 \\ \vdots \\ x'_{N-2} \\ x'_{N-1} \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \\ 0 \\ \vdots \\ 0 \\ x_{N-1} \end{bmatrix},$$

is the linear system of equations solving the new position of particles, x'.

b) Write a computer program that solves this problem using the Gauss Elimination method taking number of particles, N and time, t as the inputs.

c) Write a computer program that solves this problem using the Gauss-Siedel method taking number of particles, N and time, t as the inputs.

- Your code should be named *e123456.ext*, where *.ext* is *.m*, *.py*, *.c*, *.cpp* etc. depending on your language of preference,

- Your code should benefit from the special structure of the system and should avoid any redundant **operation** and **storage**.
- The inputs should be taken from the command prompt similar to the following:

  Enter the number of particles: < user input >

  Enter the time instant: < user input >

  Enter the tolerance to terminate: < user input > (*for Gauss-Siedel only*)

  Enter the maximum number of iterations to terminate: < user input > (*for Gauss-Siedel only*)

Present your results in a short report (a few pages of a text document only, saved as a .pdf document) which should include the following:

- A basic introduction paragraph,
- Necessary formulations and hand calculations to write your code, and your derivation in part (a),
- Your numerical results i.e. plots, tables etc.,
- Your line plots should include the evolution of the solution by plotting the position of particles, x' at different time instants, t for the same number of particles,N,
- Your line plots should include the evolution of the solution by plotting the position of particles, x' at different number of particles, N at the same time,t
- Discussion of the results and conclusion,
- Appendix section including your code and executable file in the case programming languages such as C/C++, Fortran etc. are used.