

*Cambridge Advanced Subsidiary Level Notes*  
Mathematics Cheatsheet

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# 1 Probability and Statistics

## Discrete random variables

Discrete random variables are those where events are separate and separate, *discrete*.

- Sum of all probabilities

$$\sum(P(X = x)) = 1$$

- Expectation (mean)

$$E(X) = \mu = \sum(x \cdot P(X = x))$$

- Variance

$$\text{Var}(X) = \sigma^2 = E(X^2) - E(X)^2$$

$$\text{where } E(X^2) = \sum(x^2 \cdot P(X = x))$$

## Binomial distribution

When there are a fixed number of trials  $n$ , each with only two possible outcomes where success has probability  $p$ , the discrete random variable  $X$  follows a binomial distribution  $X \sim B(n, p)$ .

- The probability of  $r$  successes

$$P(X = r) = \binom{n}{r} (p^r)(1-p)^{n-r}$$

- Expectation

$$E(X) = np$$

- Variance

$$\text{Var}(X) = np(1-p)$$

## Geometrical distribution

Used when the number of trials  $X$  until the first success is counted. Trials must be independent and probability of success is  $p$ .  $X \sim \text{Geo}(p)$ .

- Probability that success occurs on the  $r$ th trial

$$P(X = r) = p(1-p)^{r-1}$$

- Failure for  $k$  trials

$$P(X > k) = (1-p)^k$$

- Mean

$$E(X) = p^{-1}$$

## Normal distribution

A continuous distribution used for data that clusters around the mean in a “bell curve”, denoted  $X \sim N(\mu, \sigma^2)$ .

- To standardise the variable  $X$  to the standard variable  $Z$  such that  $Z \sim N(0, 1)$ .

$$Z = \frac{X - \mu}{\sigma}$$

- The curve is symmetrical, and so the following properties are derived

$$P(Z < -a) = P(Z > a) = 1 - \Phi(a)$$

$$P(Z > -a) = \Phi(a)$$

## Normal approximation to the binomial

- The approximation is valid when

$$np > 5 \text{ and } n(1-p) > 5$$

- Continuity must be corrected as half a unit above or below the given argument must be taken as a discrete variable is being converted to continuous.