International Journal of Thermal Sciences 139 (2019) 424–432

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

A review of the models using the Cattaneo and Vernotte hyperbolic heat equation and their experimental validation

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ARTICLE INFO

Keywords:
Non-Fourier
Cattaneo
Vernotte
Double phase lag
Hyperbolic heat equation
Bio heat transfer

ABSTRACT

Models using Cattaneo and Vernotte hyperbolic heat equation or derived from it (Double Phase Lag model and their various versions) are very common in the present thermal literature, especially for simulating heat transfer at the meso scale, such as in bio heat transfer. Such papers refer to so-called experimental validations made in several previous articles. We show in this short review that the corresponding experiments were biased, because of a deficient methodological approach based on too simple assumptions or on poor data reduction techniques.

1. Introduction: non-Fourier models for heat conduction

There are presently many papers published in the heat transfer literature that deal with « Dual Phase Lag » models or using the Cattaneo-Vernotte term, an extra hyberbolic term to « correct » the Fourier heat equation. These models were originally designed for very short times and distances. They are based on a phenomenological adhoc correction for the Fourier constitutive law, that is the definition of the heat flux. They also seem to be very popular within the domain of bio heat transfer.

It will be shown here that papers dealing with this subject refer either to models that are not relevant with the corresponding experiments or to ill-designed parameter estimation methods. We will first present a short review of these models, and then scrutinize in which conditions the authors claim they have validated them for use in bio heat transfer, beginning with the experimental results.

The final part of this review will concern a paper devoted to a Fourier based model for one-dimensional thermal diffusion in materials with a non-homogeneous inner structure. This one uses time dependent conductivities and volumetric heat capacities.

1.1. The Cattaneo-Vernotte model

We remind the definition of the heat flux $\boldsymbol{q}$ proposed by Cattaneo [1] and Vernotte [2]:

$$\boldsymbol{q} + \tau \frac{\partial \boldsymbol{q}}{\partial t} = -k \, \nabla T \tag{1}$$

k is the thermal conductivity of the medium while τ is an extra thermophysical property of the medium, a relaxation time, specific to this model which is denoted CV hereafter. As a consequence, a local heat balance on a homogeneous medium with no thermal dependance nor velocity, yields the CV heat equation:

$$\rho c \frac{\partial T}{\partial t} = -\nabla . \boldsymbol{q} \tag{2a}$$

$$\Rightarrow \rho c \left(\frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} \right) = k \nabla^2 T \tag{2b}$$

where ρ is the density and c the specific heat of the medium.

Let us note that the classical Fourier model corresponds to a zero relaxation time in equation (1), with a heat equation of parabolic nature, that is of the diffusion type. As soon as τ departs from zero, equation (2b) becomes an hyperbolic partial differential equation, that is a wave equation:

$$\frac{1}{a} \frac{\partial T}{\partial t} + \frac{1}{C^2} \frac{\partial^2 T}{\partial t^2} = \nabla^2 T \tag{3}$$

where celerity C depends on τ and on the thermal diffusivity of the medium $a = k/\rho c$:

$$C = \sqrt{a/\tau} \tag{4}$$

The Laplace transform of equation (1) yields, for a zero initial temperature field:

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https://doi.org/10.1016/j.ijthermalsci.2019.02.021
Received 2 November 2018; Received in revised form 11 February 2019; Accepted 12 February 2019
Available online 27 February 2019
1290-0729/

$$\bar{\boldsymbol{q}} = -\frac{k}{1+p\tau}\nabla\bar{T}$$
$$\text{where } \bar{\psi}(p;\boldsymbol{r}) \equiv L[\psi(t,\boldsymbol{r})] \equiv \int_0^\infty \psi(t,\boldsymbol{r})\exp(-pt)\mathrm{d}t \tag{5}$$

In equation (5), the upper bar on any function ψ of time t designates its Laplace transform also noted $L[\psi]$, variable p being the Laplace parameter and $\boldsymbol{r}$ being the vector of space coordinates.

Returning to the time domain, a convolution product appears in the CV definition of the heat flux $\boldsymbol{q}$, which denotes a memory effect for the system:

$$\boldsymbol{q}(t,\boldsymbol{r}) = -\int_0^t f(t-t')\nabla T(t',\boldsymbol{r})\mathrm{d}t' \tag{6a}$$

$$\text{with } f(t) = \frac{k}{\tau}\exp\left(-\frac{t}{\tau}\right) \tag{6b}$$

This CV heat equation has been introduced because of the apparent « paradox » of an infinite value for the propagation velocity C, especially for « short times » after the initial thermal excitation of a material, while the Fourier heat equation would remain valid for long times only.

To our knowledge, the CV heat equation has never been validated experimentally on classical solid homogeneous materials. Some experimental works [3,4] showed that for very short times (10^{-12} to 10^{-13} s, that is for pico second laser excitations or shorter durations), and consequently for very small distances, the Fourier heat equation is not valid any more and 2 temperatures (electrons and lattice) have to be introduced to describe the transfer of heat.

1.2. The dual phase lag model

Another non-Fourier constitutive equation, the dual phase lag model (DPL), has been introduced in 1995 by J. Zhou [5]. Its constitutive equation is:

$$\boldsymbol{q}(t+\tau_q,\boldsymbol{r}) = -k\nabla T(t+\tau_T,\boldsymbol{r}) \tag{7}$$

It involves two time lags, one for the flux, τ_q and the other for temperature, τ_T. In fact J. Zhou made a first order Taylor expansion with respect to τ_q in the left term of equation (7) and with respect to τ_T in its right term, to obtain the linearized DPL model:

$$\boldsymbol{q}(t,\boldsymbol{r}) + \tau_q\frac{\partial\boldsymbol{q}}{\partial t} = -k\nabla T(t,\boldsymbol{r}) + \tau_T\frac{\partial T}{\partial t}(t,\boldsymbol{r}) \tag{8}$$

This development was made with the assumption that both τ_q and τ_T are small (and, implicitely, of the same order of magnitude) but Zhou did not mention how small they should be: it should be small with a reference quantity.

It is easy to show, using a Laplace transformation of (8):

$$\bar{\boldsymbol{q}} = -k\frac{1+p\tau_T}{1+p\tau_q}\nabla\bar{T} \tag{9}$$

Returning to the time domain, a convolution product appears in the constitutive equation of the DPL model:

$$\boldsymbol{q}(t,\boldsymbol{r}) = -k\frac{\tau_T}{\tau_q}\nabla T(t,\boldsymbol{r}) - \int_0^t g(t-t')\nabla T(t',\boldsymbol{r})\mathrm{d}t'$$
$$\text{with } g(t) = k\left(1-\frac{\tau_T}{\tau_q}\right)\exp\left(-\frac{t}{\tau_q}\right) \tag{10}$$

The corresponding DPL heat equation can be derived from the local heat balance (2a) and from equation (8):

$$\rho c\left(\frac{\partial T}{\partial t} + \tau_q\frac{\partial^2 T}{\partial t^2}\right) = k\nabla^2 T + k\tau_T\nabla^2\left(\frac{\partial T}{\partial t}\right) \tag{11}$$

Of course, the Fourier model is recovered if $\tau_q = \tau_T = 0$, which was already apparent in equation (7) while the CV model corresponds to $\tau_T = 0$. Let us remark that it should be natural to have the following restriction for the values of the two time lags $\tau_T \le \tau_q$, because, in the opposite case, equation (7) would violate the causality principle with a flux at a given time that would depend on a temperature at a later time.

2. Review of experimental papers trying to validate non-Fourier models for bio heat transfer

The experiment-based papers we have decided to review next tried to validate a non-Fourier model for different classes of material samples. It will be shown that they suffer from serious flaws that do not allow them to reach their objective.

2.1. Kaminski [6]

This paper deals with the CV model given in equations (3) and (6a), with an attempt to estimate the relaxation time τ from temperature measurements in materials with « nonhomogeneous inner structure », without any clear definition of this term. The morphological structure of the tested samples seems to be granular porous beds of given void fraction (« H acid », sodium bicarbonate, sand, glass ballotini, « ion exchanger »). The setup, see Fig. 1, consisted of a sample containing a line electrical heater and a thermocouple located in a needle parallel to it, at a distance x_{TC}.

This distance was equal to 6.8 and 16.8 mm in the first and second experiments respectively. In order to estimate the relaxation time, the author introduced an ill-defined penetration time t_p, corresponding to the time for the temperature signal to depart from its initial value before the start of the excitation, that was supposed to be a step thermal power in time. Once this penetration time has been measured, the relaxation time was calculated using the celerity C of the « heat wave » and equation (4):

$$x_{TC} = C\ t_p \Rightarrow \tau = a\ t_p^2/x_{TC}^2 \tag{12ab}$$

This required the determination of the diffusivity that was estimated using the Fourier model for a line heat source, which is not consistent with the purpose of the author: the two parameters τ and a should have been estimated using the solution of CV model corresponding to this geometry and excitation only.

The authors measured values of τ ranging from 11 to 54 s for the considered samples but did not show the corresponding measured temperature responses in this paper.

The major bias in these experiments results from the definition of the penetration time t_p that is specific to the location of the

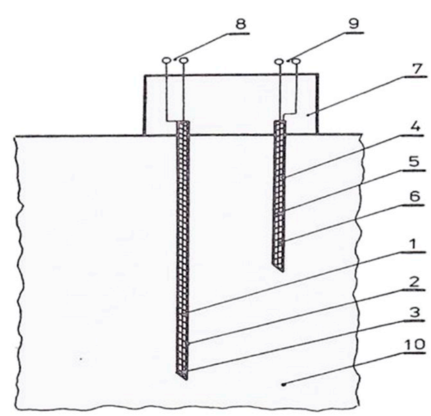

Fig. 1. Setup used by Kaminski [6]: (1) resistance wire, (2) needle, (3) electric insulation, (4) needle, (5) thermocouple, (6) insulation, (7) support, (8) to power supply and stabilizer, (9) to amplifier and recorder, (10) material. Reprinted by permission from: [ASME] [Journal of Heat Transfer] [Hyperbolic heat conduction equation for materials with a non-homogeneous inner structure, W. Kaminski] (1990).

thermocouple. As there is an asymptotic zero level for time going to zero in the Fourier solution, t_p is the time where the signal gets larger than its measurement noise, that is a threshold that can be quantified by its standard deviation. So the penetration time depends on the level of the forced excitation, the step of thermal power here: an increase of this power will allow the temperature signal to gets larger than the noise level at an earlier time, with a corresponding decrease for t_p.

Other biases are the definition of the time $t = 0$ that depends on the inertia of both the heater and of the thermocouple sheaths and on the uncertainty in their exact locations, together with the intrusive character of the thermocouples. These points have not been discussed in the paper, as well as other possible sources of error: thermal contact resistances between both actuator and sensor with the porous bed, validity of the 1D model that is used for estimating the diffusivity (3D conduction can take place because of the finite length of the resistance).

Several weaknesses of this paper were pointed out in Refs. [7,8].

2.2. Mitra et al. [9]

This paper also deals with an attempt to validate the CV model on processed meat samples. The principle of the first experiment consisted in bringing into contact 2 identical meat samples initially at two different temperatures (room temperature and a cold temperature), with 2 thermocouples embedded in the cold sample at 2 different distances from the contact plane, see Fig. 2a.

Three other similar contact experiments involving 3 meat samples or an aluminium plate have also been made, see Fig. 2a and b. So, contrary to Kaminski's experiment (forced thermal regime generated by a step in thermal power), the transient character of the recorded temperature signal stemmed from a relaxation of the temperature field (a free regime) in each of the materials brought into contact.

The relaxation times (of the order of 15 s for the 4 experiments) were measured exactly in the same way as in Ref. [6], that is using a penetration time. So, they suffer from the same bias: penetration times depend both on the measurement noise and on the initial temperature difference between the two bodies that are brought into contact.

The authors plotted together the thermal responses of the temperature sensors and the Fourier theoretical responses valid for instantaneous contacts between semiinfinite media, with an important time lag for the measurements.

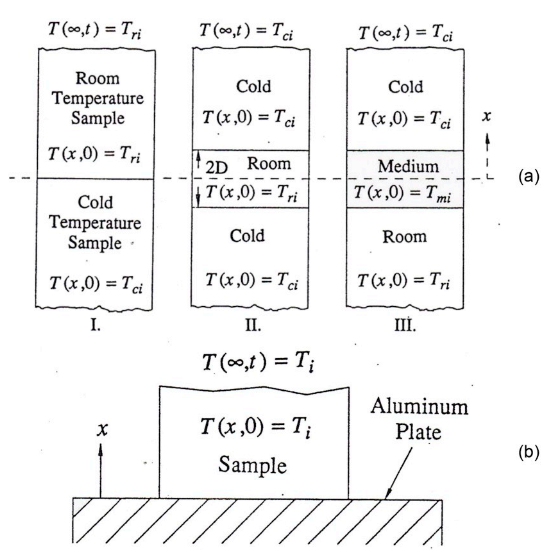

Fig. 2. Setup used by Mitra et al. [9] (a) Schematic of the experimental conditions for Experiments I, II and II (b) Schematic of the experimental conditions for Experiment IV Reprinted by permission from: [ASME] [Journal of Heat Transfer] [Experimental evidence of hyperbolic heat conduction in processed meat, K. Mitra, S. Kumar, A. Vedavarz, M.K. Moallemi] (1995).

Many questions arise, first concerning the choice of the initial time: the experimental procedure of bringing two pieces of meat into contact was not described and the contact model used was too simple since the samples were soft materials whose thicknesses and structures probably varied once a pressure had been applied. In the same way the positions of the thermocouple junctions may have moved during this mechanical operation. Another question arises: the cold samples were removed from a cold chamber before the experiment. In this lapse of time an internal temperature gradient may have built up inside: this could have been checked by recording the temperatures before the experiment, in order to see whether a corresponding relaxation appeared in the monitored samples.

2.3. Roetzel et al. [7]

The first two parts of this article are analyzed here: it contains first a balanced review of 4 papers of the literature on the CV model and on the attempts made for its experimental validation/invalidation. The first two [6,9] are analyzed above, while the last two conclude on the validity of Fourier law and on the absence of hyperbolic effects after experiments on wet sand [10], and on dry sand and processed meat [8]. We sum up here the main results of these last two papers before returning to the analysis of the last part of [7].

2.3.1. Graβman and Peters [10]

Graβman and Peters [3] constructed an instrumented cylindrical cell, with PVC walls and lids, containing the porous material for which the validity of the Fourier law had to be tested. It was heated by Joule effect in a metallic wire located on the axis of the cell and with NTC thermistances located at different radii in the central horizontal plane in Fig. 3. So, in this plane, a corresponding analytical model was found: it was 1D, in the radial direction, because of angular symmetry, and transient. It took into account the inertia of the heated wire and linear heat losses with the lateral wall and the outside environment. It was verified using a finite difference code. Experiments were made with wet sand, with calculation of the volumetric heat capacity of the porous medium by a mixing law and measurement of its equivalent conductivity by a 1D steady state experiment in the same cell. The authors used a Joule heating composed of a succession of door functions, with relaxation regimes in between (on/off switchings). The simulated temperature curves and the experimental one overlap very well, once the previously estimated conductivity has been modified, in order to take heat losses through the lids into account: these could not be considered in the previous 1D steady experiment. No phase lag appeared, even for the fastest temperature responses.

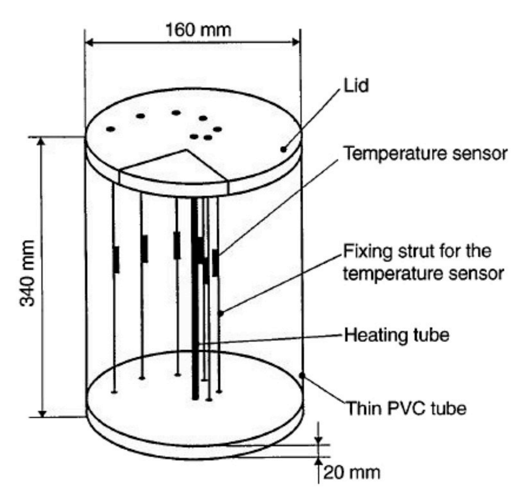

Fig. 3. Setup used by Graβman and Peters [10] Reprinted by permission from: [Springer] [Heat and Mass Transfer] [Experimental investigation of heat conduction in wet sand, A. Grassman and F. Peters] (1999).

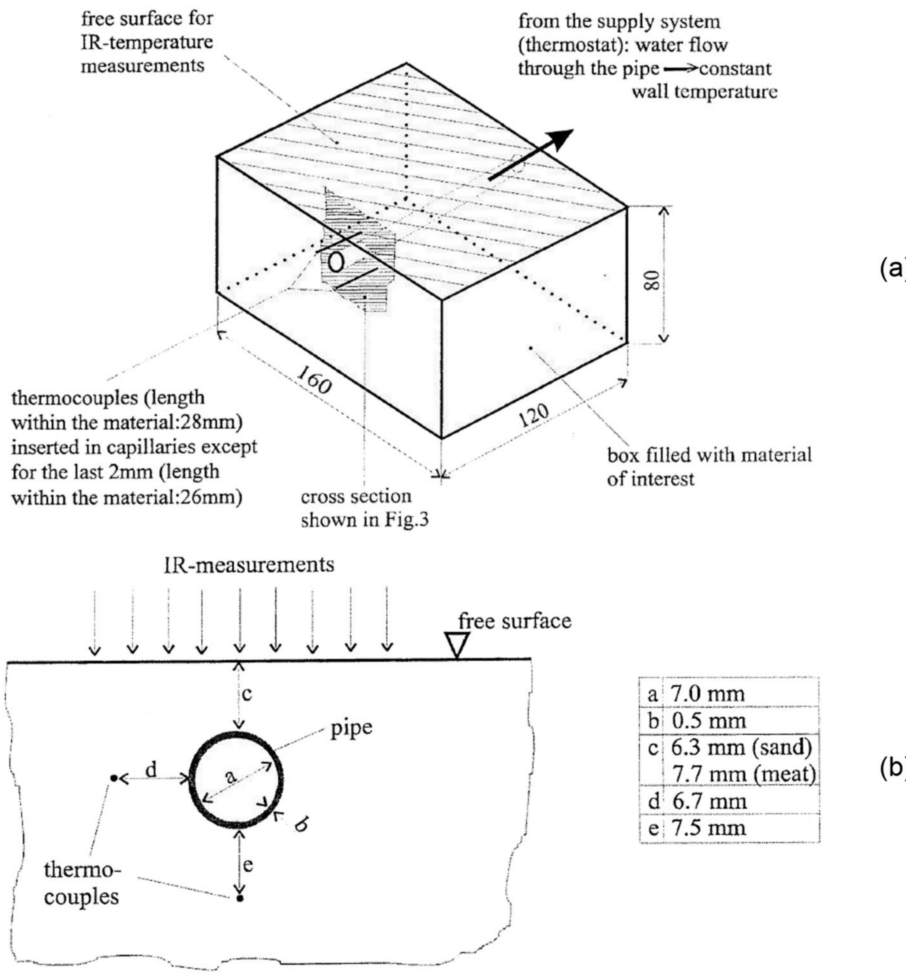

Fig. 4. a,b – Setup used by Herwig and Becker [8]. (a) Sketch of the experimental arrangement. (b) Geometrical details of the experimental arrangement Reprinted by permission from: [Springer] [Heat and Mass Transfer] [Experimental evidence about the controversy concerning Fourier or non-Fourier heat conduction in materials with a nonhomoneous inner structure, H. Herwig, K. Beckert] (2000).

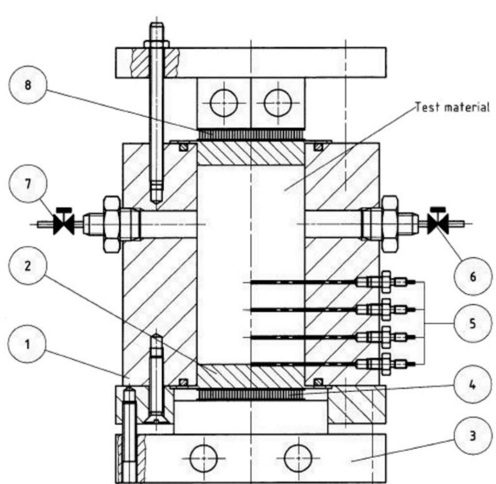

Fig. 5. Setup used by Roetzel et al. [7]: the test cell for experiment.

2.3.2. *Herwig and Beckert [8]*

Herwig and Beckert [8] tested the Fourier law in a box filled with the material of interest, se Fig. 4a. A copper pipe was set below the free upper surface. At time $t = 0$, a valve was open and water started to flow at a temperature lower than the initial temperature in the system. So, the thermal excitation was closer to a uniform temperature jump, which was approached in Mitra et al. [9] than to a uniform flux (or more precisely a uniform volumetric generated power inside a thermal actuator) as in the preceding work [10]. Two thermocouples were located in the material and an infrared camera was able to monitor the 2D temperature field of its upper free surface, see Fig. 4b.

The authors used processed meat and dried sand as filling material in the box. The thermocouple responses did not exhibit any time lag once the water flow valve was open. The same was true for the temperature measured by the infrared camera at 3 different axial locations on the free surface. The thermal diffusivity was estimated, for both dry sand and processed meat, in order to minimize the temperature residuals (curve fitting) between the two thermocouple measurements and the corresponding output of a numerical simulation code. The estimated diffusivities, for processed meat, were consistent with the diffusivities used in Mitra's study [9].

2.3.3. *Experimental study of Roetzel et al.'s paper [7]*

In the second part of Roetzel et al.'s paper [7], the authors constructed a vertical cylindrical box, see Fig. 5.

This box was filled with the material to be characterized, and the bottom face was stimulated by a Peltier element (4) in harmonic regime. Its top surface was cooled by another Peltier cell (8), using the same thermostatic bath as the cooling water of the Peltier cell. Parallel thermocouples (5) were inserted in planes normal to the axis, with hot junctions at different axial positions. The authors constructed the analytical steady harmonic solution (complex temperature) of the CV heat equation model in a 1D semi infinite medium, where the two parameters, the relaxation time τ and the diffusivity a, were present in both the amplitude ratio (attenuation) and in the phase shift together with the angular frequency of the source. They used a data acquisition system to measure the attenuation B and the phase shift G of the signal and a closed form formula allowed them to retrieve both parameters using one frequency and the output of two thermocouples. They got values of τ close to 2 s for both dry sand and processed meat.

The following remarks can be made about these measurements and their parameter estimation part:

1) The model used is based on a 1D heat transfer assumption in the sample to estimate its two parameters, using the well known Angström bar method [11] for measuring thermal diffusivity. This method relies on a very strong assumption, that is the absence of losses in the transverse direction, and is not used for estimating the thermal diffusivity anymore. However, no thermocouple was inserted with off axis junctions to check the effect of a 2D transfer caused by simultaneous diffusion in the radial direction. This could have happened here because the metallic sheathed thermocouples were set in a direction perpendicular to the main (axial) temperature gradient, contrary to the arrangement made by Graβman and Peters [10]. So they could have offered a preferential path for heat conduction in the radial direction. So this model bias has probably happened in the case of process meat since the characteristic 1D diffusion time based on the radius of the sample $R = 20$ mm is $t_c = 0.132\,R^2/a = 400$ s [12] for a period of the harmonic source shown in the paper close to this value (about 7 mn).
2) No temperature residual plot, showing the difference between the experimental signal and its recalculated values using the analytical model fed with the estimated parameters has been shown: it would have been useful to check the 1D hypothesis at least. In a similar way, replication of the estimation for different pairs of sensors would have allowed to check the relevance of the model.
3) The value of 2 seconds that was found for τ is two orders of magnitude smaller than the period of the excitation, so a sound error analysis should have been made to assess the values of its uncertainty. The error analysis given in equations (20) and (21), whose numerical values were not given in the paper, is deficient for two reasons: the relative errors in B and G are correlated since they stem from the same measurements and the authors forgot to transform the minus sign into a plus sign when they passed from the logarithmic differential of a ratio to the corresponding relative error (if $z = y/x$, then $\Delta z/z = \Delta y/y + \Delta x/x$ and not $\Delta z/z = \Delta y/y - \Delta x/x$).

2.4. Interpretation of the experiments of Andrä et al. (13) by Liu and Chen [14]

Andrä et al. [13] made transient temperature measurements in a medium composed of two regions: a cylindrical region (carrageenan, a sulfated polysaccharide) containing magnetic particles heated by an alternating magnetic field, that was embedded in an extended muscle tissue from cow. This system was used to model magnetic hyperthermia therapy for small breast carcinomas, see Fig. 6a. Thermocouples, whose junctions were located in both regions, allowed transient temperature measurements, see Fig. 6b.

A corresponding Fourier type temperature field has been modeled, where the heated region (a step heating) was given a spherical shape. Comparison between the measured and modeled 1D (radial) transient temperatures were made, with small differences during the transient phase only. The authors explained the systematic deviations (measured temperatures lower than simulated ones) by errors in the thermocouple positions. Another source of error, apart from the spherical shape assumption that can modify the temperature field for short times, can stem from the different thermopysical properties of the two materials that were taken from the literature, with a series model for the

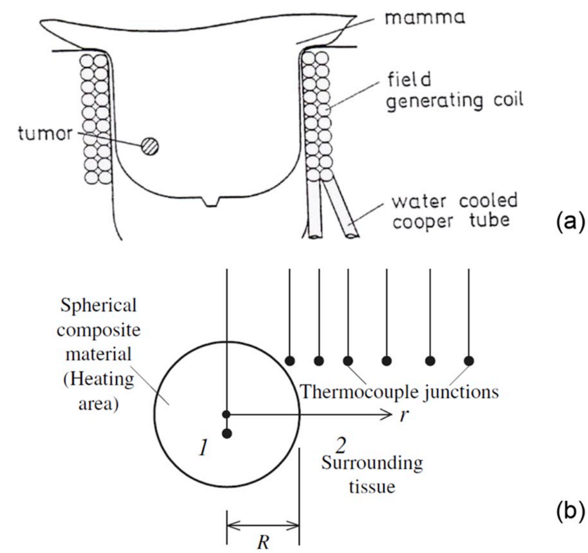

Fig. 6. Setup used by Andrä et al. [13] and investigated by Liu and Chen [14] (a) Scheme of localized magnetic hyperthermia applied to a breast carcinoma. The tumor region contains injected magnetic particles, from Ref. [13]. (b) Illustration for model comparison, from Ref. [14].

conductivity of the heated region. The authors concluded that there was a agreement between their temperature measurements and the outputs of their model, which was based on the classical Fourier law.

Liu and Chen [14] revisited the measurements of Andrä et al., in order to compare them to the output of a DPL heat transfer model. Since this model needed 2 lag times (τ_q and τ_T) per region, that is 4 parameters, a sound inverse technique would have consisted in implementing a non linear least square parameter estimation technique to all the transient temperature measurement points of [13], that is the 44 measurement made in the non heated region. However, the authors used only 4 transient measurements per radius, to get one set of 4 relaxation time lags per radius.

They used therefore what is called « exact matching » in statistical parameter estimation [15,16], which means that they solved 4 equations with 4 unknowns per radius. This resulted in residuals equal to zero for each radius: there was no room left for the effect of measurement noise, for the error caused by uncertainty in the « classical » thermophysical properties of both materials as well as for the error on the precise location of the thermocouples. Under these conditions, no wonder the DPL model outputs went precisely through the experimental points.

The authors found τ_q between 7.4 and 8.9 s and τ_T between 14.5 and 21.4 s. Let us note that these values violate the causality principle ($\tau_T \leq \tau_q$) already mentioned at the end of section 1 above.

2.5. Jaunich et al. [17]

These researchers, which were interested in laser induced hyperthermia made experiments with single- and multi-layer tissues phantoms that simulated skin, with inhomogeneities simulating a tumor, and freshly excised mouse skin tissue samples. These phantoms were composed of different polymer resins with India ink as a radiation absorber and Titanium dioxide particles as scatterers. Inhomogeneities have been drilled in the tissue phantoms, but no information was given about their composition in the paper. A focused laser beam, as well as a collimated beam, was used in order to obtain a higher required temperature rise at the region of interest for the former type of excitation, see Fig. 8.

The temperature was measured axially by embedded thermocouples while an infrared camera was said to give some informations about its radial distribution.

Both the Fourier and Cattaneo Vernotte forms of the heat equation were solved numerically, with the volumic source term calculated through the solution of the radiative transfer equation (transient discrete ordinate method), with absorption and scattering, but without emission, for a given distribution of the absorbed light of the laser beam.

The authors showed simulated radial temperature and axial distributions and compared them with several in and off-axis point measurements. This was made for discrete values of the unique relaxation time τ of the CV model (from 10 to 20 s) or of the Fourier model (τ = 0 s). However, the way the off-axis points have been measured is unclear: the presence of corresponding thermocouple positions is not given and, transformation of the infrared camera signal into a local inner, and not surface temperature, for rather absorbing materials in its spectral interval [8–14 μm] is not explained, even if this camera seems to have been used for measuring temperatures in the top circular surface of the tissue. The corresponding temperature histories are also given for different values of the irradiation times (8–17 s). In all presented figures, the temperatures output of the Fourier model ly below the CV corresponding simulations and below the experimental points. However, the fit (in terms of shape of the space or time distributions) between CV and measured temperatures is not really good, which is not surprising because of the large number of not precisely known parameters present in the numerical model.

2.6. *Sahoo et al. [18]*

These authors made an experiment where a basin containing a nanoparticle (gold « mesoflowers ») embedded collagen gel was heated by a near infrared laser, see Fig. 8.

Several 1 mm sheathed thermocouples were inserted in- and off-axis and the step heat source space distribution in the simulation model took into account absorption and scattering of the laser beam through a simple Beer's type law. The transient temperature signal of the thermocouples were compared to the corresponding output of the numerical solution of different versions of the DPL heat transfer model, see equation (2a) with an extra volumetric heat source and (7). Its first order linear version (8) was implemented, as well as higher order series expansion of the two terms of (7), the values of the two lag times τ_q and τ_T being derived from Ref. [14].

It should be noted that this model seems to have been solved in rectangular 2D coordinates, while the axis of symmetry of the laser beam should have defined the axisymmetrical cylindrical system of coordinates of the right numerical solution of direct problem.

The authors found that the Fourier model underestimated the experimental points more than the simpler linearized DPL model. However higher order versions of the DPL model, with different forms of the laser heat absorption and scattering model and different values of the 4 penetration times could better fit the experimental points.

Regarding the experiment, a possible bias was derived from the semi transparent character of the medium used, with a possible different absorption of the laser radiation by the thermocouple junctions that may create a local temperature difference with the medium at short times.

Regarding the model output, there are many « supposed to be known » parameters in the different forms of the DPL model used: the heat exchange coefficient at the free surface of the basin, the thermal diffusivity and volumetric heat of the homogenized composite medium, the exact location of the thermocouples, the 2 DPL time lags, the 3 optical parameters of the heat generation term, the intensity of the laser, whose increase would bring the « classical Fourier based Pennes Model » at the same location as the experimental points in Fig. 7 of the paper. In a similar way, the very simple solution of the coupled conduction-radiation problem, that does not use any radiative transfer equation while some scattering is present, could be questioned.

So, evidence of a non-Fourier behaviour is impossible to be brought to the fore by this paper.

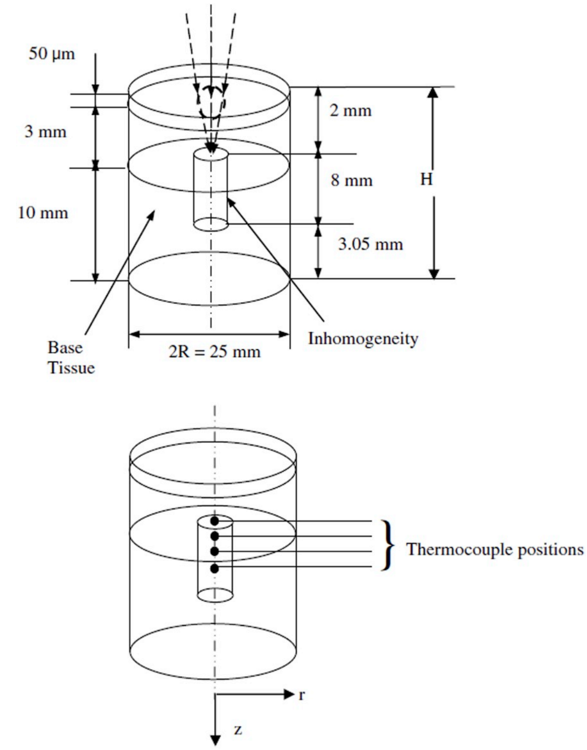

Fig. 7. Setup used by Jaunich et al. [17]: Schematic of tissue phantom containing inhomogeneity (focused beam technique.

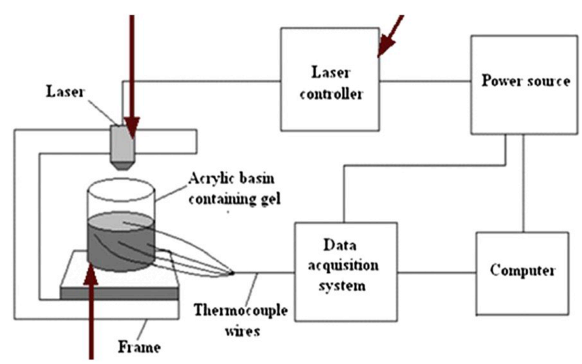

Fig. 8. Setup used by Sahoo et al. [18]: Schematic diagram of experimental setup (the full arrows of the original figure were connected to photographs of the basin, of the laser and of its controller).

3. Further remarks on the validity of the Fourier law for heat diffusion at the meso-scale

The above review has shown that non-Fourier models have not been validated at the meso-scale for heat transfer in heterogeneous materials. One can wonder whether Fourier's law applied to each of its constituents could lead to a global behaviour of the corresponding system that differs from that of an equivalent classical equivalent homogeneous medium having a given uniform volumetric heat and conductivity. A response is possible for transient heat transfer in a multilayer stack.

If the material is not homogeneous, but 1D periodical, it has been shown that it can be replaced by an equivalent homogeneous material that obeys Fourier's law, as soon as the number of unit cells is large enough [19].

If it is not the case, a non local approach has shown analytically, in the same paper, that a stack composed of a limited number of different materials can be replaced by an equivalent material system, in terms of inlet and outlet temperatures and fluxes.

Table 1
Synthesis of experiment-based papers that claim to validate a hyperbolic model (CV or DPL).

First author, paper number and year	Tested materials	Excitation and temperature measurement	Principle of validation test of CV or DPL model	Main remarks
Kaminski [6] 1990	sand and various materials	linear Joule heating and transient thermocouple responses at an off-axis location	observation of the time lag in the measured temperature versus time curve to retrieve the CV relaxation time	The observed lag time depends on the levels of the excitation and on the measurement noise.
Mitra [9] 1995	processed meat	Instantaneous contact between materials at different initial temperatures with thermocouple measurements	observation of the time lag in the measured temperature versus time curve to retrieve the CV relaxation time	Same remarks as in Ref. [6]. The initial temperature field in each material is not controlled and the latter deformations of the system are not taken into account.
Roetzel [7] 2003	processed meat, synthetic sand, sodium bicarbonate and various porous materials under variable gas pressure	Angström bar type experiment with planar Peltier heating and cooling and thermocouple measurements (harmonic regime)	non linear estimation of both CV relaxation time and thermal diffusivity using amplitude ratio and phase shift	The 2D effects are not studied nor taken into account in the model. The error analysis is deficient.
Liu [14] 2010 (discussion of Andrä [13] 1999 experiments)	muscle tissue from cow	heating by a radio frequency field in a spherical region containing magnetic particles and transient off- axis thermocouple responses in outer region	estimation of the 2 phase lags per region of the 2D (radial and axial) DPL model to get the best fit for the measured temperatures	Non consistent estimation technique: the phase lags depend on the chosen measurement radii (exact matching)
Jaunich [17], 2008	skin phantoms and mouse skin tissues	laser beam heating and in and off-axis thermocouple and infrared camera measurements	adjustment of the relaxation time of the 2D (radial and axial) CV model to get the best fit for the measured temperatures	The final temperature solution of the CV heat equation uses 2 other optical models (laser beam time and space variation, radiative transfer equation) for 2 or 4 regions (case of the 3 layer phantom with 1 embedded heterogeneity) which depend on many not precisely known parameters that are retrieved from the literature. So, an extensive sensitivity study is lacking.
Sahoo [18]	collagen gel with embedded gold particles	laser beam heating and in and off-axis thermocouple measurements	adjustment of the 2 phase lags of various order expansions of the 2D (radial and axial) DPL model to get the best fit for the measured temperatures	Same type of remark as for [17]. A possible different absorption of the laser beam by the thermocouple junctions, leading to biased temperature measurements, is not considered.

This equivalent material is characterized by a heat flux that is defined through a convolution product, different from equations (6) and (10), between a time dependent conductivity and a time derivative of the temperature gradient if the stack of thickness e presents a material symmetry with respect to the $x = e/2$ plane, that is if inlet ($x = 0$) and outlet ($x = e$) can be switched. In that case, the constitutive law, purely based on Fourier heat transfer in each local internal layer of the stack is:

$$q(t, x) = -\int_0^t k(t-t') \frac{\partial^2 T}{\partial x\, \partial t}(t', \boldsymbol{x})\, \mathrm{d}t' \tag{13}$$

with the conductivity function $k(t)$ depending not only on time t but also on the thermopysical properties and thicknesses of each layer of the stack.

The related heat equation is:

$$\int_0^t \rho c(t - t') \frac{\partial^2 T}{\partial t^2}(t', x)\, \mathrm{d}t' = \int_0^t k(t - t') \frac{\partial^3 T}{\partial x^2\, \partial t}(t', x)\, \mathrm{d}t' \tag{14}$$

with the volumetric heat capacity function $\rho c(t)$ depending on the same quantities as $k(t)$.

Let us note that these equations only govern the temperature of the equivalent material and yield the exact temperatures and fluxes at the boundaries of the domain ($x = 0$ and outlet $x = e$) only.

In the case of a non symmetrical stack of layers, an additional convolutive term has to be added to the constitutive equation:

$$q(t, x) = -\int_0^t k(t - t') \frac{\partial^2 T}{\partial x \partial t}(t', \boldsymbol{x})\mathrm{d}t' - \int_0^t K(t - t') \frac{\partial T}{\partial t}(t', \boldsymbol{x})\mathrm{d}t' \tag{15}$$

where function $K(t)$ plays the role of a pseudo conduction velocity.

4. Synthesis of the experimental results and conclusion

In the above review, we have shown that all the experiment-based papers that aimed at validating a non-Fourier model (CV or DPL) for different classes of material samples presented many flaws and methodological biases. The main features of these papers are summed up in Table 1 below.

These flaws are listed below, in a non exhaustive way:

- experimental boundary conditions that were not validated by direct simulation (neglected losses with unvalidable 1D assumptions), and uncertain quantification of the experimental source term,
- origin of time (start of the thermal source creating the transient transfer) not well documented for the experimental configuration,
- assumption of a uniform initial temperature field that was not confirmed by a temperature recording before time $t = 0$,
- errors in the « imported parameters », derived from the values of the literature (thermophysical properties) that were not taken into account,
- ignorance of modern parameter estimation techniques, based on least squares and analysis of the sensitivity coefficients, with absence of calculation of the matrix of variance-covariance of the parameters that are looked for [20,21]. This can lead to excellent residuals but to biased estimations because of non linearly independent sensitivity coefficients, especially if the number of parameters that are looked for is numerous,
- non-existent or deficient analysis of the errors made in the reduction of the experimental measurements.

So, all the above points have to be checked, in order to make thermal measurements and corresponding models agree using unbiased data reduction techniques. This is particularly true for bio heat transfer, where measurements are difficult because of the specific nature of the tissues involved. As a consequence, the analysis of the deviations between model outputs and measurement points deserve to be analyzed with a very critical hindsight.

We also note that two experimental studies detailed above, Herwig and Beckert [8], for processed meat and dried sand, and Graβman and Peters [10], for wet sand, with two completely different types of thermal excitation and geometry, and where no obvious model or measurement bias appeared, concluded that the Fourier law explained their measurements in a satisfactory way.

For a more general point of view, a first conclusion of this review is that before proposing a new heat transfer model and to « measure » its coefficients, it is compulsory to prove that the classical model cannot explain the experimental measurements.

Another conclusion is that the experiment allowing to compare these different models on a sound basis remains to be made and the points presented above can be used as a roadmap for following a rigorous procedure.

Let us finally note that Fourier's constitutive law has proved to be remarkably robust in many fields of application, provided that the right measurements and their critical analysis have been made.

Acknowledgements

The author wants to thank the American Society of Mechanical Engineers for its permission to reproduce Fig. 1 from [Hyperbolic heat conduction equation for materials with a non-homogeneous inner structure, W. Kaminski, Journal of Heat Transfer 112 (1990)] and Fig. 1a and b from [Experimental evidence of hyperbolic heat conduction in processed meat, K. Mitra, S. Kumar, A. Vedavarz, M.K. Moallemi Journal of Heat Transfer 117 (1995)].

The author wants to thank Springer Nature for its permission to reproduce Fig. 1 from [Experimental investigation of heat conduction in wet sand, A. Grassman and F. Peters, Heat an Mass Transfer 35 (1999)] and Fig. 2 an 3 from [Experimental evidence about the controversy concerning Fourier or non-Fourier heat conduction in materials with a nonhomoneous inner structure, H. Herwig and K. Becker, Heat and Mass Transfer 36 (2000)].

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijthermalsci.2019.02.021.

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