Uncertainty quantification in Bayesian Neural Networks using cubature rules

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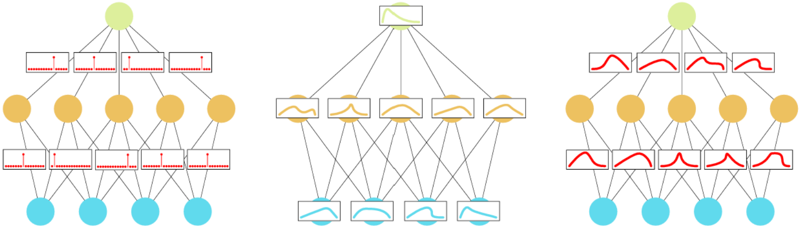
April 26



(a) (b) (c)

Bayesian Neural Networks

What is a Bayesian Neural Network?

A Bayesian Neural Network is a neural networks where the learnable parameters θ_i **are probability distributions** instead of scalars.
e.g. the weights could be Normal distributed like so,

$$w_i = \mathcal{N}(\mu_i, \sigma_i^2) \tag{1}$$

- The activation functions could be stochastic too!
- **Ensemble learning** takes place in a BNN by letting the model train with multiple distributions of the learnable parameters θ

How does it work?

- Choose a **Stochastic model** i.e. prior distributions for $p(\theta)$ and $p(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta})$
- Obtain the posterior probability

$$p(\boldsymbol{\theta}|D) = \frac{p(D_y|D_x, \boldsymbol{\theta})p(\boldsymbol{\theta})}{\int_\theta p(D_y|D_x, \boldsymbol{\theta}')p(\boldsymbol{\theta}')d\boldsymbol{\theta}'} \tag{2}$$

where,
D_y = Data labels
D_x = Data inputs

- Computing the integral, $\int_\theta p(D_y|D_x, \boldsymbol{\theta}')p(\boldsymbol{\theta}')d\boldsymbol{\theta}'$ is very difficult.
- To address this, two broad approaches are followed:
  1. MCMC (Markov chain monte carlo)
  2. Variational inference

Bayesian Neural Networks

Motivation

The main goal is to get a better idea about the **uncertainty**. For example, if all the results of the models are vastly different, there's more uncertainty thereby providing a way to *preemptively* assess **generalizability**.

Uncertainty quantification

Some techniques:

- Particle filtering
- Variational inference
- Cubature rules*
- and so on...