Probabilities

Single events

When we do an experiment, many different outcomes may happen. The *likelyhood* of a certain outcome is the probability of that outcome. It is measured from a scale of 0 to 1, where a probability of 0 means that that outcome is impossible, and a probability of 1 means that outcome is the only one that is possible.

In short, the probability of an event is written as $P(\text{event})$. Some examples follow:

$$P(\text{the day after Friday will be Saturday}) = 1$$
$$P(\text{the Sun will rise in the west tomorrow}) = 0$$
$$P(\text{a pregnant mother will have a daughter}) = 0.5 = \frac{1}{2}$$

Let us consider the example of a coin toss. In a normal coin, there are two possible outcomes: heads or tails. To *experimentally* find the probability of either outcome, we will toss the coin many times. We will record the outcomes in a table like below:

Toss number	Outcome
1	H
2	T
...	...

To find the probability of an outcome, the general formula for applying experimental results is:

$$P(\text{outcome}) = \frac{\text{number of times outcome has occured}}{\text{total attempts}}$$

So in our example, if we see that, out of 100 total attempts, there have been 50 heads and 50 tails:

$$P(\text{H}) = \frac{50}{100} = 0.5$$

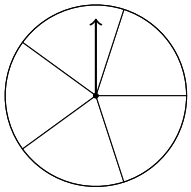
and,

$$P(\text{T}) = \frac{50}{100} = 0.5$$

Notice that, out of two probable outcomes, both have the same probability of ocurring. This means the coin is random, i.e., if the coin is tossed there is an equal probability of either outcome. Also notice that, the sum of all the outcomes equals 1: $0.5 + 0.5 = 1$. Furthermore, in such a situation, where probabilities of all outcomes are equal, the outcomes are said to be perfectly random.

However, if the result of our experiment yielded 75 heads and 25 tails, $P(\text{H}) = \frac{3}{4}$ and $P(\text{T}) = \frac{1}{4}$. The sum of all outcomes is still 1, yet the outcomes are not all equal, meaning the coin is unfair and hence biased.

Problem. In an experiment, a spinner of the following nature was used:



Here, the arrow can be spun and it will land on one of the five segments. Each of the segments corresponds to a fruit: apple, banana, mango, jackfruit and lychee. The spinner was spun 50 times, and it yielded the following results:

outcome	number of times
apple	10
banana	10
mango	10
jackfruit	10
lychee	10

Solve:

1. Find the probability of the arrow landing on each of the fruits.
2. Is the spinner fair or biased, explain.

Multiple events

Finding the probabilities of multiple events, happening one after another can involve two situations, where the events are dependent on each other and where the events are independent.

Independent events

This involves events where no matter how many events occur, the probabilities of the individual outcomes are unaffected. An example is multiple coin tosses. No matter how many times the coin is tossed, the probability of the outcome being heads or tails is the same.

To find the probabilities of multiple coin tosses, we may use tree diagrams.