Video Frame Interpolation using Pyramid Representation

Jan Murić, Paulo Erak

CISUC - Centre for Informatics and Systems of the University of Coimbra; University of Coimbra,
Portugal
{uc2024189891, uc2024164543}@student.uc.pt

Abstract

The primary goal of this paper is synthesizing nonexistent intermediate frames between given input frames. By achieving this, we can enhance video quality in several ways, such as increasing frame rates, creating slow-motion effects, improving video compression, and generating animations from fewer frames. One of the possible ways to achieve this is through calculation and use of optical flow. This paper focuses on estimating optical flows from different levels of feature pyramid representation of our inputs. We then combine them, use them for feature warping and in final construction of an intermediate frame. We use convolutional models for feature extraction, flow estimation, creation of the intermediate frame and its refinement. Through this approach we try to minimize artifacts and improve the visual quality of synthesized frames. We provide insight into related work that inspired this paper, reflect on problems encountered during development and possible improvements that can be made to our model. Source codes are available at: https://github.com/MuricJ/AI video interpolator.g it

26 1 Introduction

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Video frame interpolation is a form of video processing which aims to generate intermediate frames between two given input frames. By achieving this, we can enhance video quality in several ways, such as increasing frame rates, creating slow-motion effects, improving video compression, and generating animations from fewer frames. Video frame interpolation has significant applications in entertainment, media editing, and machine learning tasks, making it an essential research area in computer vision.

The area of video frame interpolation is a very active relative in the last eight years. Articles specialize in a wide range of improvements: focus on smaller and faster-moving objects (Wu, 2023), frame interpolation for large movements (Reda, 2022), unsupervised learning for use of smaller datasets (Simon Meister, 2017), lightweight models (Hahm,

43 2023), use of blurry input frames (W. Shen, 2020), etc. This 44 range shows that there are many ways in which the field can 45 advance, and that future work seems promising. There is 46 also a healthy split between advancement of well-47 established ideas and concepts such as optical flow (Hahm, 48 2023) (Simon Meister, 2017) (Reda, 2022) and moving 49 away from them in search of newer, alternative methods 50 (Ziwei Liu, 2017).

During our exploration of the topic, and subsequent read-52 ing of research papers on the matter, we found the well-53 established idea of optical flow intriguing and quite intui-54 tive. Optical flow being the distribution of the apparent 55 velocities of objects in an image. By estimating optical flow 56 between video frames, we can measure the velocities of 57 objects in the video. This gives us an idea of the location of 58 the object in the next frame. This concept seems intuitive 59 and easy to follow. The truth is, however, that optical flow 60 between two frames is often hard to properly calculate.

Several research papers describe unique methods for ad-62 dressing challenges connected to optical flow estimation. We found the method used in FILM (Reda, 2022) particu-64 larly intriguing and inspiring. The method used in FILM is 65 made to combat blurriness that usually comes with large 66 motions between frames. FILM method uses a pyramidal 67 representation of input frames (image pyramids) where each 68 layer represents smaller and smaller resolutions of input 69 frames. This provides us with a range of scales which offer 70 a coarser or finer look into the frames. Features are extract-71 ed at each pyramid level using a specially made feature 72 extractor that shares weights between levels, turning the 73 image pyramids into feature pyramids. Starting from the 74 coarsest level of feature pyramids, optical flow is estimated, 75 upscaled and sent to the next layer for further optical flow 76 refinement. The feature pyramid is then warped on each level using the optical flow estimated on that level. Warped 78 features and optical flows are jointly sent to the decoder for 79 fusion and refinement producing an intermediate frame. 80 Having the method's original goal in mind, this layered 81 approach to the problem makes sense. Particularly because 82 large movement can be seen and addressed on lower resolu-83 tions and then further refined on higher resolutions where 84 more details become apparent. Extreme motions still prove 85 to cause problems causing un-natural deformations. Using 86 this method as an inspiration we strive to make our model 143 (Reda, 2022). Bidirectional flow estimation is often consid-87 achieve visually pleasing intermediate frames.

89 procedure and inputs into the decoder. We assume that by 146 input frame to the second input frame and an optical flow 90 using inversed flow on extracted features in the first part of 147 from the second input frame to the first input frame. "The 96 loss functions, and differences in the resources available for 153 can be calculated for each level of the pyramid. 97 training and testing our model compared to those in the 154 98 original inspirational work (Reda, 2022), are discussed in 155 tained, they can be used in the process of warping input 99 more detail later in the paper.

101 work in greater detail, describe the materials used and their 158 warping stands by itself. Warped images usually do not 102 sources, explain the methods employed along with the ra- 159 provide a viable intermediate frame. Combinations of 103 tionale behind their use and their implementation, present 160 warped images give clues to the model in what way the final 104 the experimental setups and results, and conclude with our 161 intermediate frame should be constructed. 105 final thoughts on the project.

Related Work 106 **2**

Video frame interpolation is a long-standing idea that has 108 inspired different approaches to its execution.

Pyramidal representation is presented as a good starting 110 point for image processing tasks (Hahm, 2023). In pyrami-111 dal representation an image is subjected to repeated smooth-112 ing and subsampling. Lowpass pyramidal representation generates a pyramidal representation of an image where 114 each layer presents that image at different resolutions. Low-115 er resolutions provide a coarser overview, and higher resolu-116 tions provide finer overview. This should work in favour of larger motions (Reda, 2022) because larger motions can be 118 detected at lower resolutions and subsequently refined on 119 finer levels of the pyramid. Using original pixels from input 120 images and going forward through any kind of model would be cumbersome. Instead of using pixels, features are extracted on each level of the pyramidal representation. Feature extraction is carried out using convolutional layers. Extracted features are used in later calculations and image synthesis (Hahm, 2023) (Reda, 2022). The result of extract-126 ing features on each level of the image pyramid is the feature pyramid.

One of the more prominent approaches to video frame in-129 terpolation uses optical flow, a concept that stems from 130 psychology (Gibson, 1950). Optical flow is described in 131 literature as the distribution of the apparent velocities of 132 objects in an image or in our context between images. Dis-133 placement vectors are calculated between two or more input 134 images and represent the direction and amount of amount of 135 movement of objects between frames. Provided the correct 136 estimation of optical flow, we can, in theory, anticipate 138 multiplication factor between 0 and 1. Calculating optical 197 calculation starts from the coarsest level. Model predicts 139 flow is a non-trivial matter and many papers focus on refin- $\frac{137}{198}$ task-oriented flows, Wt \rightarrow 0 and Wt \rightarrow 1, from the intermedi-140 ing calculations (W. Shen, 2020) (Liu, 2020) (Hahm, 199 ate frame to the input frames. Task oriented flow is comput-141 2023) or finding ways around problems connected to the 200 ed at each level as the sum of the upsampled flow from the 142 calculations (Ziwei Liu, 2017) (Simon Meister, 2017)

144 ered as a good practice (Simon Meister, 2017) (Liu, 2020). We propose a similar model with changes to the warping 145 Bidirectional flow considers an optical flow from the first training we can make our model more robust to subtle dif- 148 advantage of using bidirectional optical flow is that the ferences between optical flows from one frame to another. 149 extracted features are screened twice, which ensures the We also send the original non-warped features into the de- 150 quality of the features and prevents the subsequent work coder to give it more context on unmodified data. Other 151 from being affected by the quality of the features." (Jiang, modifications, including the dampening of optical flows, 152 2023). If pyramidal representation is used, the optical flow

In the situation where the proper optical flows are ob-156 images. Warping requires a specially built function and In the remainder of this paper, we will discuss related 157 can't be easily understood by a neural network. This is why

Decoder stands at the end of the model, and it is respon-163 sible for intermediate image synthesis. The decoder will be 164 fed information that was calculated throughout earlier parts 165 of the video interpolation process. Depending on the method 166 the decoder could be fed warped frames, initial estimate of 167 intermediate frame, original input frames, scaled bidirec-168 tional flow and warped context features (Hahm, 2023), 169 warped features and optical flows (Reda, 2022), warped 170 input frames and feature pyramids (Liu, 2020), etc. The 171 general rule is that we need to provide enough information 172 to the decoder for it to be able to construct a viable interme-173 diate frame. The quantity and quality of the information we 174 feed the decoder impacts results greatly.

The main model that served as our inspiration is the mod-176 el used in FILM: Frame Interpolation for Large Motion 177 (Reda, 2022). The model and reasoning behind it intrigued 178 us and gave us ideas for development of our version of the 179 model. FILM model takes two frames as inputs and makes a 180 pyramidal representation of them. Each layer of the image 181 pyramid is subjected to feature extraction using a UNet-182 style encoder that allows weight sharing across the scales. 183 Each scale has its features extracted at multiple levels of 184 depth. Features extracted on level l+1 are pulled through the 185 average pool layer of stride 2 before being processed on 186 level l. More features are extracted at deeper levels; UNet 187 encoders dedicated to deeper depths have a greater number 188 of output channels than the encoders at shallower levels. 189 UNet encoders that are on the same depth level share 190 weights between scales. The last part of feature extraction is 191 concatenation of feature maps that share spatial dimensions 192 but are of different depth levels. This process creates a 193 "scale-agnostic" feature pyramid.

With the extracted feature pyramids FILM model contin-195 ues to the next stage, calculation of a bidirectional motion at positions of objects in moments between frames using a 196 each pyramid level. The process of bidirectional motion 201 coarser level l+1 of the feature pyramid and predicted resid- 258 202 ual. Predicted residual is calculated by the stack of convolu- 259 203 tional layers. The stack of convolutional layers is fed con- 260 204 catenation of original features and warped features. Original 261 This stands in contrast to to FILM as it strays away from the 205 features meaning features of one of the input frames ex- 262 assumption that the feature extractor should share weights 206 tracted at that level. Warped features are features of the 263 on all pyramid levels. In our testing we found that the Unet-207 other input frame that have been warped to resemble origi- 264 style decoder struggles to learn the identity with FILM's 208 nal features. The convolutional stack learns to output resid- 265 encoder. To combat this issue, the first layer of the encoder 209 ual flows which supplement the upsampled flow and 266 is not constrained by the lower downsampled layers. Subse-210 through their summation we obtain optical flows for level 1. 267 quent feature extractor layers follow a weight sharing archi-211 "This process is based on the intuition that large motion at 268 tecture and can be expressed as follows in the second layer: 212 finer scales should be the same as small motion at coarser 269 213 scales." (Reda, 2022) Our model follows these same princi- 270 214 ples and methods with one exception. Our stack of convolu- 271 215 tional layers use dampers which reduce the residuals for a 272 and in the third layer: 216 constant factor. Dampers increases stability and consequent- 273 217 ly speed up training of the model.

After obtaining the oriented flows, FILM model creates 275 219 backward warped feature maps of both frames on each lev- 276 Finally, to get the final features, masks from layers with 220 el.

Warped features and task-oriented flows are concatenated 278 ing that features are size-agnostic: 222 at each level and sent to the UNet-like decoder for the final 279 223 part called Fusion. In Fusion, information from coarser level 280 224 $\bar{l}+1$ is upsampled and concatenated to the input information 281 225 from level l. Joint information from levels l and l+1 is fed to 282 226 the stack of convolutional layers. Output from the stack of 283 227 convolutional layers is propagated to the finer level l-1. This 284 228 process repeats until model reaches the finest level. Output 285 229 from the finest level represents the final version of interme- 286 Observe that increasing i is equivalent to going deeper in the 230 diate frame.

232 and therefore it is assumed that the reader is familiar with 289 equivalent to starting the feature extraction with an image 233 FILM's working principles. We implement changes in fea- 290 which has been downsampled by a factor of two j times. 234 ture extraction, flow estimation, warping of input frames 291 Therefore, it's easy to see that concatenating the channels in 235 and change inputs into the decoder for frame synthesis. 292 the above proposed fashion is valid dimension-wise, as the 236 Changes are also made to hyperparameters and contribu- 293 sums of the indexes of every feature f_ij in one level P_k is 237 tions of L1, perceptual and style loss to total loss.

238 **3** Methods

In this section we will describe our model and the key 297 240 differences from FILM, as well as the reasoning behind the 298 In the first step of our model, we calculate the features 241 changes. It takes two inputs images x_1 and x_2 and tries to 299 $P(x_1)$ and $P(x_2)$ of both of the input frames and pass them 242 predict the output image y.

245 aims to extract features in a scale-agnostic manner. In the 303 using residual accumulation in exactly the same fashion as 246 original FILM model this is achieved using weight sharing.

Let D(x) and U(x) be the 2x downsample and upsample ³⁰⁶ 249 operators (we use max pooling and bilinear interpolation, 307 and 250 respectively). For both input frames $x \in \{x_1, x_2\}$ we calcu-308 251 late $D_n(x) = D(D_{n-1}(x))$ and $U_n(x) = U(U_{n-1}(x))$ 309 251 late $D_n(x) = D(D_{n-1}(x))$ 252 where $D_0(x) = U_0(x) = x$.

255 tional block with *in* input channels and out output channels 313 the flow by dividing it with image dimensions s = (v, w). 256 to the input x. The first level of features is extracted as fol- 314 In this way, a flow vector v = (a, b) would be equivalent to 257 lows

$$f_{1i} = \text{conv}_i(x, \text{in_channels,B}) \text{ for all } \forall i \in \{0 \dots 4\}$$

$$f_{2i} = \text{conv}_5(D(f_{1i}), B, 2B) \forall i \in \{0 \dots 3\}$$
 (1)

$$f_{3i} = \text{conv}_6(D(f_{2i}), 2B, 4B) \forall i \in \{0 \dots 2\}$$
 (2)

277 equal spatial dimensions are concatenated, as we are assum-

$$P1 = f_{10}$$

$$P2 = \operatorname{concat}(f_{20}, f_{11})$$

$$P3 = \operatorname{concat}(f_{30}, f_{21}, f_{12})$$

$$P4 = \operatorname{concat}(f_{31}, f_{22}, f_{13})$$

$$P5 = \operatorname{concat}(f_{32}, f_{23}, f_{14})$$
(3)

287 feature extractor, where each layer is pooled and has the This paper tries to recreate and improve the FILM model 288 spatial dimension reduced by two, whereas increasing j is 294 always equal the feature level k = i + j. P(x) =295 (P1, P2, P3, P4, P5) where P(x) is called the feature pyra-296 mid of the image x.

300 to second part of the model - the flow accumulator. The 301 feature pyramids extracted from the encoder are used to The model starts with a hierarchical feature extractor that 302 compute the optical flow between the two input frames 304 the FILM model. Let

$$F_k(Pi, Pj) = \sum r (Pi_m, Pj_m)$$
 (4)

$$F(Pi, Pj) = (F5, F4, F3, F2, F1)$$
 (5)

311 The optical flow in FILM is expressed as an absolute pixel 254 Let conv_n(x,in,out) be a function that applies a convolu- 312 displacement in x and y dimensions, however we normalize 315 a pixel displacement of length $\sqrt{((av)^2 + (wb)^2)}$. We ob-

316 serve that the maximal possible norm $\|\mathbf{v}\|$ is equal to $\sqrt{2}$ 374 317 which helps keep flow values from exploding or causing 375 318 errors with numerical precision. In a 1000x1000 static im- 376 319 age with a maximal displacement of 5% (or 50 pixels) the 377 320 largest flow in pixel terms is 50 and the smallest is 0 (static 378 which is the unconstrained transformation. We use the for-322 ence of normalized flows is only 0.05. Furthermore, having 380 of training to constrain the possible values of F_{12} and F_{21} . 323 a normalized flow value is beneficial in the context of a 381 With this constraint the network quickly learns to superim-325 scaled in the residual calculation, it must be multiplied by 383 decoder. While this constraint limits the expressive power of 326 the upscale factor to maintain consistency, as its value is 384 the network to linear motion, it can be easily removed after 327 dependent on the image resolution. This may also pose a 385 a sufficiently good flow estimator has been learned by re-328 problem for the convolutional block responsible for calcu- 386 placing the first warping regime with the second on an al-329 lating the flow residual, as it must learn to calculate the flow 387 ready trained network and continuing the training. This is 330 residual that is nominally different at different pyramid 388 valid because under the assumption (8), both warp regimes 331 levels, but still corresponds to the same spatial displace- 389 are are equivalent, but the second one removes the con-332 ment.

336 and define the dampened flow as:

$$F'^{(P_i,P_j)} = (d_f F_5, d_f F_4, d_f F_3, d_f F_2, d_f F_1)$$
 (6)

340 where d_f is a number such that in the initial stage of training 398 decoder. To allow the reconstruction of the original features, 341 F' is close to 0. In the case of Xavier initialization (Glorot, 399 we concatenate the original feature pyramid to the decoder 342 2010) which we used, $d_f = 0.01$ was sufficent. The idea 400 input, as well as the warped feature pyramid together with 343 behind this modification is simple: in the initial stages of 401 the flow estimates. As such, on every level i of our decoder, 344 training we assert that the zero function is a good approxi- 402 the input is: 345 mation for the flow. Since we are dealing with frame inter-346 polation of natural videos, the difference between two 347 frames is expected to be on the order of 1/60 to 1/24 sec-348 onds, where we do not expect to find large visual displace-349 ments between the two frames. Instead of starting with a 350 large, normalized flow value that may, in the initial stages 351 of training, distort the image beyond recognition, we let the 352 warp stay small and allow it to move out of the approximate 407 half in every step and at the final step the output is finally 353 solution as the training progresses.

355 After calculating $P(x_1)$ and $P(x_2)$ and the dampened flows 356 in both directions $F_{12} = F'(P(x_1), P(x_2))$, $F_{21} =$ 357 $F'(P(x_2), P(x_1))$, where F_{12} represents the flow that needs 358 to be applied to x_1 to transform into x_2 and F_{21} represents 411 360 x_1 , we apply the warp as follows:

$$P1_{w} = \text{warp}(P(x_{1}), -0.5F_{21})$$

$$P2_{w} = \text{warp}(P(x_{2}), -0.5F_{12})$$
(7)

369 Note that the above transformation implicitly assumes that

$$F_{12} = -F_{21} \tag{8}$$

373 because under that assumption

$$P1_w = warp(P(x_1), 0.5F_{12}) P2_w = warp(P(x_2), 0.5F_{21})$$
(9)

part of the image), however in the same image the differ- 379 mer method of the warp transformation in the initial stages weight-sharing pyramid representation. As the flow is up-382 pose the features over each other before passing them to the 390 straint.

334 In trying to stabilize the learning process with normalized 392 The last part of our model is a decoder. In the original FILM optical flow, we also propose to use a damping factor d_f 393 model, the decoder is a Unet-style decoder that takes only 394 the warped features and the flow estimates to reconstruct the 395 original image. We found that this decoding scheme is too 396 reliant on a very accurate flow estimator, as there is no in-397 formation flow from the original unwarped image to the

$$\begin{aligned} \operatorname{dec}(i) &= \operatorname{conv}(\operatorname{concat}(P1_i, P2_i, & & \\ & P1i_w, P2i_w, & & \\ & F_{12i}, F_{21i}, & & \\ & U(\operatorname{dec}(i-1)))) \end{aligned} \tag{10}$$

405 where dec(0) is an empty tensor and out = conv(dec(5))406 is the final output of our model. The convolutional channels 408 passed through the last convolutional layer with kernel size 409 1 to produce the RGB channels.

Materials

We used the Vimeo90K (Xue, 2019) to train our model 359 the flow that needs to be applied to x_2 to transform it into 412 and we used the factor B=24 for the base number of chan-413 nels, which means the model has 33.4M parameters. We 414 trained the model on 256x256 image crops with the Adam 415 optimizer at batch size 7 using a learning rate of 1e-4 and 416 reducing it after 4 epochs to 5e-5 and then training for 4 417 more epochs. After this, the model was trained for 1 epoch 365 Where warp() applies a bilinear resample warp to every 418 on full-resolution Vimeo90K for better noise resilience. As 366 level of the feature pyramid with their respective flow ap- 419 it's a fully convolutional model, it's resolution-independent. proximations and constructs a new warped feature pyramid. 420 The model was implemented in Pytorch and trained on a 421 single laptop RTX 4060 GPU.

> 423 We minimized L1 loss and when the result was satisfactory 424 we switched to the style loss proposed in the FILM paper.

Results and discussion

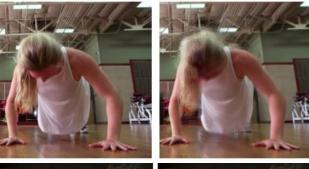






Figure 1: On the left are the original intermediate frames and on the right are the frames generated by our model using L1 loss.

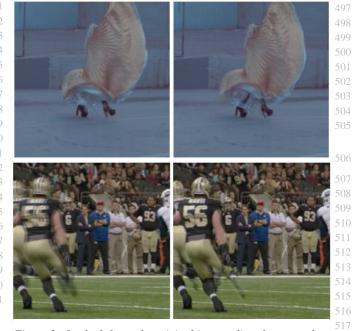


Figure 2: On the left are the original intermediate frames and on the right are the frames generated by our model using L1 and style loss.

We trained the network with L1 for 8 epochs, which is in 474 total only ~50K iterations at our batch size, compared to 475 FILM which was trained for 3M iterations. Due to computa- 522 Gibson, J. J. (1950). The perception of the visual world. 476 tional constraints, training at a high resolution or with a 523 477 large batch size wasn't feasible.

We found that the L1 model gave good results for video 80 frames that do not exhibit large motion. In frames that ex-81 hibited large motion, the model would produce blurry imag-82 es. This is a known limitation of using L1 loss.

	Vimeo90K
	PSNR (mean)
Our model L1 loss	33.0361
Our model L1 + style loss	33.3934

Table 1: Comparison of PSNR for two iterations of our model on Vimeo90k dataset.

Using style loss, the model produces sharper images with 487 fewer blurry regions, even with motion that would have 488 otherwise caused blurring. While the intermediate frames 489 did look better when viewed as stills, we found that using 490 style loss for the purpose of video interpolation has multiple problems. For one, minimizing style loss makes the model 492 more likely to produce outputs that are very similar copies of one of the input frames. This causes interpolated videos 494 to look choppy, because the intermediate frame does not 495 really represent a frame that is temporally in between the 496 two input frames. Furthermore, we found that artefacts pro-497 duced by models minimized with style loss tend to be more 498 visually intrusive as they tend to have sharp edges and high 499 contrast, whereas most of L1 artefacts were happening 500 strictly in regions with high motion and were typically blur-501 ry – in a way resembling motion blur that naturally occurs 502 when recording fast motion with a high exposure time. Because of this, we consider L1 to be superior when interlac-504 ing original video frames with generated ones.

Conclusion

In this paper we gave insights into the topic of Video 508 Frame Interpolation and the current state of the field, as well 509 as possible ways in which the field can advance. We sum-510 marized and discussed key concepts and past works that have been crucial to our project. We explored ways of modi-512 fying existing methods of video frame interpolation, specifi-513 cally FILM model (Reda, 2022). We presented our modifi-514 cations as well as reasonings behind them. We presented 515 and discussed our results obtained on our model using two 516 versions of a loss function. We found that we were able to 517 train a model for only 8 epochs with damper and normalized 518 flow and got visually pleasing results. Further work on 519 training the model with higher computational power is nec-520 essary to find the performance ceiling of our model.

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