Introduction to

Information Retrieval

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Lecture 21: Link Analysis

Outline

❶ Recap

❷ Anchor Text

❸ Citation Analysis

❹ PageRank

❺ HITS: Hubs & Authorities

Take-away today

- Anchor text: What exactly are links on the web and why are they important for IR?
- Citation analysis: the mathematical foundation of PageRank and link-based ranking
- PageRank : the original algorithm that was used for link-based ranking on the web
- Hubs & Authorities: an alternative link-based ranking algorithm

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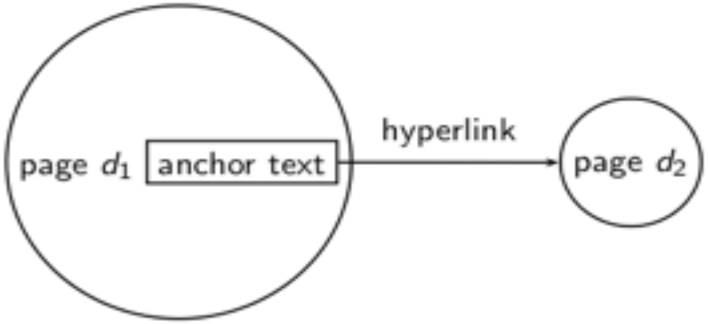
2. Anchor Text
3. Citation Analysis
4. PageRank
5. HITS: Hubs & Authorities

The web as a directed graph

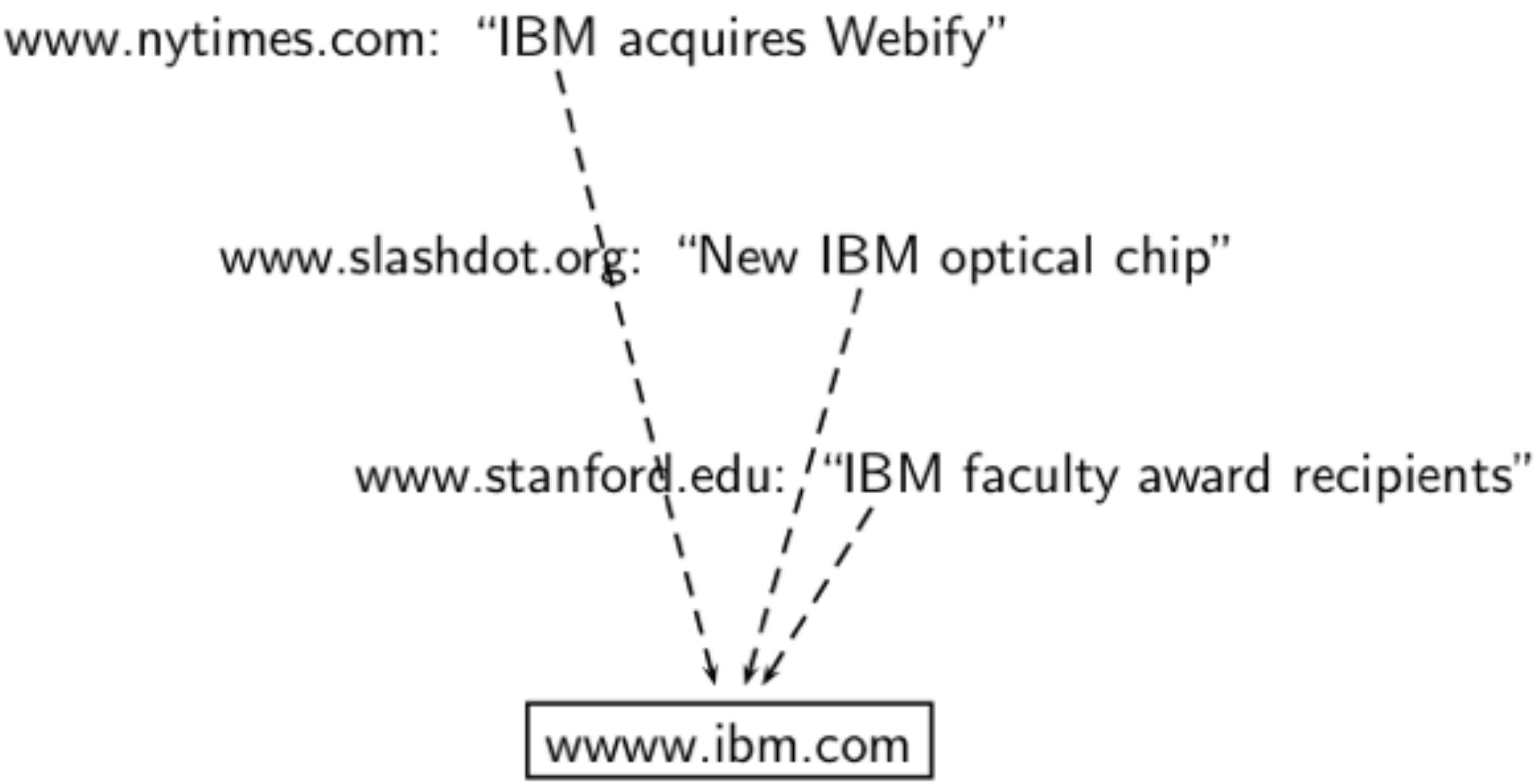


- Assumption 1: A hyperlink is a quality signal.
  - The hyperlink $d_1 \rightarrow d_2$ indicates that d_1's author deems d_2 high-quality and relevant.
- Assumption 2: The anchor text describes the content of d_2.
  - We use anchor text somewhat loosely here for: the text surrounding the hyperlink .
  - Example: "You can find cheap cars <a href =http://…>here . "
  - Anchor text: "You can find cheap cars here"

[text of d_2] only vs. [text of d_2] + [anchor text → d_2]

- Searching on [text of d_2] + [anchor text → d_2] is often more effective than searching on [text of d_2] only.
- Example: Query *IBM*
  - Matches IBM's copyright page
  - Matches many spam pages
  - Matches IBM wikipedia article
  - May not match IBM home page!
  - … if IBM home page is mostly graphics
- Searching on [anchor text → d_2] is better for the query *IBM*.
  - In this representation, the page with most occurences of *IBM* is www.ibm.com

Anchor text containing *IBM* pointing to www.ibm.com



Indexing anchor text

- Thus: Anchor text is often a better description of a page's content than the page itself.
- Anchor text can be weighted more highly than document text.

  (based on Assumption 1&2)

Exercise: Assumptions underlying PageRank

- Assumption 1: A link on the web is a quality signal – the
  author of the link thinks that the linked-to page is high-quality.
- Assumption 2: The anchor text describes the content of the linked-to page.
- Is assumption 1 true in general?
- Is assumption 2 true in general?

Google bombs

- A Google bomb is a search with “bad” results due to maliciously manipulated anchor text.
- Google introduced a new weighting function in January 2007

  that fixed many Google bombs.
- Still some remnants: [dangerous cult] on Google, Bing, Yahoo

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Origins of PageRank: Citation analysis (1)

- Citation analysis: analysis of citations in the scientific literature.
- Example citation: "Miller (2001) has shown that physical activity alters the metabolism of estrogens."
- We can view "Miller (2001)" as a hyperlink linking two scientific articles.
- One application of these "hyperlinks" in the scientific literature:
  - Measure the similarity of two articles by the overlap of other articles citing them.
  - This is called cocitation similarity.
  - Cocitation similarity on the web: Google's "find pages like this" or "Similar" feature.

Origins of PageRank: Citation analysis (2)

- Another application: Citation frequency can be used to measure the impact of an article .
  - Simplest measure: Each article gets one vote – not very accurate.
- On the web: citation frequency = inlink count
  - A high inlink count does not necessarily mean high quality ...
  - ... mainly because of link spam.
- Better measure: weighted citation frequency or citation rank
  - An article's vote is weighted according to its citation impact.

Origins of PageRank: Citation analysis (3)

- Better measure: weighted citation frequency or citation rank.
- This is basically PageRank.
- PageRank was invented in the context of citation analysis by Pinsker and Narin in the 1960s.

Origins of PageRank: Summary

- We can use the same formal representation for
  - citations in the scientific literature
  - hyperlinks on the web
- Appropriately weighted citation frequency is an excellent measure of quality ...
  - ... both for web pages and for scientific publications.
- Next: PageRank algorithm for computing weighted citation frequency on the web.

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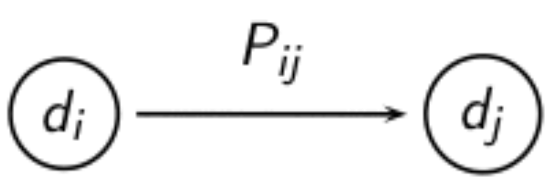
Model behind PageRank: Random walk

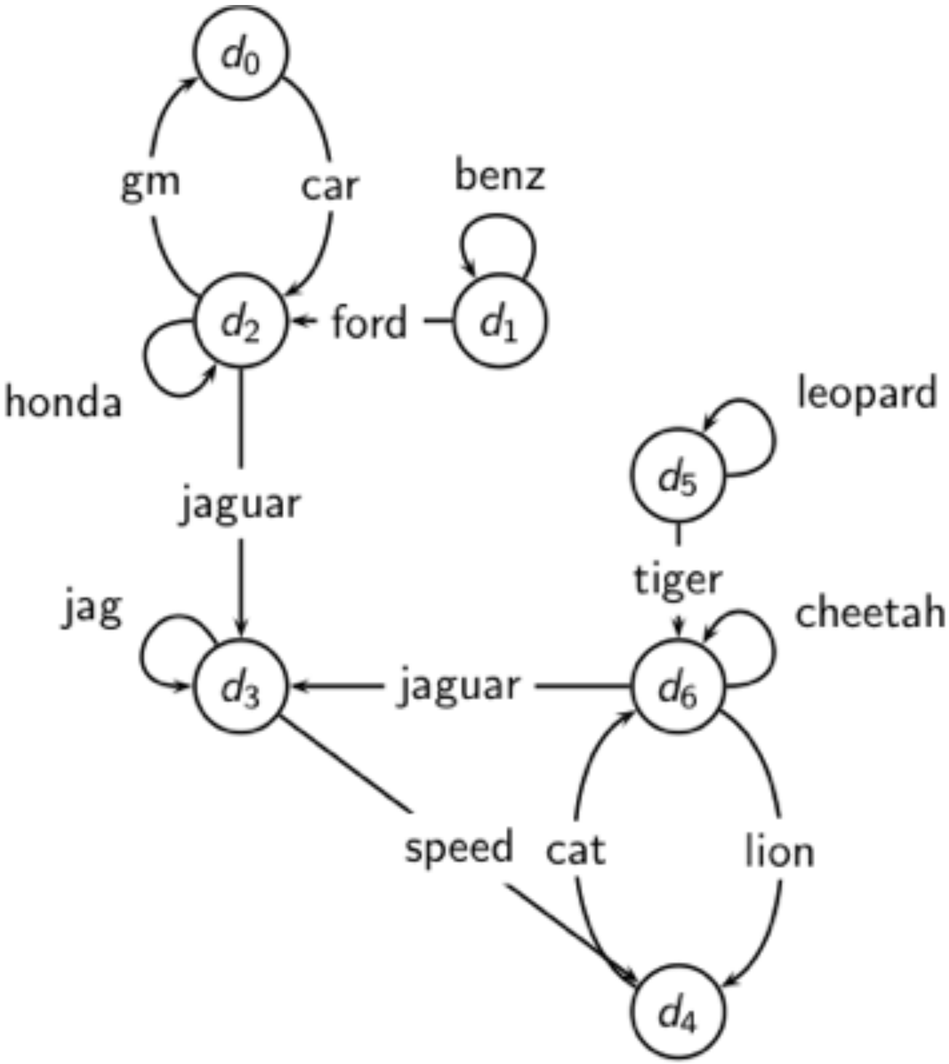
- Imagine a web surfer doing a random walk on the web
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably
- In the steady state, each page has a long-term visit rate.
- This long-term visit rate is the page's PageRank.
- PageRank = long-term visit rate = steady state probability.

Formalization of random walk: Markov chains

- A Markov chain consists of N states, plus an $N \times N$ transition probability matrix P.
- state = page
- At each step, we are on exactly one of the pages.
- For $1 \le i, j \le N$, the matrix entry P_{ij} tells us the probability of j being the next page, given we are currently on page i.
- Clearly, for all i, $\sum_{j=1}^{N} P_{ij} = 1$



Example web graph



Link matrix for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	1	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
d_6	0	0	0	1	1	0	1

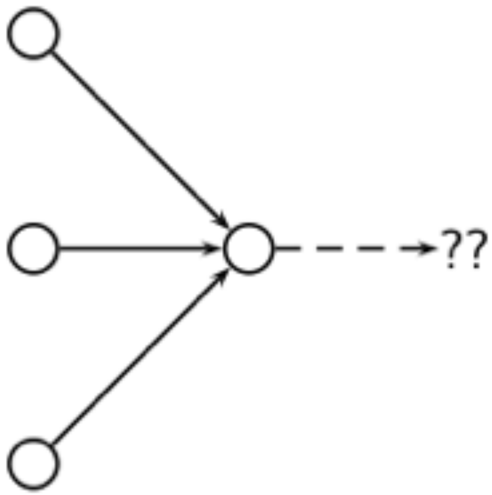
Transition probability matrix P for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Long-term visit rate

- Recall: PageRank = long-term visit rate.
- Long-term visit rate of page *d* is the probability that a web surfer is at page *d* at a given point in time.
- Next: what properties must hold of the web graph for the long-term visit rate to be well defined?
- The web graph must correspond to an ergodic Markov chain.
- First a special case: The web graph must not contain dead ends.

Dead ends



- The web is full of dead ends.
- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

Teleporting – to get us of dead ends

- At a dead end, jump to a random web page with prob.

  $1/N$ At a non-dead end, with probability 10%, jump to a random web page (to each with a probability of $0.1/N$).
- With remaining probability (90%), go out on a random hyperlink.
  - For example, if the page has 4 outgoing links: randomly choose one with probability (1-0.10)/4=0.225
- 10% is a parameter, the teleportation rate.

Teleporting – to get us of dead ends

- At a dead end, jump to a random web page with prob, 1/ N .
- At a non-dead end, with probability 10%, jump to a random web page (to each with a probability of 0.1/N).
- With remaining probability (90%), go out on a random hyperlink.
  - randomly choose one with probability 0.9/N
- 10% is a parameter, the teleportation rate.
- Note: “jumping” from dead end is independent of teleportation rate.

Result of teleporting

- With teleporting, we cannot get stuck in a dead end.
- But even without dead ends, a graph may not have well-defined long-term visit rates.
- More generally, we require that the Markov chain be ergodic.

Ergodic Markov chains

- A Markov chain is ergodic if it is irreducible and aperiodic.
- Irreducibility. Roughly: there is a path from any other page.
- Aperiodicity. Roughly: The pages cannot be partitioned such that the random walker visits the partitions sequentially.

Ergodic Markov chains

- Theorem: For any ergodic Markov chain, there is a unique long-term visit rate for each state.
- This is the steady-state probability distribution.
- Over a long time period, we visit each state in proportion to this rate.
- It doesn't matter where we start.
- Teleporting makes the web graph ergodic.
- $\Longrightarrow$ Web-graph+teleporting has a steady-state probability distribution.
- $\Longrightarrow$ Each page in the web-graph+teleporting has a PageRank.

Formalization of "visit": Probability vector

- A probability (row) vector $\vec{x} = (x_1, \ldots, x_N)$ tells us where the random walk is at any point.
- Example

(	0	0	0	…	1	…	0	0	0	)
	1	2	3	…	i	…	N-2	N-1	N	

- More generally: the random walk is on the page i with probability x_i.
- Example:

(	0.05	0.01	0.0	…	0.2	…	0.01	0.05	0.03	)
	1	2	3	…	i	…	N-2	N-1	N	

- $\Sigma\, x_i = 1$

Change in probability vector

- If the probability vector is $\vec{x} = (x_1, \ldots, x_N)$, at this step, what is it at the next step?
- Recall that row i of the transition probability matrix P tells us where we go next from state i.

Change in probability vector

- If the probability vector is $\vec{x} = (x_1, \ldots, x_N)$, at this step, what is it at the next step?
- Recall that row i of the transition probability matrix P tells us where we go next from state i.
- So from $\vec{x}$, our next state is distributed as $\vec{x}P$.
- For instance, consider x_i the probability of being on page i.
- The multiplication of the vector $\vec{x}$ with the matrix P (see example on board) multiplies x_i with the probability x_i
- move from page i to page j (corresponding to the

Steady state in vector notation

- Computing $\vec{x}P$ amounts to computing the probabilities of being on each page i after the random walker has made his/her first move.
- The multiplication of the vector $\vec{x}$ with the matrix P (see example on board) multiplies x_i with the probability p_{ij} to move from page i to page j.
- The values p_{ij} with j ranging from 1 to N, orrespond to the probabilities in the first row of the matrix P).

Result of our computation

If the original vector was the vector $\vec{x}_0$ then we call the result of the multiplication $\vec{x}_1$.
This is the result of the first step in our random walk recording the new probabilities of being on page i. On the next step, we multiply $\vec{x}_1$ again with the matrix P, resulting in the vector $\vec{x}_2$ recording the new probabilities of being on page i. Repeating this m steps, gives the vector $\vec{x}_m$.

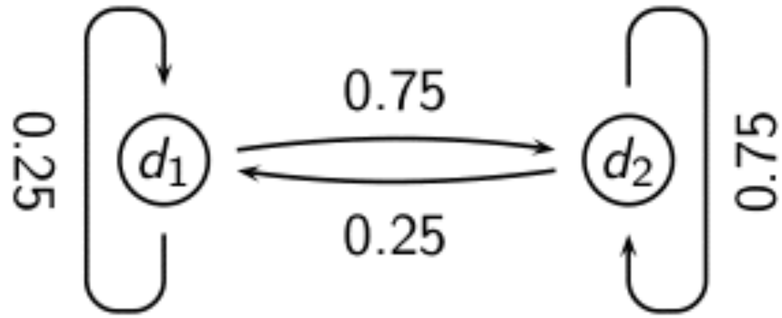
Claim: as m grows, $\vec{x}_m$ "stabilizes", i.e. the differences between Its coordinates and the coordinates of $\vec{x}_{m-1}$ become arbitrarily small for m large enough (convergence to a steady state).

Steady state in vector notation

- The steady state in vector notation is simply a vector $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_N)$ of probabilities.
- (We use $\vec{\pi}$ to distinguish it from the notation for the probability vector $\vec{x}$.)
- π is the long-term visit rate (or PageRank) of page *i*.
- So we can think of PageRank as a very long vector – one entry per page.
- It represents the stabilization of the probabilities (converging to a fixed value) over the duration of the walk.

Steady-state distribution: Example

- What is the PageRank / steady state in this example?



Steady-state distribution: Example

	x_1	x_2		
			$P_{11} = 0.25$ $P_{21} = 0.25$	$P_{12} = 0.75$ $P_{22} = 0.75$
t_0	0.25	0.75		
t_1				

PageRank vector $\vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

Steady-state distribution: Example

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t_0	0.25	0.75	0.25	0.75
t_1				

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

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t_0	0.25	0.75	0.25	0.75
t_1	0.25	0.75		

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

Steady-state distribution: Example

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			$P_{21} = 0.25$	$P_{22} = 0.75$
t_0	0.25	0.75	0.25	0.75
t_1	0.25	0.75		*(convergence)*

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

How do we compute the steady state vector?

- In other words: how do we compute PageRank?

How do we compute the steady state vector?

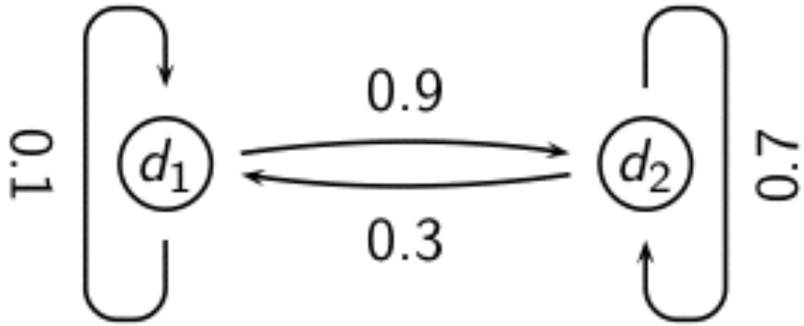
- In other words: how do we compute PageRank?
- Recall: $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_N)$ is the PageRank vector, the vector of steady-state probabilities ...
- … and if the distribution in this step is $\vec{x}$, then the distribution in the next step is $\vec{x}P$.
- But $\vec{\pi}$ is the steady state!
- So: $\vec{\pi} = \vec{\pi} P$
- Solving this matrix equation gives us $\vec{\pi}$.
- $\vec{\pi}$ is the principal left eigenvector for P …
- … that is, $\vec{\pi}$ is the left eigenvector with the largest eigenvalue.
- All transition probability matrices have largest eigenvalue

One way of computing the PageRank $\vec{\pi}$

- Start with any distribution $\vec{x}$, e.g., uniform distribution
- After one step, we're at $\vec{x}P$.
- After two steps, we're at $\vec{x}P^2$.
- After k steps, we're at $\vec{x}P^k$.
- Algorithm: multiply $\vec{x}$ by increasing powers of P until convergence.
- This is called the power method.
- Recall: regardless of where we start, we eventually reach the steady state $\vec{\pi}$.
- Thus: we will eventually (as a limit (!)) reach the steady state.

Power method: Example

- What is the PageRank / steady state in this example?



Computing PageRank: Power Example

	x_1	x_2		
			$P_{11} = 0.1$ $\quad P_{12} = 0.9$ $P_{21} = 0.3$ $\quad P_{22} = 0.7$	
t_0	*0*	*1*		$= \vec{x}P$
t_1				$= \vec{x}P^2$
t_2				$= \vec{x}P^3$
t_3				$= \vec{x}P^4$
				$\ldots$
t_∞				$= \vec{x}P^\infty$

Computing PageRank: Power Example

	x_1	x_2			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1					$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					$\vec{\dots}$
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Computing PageRank: Power Example

	x_1	x_2			
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t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7			$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					$\vec{\ldots}$
t_∞					$= \vec{x}P^\infty$

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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
t_∞					$= \vec{x}P^\infty$

Computing PageRank: Power Example

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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
t_2	*0.24*	*0.76*			$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
t_∞					$= \vec{x}P^\infty$

Computing PageRank: Power Example

	x_1	x_2			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
t_∞					$= \vec{x}P^\infty$

Computing PageRank: Power Example

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t_3	0.252	0.748			$= \vec{x}P^4$
					. . .
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t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
					. . .
t_∞					$= \vec{x}P^\infty$

Computing PageRank: Power Example

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			. . .		. . .
t_∞					$= \vec{x}P^\infty$

Computing PageRank: Power Example

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t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
				. . .	. . .
t_∞	0.25	0.75			$= \vec{x}P^\infty$

Computing PageRank: Power Example

	x_1	x_2			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
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t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
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t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
			. . .		. . .
t_∞	0.25	0.75	0.25	0.75	$= \vec{x}P^\infty$

Computing PageRank: Power Example

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t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
t_∞	0.25	0.75	0.25	0.75	$= \vec{x}P^\infty$

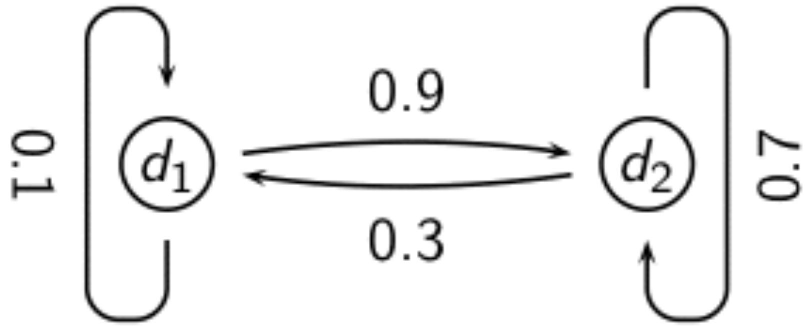
PageRank vector $= \pi = (\pi_1, \pi_2) = (0.25, 0.75)$

Exercise

Compute the steady state vector by solving the equation $\vec{\pi} = \vec{\pi} P$ for the vector $\vec{\pi}$

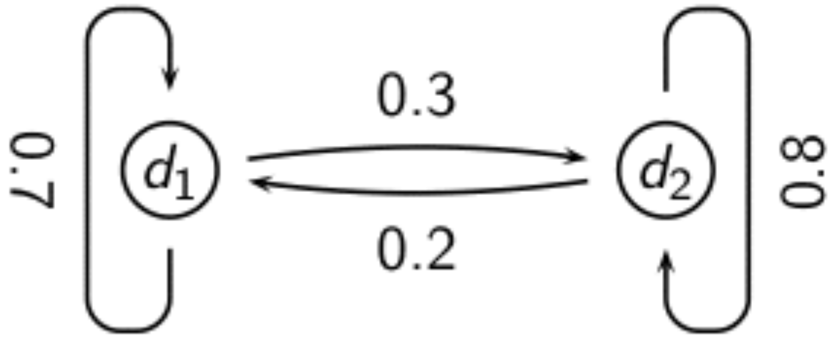
Power method: Example

- What is the PageRank / steady state in this example?



- The steady state distribution (= the PageRanks) in this example are 0.25 for d_1 and 0.75 for d_2.

Exercise: Compute PageRank using power method



Solution

	x_1	x_2	
			$P_{11} = 0.7$ $P_{12} = 0.3$ $P_{21} = 0.2$ $P_{22} = 0.8$
t_0	0	1	
t_1			
t_2			
t_3			
t_∞			

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1				
t_2				
t_3				
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8		
t_2				
t_3				
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2				
t_3				
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7		
t_3				
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	*0*	*1*	*0.2*	*0.8*
t_1	*0.2*	*0.8*	*0.3*	*0.7*
t_2	*0.3*	*0.7*	*0.35*	*0.65*
t_3				
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7	0.35	0.65
t_3	0.35	0.65		
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7	0.35	0.65
t_3	0.35	0.65	0.375	0.625
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7	0.35	0.65
t_3	0.35	0.65	0.375	0.625
t_∞				

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7	0.35	0.65
t_3	0.35	0.65	0.375	0.625
t_∞	0.4	0.6		

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

Solution

	x_1	x_2		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7	0.35	0.65
t_3	0.35	0.65	0.375	0.625
			. . .	
t_∞	0.4	0.6	0.4	0.6

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

Exercise

Compute the steady state vector by solving the equation $\vec{\pi} = \vec{\pi} P$ for the vector $\vec{\pi}$

PageRank summary

- Preprocessing
  - Given graph of links, build matrix *P*
  - Apply teleportation
  - From modified matrix, compute $\vec{\pi}$
  - $\vec{\pi}_i$ is the PageRank of page *i*.
- Query processing
  - Retrieve pages satisfying the query
  - Rank them by their PageRank
  - Return reranked list to the user

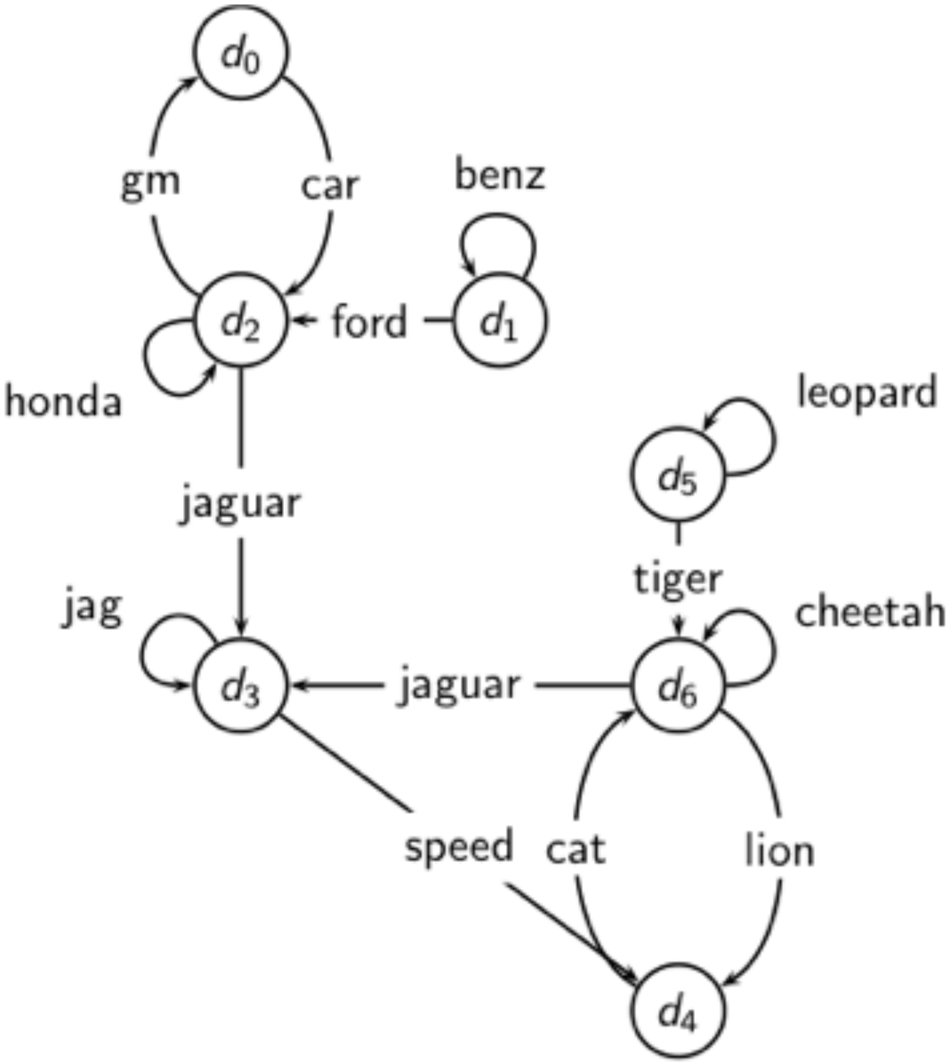
PageRank issues

- Real surfers are not random surfers.
  - Examples of nonrandom surfing: back button, short vs. long paths, bookmarks, directories – and search!
  - → Markov model is not a good model of surfing.
  - But it's good enough as a model for our purposes.
- Simple PageRank ranking (as described on previous slide) produces bad results for many pages.
  - Consider the query [video service].
  - The Yahoo home page (i) has a very high PageRank and (ii) contains both *video* and *service*.
  - If we rank all Boolean hits according to PageRank, then the Yahoo home page would be top-ranked.
  - Clearly not desirable.

PageRank issues

- In practice: rank according to weighted combination of raw text match, anchor text match, PageRank & other factors.

Example web graph



Transition (probability) matrix

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Transition matrix with teleporting

The adjacency matrix A of the web-graph is obtained thus:

if there is a hyperlink from page i to page j, then:

$A_{ij} = 1$, otherwise $A_{ij} = 0$.

Derive the transition probability matrix *P* from matrix *A*
[α is the probability that "teleportation" happens]

1) If a row of *A* has only zeros: replace each element by 1/N
2) For all other rows: divide each 1 in *A* by the number of 1's in its row.
3) Multiply the resulting matrix by 1 −α
4) Add α/*N* to every entry of the resulting matrix

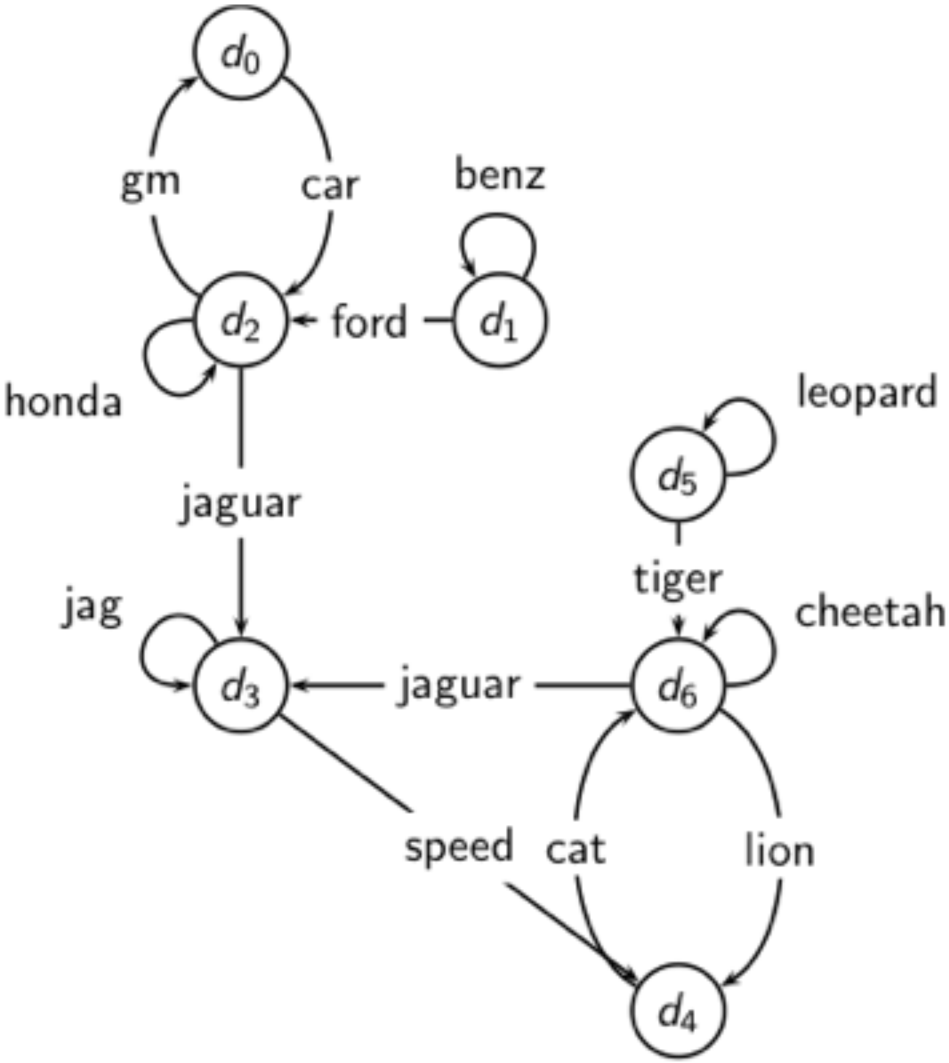
Transition matrix with teleporting

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.02	0.02	0.88	0.02	0.02	0.02	0.02
d_1	0.02	0.45	0.45	0.02	0.02	0.02	0.02
d_2	0.31	0.02	0.31	0.31	0.02	0.02	0.02
d_3	0.02	0.02	0.02	0.45	0.45	0.02	0.02
d_4	0.02	0.02	0.02	0.02	0.02	0.02	0.88
d_5	0.02	0.02	0.02	0.02	0.02	0.45	0.45
d_6	0.02	0.02	0.02	0.31	0.31	0.02	0.31

Power method vectors $\vec{x}P^k$

	$\vec{x}$	$\vec{x}P^1$	$\vec{x}P^2$	$\vec{x}P^3$	$\vec{x}P^4$	$\vec{x}P^5$	$\vec{x}P^6$	$\vec{x}P^7$	$\vec{x}P^8$	$\vec{x}P^9$	$\vec{x}P^{10}$	$\vec{x}P^{11}$	$\vec{x}P^{12}$	$\vec{x}P^{13}$
d_0	0.14	0.06	0.09	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
d_1	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_2	0.14	0.25	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11
d_3	0.14	0.16	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25
d_4	0.14	0.12	0.16	0.19	0.19	0.20	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
d_5	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_6	0.14	0.25	0.23	0.25	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.31	0.31

Example web graph



	PageRank
d $_0$	***0.05***
d $_1$	***0.04***
d $_2$	***0.11***
d $_3$	***0.25***
d $_4$	***0.21***
d $_5$	***0.04***
d $_6$	***0.31***

How important is PageRank?

- Frequent claim: PageRank is the most important component of web ranking.
- The reality:
  - There are several components that are at least as important: e.g., anchor text, phrases, proximity, tiered indexes ...
  - Rumor has it that PageRank in his original form (as presented here) now has a negligible impact on ranking!
  - However, variants of a page's PageRank are still an essential part of ranking.
  - Adressing link spam is difficult and crucial.