1.05 Portfolio Mathematics

Question 1

An analyst estimates the following joint probability function of returns for two assets, X and Y, under three scenarios:

Scenario	Return on X (%)	Return on Y (%)	Probability
1	12	7	0.28
2	9	17	0.51
3	7	2	0.21

Based only on this information, the covariance of returns of the two assets is *closest* to:

A. −1.63

B. 0.39

C. 6.53

Question 2

An analyst estimates a joint probability function of two assets' returns during good, average, and poor business conditions:

Business conditions		Good	Average	Poor
	Returns	R_Y = 17%	R_Y = 10%	R_Y = 6%
Good	R_X = 21%	0.15	0	0
Average	R_X = 13%	0	0.65	0
Poor	R_X = 11%	0	0	0.20

The covariance of returns is *closest* to:

A. 9.8

B. 10.70

C. 20.23

Question 3

All else equal, if a portfolio consists of two stocks, the portfolio will have the *least* risk if the correlation between the two stocks is:

A. less than zero.

B. equal to zero.

C. greater than zero.

Question 4

A portfolio manager gathers the following information about a portfolio:

	Bonds	Stocks	Real Estate
Expected return	4%	8%	2%
Portfolio weight	35%	50%	15%

Covariance Matrix

	Bonds	Stocks	Real Estate
Bonds	80	100	45
Stocks	100	300	185
Real estate	45	185	180

If the benchmark has an expected return of 6% and standard deviation of 12.5%, based on only this information, the portfolio's risk is *most likely*:
A. less than the benchmark's.
B. the same as the benchmark's.
C. more than the benchmark's.

Question 5
An investor has a safety-first optimal portfolio of €40 million with an expected annual return of 15% and a standard deviation of 17%. The investor plans to withdraw €1 million in one year without reducing the initial principal (assume that the initial principal is €40 million). The probability that the investor's actual return will fall below the shortfall level is *closest* to:
A. 16.60%
B. 22.96%
C. 77.04%

Question 6
An investor invests €1,000,000 today and plans to withdraw €50,000 one year from today. None of the withdrawal can be from principal. An advisor recommends three different portfolio allocations:

Allocation	Expected Annual Return	Standard Deviation
1	7.0%	9.5%
2	9.0%	11.0%
3	11.0%	12.5%

The optimal allocation, according to Roy's safety-first criterion, has a safety-first ratio *closest* to:
A. 0.21
B. 0.36
C. 0.48

Question 7

A portfolio comprises three equally weighted assets. The covariance matrix for the three assets' returns is shown below:

	Asset A	Asset B	Asset C
Asset A	200		
Asset B	100	75	
Asset C	125	90	210

All else equal, if the covariance between Asset A and Asset C increases to 150 and the covariance between Asset B and Asset C is simultaneously halved, the overall portfolio variance will be *closest* to:

A. 117

B. 124

C. 196