Семинар 5
SVM и
Логистическая регрессия

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ЗАДАЧА 3.5

$$\mathrm{p}(y = 1|x) = \frac{1}{1 + e^{-\langle w,x \rangle}}$$

ЗАДАЧА 3.5

$$e^{-\langle w,x\rangle} = \frac{1}{\mathrm{p}(y=1|x)} - 1 = \frac{1-p(y=1|x)}{p(y=1|x)} = \frac{p(y=0|x)}{p(y=1|x)}$$

ЗАДАЧА 3.5

$$-\langle w, x\rangle = log\left(\frac{p(y=0|x)}{\mathrm{p}(y=1|x)}\right)$$

ЗАДАЧА 3.5

$$\langle w, x \rangle = log\left(\frac{p(y=1|x)}{\mathrm{p}(y=0|x)}\right)$$

ЗАДАЧА 3.6

$$L(y, b) = ([y = 1] - b)^2$$

$$\frac{1}{n}\sum_{i=1}^{n} L(y_i, b_i) \to \min_b$$

ЗАДАЧА 3.6

Согласно ЗБЧ (закон больших чисел):

$$\frac{1}{n}\sum_{i=1}^{n} L(y_i, b_i) \xrightarrow{P} \mathbb{E}(L(y, b|x))$$

Сходимость по вероятности означает, что:

$$\lim_{n\to\infty} \mathbb{P}\left(\left|\frac{1}{n}\sum_{i=1}^{n} L(y_i, b_i) - \mathbb{E}(L(y, b|x))\right| > \varepsilon\right) = 0\,, \forall\, \varepsilon > 0$$

ЗАДАЧА 3.6

Будем минимизировать математическое ожидание:

$$\mathbb{E}(L(y, b|x) = p(y = 1|x) * (b - 1)^2 + p(y = 0|x) * (b - 0)^2$$

ЗАДАЧА 3.6

$$p(y = 1|x) = 1 - p(y = 0|x) = p$$

$$\mathbb{E}(L(y, b|x) = p * (b - 1)^2 + (1 - p) * b^2 \to \min_b$$

ЗАДАЧА 3.6

$$\frac{\partial \mathbb{E}(L(y,b)|x)}{\partial b} = 2p(b-1) + 2(1-p)b = 2b - 2p = 0$$

ЗАДАЧА 3.6

$$b = p = p(y = 1|x)$$

ЗАДАЧА 3.4

$$L = -\sum_{i=1}^{100} (y_i \ln p_i + (1 - y_i) \ln(1 - p_i)) \to \min$$

$$p = \frac{1}{1 + e^{-w_0 - w_1 x}}$$

ЗАДАЧА 3.4

$$1 - p = 1 - \frac{1}{1+e} = \frac{1+e-1}{1+e} = \frac{e}{1+e}$$

$$L$$

$$= -\sum_{i=1}^{100} \left(y_i \ln\left(\frac{1}{1+e^{-w_0-w_1x_i}}\right) + (1-y_i) \ln\left(\frac{e^{-w_0-w_1x_i}}{1+e^{-w_0-w_1x_i}}\right)\right)$$

ЗАДАЧА 3.4

$$L = \sum_{i=1}^{100} (y_i \ln(1 + e^{-w_0 - w_1 x_i}) - (1 - y_i)(-w_0 - w_1 x_i) + (1 - y_i) \ln(1 + e^{-w_0 - w_1 x_i}))$$

ЗАДАЧА 3.4

$$L = \sum_{i=1}^{100} \left((1 - y_i)(w_0 + w_1 x_i) + \ln(1 + e^{-w_0 - w_1 x_i}) \right) \to \min_{w_0, w_1}$$

ЗАДАЧА 3.4

$$\frac{\partial L}{\partial w_0} = \sum_{i=1}^{100} \left(1 - y_i + \frac{1}{1 + e^{-w_0 - w_1 x_i}} (-1) e^{-w_0 - w_1 x_i}\right) = 0$$

$$\frac{\partial L}{\partial w_1} = \sum_{i=1}^{100} \left((1 - y_i) x_i + \frac{1}{1 + e^{-w_0 - w_1 x_i}} (-x_i) e^{-w_0 - w_1 x_i}\right) = 0$$

ЗАДАЧА 3.4

$y = 1, x = 1\ -12$ наблюдений

$$\frac{\partial L}{\partial w_0} = -\frac{12}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1}$$

$$\frac{\partial L}{\partial w_1} = -\frac{12}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1}$$

ЗАДАЧА 3.4

$y = 0, x = 1$ – 36 наблюдений

$$\frac{\partial L}{\partial w_0} = 36 - \frac{36}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1}$$

$$\frac{\partial L}{\partial w_1} = 36 - \frac{36}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1}$$

ЗАДАЧА 3.4

$y = 1, x = 0$ – 32 наблюдения

$$\frac{\partial L}{\partial w_0} = -\frac{32}{1 + e^{-w_0}} e^{-w_0}$$

$$\frac{\partial L}{\partial w_1} = 0$$

ЗАДАЧА 3.4

$y = 0, x = 0$ – 20 наблюдений

$$\frac{\partial L}{\partial w_0} = 20 - \frac{20}{1 + e^{-w_0}} e^{-w_0}$$

$$\frac{\partial L}{\partial w_1} = 0$$

ЗАДАЧА 3.4

$$\frac{\partial L}{\partial w_0} = 56 - \frac{48}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1} - \frac{52}{1 + e^{-w_0}} e^{-w_0} = 0$$

$$\frac{\partial L}{\partial w_1} = 36 - \frac{48}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1} = 0$$

ЗАДАЧА 3.4

$$\frac{48}{1+e^{-w_0-w_1}}e^{-w_0-w_1} = t \Leftrightarrow t = 36$$

$$\frac{\partial L}{\partial w_0} = 56 - 36 - \frac{52}{1+e^{-w_0}}e^{-w_0} = 0$$

ЗАДАЧА 3.4

$$52e^{-w_0} = 20 + 20e^{-w_0}$$

$$e^{-w_0} = \frac{5}{8}$$

$$w_0 = \ln\frac{8}{5} \approx 0.47$$

ЗАДАЧА 3.4

$$\frac{48}{1 + e^{-w_0 - w_1}} e^{-w_0 - w_1} = 36$$

$$48e^{-w_0 - w_1} = 36 + 36e^{-w_0 - w_1}$$

$$-w_0 - w_1 = \ln 3$$

$$w_1 = -\ln\frac{8}{5} - \ln 3 = \ln\frac{5}{24} \approx -1.57$$

ЗАДАЧА 3.4

$$p = \frac{1}{1 + e^{-w_0 - w_1 x}}$$

$$p = \frac{1}{1 + e^{\ln\frac{5}{8}}} = \frac{8}{13} = \frac{32}{32 + 20}$$