Multiobjective Decomposition-based Mallows models Estimation of Distribution Algorithm. A case of study for Permutation Flowshop Scheduling Problem

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1. Supplementary results

Table 1 present the average HV results obtained by MOEA/D and MEDA/D-MK for the 110 Taillard test instances optimizing the objectives Total Flowtime (TFT), and the makespan (C_{max}). For each algorithm configuration and problem instance, we have executed 10 independent runs. Next, we have used the Kruskall-Wallis statistical test. The Kruskall-Wallis test is the main statistical method used in the determination if two samples are from the same population, which is an alternative to the one-way independent-samples Analysis of Variance (ANOVA). Therefore, we applied the Kruskall-Wallis test (at 5% significance level) to check if the results obtained have a statistical difference. Next, the *post-hoc* test *Nemenyi* is applied to check (if they have achieved significant differences) which one(s) have produced the best results. Thus, the best ranked algorithm(s) is(are) highlighted in boldface.

Table 1: Average normalized HV results

Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_001	0.9020	**0.9367**	0.8722	**0.9276**
Ta_002	0.6969	**0.7767**	0.7280	**0.7743**
Ta_003	0.7602	**0.9107**	0.7111	0.8775
Ta_004	0.8541	**0.9077**	0.8412	0.8944
Ta_005	**0.8005**	**0.8169**	0.7440	0.7560
Ta_006	0.7015	**0.7371**	**0.7344**	**0.7440**
Ta_007	0.7619	**0.9393**	0.7312	0.7681
Ta_008	0.8893	**0.9442**	0.8256	0.9002
Ta_009	0.7704	**0.8242**	0.6999	0.7811
Ta_010	0.8058	**0.8736**	0.7637	0.8116
Ta_011	0.7366	**0.8338**	0.6702	0.7757
Ta_012	0.7140	**0.8043**	0.6290	0.7478

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Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_013	0.8347	**0.8926**	0.6708	0.8009
Ta_014	0.7929	**0.8457**	0.7121	0.8012
Ta_015	0.7782	**0.8668**	0.6966	**0.8572**
Ta_016	0.8290	**0.8830**	0.7929	0.8553
Ta_017	**0.8137**	**0.8487**	0.7704	0.7828
Ta_018	0.8353	**0.8925**	0.8086	**0.9073**
Ta_019	0.8345	**0.8923**	0.7302	**0.8448**
Ta_020	0.8247	**0.9035**	0.7432	0.8248
Ta_021	0.6278	**0.6860**	0.5342	0.6794
Ta_022	0.8160	**0.8647**	0.6870	0.8121
Ta_023	0.7940	**0.8504**	0.6598	0.7377
Ta_024	0.8768	**0.9357**	0.8710	**0.9234**
Ta_025	0.7830	**0.8561**	0.6669	0.6970
Ta_026	0.8183	**0.8762**	0.7463	0.8060
Ta_027	0.7607	**0.8578**	0.7194	0.8262
Ta_028	0.8082	**0.8522**	0.6665	0.7503
Ta_029	0.8017	**0.8405**	0.7442	**0.8279**
Ta_030	0.7597	**0.8500**	0.6584	0.7957
Ta_031	0.9057	**0.9509**	0.8853	0.9178
Ta_032	0.8075	**0.8885**	0.6935	0.7586
Ta_033	0.9076	**0.9589**	0.7668	0.8440
Ta_034	0.8505	**0.9401**	0.6435	0.7883
Ta_035	0.9184	**0.9763**	0.7974	0.9369
Ta_036	0.8729	**0.9330**	0.6318	0.7317
Ta_037	0.8334	**0.9222**	0.7542	0.8375
Ta_038	0.8726	0.8955	0.8630	**0.9198**
Ta_039	0.7799	**0.8972**	0.5733	0.6997
Ta_040	0.9010	**0.9593**	0.7914	0.9082
Ta_041	0.7539	**0.8701**	0.6074	0.6836
Ta_042	0.6859	**0.8281**	0.5166	0.6305
Ta_043	0.6902	**0.8274**	0.5331	0.6028
Ta_044	0.8043	**0.8786**	0.6765	0.7206
Ta_045	0.7657	**0.8601**	0.5883	0.6542
Ta_046	0.7645	**0.8680**	0.5779	0.6524
Ta_047	**0.7819**	**0.8725**	0.6172	0.6571
Ta_048	0.7800	**0.8497**	0.5948	0.6572
Ta_049	0.7795	**0.8431**	0.6390	0.6996
Ta_050	0.7472	**0.8507**	0.5243	0.6008
Ta_051	0.7544	**0.8551**	0.5427	0.6306
Ta_052	**0.7552**	**0.8110**	0.5567	0.5448
Ta_053	0.7375	**0.8282**	0.5507	0.6509
Ta_054	0.7240	**0.8767**	0.5419	0.6592
Ta_055	0.6776	**0.7781**	0.4816	0.5904
Ta_056	0.7547	**0.8461**	0.5525	0.6542
Ta_057	0.7155	**0.7632**	0.4625	0.6252
Ta_058	0.7552	**0.8996**	0.6160	0.6876
Ta_059	0.7243	**0.8311**	0.6033	0.6560

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Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_060	0.8181	**0.9186**	0.7115	0.7778
Ta_061	**0.9676**	**0.9890**	0.7784	0.8738
Ta_062	0.8840	**0.9041**	0.8442	0.8611
Ta_063	0.7648	**0.8273**	0.6537	0.7101
Ta_064	0.9257	**0.9477**	0.7808	0.8772
Ta_065	0.9368	**0.9655**	0.8312	0.9007
Ta_066	0.8028	**0.8678**	0.7274	0.7580
Ta_067	0.8737	**0.9009**	0.6841	0.7970
Ta_068	0.7855	**0.8625**	0.5837	0.7111
Ta_069	0.6688	**0.7139**	0.6573	**0.7187**
Ta_070	0.8608	**0.8945**	0.7112	0.8134
Ta_071	0.7616	**0.8283**	0.5507	0.6120
Ta_072	**0.8054**	**0.8733**	0.6517	0.6811
Ta_073	0.8426	**0.8989**	0.5272	0.6109
Ta_074	0.7137	**0.8223**	0.5103	0.5889
Ta_075	0.7476	**0.8168**	0.5193	0.6018
Ta_076	0.8257	**0.9061**	0.4714	0.5998
Ta_077	0.7615	**0.8201**	0.4942	0.5613
Ta_078	0.7622	**0.8359**	0.4558	0.5715
Ta_079	**0.6975**	**0.7656**	0.4514	0.5233
Ta_080	0.6758	**0.7021**	0.6256	**0.7003**
Ta_081	0.7755	**0.8519**	0.4619	0.5377
Ta_082	0.7195	**0.8296**	0.3618	0.4626
Ta_083	0.6744	**0.7889**	0.4346	0.5239
Ta_084	0.7114	**0.8095**	0.4052	0.4657
Ta_085	0.7533	**0.8610**	0.4838	0.5634
Ta_086	**0.7373**	**0.8349**	0.4327	0.4724
Ta_087	**0.7429**	**0.8115**	0.4567	0.4783
Ta_088	0.7171	**0.8564**	0.4414	0.5152
Ta_089	**0.7674**	**0.8891**	0.5681	0.5549
Ta_090	0.8332	**0.8915**	0.5853	0.6159
Ta_091	**0.9553**	**0.9663**	0.5965	0.6934
Ta_092	**0.8162**	**0.8747**	0.5612	0.5728
Ta_093	**0.7256**	**0.7531**	0.5720	0.5916
Ta_094	**0.7837**	**0.8160**	0.6746	0.6467
Ta_095	**0.8969**	**0.9153**	0.4887	0.5308
Ta_096	**0.8871**	**0.8790**	0.5846	0.6101
Ta_097	**0.7482**	**0.7618**	0.5677	0.6080
Ta_098	**0.7778**	**0.8102**	0.4530	0.5061
Ta_099	0.9074	**0.9613**	0.5325	0.6161
Ta_100	0.5293	**0.5434**	0.4528	0.4627
Ta_101	**0.7336**	**0.7892**	0.4480	0.4422
Ta_102	**0.7577**	**0.8340**	0.3980	0.4649
Ta_103	0.7396	**0.8493**	0.4586	0.4931
Ta_104	0.7635	**0.8410**	0.4473	0.5238
Ta_105	0.6989	**0.7957**	0.3811	0.4438
Ta_106	**0.7797**	**0.8316**	0.4561	0.4811

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Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_107	**0.7402**	**0.8024**	0.3574	0.4036
Ta_108	**0.6983**	**0.7760**	0.4738	0.4960
Ta_109	**0.7441**	**0.8242**	0.3409	0.3954
Ta_110	**0.7056**	**0.8095**	0.4271	0.4321

The results in Table 1 show that, overall, MEDA/D-MK using the scalarizing function *Weighted Sum* (MEDA/D-MK^w) obtain the best results. MEDA/D-MK^w significantly outperforms the other three algorithm configurations in 78 of the 110 test instances. As Miettinen [**?**] argued, the *Weighted sum* approach is good at convex (concave) bi-objectives problems, while *Tchebycheff* approach is useful when the problem is non-convex. This is also confirmed in our results, in which *Weighted sum* have outperformed *Tchebycheff*. Therefore, due to these results, we have used the *Weighted sum* for the remaining experimental studies.

Moreover, these results contemplate that, MEDA/D-MK significantly outperforms MOEA/D (using the tailored genetic operators for the objectives TFT and C_{max} in most of the test instances.

The execution time analysis was performed on a PC with Intel Xeon E5-620 2.4 GHz processor and 12 GB memory. Table 2 indicates that MEDA/D-MK consumes more CPU time than MOEA/D. This is because the Mallows Model EDA components (e.g., learning and sampling steps) involves more computational overhead than crossover and mutation operators. Regarding that, the computational cost of the sampling in the MEDA/D-MK is $\mathcal{O}(n^2)$. This should explain why the difference between both algorithms increases as the problem size increases.

Table 2: Average CPU time (in seconds) used by MOEA/D^T and MEDA/D^T

Taillard Instance	MOEA/D		MEDA/D-MK	
	mean	std ved	mean	std dev
Ta_001	73.05	8.83	93.15	2.84
Ta_002	72.19	6.59	92.33	1.46
Ta_003	79.95	4.34	91.09	4.05
Ta_004	73.04	5.00	92.89	3.45
Ta_005	74.96	3.14	94.97	4.18
Ta_006	75.73	7.71	91.86	2.42
Ta_007	79.01	3.29	93.39	2.14
Ta_008	72.07	5.40	91.31	2.20
Ta_009	71.93	5.60	93.37	4.30
Ta_010	72.37	4.42	92.02	3.78
Ta_011	89.59	6.89	101.42	1.88
Ta_012	83.29	5.76	102.44	3.13
Ta_013	81.65	3.54	96.30	5.18
Ta_014	84.19	1.60	103.58	2.19
Ta_015	85.60	2.93	103.31	1.50
Ta_016	87.04	3.31	102.10	2.60
Ta_017	87.01	4.94	97.61	3.42
Ta_018	83.59	1.42	98.93	1.70
Ta_019	80.57	3.50	103.25	2.09
Ta_020	88.45	2.90	101.50	2.23
Ta_021	100.51	3.47	117.04	3.62
Ta_022	99.47	2.23	118.18	1.90

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Taillard Instance	MOEA/D		MEDA/D-MK	
	mean	std ved	mean	std dev
Ta_023	100.36	3.12	121.18	2.53
Ta_024	101.67	2.25	117.09	2.45
Ta_025	105.48	3.75	117.78	2.34
Ta_026	103.58	2.74	115.64	2.25
Ta_027	101.43	2.56	119.57	3.51
Ta_028	99.38	4.37	118.92	3.08
Ta_029	100.44	4.75	118.43	2.62
Ta_030	105.53	3.81	116.99	2.37
Ta_031	217.35	6.43	303.42	5.98
Ta_032	228.03	5.97	312.20	2.89
Ta_033	224.24	6.10	307.18	7.81
Ta_034	221.43	5.89	305.71	4.49
Ta_035	223.08	3.24	309.09	4.70
Ta_036	225.31	6.49	310.94	3.52
Ta_037	222.35	3.13	306.17	4.75
Ta_038	221.72	7.12	309.08	4.10
Ta_039	228.40	6.00	305.58	8.07
Ta_040	223.07	2.63	305.41	4.81
Ta_041	271.91	3.51	368.68	4.41
Ta_042	279.66	6.22	366.26	3.21
Ta_043	277.87	2.01	366.54	2.19
Ta_044	278.06	5.95	366.01	6.25
Ta_045	278.01	4.37	366.33	4.98
Ta_046	282.20	9.51	369.12	5.05
Ta_047	275.67	4.01	362.17	4.25
Ta_048	279.91	3.59	368.40	2.97
Ta_049	279.47	4.64	367.16	4.00
Ta_050	276.11	9.23	367.18	8.11
Ta_051	389.90	3.72	480.24	2.74
Ta_052	393.41	3.12	482.20	2.18
Ta_053	389.67	4.35	478.65	2.48
Ta_054	389.59	4.45	476.73	3.43
Ta_055	399.29	5.85	480.22	4.41
Ta_056	394.70	5.63	481.69	5.48
Ta_057	394.38	5.64	477.36	6.18
Ta_058	392.12	5.31	477.43	3.46
Ta_059	396.09	7.70	478.83	4.29
Ta_060	391.73	3.22	477.30	8.15
Ta_061	548.77	6.86	892.75	9.54
Ta_062	564.76	14.99	880.62	9.46
Ta_063	547.39	6.97	883.68	8.30
Ta_064	549.89	8.52	892.55	13.22
Ta_065	561.64	12.25	867.95	10.37
Ta_066	552.11	5.90	869.71	12.62
Ta_067	549.52	5.24	889.50	5.13
Ta_068	557.51	9.90	874.56	15.09
Ta_069	547.54	6.34	884.94	5.10

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Table 2 – continued from previous page

Taillard Instance	MOEA/D		MEDA/D-MK	
	mean	std ved	mean	std dev
Ta_070	550.07	9.65	880.50	3.91
Ta_071	775.07	6.93	1118.42	8.04
Ta_072	764.39	7.49	1106.47	6.80
Ta_073	782.00	7.33	1114.78	14.42
Ta_074	762.59	5.81	1114.07	8.07
Ta_075	787.13	9.43	1127.19	5.05
Ta_076	774.33	6.09	1110.00	6.41
Ta_077	772.64	6.65	1125.14	8.99
Ta_078	778.16	4.80	1119.39	10.38
Ta_079	780.24	5.43	1124.94	10.05
Ta_080	765.56	3.09	1123.50	5.14
Ta_081	1241.83	8.96	1586.35	9.08
Ta_082	1242.28	4.55	1586.54	9.09
Ta_083	1243.11	5.30	1598.08	9.31
Ta_084	1232.42	7.14	1573.39	7.75
Ta_085	1251.57	5.90	1589.51	10.26
Ta_086	1246.92	9.88	1585.96	6.85
Ta_087	1232.00	6.84	1575.23	10.91
Ta_088	1253.64	6.89	1593.02	9.83
Ta_089	1245.93	6.68	1586.87	11.24
Ta_090	1241.25	8.28	1587.45	8.55
Ta_091	2491.88	27.21	3789.63	25.26
Ta_092	2477.69	15.23	3760.98	22.98
Ta_093	2492.70	8.00	3786.04	14.15
Ta_094	2457.41	10.34	3787.36	41.81
Ta_095	2482.50	17.15	3749.81	22.27
Ta_096	2480.92	6.24	3739.61	13.52
Ta_097	2461.74	16.93	3792.48	24.49
Ta_098	2471.82	8.56	3731.21	19.35
Ta_099	2450.06	11.49	3724.39	11.58
Ta_100	2469.99	15.26	3754.15	19.43
Ta_101	4293.12	15.47	5639.96	24.15
Ta_102	4311.73	34.07	5584.47	20.77
Ta_103	4286.46	32.44	5571.16	20.14
Ta_104	4244.79	20.87	5544.95	32.86
Ta_105	4295.91	15.80	5600.41	25.71
Ta_106	4289.32	12.25	5578.47	23.18
Ta_107	4260.43	11.62	5582.55	15.94
Ta_108	4313.17	33.16	5597.05	28.62
Ta_109	4294.80	13.40	5614.86	29.65
Ta_110	3501.59	607.55	5562.85	23.67