1.04 Probability Trees and Conditional Expectations

Question 1

An analyst has estimated the probabilities of two unrelated events. Which probability rule is *most appropriate* for calculating the joint probability of these two events?

A. Addition

B. Multiplication

C. Total probability

Question 2

wo research analysts asked a mutual fund manager the following questions:

> Analyst A: "What is the probability that the fund will have a zero percent return this year?"
>
> Analyst B: "What is the probability that the fund will have a negative return this year?"

The two events described by the analysts are *most likely*:

A. mutually exclusive and exhaustive.

B. mutually exclusive and not exhaustive.

C. neither mutually exclusive nor exhaustive.

Question 3

An investment advisor uses several criteria to screen a list of 200 mutual funds for a client.

Criterion number	Description	Fraction of the 200 funds meeting criterion
1	Environmentally sustainable	0.80
2	Large-cap value	0.50
3	Multisector	0.40
4	Low turnover	0.25
5	Contains bonds	0.25

A mutual fund must meet the first four criteria in order to pass the screening. If the criteria are independent, the number of funds that pass the screen is *closest* to:

A. 2

B. 4

C. 8

Question 4

A and B are two events with the following probabilities:

Probability of *A*	*P*(*A*)	0.31
Probability of *B*	*P*(*B*)	0.90
Probability of *A* given *B*	*P*(*A*|*B*)	0.28

The joint probability of A and B is *closest* to:
A. 0.0868
B. 0.2520
C. 0.2790

Question 5
A regulator devised a test to determine if distressed banks will go bankrupt and has estimated the following related probabilities:

Probability that a bank does not pass the test	0.6200
Joint probability that a bank goes bankrupt and passes the test	0.0500
Probability that a bank goes bankrupt given that it does not pass the test	0.7903

The probability that a bank goes bankrupt is *closest* to:
A. 0.06
B. 0.49
C. 0.54

Question 6
A money management firm surveyed individuals about their attitudes toward investing. A partial summary of the data is shown below:

Event	Probability
Investor is an active investor	0.45
Portfolio holds equities, given that the investor is an active investor	0.75
Portfolio does not hold equities, given that the investor is not an active investor	0.80

If a randomly selected individual's portfolio holds equities, the probability that the investor is not an active investor is *closest* to:
A. 11.0%
B. 24.6%
C. 44.8%

Question 7

Two events, A and B, are not independent: B's probability depends on whether A occurs. If the unconditional probability of A is known, which of the following probability rules is *least appropriate* to estimate the unconditional probability of B?

A. Addition rule

B. Multiplication rule

C. Total probability rule

Question 8

An analyst's estimates concerning a company's change in EPS are:

- *P*(Positive change in EPS) = 0.50
- *P*(No change in EPS) = 0.20
- *P*(Negative change in EPS) = 0.30

The analyst has heard rumors that the company may increase its dividend payout this year and estimates the following conditional probabilities of a dividend increase given the change in EPS:

- Probability (Increased dividend | Positive change in EPS) = 0.60
- Probability (Increased dividend | No change in EPS) = 0.30
- Probability (Increased dividend | Negative change in EPS) = 0.10

If the company announces a dividend increase, the revised probability that there was a negative change to its EPS is *closest* to:

A. 0.030

B. 0.077

C. 0.390

Question 9

The probability that the price of natural gas will decline below $4.00 per MMBtu (million British thermal units) this year is 30%. The probability of a producer suspending production given that the natural gas price drops below $4.00 per MMBtu is 60%. The probability of both the natural gas price dropping below $4.00 per MMBtu and the producer suspending production is *closest* to:

A. 0.18

B. 0.72

C. 0.90

Question 10

An analyst determines that the probabilities of two events are as follows:

Event	Description	Probability
A	Price of oil increases	0.33
B	S&P 500 Index increases	0.49

Assuming the events are independent but not mutually exclusive, the probability that at least one of the two events will occur is *closest* to:

A. 0.16
B. 0.66
C. 0.82

Question 11
An analyst estimates the probability distribution for a company's upcoming quarterly EBITDA:

Probability	EBITDA ($ billions)
0.5	2.05
0.2	2.20
0.3	2.40

The standard deviation of the distribution (in $ billions) is *closest* to:
A. 0.147
B. 0.150
C. 0.152

Question 12
The following are probabilities associated with two events: a recession and a central bank lowering interest rates:

Probability that central bank lowers rates	0.7700
Probability of a recession given that the central bank lowers rates	0.7000
Probability of a recession given that the central bank does not lower rates	0.5625

The probability of a recession is *closest* to:
A. 0.40
B. 0.54
C. 0.67

Question 13
An analyst designates the following events as possible return outcomes for a portfolio's value at the end of one year, estimating their probabilities as follows:

Event	Return Range (%)	Probability
V	$-16 \le X < -12$	0.17
W	$-12 \le X < -5$	0.21
X	$-5 \le X < 0$	0.34
Y	$0 \le X < 5$	0.14
Z	$5 \le X < 12$	0.12

It is *least appropriate* to conclude that the events are:
A. dependent.
B. exhaustive.
C. mutually exclusive.

Question 14
The probability that a company will win a government contract bid is 0.2. If it wins the bid, there is a 0.6 probability that the company will beat earnings this year. The unconditional probability that the company will beat earnings is 0.36. If the company did beat earnings at year-end, then the probability that the company won the bid is *closest* to:
A. 0.20
B. 0.33
C. 0.40

Question 15
An investor holds two funds: Fund A and Fund B. The investor wants to forecast the probability that Fund A will outperform Fund B each year for the next two years. If each year's performance is independent, which probability rule should the analyst use?
A. Addition rule
B. Multiplication rule
C. Total probability rule

Question 16
A and B are two events with the following probabilities:

Probability of *B*	$P(B)$	0.23
Probability of *A* and *B*	$P(AB)$	0.17
Probability of *A* or *B*	$P(A \text{ or } B)$	0.46

The probability of A occurring is *closest* to:
A. 0.23
B. 0.29
C. 0.40

Question 17

A and B are two events with the following probabilities:

Probability of *A*	*P*(*A*)	0.48
Probability of *B*	*P*(*B*)	0.25
Probability of *A* and *B*	*P*(*AB*)	0.06

The probability of A or B is *closest* to:

A. 0.12

B. 0.67

C. 0.73

Question 18

An analyst studies 10 years of historical data on stock splits for companies listed on the Toronto Stock Exchange (TSE) to determine the probability that a firm listed on the TSE will have a stock split this year. The probability derived from this method is *best* described as:

A. a priori.

B. empirical.

C. subjective.

Question 19

Fifty mutual funds are each subjected to two tests. The first tests a fund's 5-year average return, and the second tests its 5-year average expense ratio. The results are:

- 16 funds pass both tests,
- 31 funds pass the first test, and
- 20 funds pass the second test.

Given that a fund passed the first test, the probability that it also passed the second test is *closest* to:

A. 0.248

B. 0.516

C. 0.700

Question 20

An analyst studies events that may happen to companies in their first year of operation:

- a major cyberattack on a small (S_1), medium (S_2), or large (S_3) company, and
- bankruptcy (*T*).

The analyst estimates the following probabilities:

Joint probability of *T* and S_1	$P(TS_1)$	0.340
Joint probability of *T* and S_2	$P(TS_2)$	0.390
Probability of *T*	*P*(*T*)	0.908
Probability of *T* given S_3	$P(T\|S_3)$	0.890

If there is no probability of a bankruptcy without a cyberattack, then the probability that a large company experiences a cyberattack in its first year of operation is *closest* to:
A. 0.176
B. 0.200
C. 0.377

Question 21
There is a 0.6 probability that a portfolio manager will add a new asset to an existing portfolio.

- If the manager adds the asset, then there is a 0.9 probability that the overall portfolio will beat its benchmark.
- If the manager does not add the asset, then there is a 0.7 probability that the portfolio will beat its benchmark.

If the portfolio beats its benchmark at the end of the year, the probability that the manager bought the new asset is *closest* to:
A. 0.60
B. 0.66
C. 0.82

Question 22
The expected value of a random variable is *most likely* a(n):
A. equally weighted average of possible outcomes.
B. probability-weighted average of possible outcomes.
C. equally weighted average of historical observations.

Question 23
Within a certain stock index, 70% of the securities are technology stocks, 50% of the securities are growth stocks, and 40% of the securities are both technology stocks and growth stocks. The categories are not mutually exclusive. If one security is selected randomly, the probability that it is either a technology stock or growth stock is *closest* to:
A. 0.50
B. 0.80
C. 0.85

Question 24
Three companies make a specialty component used in the automobile industry. The relevant information for each company is shown below:

	Market Share (%)	Defective Components (%)
Company A	58	3
Company B	31	2
Company C	11	4

If a component is randomly selected and found to be defective, the probability that it was manufactured by Company C is *closest* to:
A. 0.004
B. 0.028
C. 0.157

Question 25
An analyst has four independent criteria for selecting stocks and assumes the probabilities of meeting each criterion apply to every stock as follows:

Criterion	Probability of Meeting Criterion
A	0.451
B	0.287
C	0.193
D	0.554

Under the analyst's assumptions, the probability of a stock meeting the first three criteria is *closest* to:
A. 0.014
B. 0.025
C. 0.931

Question 26
A distressed company may default on existing debt of £10 million depending on whether it declares bankruptcy. The company has a 0.8 probability of going bankrupt.

- If the company goes bankrupt, then its lender has
    - a 0.6 probability of recovering £3 million and
    - a 0.4 probability of recovering nothing.
- If the company does not go bankrupt, then its lender has
    - a 0.7 probability of recovering the full loan and
    - a 0.3 probability of recovering £9 million.

The lender's expected recovery amount on the loan is *closest* to:
A. £1.44 million.
B. £3.38 million.
C. £5.58 million.

Question 27
Which of the following statements concerning properties of probability is *least likely* to be accurate?
A. An event can have an infinite number of outcomes.
B. The sum of probabilities for two mutually exclusive events equals 1.
C. If P(A | B) is greater than 0, then A and B cannot be mutually exclusive.

Question 28

An analyst considers the impact of a company's earnings-per-share on its CEO's compensation and estimates the following probabilities:

Event	Probability
Probability that company's EPS does *not* increase	0.60
Probability that CEO's compensation does *not* increase given that EPS does *not* increase	0.90
Probability that CEO's compensation increases given that EPS increases	0.80

Based on the information given, a calculated probability of 0.38 *most likely* represents the:

A. total probability of an increase in compensation.

B. joint probability of an increase in compensation and in EPS.

C. conditional probability that compensation increased given that EPS does not increase.

Question 29

An analyst wants to estimate the unconditional probability that a company will default on its bonds, which depends on whether its revenue growth is positive. If the analyst can estimate the probability that revenue growth will be positive, then the *most appropriate* probability rule to estimate the unconditional probability of default is the:

A. addition rule.

B. multiplication rule.

C. total probability rule.

Question 30

A trader owns a share of stock and estimates that there is a:

- 0.6 probability of the stock closing at a higher price today,
- 0.9 probability that the trader will sell the stock given that the stock closes higher, and
- 0.2 probability that the trader will sell the stock given that the stock does not close higher.

The expected probability that the trader will sell the stock is *closest* to:

A. 0.54

B. 0.60

C. 0.62

Question 31

For two events, A and B, which of the following *best* describes a property of conditional probability?

A. A and B are mutually exclusive events.

B. Conditional probability can be higher than unconditional probability.

C. The probability of A given B cannot equal the probability of B given A.

Question 32

A fund has 200 bonds. The fund's previous manager added some of the bonds to the funds; the remainder were added by the current manager. Forty of the 200 bonds are investment grade; the remainder are high-yield bonds. The current manager added 35% of the investment-grade bonds and 80% of the high-yield bonds to the fund. The probability that a randomly selected security is either an investment-grade bond or any bond that was not added by the current manager is *closest* to:

A. 20%
B. 36%
C. 49%

Question 33

An analyst who is evaluating the probability that a company will fire its CEO if EPS decreases this year gathers the following relevant estimates:

Probability that EPS decreases	0.35
Probability that company fires CEO if EPS decreases	0.85
Probability that company does not fire CEO if EPS does not decrease	0.75

The analyst later learns that the company did not fire the CEO after reporting EPS. The updated probability that the company's EPS decreased is *closest* to:

A. 0.05
B. 0.10
C. 0.15

Question 34

A mutual fund contains 250 constituent companies.

- 12% of the companies are categorized as healthcare stocks.
- 16% of the companies are categorized as small-cap stocks.
- 4% of the companies are categorized as both healthcare and small-cap stocks.

If the categories are not mutually exclusive, then the probability that any constituent company is a healthcare or small-cap stock is *closest* to:

A. 24.0%
B. 26.1%
C. 28.0%

Question 35

An analyst compiles the following covariance matrix for two assets:

	Asset A	Asset B
Asset A	22.32	3.02
Asset B	3.02	31.95

A portfolio is created from these two assets, with 31% invested in Asset B. The portfolio's variance is *closest* to:
A. 4.08
B. 14.99
C. 25.31

Question 36
An analyst compiles information on a company's estimated future earnings per share (EPS):

Probability	EPS (EUR)
0.05	1.25
0.10	1.30
0.65	1.35
0.20	1.40

If the company's EPS amount is treated as a random variable, its standard deviation is *closest* to:
A. EUR 0.0354
B. EUR 0.0433
C. EUR 0.0612

Question 37
An investor estimates a covariance matrix for the relative price movements of Asset A and Asset B that is shown below:

	Asset A	**Asset B**
Asset A	900	30
Asset B	30	400

If the investor creates a portfolio with 35% invested in Asset A and 65% invested in Asset B, the portfolio's standard deviation is *closest* to:
A. 16.7%
B. 17.1%
C. 24.3%

Question 38
An analyst estimates the probability of future returns on an asset as follows:

Probability	Return
0.17	0.07
0.26	0.11
0.57	0.15

If the asset's return is a random variable, its variance is *closest* to:

A. 0.0009
B. 0.0012
C. 0.0013

Question 39
For two events A and B, one calculates the probability of (A or B) by adding the probability of A to the probability of B, then:
A. subtracting the conditional probability of A given B [ie, P(A|B)].
B. subtracting the probability of A and B [ie, P(AB)] to avoid counting it twice.
C. adding the probability of A and B [ie, P(AB)] in case the events are not mutually exclusive.

Question 40
There is an 80% chance that a hedge fund's return will exceed its benchmark this year. There is a 90% chance that the fund manager will earn higher total compensation than last year if the fund's return exceeds the benchmark. Suppose that the overall probability that the manager will earn higher total compensation this year is 73%. Which of the following statements is *most accurate*?
A. The conditional probability of the fund return exceeding the benchmark is 0.80.
B. The unconditional probability of earning higher compensation this year is 0.73.
C. The conditional probability of earning higher total compensation this year given that returns exceed the benchmark is 0.72.

Question 41
An analyst determines that two events are independent but not mutually exclusive: a rise in stock prices (S) and a rise in the price of West Texas intermediate crude oil (*T*). If $P(S|T) = 0.54$, $P(S \text{ or } T) = 0.655$, and $P(T) = 0.25$, then the probability of events S and T both occurring is closest to:
A. 0
B. 0.063
C. 0.135

Question 42
An analyst compiles the following data about an equity-indexed annuity that pays one of five possible returns to investors each year:

Return (%)	Estimated Future Probability	Historical Occurances Last 10 Years
1	0.13	2
2	0.21	1
3	0.41	2
4	0.14	4
5	0.11	1

The expected value of the asset's return is *closest* to:

A. 2.89%
B. 3.00%
C. 3.10%

Question 43
An analyst identifies two events and estimates probabilities as follows:

Probability of *X*	0.5100
Probability of *Y*	0.3600
Probability of *X* given *Y*	0.7222

Events X and Y are *most likely*:
A. dependent.
B. mutually exclusive.
C. perfectly correlated.

Question 44
A portfolio comprises two stocks, A and B, with the following characteristics:

All values expressed as %	A	B
Portfolio weight (w)	40	60
Return	3	6
Standard deviation (σ)	5	10

If the portfolio standard deviation equals 5.40%, the correlation between Stocks A and B is *closest* to:
A. −0.4517
B. −0.0023
C. 0.1733

Question 45
An analyst estimates the probability that:
- a carbon tax will be enacted is 0.40.
- oil prices will fall given that a carbon tax is enacted is 0.75.
- either a carbon tax will be enacted or oil prices will fall is 0.80.

The probability that oil prices will fall is *closest* to:
A. 0.10
B. 0.40
C. 0.70

Question 46

A company generates substantial income from a major foreign subsidiary. The company's net profitability is impacted by changes to the foreign exchange rate. There is a

- 0.3 probability that the foreign currency will depreciate relative to the company's domestic currency.
- 0.2 probability of the company recording a net loss this year given that the foreign currency depreciates.
- 0.1 probability of the company recording a net loss this year given that the foreign currency does not depreciate.

At year-end, the company did not record a net loss. The probability that the foreign currency did not depreciate is *closest* to:

A. 0.70
B. 0.72
C. 0.87

Question 47

An analyst has developed a screening test to determine whether a company's earnings will decrease. If the company does not pass the test, it is likely that its earnings will decrease in the

Event	Probability
Probability of an earnings decrease in the next quarter	0.47
Probability of passing the test	0.62
Probability of an earnings decrease in the next quarter given it did *not* pass the test	0.90

Based on this information, the probability that the company's earnings will decrease in the next quarter given that the company passes the test is *closest* to:

A. 0.13
B. 0.21
C. 0.34