Multiobjective Decomposition-based Mallows models Estimation of Distribution Algorithm. A case of study for Permutation Flowshop Scheduling Problem

Murilo Zangari[a,*], Alexander Mendiburu[b], Roberto Santana[c], Aurora Pozo[a]

[a]*Computer Science Department, Federal University of Paraná (UFPR), Brazil. PO 19081, ZIP Code: 81531-970, Curitiba, Brazil*
[b]*Intelligent Systems Group, Department of Computer Architecture and Technology, University of the Basque Country (UPV/EHU), Paseo Manuel de Lardizabal 1, 20080 San Sebastián, Guipúzcoa, Spain*
[c]*Intelligent Systems Group, Department of Computer Science and Artificial Intelligence, University of the Basque Country (UPV/EHU), Paseo Manuel de Lardizabal 1, 20080 San Sebastián, Guipúzcoa, Spain*

1. Supplementary results

Table 1 present the average *HV* results obtained by MOEA/D and MEDA/D-MK for the 110 Taillard test instances optimizing the objectives Total Flowtime (TFT), and the makespan (C_{max}). For each algorithm configuration and problem instance, we have executed 10 independent runs. Next, we have used the Kruskall-Wallis statistical test. The Kruskall-Wallis test is the main statistical method used in the determination if two samples are from the same population, which is an alternative to the one-way independent-samples Analysis of Variance (ANOVA). Therefore, we applied the Kruskall-Wallis test (at 5% significance level) to check if the results obtained have a statistical difference. Next, the *post-hoc* test *Nemenyi* is applied to check (if they have achieved significant differences) which one(s) have produced the best results. Thus, the best ranked algorithm(s) is(are) highlighted in boldface.

Table 1: Average normalized HV results

Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_001	0.9020	**0.9367**	0.8722	**0.9276**
Ta_002	0.6969	**0.7767**	0.7280	**0.7743**
Ta_003	0.7602	**0.9107**	0.7111	0.8775
Ta_004	0.8541	**0.9077**	0.8412	0.8944
Ta_005	**0.8005**	**0.8169**	0.7440	0.7560
Ta_006	0.7015	**0.7371**	**0.7344**	**0.7440**
Ta_007	0.7619	**0.9393**	0.7312	0.7681
Ta_008	0.8893	**0.9442**	0.8256	0.9002
Ta_009	0.7704	**0.8242**	0.6999	0.7811
Ta_010	0.8058	**0.8736**	0.7637	0.8116
Ta_011	0.7366	**0.8338**	0.6702	0.7757
Ta_012	0.7140	**0.8043**	0.6290	0.7478

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*Corresponding author
Email addresses: murilo.zangari@gmail.com (Murilo Zangari), alexander.mendiburu@ehu.es (Alexander Mendiburu), roberto.santana@ehu.es (Roberto Santana), aurora@inf.ufpr.br (Aurora Pozo)

Table 1 – continued from previous page

Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_013	0.8347	**0.8926**	0.6708	0.8009
Ta_014	0.7929	**0.8457**	0.7121	0.8012
Ta_015	0.7782	**0.8668**	0.6966	**0.8572**
Ta_016	0.8290	**0.8830**	0.7929	0.8553
Ta_017	**0.8137**	**0.8487**	0.7704	0.7828
Ta_018	0.8353	**0.8925**	0.8086	**0.9073**
Ta_019	0.8345	**0.8923**	0.7302	**0.8448**
Ta_020	0.8247	**0.9035**	0.7432	0.8248
Ta_021	0.6278	**0.6860**	0.5342	0.6794
Ta_022	0.8160	**0.8647**	0.6870	0.8121
Ta_023	0.7940	**0.8504**	0.6598	0.7377
Ta_024	0.8768	**0.9357**	0.8710	**0.9234**
Ta_025	0.7830	**0.8561**	0.6669	0.6970
Ta_026	0.8183	**0.8762**	0.7463	0.8060
Ta_027	0.7607	**0.8578**	0.7194	0.8262
Ta_028	0.8082	**0.8522**	0.6665	0.7503
Ta_029	0.8017	**0.8405**	0.7442	**0.8279**
Ta_030	0.7597	**0.8500**	0.6584	0.7957
Ta_031	0.9057	**0.9509**	0.8853	0.9178
Ta_032	0.8075	**0.8885**	0.6935	0.7586
Ta_033	0.9076	**0.9589**	0.7668	0.8440
Ta_034	0.8505	**0.9401**	0.6435	0.7883
Ta_035	0.9184	**0.9763**	0.7974	0.9369
Ta_036	0.8729	**0.9330**	0.6318	0.7317
Ta_037	0.8334	**0.9222**	0.7542	0.8375
Ta_038	0.8726	0.8955	0.8630	**0.9198**
Ta_039	0.7799	**0.8972**	0.5733	0.6997
Ta_040	0.9010	**0.9593**	0.7914	0.9082
Ta_041	0.7539	**0.8701**	0.6074	0.6836
Ta_042	0.6859	**0.8281**	0.5166	0.6305
Ta_043	0.6902	**0.8274**	0.5331	0.6028
Ta_044	0.8043	**0.8786**	0.6765	0.7206
Ta_045	0.7657	**0.8601**	0.5883	0.6542
Ta_046	0.7645	**0.8680**	0.5779	0.6524
Ta_047	**0.7819**	**0.8725**	0.6172	0.6571
Ta_048	0.7800	**0.8497**	0.5948	0.6572
Ta_049	0.7795	**0.8431**	0.6390	0.6996
Ta_050	0.7472	**0.8507**	0.5243	0.6008
Ta_051	0.7544	**0.8551**	0.5427	0.6306
Ta_052	**0.7552**	**0.8110**	0.5567	0.5448
Ta_053	0.7375	**0.8282**	0.5507	0.6509
Ta_054	0.7240	**0.8767**	0.5419	0.6592
Ta_055	0.6776	**0.7781**	0.4816	0.5904
Ta_056	0.7547	**0.8461**	0.5525	0.6542
Ta_057	0.7155	**0.7632**	0.4625	0.6252
Ta_058	0.7552	**0.8996**	0.6160	0.6876
Ta_059	0.7243	**0.8311**	0.6033	0.6560

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Table 1 – continued from previous page

Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_060	0.8181	**0.9186**	0.7115	0.7778
Ta_061	**0.9676**	**0.9890**	0.7784	0.8738
Ta_062	0.8840	**0.9041**	0.8442	0.8611
Ta_063	0.7648	**0.8273**	0.6537	0.7101
Ta_064	0.9257	**0.9477**	0.7808	0.8772
Ta_065	0.9368	**0.9655**	0.8312	0.9007
Ta_066	0.8028	**0.8678**	0.7274	0.7580
Ta_067	0.8737	**0.9009**	0.6841	0.7970
Ta_068	0.7855	**0.8625**	0.5837	0.7111
Ta_069	0.6688	**0.7139**	0.6573	**0.7187**
Ta_070	0.8608	**0.8945**	0.7112	0.8134
Ta_071	0.7616	**0.8283**	0.5507	0.6120
Ta_072	**0.8054**	**0.8733**	0.6517	0.6811
Ta_073	0.8426	**0.8989**	0.5272	0.6109
Ta_074	0.7137	**0.8223**	0.5103	0.5889
Ta_075	0.7476	**0.8168**	0.5193	0.6018
Ta_076	0.8257	**0.9061**	0.4714	0.5998
Ta_077	0.7615	**0.8201**	0.4942	0.5613
Ta_078	0.7622	**0.8359**	0.4558	0.5715
Ta_079	**0.6975**	**0.7656**	0.4514	0.5233
Ta_080	0.6758	**0.7021**	0.6256	**0.7003**
Ta_081	0.7755	**0.8519**	0.4619	0.5377
Ta_082	0.7195	**0.8296**	0.3618	0.4626
Ta_083	0.6744	**0.7889**	0.4346	0.5239
Ta_084	0.7114	**0.8095**	0.4052	0.4657
Ta_085	0.7533	**0.8610**	0.4838	0.5634
Ta_086	**0.7373**	**0.8349**	0.4327	0.4724
Ta_087	**0.7429**	**0.8115**	0.4567	0.4783
Ta_088	0.7171	**0.8564**	0.4414	0.5152
Ta_089	**0.7674**	**0.8891**	0.5681	0.5549
Ta_090	0.8332	**0.8915**	0.5853	0.6159
Ta_091	**0.9553**	**0.9663**	0.5965	0.6934
Ta_092	**0.8162**	**0.8747**	0.5612	0.5728
Ta_093	**0.7256**	**0.7531**	0.5720	0.5916
Ta_094	**0.7837**	**0.8160**	0.6746	0.6467
Ta_095	**0.8969**	**0.9153**	0.4887	0.5308
Ta_096	**0.8871**	**0.8790**	0.5846	0.6101
Ta_097	**0.7482**	**0.7618**	0.5677	0.6080
Ta_098	**0.7778**	**0.8102**	0.4530	0.5061
Ta_099	0.9074	**0.9613**	0.5325	0.6161
Ta_100	0.5293	**0.5434**	0.4528	0.4627
Ta_101	**0.7336**	**0.7892**	0.4480	0.4422
Ta_102	**0.7577**	**0.8340**	0.3980	0.4649
Ta_103	0.7396	**0.8493**	0.4586	0.4931
Ta_104	0.7635	**0.8410**	0.4473	0.5238
Ta_105	0.6989	**0.7957**	0.3811	0.4438
Ta_106	**0.7797**	**0.8316**	0.4561	0.4811

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Taillard Instance	*Weighted Sum*		*Tchebycheff*	
	MOEA/D	MEDA/D-MK	MOEA/D	MEDA/D-MK
Ta_107	**0.7402**	**0.8024**	0.3574	0.4036
Ta_108	**0.6983**	**0.7760**	0.4738	0.4960
Ta_109	**0.7441**	**0.8242**	0.3409	0.3954
Ta_110	**0.7056**	**0.8095**	0.4271	0.4321

The results in Table 1 show that, overall, MEDA/D-MK using the scalarizing function *Weighted Sum* (MEDA/D-MK^w) obtain the best results. MEDA/D-MK^w significantly outperforms the other three algorithm configurations in 78 of the 110 test instances. As Miettinen [1] argued, the *Weighted sum* approach is good at convex (concave) bi-objectives problems, while *Tchebycheff* approach is useful when the problem is non-convex. This is also confirmed in our results, in which *Weighted sum* have outperformed *Tchebycheff*. Therefore, due to these results, we have used the *Weighted sum* for the remaining experimental studies.

Moreover, these results contemplate that, MEDA/D-MK significantly outperforms MOEA/D (using the tailored genetic operators for the objectives TFT and C_{max} in most of the test instances.

References

[1] K. Miettinen, Nonlinear multiobjective optimization, vol. 12, Springer Science & Business Media, 2012.