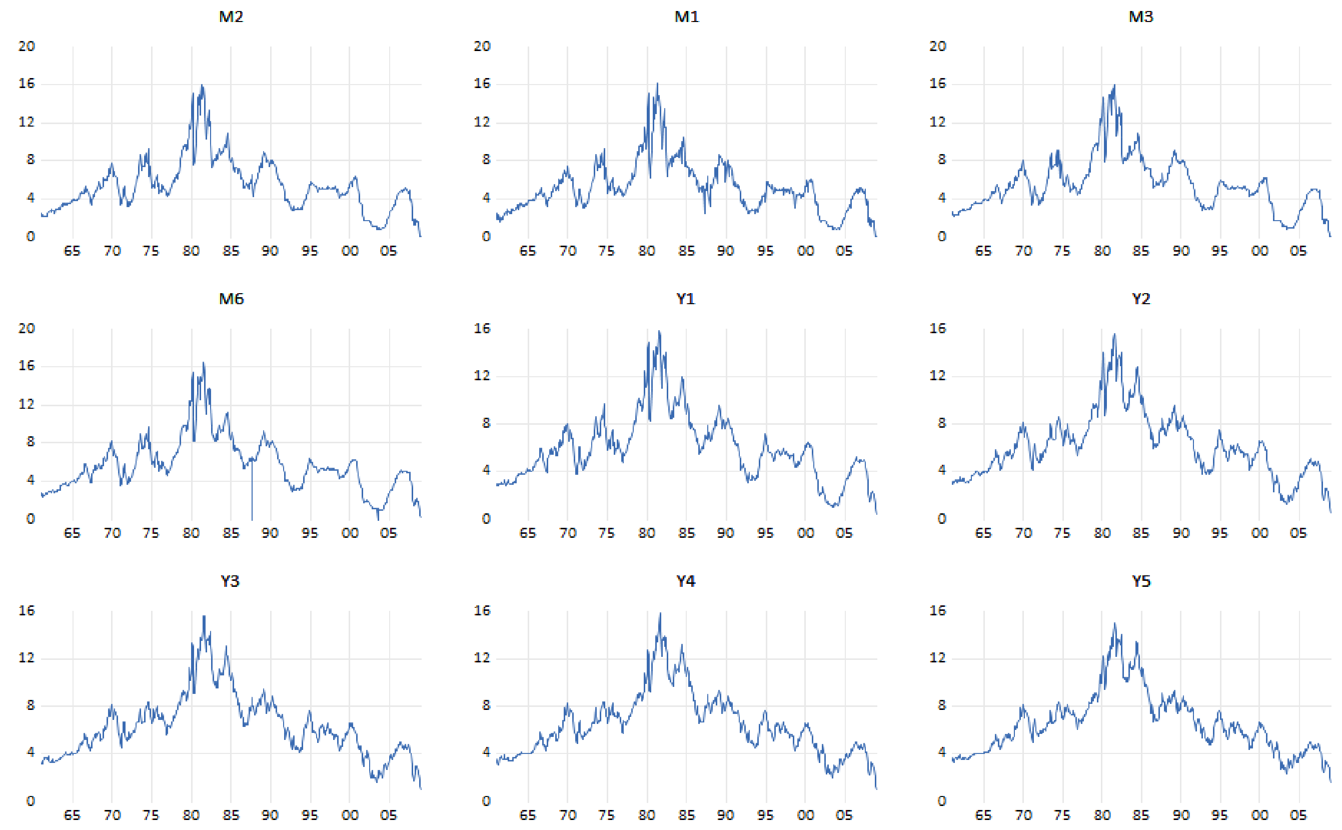
ANALYSIS OF THE SERIES ON THE US TERM STRUCTURE



Before checking for a unit root, we graph them to see or to have an idea how would our series look like and behave graphically. Series that have unit root lie outside of the stationary and ergodic processes. They are not mean reverting and their variances depend on time. The graphs we have above seem to follow this pattern, their values do not go around the a mean and the variance is not constant, therefore the series we have, **look** non-stationary. This, in turn, might lead to a spurious relationship. This interpretation goes for all the series above since all of them look really similar to each other with have almost the same behaviour. That was actually expected since the paper that is provided as a supporting material for this project suggests that in general the treasury yields and interest rates are considered to be I(1) series. As a quick remainder, we can add here that integrated processes are those that have autocovariances greater than 0 and finite where the past shocks never die out but instead accumulate over time.

Since the data generating process of our series do not evolve around 0, we will add a constant while conducting the unit root test. It therefore will have an intercept and we will use Bayes information (SIC) criteria to choose the lags and will use Augmented Dickey-Fuller test for the unit root.

The ADF test on the series **M1** is run and the result is:

Null Hypothesis: M1 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.959225	0.0395
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M1)
Method: Least Squares
Date: 11/07/20 Time: 17:49
Sample (adjusted): 1961M02 2008M12
Included observations: 575 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1(-1)	-0.032005	0.010815	-2.959225	0.0032
C	0.163464	0.063216	2.585803	0.0100

R-squared	0.015053	Mean dependent var	-0.003335
Adjusted R-squared	0.013334	S.D. dependent var	0.690930
S.E. of regression	0.686308	Akaike info criterion	2.088491
Sum squared resid	269.8935	Schwarz criterion	2.103637
Log likelihood	-598.4411	Hannan-Quinn criter.	2.094398
F-statistic	8.757015	Durbin-Watson stat	2.152001
Prob(F-statistic)	0.003212		

We can see that the P value of the constant is significant, so we did good by adding an intercept in our analysis. We do not consider the P values attached to the variables M1(-1) and C, since the Dickey-Fuller test has its own distribution which EViews provide us. The t-stats which is -2.9592(coefficient of M1(-1)/ it's standard error) that is smaller than the %1 percent level in absolute value, it falls within the non-rejection region (The null hypothesis is that M1 has a unit root) as we expected from its graph above. M1 has a unit root, we fail to reject the null hypothesis at 1 percent level. It is not stationary in levels and M1 is I (1). The SIC criteria did not choose any lag for us, probably that is the best parsimonious model for this series. However, it is important to note that: we generally use %5 as a threshold but as we will see for the other series and since they are interest rates, given the dynamics of the other series it makes sense to consider it as an I(1).

The test on the series **M2** is run and the result is:

Null Hypothesis: M2 has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.125743	0.2347
Test critical values:	1% level	-3.441533	
	5% level	-2.866365	
	10% level	-2.569399	

*MacKinnon (1996) one-sided p-values.

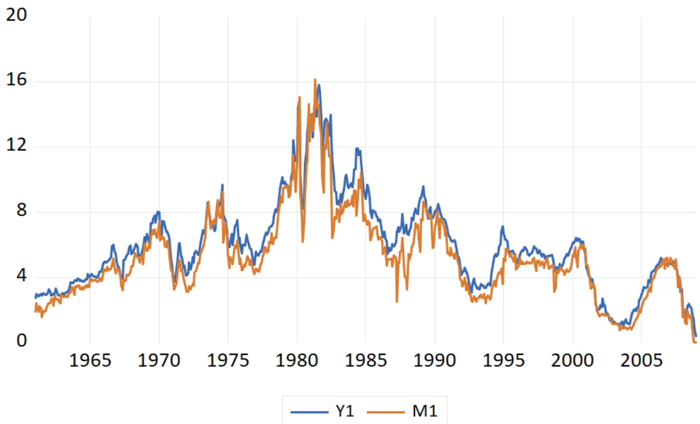
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M2)
Method: Least Squares
Date: 11/07/20 Time: 20:23
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M2(-1)	-0.017114	0.008051	-2.125743	0.0340
D(M2(-1))	0.170229	0.041582	4.093832	0.0000
D(M2(-2))	-0.116236	0.041702	-2.787311	0.0055
C	0.089804	0.049104	1.828877	0.0679

R-squared	0.044108	Mean dependent var	-0.003939
Adjusted R-squared	0.039069	S.D. dependent var	0.533851
S.E. of regression	0.523319	Akaike info criterion	1.549706
Sum squared resid	155.8280	Schwarz criterion	1.580078
Log likelihood	-439.9906	Hannan-Quinn criter.	1.561553
F-statistic	8.751929	Durbin-Watson stat	1.986254
Prob(F-statistic)	0.000011		

This time Bayes information criteria chose two lags and t-stats for this test is -2.125 which is well within the rejection region at all critical value levels, so the null hypothesis that the M2 series have a unit root is not rejected. Therefore, M2 is not stationary in levels and it is of I (1). So except the series M1, all the series have a unit root at 5% level.

COINTEGRATION REGRESSIONS



It looks like there is a high correlation between the two series, since none of them are stationary, meaning that none of them have constant mean and variance, the regression result might be spurious .After having checked our series if they had a unit root and saw that all of them have it, we need to see the relationship between them to have an idea about them. Since, an ordinary regression analysis between non-stationary series might be spurious, we need another test that takes into account this fact. But let's firstly see an example and how it looks like;

Dependent Variable: M1
Method: Least Squares
Date: 11/08/20 Time: 13:32
Sample: 1961M01 2008M12
Included observations: 576

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.288976	0.063052	-4.583141	0.0000
Y1	0.919597	0.009560	96.18959	0.0000

R-squared	0.941586	Mean dependent var	5.202562
Adjusted R-squared	0.941484	S.D. dependent var	2.655165
S.E. of regression	0.642285	Akaike info criterion	1.955895
Sum squared resid	236.7919	Schwarz criterion	1.971021
Log likelihood	-561.2979	Hannan-Quinn criter.	1.961794
F-statistic	9252.438	Durbin-Watson stat	0.774853
Prob(F-statistic)	0.000000		

When we regress M1 on a constant and Y1, we find that the t-statistic is -4.58 and P value is 0.000, therefore the relationship is significant. However, it might be a spurious relationship. The R square is higher than Durbin-Watson stat which is a sign of a spurious relationship and we know that our series are non-stationary, that's is why we should do a cointegration test which will allow us to see the long run relationship between the series. Two sets of variables are cointegrated if a linear combination of those variables have a lower order integration. Notice that M1 and Y1 is of I(1), so we need a number Beta such that when we multiply Yt by it, the distance between M1 and Y1 will be constant and the difference between them will be a stationary series of I(0).

So, the theory (Hall et all) says that, the spreads between r(k,t) and r(1,t) are the stationary linear combinations of X(t) that we are looking for. The spread is R(i,t) -R(j,t)=S(i,j,t). The model predicts that any yield series is cointegrated with the one period yield, so that if we were to consider a set of

n yield series(which included the one period yield), then each of the (n-1), n dimensional spread vectors is cointegrating for the vector x(t)=[R(1,t),R(K2,t),R(K3,t),….,R(Kn,t)] transposed where Ks are the maturities of the other (n-1) bills. As these spread vectors are linearly independent, the cointegration space has rank (n-1).

Since we have 9 series, according to the theory, we should have 8 cointegrating spread vectors. Let's see what the Johanson cointegration test tells about it. Before doing it, we use Bayesian information criteria (SIC) to decide about the model and number of lags which it chooses the model 2 with 2 lagged values. Now we can run the test. The test is run in levels but we also now that the log-transformation of the raw variables can be used.

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.351660	796.6666	208.4374	0.0000
At most 1 *	0.233595	548.3628	169.5991	0.0000
At most 2 *	0.172224	395.9193	134.6780	0.0000
At most 3 *	0.134907	287.6148	103.8473	0.0000
At most 4 *	0.130491	204.5764	76.97277	0.0000
At most 5 *	0.096233	124.4558	54.07904	0.0000
At most 6 *	0.069154	66.47747	35.19275	0.0000
At most 7 *	0.038675	25.41518	20.26184	0.0089
At most 8	0.004900	2.814507	9.164546	0.6159

Trace test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.351660	248.3038	59.24000	0.0000
At most 1 *	0.233595	152.4435	53.18784	0.0000
At most 2 *	0.172224	108.3045	47.07897	0.0000
At most 3 *	0.134907	83.03841	40.95680	0.0000
At most 4 *	0.130491	80.12062	34.80587	0.0000
At most 5 *	0.096233	57.97830	28.58808	0.0000
At most 6 *	0.069154	41.06229	22.29962	0.0000
At most 7 *	0.038675	22.60067	15.89210	0.0038
At most 8	0.004900	2.814507	9.164546	0.6159

Max-eigenvalue test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

We are interested in Trace and Max-Eigen Statistic. What None implies is that ''**there is no cointegration''.** The trace statistic is greater than the critical value and the p value is lower than %5 so we reject the H0 that **there is no cointegration.** At most 1 says that there is **at most one cointegration equation.** Well, we check the trace statistic which is way beyond the critical value of %5 and p value is 0, so we reject the H0 that **there is at most one cointegration.** The H0 is rejected until and included at most 7 equations, while we fail to reject at most 8 cointegration relations. This fits the theory. We have 8 cointegrating equations, since we had 9 series, the theory seems to be approved by the test. So, here we have n-1 linearly independent spread vectors defined using one-period will comprise a basis for the cointegration space.

This means that if we consider all the spreads containing M1, and they are all stationary, the n-1 spreads will be the cointegrating vectors. For example, let's consider Y1-m1 and suppose that the spreads between them is stationary. So, the n-dimensional vector (-1, 0, 0, 0, 1, 0, 0, 0, 0) is a cointegration vector. If we do the analysis on all the spreads and if they all turn out to be stationary, they are basis for all the cointegrating vectors, or in other words, the cointegrating vectors are linear combinations of these spread vectors.

To see how all the cointegration relations look like, using information criteria, we find that SIC selects only one lag. Our cointegration rank is 8 as shown before and the model is the second one with an intercept but not a trend.

Vector Error Correction Estimates
Date: 11/20/20 Time: 20:32
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	CointEq2	CointEq3	CointEq4	CointEq5	CointEq6	CointEq7	CointEq8
M1(-1)	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
M2(-1)	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
M3(-1)	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000
M6(-1)	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000
Y1(-1)	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000
Y2(-1)	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000
Y3(-1)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000
Y4(-1)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000
Y5(-1)	-0.888461	-0.945317	-0.970629	-0.988963	-0.998824	-1.021621	-1.013705	-1.010163
	(0.07558)	(0.07507)	(0.07459)	(0.07334)	(0.06086)	(0.03662)	(0.02134)	(0.00953)
	[-11.7548]	[-12.5917]	[-13.0121]	[-13.4841]	[-16.4107]	[-27.8996]	[-47.5085]	[-105.960]
C	0.630206	0.769154	0.802416	0.731441	0.588322	0.534519	0.311498	0.153240
	(0.53326)	(0.52968)	(0.52629)	(0.51746)	(0.42942)	(0.25835)	(0.15054)	(0.06726)
	[1.18179]	[1.45211]	[1.52466]	[1.41351]	[1.37004]	[2.06896]	[2.06916]	[2.27825]

Here, each column is a cointegration vector. In the long run the series are related with equations above. For example, **ECMt** = **M1t** – 0.888 **Y5t** + 0.630 becomes **M1t** = 0.888 **Y1t** - 0.630+ **ut** where ut is the stationary error term. M1 and Y5 move with this relation in the long run. M2 and Y5 is the same, the interpretation goes for all.

Error Correction:	D(M1)	D(M2)	D(M3)	D(M6)	D(Y1)	D(Y2)	D(Y3)	D(Y4)	D(Y5)
CointEq1	-1.014230	-0.125796	-0.126877	0.018695	-0.064095	-0.029884	-0.057521	-0.030110	-0.036744
	(0.10802)	(0.09185)	(0.09415)	(0.10810)	(0.09231)	(0.08197)	(0.07690)	(0.07354)	(0.06868)
	[-9.38932]	[-1.36956]	[-1.34760]	[0.17294]	[-0.69436]	[-0.36456]	[-0.74803]	[-0.40944]	[-0.53500]
CointEq2	1.133626	-0.746359	0.234861	-0.173129	-0.070250	-0.092948	-0.099202	-0.097684	-0.015489
	(0.29457)	(0.25048)	(0.25675)	(0.29479)	(0.25173)	(0.22354)	(0.20970)	(0.20054)	(0.18729)
	[3.84843]	[-2.97974]	[0.91475]	[-0.58728]	[-0.27907]	[-0.41581]	[-0.47307]	[-0.48710]	[-0.08270]
CointEq3	-0.404309	0.782295	-0.341694	0.596852	0.315734	0.306768	0.361564	0.381262	0.268916
	(0.29131)	(0.24770)	(0.25391)	(0.29153)	(0.24894)	(0.22106)	(0.20738)	(0.19832)	(0.18522)
	[-1.38791]	[3.15817]	[-1.34575]	[2.04730]	[1.26832]	[1.38771]	[1.74352]	[1.92244]	[1.45190]
CointEq4	0.296967	0.218314	0.219047	-0.777197	0.109555	0.055645	0.059753	0.040554	0.060684
	(0.10321)	(0.08776)	(0.08996)	(0.10329)	(0.08820)	(0.07832)	(0.07347)	(0.07027)	(0.06562)
	[2.87727]	[2.48755]	[2.43494]	[-7.52435]	[1.24212]	[0.71046]	[0.81325]	[0.57714]	[0.92473]
CointEq5	-0.075716	-0.326306	-0.133548	0.214693	-0.769849	-0.383880	-0.540369	-0.587700	-0.547713
	(0.20896)	(0.17769)	(0.18214)	(0.20913)	(0.17857)	(0.15857)	(0.14876)	(0.14226)	(0.13286)
	[-0.36234]	[-1.83641]	[-0.73324]	[1.02662]	[-4.31115]	[-2.42082]	[-3.63254]	[-4.13109]	[-4.12241]
CointEq6	0.170246	0.423764	0.402909	0.543117	0.716389	0.183466	0.656235	0.480826	0.440261
	(0.32582)	(0.27706)	(0.28399)	(0.32608)	(0.27844)	(0.24725)	(0.23195)	(0.22182)	(0.20716)
	[0.52251]	[1.52953]	[1.41874]	[1.66562]	[2.57291]	[0.74201]	[2.82923]	[2.16763]	[2.12519]
CointEq7	-0.494879	-0.480804	-0.352663	-0.555003	-0.065006	0.009267	-0.442249	0.226148	0.030559
	(0.40247)	(0.34223)	(0.35080)	(0.40278)	(0.34393)	(0.30542)	(0.28651)	(0.27400)	(0.25590)
	[-1.22960]	[-1.40491]	[-1.00532]	[-1.37793]	[-0.18901]	[0.03034]	[-1.54356]	[0.82535]	[0.11942]
CointEq8	0.531099	0.265780	-0.100697	-0.048171	-0.259783	-0.308158	-0.216460	-0.892340	-0.334446
	(0.37754)	(0.32103)	(0.32907)	(0.37783)	(0.32263)	(0.28650)	(0.26877)	(0.25703)	(0.24005)
	[1.40672]	[0.82789]	[-0.30600]	[-0.12749]	[-0.80520]	[-1.07559]	[-0.80538]	[-3.47172]	[-1.39325]

The Second part of the output shows the results from the second step Var in first differences, including the error correction terms. CointEg,js are the adjustment coefficients. When the series are cointegrated, we need also the adjustment coefficients. M1 will adjust in the long run with its coefficient which is -1.014230.

Here, we can have a clear look at the adjustment coefficients as equations;

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D(M1) = - 1.01423015399*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) + 1.13362637933*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) - 0.404308565142*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) + 0.296967009878*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) - 0.075715633458*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.170245520025*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) - 0.494879242522*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) + 0.531098592817*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) - 0.0444706586516*D(M1(-1)) - 0.155683613799*D(M2(-1)) + 0.476463855661*D(M3(-1)) - 0.123641736816*D(M6(-1)) - 0.25736405489*D(Y1(-1)) + 0.403635618684*D(Y2(-1)) + 0.216563641807*D(Y3(-1)) + 0.438549038915*D(Y4(-1)) - 0.619524243442*D(Y5(-1))

D(M2) = - 0.12579580784*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) - 0.746358661173*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) + 0.782294729219*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) + 0.218314459536*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) - 0.326305534095*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.423764060026*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) - 0.480803689961*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) + 0.265780045494*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) + 0.0509890405648*D(M1(-1)) - 0.104152154399*D(M2(-1)) + 0.16054226946*D(M3(-1)) - 0.0813491615675*D(M6(-1)) - 0.0624783500554*D(Y1(-1)) + 0.341846243408*D(Y2(-1)) + 0.321242390991*D(Y3(-1)) + 0.185744532051*D(Y4(-1)) - 0.578094273455*D(Y5(-1))

D(M3) = - 0.126877247633*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) + 0.234861203322*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) - 0.341693977504*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) + 0.21904742332*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) - 0.133548314561*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.402908691908*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) - 0.352663338709*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) - 0.10069672505*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) + 0.0524027061803*D(M1(-1)) - 0.127906776483*D(M2(-1)) + 0.110159573059*D(M3(-1)) - 0.0292051461234*D(M6(-1)) - 0.187150352387*D(Y1(-1)) + 0.451940176662*D(Y2(-1)) + 0.283142198826*D(Y3(-1)) + 0.278004997234*D(Y4(-1)) - 0.635357504023*D(Y5(-1))

D(M6) = 0.0186954667965*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) - 0.173128566376*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) + 0.596851931109*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) - 0.777196743934*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) + 0.214692592453*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.543117463308*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) - 0.555003050971*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) - 0.0481713662729*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) + 0.018445904106*D(M1(-1)) - 0.0513230421647*D(M2(-1)) + 0.0770488548203*D(M3(-1)) - 0.00112540814575*D(M6(-1)) - 0.137721002568*D(Y1(-1)) + 0.0184308205153*D(Y2(-1)) + 0.718639917867*D(Y3(-1)) + 0.189795153919*D(Y4(-1)) - 0.638334537441*D(Y5(-1))

D(Y1) = - 0.0640954026915*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) - 0.0702496841097*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) + 0.315734230965*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) + 0.109554961999*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) - 0.769849485451*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.716388625766*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) - 0.0650057568378*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) - 0.259782673618*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) + 0.0354901109376*D(M1(-1)) - 0.10425534644*D(M2(-1)) + 0.0477188462062*D(M3(-1)) + 0.0325727932054*D(M6(-1)) - 0.030298323227*D(Y1(-1)) - 0.0749718534603*D(Y2(-1)) + 0.640234869093*D(Y3(-1)) + 0.254800828303*D(Y4(-1)) - 0.615765097589*D(Y5(-1))

D(Y2) = - 0.0298837889803*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) - 0.0929483398984*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) + 0.306768420435*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) + 0.0556450522133*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) - 0.383879806989*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.183466147597*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) + 0.00926728356182*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) - 0.308157580802*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) + 0.0246805471993*D(M1(-1)) + 0.0356967119349*D(M2(-1)) - 0.139591639041*D(M3(-1)) + 0.0263768739061*D(M6(-1)) + 0.0921704512751*D(Y1(-1)) - 0.240197974987*D(Y2(-1)) + 0.711137901737*D(Y3(-1)) + 0.229656590831*D(Y4(-1)) - 0.571329642477*D(Y5(-1))

D(Y3) = - 0.0575212583232*( M1(-1) - 0.888460729483*Y5(-1) + 0.630206452789 ) - 0.0992022369155*( M2(-1) - 0.945316945757*Y5(-1) + 0.769154265328 ) + 0.361563987304*( M3(-1) - 0.970629291564*Y5(-1) + 0.802416466384 ) + 0.0597527033167*( M6(-1) - 0.988963346964*Y5(-1) + 0.731441230724 ) - 0.540369053172*( Y1(-1) - 0.998823699137*Y5(-1) + 0.588322282034 ) + 0.656235151929*( Y2(-1) - 1.02162064431*Y5(-1) + 0.534519050156 ) - 0.442248512988*( Y3(-1) - 1.01370530903*Y5(-1) + 0.311498111689 ) - 0.216459743414*( Y4(-1) - 1.01016252605*Y5(-1) + 0.153240353437 ) + 0.0310408999071*D(M1(-1)) + 0.0667247421512*D(M2(-1)) - 0.240610859502*D(M3(-1)) + 0.00629020118856*D(M6(-1)) + 0.146078908552*D(Y1(-1)) - 0.0688319780971*D(Y2(-1)) + 0.481234457896*D(Y3(-1)) + 0.00277019718184*D(Y4(-1)) - 0.33226058712*D(Y5(-1))
```

Now, we analyse the pairs Y1-M1. They are cointegrated and their cointegration rank is 1. The VEC with 1 cointegration rank and model 2 is:

Vector Error Correction Estimates
Date: 11/20/20 Time: 21:02
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
Y1(-1)	1.000000	
M1(-1)	-1.083608 (0.02696) [-40.1991]	
C	-0.332906 (0.15729) [-2.11652]	
Error Correction:	**D(Y1)**	**D(M1)**
CointEq1	-0.019925 (0.03594) [-0.55435]	0.277318 (0.04449) [6.23268]
D(Y1(-1))	0.145107 (0.05684) [2.55271]	0.403331 (0.07037) [5.73189]
D(Y1(-2))	-0.111014 (0.05632) [-1.97114]	-0.173028 (0.06972) [-2.48187]
D(M1(-1))	-0.003890 (0.04720) [-0.08241]	-0.151731 (0.05843) [-2.59698]
D(M1(-2))	0.029184 (0.04237) [0.68885]	0.044225 (0.05244) [0.84327]

M1(-1) is not exactly 1, so we can impose a restriction. We can also restrict CointEq1 to be 0 since it is very close to zero. Restriction $A1$=0 has a very important consequence: it means that variable Y1 does not adjust toward the equilibrium in ECM, and all the adjustment is made by M1.

Vector Error Correction Estimates
Date: 11/20/20 Time: 21:26
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:	
B(1,1)=1, B(1,2)=-1, A(1,1)=0	
Convergence achieved after 2 iterations.	
Restrictions identify all cointegrating vectors	
LR test for binding restrictions (rank = 1):	
Chi-square(2)	10.29132
Probability	0.005825

Cointegrating Eq:	CointEq1
Y1(-1)	1.000000
M1(-1)	-1.000000
C	-0.770681 (0.07608) [-10.1304]

Error Correction:	D(Y1)	D(M1)
CointEq1	0.000000 (0.00000) [NA]	0.272013 (0.03540) [7.68359]
D(Y1(-1))	0.160692 (0.05665) [2.83675]	0.431863 (0.07085) [6.09564]
D(Y1(-2))	-0.101078 (0.05640) [-1.79217]	-0.161697 (0.07054) [-2.29232]
D(M1(-1))	-0.018453 (0.04627) [-0.39878]	-0.190749 (0.05787) [-3.29594]

We imposed the restriction that B(1,1)=1,B(2,1)=-1 and A(1,1)=0 but the P value turned out be smaller than %5 level so we reject the validity of this restriction. So, we go back to unrestricted model ; Y1= 1.0836M1+0.33+ut is the cointegration relation between Y1 and M1. That's how they move in the long run. Y1 will adjust in the long run with the coefficient -0.019925. As we can see, the adjustment speed of M1 is much higher than Y1. The effect of M1 and M2's effect on Y1 is very small. While Y1's effects on M1 are higher. What does it mean in economic terms? It means that the short run interest rates(M) are adjusting themselves with respect to long run interest rates. Here, the spread is not a cointegration vector. The long run interest rates are adjusting with a very low speed and also the effect of short run interest rates on long run are much lower. So, we cannot say that this result is exactly what PEH says but it is somehow consistent.

I would like conclude this analysis with a Granger Causality test even if it is not requested. We sometimes are interested in testing if one variable anticipates the movement of another one. For example, if *x* anticipates *y*. This could be done by testing if the coefficients of past *x* are significantly different in the equation for *y*.
This may be interesting when forecasting; interpreting this as causality however requires additional assumptions.

VEC Granger Causality/Block Exogeneity Wald Tests
Date: 11/20/20 Time: 21:46
Sample: 1961M01 2008M12
Included observations: 573

Dependent variable: D(Y1)

Excluded	Chi-sq	df	Prob.
D(M1)	0.662396	2	0.7181
All	0.662396	2	0.7181

Dependent variable: D(M1)

Excluded	Chi-sq	df	Prob.
D(Y1)	46.61604	2	0.0000
All	46.61604	2	0.0000

The output is divided on two parts. So, when Y1 is dependent variable and M1 is excluded, the question is ''M1 does not cause Y''. P value is above 5%, 0.7181 so M1 does not cause Y1.

When M1 is dependent and Y1 is excluded, the question is ''Y1 does not cause M1''. P value is 0. We reject the hypothesis that Y does not cause M1. So, Y granger causes M1. We know granger causality is not a real causality but this is not the place to discuss about it.