# Multiobjective Decomposition-based Mallows models Estimation of Distribution Algorithm. A case of study for Permutation Flowshop Scheduling Problem

Murilo Zangari[a,*], Alexander Mendiburu[b], Roberto Santana[c], Aurora Pozo[a]

[a] *Computer Science Department, Federal University of Paraná (UFPR), Brazil. PO 19081, ZIP Code: 81531-970, Curitiba, Brazil*

[b] *Intelligent Systems Group, Department of Computer Architecture and Technology, University of the Basque Country (UPV/EHU), Paseo Manuel de Lardizabal 1, 20080 San Sebastián, Guipúzcoa, Spain*

[c] *Intelligent Systems Group, Department of Computer Science and Artificial Intelligence, University of the Basque Country (UPV/EHU), Paseo Manuel de Lardizabal 1, 20080 San Sebastián, Guipúzcoa, Spain*

## 1. Supplementary results

Table 1 present the average *HV* results obtained by MOEA/D and MEDA/D-MK, using *Weighted Sum* and *Tchebycheff* scalarizing functions, for the 110 Taillard test instances optimizing the objectives Total Flowtime (TFT), and the makespan ($C_{max}$). For each algorithm configuration and problem instance, we have executed 10 independent runs. Next, we have used the Kruskall-Wallis statistical test. The Kruskall-Wallis test is the main statistical method used in the determination if two samples are from the same population, which is an alternative to the one-way independent-samples Analysis of Variance (ANOVA). Therefore, we applied the Kruskall-Wallis test (at 5% significance level) to check if the results obtained have a statistical difference. Next, the *post-hoc* test *Nemenyi* is applied to check (if they have achieved significant differences) which one(s) have produced the best results. Thus, the best ranked algorithm(s) is(are) highlighted in boldface.

Table 1: Average normalized HV results

| Taillard Instance | *Weighted Sum* | | *Tchebycheff* | |
|---|---|---|---|---|
| | MOEA/D | MEDA/D-MK | MOEA/D | MEDA/D-MK |
| Ta_001 | 0.9020 | **0.9367** | 0.8722 | **0.9276** |
| Ta_002 | 0.6969 | **0.7767** | 0.7280 | **0.7743** |
| Ta_003 | 0.7602 | **0.9107** | 0.7111 | 0.8775 |
| Ta_004 | 0.8541 | **0.9077** | 0.8412 | 0.8944 |
| Ta_005 | **0.8005** | **0.8169** | 0.7440 | 0.7560 |
| Ta_006 | 0.7015 | **0.7371** | **0.7344** | **0.7440** |
| Ta_007 | 0.7619 | **0.9393** | 0.7312 | 0.7681 |
| Ta_008 | 0.8893 | **0.9442** | 0.8256 | 0.9002 |
| Ta_009 | 0.7704 | **0.8242** | 0.6999 | 0.7811 |
| Ta_010 | 0.8058 | **0.8736** | 0.7637 | 0.8116 |
| Ta_011 | 0.7366 | **0.8338** | 0.6702 | 0.7757 |

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*Corresponding author

*Email addresses:* murilo.zangari@gmail.com (Murilo Zangari), alexander.mendiburu@ehu.es (Alexander Mendiburu), roberto.santana@ehu.es (Roberto Santana), aurora@inf.ufpr.br (Aurora Pozo)

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| Taillard Instance | *Weighted Sum* | | *Tchebycheff* | |
|---|---|---|---|---|
| | MOEA/D | MEDA/D-MK | MOEA/D | MEDA/D-MK |
| Ta_012 | 0.7140 | **0.8043** | 0.6290 | 0.7478 |
| Ta_013 | 0.8347 | **0.8926** | 0.6708 | 0.8009 |
| Ta_014 | 0.7929 | **0.8457** | 0.7121 | 0.8012 |
| Ta_015 | 0.7782 | **0.8668** | 0.6966 | **0.8572** |
| Ta_016 | 0.8290 | **0.8830** | 0.7929 | 0.8553 |
| Ta_017 | **0.8137** | **0.8487** | 0.7704 | 0.7828 |
| Ta_018 | 0.8353 | **0.8925** | 0.8086 | **0.9073** |
| Ta_019 | 0.8345 | **0.8923** | 0.7302 | **0.8448** |
| Ta_020 | 0.8247 | **0.9035** | 0.7432 | 0.8248 |
| Ta_021 | 0.6278 | **0.6860** | 0.5342 | 0.6794 |
| Ta_022 | 0.8160 | **0.8647** | 0.6870 | 0.8121 |
| Ta_023 | 0.7940 | **0.8504** | 0.6598 | 0.7377 |
| Ta_024 | 0.8768 | **0.9357** | 0.8710 | **0.9234** |
| Ta_025 | 0.7830 | **0.8561** | 0.6669 | 0.6970 |
| Ta_026 | 0.8183 | **0.8762** | 0.7463 | 0.8060 |
| Ta_027 | 0.7607 | **0.8578** | 0.7194 | 0.8262 |
| Ta_028 | 0.8082 | **0.8522** | 0.6665 | 0.7503 |
| Ta_029 | 0.8017 | **0.8405** | 0.7442 | **0.8279** |
| Ta_030 | 0.7597 | **0.8500** | 0.6584 | 0.7957 |
| Ta_031 | 0.9057 | **0.9509** | 0.8853 | 0.9178 |
| Ta_032 | 0.8075 | **0.8885** | 0.6935 | 0.7586 |
| Ta_033 | 0.9076 | **0.9589** | 0.7668 | 0.8440 |
| Ta_034 | 0.8505 | **0.9401** | 0.6435 | 0.7883 |
| Ta_035 | 0.9184 | **0.9763** | 0.7974 | 0.9369 |
| Ta_036 | 0.8729 | **0.9330** | 0.6318 | 0.7317 |
| Ta_037 | 0.8334 | **0.9222** | 0.7542 | 0.8375 |
| Ta_038 | 0.8726 | 0.8955 | 0.8630 | **0.9198** |
| Ta_039 | 0.7799 | **0.8972** | 0.5733 | 0.6997 |
| Ta_040 | 0.9010 | **0.9593** | 0.7914 | 0.9082 |
| Ta_041 | 0.7539 | **0.8701** | 0.6074 | 0.6836 |
| Ta_042 | 0.6859 | **0.8281** | 0.5166 | 0.6305 |
| Ta_043 | 0.6902 | **0.8274** | 0.5331 | 0.6028 |
| Ta_044 | 0.8043 | **0.8786** | 0.6765 | 0.7206 |
| Ta_045 | 0.7657 | **0.8601** | 0.5883 | 0.6542 |
| Ta_046 | 0.7645 | **0.8680** | 0.5779 | 0.6524 |
| Ta_047 | **0.7819** | **0.8725** | 0.6172 | 0.6571 |
| Ta_048 | 0.7800 | **0.8497** | 0.5948 | 0.6572 |
| Ta_049 | 0.7795 | **0.8431** | 0.6390 | 0.6996 |
| Ta_050 | 0.7472 | **0.8507** | 0.5243 | 0.6008 |
| Ta_051 | 0.7544 | **0.8551** | 0.5427 | 0.6306 |
| Ta_052 | **0.7552** | **0.8110** | 0.5567 | 0.5448 |
| Ta_053 | 0.7375 | **0.8282** | 0.5507 | 0.6509 |
| Ta_054 | 0.7240 | **0.8767** | 0.5419 | 0.6592 |
| Ta_055 | 0.6776 | **0.7781** | 0.4816 | 0.5904 |
| Ta_056 | 0.7547 | **0.8461** | 0.5525 | 0.6542 |
| Ta_057 | 0.7155 | **0.7632** | 0.4625 | 0.6252 |
| Ta_058 | 0.7552 | **0.8996** | 0.6160 | 0.6876 |

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**Table 1 – continued from previous page**

| Taillard Instance | *Weighted Sum* | | *Tchebycheff* | |
|---|---|---|---|---|
| | MOEA/D | MEDA/D-MK | MOEA/D | MEDA/D-MK |
| Ta_059 | 0.7243 | **0.8311** | 0.6033 | 0.6560 |
| Ta_060 | 0.8181 | **0.9186** | 0.7115 | 0.7778 |
| Ta_061 | **0.9676** | **0.9890** | 0.7784 | 0.8738 |
| Ta_062 | 0.8840 | **0.9041** | 0.8442 | 0.8611 |
| Ta_063 | 0.7648 | **0.8273** | 0.6537 | 0.7101 |
| Ta_064 | 0.9257 | **0.9477** | 0.7808 | 0.8772 |
| Ta_065 | 0.9368 | **0.9655** | 0.8312 | 0.9007 |
| Ta_066 | 0.8028 | **0.8678** | 0.7274 | 0.7580 |
| Ta_067 | 0.8737 | **0.9009** | 0.6841 | 0.7970 |
| Ta_068 | 0.7855 | **0.8625** | 0.5837 | 0.7111 |
| Ta_069 | 0.6688 | **0.7139** | 0.6573 | **0.7187** |
| Ta_070 | 0.8608 | **0.8945** | 0.7112 | 0.8134 |
| Ta_071 | 0.7616 | **0.8283** | 0.5507 | 0.6120 |
| Ta_072 | **0.8054** | **0.8733** | 0.6517 | 0.6811 |
| Ta_073 | 0.8426 | **0.8989** | 0.5272 | 0.6109 |
| Ta_074 | 0.7137 | **0.8223** | 0.5103 | 0.5889 |
| Ta_075 | 0.7476 | **0.8168** | 0.5193 | 0.6018 |
| Ta_076 | 0.8257 | **0.9061** | 0.4714 | 0.5998 |
| Ta_077 | 0.7615 | **0.8201** | 0.4942 | 0.5613 |
| Ta_078 | 0.7622 | **0.8359** | 0.4558 | 0.5715 |
| Ta_079 | **0.6975** | **0.7656** | 0.4514 | 0.5233 |
| Ta_080 | 0.6758 | **0.7021** | 0.6256 | **0.7003** |
| Ta_081 | 0.7755 | **0.8519** | 0.4619 | 0.5377 |
| Ta_082 | 0.7195 | **0.8296** | 0.3618 | 0.4626 |
| Ta_083 | 0.6744 | **0.7889** | 0.4346 | 0.5239 |
| Ta_084 | 0.7114 | **0.8095** | 0.4052 | 0.4657 |
| Ta_085 | 0.7533 | **0.8610** | 0.4838 | 0.5634 |
| Ta_086 | **0.7373** | **0.8349** | 0.4327 | 0.4724 |
| Ta_087 | **0.7429** | **0.8115** | 0.4567 | 0.4783 |
| Ta_088 | 0.7171 | **0.8564** | 0.4414 | 0.5152 |
| Ta_089 | **0.7674** | **0.8891** | 0.5681 | 0.5549 |
| Ta_090 | 0.8332 | **0.8915** | 0.5853 | 0.6159 |
| Ta_091 | **0.9553** | **0.9663** | 0.5965 | 0.6934 |
| Ta_092 | **0.8162** | **0.8747** | 0.5612 | 0.5728 |
| Ta_093 | **0.7256** | **0.7531** | 0.5720 | 0.5916 |
| Ta_094 | **0.7837** | **0.8160** | 0.6746 | 0.6467 |
| Ta_095 | **0.8969** | **0.9153** | 0.4887 | 0.5308 |
| Ta_096 | **0.8871** | **0.8790** | 0.5846 | 0.6101 |
| Ta_097 | **0.7482** | **0.7618** | 0.5677 | 0.6080 |
| Ta_098 | **0.7778** | **0.8102** | 0.4530 | 0.5061 |
| Ta_099 | 0.9074 | **0.9613** | 0.5325 | 0.6161 |
| Ta_100 | 0.5293 | **0.5434** | 0.4528 | 0.4627 |
| Ta_101 | **0.7336** | **0.7892** | 0.4480 | 0.4422 |
| Ta_102 | **0.7577** | **0.8340** | 0.3980 | 0.4649 |
| Ta_103 | 0.7396 | **0.8493** | 0.4586 | 0.4931 |
| Ta_104 | 0.7635 | **0.8410** | 0.4473 | 0.5238 |
| Ta_105 | 0.6989 | **0.7957** | 0.3811 | 0.4438 |

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**Table 1 – continued from previous page**

| Taillard Instance | *Weighted Sum* | | *Tchebycheff* | |
|---|---|---|---|---|
| | MOEA/D | MEDA/D-MK | MOEA/D | MEDA/D-MK |
| Ta_106 | **0.7797** | **0.8316** | 0.4561 | 0.4811 |
| Ta_107 | **0.7402** | **0.8024** | 0.3574 | 0.4036 |
| Ta_108 | **0.6983** | **0.7760** | 0.4738 | 0.4960 |
| Ta_109 | **0.7441** | **0.8242** | 0.3409 | 0.3954 |
| Ta_110 | **0.7056** | **0.8095** | 0.4271 | 0.4321 |

The results in Table 1 show that, overall, MEDA/D-MK using the scalarizing function *Weighted Sum* (MEDA/D-MK$^w$) achieves the best results. MEDA/D-MK$^w$ significantly outperforms the other three algorithm configurations in 78 of the 110 test instances. As Miettinen [2] argued, the *Weighted sum* approach is good at convex (concave) bi-objectives problems, while *Tchebycheff* approach is useful when the problem is non-convex. This is also confirmed in our results, in which *Weighted sum* have outperformed *Tchebycheff*.

Moreover, these results contemplate that, MEDA/D-MK significantly outperforms MOEA/D, using the tailored genetic operators for the objectives $TFT$ and $C_{max}$, in most of the test instances.

The execution time analysis was performed on a PC with Intel Xeon E5-620 2.4 GHz processor and 12 GB memory. Table 2 indicates that MEDA/D-MK consumes more CPU time than MOEA/D. This is because the Mallows Model EDA components, learning and sampling steps, involve more computational overhead than crossover and mutation operators. Regarding that, the computational cost of the sampling in the MEDA/D-MK is $\mathcal{O}(n^2)$. This should explain why the difference between both algorithms increases as the problem size increases.

Table 2: Average CPU time (in seconds) used by MOEA/D$^T$ and MEDA/D$^T$

| Taillard Instance | MOEA/D | | MEDA/D-MK | |
|---|---|---|---|---|
| | mean | std ved | mean | std dev |
| Ta_001 | 73.05 | 8.83 | 93.15 | 2.84 |
| Ta_002 | 72.19 | 6.59 | 92.33 | 1.46 |
| Ta_003 | 79.95 | 4.34 | 91.09 | 4.05 |
| Ta_004 | 73.04 | 5.00 | 92.89 | 3.45 |
| Ta_005 | 74.96 | 3.14 | 94.97 | 4.18 |
| Ta_006 | 75.73 | 7.71 | 91.86 | 2.42 |
| Ta_007 | 79.01 | 3.29 | 93.39 | 2.14 |
| Ta_008 | 72.07 | 5.40 | 91.31 | 2.20 |
| Ta_009 | 71.93 | 5.60 | 93.37 | 4.30 |
| Ta_010 | 72.37 | 4.42 | 92.02 | 3.78 |
| Ta_011 | 89.59 | 6.89 | 101.42 | 1.88 |
| Ta_012 | 83.29 | 5.76 | 102.44 | 3.13 |
| Ta_013 | 81.65 | 3.54 | 96.30 | 5.18 |
| Ta_014 | 84.19 | 1.60 | 103.58 | 2.19 |
| Ta_015 | 85.60 | 2.93 | 103.31 | 1.50 |
| Ta_016 | 87.04 | 3.31 | 102.10 | 2.60 |
| Ta_017 | 87.01 | 4.94 | 97.61 | 3.42 |
| Ta_018 | 83.59 | 1.42 | 98.93 | 1.70 |
| Ta_019 | 80.57 | 3.50 | 103.25 | 2.09 |
| Ta_020 | 88.45 | 2.90 | 101.50 | 2.23 |
| Ta_021 | 100.51 | 3.47 | 117.04 | 3.62 |
| Ta_022 | 99.47 | 2.23 | 118.18 | 1.90 |

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**Table 2 – continued from previous page**

| Taillard Instance | MOEA/D | | MEDA/D-MK | |
|---|---|---|---|---|
| | mean | std ved | mean | std dev |
| Ta_023 | 100.36 | 3.12 | 121.18 | 2.53 |
| Ta_024 | 101.67 | 2.25 | 117.09 | 2.45 |
| Ta_025 | 105.48 | 3.75 | 117.78 | 2.34 |
| Ta_026 | 103.58 | 2.74 | 115.64 | 2.25 |
| Ta_027 | 101.43 | 2.56 | 119.57 | 3.51 |
| Ta_028 | 99.38 | 4.37 | 118.92 | 3.08 |
| Ta_029 | 100.44 | 4.75 | 118.43 | 2.62 |
| Ta_030 | 105.53 | 3.81 | 116.99 | 2.37 |
| Ta_031 | 217.35 | 6.43 | 303.42 | 5.98 |
| Ta_032 | 228.03 | 5.97 | 312.20 | 2.89 |
| Ta_033 | 224.24 | 6.10 | 307.18 | 7.81 |
| Ta_034 | 221.43 | 5.89 | 305.71 | 4.49 |
| Ta_035 | 223.08 | 3.24 | 309.09 | 4.70 |
| Ta_036 | 225.31 | 6.49 | 310.94 | 3.52 |
| Ta_037 | 222.35 | 3.13 | 306.17 | 4.75 |
| Ta_038 | 221.72 | 7.12 | 309.08 | 4.10 |
| Ta_039 | 228.40 | 6.00 | 305.58 | 8.07 |
| Ta_040 | 223.07 | 2.63 | 305.41 | 4.81 |
| Ta_041 | 271.91 | 3.51 | 368.68 | 4.41 |
| Ta_042 | 279.66 | 6.22 | 366.26 | 3.21 |
| Ta_043 | 277.87 | 2.01 | 366.54 | 2.19 |
| Ta_044 | 278.06 | 5.95 | 366.01 | 6.25 |
| Ta_045 | 278.01 | 4.37 | 366.33 | 4.98 |
| Ta_046 | 282.20 | 9.51 | 369.12 | 5.05 |
| Ta_047 | 275.67 | 4.01 | 362.17 | 4.25 |
| Ta_048 | 279.91 | 3.59 | 368.40 | 2.97 |
| Ta_049 | 279.47 | 4.64 | 367.16 | 4.00 |
| Ta_050 | 276.11 | 9.23 | 367.18 | 8.11 |
| Ta_051 | 389.90 | 3.72 | 480.24 | 2.74 |
| Ta_052 | 393.41 | 3.12 | 482.20 | 2.18 |
| Ta_053 | 389.67 | 4.35 | 478.65 | 2.48 |
| Ta_054 | 389.59 | 4.45 | 476.73 | 3.43 |
| Ta_055 | 399.29 | 5.85 | 480.22 | 4.41 |
| Ta_056 | 394.70 | 5.63 | 481.69 | 5.48 |
| Ta_057 | 394.38 | 5.64 | 477.36 | 6.18 |
| Ta_058 | 392.12 | 5.31 | 477.43 | 3.46 |
| Ta_059 | 396.09 | 7.70 | 478.83 | 4.29 |
| Ta_060 | 391.73 | 3.22 | 477.30 | 8.15 |
| Ta_061 | 548.77 | 6.86 | 892.75 | 9.54 |
| Ta_062 | 564.76 | 14.99 | 880.62 | 9.46 |
| Ta_063 | 547.39 | 6.97 | 883.68 | 8.30 |
| Ta_064 | 549.89 | 8.52 | 892.55 | 13.22 |
| Ta_065 | 561.64 | 12.25 | 867.95 | 10.37 |
| Ta_066 | 552.11 | 5.90 | 869.71 | 12.62 |
| Ta_067 | 549.52 | 5.24 | 889.50 | 5.13 |
| Ta_068 | 557.51 | 9.90 | 874.56 | 15.09 |
| Ta_069 | 547.54 | 6.34 | 884.94 | 5.10 |

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**Table 2 – continued from previous page**

| Taillard Instance | MOEA/D | | MEDA/D-MK | |
|---|---|---|---|---|
| | mean | std ved | mean | std dev |
| Ta_070 | 550.07 | 9.65 | 880.50 | 3.91 |
| Ta_071 | 775.07 | 6.93 | 1118.42 | 8.04 |
| Ta_072 | 764.39 | 7.49 | 1106.47 | 6.80 |
| Ta_073 | 782.00 | 7.33 | 1114.78 | 14.42 |
| Ta_074 | 762.59 | 5.81 | 1114.07 | 8.07 |
| Ta_075 | 787.13 | 9.43 | 1127.19 | 5.05 |
| Ta_076 | 774.33 | 6.09 | 1110.00 | 6.41 |
| Ta_077 | 772.64 | 6.65 | 1125.14 | 8.99 |
| Ta_078 | 778.16 | 4.80 | 1119.39 | 10.38 |
| Ta_079 | 780.24 | 5.43 | 1124.94 | 10.05 |
| Ta_080 | 765.56 | 3.09 | 1123.50 | 5.14 |
| Ta_081 | 1241.83 | 8.96 | 1586.35 | 9.08 |
| Ta_082 | 1242.28 | 4.55 | 1586.54 | 9.09 |
| Ta_083 | 1243.11 | 5.30 | 1598.08 | 9.31 |
| Ta_084 | 1232.42 | 7.14 | 1573.39 | 7.75 |
| Ta_085 | 1251.57 | 5.90 | 1589.51 | 10.26 |
| Ta_086 | 1246.92 | 9.88 | 1585.96 | 6.85 |
| Ta_087 | 1232.00 | 6.84 | 1575.23 | 10.91 |
| Ta_088 | 1253.64 | 6.89 | 1593.02 | 9.83 |
| Ta_089 | 1245.93 | 6.68 | 1586.87 | 11.24 |
| Ta_090 | 1241.25 | 8.28 | 1587.45 | 8.55 |
| Ta_091 | 2491.88 | 27.21 | 3789.63 | 25.26 |
| Ta_092 | 2477.69 | 15.23 | 3760.98 | 22.98 |
| Ta_093 | 2492.70 | 8.00 | 3786.04 | 14.15 |
| Ta_094 | 2457.41 | 10.34 | 3787.36 | 41.81 |
| Ta_095 | 2482.50 | 17.15 | 3749.81 | 22.27 |
| Ta_096 | 2480.92 | 6.24 | 3739.61 | 13.52 |
| Ta_097 | 2461.74 | 16.93 | 3792.48 | 24.49 |
| Ta_098 | 2471.82 | 8.56 | 3731.21 | 19.35 |
| Ta_099 | 2450.06 | 11.49 | 3724.39 | 11.58 |
| Ta_100 | 2469.99 | 15.26 | 3754.15 | 19.43 |
| Ta_101 | 4293.12 | 15.47 | 5639.96 | 24.15 |
| Ta_102 | 4311.73 | 34.07 | 5584.47 | 20.77 |
| Ta_103 | 4286.46 | 32.44 | 5571.16 | 20.14 |
| Ta_104 | 4244.79 | 20.87 | 5544.95 | 32.86 |
| Ta_105 | 4295.91 | 15.80 | 5600.41 | 25.71 |
| Ta_106 | 4289.32 | 12.25 | 5578.47 | 23.18 |
| Ta_107 | 4260.43 | 11.62 | 5582.55 | 15.94 |
| Ta_108 | 4313.17 | 33.16 | 5597.05 | 28.62 |
| Ta_109 | 4294.80 | 13.40 | 5614.86 | 29.65 |
| Ta_110 | 3501.59 | 607.55 | 5562.85 | 23.67 |

We have produced our reference sets (best-known) for each test instance. Table 3 presents the $HV$ results obtained by MEDA/D-MK compared to the reference best-known sets from Dubois-Lacoste et al. (TP+PLS) [1] and Minella et. al [3] for the 110 Taillard test instances optimizing the objectives Total Flowtime (TFT), and the makespan ($C_{max}$).

Table 3: Hypervolume obtained from the reference sets

| Taillard Instance | TP+PLS | Minella et al. | MEDA/D-MK |
|---|---|---|---|
| 20 × 5 | | | |
| Ta_001 | **0.436** | **0.436** | **0.436** |
| Ta_002 | **0.492** | 0.464 | 0.461 |
| Ta_003 | **0.899** | 0.894 | 0.895 |
| Ta_004 | **0.861** | 0.859 | 0.849 |
| Ta_005 | 0.629 | **0.644** | **0.644** |
| Ta_006 | **0.589** | **0.589** | 0.576 |
| Ta_007 | **0.816** | 0.792 | 0.792 |
| Ta_008 | 0.884 | **0.882** | **0.882** |
| Ta_009 | 0.785 | 0.782 | **0.785** |
| Ta_010 | 0.658 | **0.659** | **0.659** |
| 20 × 10 | | | |
| Ta_011 | 0.726 | **0.739** | **0.739** |
| Ta_012 | 0.766 | 0.772 | **0.774** |
| Ta_013 | 0.845 | 0.846 | **0.847** |
| Ta_014 | 0.787 | 0.775 | **0.789** |
| Ta_015 | 0.612 | 0.613 | **0.614** |
| Ta_016 | **0.850** | 0.845 | 0.844 |
| Ta_017 | **0.713** | **0.713** | **0.713** |
| Ta_018 | **0.840** | 0.828 | **0.840** |
| Ta_019 | **0.943** | **0.943** | **0.943** |
| Ta_020 | 0.853 | 0.849 | **0.854** |
| 20 × 20 | | | |
| Ta_021 | **0.643** | 0.628 | 0.619 |
| Ta_022 | 0.773 | **0.779** | 0.778 |
| Ta_023 | 0.808 | 0.807 | **0.809** |
| Ta_024 | **0.874** | 0.864 | **0.874** |
| Ta_025 | **0.821** | 0.818 | **0.821** |
| Ta_026 | 0.867 | **0.878** | 0.876 |
| Ta_027 | **0.827** | 0.804 | 0.822 |
| Ta_028 | 0.824 | 0.823 | **0.826** |
| Ta_029 | **0.767** | **0.767** | **0.767** |
| Ta_030 | 0.764 | 0.777 | **0.781** |
| 50 × 5 | | | |
| Ta_031 | **0.954** | 0.771 | 0.864 |
| Ta_032 | **0.953** | 0.865 | 0.911 |
| Ta_033 | **0.922** | 0.806 | 0.876 |
| Ta_034 | **0.976** | 0.893 | 0.942 |
| Ta_035 | **0.985** | 0.910 | 0.970 |
| Ta_036 | **0.953** | 0.909 | 0.936 |
| Ta_037 | **0.832** | 0.760 | 0.752 |
| Ta_038 | **0.904** | 0.782 | 0.851 |
| Ta_039 | **0.927** | 0.880 | 0.894 |
| Ta_040 | **0.964** | 0.788 | 0.912 |
| 50 × 10 | | | |
| Ta_041 | **0.919** | 0.702 | 0.827 |
| Ta_042 | **0.802** | 0.674 | 0.764 |

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**Table 3 – continued from previous page**

| Taillard Instance | TP+PLS | Minella et al. | MEDA/D-MK |
|---|---|---|---|
| Ta_043 | **0.866** | 0.696 | 0.832 |
| Ta_044 | **0.960** | 0.788 | 0.923 |
| Ta_045 | **0.890** | 0.649 | 0.855 |
| Ta_046 | **0.874** | 0.686 | 0.866 |
| Ta_047 | **0.906** | 0.731 | 0.864 |
| Ta_048 | **0.879** | 0.714 | 0.770 |
| Ta_049 | **0.896** | 0.707 | 0.794 |
| Ta_050 | 0.912 | 0.737 | 0.871 |
| 50 × 20 | | | |
| Ta_051 | **0.906** | 0.615 | 0.832 |
| Ta_052 | **0.852** | 0.594 | 0.816 |
| Ta_053 | **0.890** | 0.612 | 0.825 |
| Ta_054 | **0.863** | 0.656 | 0.810 |
| Ta_055 | **0.778** | 0.516 | 0.688 |
| Ta_056 | **0.902** | 0.708 | 0.817 |
| Ta_057 | **0.888** | 0.616 | 0.801 |
| Ta_058 | **0.918** | 0.726 | 0.857 |
| Ta_059 | **0.813** | 0.585 | 0.705 |
| Ta_060 | **0.913** | 0.695 | 0.860 |
| 100 × 5 | | | |
| Ta_061 | **0.978** | 0.749 | 0.925 |
| Ta_062 | **0.985** | 0.880 | 0.843 |
| Ta_063 | **0.970** | 0.835 | 0.884 |
| Ta_064 | 0.964 | 0.877 | **0.966** |
| Ta_065 | **0.992** | 0.888 | **0.992** |
| Ta_066 | **0.988** | 0.896 | 0.932 |
| Ta_067 | 0.878 | 0.654 | **0.937** |
| Ta_068 | 0.935 | 0.842 | **0.946** |
| Ta_069 | **0.911** | 0.664 | 0.753 |
| Ta_070 | **0.962** | 0.856 | 0.943 |
| 100 × 10 | | | |
| Ta_071 | **0.909** | 0.762 | 0.844 |
| Ta_072 | **0.933** | 0.779 | 0.856 |
| Ta_073 | **0.965** | 0.766 | 0.851 |
| Ta_074 | **0.819** | 0.564 | 0.804 |
| Ta_075 | **0.942** | 0.764 | 0.793 |
| Ta_076 | **0.976** | 0.755 | 0.905 |
| Ta_077 | **0.898** | 0.745 | 0.770 |
| Ta_078 | 0.840 | 0.679 | **0.844** |
| Ta_079 | **0.892** | 0.708 | 0.857 |
| Ta_080 | **0.857** | 0.610 | 0.620 |
| 100 × 20 | | | |
| Ta_081 | 0.904 | 0.692 | **0.933** |
| Ta_082 | **0.859** | 0.534 | 0.784 |
| Ta_083 | **0.893** | 0.586 | 0.840 |
| Ta_084 | **0.908** | 0.652 | 0.827 |
| Ta_085 | **0.879** | 0.540 | 0.847 |
| Ta_086 | **0.862** | 0.476 | 0.826 |

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**Table 3 – continued from previous page**

| Taillard Instance | TP+PLS | Minella et al. | MEDA/D-MK |
|---|---|---|---|
| Ta_087 | **0.811** | 0.484 | 0.802 |
| Ta_088 | 0.863 | 0.556 | **0.892** |
| Ta_089 | **0.906** | 0.598 | 0.853 |
| Ta_090 | **0.867** | 0.547 | 0.843 |
| $200 \times 10$ | | | |
| Ta_091 | 0.854 | 0.622 | **0.947** |
| Ta_092 | 0.823 | 0.715 | **0.903** |
| Ta_093 | 0.713 | 0.637 | **0.793** |
| Ta_094 | 0.767 | 0.734 | **0.875** |
| Ta_095 | 0.901 | 0.750 | **0.967** |
| Ta_096 | 0.896 | 0.768 | **0.899** |
| Ta_097 | **0.855** | 0.718 | 0.827 |
| Ta_098 | 0.836 | 0.767 | **0.882** |
| Ta_099 | 0.911 | 0.818 | **0.962** |
| Ta_100 | 0.714 | 0.679 | **0.758** |
| $200 \times 20$ | | | |
| Ta_101 | 0.820 | 0.611 | **0.918** |
| Ta_102 | 0.792 | 0.527 | **0.882** |
| Ta_103 | 0.845 | 0.576 | **0.901** |
| Ta_104 | 0.805 | 0.580 | **0.850** |
| Ta_105 | 0.775 | 0.562 | **0.782** |
| Ta_106 | 0.802 | 0.560 | **0.914** |
| Ta_107 | 0.740 | 0.450 | **0.859** |
| Ta_108 | **0.873** | 0.564 | 0.769 |
| Ta_109 | 0.862 | 0.632 | **0.918** |
| Ta_110 | **0.858** | 0.525 | 0.845 |

The results show that MEDA/D-MK achieves competitive results. Our approach achieves the best results, in most of the cases, for the groups $20 \times 10, 200 \times 10$, and $200 \times 20$. The best-known sets from [1] achieves the best $HV$ values for 50 and 100 jobs. Our best-known approximated Pareto fronts are available on-line[1] for future comparison to other approaches.

## References


[1] J. Dubois-Lacoste, M. López-Ibáñez, T. Stützle, A hybrid TP+PLS algorithm for bi-objective flow-shop scheduling problems, Computers & Operations Research 38 (8) (2011) 1219–1236.

[2] K. Miettinen, Nonlinear multiobjective optimization, vol. 12, Springer Science & Business Media, 2012.

[3] G. Minella, R. Ruiz, M. Ciavotta, Restarted Iterated Pareto Greedy algorithm for multi-objective flowshop scheduling problems, Computers & Operations Research 38 (11) (2011) 1521–1533.


[1] Available at at https://github.com/MuriloZangari/supplementary_results_mopfsp